Probability distributions play a critical role in the field of statistics, serving as essential tools for modeling and understanding the probability of different possible values of a variable. Graphs and probability tables are frequently used to illustrate probability distributions. Among the multitude of distributions available, the Normal Probability Distribution, the Gamma Probability Distribution, and the Beta Probability Distribution are very important and are widely applied models with unique properties and uses.

Normal Probability Distribution is the most commonly used distribution for continuous probability. The most notable mention of this distribution is its symmetry about the mean which means that the data near the mean occur more frequently compared to the data farther from the mean and its bell-shape. A Normal Probability Distribution is said to occur to a random variable Y if and only if and only if the density function of Y is

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/(2\sigma^2)},$$

for $\sigma > 0$, -∞ $< \mu < \infty$, and -∞ $< y < \infty$. The normal density functions contain two parameters, μ and σ. However if Y is a normally distributed random variable, then

$$E(Y) = \mu \text{ and } V(Y) = \sigma^2 .$$

The parameter μ locates the center of the distribution while σ measures the spread of this distribution. The purpose of the Normal distribution is to provide a mathematical model for describing the distribution of a continuous random variable. It is particularly valuable in quality control, sales forecasting, and financial forecasting, offering a standardized structure to analyze and make predictions about data that exhibit a bell-shaped, symmetric distribution around a central mean with a known standard deviation. For quality control, companies monitor the quality of their products by using statistical process control techniques like measuring the mean and standard deviation of their progress thus allowing them to use Normal Distribution to set control limits. For sales forecasting, companies can calculate the probability of reaching certain sales levels and adjust sales objectives accordingly by estimating the mean and standard deviation of sales. For financial analysis, Normal Distribution is used to estimate the return expected and the risk associated with different investments.

Gamma Probability Distribution is a continuous probability distribution that is commonly used to model the time until a specified event occurs, such as the lengths of time between arrivals at a supermarket checkout queue. A Gamma Probability Distribution is said to occur to a random variable Y if and only if the density function of Y is

$$f(x) = \begin{cases} \frac{y^{\alpha-1}e^{-y/\beta}}{\beta^\alpha \Gamma(\alpha)}, & 0 < y < \infty \\ 0 & , \; elsewhere \end{cases}$$

where

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy .$$

The quantity $\Gamma(\alpha)$ is known as the gamma function. The values of α dictates the shape of the gamma density. It sometimes referred as the shape parameter associated with a gamma distribution. The parameter β is generally referred as the scale parameter because a random variable that has a gamma distribution with the same shape parameter but with a different scale parameter is produced when a gamma-distributed random variable is multiplied by a positive constant. The gamma distribution has been applied to simulate the amount of rain and insurance claims. This means that, like how the exponential distribution produces a Poisson process, the total number of insurance claims and the quantity of rainfall that accumulates in a reservoir are both modeled by gamma processes. The gamma distribution is used in wireless communication to simulate the multi-path fading of signal strength. In oncology, the age distribution of cancer incidence frequently resembles the gamma distribution, where the number of driving events and the interval between them are predicted by the shape and scale factors, respectively. A constitutively expressed protein's copy number in bacteria typically follows the gamma distribution, whose scale and shape parameters are the average number of bursts per cell cycle and the average number of protein molecules generated by a single mRNA over its lifetime, respectively.

The Beta Probability Distribution is a continuous probability distribution defined on the interval [0, 1]. It is commonly used to model random variables that represent proportions or probabilities. A Beta Probability Distribution is said to occur to a random variable Y if and only if the density function of Y is

$$f(y) = \begin{cases} \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)}, & 0 < y < 1 \\ 0 & , \; elsewhere \end{cases}$$

where

$$B(\alpha,\beta) = \int_0^1 y^{\alpha-1}(1-y)^{\beta-1}dy = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

and $\alpha > 0$ and $\beta > 0$. For different various values of the parameters of α and β, the graphs of beta density functions differ. The cumulative distribution function for the beta random variable is commonly called the incomplete beta function and is

$$F(y) = \int_0^y \frac{t^{\alpha-1}(1-t)^{\beta-1}}{B(\alpha,\beta)}dt = I_y(\alpha,\beta).$$

$I_y(\alpha,\beta)$ is related to the binomial probability function when α and β are positive integers. Beta probability distribution is most frequently used to simulate the uncertainty surrounding the likelihood that a random experiment will succeed. The "beta distribution" is a three-point technique used in project management that acknowledges the uncertainty in project time estimation. Together with the fundamental statistics, it offers strong mathematical tools for calculating the confidence intervals for the anticipated completion time. Additionally, the beta distribution is employed in PERT, yielding a nearly normal bell-shaped curve. As an illustration, the beta distribution is employed in Bayesian analysis to specify the starting knowledge of the success probability that aids in the successful completion of the given task. It is an appropriate technique for proportions and percentages that exhibit random behavior.

In conclusion, probability distributions are fundamental tools in the field of statistics, providing a mathematical framework for modeling and understanding the likelihood of different outcomes. The Normal Probability Distribution, characterized by its bell-shaped curve and symmetry around the mean, plays a crucial role in various applications such as quality control, sales forecasting, and financial analysis. Its standardized structure allows for the analysis and prediction of data exhibiting a bell-shaped, symmetric distribution around a central mean with a known standard deviation. The Gamma Probability Distribution, a continuous distribution commonly employed to model the time until a specified event occurs, finds applications in diverse fields, including simulating rainfall, insurance claims, wireless communication, oncology, and protein copy number distribution. The gamma distribution's shape parameter influences the distribution's form, while the scale parameter determines the spread. The Beta Probability Distribution, defined on the interval [0, 1], is frequently used to model proportions or probabilities. Its flexibility makes it suitable for various applications, such as project management, where it is utilized to account for uncertainty in time estimation, and Bayesian analysis, where it specifies the initial knowledge of success probability. In summary, these probability distributions contribute significantly to statistical modeling and analysis, providing valuable tools for understanding and predicting real-world phenomena in diverse fields.

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