1. Determine the values $\int_1^2 e^x \sin(4x)dx$ with $h = 0.1$ by

a. Use the composite trapezoidal rule

b. Use the composite Simpsons' method

c. Use the composite midpoint rule

$$h = \frac{b-a}{2n} \Rightarrow n = 5$$

(a)

$$\int_1^2 f(x)\,dx = \frac{h}{2}\left[f(1) + 2\sum_{i=1}^{2n-1} f(x_i) + f(2)\right] - \frac{b-a}{12} h^2 f''(\xi)$$

$$\approx \underline{0.396}$$

(b)

$$\int_1^2 f(x)\,dx = \frac{h}{3}\left[f(1) + 2\sum_{i=2}^{n} f(x_{2i-2}) + 4\sum_{i=1}^{n} f(x_{2i-1}) + f(2)\right] - \frac{b-a}{180} h^4 f^{iv}(\xi)$$

$$\approx \underline{0.386}$$

(c)

$$\int_1^2 f(x)\,dx = \frac{h}{2}\left[\sum_{i=1}^{n} f(x_{2i-1})\right] + \frac{b-a}{6} h^2 f''(\xi)$$

$$\approx \underline{0.365}$$

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PS C:\Users\afatf\Desktop\E94116067_numerical_hw4>
a: 0.396
b: 0.386
c: 0.365
```

2. Approximate $\int_1^{1.5} x^2 \ln x dx$ using Gaussian Quadrature with $n=3$ and $n=4$. Then compare the result to the exact value of the integral.

exact ≈ 0.19226

$n=3$: $x_1 = -x_3 = -0.775$, $x_2 = 0$

$c_1 = c_3 = 0.556$, $c_2 = 0.889$

$$\frac{x-a}{b-a} = \frac{\eta+1}{2} \Rightarrow x = \frac{a+b}{2} + \frac{b-a}{2}\eta$$

$$\Rightarrow \int_1^{1.5} f(x)\,dx = \frac{b-a}{2} \sum_{i=1}^{n} c_i f\left(\frac{a+b}{2} + \frac{b-a}{2}\eta_i\right) \approx \underline{0.19238}$$

error : $\left|0.19238 - 0.19226\right| = \underline{1.2 \times 10^{-4}}$

$n=4$: $x_1 = -x_4 = -0.861$, $x_2 = -x_3 = 0.340$

$c_1 = c_4 = 0.348$, $c_2 = c_3 = 0.652$

$$\Rightarrow \int_1^{1.5} f(x)\,dx = \frac{b-a}{2} \sum_{i=1}^{n} c_i f\left(\frac{a+b}{2} + \frac{b-a}{2}\eta\right) \approx \underline{0.19226}$$

$$\text{error} = \left| 0.19226 - 0.19226 \right| = \underline{0}$$

```
PS C:\Users\afatf\Desktop\E94116067_numerical_hw4>
n = 3: 0.19238 , Error: 0.00012
n = 4: 0.19226 , Error: 0.00000
```

3. Approximate $\int_0^{\pi/4} \int_{\sin x}^{\cos x} (2y\sin x + \cos^2 x)\,dy\,dx$ using

a. Simpson's rule for $n=4$ and $m=4$

b. Gaussian Quadrature, $n=3$ and $m=3$

c. Compare these results with the exact value.

(a)

$$\Rightarrow \frac{h}{9}\left[k_0 f(x_0,y_0) + 2\sum_{i=2}^{n} k_{2i-2} f(x_{2i-2}, y_0) + 4\sum_{i=1}^{n} k_{2i-1} f(x_{2i-1}, y_0) + k_{2n} f(x_{2n}, y_0)\right]$$

$$+ \frac{2h}{9}\sum_{j=2}^{m}\left[k_0 f(x_0, y_{2j-2}) + 2\sum_{i=2}^{n} k_{2i-2} f(x_{2i-2}, y_{2j-2}) + 4\sum_{i=1}^{n} k_{2i-1} f(x_{2i-1}, y_{2j-2}) + k_{2n} f(x_{2n}, y_{2j-2})\right]$$

$$+ \frac{4h}{9}\sum_{j=1}^{m}\left[k_0 f(x_0, y_{2j-1}) + 2\sum_{i=2}^{n} k_{2i-2} f(x_{2i-2}, y_{2j-1}) + 4\sum_{i=1}^{n} k_{2i-1} f(x_{2i-1}, y_{2j-1}) + k_{2n} f(x_{2n}, y_{2j-1})\right]$$

$$+ \frac{h}{9}\left[k_0 f(x_0, y_{2m}) + 2\sum_{i=2}^{n} k_{2i-2} f(x_{2i-2}, y_{2m}) + 4\sum_{i=1}^{n} k_{2i-1} f(x_{2i-1}, y_{2m}) + k_{2n} f(x_{2n}, y_{2m})\right]$$

$$\approx \underline{0.51185}$$

(b)

$$n = m = 3 \Rightarrow \quad x_1 = -x_3 = -0.775 \qquad x_2 = 0$$

$$c_1 = c_3 = 0.556 \qquad c_2 = 0.889$$

$$\Rightarrow \frac{b-a}{2} \sum_{i=1}^{n}\sum_{j=1}^{m} c_i c_j G_1(\xi_i) f\left(\frac{b-a}{2}\xi_i + \frac{a+b}{2},\ G_1(\xi_i)\eta_j + G_2(\xi_i)\right)$$

$$\approx \underline{0.51230}$$

(c)

exact ≈ 0.51185

Simpson : $\left| 0.51185 - 0.51185 \right| = \underline{0}$

Gauss : $\left| 0.51230 - 0.51185 \right| = \underline{4.5 \times 10^{-4}}$

```
PS C:\Users\afatf\Desktop\E94116067_numerical_hw4>
a: 0.51185
b: 0.51230
c: Simpson: 0.00000 , Gauss: 0.00045
```

4. Use the composite Simpson's rule and $n=4$ to approximate the improper integral a) $\int_0^1 x^{-1/4} \sin x dx$, b) $\int_1^{\infty} x^{-4} \sin x dx$ by use the transform $t = x^{-1}$

(a)

$t = x^{-1} \Rightarrow x = t^{-1} \Rightarrow dx = -t^{-2} dt \quad \Big|_0^1 \Rightarrow \Big|_{\infty}^1$

$\Rightarrow \int_{\infty}^{1} -t^{-\frac{7}{4}} \sin(t^{-1}) dt = \int_{\infty}^{1} -\frac{\sin(t^{-1})}{t^{\frac{7}{4}}} dt$

$\Rightarrow \frac{h}{3} \left[f(t_0) + 2 \sum_{i=2}^{n} f(t_{2i-2}) + 4 \sum_{i=1}^{n} f(t_{2i-1}) + f(t_{2n}) \right] \approx \underline{0.5574}$

(b)

$$\int_{1}^{\infty} \Rightarrow \int_{1}^{0}$$

$$\Rightarrow \int_{1}^{0} -t^{2}\sin(t^{-1})\,dt$$

$$\Rightarrow \frac{h}{3}\left[f(t_0) + 2\sum_{i=2}^{n} f(t_{2i-2}) + 4\sum_{i=1}^{n} f(t_{2i-1}) + f(t_{2n})\right] \approx \underline{0.2901}$$

```
PS C:\Users\afatf\Desktop\E94116067_numerical_hw4>
a: 0.5574
b: 0.2901
```