ADSP HW1

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游家權

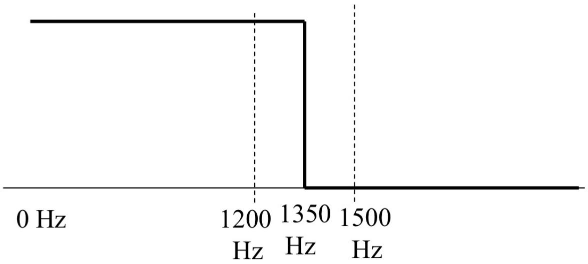
(1) Design a Mini-max **lowpass** FIR filter such that (40 scores)

① Filter length = 17, ② Sampling frequency f_s = 6000Hz,

③ Pass Band 0~1200Hz ④ Transition band: 1200~1500 Hz,

⑤ Weighting function: $W(F)$ = 1 for passband, $W(F)$ = 0.6 for stop band .

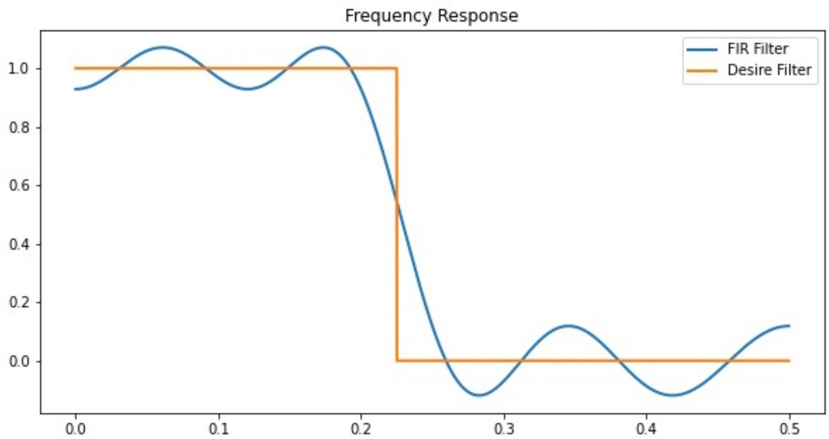
⑥ Set Δ = 0.0001 in Step 5.



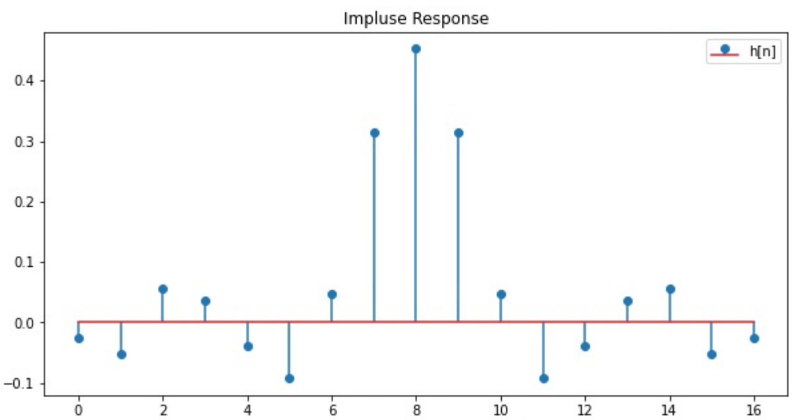
※ The code should be handed out by NTUCool, too.

Show (a) the frequency response, (b) the impulse response $h[n]$, and (c) the maximal error for each iteration.

(a) Frequecy Response



(b) Impulse Response



(c) Maximum Error for each iteration

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1st_iteration_MaxErr = 0.12362984401511301
2nd_iteration_MaxErr = 0.07704144356675102
3rd_iteration_MaxErr = 0.07127611617996485
4th_iteration_MaxErr = 0.0712072852346789
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(2) (a) How do we convert convolution into addition?

(b) From the view point of implementation, what are the disadvantages of the discrete Fourier transform? (10 scores)

(a) 先將等號兩邊做 Laplace Transform 再取 log.

e.g. $z(t) = x(t) * y(t) \Rightarrow Z(s) = X(s) \cdot Y(s) \Rightarrow \log(Z(s)) = \log(X(s)) + \log(Y(s))$

(b) ① 運算上要處理複数乘法，而一個複数乘法等同於四個實数乘法，故運算量較大

② 運算上要處理無理数，因為其值是 π 的倍数，無理数的数位處理比較麻煩

(3) How do we <u>implement</u> $y[n] = x[n] * (0.8^n u[n] - 0.6^n u[n])$ <u>efficiently</u> where * means convolution and $u[n]$ is the unit step function? (10 scores)

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n} = \sum_{n=-\infty}^{\infty} [0.8^n u[n] - 0.6^n u[n]] z^{-n} = \sum_{n=0}^{\infty} (0.8^n - 0.6^n) z^{-n}$$

$$= \sum_{n=0}^{\infty} 0.8^n z^{-n} - \sum_{n=0}^{\infty} 0.6^n z^{-n}$$

$$= \frac{1}{1-0.8z^{-1}} - \frac{1}{1-0.6z^{-1}} = \frac{1-\frac{0.6}{z} - (1-\frac{0.8}{z})}{(1-\frac{0.8}{z})(1-\frac{0.6}{z})} = \frac{\frac{0.2}{z}}{1-\frac{1.4}{z}+\frac{0.48}{z^2}}$$

$$Y(z) = X(z) \cdot H(z) = X(z) \cdot \frac{0.2z^{-1}}{1-1.4z^{-1}+0.48z^{-2}}$$

$$Y(z) = X(z) + \left[1 - \frac{1-1.4z^{-1}+0.48z^{-2}}{0.2z^{-1}}\right] Y(z)$$

$$= X(z) + \left[\frac{0.2z^{-1} - 1 + 1.4z^{-1} - 0.48z^{-2}}{0.2z^{-1}}\right] Y(z)$$

$$= X(z) + \frac{-0.48z^{-2} + 1.6z^{-1} - 1}{0.2z^{-1}} Y(z)$$

$$\Rightarrow 0.2z^{-1}Y(z) = 0.2z^{-1}X(z) - Y(z) + 1.6z^{-1}Y(z) - 0.48z^{-2}Y(z)$$

$$\Rightarrow Y(z) = 0.2z^{-1}X(z) + 1.4z^{-1}Y(z) - 0.48z^{-2}Y(z)$$

$$\Rightarrow \underline{y[n] = 0.2x[n-1] + 1.4y[n-1] - 0.48y[n-2]} \ \#$$

(4) Why (a) the step invariance method and (b) the bilinear transform can reduce or avoid the aliasing effect in IIR filter design? (10 scores)

(a) Step invarience 透過積分的方式將高頻的能量壓下來，故能降低常常在高頻部份出現的 aliasing effect.

(b) Bilinear transform 將整個 $-\infty \sim \infty$ 的頻域 mapping 到 $-\frac{f_s}{2}, \frac{f_s}{2}$ 之間，使 aliasing effect 完全消失

(5) Suppose that $x[n] = y(0.002n)$ and the length of $x[n]$ is 2000. If $X[m]$ is the FFT of $x[n]$, which frequency do (a) $X[300]$ and (b) $X[1800]$ correspond to? (10 scores)

$f_s = \frac{1}{0.002} = 500\,Hz$, $N = 2000$ By: $f = m \cdot \frac{f_s}{N}$

(a) $f = 300 \cdot \frac{500}{2000} = \underline{75\,Hz}$ #

(b) 因為 $1800 > \frac{N}{2} = 1000$，故 $f = 1800 \cdot \frac{500}{2000} - 500 = \underline{-50\,Hz}$ #

(6) Suppose that we want to design a 25-point lowpass filter where $F < 0.25$ is the passband. Which one has the least error in the passband? W(F) means the weight function. (10 scores)

(a) transition : $0.23 < F < 0.27$, $W(F) = 0.5$ for $F < 0.23$, $W(F) = 1$ for $F > 0.27$;
(b) transition : $0.2 < F < 0.3$, $W(F) = 2$ for $F < 0.2$, $W(F) = 2$ for $F > 0.3$;
(c) transition : $0.2 < F < 0.3$, $W(F) = 1$ for $F < 0.2$, $W(F) = 0.5$ for $F > 0.3$;
(d) transition : $0.23 < F < 0.27$, $W(F) = 3$ for $F < 0.23$, $W(F) = 2$ for $F > 0.27$.

Ans: C # 因選項C有較寬的 transition band，且在 pass band 的權重也比較高

(7) Use the MSE method to design the 5-point FIR filter that approximates the lowpass filter of $H_d(F) = 1$ for $|F| < 0.3$ and $H_d(F) = 0$ for $0.3 < |F| < 0.5$.

(10 scores)

By ADSP2 P.50

$$S[0] = \int_{-1/2}^{1/2} H_d(F)\,dF = \underline{0.6}$$

$$S[n] = 2\int_{-1/2}^{1/2} \cos(2\pi nF) H_d(F)\,dF$$

$$S[1] = 2\int_{-1/2}^{1/2} \cos(2\pi F) H_d(F)\,dF = 2\int_{-0.3}^{0.3} \cos(2\pi F)\,dF = 2\cdot\left[\sin(2\pi F)\cdot\frac{1}{2\pi}\right]\Big|_{-0.3}^{0.3} = \frac{1}{\pi}\left[\sin(0.6\pi) - \sin(-0.6\pi)\right] = \frac{2}{\pi}\cdot\sin(0.6\pi) = 0.\underline{6055}$$

$$S[2] = 2\int_{-0.3}^{0.3} \cos(4\pi F)\,dF = 2\cdot\left[\sin(4\pi F)\cdot\frac{1}{4\pi}\right]\Big|_{-0.3}^{0.3} = \frac{1}{\pi}\cdot\sin(1.2\pi) = \underline{-0.1871}$$

$S[0]$	$S[1]$	$S[2]$
0.6	0.6055	-0.1871

$$\frac{S[n]}{2} = r[n]$$

$r[0]$	$r[1]$	$r[2]$
0.6	0.3025	-0.09355

$$\boxed{\begin{aligned} h[0] &= -0.0935 \\ h[1] &= 0.3025 \\ h[2] &= 0.6 \\ h[3] &= 0.3025 \\ h[4] &= -0.0935 \end{aligned}}$$

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学号尾数2的Bonus Question:

如果將pass band与stop band的weight都乘上某个常数,是否會影响結果?

Ans: 不影响。# 這等同於在A matrix的最後一行乘上某個常數,做A^{-1}之後,結果还是一樣