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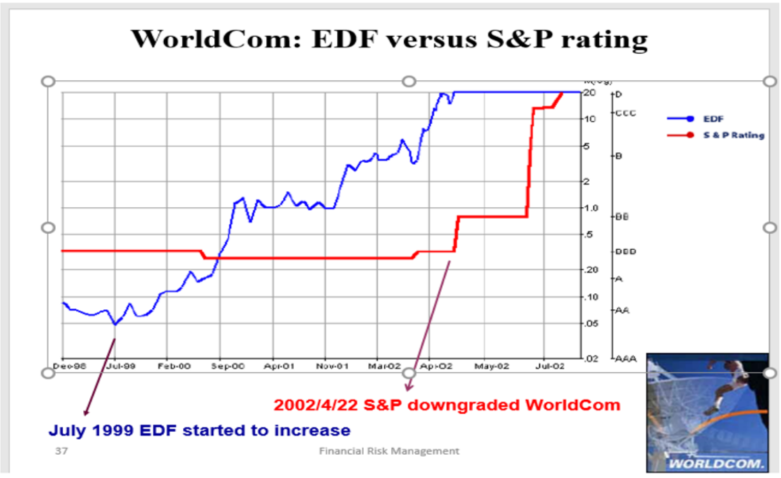
[Part I: Python for finance]

Part I aims to help you practice the "Merton Distance to Default Model." You are required to use Python to compute the Merton Distance to Default for WorldCom case you see in the lecture slides. EDF (blue line)

Required: [Note: You need to submit your Python code] Compute the expected default probability (EDF) for WorldCom (MCI Inc.) on two dates based on KMV-Merton model.

(1.) Date 1: October 1, 2001
(2.) Date 2: July 1, 2002
(3.) Compute the expected default probability (EDF) for WorldCom (MCI Inc.) from January 1999 to June, 2002, based on KMV-Merton model



Code

```
import warnings
import numpy as np
import pandas as pd
import datetime as dt
import matplotlib.pyplot as plt
import matplotlib.dates as mdates

from scipy.stats import norm
from scipy.optimize import fsolve
warnings.filterwarnings("ignore")


class Merton_KMV():
    def __init__(self,D,E,t,r,sigma_e,x):
        self.D = D
```

```
        self.E = E
        self.t = t
        self.r = r
        self.sigma_e = sigma_e
        self.x = x
    def d1(self,A,D,sigma_a,r,t):
        self.d1_v = ( np.log(A/D) + ( r + sigma_a**2 /2 ) * t ) /
( sigma_a*np.sqrt(t) )
        return self.d1_v
    def d2(self,A,D,sigma_a,r,t):
        self.d2_v = self.d1_v - sigma_a*np.sqrt(t)
        return self.d2_v
    def func(self,x):
        sigma_a = x[0]
        a =  x[1]
        d1_value = self.d1(A=a, D=self.D , sigma_a=sigma_a , r=self.r ,
t=self.t)
        d2_value = self.d2(A=a, D=self.D , sigma_a=sigma_a , r=self.r ,
t=self.t)
        return [ (a*norm.cdf(d1_value) - np.exp(-
self.r*self.t)*self.D*norm.cdf(d2_value) - self.E) , ((a/self.E) *
norm.cdf(d1_value) * sigma_a - self.sigma_e )]
    def fsolve(self):
        root = fsolve( self.func, x0=self.x)
        return root


def KMV_df(df,trading_days):
    """
    build a df that content the parameters for calculating firm asset value
and sigma_a
    """
    df['RET'] = np.log( df['RET'].values + 1)
    n=trading_days
    sigma_e = []
    df['KMV debt'] = df['DLC'] + 0.5 * df['DLTT']
    for i in range(df.shape[0]+1,n,-1):
        e_values = np.std( df['RET'][-(n-i):i].values ) * np.sqrt(n)
        sigma_e.append(e_values)
```

```
    df = df[n-1:]
    df['sigma_e'] = list(reversed(sigma_e))

    df = df [['DATE','RET','me','ir','KMV debt','sigma_e']]
    df = df.reset_index(drop=True)
    return df

def d2(A,D,sigma_a,r,t):
    d2_v =  ( np.log(A/D) + (r-0.5*sigma_a**2) * t ) / ( sigma_a*np.sqrt(t) )
    return d2_v

def d1(A,D,sigma_a,r,t):
    d1_v =  ( np.log(A/D) + (r+0.5*sigma_a**2) * t ) / ( sigma_a*np.sqrt(t) )
    return d1_v

def kmv(kmv_df):
    sigma_list = []
    A_list = []
    EDP_list = []
    d2_list = []
    d1_list = []
    for i in range(kmv_df.shape[0]):
        t = 1
        d = kmv_df['KMV debt'][i]
        e = kmv_df['me'][i]
        r = kmv_df['ir'][i]
        sigma_e = kmv_df['sigma_e'][i]

        model = Merton_KMV(D=d,E=e,t=t,r=r,sigma_e=sigma_e,x=[ sigma_e ,
e+d ])
        ans = model.fsolve()

        sigma_list.append(ans[0])
        A_list.append(ans[1])

        d2_vlaue = d2(A=ans[1],D=d, sigma_a=ans[0],r=r,t=t)
        d2_list.append(d2_vlaue)
```

```python
        d1_vlaue = d1(A=ans[1],D=d, sigma_a=ans[0] ,r=r,t=t)
        d1_list.append(d1_vlaue)
        EDP_list.append( norm.cdf(-1*d2_vlaue) )

    kmv_df['sigma_a'] = sigma_list
    kmv_df['A'] = A_list
    kmv_df['d1'] = d1_list
    kmv_df['d2'] = d2_list
    kmv_df['EDP'] = EDP_list
    return kmv_df
```

for

(1.) Date 1: October 1, 2001

(2.) Date 2: July 1, 2002

```python
df_1 = pd.read_csv(r'/Users/chen-lichiang/Desktop/data_20020701.csv')
df_2 = pd.read_csv(r'/Users/chen-lichiang/Desktop/data_20011001.csv')


kmv_df_1 = KMV_df(df=df_1,trading_days=df_1.shape[0])
kmv_df_1 = kmv(kmv_df=kmv_df_1)


kmv_df_2 = KMV_df(df=df_2,trading_days=df_2.shape[0])
kmv_df_2 = kmv(kmv_df=kmv_df_2)

print("----[ Part I: Python for finance -(1,2) ]---\n")

print("------- EDP on date :"+str(kmv_df_1['DATE'][0])+"----------\n")
print(kmv_df_1,"\n")
print("------ EDP on date :"+str(kmv_df_2['DATE'][0])+"-----------\n")
print(kmv_df_2,'\n')
```

output:

Ans

```
(base) jiangchenlide-MacBook-Pro:School chen-lichiang$ python -u "/Users/chen-lichiang/Desktop/python/School/KMV Merton/KMV.py"
----------------------------[ Part I: Python for finance -(1,2) ]----------------

---------------------------- EDP on date :2002/6/25-------------------------

        DATE       RET           me     ir  KMV debt   sigma_e   sigma_a             A        d1       d2       EDP
0  2002/6/25 -0.092019  2459.153831  0.022   12438.5  1.019982  0.223974  14200.473456  0.801704  0.57773  0.281723

---------------------------- EDP on date :2001/9/28-------------------------

        DATE       RET           me      ir  KMV debt   sigma_e  sigma_a             A        d1        d2       EDP
0  2001/9/28  0.037945  44605.46645  0.0282   13447.0  0.735922  0.57045  57656.496838  2.886586  2.316135  0.010275
```

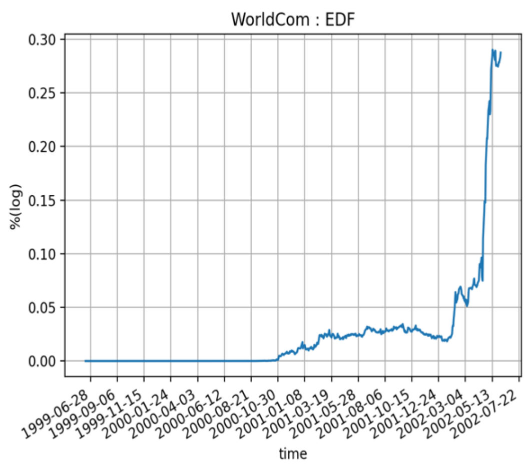
(3.) Compute the expected default probability (EDF) for WorldCom (MCI Inc.) from January 1999 to June, 2002, based on KMV-Merton model

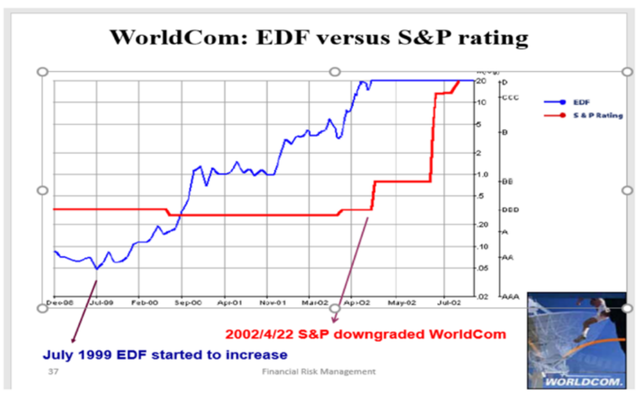
```python
print("---------[ Part I: Python for finance -( bonus )]------------\n")
df_3 = pd.read_excel(r'/Users/chen-lichiang/Desktop/data_bonus.xls')
kmv_df = KMV_df(df=df_3,trading_days=365)
kmv_df = kmv(kmv_df=kmv_df)
kmv_df = kmv_df[:-35]
days = kmv_df['DATE']
print(kmv_df)

plt.figure()
plt.title("WorldCom : EDF")
plt.plot(days,kmv_df['EDP'])
plt.gca().xaxis.set_major_locator(mdates.DayLocator(interval=70))
plt.gcf().autofmt_xdate()
plt.xlabel('time')
plt.ylabel('%(log)')
plt.grid()
plt.show()
```

Ans for Bonus

output :

```
(base) jiangchenlide-MacBook-Pro:School chen-lichiang$ python -u "/Users/chen-lichiang/Desktop/python/School/KMV Merton/KMV.py"
---------------------------[ Part I: Python for finance -( bonus )]------------

          DATE       RET             me      ir  KMV debt   sigma_e   sigma_a              A        d1        d2           EDP
0   1999-06-15  0.004873  167459.130000  0.0510   13172.5  0.495918  0.461427  179976.675775  6.007780  5.546353  1.458447e-08
1   1999-06-16  0.026723  171994.481438  0.0510   13172.5  0.494681  0.461122  184512.027216  6.065428  5.604306  1.045453e-08
2   1999-06-17  0.010760  173855.138437  0.0510   13172.5  0.494936  0.461695  186372.684217  6.080206  5.618512  9.630470e-09
3   1999-06-18  0.019212  177227.579250  0.0510   13172.5  0.495010  0.462355  189745.125031  6.110971  5.648616  8.087229e-09
4   1999-06-21  0.015625  180018.564750  0.0510   13172.5  0.495870  0.463632  192536.110532  6.126902  5.663270  7.425754e-09
..         ...       ...            ...     ...       ...       ...       ...            ...       ...       ...           ...
741 2002-05-29 -0.022347    5244.219663  0.0235   12438.5  1.074173  0.401461   16706.362425  0.994066  0.592605  2.767228e-01
742 2002-05-30 -0.058156    4947.935993  0.0235   12438.5  1.071089  0.386489   16425.371285  0.973423  0.586934  2.786239e-01
743 2002-05-31 -0.006006    4918.307661  0.0235   12438.5  1.069914  0.384461   16399.787689  0.972471  0.588010  2.782627e-01
744 2002-06-03 -0.068563    4592.395659  0.0220   12438.5  1.070917  0.369450   16097.355244  0.942226  0.572776  2.833983e-01
745 2002-06-04 -0.066691    4296.112341  0.0220   12438.5  1.070982  0.354885   15810.698279  0.915400  0.560515  2.875642e-01
```





[Part II: Academic paper]

Task : Provide your opinions for the discussion of the advantages and the disadvantages of using Merton Distance to Default Model to measure the default risk

[1] Forecasting default with the Merton distance to default model.

- Bharath, S. T., & Shumway, T. (2008).

In this Paper, it discuss the Merton DD model does not produce a sufficient for the probability of default , but it’ s still functional for forecasting defaults .

(一.) Basic explain of Merton DD :

One innovation forecasting model which has been widely applied in both academic research and practice, is a particular application of Merton(1974) that was developed by the proprietors of the KMV corporation . However, Merton invented this model for solving the problem of pricing option by Brownian motion, and than modified to the corporate finance area , analogy call option to equality value .

1.first let's plot a long call option payoff and discuss the connection between call option value and equity value

(1.) St : Firm Asset Value (St = Asset)

(2.) K : Debt (K = D)

(3.) T : Debt Duration

Share holder --> long side

Creditor --> short side

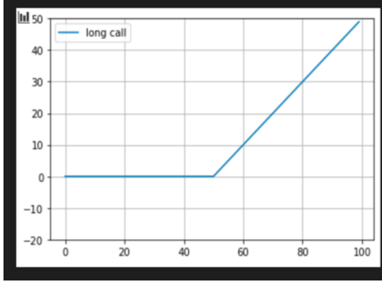
When equity value is long call , share holder borrower D from creditor , is like having the firm asset value give to the creditor in this periods , however when the expire date come ,

if

firm value > Debt , share holder exercise the option , payoff : Firm Value - D firm value < Debt , share holder will not exercise the option

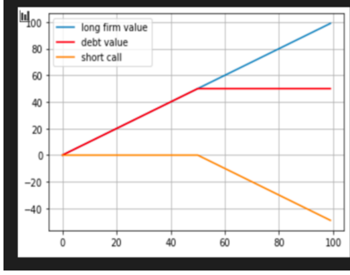
Equity value to call option

```
st = np.arange(0,100)
k  = 50
pay_off = [ np.maximum(0,st[i]-k)  for i in range(len(st)) ]

plt.figure()
plt.plot(pay_off,label='long call')
plt.grid()
plt.legend()
plt.ylim(-20,50)
plt.show()
```



Debt value = short call + long firm value

```
st = np.arange(0,100)
k  = 50
pay_off = [ -np.maximum(0,st[i]-k)  for i in range(len(st)) ]

plt.figure()
plt.plot(st,label='long firm value')
plt.plot(st+pay_off,label='debt value',color='r')
plt.plot(pay_off,label='short call')
plt.grid()
plt.legend()
plt.show()
```



we can also analog equity value to put option , and we call easily derive by transfer above call option to put option by put-call parity .

In the Merton model , we all know that when we derive our pricing model, we can write our Call option value in the form of partial differential equation , and when the PDE subject to the boundary condition , the PDE is the form of Feynmen-Kac , we can derive the call option value in the according equation

C = e**(-r*t) * E[max(S_T-K,0)]

And we also know that the when we solving the above equation we can write Call option into the form of

C = St*N(d1) -PV(K)*N(d2)

And N(d2) is the probability of P(St>K) , when we analogy call option to equality value the N(d2) will be P(A>D) , is the probability of not default , if we do a little math we can have our default probability 1-N(d2) .

(二.) Test Hypothesis

(1.) whether the probability of default given is the Merton DD model is a sufficient statistic for forecasting bankruptcy .

(2.) whether a sufficient statistic for default probability can be calculated without considering the Merton DD model' s functional from .

(3.) whether the forecasting ability of the Merton DD model is sensitive to the manner in which total firm value and firm volatility are calculated

Result :

(1.) → easily reject 1

(2.) → find some evidence to support

(3.) → find some evidence to against

(三.) Methodology, Result and, Take out

(1.) naïve alternative model

In order to test second hypothesis , the paper construct another model , a naïve alternative , basically it simply the procedure of computer the default probability by using a approximately parameters , it approximately the market value of each firm’ s debt with the face value of its debt , same as the firm value sigma , the paper also approximately the value of sigma v . this naïve alternative constructed a predictor that is extremely easily to calculate , and it may have significant predictive power .

(2.) Hazard model

Hazard models have recently been applied by a number of authors and probably represent the state of the art in default forecasting with reduced-form models. Proportional hazard models make the assumption that the hazard rate or the probability of default at time t conditional on survival until time t is

$$\lambda(t) = \phi(t)[exp(x(t)\mathrm{B})]$$

$\phi(t)$ → baseline, hazard rate

$exp(x(t)\mathrm{B})$ → the expected time to default to vary across firms

Our first hypothesis that , Merton DD is a sufficient statistic for forecasting the default probability , implies that no other variable in a hazard model should be a statistically significant covariate .

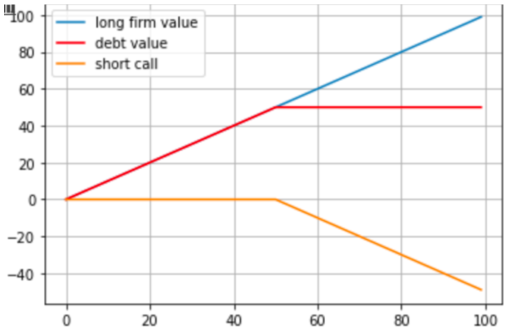
(3.) result and takeout

The paper showing the result that the Merton DD is not a sufficient statistic for default probability , but still a useful variable for forecasting default . Beside the functional from suggested by the Merton model is very useful , In the paper ,it construct lots of parameters with approximately ,and still got a useful naïve predictor , therefore next time when we construct a DP for a finance problem , we should consider whether it could link to the Merton DD model .

[2] Banks’ risk dynamics and distance to default

- Nagel, S., & Purnanandam, A. (2020).

This paper adapt structural models of default risk to take into account the special nature of bank assets . Typical bank assets are risky claims , which implies that they embed a short put option on the borrower’ s assets .



Due to the payoff non-linearity by the risky debt claim , bank asset volatility rises following negative shocks to borrower asset values , and because of this problem and others assumption of structural model , it can severely understate bank' s default risk . Therefore , this paper propose a modification of the Merton Model that takes into account the capped upside of bank assets.

I think the most important modification of this model is the log-normal distribution assumption not to the asset of the bank , but to the assets of the bank' s borrowers that serve as loan collateral . Just like I mention earlier , Bank' s assets are risky debt claims with capped upside and hence the asset payoff is nonlinear , with embed option , A bad shock to asset values therefore reduces the distance to default much more than it would in the standard model , because when A draw down a lot sigma A increase a lot , therefore according to the equation the distance to default will reduce .

when we using the model of Merton DD model we should exam whether the assu mption of the log-normally distributed is appropriate , otherwise our EDP might be much bias than we thought to the real value .

[3] Default risk in equity return

- Vassalou, M., & Xing, Y. (2004)

(一.)Introduction

This paper is the first study using Merton' s option pricing model to compute default measures for individual firms and the default risk relation to the equity returns . this paper also test whether the Fama-French factors SMB and HNL contain default related information , because author believe that size and book-to-market contain tremendous information about default risk .

(二.)The Two Test Hypothesis

(1.) whether default risk is priced
(2.) whether the FF factors SMB and HML proxy for default risk

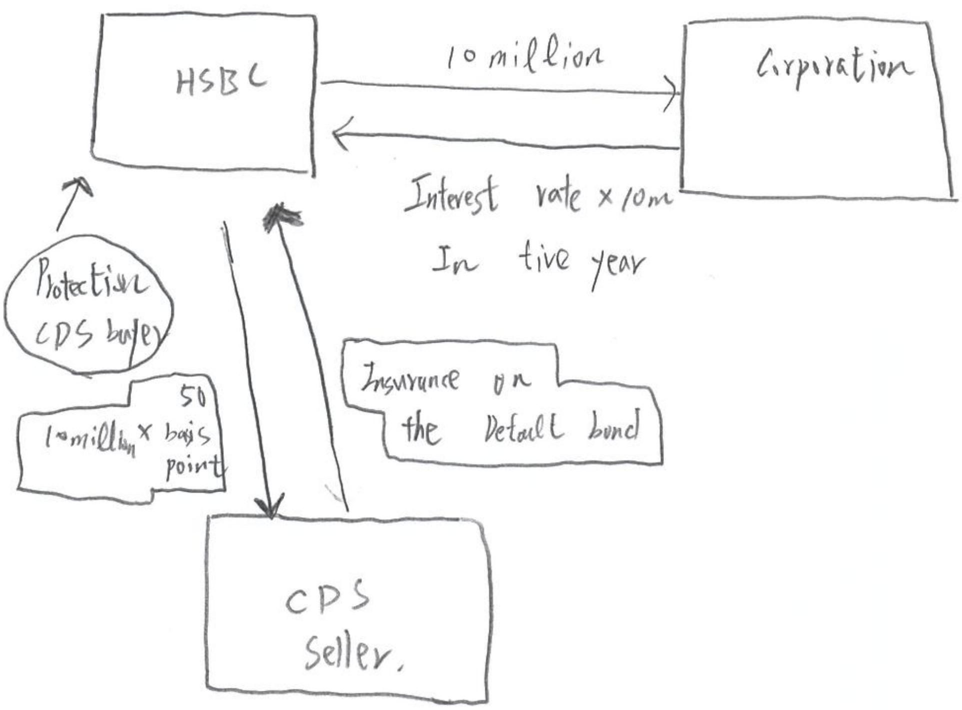
The test result show that default risk is systematic and it is priced in the cross section of equity returns , and Table 2 shows that default risk correlated with EMKT and SMB , but

SMB is not a priced factor , this mean that , although SMB and HML contain some default-related information , this is not the reason that the Fama-French model is able to explain the cross section of equity return .Beside this paper also show that , high-default-risk firms earn higher returns than low default firms , only if they are small in size and/or high BM. If we going to pricing equity value , default risk is might contain useful information for us .

(Part Ⅲ) - (1). The bank of HSBC makes a USD 10 million five-year loan and wants to offset the credit exposure to the obigor, A five year Credit Default swap with the loan as the reference asset trade on the market at a swap premium of 50 basis points annually

Ans (Part Ⅲ) - (1)

①

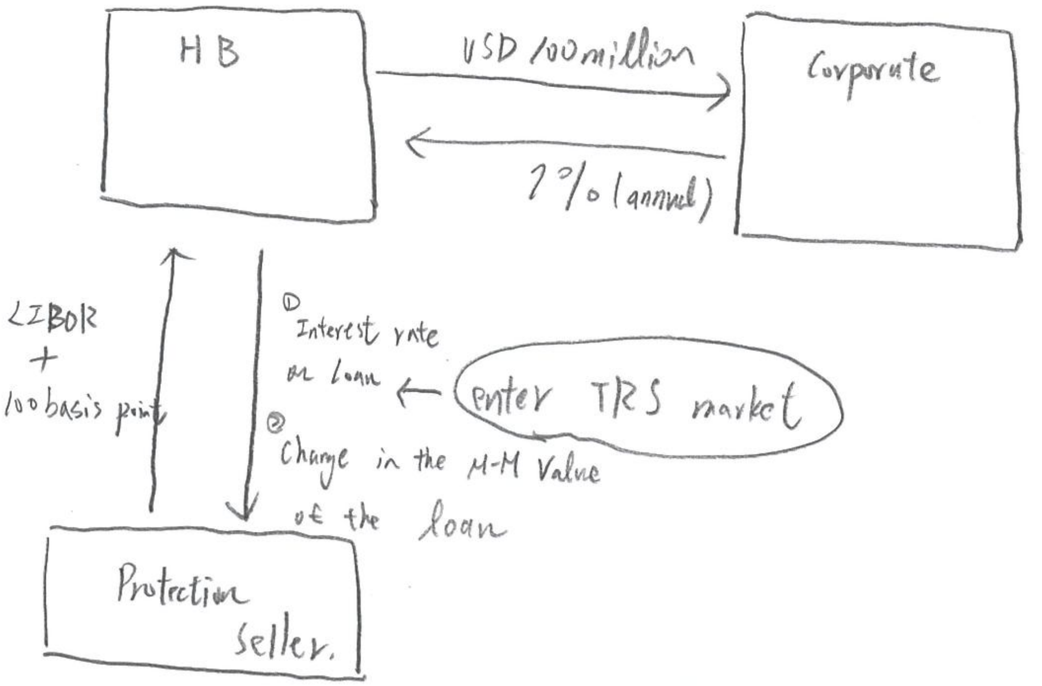


⇒ HSBC will be Protection CDS buyer & paid 100 million × 50 basis point annually to the CDS seller, and If the Corporation Default, the loss cost by the Corporation will be paid by the CDS seller.

②

CDS can not entirely eliminate the credit risk, because CDS seller will still have a chance that will not paid the loss caused by bond default, but the credit risk indeed will be lower.

(Part Ⅲ-12)



⇒ Settlement payments are made annulley, what is the Cash flow for Helmon Bank on the first settlement date if the M-M value of the loan falls by 10% and LIBOR is 2%.

		Cash flow
① outflow :	$7\% \times 100$ million =	7 m
② outflow :	$\left(\frac{(1-0.1)\times 100 - 100}{100}\right) \times 100$ =	−10 m
③ Inflow :	$(2\% + 1\%) \times 100$ =	3 m

⇒ $-7m - (-10m) + 3m = \underline{6m}$ # Ans

[Part III]

(3) Illustrate the framework CDO and discuss the extent to which the default correlation affects senior tranche of CDO.

— Ans

(一) Collateralized debt obigation

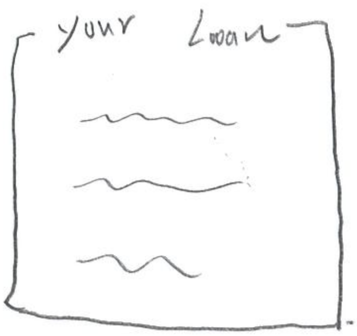
⇒ A CDO is a type of structured asset - backed security. Originally developed as instruments for the Corporate debt markets, after 2002 CDOs became vechicles for refinancing MBS.

⇒ First we gona explain Mortgage back security a little bit.

⇒ MBS invented in 1981, and below is how MBS created

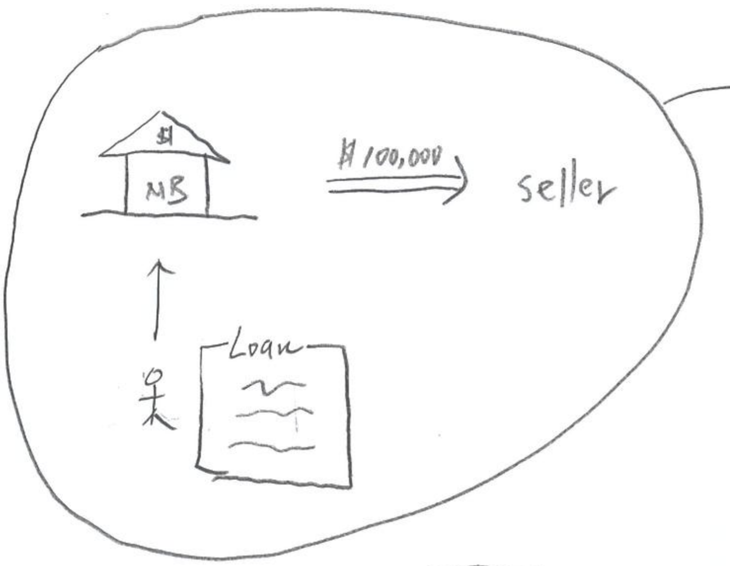
want to buy a house, but didn't have enough money, so he go to Neighbor hood bank to get a Loan.

The NB look at your { finance, Income, Credit score, DTI } ⇒ { your Loan, $ 100,000, 5% Interest, 30 - years.

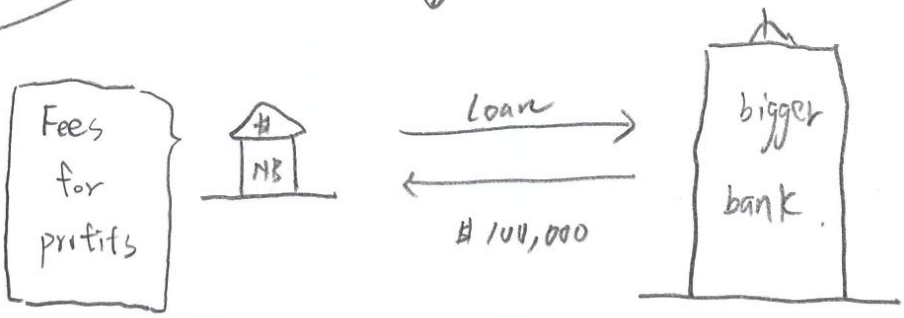
⇒ 

⇒ get approved by company ⇒ become $\boxed{\text{Conforming Loan}}$

⇒ After getting your Loan you will be able to buy the house, however, it the NB (neighbor hood bank) tried to make Instance Investment of $100,000, They would like to sell the Loan, and get the Money back.



NB tried to sell loan to a bigger bank

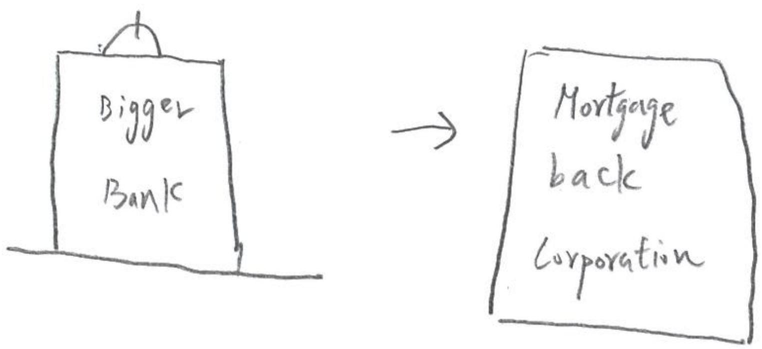


⇒ The more Loan the NB sell, the much more profit that the NB get.

⇒ and how does Bigger bank make money,
assume bigger bank have

Loan		All — Loan
$100,000	× 1000 ⟶	100 Million
5% Ir		5% Ir
30 - year		⇒ 5M/year

bigger bank create a Corporation base on the asset of all the Loan he bought. In this scenario, The Corporation Worth 100 Millions dollars and generate 5M/year. The bigger bank thought lots of people will want to invest in this kind of company.



share's × 1,000,000
sell to Investor

⇒ Each of share entitle for 1/1M profit.

① each loan will pay bank $100,000, All the Loan = 100$/M

② IR 5% ⇒ 5% × 100M = 5M/year.

$$\Rightarrow 1 \text{ share} = \frac{100,000 \times 1000}{1,000,000} + 5 = 105\ \$ \text{ per share.}$$

⇒ In this case, Bigger bank make 105 − 100 per. share and the Investor will get 100 + 30 years × 5/years = 250 $, This is how Bigger bank make money through MBS.

⇒ however, In real world, some of the Mortgage will default, (Generally 20% ~25% Interste rate), & this kind of action will affect the return of MBS, In 2008 financial crisis, MB & BB know some of the loan will default, but if most of the loan do not default, Investor still get a good amount of returns, BB put bad loans together create a much more risky but higher return MBS, Investor do not know how bad the loans were, because of the credit agency didn't do their job well (Agency paid by the bank), and the rest is the history.

⇒ CDOs & credit default swaps.

like I said earlier, CDOS is a type of MSB but have a system call Tranches { equity / mezzanine / senior.

⇒ {
equity → 7.5% → higher risk, higher returns.
mezzanine → 5%
senior → 3.13% → low risk, low returns.

* For example, A MBS \$100M $\xrightarrow[\substack{\uparrow \\ \text{if no one default}}]{}$ 5M/year

Equity → 7.5% → 30M (300,000 shares)

⇒ 30M × 7.5% = 2.25M

Mezzanine → 5% → 30M (300,000 shares)

⇒ 30M × 5% = 1.5M

Senior → 3.13% → 40M (400,000 shares)

⇒ 40M × 3.13% = 1.25M

* however, if today 80% of the loan default, & it only generate 1M/year. than

Equity → 0
Mezzanine → 0
Senior → 1M.

* Credit default Swap for hedging the risk of default.

Equity → expensive CDS → (Just like deep OTM put is the most expensive one to prevent Market Crash)

mezzanine → *

Senior → cheap CDS

7 Senior Trench of CDO basically represent the majority of the people whether they "can" pay their loan or not, so if someone bet on the senior. Trench of CDO & buy CDS on senior trench basically is betting against the Country's economic.