Instructions

- Please upload your solutions into the appropriate slot on Crowdmark.
- The **coverage** for this assignment is up to section 3.2 (inclusive). Your solutions should not use material from any later sections. You are allowed to use material from earlier sections. You are also allowed to use any results that appear in the Weekly Practice Problems (but please make sure to clearly cite them).
- You can earn a 0.25 course grade bonus for typesetting your solutions in LaTeX (or equivalent typesetting software). Please see the course outline for more details.

Problems

Q1. A *rhombus* is a parallelogram with sides of equal length. Let $\vec{u}$ and $\vec{w}$ be non-zero vectors in $\mathbb{R}^3$ which are not parallel. Prove that the parallelogram defined by vectors $\vec{u}$ and $\vec{w}$ is a rhombus if and only if the diagonals of the parallelogram are orthogonal.

Q2. (a) Prove that $\begin{bmatrix}1\\2\\2\\3\end{bmatrix} \notin \operatorname{Span}\left\{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\2\\2\\0\end{bmatrix}\right\}$ in $\mathbb{R}^4$.

(b) Prove that $\operatorname{Span}\left\{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\2\\2\\0\end{bmatrix}\right\} = \operatorname{Span}\left\{\begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \begin{bmatrix}2\\0\\0\\2\end{bmatrix}, \begin{bmatrix}0\\1\\1\\0\end{bmatrix}\right\}$ in $\mathbb{R}^4$. (Remember that to prove two sets are equal, you must show that they are subsets of each other.)

Q3. (a) Let $\vec{v}_1, \ldots, \vec{v}_k \in \mathbb{R}^n$. Suppose that $\vec{x} \in \mathbb{R}^n$ is orthogonal to $\vec{v}_i$ for all $i = 1, \ldots, k$. Prove that if $\vec{x} \in \operatorname{Span}\{\vec{v}_1, \ldots, \vec{v}_k\}$, then $\vec{x} = \vec{0}$.

(b) Prove that $\mathbb{R}^3$ cannot be spanned by one vector—that is, prove that $\mathbb{R}^3 \neq \operatorname{Span}\{\vec{v}\}$ for any $\vec{v} \in \mathbb{R}^3$.

(c) Prove that $\mathbb{R}^3$ cannot be spanned by two vectors—that is, prove that $\mathbb{R}^3 \neq \operatorname{Span}\{\vec{v}_1, \vec{v}_2\}$ for any $\vec{v}_1, \vec{v}_2 \in \mathbb{R}^3$. [**Hint:** Suppose, for a contradiction, that $\mathbb{R}^3 = \operatorname{Span}\{\vec{v}_1, \vec{v}_2\}$. Then think about using part (a). One of the Week 1 Practice Problems will be useful.]

Q4. Let $\vec{a}, \vec{b} \in \mathbb{R}^3$. Let $\vec{d}, \vec{v}$, and $\vec{w}$ be non-zero vectors in $\mathbb{R}^3$ with $\vec{v} \neq c\vec{w}$ for all $c \in \mathbb{R}$. Consider the line $\mathcal{L} = \{\vec{a} + t\vec{d} : t \in \mathbb{R}\}$ and the plane $\mathcal{P} = \{\vec{b} + r\vec{v} + s\vec{w} : r, s \in \mathbb{R}\}$. Prove that $\mathcal{L} \subseteq \mathcal{P}$ if and only if $\vec{a} \in \mathcal{P}$ and $\vec{d} \in \operatorname{Span}\{\vec{v}, \vec{w}\}$.