

Question 1.

(a). MGU: $\theta = \{x/A, y/A, z/B\}$

(b). MGU doesn't exist.

(c). MGU: $\theta = \{x/B, y/A\}$

(d). MGU: $\theta = \{x/John, y/John\}$

(e). MGU doesn't exist.

Question 2.

(a) i. $\forall f (Is_food(f)) \Rightarrow Likes(John, f)$

ii. $Is_food(Apples)$

iii. $Is_food(Chicken)$

iv. $\forall f (\exists p (Eats(p, f) \wedge \neg Killed(p, f))) \Rightarrow Is_food(f)$

v. $\forall p [\exists t (Killed(p, t))] \Rightarrow \neg Alive(p)$

vi. $Eats(Bill, Peanuts) \wedge Alive(Bill)$

vii. $\forall t Eats(Bill, t) \Rightarrow Eats(Sue, t)$

(b) i. $\forall f (Is_food(f)) \Rightarrow Likes(John, f)$

$\hookrightarrow \forall f (\neg Is_food(f) \vee Likes(John, f))$

$\hookrightarrow \neg Is_food(f) \vee Likes(John, f)$

ii. $Is_food(Apple)$

iii. $Is_food(Chicken)$

iv. $\forall f (\exists p (Eats(p, f) \wedge \neg Killed(p, f))) \Rightarrow Is_food(f)$

$\hookrightarrow \forall f ([\neg \exists p (Eats(p, f) \wedge \neg Killed(p, f))] \vee Is_food(f))$

$\hookrightarrow \forall f (\forall p \neg Eats(p, f) \vee Killed(p, f) \vee Is_food(f))$

$\hookrightarrow \neg Eats(p, f) \vee Killed(p, f) \vee Is_food(f)$

v. $\forall p [\exists t (Killed(p, t))] \Rightarrow \neg Alive(p)$

$\hookrightarrow \forall p [\neg \exists t Killed(p, t) \vee \neg Alive(p)]$

$\hookrightarrow \forall p [\forall t \neg Killed(p, t) \vee \neg Alive(p)]$

$$\hookrightarrow \neg \text{Killed}(P, t) \vee \neg \text{Alive}(P)$$

$$\text{vi. Eats}(\text{Bill}, \text{Peanuts}) \wedge \text{Alive}(\text{Bill})$$

$$\text{vii. } \forall t \text{ Eats}(\text{Bill}, t) \Rightarrow \text{Eats}(\text{Sue}, t)$$

$$\hookrightarrow \forall t \neg \text{Eats}(\text{Bill}, t) \vee \text{Eats}(\text{Sue}, t)$$

$$\hookrightarrow \neg \text{Eats}(\text{Bill}, t) \vee \text{Eats}(\text{Sue}, t)$$

Thus, the CNF is:

$$(\neg \text{Is-food}(f_1) \vee \text{Likes}(\text{John}, f_1)) \wedge \text{Is-food}(\text{Apples}) \wedge \text{Is-food}(\text{Chicken})$$

$$\wedge (\neg \text{Eats}(P_2, f_2) \vee \text{Killed}(P_1, f_2) \vee \text{Is-food}(f_2))$$

$$\wedge (\neg \text{Killed}(P_3, t_3) \vee \neg \text{Alive}(P_3))$$

$$\wedge \text{Eats}(\text{Bill}, \text{Peanuts}) \wedge \text{Alive}(\text{Bill})$$

$$\wedge (\neg \text{Eats}(\text{Bill}, t_4) \vee \text{Eats}(\text{Sue}, t_4))$$

$$(c) \Delta: 1. \neg \text{Is-food}(f_1) \vee \text{Likes}(\text{John}, f_1)$$

$$2. \text{Is-food}(\text{Apples})$$

$$3. \text{Is-food}(\text{Chicken})$$

$$4. \neg \text{Eats}(P_2, f_2) \vee \text{Killed}(P_1, f_2) \vee \text{Is-food}(f_2)$$

$$5. \neg \text{Killed}(P_3, t_3) \vee \neg \text{Alive}(P_3)$$

$$6. \text{Eats}(\text{Bill}, \text{Peanuts})$$

$$7. \text{Alive}(\text{Bill})$$

$$8. \neg \text{Eats}(\text{Bill}, t_4) \vee \text{Eats}(\text{Sue}, t_4)$$

$$9. \text{Killed}(\text{Bill}, \text{Peanuts}) \vee \text{Is-food}(\text{Peanuts}) \text{ from 4, 6, } \theta = \{P_2/\text{Bill}, f_2/\text{Peanuts}\}$$

$$10. \neg \text{Alive}(\text{Bill}) \vee \text{Is-food}(\text{Peanuts}) \text{ from 5, 9, } \theta = \{P_3/\text{Bill}, t_3/\text{Peanuts}\}$$

$$11. \text{Is-food}(\text{Peanuts}) \text{ from 7, 10, } \theta = \{\}$$

$$12. \text{Likes}(\text{John}, \text{Peanuts}) \text{ from 1, 11, } \theta = \{f_1/\text{Peanuts}\}$$

By 12, we see that John likes Peanuts.

- (d). Δ :
1. $\neg \text{Is_food}(f_1) \vee \text{Likes}(\text{John}, f_1)$
 2. $\text{Is_food}(\text{Apples})$
 3. $\text{Is_food}(\text{Chicken})$
 4. $\neg \text{Eats}(P_2, f_2) \vee \text{Killed}(P_2, f_2) \vee \text{Is_food}(f_2)$
 5. $\neg \text{Killed}(P_3, t_3) \vee \neg \text{Alive}(P_3)$
 6. $\text{Eats}(\text{Bill}, \text{Peanuts})$
 7. $\text{Alive}(\text{Bill})$
 8. $\neg \text{Eats}(\text{Bill}, t_4) \vee \text{Eats}(\text{Sue}, t_4)$
 9. $\text{Eats}(\text{Sue}, \text{Peanuts})$ by 6, 8. $\theta = \{t_4 / \text{Peanuts}\}$
- Thus by 9, we see Sue eats Peanuts.

(e). Three new rules/facts:

$$\textcircled{1} \forall P \forall t \neg \text{Eats}(P, t) \Rightarrow \text{Die}(P)$$

$$\hookrightarrow \forall P \forall t \text{Eats}(P, t) \vee \text{Die}(P)$$

$$\hookrightarrow \text{Eats}(P, t) \vee \text{Die}(P)$$

$$\textcircled{2} \forall P \text{Die}(P) \Rightarrow \neg \text{Alive}(P)$$

$$\hookrightarrow \forall P \neg \text{Die}(P) \vee \neg \text{Alive}(P)$$

$$\hookrightarrow \neg \text{Die}(P) \vee \neg \text{Alive}(P)$$

$$\textcircled{3} \text{Alive}(\text{Bill})$$

- Δ :
1. $\neg \text{Is_food}(f_1) \vee \text{Likes}(\text{John}, f_1)$
 2. $\text{Is_food}(\text{Apples})$
 3. $\text{Is_food}(\text{Chicken})$
 4. $\neg \text{Eats}(P_2, f_2) \vee \text{Killed}(P_2, f_2) \vee \text{Is_food}(f_2)$
 5. $\neg \text{Killed}(P_3, t_3) \vee \neg \text{Alive}(P_3)$
 6. $\neg \text{Eats}(\text{Bill}, t_4) \vee \text{Eats}(\text{Sue}, t_4)$
 7. $\text{Eats}(P_5, t_5) \vee \text{Die}(P_5)$
 8. $\neg \text{Die}(P_6) \vee \neg \text{Alive}(P_6)$
 9. $\text{Alive}(\text{Bill})$

10. $\neg \text{Die}(\text{Bill})$ by 8, 9, $\theta = \{P_6 / \text{Bill}\}$

11. $\neg \text{Killed}(\text{Bill}, t_3)$ by 5, 9, $\theta = \{P_3 / \text{Bill}\}$

12. $\text{Eats}(\text{Bill}, t_5)$ by 7, 10, $\theta = \{P_5 / \text{Bill}\}$

13. $\neg \text{Eats}(\text{Bill}, f_2) \vee \text{Is-food}(f_2)$ by 4, 11, $\theta = \{P_2 / \text{Bill}, t_3 / f_2\}$

\vdots

After trying all combinations of resolution, we cannot get an explicit term $\text{Eats}(\text{Sue}, \text{food})$, where food is some known constant.

Thus, what does sue eat is undecidable.

Further explanation: From the context, we can only observe that Bill does eat something, so that he won't die, and hence Sue also eats something. However, we are not able to determine a unifier θ to substitute t_5 by a constant in $\text{Eats}(\text{Bill}, t_5)$. Thus we only know Sue eats something (as Bill eats t_5), but we do not know exactly what she eats.