Homework #2 (100 Points)

Due 03:00pm on Tuesday 02/09/2016 through gsubmit.

Feel free to make assumptions, if you feel that such assumptions are justified or necessary. Please state your assumptions clearly. Unreasonable assumptions lead to unreasonable grades!

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1. The availability of a web server is rated at 96%. Answer the following questions:

   a. What is the probability that in a sequence of 20 connections to the web server, no connections will be refused.
   b. What is the probability that in a sequence of 20 connections to the web server, exactly one connection will be refused.
   c. What is the probability that in a sequence of 20 connections to the web server, at most three connections will be refused.
   d. What is the probability that the web server will be able to successfully respond to 4 (or more) consecutive requests for connections.
   e. What is the average number of requests that will be successfully served before a connection is refused?
   f. What is the average number of requests that will be successfully served out of a set of 50 requests for connections?
   g. What is the probability that out of a very large number of requests at least 95% of the requests will be processed successfully?
   h. Which of the above questions refers to the reliability of the web server?

2. The arrival of packets at an Ethernet adapter of a web server is described by a Poisson process with a rate of 200 packets per second. Answer the following questions:

   a. Write down the formula for the probability distribution of the number of packets arriving to the adapter in 10 milliseconds.
   b. What is the probability that more than 3 packets will be received within a 10-millisecond period of time?
   c. Write down the formula for the probability distribution of the inter-arrival time of packets in milliseconds.
   d. What is the probability that two packets will be separated by more than 8 milliseconds?
   e. What is the standard deviation of the inter-arrival time of packets?

   Packets that arrive to the Ethernet adapter described above are queued up in a buffer until processed by the Interrupt Service Routine (ISR). Assuming that the ISR service time per packet is exponential with an average of 4.8 milliseconds. Answer the following questions:

   a. What is the capacity of the Ethernet adapter?
   b. Write down the probability distribution of the number of packets at the Ethernet adapter

(either queued or being transmitted).

c. How many packets do you expect to find waiting in the buffer of the adapter at any point in time (on average)?
d. What is the average waiting time in the buffer (i.e., how long is a packet buffered until the ISR starts processing it)?
e. What is the slowdown caused by queuing at the Ethernet adapter?

3. Write a Java function that returns a random value that is distributed according to an exponential distribution with a mean of T=1/Lambda.

   Hint: Given that all you have through Java is a uniform random number generator, you will need to find the relationship that maps a uniform random variable that ranges from 0 to 1 to an exponential random variable with mean T = 1/Lambda. As explained in the lecture notes, pages 9-10, one way of establishing this relationship is to equate the cumulative distribution functions for a uniformly distributed random variable (say y) and an exponentially distributed random variable (say x).

   Answer the following questions:

   a. Write a program that calls the function you wrote to generate 100 exponentially-distributed random values using Lambda = 4. Generate and plot a CDF of the exponential distribution using the 100 generated values (for plotting the CDF, you may want to use software such as excel). Compare the CDF of the random variable that you have obtained empirically with the analytical CDF of the random variable. Do they match? Explain why or why not.
   b. Let the 100 random values you obtained above represent the inter-arrival times (in seconds) of queries submitted to a server. Plot a normalized histogram showing the relative frequency (on the Y axis) of the various number of queries observed over 1-second intervals (on the X axis). Write down the formula of the distribution that best characterizes the empirical results you got in this histogram.

4. You have conducted an empirical study of the hit rates and latencies of a four-level cache memory system. The following were your findings:
   - Hit rate of accesses to L1 cache is 50%
   - Hit rate of accesses to L2 cache is 60%
   - Hit rate of accesses to L3 cache is 70%
   - All misses from L3 cache are served from the last memory level
   - The access time for hits to L1 is one clock cycle
   - The access time for hits to L2 is two clock cycles (in addition to that of accessing the L1 cache)
   - The access time for hits to L3 is seven clock cycles (in addition to that of accessing the L1 and L2 caches)
   - The access time from the last memory level is twenty clock cycles (in addition to that of accessing the L1, L2, and L3 caches)

   Let *t* be a random variable denoting the overall delay for an access to the above memory system

. Assuming that hit/miss probabilities in one level are independent from hits/misses in other levels, answer the following questions:

a. Derive the probability mass function (PMF) for the random variable t.
b. Derive the mean of the random variable t.
c. Derive the standard deviation of the random variable t.
d. Write a Java function that returns a random value that is distributed according to the PMF you obtained for part (a).

5. Write a Java function -- say Zrand() -- that returns a random value that is distributed according to a standard normal distribution.

   Hint: Given that all you have through common Java libraries is a uniform random number generator, you will need to find the relationship that maps a uniform random variable that ranges from 0 to 1 to a standard normal random variable. One way of doing this is to use the Central Limit Theorem (CLT) to generate this random value using the uniform random number generator. Recall that according to CLT, the sum of N random variables (that follow any distribution with finite moments) approaches a Normal distribution, whose mean is N times the mean of the individual random variables, and whose variance is N times the variance of the individual random variables. [Note: You may assume that aggregating 30 or more samples is "enough" for the CLT to hold.]

   a. Write a program that calls the function you wrote to generate 100 standard-normal random values. Generate and plot a CDF of the standard normal distribution using the 100 generated values (for plotting the CDF, you may want to use software such as excel). Compare the CDF of the random variable that you have obtained empirically with the CDF given in the standard normal distribution table by looking up the probability that the random variable is less than or equal to 0, 1, 2, 3, and 4. Do they match? Explain why or why not.
   b. Write a program that calls the function you wrote in part (a) to generate a normally-distributed random variable with a mean = 72 and standard deviation = 16. Use your program to compute the probability that this random variable is between 66 and 80. Compare this answer with the answer you get analytically. Show your work.