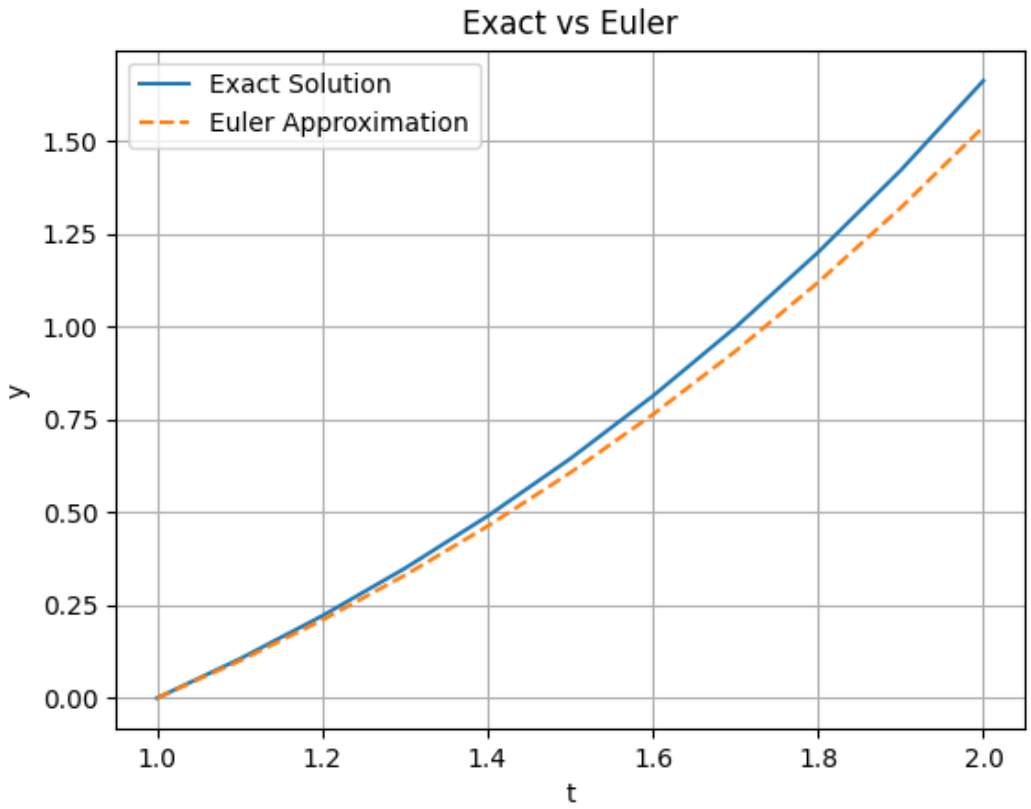
1. The initial-value problem

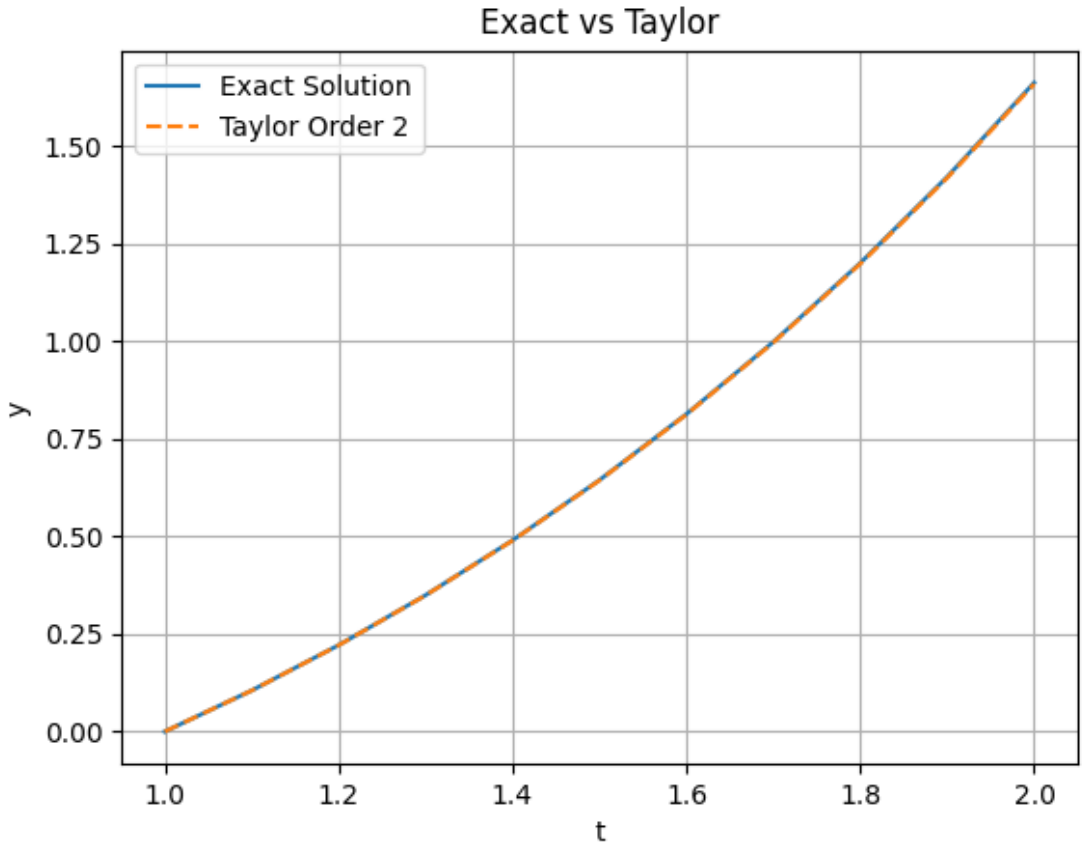
   $y' = 1 + (y/t) + (y/t)^2$, $1 \le t \le 2$, $y(1) = 0$ has the exact solution $y(t) = t\tan(\ln t)$.

a. Use Euler's method with $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

b. Use Taylor's method of order 2 with $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

```
PS C:\Users\afatf\Desktop\E94116067_numerical_hw5>
1.a
Euler Method Results:
  t  y_exact  y_euler    error
1.0 0.000000 0.000000 0.000000
1.1 0.105160 0.100000 0.005160
1.2 0.221243 0.209917 0.011325
1.3 0.349121 0.330471 0.018651
1.4 0.489682 0.462354 0.027328
1.5 0.643875 0.606285 0.037590
1.6 0.812753 0.763041 0.049711
1.7 0.997494 0.933475 0.064019
1.8 1.199439 1.118537 0.080902
1.9 1.420116 1.319293 0.100823
2.0 1.661282 1.536943 0.124338
1.b

Taylor Method (Order 2) Results:
  t  y_exact  y_Taylor2    error
1.0 0.000000   0.000000 0.000000
1.1 0.105160   0.105000 0.000160
1.2 0.221243   0.220919 0.000324
1.3 0.349121   0.348612 0.000509
1.4 0.489682   0.488954 0.000728
1.5 0.643875   0.642883 0.000993
1.6 0.812753   0.811438 0.001315
1.7 0.997494   0.995787 0.001707
1.8 1.199439   1.197252 0.002187
1.9 1.420116   1.417344 0.002772
2.0 1.661282   1.657795 0.003487
```


Exact vs Euler
Exact Solution
Euler Approximation
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
1.0
1.2
1.4
1.6
1.8
2.0
t


Exact vs Taylor
Exact Solution
Taylor Order 2
1.50
1.25
1.00
0.75
0.50
0.25
0.00
y
1.0
1.2
1.4
1.6
1.8
2.0
t

2. The system of initial-value problems

$$u_1' = 9u_1 + 24u_2 + 5\cos t - \frac{1}{3}\sin t, \quad u_1(0) = \frac{4}{3},$$

$$u_2' = -24u_1 - 52u_2 - 9\cos t + \frac{1}{3}\sin t, \quad u_2(0) = \frac{2}{3},$$

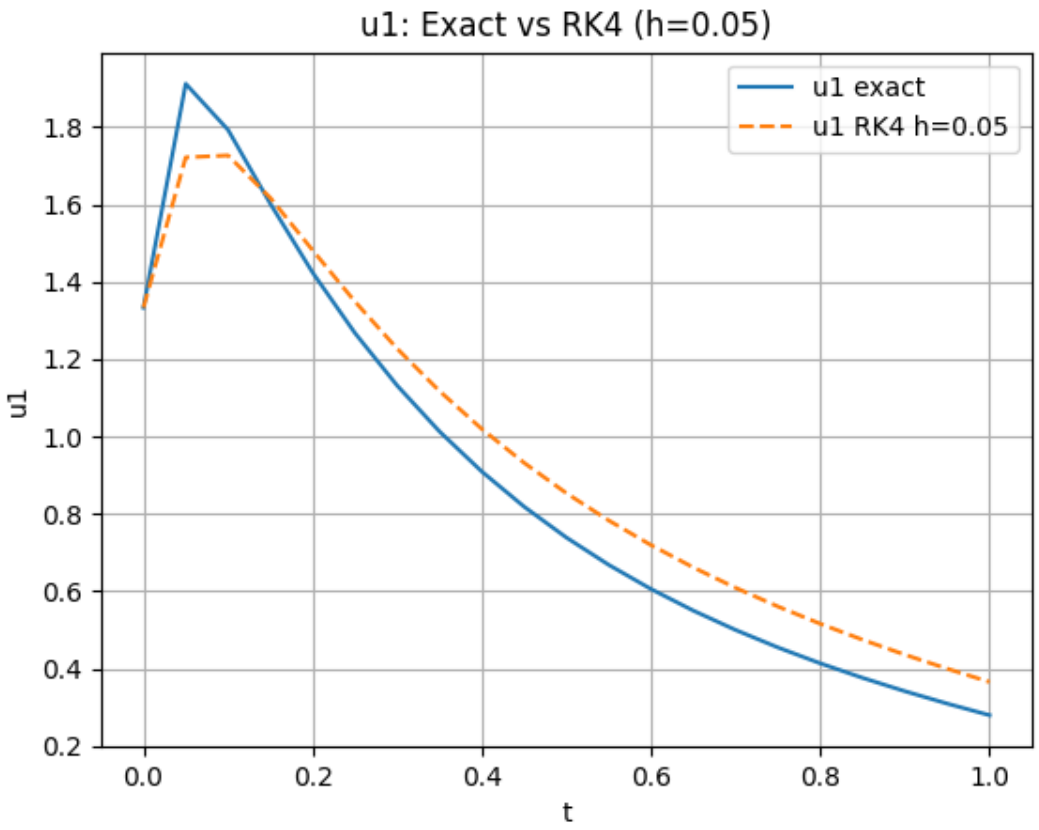
has the unique solution

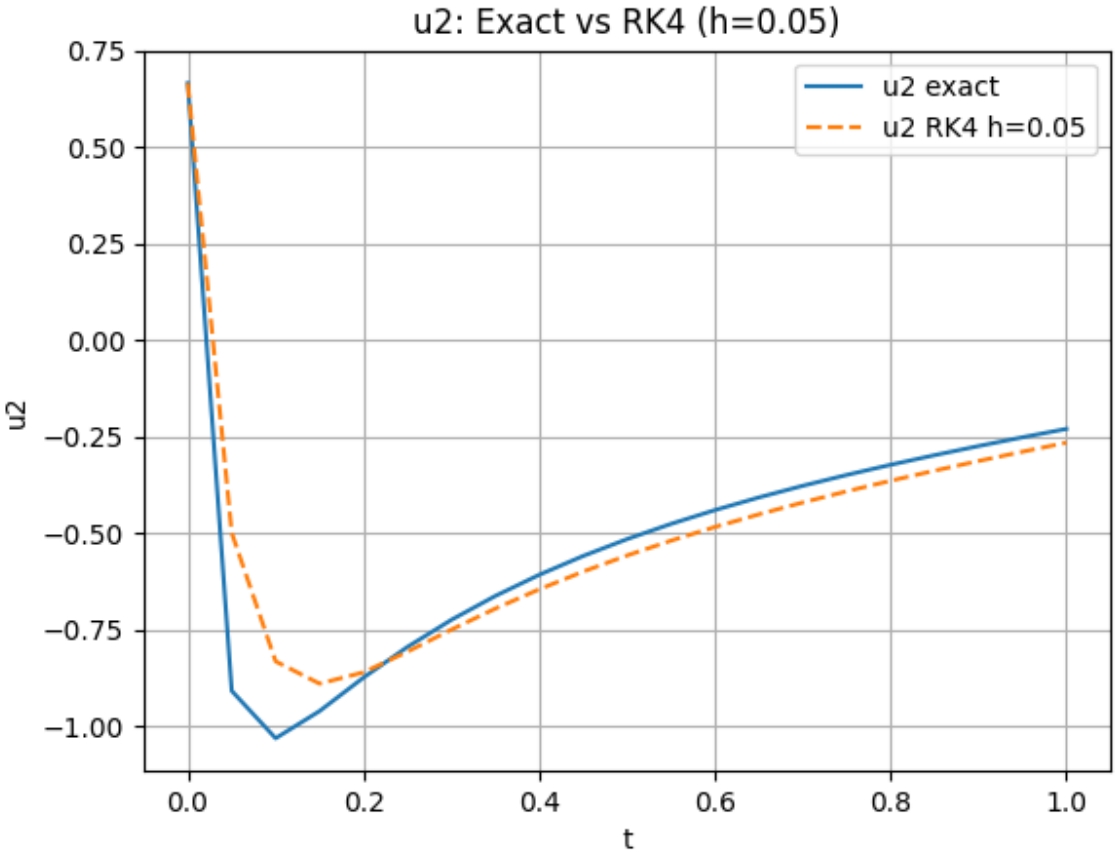
$$u_1 = 2e^{-3t} - e^{-39t} + \frac{1}{3}\cos t, \quad u_2 = -e^{-3t} + 2e^{-39t} - \frac{1}{3}\cos t.$$

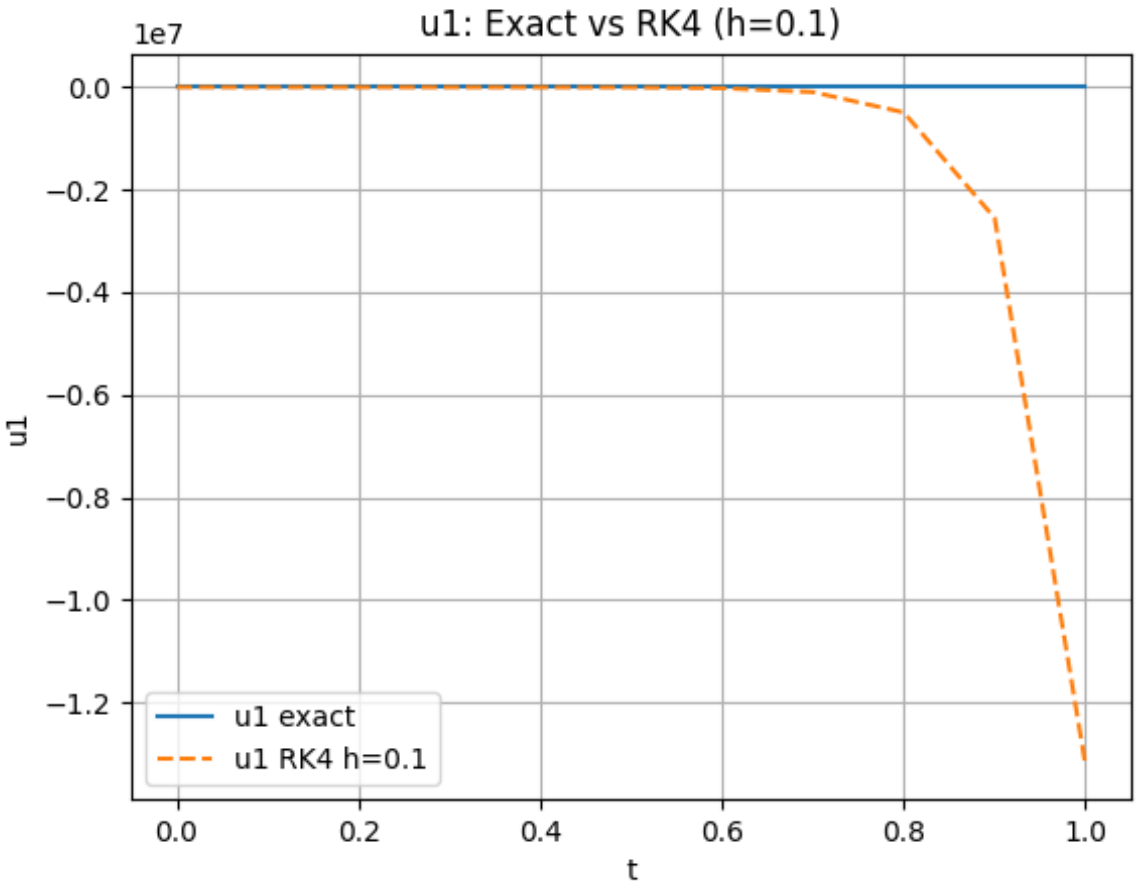
Try $h = 0.05$ and $h = 0.1$ in Runge-Kutta method, and compare their results with the exact value.

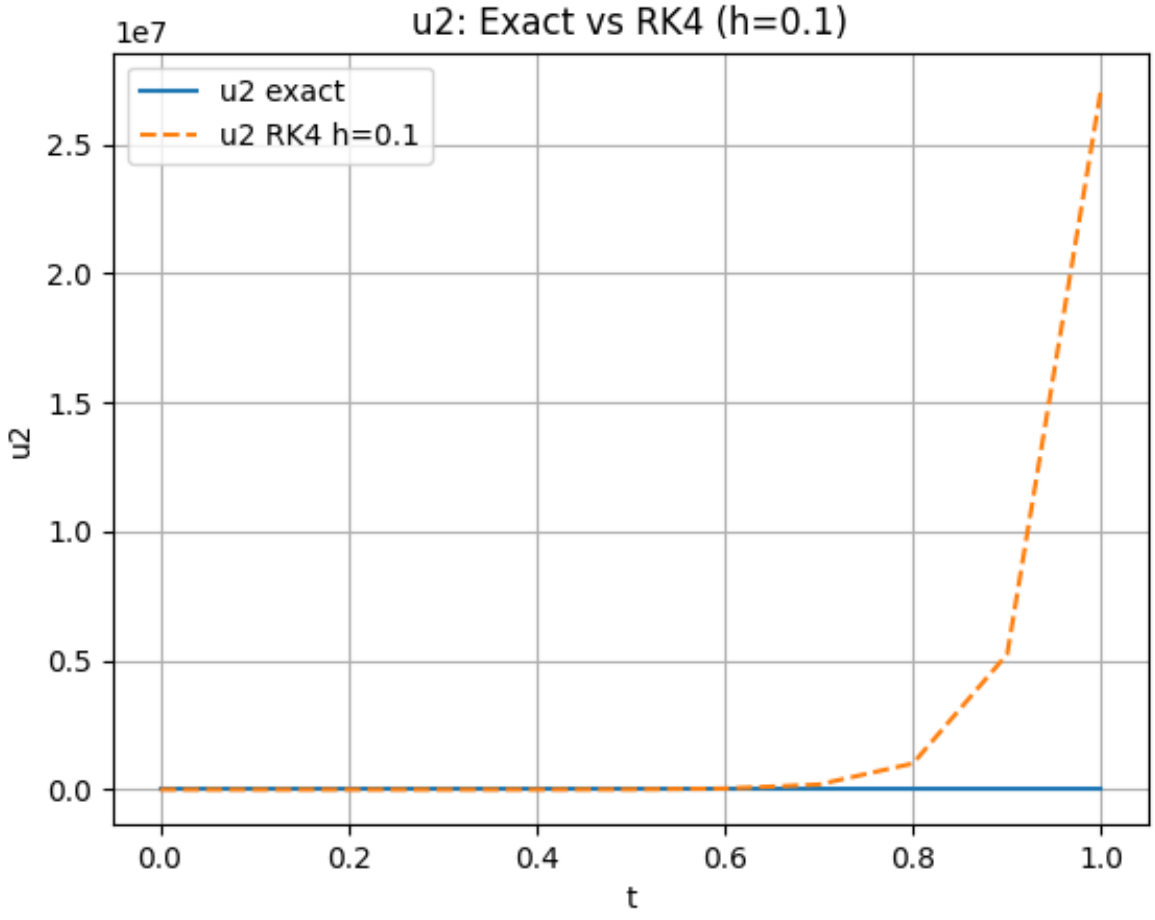
```
PS C:\Users\afatf\Desktop\E94116067_numerical_hw5> & C:/Python313/python.exe c:/

RK4 (h = 0.05)
         t   u1_exact     u1_RK4     err_u1    u2_exact     u2_RK4     err_u2
0.00000000 1.33333333 1.33333333 0.00000000  0.66666667  0.66666667 0.00000000
0.05000000 1.91205863 1.72188026 0.19017837 -0.90907659 -0.49959934 0.40947725
0.10000000 1.79306259 1.72691505 0.06614754 -1.03200245 -0.83259770 0.19940475
0.15000000 1.60196676 1.61716064 0.01519388 -0.96145871 -0.89037299 0.07108572
0.20000000 1.42390240 1.48168729 0.05778489 -0.87468103 -0.86104209 0.01363894
0.25000000 1.26764562 1.34894503 0.08129941 -0.79522077 -0.80750453 0.01228376
0.30000000 1.13157652 1.22706330 0.09548678 -0.72499857 -0.75034063 0.02534206
0.35000000 1.01299856 1.11747812 0.10447956 -0.66305963 -0.69588591 0.03282628
0.40000000 0.90940859 1.01952546 0.11011687 -0.60821421 -0.64573176 0.03751755
0.45000000 0.81862953 0.93197666 0.11334713 -0.55938925 -0.59993424 0.04054499
0.50000000 0.73878784 0.85354050 0.11475266 -0.51565767 -0.55809249 0.04243482
0.55000000 0.66827466 0.78301726 0.11474260 -0.47622475 -0.51970627 0.04348152
0.60000000 0.60570965 0.71933700 0.11362735 -0.44041076 -0.48429030 0.04387954
0.65000000 0.54990941 0.66156027 0.11165086 -0.40763534 -0.45140706 0.04377172
0.70000000 0.49986025 0.60886765 0.10900740 -0.37740382 -0.42067261 0.04326879
0.75000000 0.45469474 0.56054683 0.10585209 -0.34929551 -0.39175408 0.04245857
0.80000000 0.41367148 0.51598003 0.10230855 -0.32295352 -0.36436467 0.04141115
0.85000000 0.37615771 0.47463256 0.09847485 -0.29807605 -0.33825858 0.04018253
0.90000000 0.34161435 0.43604261 0.09442826 -0.27440884 -0.31322610 0.03881726
0.95000000 0.30958300 0.39981230 0.09022930 -0.25173868 -0.28908925 0.03735057
1.00000000 0.27967491 0.36559982 0.08592491 -0.22988784 -0.26569799 0.03581015
```

```
RK4 (h = 0.1)
         t   u1_exact            u1_RK4            err_u1    u2_exact            u2_RK4            err_u2
0.00000000 1.33333333          1.33333333        0.00000000  0.66666667        0.66666667        0.00000000
0.10000000 1.79306259         -3.05243707        4.84549966 -1.03200245        8.98930534       10.02130779
0.20000000 1.42390240        -23.84779485       25.27169725 -0.87468103       51.19270398       52.06738501
0.30000000 1.13157652       -130.16520165      131.29677817 -0.72499857      269.26919305      269.99419162
0.40000000 0.90940859       -680.23148478      681.14089337 -0.60821421     1399.36858287     1399.97679708
0.50000000 0.73878784      -3531.29958377     3532.03837161 -0.51565767     7258.24183557     7258.75749324
0.60000000 0.60570965     -18312.79504397    18313.40075362 -0.44041076    37634.95546605    37635.39587681
0.70000000 0.49986025     -94951.33186458    94951.83172483 -0.37740382   195131.87164762   195132.24905144
0.80000000 0.41367148    -492306.46541813   492306.87908961 -0.32295352  1011721.87162305  1011722.19457657
0.90000000 0.34161435   -2552513.62271973  2552513.96433408 -0.27440884  5245578.82423131  5245579.09864015
1.00000000 0.27967491  -13234278.78321734 13234279.06289225 -0.22988784 27197287.19435823 27197287.42424607
```


u1: Exact vs RK4 (h=0.05)
u1 exact
u1 RK4 h=0.05
u1
t
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0


u2: Exact vs RK4 (h=0.05)
u2 exact
u2 RK4 h=0.05
u2
t
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0


u1: Exact vs RK4 (h=0.1)
1e7
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
u1
u1 exact
u1 RK4 h=0.1
0.0
0.2
0.4
0.6
0.8
1.0
t


u2: Exact vs RK4 (h=0.1)
1e7
u2 exact
u2 RK4 h=0.1
2.5
2.0
1.5
1.0
0.5
0.0
u2
0.0
0.2
0.4
0.6
0.8
1.0
t