Lucrare de Laborator Nr. 1 "Elaborarea unui program MPI pentru generarea jocurilor bimatriceale si determinarea situatiilor Nash de echilibru in strategii pure"

Consideram un joc in urmatoarea forma normala $\Gamma = \langle I, J, A, B\rangle$,unde $I = \{1, 2, .., n\}$ este multime de indici ale liniilor, $J = \{1, 2, .., m\}$ este multimea de indici ale coloanelor $A = ||a_{ij}||_{i\in I}^{j\in J}$, $B = ||b_{ij}||_{i\in I}^{j\in J}$ sunt matricele de utilitate ale jucatorului 1 si r 2, respectiv.

1) Generarea jocurilor bimatriceale.

Vom construi un sir de numere $\mathbf{i} = i_1 i_2 ... i_j ... i_m$ si $\mathbf{j} = j_1 j_2 ... j_i ... j_n$ unde elementele $i_j = \mathbf{i}(j)$ (respectiv $j_i = \mathbf{j}(i)$) semnifica urmatoarele: daca jucatorul 2 (respectiv jucatorul 1) alege coloana $j \in J$ (alege linia $i \in I$) atunci jucatorul 1 (respectiv jucatorul 2) va alege linia $i_j \in I$ (va alege coloana $j_i \in J$). Notam $\mathbf{I} = \{\mathbf{i} = i_1 i_2 ... i_j ... i_m\}$ si $\mathbf{J} = \{\mathbf{j} = j_1 j_2 ... j_i ... j_n\}$ multimea tuturor sirurilor de acest tip, si $|\mathbf{I}| = n^m$ and $|\mathbf{J}| = m^n$

Pentru orice pereche $(\mathbf{i}, \mathbf{j})$ sa se genereze matricea $AB(\mathbf{i}, \mathbf{j}) = \|(\mathfrak{a}_{ij}, \mathfrak{b}_{ij})\|_{i\in I}^{j\in J}$, unde $\mathfrak{a}_{ij} \equiv a_{i_j j_i}$, $\mathfrak{b}_{ij} \equiv b_{i_j j_i}$.

Example 0.1 *Consideram jocul bimatriceal* $A = \begin{pmatrix} 3 & 5 & 4 \\ 6 & 7 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 5 & 1 \\ 4 & 3 & 2 \end{pmatrix}$. *Fie* $\mathbf{I} = \{\mathbf{i}^1, \mathbf{i}^2\}$ *si* $\mathbf{J} = \{\mathbf{j}^1, \mathbf{j}^2\}$, *unde* $\mathbf{i}^1 = 1_1 1_2 2_3$, $\mathbf{i}^2 = 1_1 2_2 1_3$ *si* $\mathbf{j}^1 = 1_1 2_2$ *and* $\mathbf{j}^2 = 2_1 1_2$.

- Pentru $\mathbf{i}^1 = 1_1 1_2 2_3, \mathbf{j}^1 = 1_1 2_2$:
  $\mathfrak{a}_{11} \equiv a_{i^1_{\mathbf{1}} j^1_{\mathbf{1}}} = a_{11} = 3$; $\mathfrak{a}_{12} \equiv a_{i^1_{\mathbf{2}} j^1_{\mathbf{1}}} = a_{11} = 3$; $\mathfrak{a}_{13} \equiv a_{i^1_{\mathbf{3}} j^1_{\mathbf{1}}} = a_{21} = 6$
  $\mathfrak{a}_{21} \equiv a_{i^1_{\mathbf{1}} j^1_{\mathbf{2}}} = a_{12} = 5$; $\mathfrak{a}_{22} \equiv a_{i^1_{\mathbf{2}} j^1_{\mathbf{2}}} = a_{12} = 5$; $\mathfrak{a}_{23} \equiv a_{i^1_{\mathbf{3}} j^1_{\mathbf{2}}} = a_{22} = 7$
  $\mathfrak{b}_{11} \equiv b_{i^1_{\mathbf{1}} j^1_{\mathbf{1}}} = b_{11} = 0$; $\mathfrak{b}_{12} \equiv b_{i^1_{\mathbf{2}} j^1_{\mathbf{1}}} = b_{11} = 0$; $\mathfrak{b}_{13} \equiv a_{i^1_{\mathbf{3}} j^1_{\mathbf{1}}} = b_{21} = 4$
  $\mathfrak{b}_{21} \equiv b_{i^1_{\mathbf{1}} j^1_{\mathbf{2}}} = b_{12} = 5$; $\mathfrak{b}_{22} \equiv b_{i^1_{\mathbf{2}} j^1_{\mathbf{2}}} = b_{12} = 5$; $\mathfrak{b}_{23} \equiv b_{i^1_{\mathbf{3}} j^1_{\mathbf{2}}} = b_{22} = 3$
- Pentru $\mathbf{i}^2 = 1_1 2_2 1_3, \mathbf{j}^1 = 1_1 2_2$:
  $\mathfrak{a}_{11} \equiv a_{i^2_{\mathbf{1}} j^1_{\mathbf{1}}} = a_{11} = 3$; $\mathfrak{a}_{12} \equiv a_{i^2_{\mathbf{2}} j^1_{\mathbf{1}}} = a_{21} = 6$; $a_{13} \equiv a_{i^2_{\mathbf{3}} j^1_{\mathbf{1}}} = a_{11} = 3$
  $\mathfrak{a}_{21} \equiv a_{i^2_{\mathbf{1}} j^1_{\mathbf{2}}} = a_{12} = 5$; $\mathfrak{a}_{22} \equiv a_{i^2_{\mathbf{2}} j^1_{\mathbf{2}}} = a_{22} = 7$; $\mathfrak{a}_{23} \equiv a_{i^2_{\mathbf{3}} j^1_{\mathbf{2}}} = a_{12} = 5$
  $\mathfrak{b}_{11} \equiv b_{i^2_{\mathbf{1}} j^1_{\mathbf{1}}} = b_{11} = 0$; $\mathfrak{b}_{12} \equiv b_{i^2_{\mathbf{2}} j^1_{\mathbf{1}}} = b_{21} = 4$; $\mathfrak{b}_{13} \equiv a_{i^2_{\mathbf{3}} j^1_{\mathbf{1}}} = b_{11} = 0$
  $\mathfrak{b}_{21} \equiv b_{i^2_{\mathbf{1}} j^1_{\mathbf{2}}} = b_{12} = 5$; $\mathfrak{b}_{22} \equiv b_{i^2_{\mathbf{2}} j^1_{\mathbf{2}}} = b_{22} = 3$; $\mathfrak{b}_{23} \equiv b_{i^2_{\mathbf{3}} j^1_{\mathbf{2}}} = b_{12} = 5$
- Pentrur $\mathbf{i}^1 = 1_1 1_2 2_3, \mathbf{j}^2 = 2_1 1_2$:
  $\mathfrak{a}_{11} \equiv a_{i^1_{\mathbf{1}} j^2_{\mathbf{1}}} = a_{12} = 5$; $\mathfrak{a}_{12} \equiv a_{i^1_{\mathbf{2}} j^2_{\mathbf{1}}} = a_{12} = 5$; $\mathfrak{a}_{13} \equiv a_{i^1_{\mathbf{3}} j^2_{\mathbf{1}}} = a_{22} = 7$
  $\mathfrak{a}_{21} \equiv a_{i^1_{\mathbf{1}} j^2_{\mathbf{2}}} = a_{11} = 3$; $\mathfrak{a}_{22} \equiv a_{i^1_{\mathbf{2}} j^2_{\mathbf{2}}} = a_{11} = 3$; $\mathfrak{a}_{23} \equiv a_{i^1_{\mathbf{3}} j^2_{\mathbf{2}}} = a_{21} = 6$
  $\mathfrak{b}_{11} \equiv b_{i^1_{\mathbf{1}} j^2_{\mathbf{1}}} = b_{12} = 5$; $\mathfrak{b}_{12} \equiv b_{i^1_{\mathbf{2}} j^2_{\mathbf{1}}} = b_{12} = 5$; $\mathfrak{b}_{13} \equiv a_{i^1_{\mathbf{3}} j^2_{\mathbf{1}}} = b_{22} = 3$
  $\mathfrak{b}_{21} \equiv b_{i^1_{\mathbf{1}} j^2_{\mathbf{2}}} = b_{11} = 0$; $\mathfrak{b}_{22} \equiv b_{i^1_{\mathbf{2}} j^2_{\mathbf{2}}} = b_{11} = 0$; $\mathfrak{b}_{23} \equiv b_{i^1_{\mathbf{3}} j^2_{\mathbf{2}}} = b_{21} = 4$

- For $\mathbf{i}^2 = 1_1 2_2 1_3, \mathbf{j}^2 = 2_1 1_2$:
  $\mathfrak{a}_{11} \equiv a_{i_1^2 j_1^2} = a_{12} = 5;$ $\mathfrak{a}_{12} \equiv a_{i_2^2 j_1^2} = a_{22} = 7;$ $\mathfrak{a}_{13} \equiv a_{i_3^2 j_1^2} = a_{12} = 5$
  $\mathfrak{a}_{21} \equiv a_{i_1^2 j_2^2} = a_{11} = 3;$ $\mathfrak{a}_{22} \equiv a_{i_2^2 j_2^2} = a_{21} = 6;$ $\mathfrak{a}_{23} \equiv a_{i_3^2 j_2^2} = a_{11} = 3$
  $\mathfrak{b}_{11} \equiv b_{i_1^2 j_1^2} = b_{12} = 5;$ $\mathfrak{b}_{12} \equiv b_{i_2^2 j_1^2} = b_{22} = 3;$ $\mathfrak{b}_{13} \equiv a_{i_3^2 j_1^2} = b_{12} = 5$
  $\mathfrak{b}_{21} \equiv b_{i_1^2 j_2^2} = b_{11} = 0;$ $\mathfrak{b}_{22} \equiv b_{i_2^2 j_2^2} = b_{21} = 4;$ $\mathfrak{b}_{23} \equiv b_{i_3^2 j_2^2} = b_{11} = 0$

Astfel am obtinut urmatoarele jocuri bimatriceale

$$AB\left(\mathbf{i}^1, \mathbf{j}^1\right) = \begin{pmatrix} (3,0) & (3,0) & (6,4) \\ (5,5) & (5,5) & (7,3) \end{pmatrix},$$

$$AB\left(\mathbf{i}^2, \mathbf{j}^1\right) = \begin{pmatrix} (3,0) & (6,4) & (3,0) \\ (5,5) & (7,3) & (5,5) \end{pmatrix},$$

$$AB\left(\mathbf{i}^1, \mathbf{j}^2\right) = \begin{pmatrix} (5,5) & (5,5) & (7,3) \\ (3,0) & (3,0) & (6,4) \end{pmatrix},$$

$$AB\left(\mathbf{i}^2, \mathbf{j}^2\right) = \begin{pmatrix} (5,5) & (7,3) & (5,5) \\ (3,0) & (6,4) & (3,0) \end{pmatrix}.$$

Sa se elaboreze un program MPI care va realiza urmatoarele:

- Procesul MPI cu rancul 0 va initializa matricele initiale A si B,si va transmite tuturor proceselor din comunicatorul MPI_COM_WORLD aceste matrici.

- Fiecare proces MPI cu rancul k, folosind un algoritm combinatorial, va genera perechea de siruri

$$\left(\mathbf{i}^k = i_1^k i_2^k ... i_j^k ... i_m^k,\ \mathbf{j}^k = j_1^k j_2^k ... j_i^k ... j_n^k.\right).$$

  Generarea sirurilor $\mathbf{i}$ este echivalent cu urmatoarea problema: pentru o multime de numere naturale $\{1, 2, ..., i, ...n\}$, sa se genereze siruri de lungimea m din aceste numere. De exemplu daca $I = \{1, 2\}$ si $J = \{1, 2, 3\}$ atunci

$$\mathbf{I} = \{1_1 1_2 1_3, 2_1 2_2 2_3, 1_1 1_2 2_3, 1_1 2_2 1_3, 2_1 1_2 1_3, 1_1 2_2 2_3, 2_1 1_2 2_3, 2_1 2_2 1_3\},$$

$$\mathbf{J} = \{1_1 1_2, 2_1 2_2, 3_1 3_2, 1_1 2_2, 2_1 1_2, 1_1 3_2, 3_1 1_2, 2_1 3_2, 3_1 2_2\}.$$

So, to construct the set I for $n = 2, m = 3$, we must: a) generate the strings $(1,1,1)$ and $(2,2,2)$; b) having the numbers $\{1,2\}$ to generate all the sub-strings of length 3 with the elements in this set, that is the strings $(1,1,2)$, $(1,2,1)$, $(2,1,1)$, $(1,2,2,)$, $(2,1,2)$, $(2,2,1)$. In mathematics, a multiset (or bag) is a generalization of the concept of a set that, unlike a set, allows multiple instances of the multiset's elements. For example, $\{a,a,b\}$ and $\{a,b\}$ are different multisets although they are the same set. However, order is important, so $\{a,a,b\}$ and $\{a,b,a\}$ are the different multiset. It can be easily noticed that any informational extended strategy is nothing more than a multiset, so their generation actually consists in generating multisets .

- Fiecare proces MPI cu rancul k construieste perechea de matrici (A^k, B^k) , unde $A^k = \|\mathfrak{a}_{ij}\|_{i\in I}^{j\in J}$, $B^k = \|\mathfrak{b}_{ij}\|_{i\in I}^{j\in J}$, si $\mathfrak{a}_{ij} \equiv a_{i_j j_i}$, $\mathfrak{b}_{ij} \equiv b_{i_j j_i}$.

2) Determinarea solutiilor de tip Nash pentru jocurile generate.

Vezi fisierul "Modele de programare paralela pe clustere Programare MPI.pdf", pag. 113