Question 1.

(a)

P	Q	¬P	¬Q	P⇒¬Q	Q⇒¬P
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

Thus we see P⇒¬Q is equivalent to Q⇒¬P

(b)

P	Q	¬P	¬Q	P⇒¬Q	¬Q⇒P	P⇔¬Q	P∧¬Q	¬P∧Q	(P∧¬Q)∨(¬P∧Q)
T	T	F	F	F	T	F	F	F	F
T	F	F	T	T	T	T	T	F	T
F	T	T	F	T	T	T	F	T	T
F	F	T	T	T	F	F	F	F	F

Thus we see P⇔¬Q is equivalent to (P∧¬Q)∨(¬P∧Q)

Question 2.

(a)

Smoke	Fire	¬Smoke	¬Fire	Smoke⇒Fire	¬Smoke⇒¬Fire	(Smoke⇒Fire)⇒(¬Smoke⇒¬Fire)
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Thus it's neither for (Smoke⇒Fire)⇒(¬Smoke⇒¬Fire) to be valid or unsatisfiable

(b)

Smoke	Fire	Heat	Smoke ⇒ Fire	Smoke∨Heat	(Smoke∨Heat) ⇒ Fire	(Smoke⇒Fire)⇒((Smoke∨Heat)⇒Fire)
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	T	F
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T

Thus it's neither for (Smoke⇒Fire)⇒((Smoke∨Heat)⇒Fire) to be valid or unsatisfiable

(c)

S	F	H	S∧H	(S∧H)⇒F	S⇒F	H⇒F	(S⇒F)∨(H⇒F)	((S∧H)⇒F)⇒((S⇒F)∨(H⇒F))	((S⇒F)∨(H⇒F))⇒((S∧H)⇒F)
T	T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T	T
T	F	T	T	F	F	F	F	T	T
T	F	F	F	T	F	T	T	T	T
F	T	T	F	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T	T
F	F	T	F	T	T	F	T	T	T
F	F	F	F	T	T	T	T	T	T

We see ((S⇒F)∨(H⇒F))⇔((S∧H)⇒F) is always T.

Thus, ((Smoke ∧ Heat)⇒Fire)⇔((Smoke⇒Fire)∨(Heat ⇒ Fire)) is valid.

Question 3.

(a)

KB: Δ = {Mythical $\Rightarrow$ Immortal, $\neg$Mythical $\Rightarrow$ ($\neg$Immoral $\wedge$ Mammal), (Immortal $\vee$ Mammal) $\Rightarrow$ Horned,

Horned $\Rightarrow$ Magical}

(b)

(Mythical $\Rightarrow$ Immortal) $\wedge$ ($\neg$Mythical $\Rightarrow$ ($\neg$Immoral $\wedge$ Mammal)) $\wedge$ ((Immortal $\vee$ Mammal) $\Rightarrow$ Horned) $\wedge$ (Horned $\Rightarrow$ Magical)

($\neg$Mythical $\vee$ Immortal) $\wedge$ (Mythical $\vee$ ($\neg$Immortal $\wedge$ Mammal)) $\wedge$ ($\neg$(Immortal $\vee$ Mammal) $\vee$ Horned) $\wedge$ ($\neg$Horned $\vee$ Magical)

($\neg$Mythical $\vee$ Immortal) $\wedge$ (Mythical $\vee$ $\neg$Immortal) $\wedge$ (Mythical $\vee$ Mammal) $\wedge$ (($\neg$Immortal $\wedge$ $\neg$Mammal) $\vee$ Horned)

$\wedge$ ($\neg$Horned $\vee$ Magical)

($\neg$Mythical $\vee$ Immortal) $\wedge$ (Mythical $\vee$ $\neg$Immortal) $\wedge$ (Mythical $\vee$ Mammal) $\wedge$ ($\neg$Immortal $\vee$ Horned)

$\wedge$ ($\neg$Mammal $\vee$ Horned) $\wedge$ ($\neg$Horned $\vee$ Magical)

This last CNF is what we want

(c)

From KB Δ, we have:

Δ
1. {$\neg$Mythical, Immortal}
2. {Mythical, $\neg$Immortal}
3. {Mythical, Mammal}
4. {$\neg$Immortal, Horned}
5. {$\neg$Mammal, Horned}
6. {$\neg$Horned, Magical}

Then:
7. {Immortal, Mammal} by 1,3
8. {Horned, Mammal} by 4,7
9. {Horned} by 5,8
10. {Magical} by 6,9
11. {Horned, $\neg$Mythical} by 1,4

Thus, we can prove it is magical (by 10) and Horned (by 11).
However, we cannot prove it is mythical.

Question 4:

(i) Figure 1:

We can rewrite it as

$$(((\neg A \wedge B \quad) \vee (\neg B \wedge A \quad)) \wedge ((\quad C \quad) \vee (\neg D \wedge \neg C \quad)))$$
$$\vee (((\neg A \wedge \neg B \quad) \vee (\quad A \quad)) \wedge ((\quad C \wedge \neg D \quad) \vee (D \wedge \neg C \quad)))$$

We see that for any "$\wedge$" symbol, the LHS and RHS of it does not contain the same variable. <mark>thus it is decomposable.</mark>

Then, if we set $A = True$, $B = False$, $C = True$, $D = False$, we have the most top "or" as: $\alpha = ((\neg A \wedge B) \vee (\neg B \wedge A)) \wedge ((C) \vee (\neg D \wedge \neg C))$ and $\beta = ((\neg A \wedge \neg B) \vee (A)) \wedge ((C \wedge \neg D) \vee (D \wedge \neg C))$
So both α and β are evaluated to true. Then $\alpha \wedge \beta$ is not inconsistent. Hence, <mark>it is not deterministic.</mark>

<mark>It is not smooth,</mark> as the second-to-left disjunction node in the middle level $((C) \vee (\neg D \wedge \neg C))$. Here $\alpha = C$ and $\beta = \neg D \wedge \neg C$, so $var(\alpha) = \{C\} \neq var(\beta) = \{D, C\}$

(ii). Figure 2:

<mark>It is decomposable</mark>. For the bottom-level "and" nodes, the left four has variable "A" on one side, and "B" on the other side; the right four has variable "C" on one side, and "D" on the other side. For the upper-level "and" nodes, they both have "A", "B" on one side, and "C", "D" on the other side. Thus, whenever $\alpha \wedge \beta$ occurs, $var(\alpha) \cap var(\beta) = \emptyset$.

<mark>It is not deterministic.</mark> We see the bottom-left "or" node is $(\neg A \wedge B) \vee (\neg A \wedge B)$. Here, $\alpha = (\neg A \wedge B)$ and $\beta = (\neg A \wedge B)$. If we set $A = False$ and $B = True$, then $\alpha \wedge \beta = True$. So $\alpha \wedge \beta$ is not inconsistent.

Question 5:

(a)

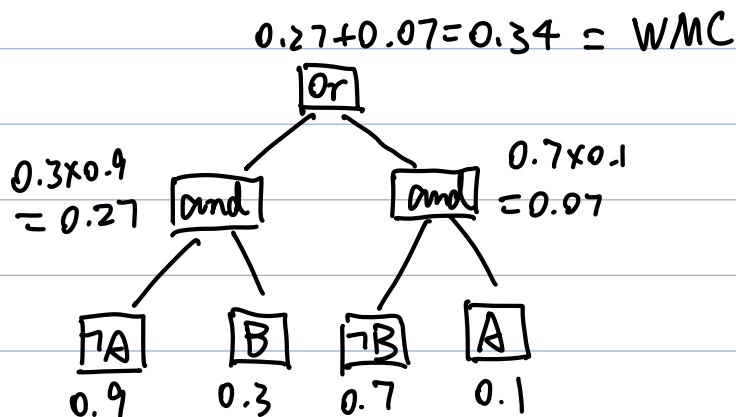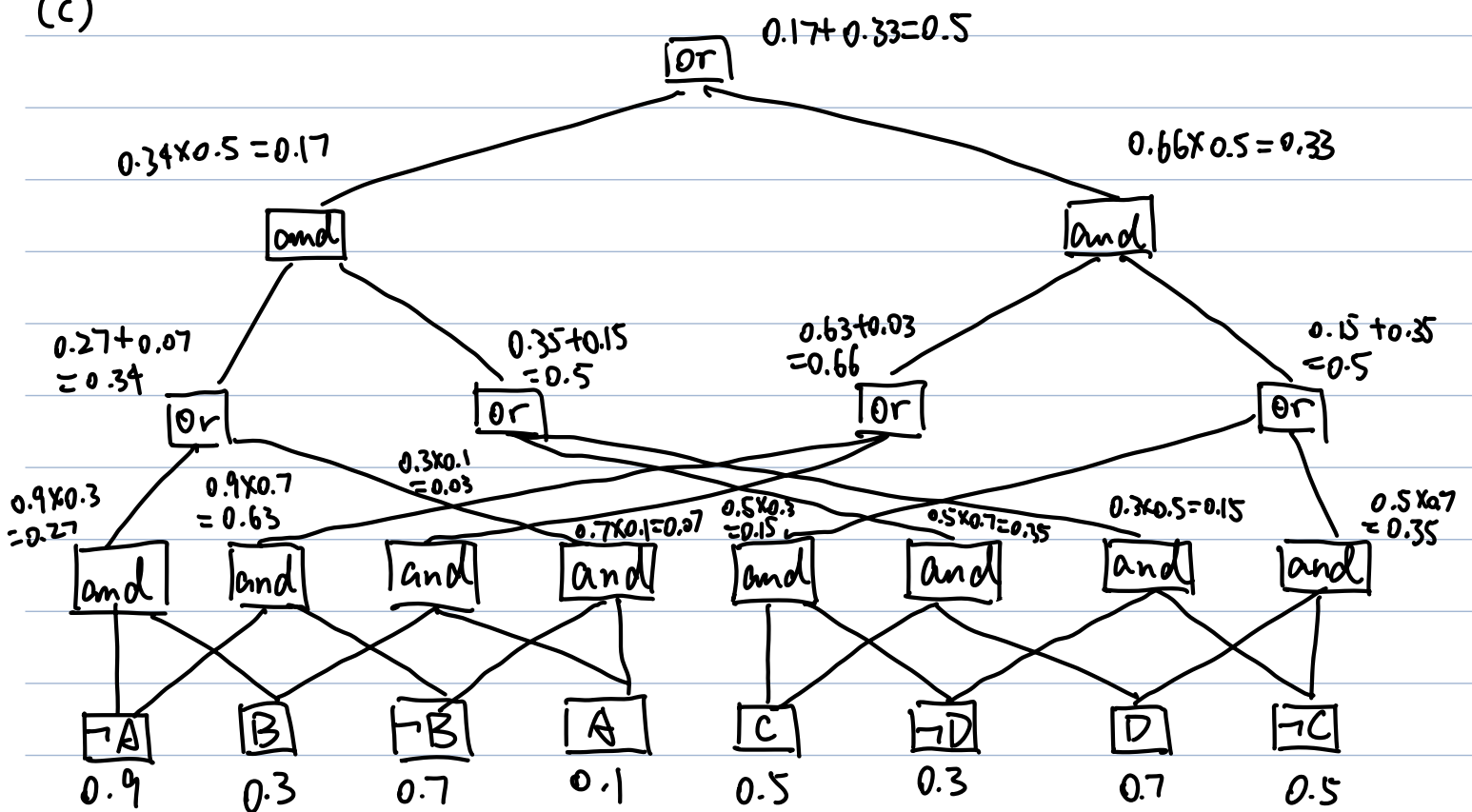| A	B	¬A	¬B	¬A∧B	¬B∧A	(¬A∧B)∨(¬B∧A)
T	T	F	F	F	F	F
T	F	F	T	F	T	T	←
F	T	T	F	T	F	T	←
F	F	T	T	F	F	F

$$w(A) \cdot w(\neg B) + w(\neg A) \cdot w(B) = 0.1 \times 0.7 + 0.9 \times 0.3 = 0.07 + 0.27 = \boxed{0.34}$$

(b). They are the same.

Here we have

$$0.27 + 0.07 = 0.34 = WMC$$

(c)

$0.17 + 0.33 = 0.5$

[or]

$0.34 \times 0.5 = 0.17$ $0.66 \times 0.5 = 0.33$

[and]   [and]

$0.27 + 0.07$ $0.35 + 0.15$ $0.63 + 0.03$ $0.15 + 0.35$
$= 0.34$ $= 0.5$ $= 0.66$ $= 0.5$

[or]   [or]   [or]   [or]

0.9×0.3 0.9×0.7 0.3×0.1 0.5×0.3 $0.3 \times 0.5 = 0.15$ 0.5×0.7
$= 0.27$ $= 0.63$ $= 0.03$ $= 0.15$ $= 0.35$
                              $0.7 \times 0.1 = 0.07$ $0.5 \times 0.7 = 0.35$

[and]   [and]   [and]   [and]   [and]   [and]   [and]   [and]

[¬A]   [B]   [¬B]   [A]   [C]   [¬D]   [D]   [¬C]

0.9 0.3 0.7 0.1 0.5 0.3 0.7 0.5

Thus, the WMC for it is 0.5.