Ch. 7 Nonlinear Regression - Problem Bank Questions

October 24, 2020

1. Consider the step function model in equation (7.5) of ISLR. State whether each of the below is true or false, and explain your answer.

(a) T/F: In step function regression, β_0 can be interpreted as the mean value for Y when $X < c_1$.
(b) T/F: $\mathbf{1}_{X_i > 0} = 1$ when $X_i = -10$.
(c) T/F: Step function regression always has lower bias than linear regression.
(d) T/F: β_j represents the average value of X_j.

2. Regression splines: For the true and false, explain your answer.

(a) T/F: As the number of knots in polynomial regression increases, the variance decreases.
(b) T/F: Polynomial regression splines can be discontinuous without constraints.
(c) T/F: Adding constraints to the piecewise polynomials that are fit increases the flexibility of the model.
(d) T/F: A cubic spline with K knots requires $3 + K$ degrees of freedom.
(e) T/F: Splines are most flexible in regions with many knots.
(f) T/F: Splines and polynomial regression models always fit well at the boundaries of the X values.

3. Smoothing Splines and Local Regression: For the true and false, explain your answer.

(a) T/F: The degrees of freedom refers to the number of coefficients in a model.
(b) T/F: The degrees of freedom is a measure of the flexibility of a model.
(c) T/F: The higher the degrees of freedom, the higher the bias of the model.
(d) T/F: The smoothing spline will have knots at the location of each of the training points.
(e) T/F: LOOCV is very expensive for smoothing splines since we have such flexible models.
(f) T/F: Local regression requires half of the training data each time we wish to make a prediction.
(g) T/F: The span in local regression plays the same role as λ in smoothing splines.

4. State whether each of the below is true or false, and explain your answer.

(a) T/F: We can determine the variance (and hence quantify uncertainty) at a given point when using an additive model
(b) T/F: The tail behavior of splines makes them a good alternative to polynomial models
(c) T/F: Step functions do not allow for interaction effects
(d) T/F: Splines have large amounts of continuity compared to other methods we know
(e) T/F: A cubic spline has more constraints than a natural cubic spline
(f) T/F: There is no close form expression to determine where to place knots
(g) T/F: The limiting behavior as lambda approaches infinity, for a smoother spline, is a linear function
(h) T/F: The coefficients of GAMs are highly interpretable

5. Consider the nonlinear models we have discussed in class:

a. Why do we say that we are not really interested in the coefficients of our nonlinear terms, and instead focus on the fitted function values?

b. How might we determine the degree of a polynomial variable or the number of knots in a spline?

c. What benefit do splines provide over piecewise polynomials? Why might we choose splines based on this?

d. What benefit do splines provide over high degree piecewise functions? Why might we choose splines based on this?

e. Since splines can have high variance at the outer range of the predictors, how can we account for this?

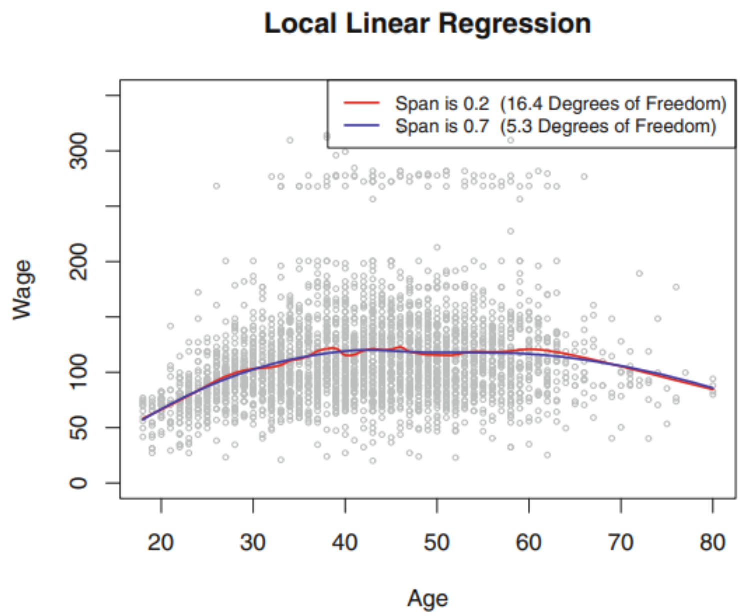
6. Consider GAMs:
    a. What is the benefit of the additivity of GAMs?
    b. When might a GAM outperform a more complex model?
    c. When might a GAM outperform a less complex model?
    d. What is the benefit of using a GAM versus using a single spline? What is the cost?
    e. What are the strengths and limitations of GAMs?

7. Smoothing Splines: answer the following questions and explain your answers. g is a smoothing spline if it minimizes

$$\sum_{i=1}^{n}(y_i - g(x_i))^2 + \lambda \int g''(t)^2 dt.$$

(a) What does the first derivative of the function g measure?
(b) What does the second derivative of the function g measure?
(c) Conceptually, what is the $\int g''(t)^2 dt$ term doing?
(d) As λ increases, what happens to the smoothness of g?
(e) What happens when $\lambda = 0$? When $\lambda = \infty$?
(f) Conceptually, what is λ controlling and how?

8. Explain how the span s controls the local linear regression fit in this plot. How would you choose the span s in practice?



9. ISL Chapter 7, Exercise 1

10. ISL Chapter 7, Exercise 5