Problem Set 3

1. (3.6 in the textbook) Let G be a connected undirected graph. Suppose that we compute a BFS tree and a DFS tree for G and end up with the same tree in either case. Show that G is a tree.

2. Let G be a directed graph and let s and t be vertices of G.

   (a) Each edge of G is colored either black or white. Decide if there's a path from s to t using at most one black edge. If there is, then find the shortest such a path. Your algorithm should run in $O(m+n)$ time.

   (b) Decide if there's a path from s to t whose length is a multiple of 5. If there is, then find the shortest such path. Your algorithm should run in $O(m+n)$ time.

   (c) The vertices of G are colored black and white with s colored black and t colored white. Alice and Bob start on s and t respectively. At each unit of time, each person can either stay where they are, or walk along an edge of G. However at a given time, they are not allowed to be on vertices of the same color. Decide if it's possible for Alice and Bob to switch places, and if so find the shortest strategy. Your algorithm should run in $O((m+n)^2)$ time.

3. Let G be a connected undirected graph. We say that an edge e of G is a bridge if removing e disconnects G, or equivalently if there are two vertices u and v such that every path between u and v uses e.

   Suppose that G has no bridges. Show that it's possible to assign a direction to each edge of G so that the resulting directed graph is strongly connected. Also give an algorithm to assign the directions which runs in linear time.

4. The goal of this problem is to apply DFS to solve Solitaire. See the notebook.