

ADSP HW5

210942152
游家權

- (2) (a) How do we use one DFT to compute the DFTs of two real signals? (b) How do we use one DFT to compute the DFTs of two real and even signals and two real and odd signals? (10 scores)

(a) 將兩個 real signal 分別放進實部與虛部

若 $f_1[n], f_2[n]$ 為 real signal

$$\text{Step 1: } f_3[n] = f_1[n] + j f_2[n]$$

$$\text{Step 2: } F_3[m] = \text{DFT}\{f_3[n]\}$$

$$\text{Step 3: } F_1[m] = \frac{F_3[m] + F_3^*[N-m]}{2}, F_2[m] = \frac{F_3[m] - F_3^*[N-m]}{2j}$$

故只需要一個 DFT 即可做完

(b) if $x_1[n], x_2[n]$ are real and even
 $x_3[n], x_4[n]$ are real and odd.

$$\begin{cases} y_1[n] = x_1[n] + x_3[n] \\ y_2[n] = x_2[n] + x_4[n] \end{cases}$$

$$y[n] = x_1[n] + x_3[n] + j(x_2[n] + x_4[n]) = y_1[n] + j y_2[n]$$

$$Y_1[m] = \frac{Y[m] + Y^*[N-m]}{2}$$

$$Y_1[m] + Y_1[N-m] = x_1[m] + x_2[N-m] + x_3[m] + x_3[N-m]$$

$$Y_1[m] = x_1[m] + x_3[m]$$

$$\begin{cases} x_1 \text{ is even: } x_1[m] = x_1[N-m] \\ x_3 \text{ is odd: } x_3[m] = -x_3[N-m] \end{cases} \Rightarrow x_1[m] = \frac{1}{2}(Y_1[m] + Y_1[N-m])$$

- (3) (a) If we denote the beginning row as the 1st row, then write the 23rd row of the 32-point Haar transform. (b) What are the most important applications of the Haar transform nowadays? (10 scores)

(a) 23rd row:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) localized spectrum analysis, edge detection

Extract local features,

Analysis of the local high frequency component

- (4) Are the following applications proper or improper to use the Walsh transform? Why? (a) LTI system analysis; (b) step-like signal expansion; (c) modulation; (d) localized feature extraction. (10 scores)

(a) improper, 因為 Walsh transform 不適合做 convolution,

(b) proper, Walsh transform 本身變化很大, 適合用於分析變化也很大的信號

(c) proper, Walsh transform 因其正交性質適合用於調變, 例如 CDMA

(d) proper,

features 通常都是變化很大的地方, 因此 Walsh transform 也適合 extract features

(5) What is the number of addition operations when we what to implement (a) the 16-point Walsh transform and (b) the 16-point NTT? (10 scores)

(a) 1st row 需15個 addition, 剩下的 row 各要 15 addition,

$$\Rightarrow 15 + 1 \times 15$$

$\Rightarrow 30$ 個 addition #

(b) 零個加法, 因為可以用 LUT

(6) What are the two main advantages of the OFDM when compared to the original FDM? (5 scores)

① OFDM 有正交的性質不同頻率的信號不會互相干擾
用內積即可還原信號

② OFDM 可以使用 FFT 的 fast algorithm 來加速.

(7) Describe two concepts that you learned from the oral presentation on 6/10.

(10 scores)

① 沒有 activation layer 的 1D CNN 可以被視為 FIR Filter
沒有 activation layer 的 RNN 可以被視為 IIR Filter

② 人的腦波能被拆成不同的 Rhythms. 各頻率的波形代表
不同的大腦活動.

(8) (a) What is the results of CDMA if there are three data $[1 \ 0 \ 1]$, $[0 \ 1 \ 0]$, $[1 \ 1 \ 0]$ and these three data are modulated by the 1st, 5th, and 10th columns (equivalent to the 1st, 5th, and 10th rows ($m = 0, 4, 9$)) of the 16-point Walsh transform?

(15 scores)

(b) Is it better to use the NTT for CDMA? Why?

(5 scores)

(a)

$$m=0: [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]$$

$$m=4: [1 \ 1 \ -1 \ -1 \ -1 \ -1 \ 1 \ 1 \ -1 \ -1 \ -1 \ -1 \ 1 \ 1 \ 1 \ 1]$$

$$m=9: [1 \ -1 \ -1 \ 1 \ 1 \ -1 \ -1 \ 1 \ -1 \ 1 \ 1 \ -1 \ -1 \ 1 \ 1 \ -1]$$

modulate $[1 \ 0 \ 1] \Rightarrow [1 \ -1 \ 1]$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

modulate $[0 \ 1 \ 0] \Rightarrow [-1 \ 1 \ -1]$

$$\Rightarrow \begin{bmatrix} -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

modulate $[1 \ 1 \ 0] \Rightarrow [1 \ 1 \ -1]$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\ -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}$$

答案:
$$\begin{bmatrix} 1 & -1 & 1 & 3 & 3 & 1 & -1 & 1 & -1 & 3 & 1 & 1 & 3 & 1 & -1 \\ 1 & -1 & -3 & -1 & -1 & -3 & -1 & 1 & -1 & -1 & -3 & -3 & -1 & 1 & -1 \\ -1 & 1 & 3 & 1 & 1 & 3 & 1 & -1 & 1 & -1 & 3 & 3 & 1 & -1 & 1 \end{bmatrix} \quad \#$$

(b) NTT 的 output 限制在 $[0, n-1]$ 之間, 故不適合用於 CDMA

學號尾數 2, 7 的 Bonus Question:

$$\begin{aligned} & (656 \times 1315) \bmod 13 \\ &= (656 \bmod 13) \times (1315 \bmod 13) \bmod 13 \\ &= (6 \times 2) \bmod 13 \\ &= \underline{12} \# \end{aligned}$$