Assignment 4 – Worksheet 1
R functions

1. Warm-up

(a) Write a function which takes a numeric vector x, and returns a named list containing the mean, median and variance of the values in x.

(b) Write a function with arguments x and n, which evaluates $\sum_{i=0}^{n} \frac{e^{-x} x^{i}}{i!}$. (Use factorial().)

(c) Write a function which goes through every entry in a list, checks whether it is a character vector (is.character()), and if so prints it (print() or cat()).

(d) Write a function with an argument k which simulates a symmetric random walk on the integers, stopping when the walk reaches k (or $-k$). A random walk on the integers is a sequence $X_1, X_2, X_3, \ldots$ with $X_0 = 0$ and $X_i = X_{i+1} + D_i$ where the D_i are independent with $P(D_i = +1) = P(D_i = -1) = 1/2$.

2. Moving Averages

a) Write a function to calculate the moving averages of length 3 of a vector $(x_1,\cdots,x_n)^T$.

(The function returns $(z_1,\cdots,z_{n-2})^T$, where

$$z_i=\frac{1}{3}(x_i+x_{i+1}+x_{i+2}),\qquad i=1,\cdots,n-2 \quad .$$

Call this function `ma3()`.

(b) Write a function which takes two arguments, x and k, and calculates the moving average of x of length k. You can use a `for()` loop, but there has to be a better way, right?

(c) How does your function behave if k is larger than (or equal to) the length of x?

(d) You can (and should) return an error in this case. Use the stop() function. Are there other choices?

(e) How does your function behave if k = 1? What should it do? Fix it if necessary.

3. Optional Plot

Take the continuous functions

$$f(x)=\begin{cases} x^2+2x+3 & \text{if } x<0 \\ x+3 & \text{if } 0\le x<2 \\ x^2+4x-7 & \text{if } 2\le x \end{cases}$$

Write a function which takes a vector and returns a vector of the values of $f(x)$. The function should be valid for inputs where $-4<x<4$.

Your function should check the input for validity and should offer the user the option of plotting the values the function returns – something like `plot=TRUE.`

4. Matrix Input

Write a function which takes a single argument – a matrix or an argument that can be coerces into a matrix. the function should return a matrix which is the same as the function argument, but ever odd number is doubled.

So, if the input is $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ the output should be $\begin{bmatrix} 2 & 2 & 6 \\ 10 & 2 & 6 \\ -2 & -2 & -6 \end{bmatrix}$.

5. Poisson process

A Poisson process of rate λ is a random vector of times $(T_1, T_2, T_3, \ldots)$ where the interarrival times $T_1, T_2 - T_1, T_3 - T_2, \ldots$ are independent exponential random variables with paramerter λ. Note that this implies $T_{i+1} > T_i$.

a) Write a function with arguments λ and M which generates a Poisson lprocess up until the time reaches M. Using the `rexp()` family in R will be helpful.

b) Generate 10,000 of these series with $\lambda = 5$ and $M = 1$. Record the lengths of each of the 10,000 vectors returned. Plot the vector lengths as a histogram. Calculate their mean and variance. How do you think the lengths are distributed? Explain.