Randomized Candidate Voting Method for Preventing Voting Manipulation

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Abstract

This paper investigates the problem of voting manipulation under the following conditions. First of all, a manipulator has complete information on other voters' preferences over candidates. Secondly, the election decides a single winner. Finally, a manipulator schemes constructive manipulation, which they want to make their most favorite candidate, A, elected. Our main contribution includes three analyses of a randomized candidate method as a potential solution to prevent voting manipulation.

Introduction

A voting manipulation problem is that a manipulator can control a winner when the gap of the total score between his target candidate and a top candidate is less than the highest point they can give. They can give the highest point to their target candidate and give 0 points to the top candidate. The Gibbard-Satterthwaite theorem proves that any deterministic voting rule cannot simultaneously satisfy the following three properties. i) Any alternative can be elected. ii) Not dictatorial: no single player can determine the winner of the game. iii) Strategy-proof: no player is better-off by misrepresenting their preference (Satterthwaite 1975). Hence, we consider a randomized voting method as a potential solution to the voting manipulation.

A brief description of the randomized candidate method introduced by Bentert and Skowron is the following. Given a set of candidates C with size m, with n number of voters, we first fix a candidate subset size, $l \leq m$, to be assigned to each voter. Each voter gives the linear order of preference. We assume no voter knows this candidate assignment of other voters, as well as score results of each candidate. In the end, we compute the total score the candidate received, divided by the number of times each candidate is ranked. The final score is n times the weighted score, and a candidate with the highest score will be elected. The algorithm assumes all m candidates are ranked at least one time since if there exists a candidate never ranked, the score computation contains a zero division. However, since $Pr(c \in C \text{ never ranked}| \text{ n voters}) = (\frac{m-l}{m})^n$, this probability is exponentially small when n is large.

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Given the randomized candidate method, our main contribution is the following analyses of the algorithm: i) The randomized candidate algorithm reduces the chances that a manipulator has an incentive to misrepresent their preference. ii) Consequences of manipulation. iii) Noise of the randomized candidate method.

Related Work

While Bentert and Skowron's research motivation is to be efficient, approximate deterministic voting methods with less information, this paper analyzes the randomized algorithm as a potential solution to the voting manipulation. Their results show that even $l = 2$, with hundreds of voters, we can approximate the actual score with exponentially small variance (Benter and Skowron 2020).

Procaccia analyzes the lottery extension, which randomly decides a winner based on the Borda count distribution. While the lottery extension is strategy-proof, choosing the same winner with the deterministic Borda method has an upper bound of $\frac{1}{2} + \Omega\frac{1}{\sqrt{m}}$ where m is the number of candidates (Procaccia 2010).

Veselova introduces the concept of a manipulator having an $incentive$ to misrepresent their preference under deterministic voting methods. Veselova defines that if there exists at least one possible situation when manipulation makes him better off, and nothing changes in all other situations, then the manipulator has an incentive to play strategically (Veselova 2020). We extend the concept of having an incentive to randomized voting methods defined in the Preliminaries section.

Ayadi et al. compares multiple scoring methods under a condition where each voter only rank their top-k candidates. Their empirical result shows that the probability harmonic scoring chooses the true winner is better than that of Borda count (Ayadi, Amor, and Lang 2020). Since Bentert and Skowron prove that the randomized algorithm approximates the actual randomized score having large number of voters with any decreasing order of the scoring method, we do not specify a scoring method in this paper.

Preliminaries

Since each voter v_i is equipped with a linear order of preference over the candidates, $pos_{v_i}(c)$ denotes the position of a candidate c in v_i's preference ranking. Bentert and Skowron denote random variable X_c as the score that candidate c receives from the randomized candidate algorithm. To be clear, X_c itself does not mean whether the score contains manipulation or not. To analyze the algorithm let us denote C_{higher} as a set of candidates having higher E[X_c] than that of the manipulator's target candidate A. Similarly, denote C_{lower} as a set of candidates having lower E[X_c] than that of the manipulator's target candidate A.

Bentert and Skowron express the expected score from the randomized candidate algorithm as follows:

$$E[X_c] = \frac{1}{\binom{m-1}{l-1}} \sum_{i=1}^{n} \sum_{j=1}^{l} \alpha_j \binom{pos_{v_i}(c)-1}{j-1} \binom{m-pos_{v_i}(c)}{l-j} \tag{1}$$

where α is a score vector, a decreasing order of points to be given to each candidate.

Adding to those terminologies, we extend Veselova's definition of $incentive$ to our case that a manipulator has an incentive to misrepresent their preference under randomized candidate method when they can achieve the following inequality:

There exists at least one $higher \in C_{higher}$ such that

$$E[X_{higher} \mid \text{misrepresent preference}] < E[X_{higher} \mid truthful]$$

and for all other $higher \in C_{higher}$,

$$E[X_{higher} \mid \text{misrepresent preference}] \leq E[X_{higher} \mid truthful] \tag{2}$$

Chances of losing an incentive to manipulate

A manipulator needs aggregated ranking information to manipulate and increase the chances that their favorite candidate A wins. Under deterministic voting methods, aggregated ranking is produced based on the total score of each candidate. Under the randomized method, ideally, they need the conditional probability of winning for each candidate under truthful voting. Although computing the probability with polynomial time is still under investigation, a manipulator can still obtain aggregated ranking based on $E[X_c]$ of each candidate $c \in C$, and they scheme to satisfy the inequality (2). Hence, we have the following result.

$$\begin{aligned} &Pr(\text{a manipulator loses an incentive to manipulate}) \\ &= \frac{\binom{pos(A)}{l} + \binom{m-pos(A)+1}{l}}{\binom{m}{l}} \end{aligned} \tag{3}$$

$\binom{pos(A)}{l}$ refers to the number of ways that candidates $\in C_{higher}$ and A occupy all the assignment. If A is in the subset, the manipulator always ranks A top. If $l = 2$, their votes is always $A \succ c \in C_{higher}$, which is thier truthful voting. If $l > 2$, no matter how the manipulator orders the assigned candidates, they cannot satisfy the inequality (2) without violating the inequality for other candidates in C_{higher}. This is same when A is not in the subset. $\binom{m-pos(A)+1}{l}$ refers to the number of ways that candidates rank lower than A and A occupy all the assignment. In this situation, the manipulator has no control to change the $E[X_c]$ $\forall c \in X_{higher}$. Intuitively speaking, in this case, the manipulator cannot reduce the gap of score between their target candidate A and $\forall c \in C_{higher}$. These cases are mutually exclusive. Hence, we sum these cases and divide by the total ways choosing l subset of candidates from m candidates. This explains the randomized candidate method can reduce the manipulator's incentive to manipulate with certain probability.

Manipulators have a price to pay

Another lemma of the randomized candidate algorithm is that a manipulation increases the expected score of a candidate $c \in C_{lower}$ and not their target candidate A. This is due to the dependency of $E[X_c]$ on $pos(c)$, clearly from the equation (1). Let $pos(c \in C_{lower} \mid \text{manipulation})$ be a position of $c \in C_{lower}$ by manipulation and $pos(c \in C_{lower} \mid \text{truthful})$ be the position by truthful preference. We know $pos(c \in C_{lower} \mid \text{manipulation}) < pos(c \in C_{lower} \mid \text{truthful})$.

The equation (1) shows that $\frac{\binom{pos_{v_i}(c)-1}{j-1}\binom{m-pos_{v_i}(c)}{l-j}}{\binom{m-1}{l-1}}$ is the probability that the candidate c is ranked at position i. Since manipulation causes $pos(c \mid \text{manipulation}) < pos(c \mid \text{truthful})$, the probability that the candidate c is ranked at a lower position, which means more favored by voters, will be increased for $pos(c \mid \text{manipulation})$, which will increase $E[X_c]$ for $c \in C_{lower}$. Moreover, since $\sum_{j=1}^{l} Pr(\text{c is ranked at j}) = 1$, the probability that the candidate c is ranked at a higher position will be decreased accordingly.As manipulation does not change $pos(A)$, the manipulator cannot increase $E[X_A]$.

Noise of the randomized method

While the randomized candidate method can reduce chances that manipulator has an incentive to manipulate, it has a cost of adding noise. Dubhashi and Ranjan prove that when random variables are negatively associated, we can still apply chernoff bounds (Dubhashi and Ranjan 1996). A voter v_i ranks a candidate in $pos_i(c)$ is the model of balls and bins, and hence X_c is negatively associated. Bentert and Skowron demonstrate that $\forall\ \delta \in$ [0,1], $Pr(|X_c - E[X_c]| \geq \delta \cdot E[X_c]) \leq 2 \cdot \exp\left(-\delta^2 E[X_c]/3\right)$ (Benter and Skowron 2020). This imply that if n is small, noise can be certain probability.

Conclusion and Future Direction

As we show already, the randomized candidate method can reduce the chances of manipulator having an incentive to misrepresent their preference. However, if the number of voters is small, the algorithm adds the cost of noise.

A possible direction of this study includes i) a manipulator schemes destructive manipulation, ii) they have multiple target of constructive manipulation.

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