

1. Given the data as listed below

x	4.0	4.2	4.5	4.7	5.1	5.5	5.9	6.3
y	102.6	113.2	130.1	142.1	167.5	195.1	224.9	256.8

- Construct the least squares approximation of degree two and compute the error.
- Construct the least squares approximation of the form be^{ax} and compute the error.
- Construct the least squares approximation of the form bx^n and compute the error.

```
PS C:\Users\afatf\Desktop\E94116067_numerical_hw8>
• (a)
a = 6.6912 , b = -1.8837 , c = 3.0864
Error: 0.0052

(b)
a = 0.3985 , b = 21.4445
Error: 94.9830

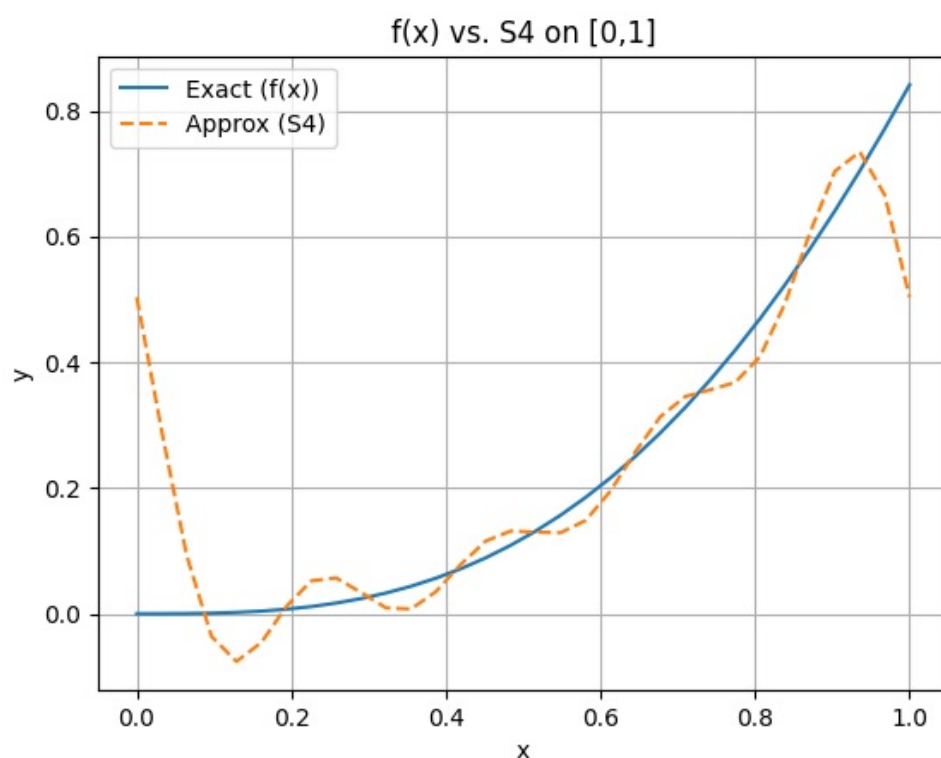
(c)
n = 2.0196 , b = 6.2390
Error: 0.0117
```

2. Find the least squares polynomial approximation of degree two on the interval $[-1,1]$ for the function $f(x) = \frac{1}{2}\cos x + \frac{1}{4}\sin 2x$

```
PS C:\Users\afatf\Desktop\E94116067_numerical_hw8>
• a = -0.2326 , b = 0.3265 , c = 0.4983
Error: 0.0032
```

3. Determine the discrete least squares trigonometric polynomial S_4 using $m=16$ for $f(x) = x^2 \sin x$ on the interval $[0,1]$.
- b. Compute $\int_0^1 S_4(x) dx$
- c. Compare the integral in part (b) to $\int_0^1 x^2 \sin x dx$
- d. Compute the error $E(S_4)$

```
PS C:\Users\afatf> & C:/Python313/python.exe
• (a)
a0 = 0.4592
a1 = -0.1468 , b1 = 0.2323
a2 = 0.0546 , b2 = -0.1249
a3 = -0.0389 , b3 = 0.0829
a4 = 0.0335 , b4 = -0.0609
(b)
Ans: 0.2296
```



(c)

x	f(x)	S4(x)	point_err
-3.1416	0.0000	0.5034	3.1416
-2.9389	0.0000	0.2910	2.9712
-2.7362	0.0003	0.0931	2.8007
-2.5335	0.0009	-0.0360	2.6303
-2.3309	0.0021	-0.0760	2.4599
-2.1282	0.0042	-0.0456	2.2895
-1.9255	0.0072	0.0114	2.1190
-1.7228	0.0114	0.0523	1.9486
-1.5201	0.0170	0.0570	1.7782
-1.3174	0.0241	0.0338	1.6078
-1.1148	0.0330	0.0093	1.4373
-0.9121	0.0437	0.0075	1.2669
-0.7094	0.0566	0.0352	1.0965
-0.5067	0.0716	0.0786	0.9261
-0.3040	0.0890	0.1156	0.7556
-0.1013	0.1089	0.1316	0.5852
0.1013	0.1315	0.1299	0.4148
0.3040	0.1568	0.1288	0.2444
0.5067	0.1849	0.1483	0.0739
0.7094	0.2161	0.1951	0.0965
0.9121	0.2503	0.2575	0.2669
1.1148	0.2876	0.3133	0.4373
1.3174	0.3282	0.3455	0.6078
1.5201	0.3720	0.3562	0.7782
1.7228	0.4190	0.3679	0.9486
1.9255	0.4695	0.4089	1.1190
2.1282	0.5232	0.4934	1.2895
2.3309	0.5803	0.6060	1.4599
2.5335	0.6407	0.7036	1.6303
2.7362	0.7044	0.7347	1.8007
2.9389	0.7713	0.6662	1.9712
3.1416	0.8415	0.5034	2.1416

f(x) Integral: 0.2232 , S4(Z) Integral: 0.2296

Absolute Error: 0.0064 , Relative Error: 2.8482%

(d)

Square Error: 0.5056