ADSP HW2

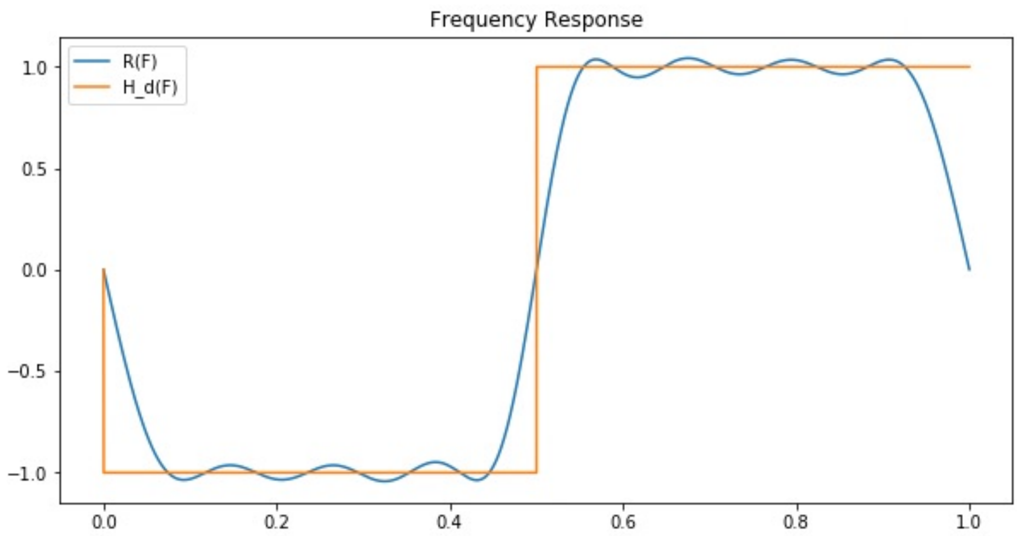
R10942152
游家權

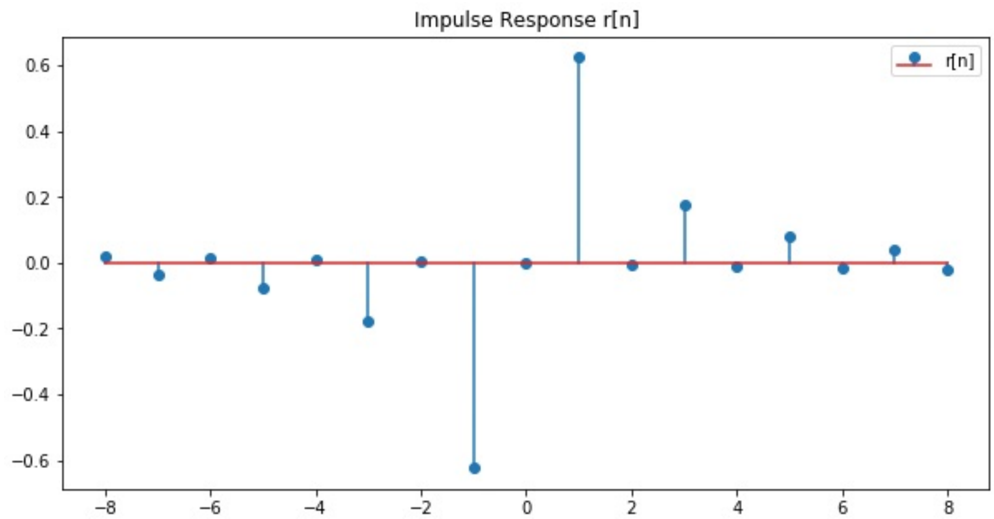
(1) Write a Matlab or Python program that uses the frequency sampling method to design a (2k+1)-point discrete Hilbert transform filter (k is an input parameter and can be any integer). (25 scores)

The transition band can be assigned to reduce the error (unnecessary to optimize). The frequency response (DTFT of $r[n]$, see pages 111 and 112, show the imaginary part) and the impulse response of the designed filter should be shown. The Matlab or Python code should be handed out by NTUCool.

set $k = 8$, transition band: $\frac{1}{2k+1}$ $\frac{k}{2k+1}$ $\frac{k+1}{2k+1}$ $\frac{2k}{2k+1}$

$-0.9j$ $-0.7j$ $0.7j$ $0.9j$





(2) (a) What are the two main advantages of the minimal phase filter compared to other IIR filters? (b) What is the advantage of the Wiener filter compared to other lowpass / highpass filters for noise removal? (c) What are the two advantages of the cepstrum compared to the equalizer for multipath problems? (15 scores)

(a) Minimal phase filter 確保了 forword 跟 inverse transform 是穩定的。另外 Minimal phase filter 也讓能量集中在 $n=0$ 附近，當 n 很大時值很接近零

(b) Wiener filter 不需要給定 stop band，而是會用機率与統計的方式找到 signal 与 noise 的关係

(c) Cepstrum 不需要測量不同路徑的延遲時間，直接觀察不同 path 在 cepstrum 上的效果並將之濾除。另外，Equalizer 的 $H(z)$ 可能是 unstable cepstrum 則沒有這種問題

(3) Suppose that an IIR filter is $H(z)=\dfrac{2z^3+4z^2+z+2}{2z^2+z+1}$

(a) Find its cepstrum.

(b) Convert the IIR filter into the minimum phase filter. (15 scores)

(a)

$$H(z)=\frac{2z^3+4z^2+z+2}{2z^2+z+1}=\frac{2z^2(z+2)(1+\frac{i}{\sqrt{2}}z^{-1})(1-\frac{i}{\sqrt{2}}z^{-1})}{2z^2(1+\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2})}$$

outside

$$=\frac{2(\underline{1-(-\frac{1}{2}z)})(1-(-\frac{i}{\sqrt{2}}z^{-1}))(1-(\frac{i}{\sqrt{2}}z^{-1}))}{(1-(-\frac{1}{4}-\frac{\sqrt{7}}{4}i)z^{-1})(1-(-\frac{1}{4}+\frac{\sqrt{7}}{4}i)z^{-1})}$$

By 投影片 ADSP6. P181

$$\hat{x}[n]=\begin{cases}\log(2), & n=0\\ -\frac{(\frac{-i}{\sqrt{2}})^n}{n}-\frac{(\frac{i}{\sqrt{2}})^n}{n}+\frac{(-\frac{1}{4}-\frac{\sqrt{7}}{4}i)}{n}+\frac{(-\frac{1}{4}+\frac{\sqrt{7}}{4}i)}{n}, & n>0\\ \frac{(-\frac{1}{2})^n}{n}, & n<0\end{cases}$$

#

(b)

$$H(z)=\frac{(z+2)(z+\frac{i}{\sqrt{2}})(z-\frac{i}{\sqrt{2}})}{(z+\frac{1}{4}+\frac{\sqrt{7}}{4}i)(z+\frac{1}{4}-\frac{\sqrt{7}}{4}i)}$$

∵只有 -2 這個 zero 在單位圓外

$$H_1(z)=\frac{(z+2)(z+\frac{i}{\sqrt{2}})(z-\frac{i}{\sqrt{2}})}{(z+\frac{1}{4}+\frac{\sqrt{7}}{4}i)(z+\frac{1}{4}-\frac{\sqrt{7}}{4}i)}\cdot -2\,\frac{z-\overline{(-2)^{-1}}}{z+2}$$

$$=\frac{-2(z-\frac{1}{2})(z+\frac{i}{\sqrt{2}})(z-\frac{i}{\sqrt{2}})}{(z+\frac{1}{4}+\frac{\sqrt{7}}{4}i)(z+\frac{1}{4}-\frac{\sqrt{7}}{4}i)}$$

#

(4) (a) Which of the following filters are always even symmetric? (b) Which of the following always filters are odd symmetric? (i) Notch filters; (ii) smoothers; (iii) edge detectors; (iv) particle filters; (v) 3 times of integrals; (vi) 2 times of differentiations. (10 scores)

Always even symmetric: (i) Notch Filters, (ii) smoothers #

Always odd symmetric: (iii) edge detector, (v) 3 times of integrals

(vi) 2 times of differentiation #

(5) Suppose that $y[n]=\alpha_1 x[n]+\alpha_2 x[n-30]+\alpha_3 x[n-40]+\alpha_4 x[n-50]$

How do we use the cepstrum and the lifter to recover $x[n]$ from $y[n]$? (10 scores)

$$P[n]=\alpha_1\delta[n]+\alpha_2\delta[n-30]+\alpha_3\delta[n-40]+\alpha_4\delta[n-50]$$

$\downarrow$ Z transform

$$P(Z)=\alpha_1\left(1+\frac{\alpha_2}{\alpha_1}Z^{-30}+\frac{\alpha_3}{\alpha_1}Z^{-40}+\frac{\alpha_4}{\alpha_1}Z^{-50}\right)$$

$\downarrow$ 取 log

$$\log(P(Z))=\log\left(1+\frac{\alpha_2}{\alpha_1}Z^{-30}+\frac{\alpha_3}{\alpha_1}Z^{-40}+\frac{\alpha_4}{\alpha_1}Z^{-50}\right)+\log\alpha_1$$

$$=\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{\alpha_2}{\alpha_1}\right)^k\cdot\frac{1}{k}\cdot Z^{-30k}\cdot\left(\frac{\alpha_3}{\alpha_1}\right)^k\cdot\frac{1}{k}Z^{-40k}\cdot\left(\frac{\alpha_4}{\alpha_1}\right)^k\cdot\frac{1}{k}\cdot Z^{-50k}+\log\alpha_1$$

$$=\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{\alpha_2}{\alpha_1}\right)^k\left(\frac{\alpha_3}{\alpha_1}\right)^k\left(\frac{\alpha_4}{\alpha_1}\right)^k\cdot\frac{1}{k^3}\cdot Z^{-120k}+\log\alpha_1$$

$$\hat{P}[n]=\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{\alpha_2}{\alpha_1}\right)^k\left(\frac{\alpha_3}{\alpha_1}\right)^k\left(\frac{\alpha_4}{\alpha_1}\right)^k\cdot\frac{1}{k^3}\cdot\delta[n-120k]$$

$\Rightarrow l(n)=0,$ for $n=120k$ #

(6) Why the Mel-frequency cepstrum is more suitable for dealing with the acoustic signal than the original cepstrum? (10 scores)

① $\sum_k |X[k]|^2 B_m[k]$ 是零的可能性降低很多，可以避免 $\log(\,) \to -\infty$

② $\sum_k |X[k]|^2 B_m[k]$ 是實數不必處理複數

③ $B_m[k]$ 較接近人耳對聲音的感知

④ 用DCT取代IFT，故計算量較小

(7) Why we rarely use $x[n] \xrightarrow{\text{DFT}} X[m] \longrightarrow Y[m] = X[m]H[m] \xrightarrow{\text{IDFT}} y[n]$

for digital filter design in practice? (5 scores)

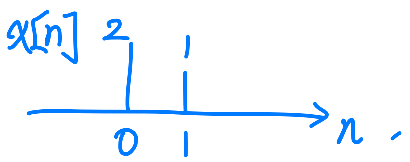
Ans: 在frequency domain做sampling會因為windows effect產生Gibbs phenomenon，使得frequency response不會跟設計的一樣

(8) (a) Which of the following vocal signal sounds louder? (b) Which of the following vocal signal propagates longer?

(i) cos(200πt), (ii) sin(600πt), (iii) cos(1800πt). (10 scores)

(a) (iii) $\cos(1800\pi t)$ # (b) (i) $\cos(200\pi t)$ #

Bonus Question for 学號尾数2



find it's cepstrum

$$X(z) = 2\cdot z^0 + 1\cdot z^{-1} = (2 + z^{-1}) = 2\left(1+\tfrac{1}{2}z^{-1}\right)$$

By formular at P.181, $a_k = \frac{1}{2}$, $b_k = 0$, $c_k = 0$, $d_k = 0$

By P.182, $$\hat{x}_n = \begin{cases} \log(2), & n=0 \\ -\dfrac{(\frac{1}{2})^n}{n}, & n>0 \\ 0, & n<0 \end{cases}$$ #