ADSP HW4

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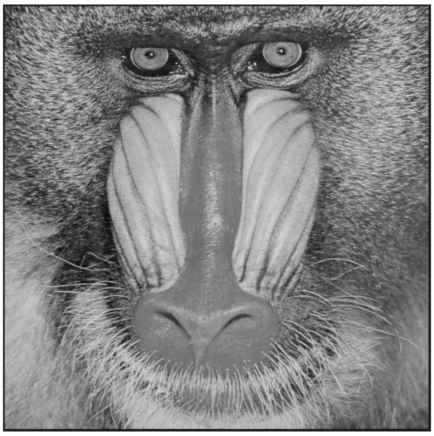
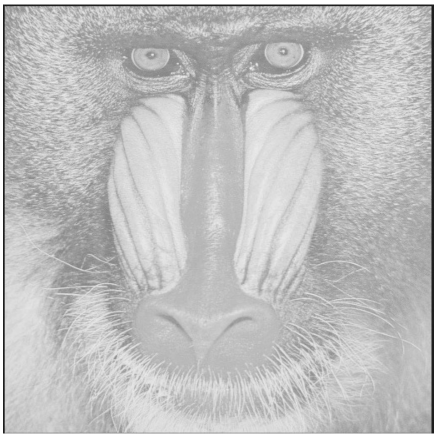
(1) Write a Matlab or Python program to measure the structural similarity (SSIM) of two images A and B. The sizes of A and B are equivalent.

SSIM(A, B, c1, c2)

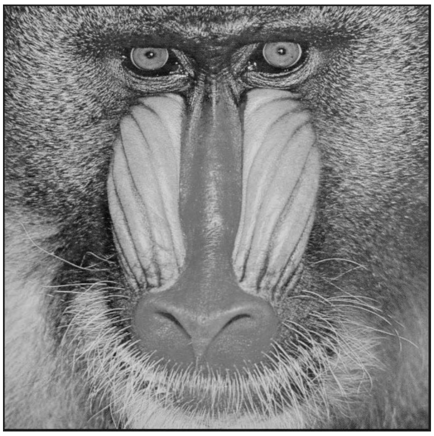
where c1 and c2 are some adjust constants.

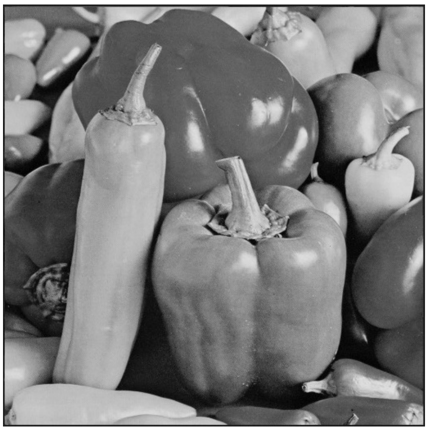
The Matlab or Python code should be handed out by NTUCool. (20 scores)





⇒ SSIM = 0.76106 #





⇒ SSIM = 0.09172 #

(2) How do we implement the following 5-point DCT with the least number of nontrivial multiplications?

$$X[m]=\sum_{n=0}^{4}\cos\left(\frac{\pi}{5}m(n+\frac{1}{2})\right)x[n] \qquad n=0,1,2,3,4 \quad m=0,1,2,3,4$$

(10 scores)

The process and the number of real multiplications should be shown.

$$F_r = \begin{array}{c|ccccc} & n=0 & 1 & 2 & 3 & 4 \\ m=0 & 1 & 1 & 1 & 1 & 1 \\ 1 & c & d & 0 & -d & -c \\ 2 & a & -b & -1 & -b & a \\ 3 & d & -c & 0 & c & -d \\ 4 & b & -a & 1 & -a & b \end{array} \quad \begin{array}{l} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}$$

$c=0.9511$, $d=0.5878$, $a=0.8090$, $b=0.3090$

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = F_r \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix},$$

$y_0 = x_0+x_1+x_2+x_3+x_4 \Rightarrow$ 0 MUL

$$\begin{bmatrix} y_1 \\ y_3 \end{bmatrix} = \begin{bmatrix} c & d \\ d & -c \end{bmatrix}\begin{bmatrix} x_0-x_4 \\ x_1-x_3 \end{bmatrix} \Rightarrow$$

By P.343, Case (3) 2 MUL

$$\begin{bmatrix} y_2 \\ y_4 \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} x_0+x_4 \\ -x_1-x_3 \end{bmatrix} + \begin{bmatrix} -x_2 \\ x_4 \end{bmatrix} \Rightarrow$$

By P.343 Case (3) 2 MUL

Ans: 共需要 2MUL+2MUL = 4MUL 共4個乘法 #

(3) Suppose that x is a complex number. What are the constraints of θ such that the multiplication of x and $\exp(j\,\theta)$ required only 2 real multiplications?

(10 scores)

$\exp(j\theta) = C + id$, where $C=\cos\theta$, $d=\sin\theta$

$$\begin{bmatrix} c & -d \\ d & c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \xrightarrow{\theta=\frac{\pi}{6}} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\text{if } \theta=\frac{2}{3}\pi, \Rightarrow \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \qquad \theta=\frac{\pi}{3} \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$\text{if } \theta=\frac{1}{4}\pi \Rightarrow \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$\boxed{\text{Ans: } \theta=\frac{i}{6}\pi \text{ 或者 } \theta=\frac{i}{4}\pi \ (\text{where } i\in\mathbb{N})}$$ #

(4) Determining the numbers of real multiplications for the (a) 220-point DFT, (b) 231-point DFT, and the (c) 245-point DFT. (15 scores)

(a) $MUL_{220} = 11 \cdot MUL_{20} + 20 \cdot MUL_{11}$

$= 11 \cdot (4MUL_5 + 5MUL_4) + 20MUL_{11}$

$= 44MUL_5 + 55MUL_4 + 20MUL_{11}$

$= 44 \times 10 + 55 \times 0 + 20 \times 40 = \underline{1240}$ #

(b) $MUL_{231} = 11 \cdot MUL_{21} + 21MUL_{11}$

$= 11 \cdot (3MUL_7 + 7MUL_3) + 21MUL_{11}$

$= 33MUL_7 + 77MUL_3 + 21MUL_{11}$

$= 33 \times 16 + 77 \times 2 + 21 \times 40 = \underline{1522}$ #

(c) $MUL_{245} = 5 \cdot MUL_{49} + 49 \cdot MUL_5$

$= 5 \cdot (7MUL_7 + 7MUL_7 + 3 \times 36) + 49 \cdot MUL_5$

$= 70MUL_7 + 540 + 49 \cdot MUL_5$

$= 70 \times 16 + 540 + 49 \times 10 = \underline{2150}$ #

(5) What are the <u>two main advantages</u> of the sectioned convolution? (10 scores)

(1) 運算時間与 input size 呈線性关係 ($\Theta(n)$)

(2) 固定L之後，每次都做固定点的DFT，讓硬件設計可以固定

(6) Suppose that a smooth filter is:

$x_s[n] = x[n] * h[n]$ $\qquad h[1] = h[-1] = 0.24$ $\qquad h[2] = h[-2] = 0.06$

$h[3] = h[-3] = 0.03$ $\qquad h[0] = 0.34$ $\qquad h[n] = 0$ otherwise

Design <u>an efficient way with least number of non-trivial real multiplications</u> to implement the above filter operation. (10 scores)

$$y[n] = x[n] * h[n] = \sum x[n-m]h[m]$$

$$= x[n+3]h[-3] + x[n+2]h[-2] + x[n+1]h[-1] + x[n]h[0]$$
$$+ x[n-3]h[3] + x[n-2]h[2] + x[n-1]h[1]$$

$$\Rightarrow \underline{0.03(x[n+3]+x[n-3]) + 0.06(x[n+2]+x[n-2])}$$
$$\underline{+0.24(x[n+1]+x[n-1])} + 0.34x[0]$$

合併

$$\Rightarrow \underbrace{0.03}_{①}[x[n+3]+x[n-3]+\underline{2}(x[n+2]+x[n-2])+\underline{2^3}(x[n+1]+x[n-1])]$$
$$+\underbrace{0.34x[0]}_{②}$$

trivial

2 MUL for each output $\Rightarrow$ <u>2N MULS</u> #

(7) Suppose that length($x[n]$) = 1100. What is the best way to implement the convolution of $x[n]$ and $y[n]$ if

(a) length($y[n]$) = 200, (b) length($y[n]$) = 20,

(c) length($y[n]$) = 7, and (d) length($y[n]$) = 2?

Also show the number of real multiplications required for each case.

(25 scores)

(a) $N=1100$, $M=200$,

if use IFFT(FFT(x[n])FFT(h[n])), $P=N+M-1=1299$

$2\cdot MUL_{1299}+3\times1299=2\cdot8252+3\times1299=\underline{20401}$

if use sectioned convolution, $L_0=550$

附近最好的点是 $P=504$, $MUL_P=2300$. $L=504-200+1=305$

$S=N/L=\lceil 1100/305\rceil=4$

\# of mul. $=S\cdot(2MUL_P+3\cdot P)=4(2\cdot2300+3\times504)$

$=\underline{24448}$

用FFT,IFFT直接做乘法次数最少 ⇒ 20401次乘法 #

(b) $N=1100$, $M=20$, $L_0=105$

附近最好的 $P=144$, $MUL_{144}=436$, $L=144-20+1=125$

$S=\left\lceil\frac{1100}{125}\right\rceil=9$

\# of MUL $=S\cdot(2MUL_P+3\cdot P)=9\times(2\times436+3\times144)$

用sectioned convolution ⟶ $=\underline{11736}$ #

(c) $N=1100,\ M=7,\ L_0=25$

附近最好的 $P=24,\ MUL_{24}=28,\ L=24-7+1=18$

$$S=\left\lceil \frac{1100}{18} \right\rceil = 62$$

$\#$ of $MUL = S\cdot(2MUL_P+3\cdot P) = 62(2\cdot 28+3\cdot 24) = \underline{7936}$ #

用sectioned convolution 共有7936个乘法 *

(d) If use Direct Implementation

$\#$ of real multiplication $= 3\times N\times M = 3\times 1100\times 2 = \underline{6600}$

If use sectioned convolution:

① $N=1100,\ M=2,\ L_0=2,\ P_0=2+2-1=3$

$S=\dfrac{1100}{2}=550$

$2S\cdot MUL_3+3S\cdot P = 2\times 550\times 2+3\times 550\times 3 = \underline{7150}$

② $P=4,\ L=4-2+1=3,\ S=N/L=\left\lceil \dfrac{1100}{3}\right\rceil = 367$

$367\times(2\cdot MUL_4+3\times 4) = \underline{4404}$ #

Ans: 4404 个实数乘法, P=4 的 sectioned convolution

學號尾數2的bonus question：What's nontrivial multiplication?

當x乘上2^k $(k \in N)$時，因為等同於bit左移或右移，實際的計算量很小，故稱為trivial multiplication，除此之外的乘法被稱為nontrivial multiplication.