Turbulence Analysis

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2023-10-26

Introduction

Turbulence is one of the fascinating topics in the research in fluid dynamics. It is characterized its chaotic motion, rapid fluctuations and lack of predictable patterns. Yet, there have been numerous attempts in scientific literature trying to model the behavior of turbulent flows, as turbulent flows are prevalent in our world and are the underlying forces that drive plenty of the physical processes, from wisps of smoking swirling up from the cigarette to mixing of chemicals in industrial processes. A better understanding and prediction of turbulent flow will help us gain a deeper insight into a wide range of applications, such as improved aerodynamics in airplane designs and better climatic modelling.

A subdomain in turbulent flow research deals with particle clustering in turbulent flow focusing on small particles' behavior in turbulent fluids. For our project, we are provided with a set of simulation results on small particle probability distribution. The outcome variable was originally a probability distribution for particle cluster volumes, but it was converted into its first four raw moments $E[X]$ to $E[X^4]$ facilitate analysis. The predictor set contains three variables:

- Reynolds number Re, which provides information on the type of flow a fluid is experiencing. A low Re corresponds with laminar flow (smooth and orderly), while a high Re corresponds with turbulent flow
- Gravitational acceleration Fr, which measures the gravitational forces particles are experiencing
- Stokes number St, where larger value corresponds with larger particle size

The main research objective of our project will be to build a viable statistical model to predict the response variable (first four raw moments of particle probability distribution) using the three predictors at hand, utilizing the data in a training set provided. Specifically, we are interested in the following:

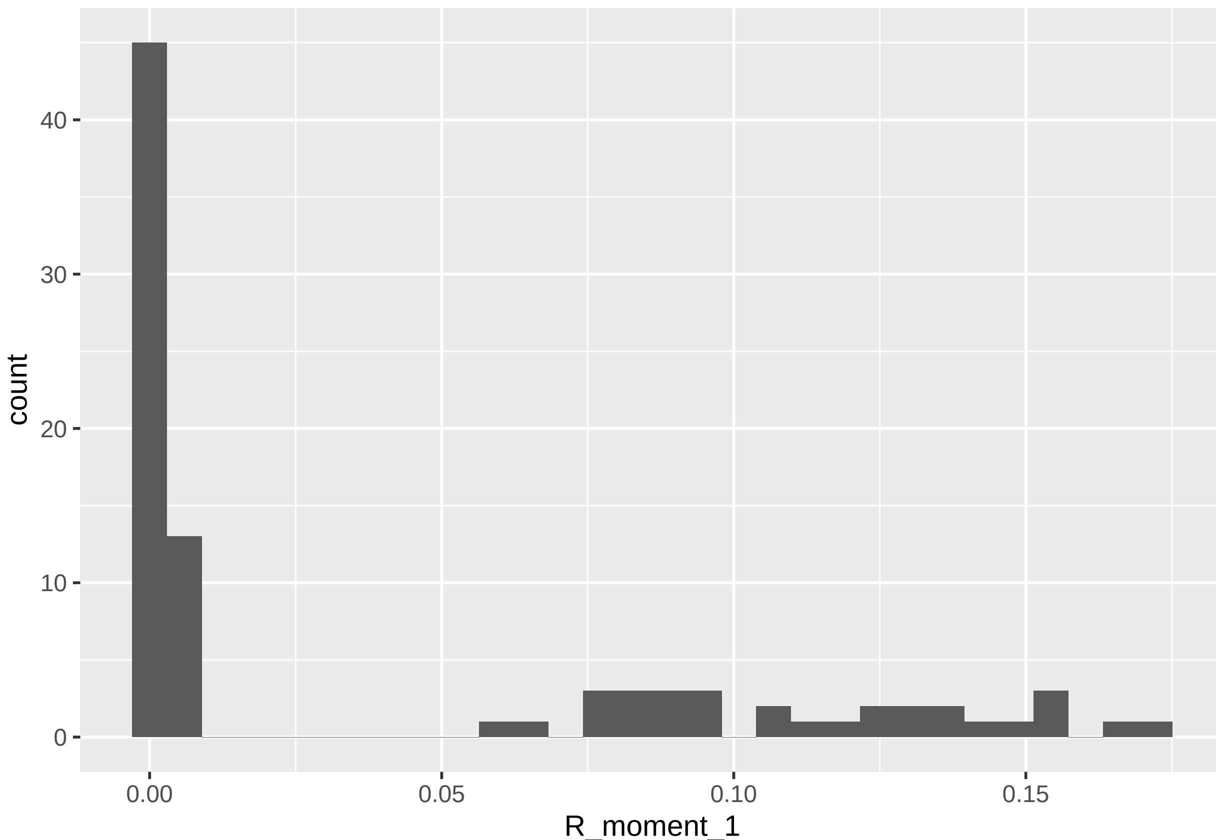
- Does there exist a significant linear relationship between the predictors and the raw four moments?
- Is there any significant interaction effects between predictors on the response variables?
- Does a linear regression model suffice? Or a more complex model is needed to better explain the relationship between the predictor and response
- Are identified effects for predictors the same for all moments, or they differ for each different moment?

Ultimately, we wish our model to capture sufficient trends in our training data, so that we can predict the four moments in our test set data as accurately as possible.

Methodology

We begin by some transformations on both predictor and response variables. For predictor variable, we first noticed that Fr only takes on 0.052, 0.3 and Inf in our training and testing data set, and directly using it is not viable since it contains infinity. Since Fr<1 corresponds with a subcritical flow while Fr > 1 corresponds with a super critical flow, we create a new categorical variable `flow` by the following:

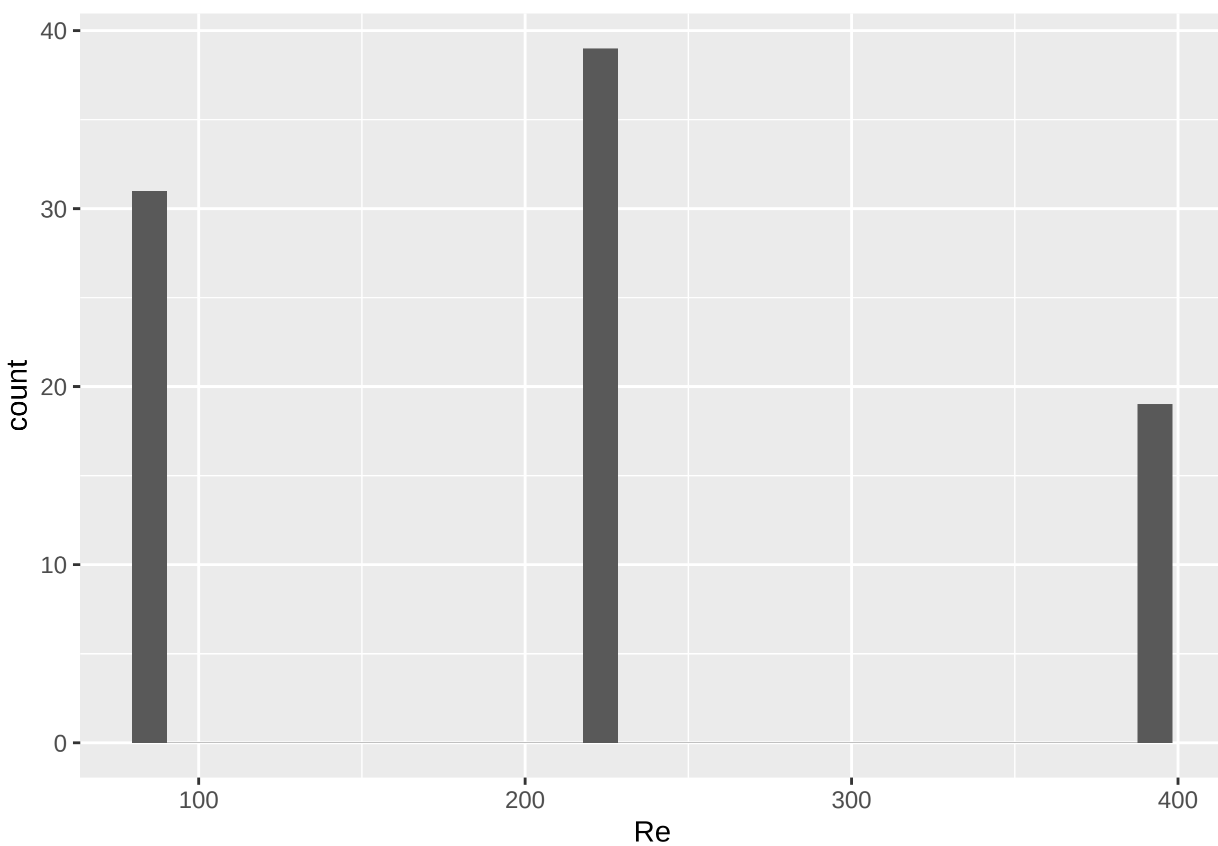
Flow	Fr
super subcritical	Fr < 0.1
subcritical	0.1 < Fr < 1
supercritical	Fr > 1

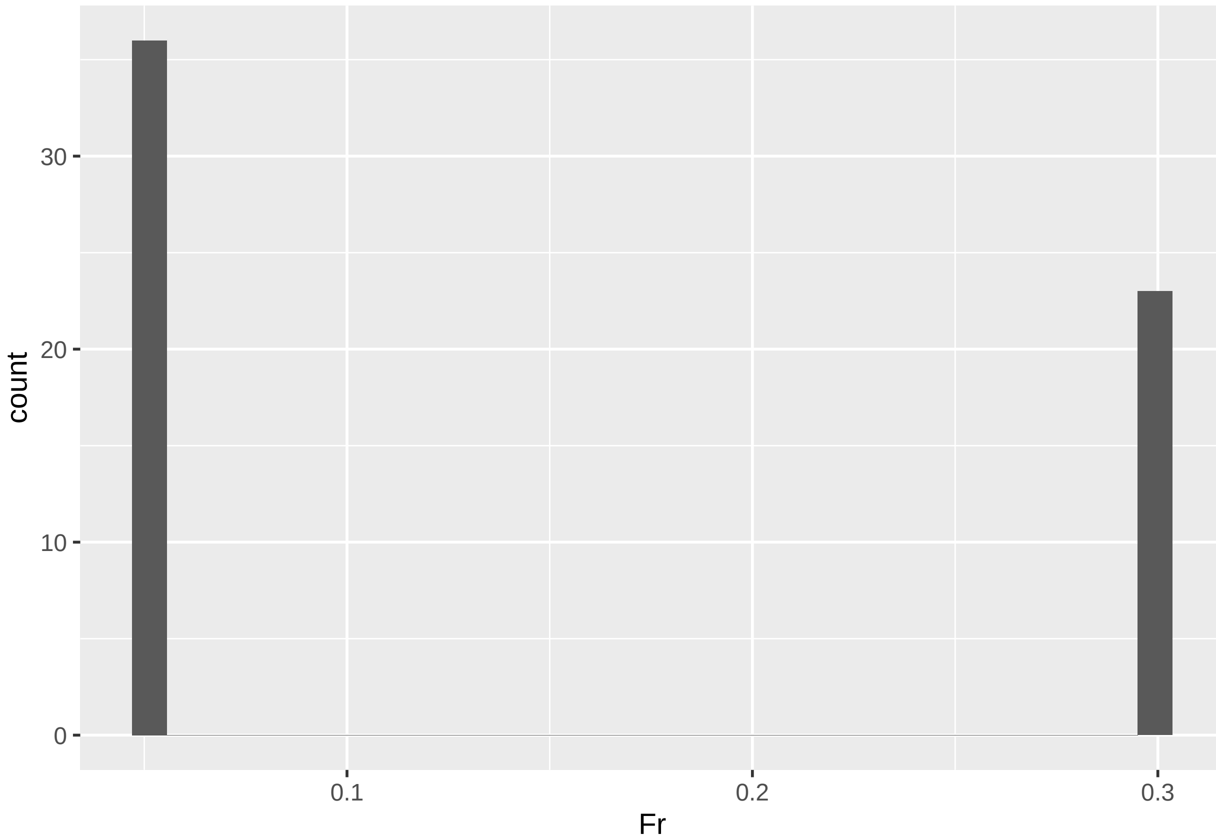


EDA

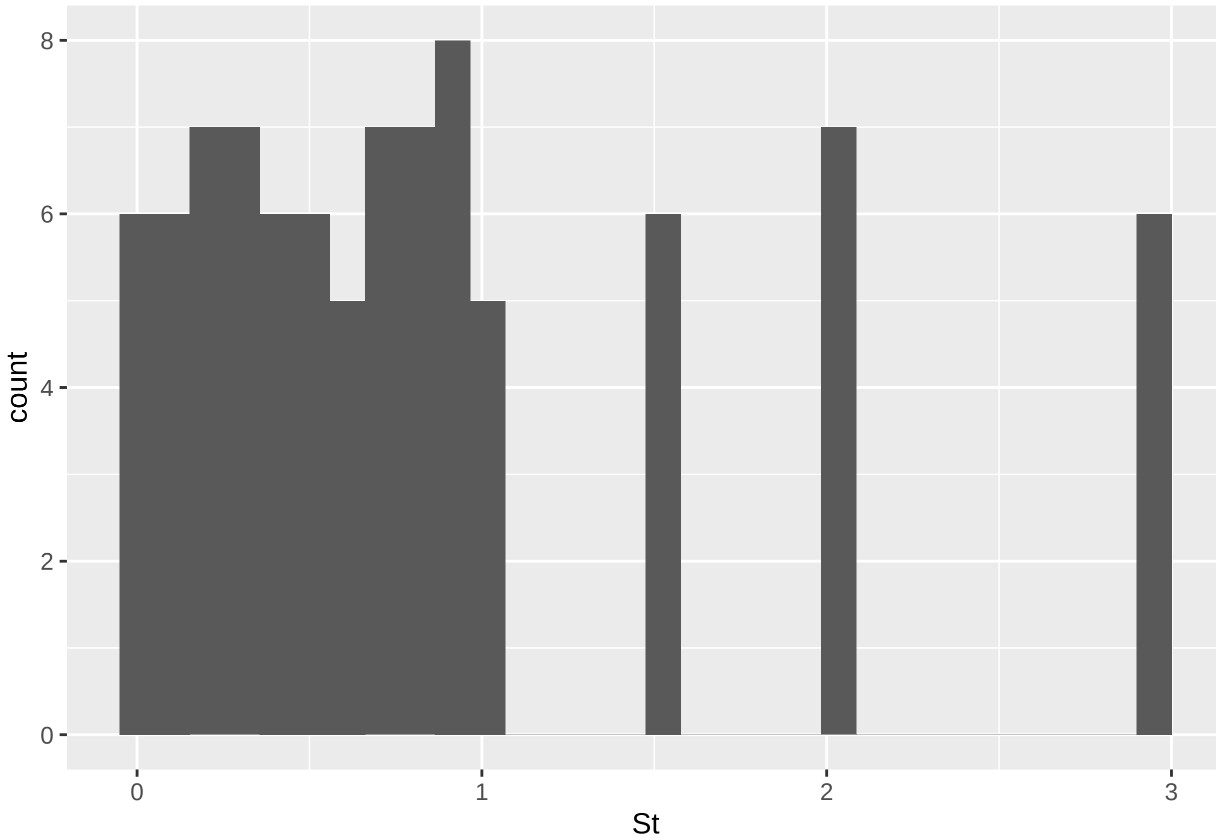
```
##        St              Re              Fr          R_moment_1
##  Min.   :0.0500   Min.   : 90.0   Min.   :0.052   Min.   :0.000222
##  1st Qu.:0.3000   1st Qu.: 90.0   1st Qu.:0.052   1st Qu.:0.002157
##  Median :0.7000   Median :224.0   Median :0.300   Median :0.002958
##  Mean   :0.8596   Mean   :214.5   Mean   :  Inf   Mean   :0.040394
##  3rd Qu.:1.0000   3rd Qu.:224.0   3rd Qu.:  Inf   3rd Qu.:0.087868
##  Max.   :3.0000   Max.   :398.0   Max.   :  Inf   Max.   :0.172340
##    R_moment_2         R_moment_3        R_moment_4           flow
##  Min.   :   0.0001   Min.   :      0   Min.   :0.000e+00   Length:89
```

```
##  1st Qu.:   0.0245   1st Qu.:      0   1st Qu.:3.000e+00   Class :character
##  Median :   0.0808   Median :      1   Median :2.100e+01   Mode  :character
##  Mean   :  92.4902   Mean   : 753370   Mean   :6.194e+09
##  3rd Qu.:   0.5345   3rd Qu.:     40   3rd Qu.:5.345e+03
##  Max.   :1044.3000   Max.   :9140000   Max.   :8.000e+10
##  log.moment.2 <- log(R_moment_2) log.moment.3 <- log(R_moment_3)
##  Min.   :-9.1805                 Min.   :-9.8759
##  1st Qu.:-3.7101                 1st Qu.:-1.4131
##  Median :-2.5157                 Median : 0.1692
##  Mean   :-1.6941                 Mean   : 2.1070
##  3rd Qu.:-0.6264                 3rd Qu.: 3.7002
##  Max.   : 6.9511                 Max.   :16.0282
##  log.moment.4 <- log(R_moment_4)    Re.fac
##  Min.   :-10.087                 Length:89
##  1st Qu.:  1.185                 Class :character
##  Median :  3.037                 Mode  :character
##  Mean   :  5.954
##  3rd Qu.:  8.584
##  Max.   : 25.105
```

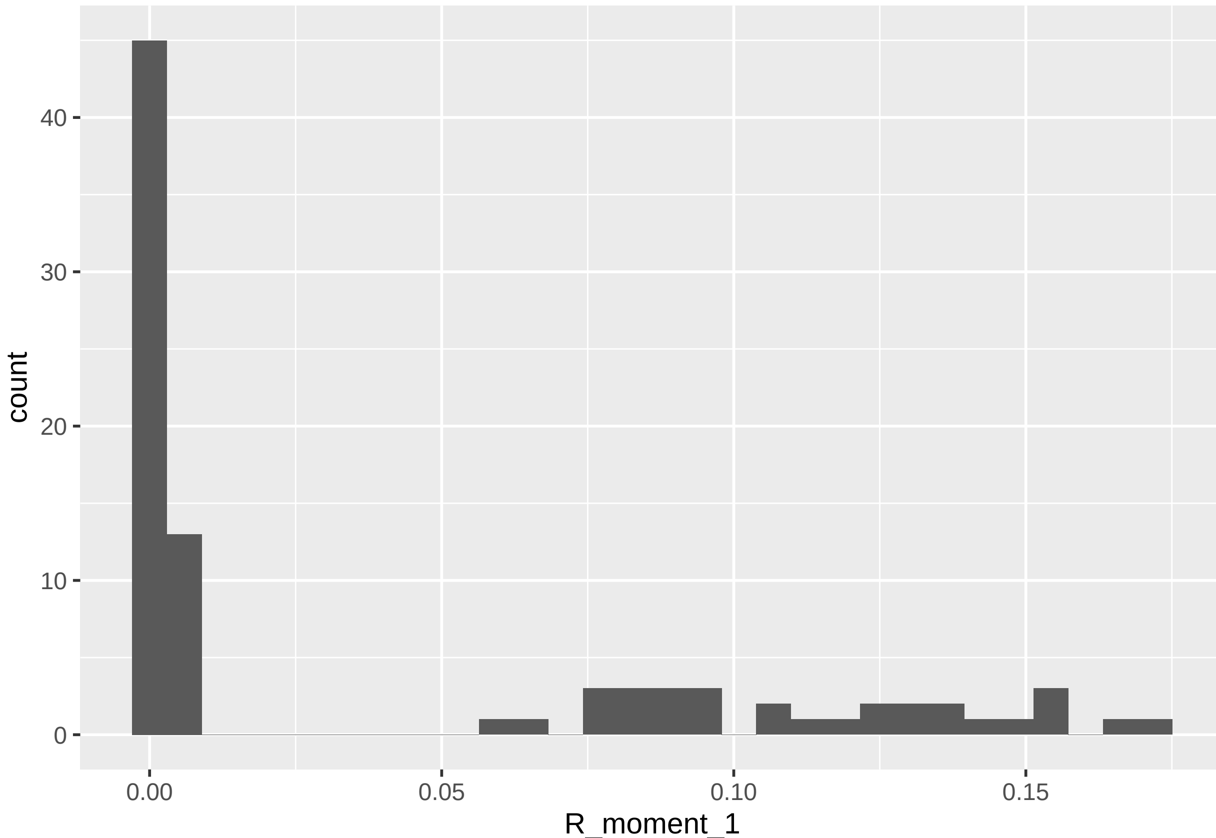


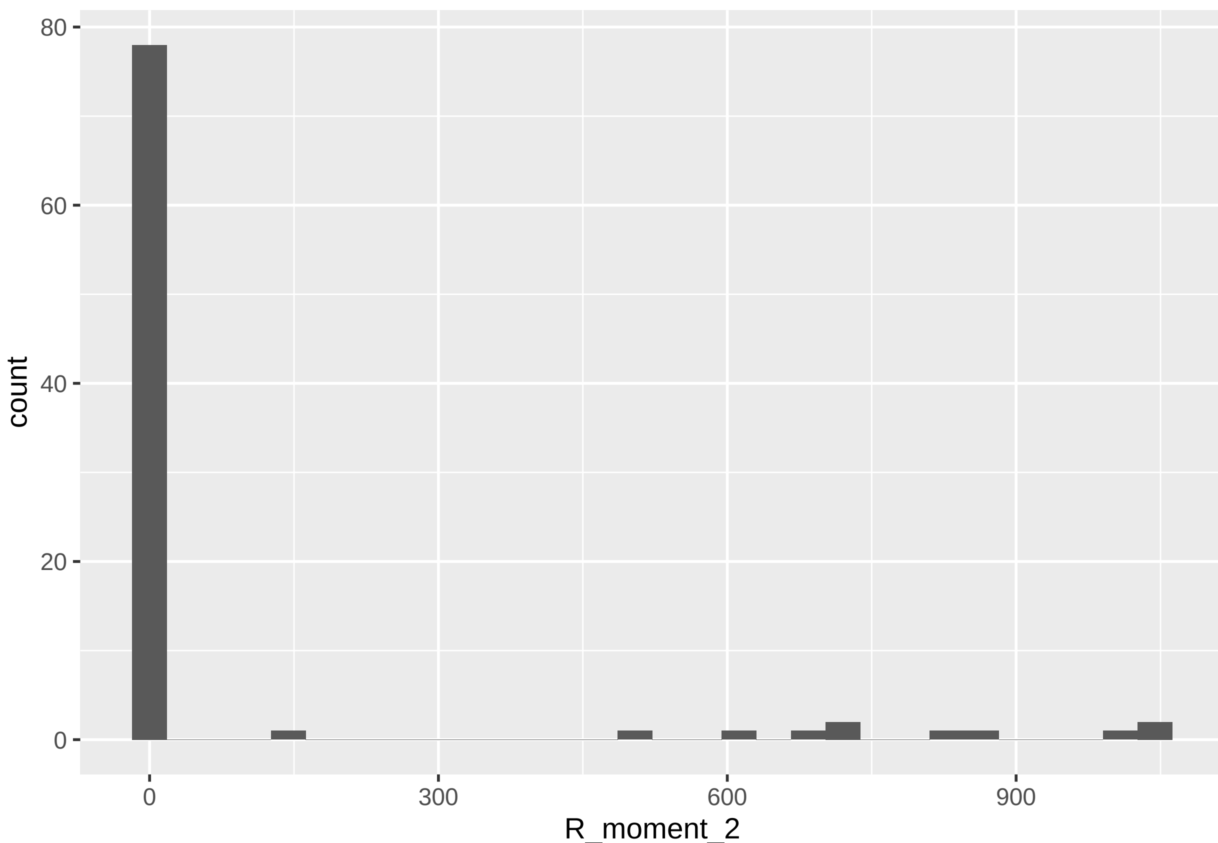


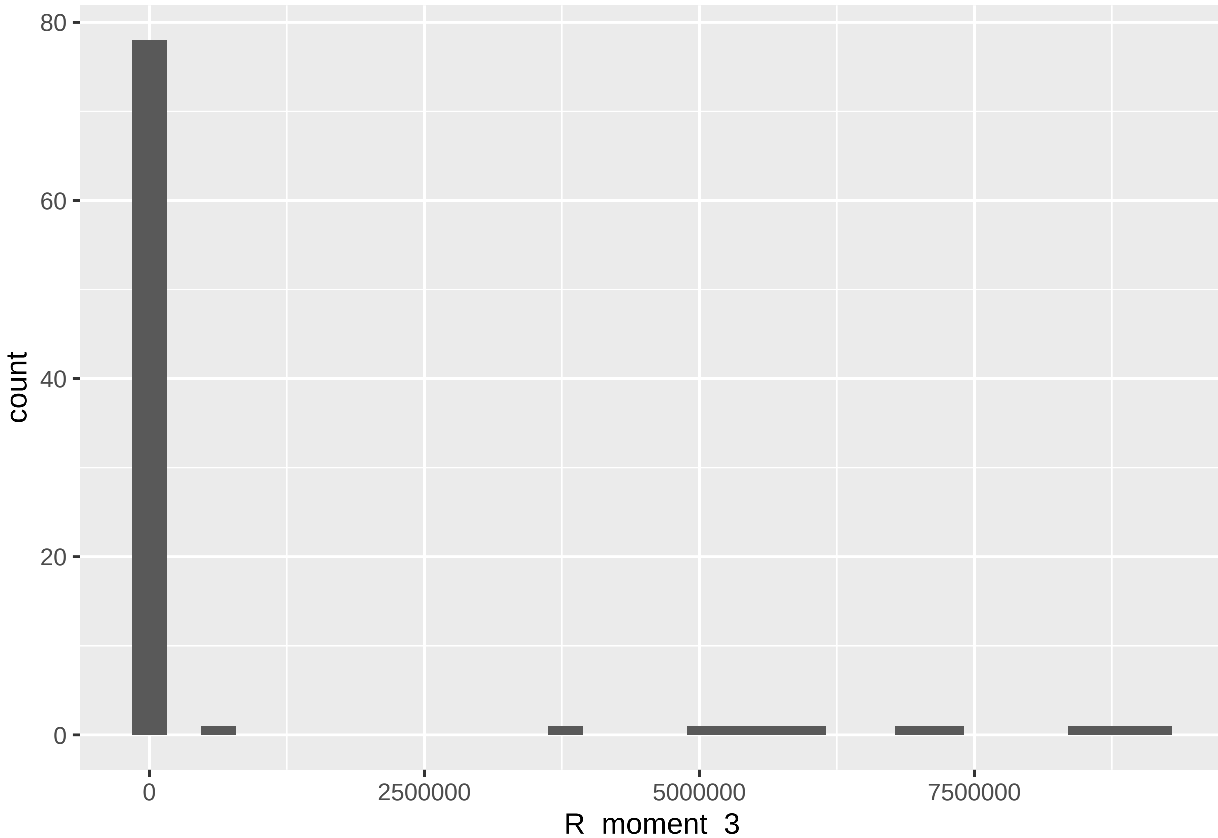
count
30
20
10
0
0.1
0.2
0.3
Fr

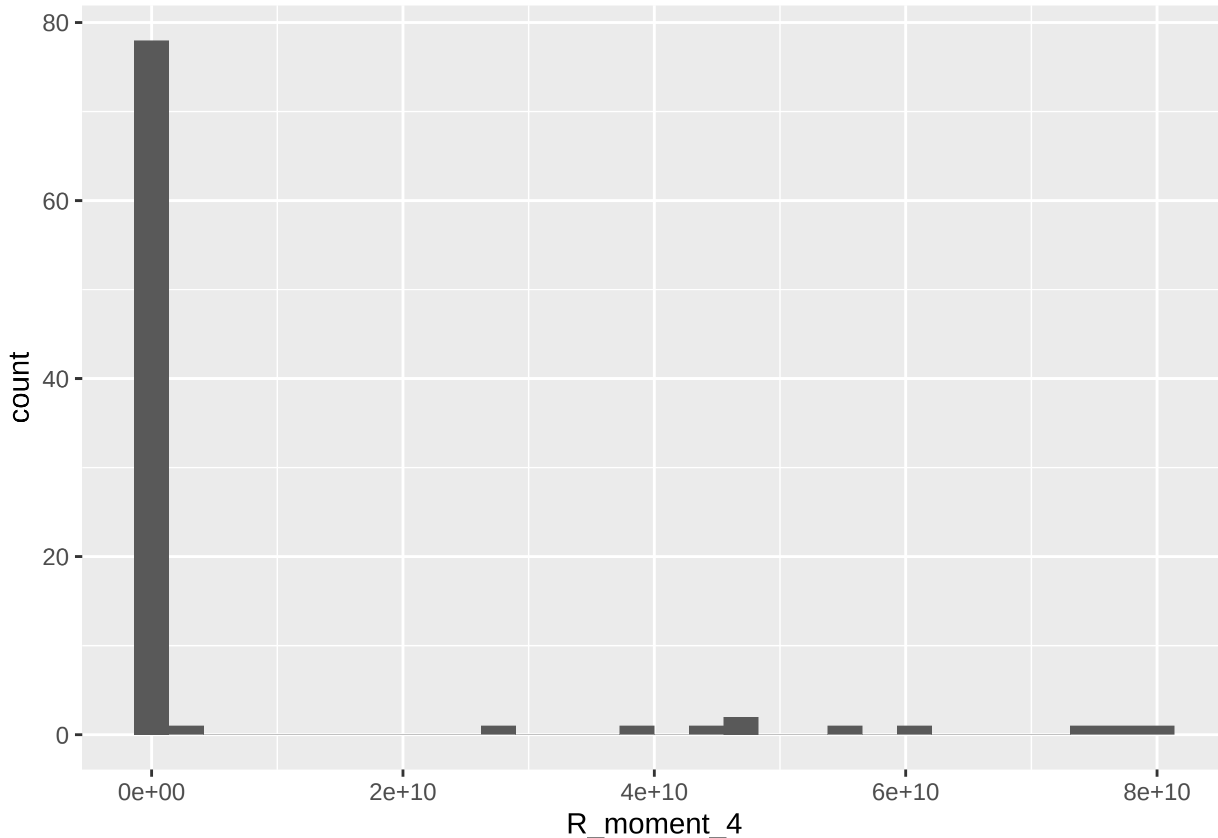


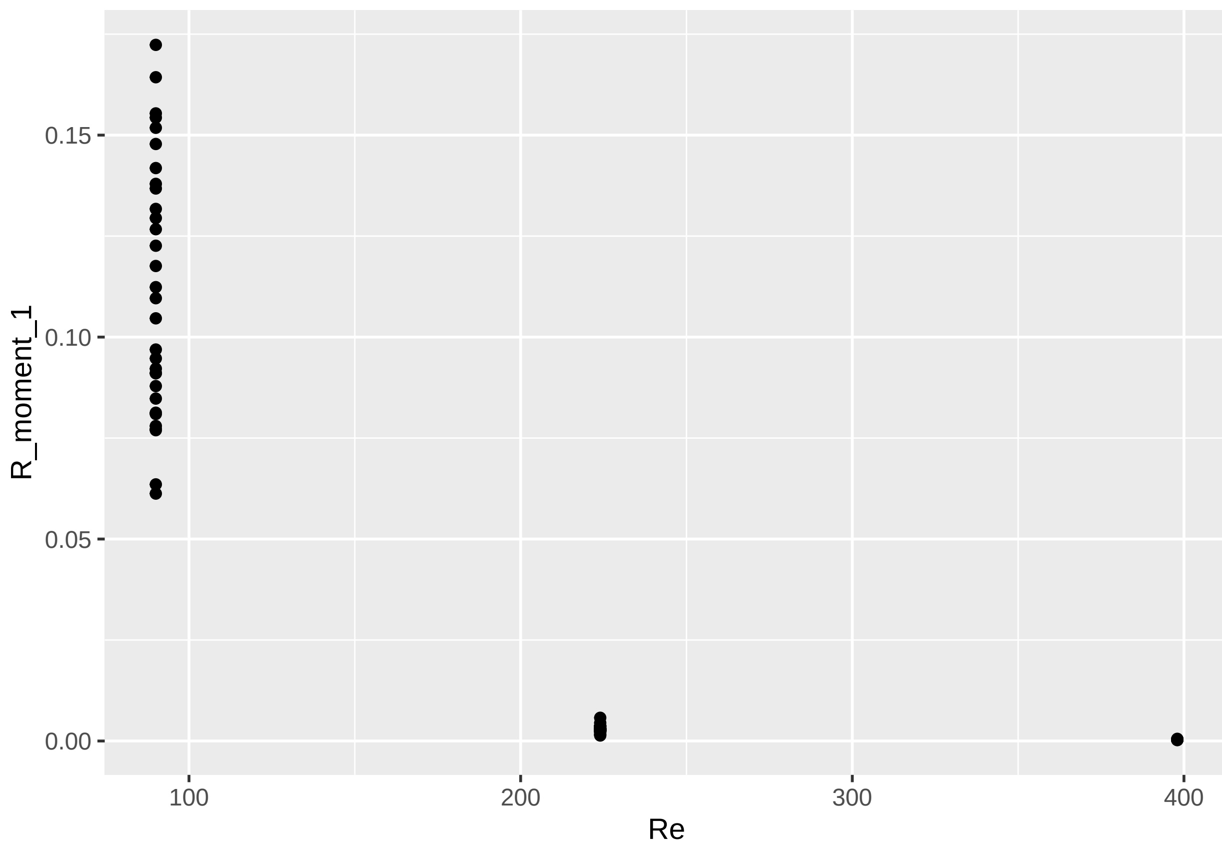
8
6
4
2
0
count
0
1
2
3
St


40
30
20
10
0
count
0.00
0.05
0.10
0.15
R_moment_1

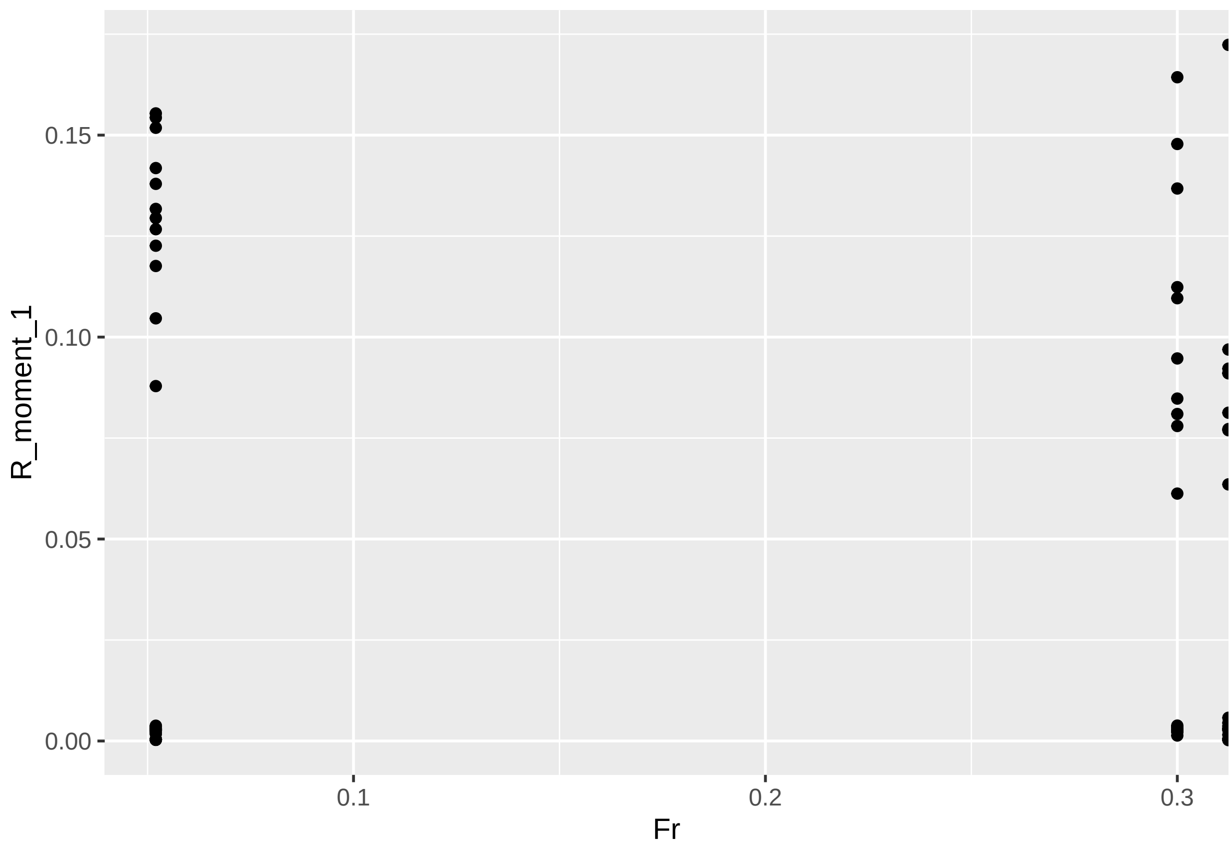

80
60
40
20
0
count
0
300
600
900
R_moment_2

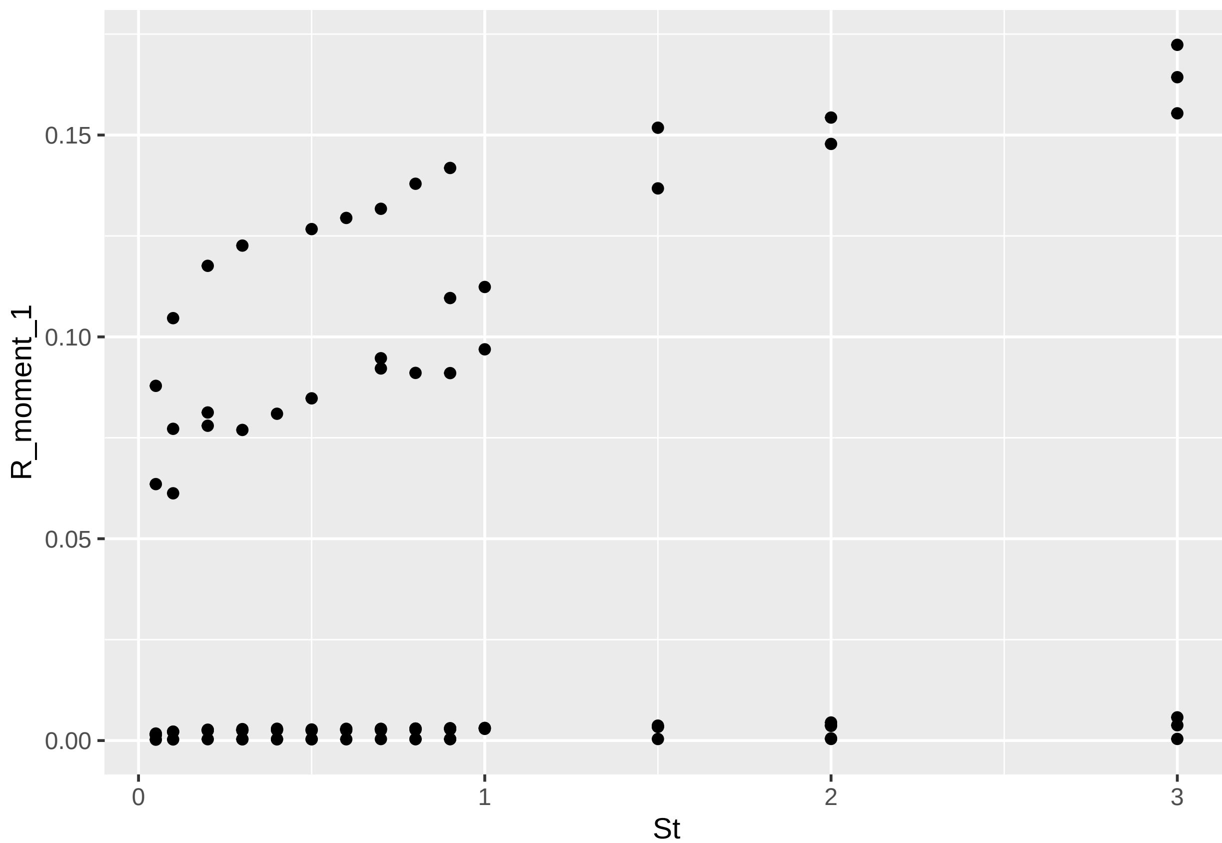

count
80
60
40
20
0
0
2500000
5000000
7500000
R_moment_3


count
80
60
40
20
0
0e+00
2e+10
4e+10
6e+10
8e+10
R_moment_4

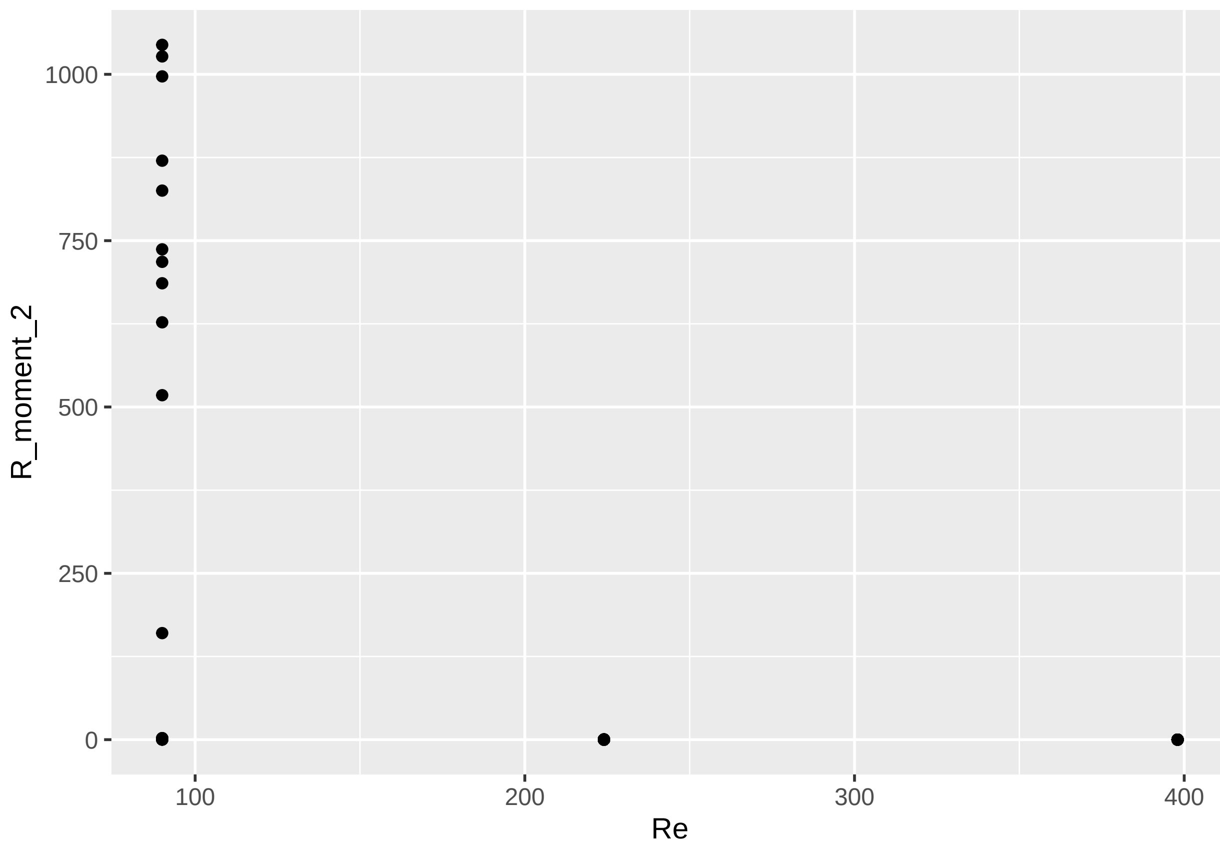


0.15
0.10
0.05
0.00
R_moment_1
100
200
300
400
Re

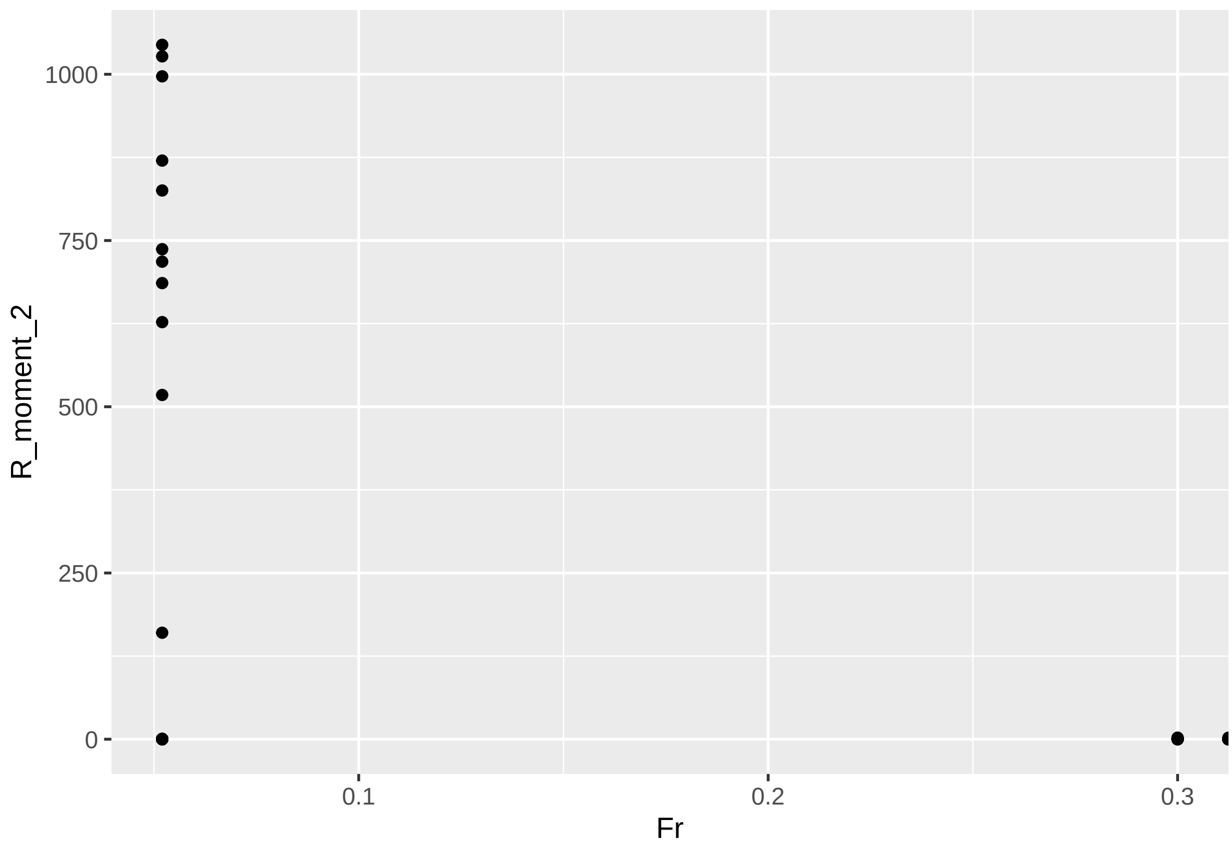

R_moment_1
0.15
0.10
0.05
0.00
0.1
0.2
0.3
Fr



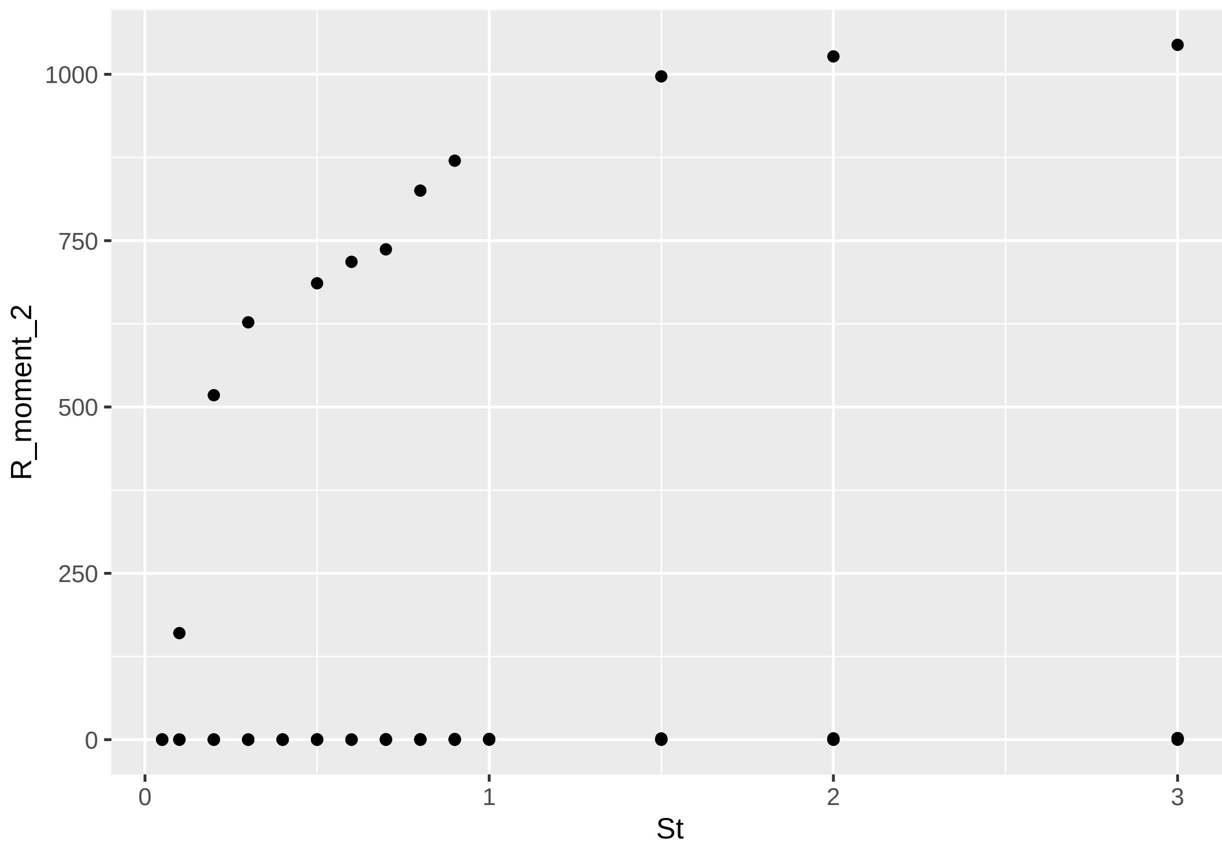
R_moment_1
0.15
0.10
0.05
0.00
0
1
2
3
St



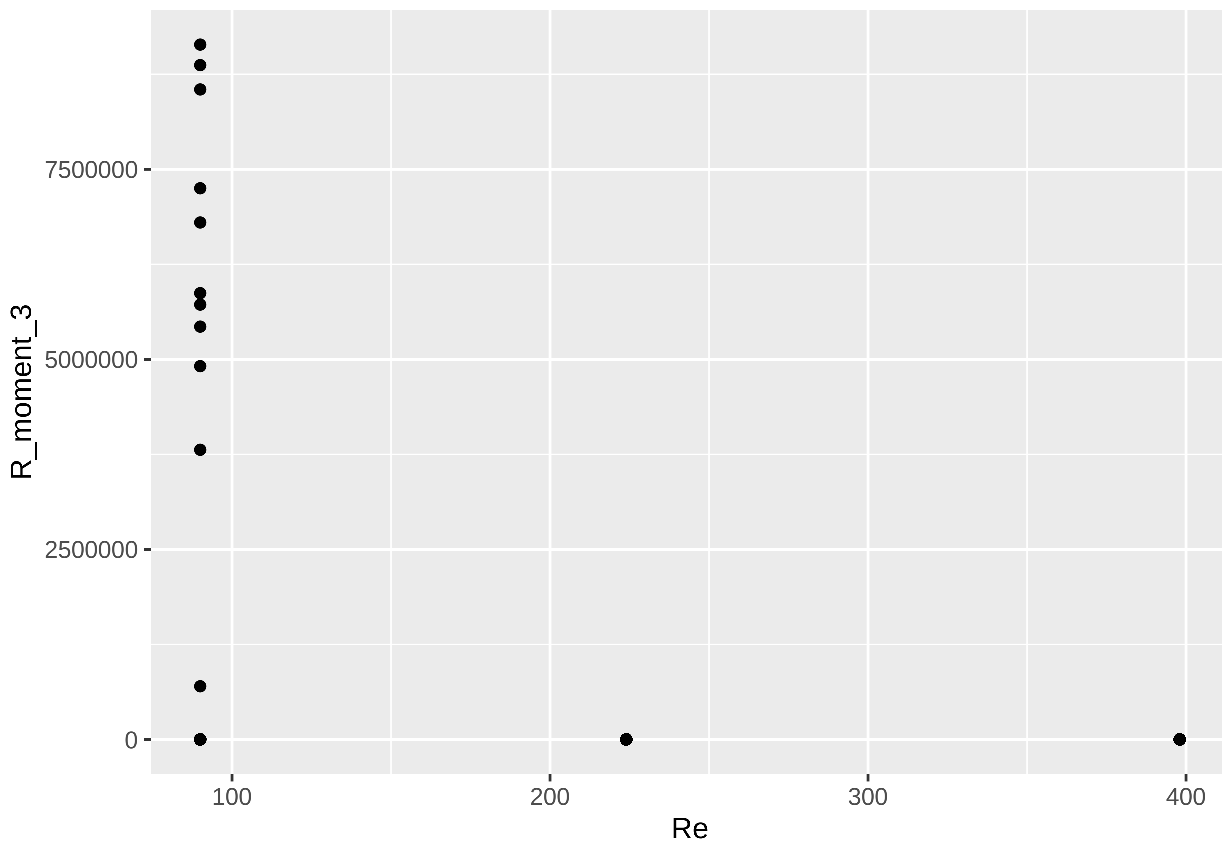
R_moment_2
1000
750
500
250
0
100
200
300
400
Re



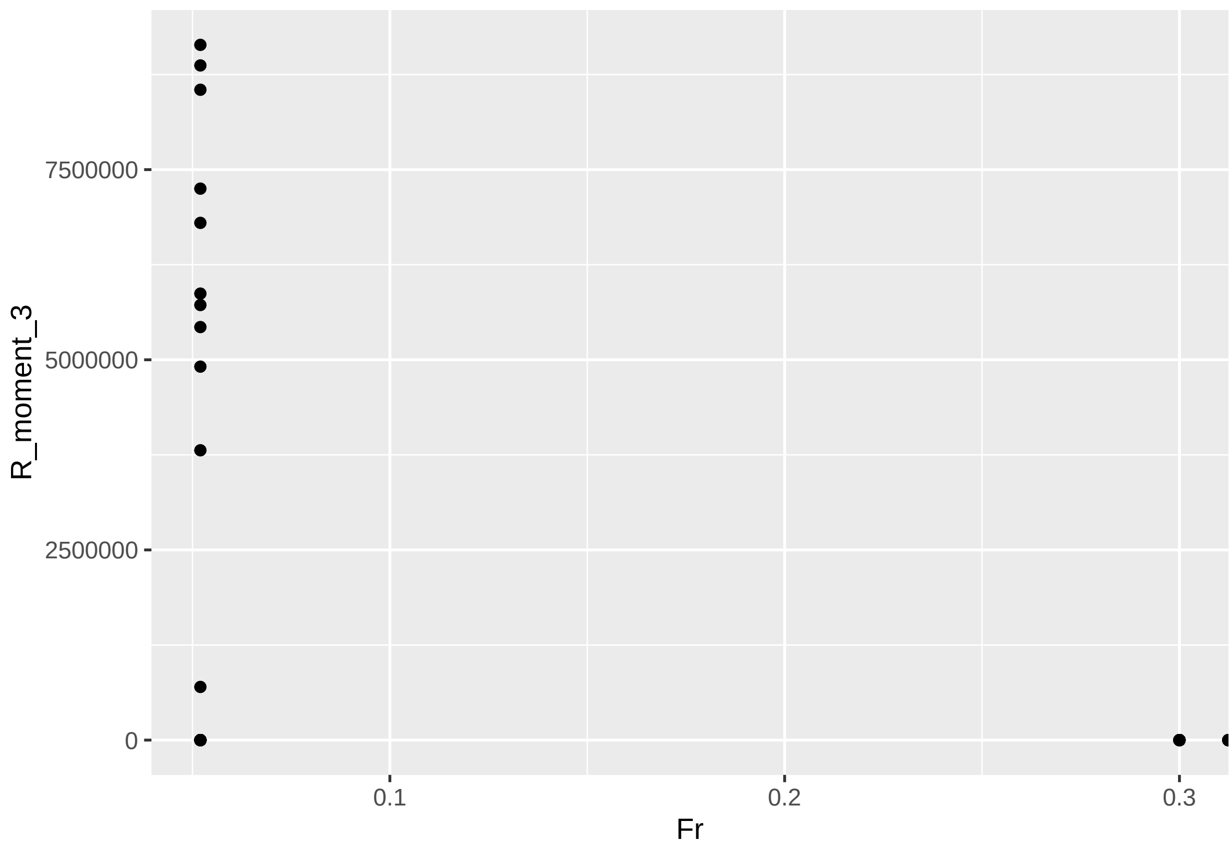
R_moment_2
1000
750
500
250
0
0.1
0.2
0.3
Fr



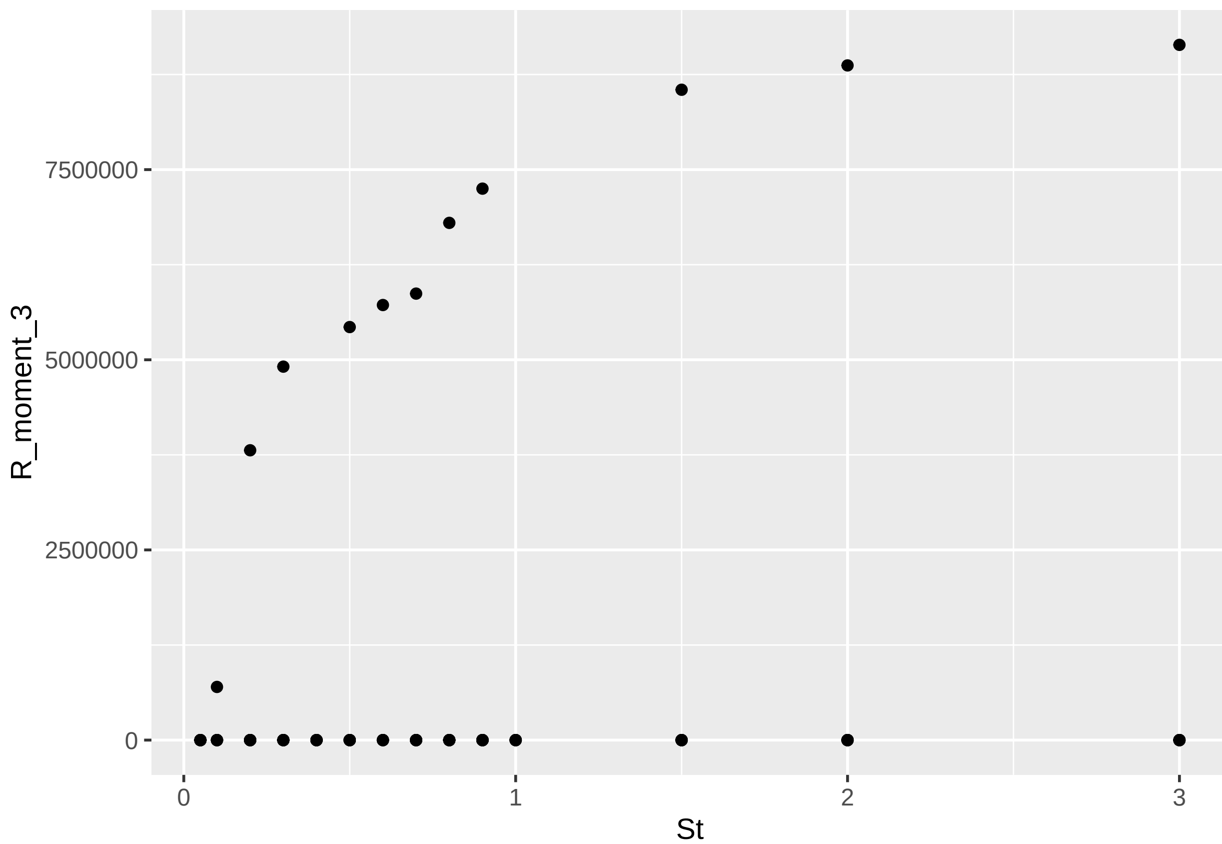
1000
750
500
250
0
R_moment_2
0
1
2
3
St



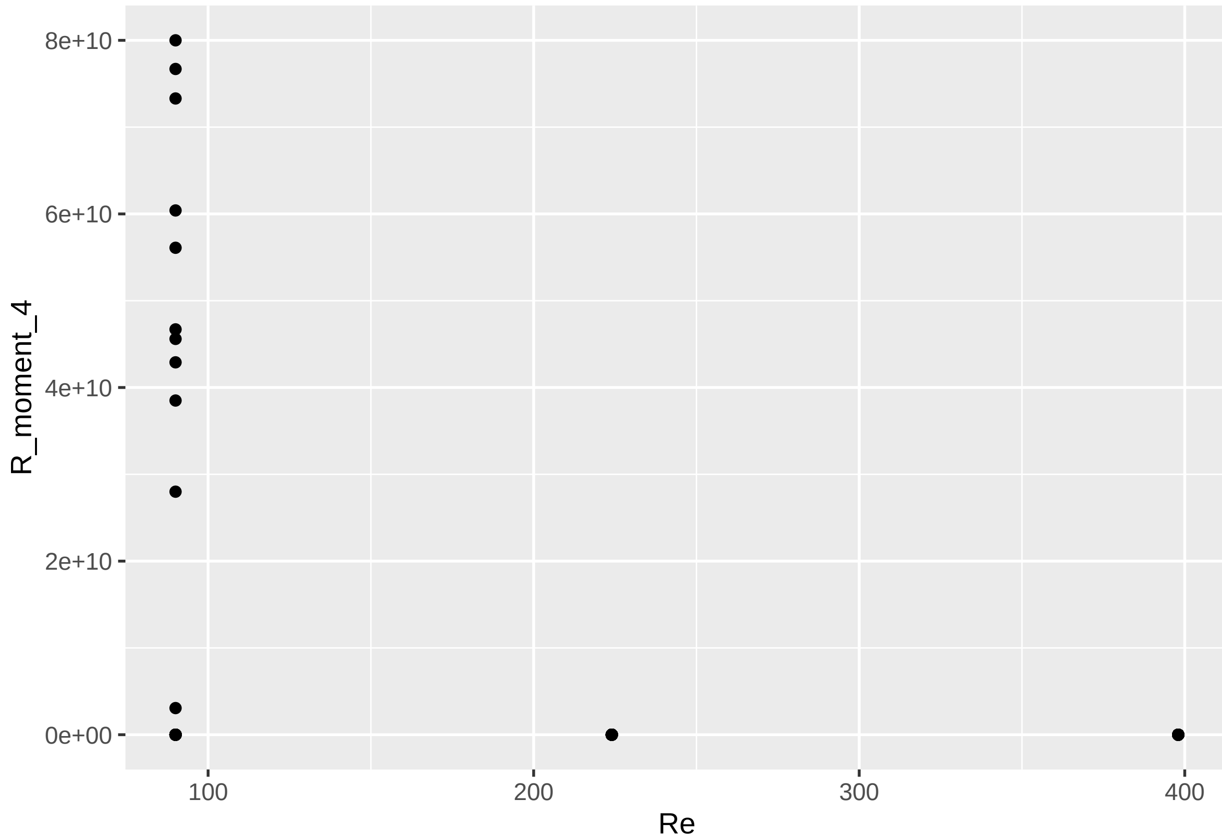
7500000
5000000
2500000
0
R_moment_3
100
200
300
400
Re



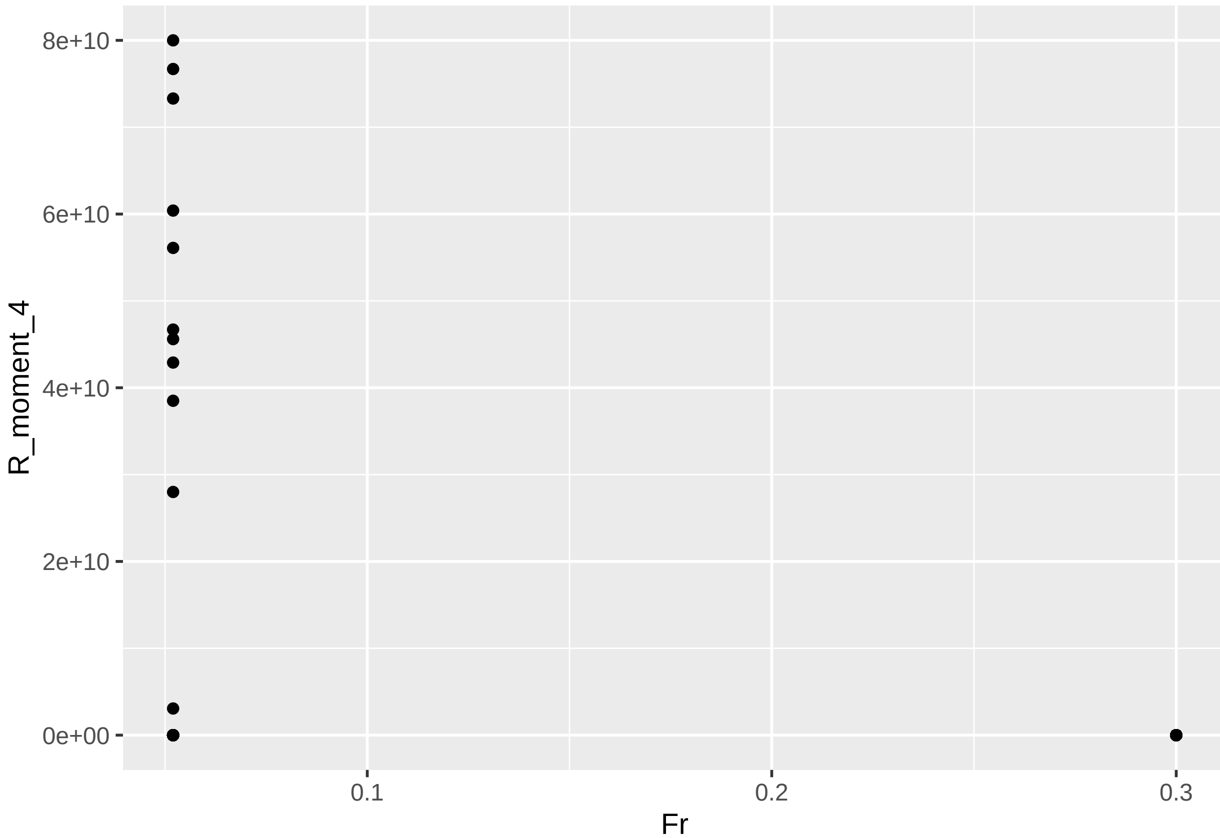
R_moment_3
7500000
5000000
2500000
0
0.1
0.2
0.3
Fr



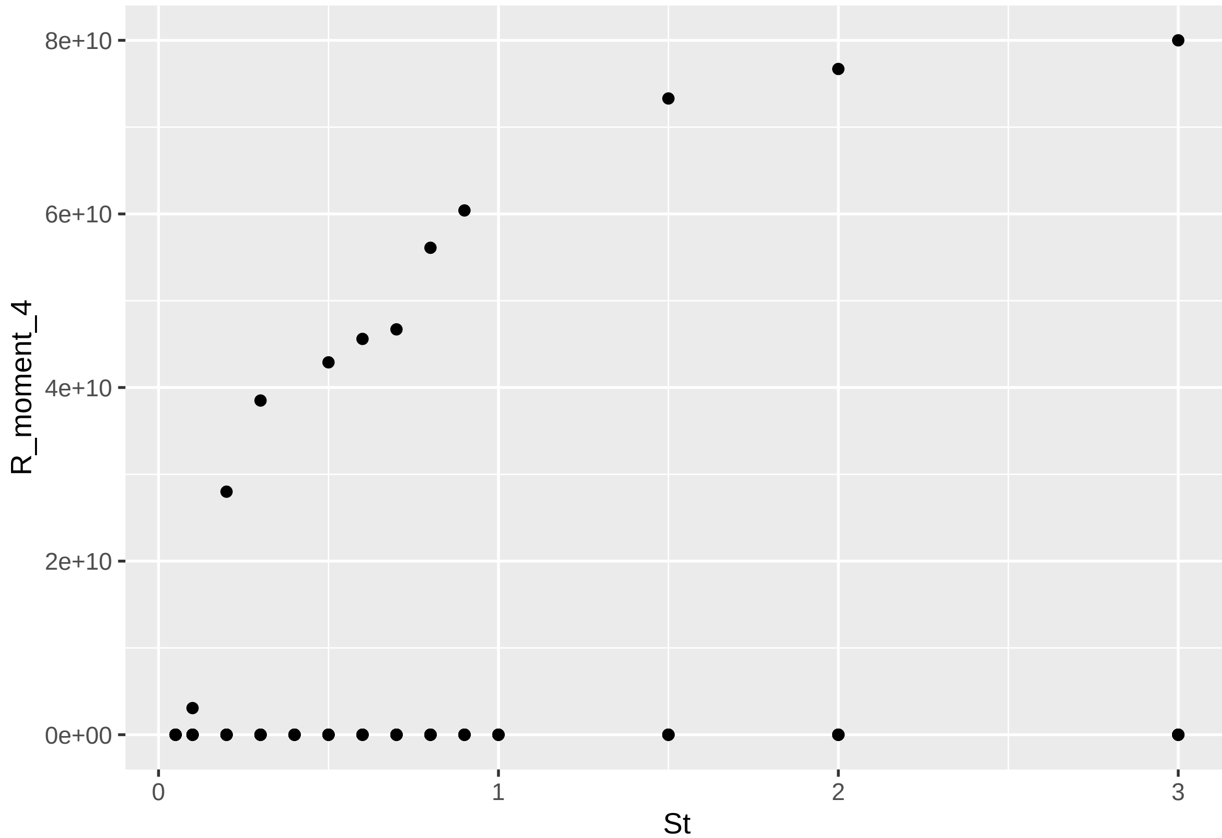
R_moment_3
7500000
5000000
2500000
0
0
1
2
3
St



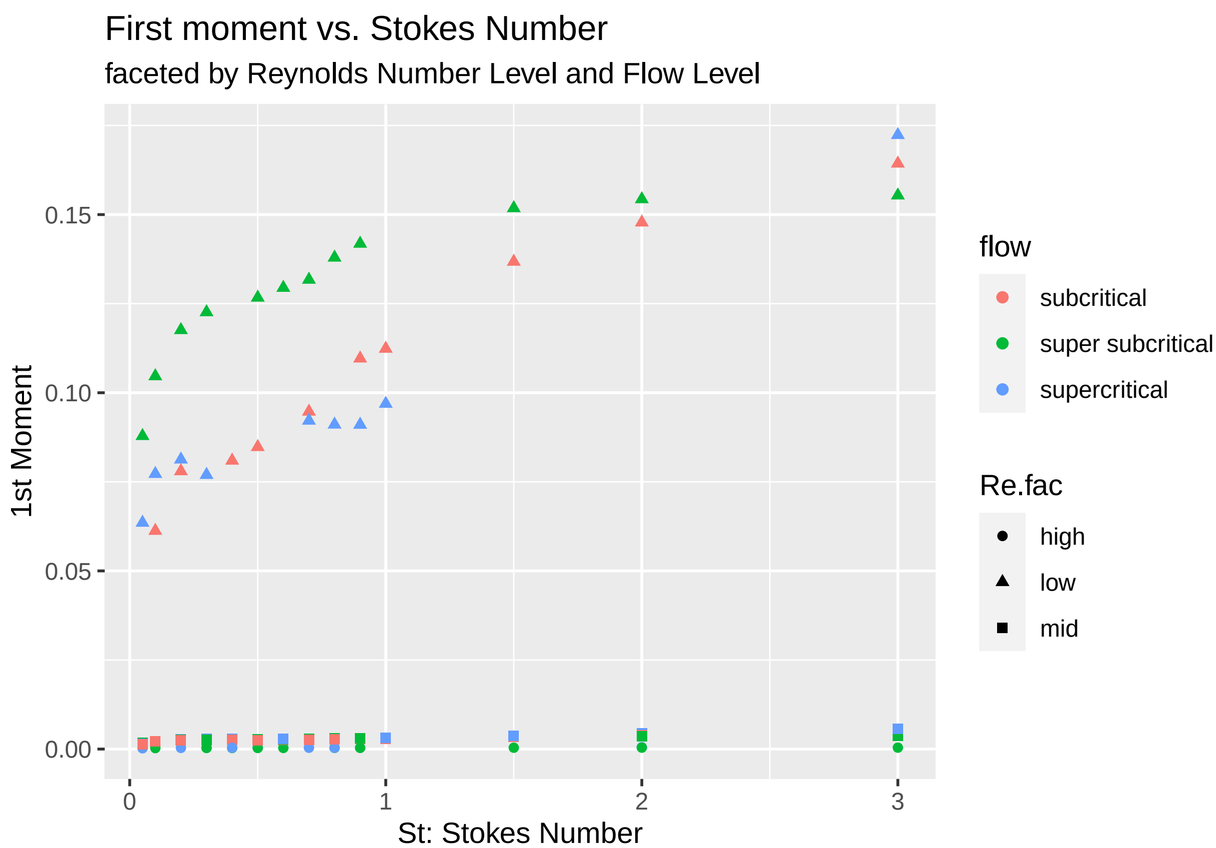
8e+10
6e+10
4e+10
2e+10
0e+00
R_moment_4
100
200
300
400
Re



8e+10
6e+10
4e+10
2e+10
0e+00
R_moment_4
0.1
0.2
0.3
Fr



The plot below suggests a very possible interaction effect between Stokes number and Reynolds number on 1st Moment:



Simple Linear Regression

- Justify making Fr as a categorical variable

Data Wrangling

First Moment Linear Fit

```
##
## Call:
## lm(formula = R_moment_1 ~ Re.fac + St + flow + Re.fac:flow, data = train)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.034040 -0.004960  0.001444  0.006424  0.050687
##
## Coefficients: (1 not defined because of singularities)
##                                Estimate Std. Error t value Pr(>|t|)
## (Intercept)                  -0.0051754  0.0069431  -0.745  0.45821
## Re.faclow                     0.0992025  0.0081623  12.154  < 2e-16 ***
## Re.facmid                    -0.0009803  0.0058722  -0.167  0.86783
## St                            0.0126502  0.0018219   6.943 9.11e-10 ***
## flowsuper subcritical        -0.0072663  0.0081572  -0.891  0.37572
```

```
## flowsupercritical               -0.0026785  0.0053280  -0.503  0.61654
## Re.faclow:flowsuper subcritical  0.0321703  0.0099707   3.226  0.00182 **
## Re.facmid:flowsuper subcritical  0.0053974  0.0080616   0.670  0.50509
## Re.faclow:flowsupercritical     -0.0076459  0.0081313  -0.940  0.34989
## Re.facmid:flowsupercritical             NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01326 on 80 degrees of freedom
## Multiple R-squared:  0.9488, Adjusted R-squared:  0.9436
## F-statistic: 185.2 on 8 and 80 DF,  p-value: < 2.2e-16
```

Using 5-fold cross-validation to estimate the test set error

```
## Linear Regression
##
## 89 samples
##  3 predictor
##
## No pre-processing
## Resampling: Cross-Validated (5 fold)
## Summary of sample sizes: 73, 72, 69, 72, 70
## Resampling results:
##
##   RMSE        Rsquared   MAE
##   0.01415704  0.9398071  0.009977374
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

Trying using polynomial terms up to degree of 5 for stokes number:

```
## Analysis of Variance Table
##
## Model 1: response ~ St + flow + Re.fac
## Model 2: response ~ poly(St, 2) + flow + Re.fac
## Model 3: response ~ poly(St, 3) + flow + Re.fac
## Model 4: response ~ poly(St, 4) + flow + Re.fac
## Model 5: response ~ poly(St, 5) + flow + Re.fac
##   Res.Df      RSS Df  Sum of Sq      F Pr(>F)
## 1     83 0.019399
## 2     82 0.019352  1 4.7187e-05 0.1959 0.6593
## 3     81 0.019180  1 1.7206e-04 0.7142 0.4006
## 4     80 0.019134  1 4.5704e-05 0.1897 0.6643
## 5     79 0.019031  1 1.0305e-04 0.4278 0.5150
```

Judging from the p value for the associated F-statistics, only the first order term is necessary.

Moments 2-4:

```
##
## Call:
## lm(formula = log(R_moment_2) ~ Re + St + flow, data = train)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.7541 -1.0168 -0.3029  0.8348  3.4238
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)            1.433205   0.550793   2.602  0.01095 *
## Re                    -0.025963   0.001856 -13.989  < 2e-16 ***
## St                     0.733752   0.258191   2.842  0.00563 **
## flowsuper subcritical  3.700934   0.520442   7.111 3.52e-10 ***
## flowsupercritical      0.929358   0.542426   1.713  0.09034 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.9 on 84 degrees of freedom
## Multiple R-squared:  0.7499, Adjusted R-squared:  0.738
## F-statistic: 62.98 on 4 and 84 DF,  p-value: < 2.2e-16
```

```
##
## Call:
## lm(formula = log(R_moment_3) ~ Re + St + flow, data = train)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.5905 -1.9037 -0.4285  1.7964  5.9281
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)            5.139254   0.948688   5.417 5.66e-07 ***
## Re                    -0.033938   0.003197 -10.616  < 2e-16 ***
## St                     0.964896   0.444709   2.170   0.0329 *
## flowsuper subcritical  7.104356   0.896411   7.925 8.56e-12 ***
## flowsupercritical      1.611925   0.934277   1.725   0.0881 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.272 on 84 degrees of freedom
## Multiple R-squared:  0.683,  Adjusted R-squared:  0.6679
## F-statistic: 45.25 on 4 and 84 DF,  p-value: < 2.2e-16
```

```
##
## Call:
## lm(formula = log(R_moment_4) ~ Re + St + flow, data = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.0985  -2.8732  -0.7093   2.6849   8.3406
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)            8.999300   1.334956   6.741 1.86e-09 ***
## Re                    -0.042210   0.004498  -9.383 1.00e-14 ***
## St                     1.152984   0.625777   1.842   0.0689 .
```

```
## flowsuper subcritical 10.487017   1.261394   8.314 1.42e-12 ***
## flowsupercritical      2.299173   1.314678   1.749   0.0840 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.605 on 84 degrees of freedom
## Multiple R-squared:  0.6607, Adjusted R-squared:  0.6445
## F-statistic: 40.89 on 4 and 84 DF,  p-value: < 2.2e-16
```

Considering a simple linear regression on the first moment: we have a 0.94 adjusted R squared value and non significant F-statistics; however the residual vs fitted values plot indicates a obvious non-linear trend, which suggests that the linearity assumption is violated.

Ridge Regression