Problem 14-9

Problem Scenario / Setup :

- String of n characters, Break points list L
- Example:
  - String length n = 20
  - Break points L = [2, 8, 10]

- Cost of breaking string is proportional to current length:
  - Break 20 char string costs 20 time units.
  - Break 12 char string costs 12 time units.
- Breaks reduce size of substrings
- Order of breaks determines the sizes of substrings during each step

- Example 1

[1 2][3 4 5 6 7 8][9 10] 11 12 13 14 15 16 17 18 19 20]

  - Total Cost: 20 + 18 + 12 = 50

- Example 2

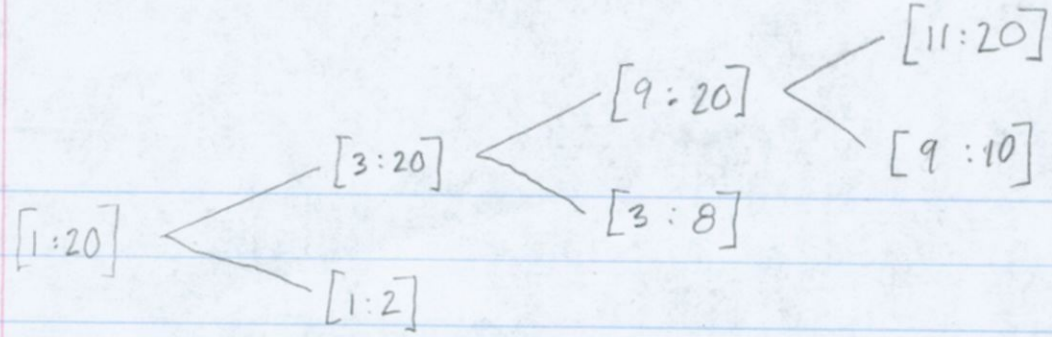
[[1 2] 3 4 5 6 7 8][9 10][11 12 13 14 15 16 17 18 19 20]

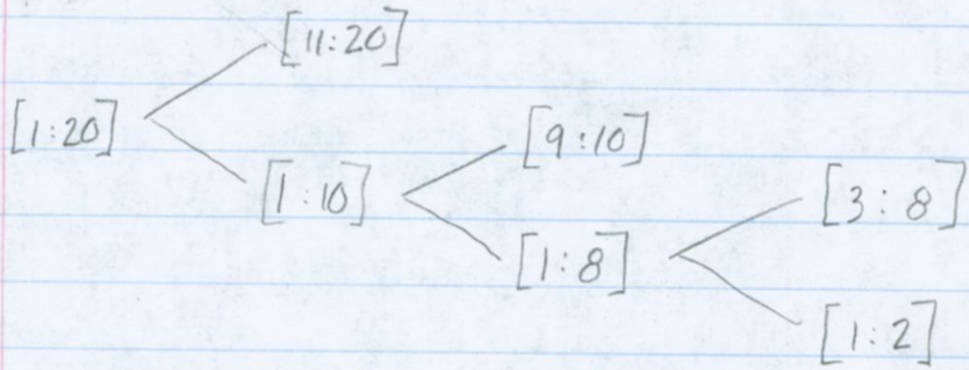
  - Total Cost: 20 + 10 + 8 = 38

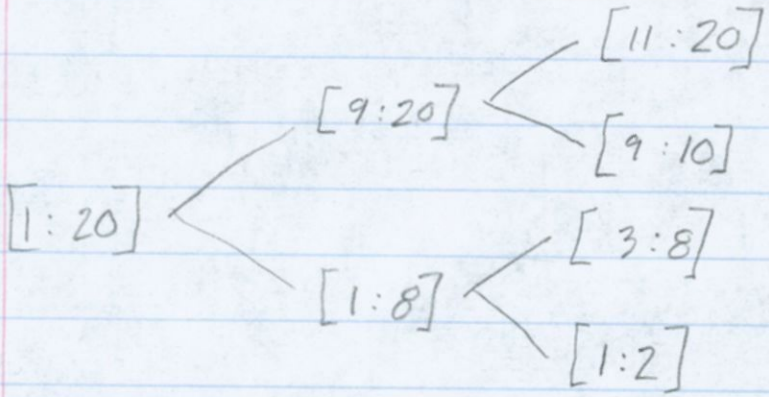
- Example 3

[[1 2][3 4 5 6 7 8][9 10][11 12 13 14 15 16 17 18 19 20]

  - Total Cost: 20 + 8 + 12 = 40


[11:20]
[9:20]
[9:10]
[3:20]
[3:8]
[1:20]
[1:2]


[11:20]
[1:20]
[9:10]
[1:10]
[3:8]
[1:8]
[1:2]


[11:20]
[9:20]
[9:10]
[1:20]
[3:8]
[1:8]
[1:2]

Dynamic Setup - Bottom up

$n = 20$

$L = [0, 2, 8, 10, 20]$
$\quad\quad 0\ \ 1\ \ 2\ \ 3\ \ 4$

0, 20 add to complete edge substrings

$dp[i][i] = 0$

$dp[i][j] = \infty$ for $i < j$

j \ i	0	1	2	3	4
0	0	0	8	18	38
1	∞	0	0	8	26
2	∞	∞	0	0	12
3	∞	∞	∞	0	0
4	∞	∞	∞	∞	0

Substrings of Length 2 : $dp[i][i+2]$

Break at $K = 1$

$dp[0][2] = dp[0][1] + dp[1][2] + (L[2] - L[0])$

$\quad\quad\quad\quad\quad\quad 0 \quad + \quad 0 \quad + \quad 8$

$dp[0][2] = 8$

Break at $K = 2$

$dp[1][3] = dp[1][2] + dp[2][3] + (L[3] - L[1])$

$\quad\quad\quad\quad\quad\quad 0 \quad + \quad 0 \quad + \quad 8$

$dp[1][3] = 8$

Break at $K = 3$

$dp[2][4] = dp[2][3] + dp[3][4] + (L[4] - L[2])$

$\quad\quad\quad\quad\quad\quad 0 \quad + \quad 0 \quad + \quad 12$

$dp[2][4] = 12$

Substrings of Length 3 : $dp[i][i+3]$

$dp[0][3]$ - possible breaks $K = 1, 2$

$K = 1$: $dp[0][3] = dp[0][1] + dp[1][3] + (L[3] - L[0])$

$0 + 8 + 10$

$dp[0][3] = 18$

$K = 2$: $dp[0][3] = dp[0][2] + dp[2][3] + (L[3] - L[0])$

$8 + 0 + 10$

$dp[0][3] = 18$

$dp[1][4]$ - possible breaks $K = 2, 3$

$K = 2$: $dp[1][4] = dp[1][2] + dp[2][4] + (L[4] - L[1])$

$0 + 12 + 18$

$dp[1][4] = 30$

$K = 3$: $dp[1][4] = dp[1][3] + dp[3][4] + (L[4] - L[1])$

$8 + 0 + 18$

$dp[1][4] = 26$

Substrings of Length 4 : $dp[i][i+4]$

$dp[0][4]$ - possible breaks $K = 1, 2, 3$

$K = 1$:

$$dp[0][4] = dp[0][1] + dp[1][4] + (L[4] - L[0])$$
$$0 + 26 + 20$$

$K = 2$:

$$dp[0][4] = dp[0][2] + dp[2][4] + (L[4] - L[0])$$
$$8 + 12 + 20$$

$K = 3$:

$$dp[0][4] = dp[0][3] + dp[3][4] + (L[4] - L[0])$$
$$18 + 0 + 20$$

$$dp[0][4] = \min(46, 40, 38) = 38$$