Calibration Quality

Comparing single trial registration matrix to “ground truth” registration matrix obtained from a combined 24 trials

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~~Kuka→ Aruco~~ Aruco→KUKA Registration

- 29 calibration trials captures→ **5 for repeatability** +24 for registration matrix
- JSON Calibration
- Get registration matrix (source=JSON, target=aruco X24 trials)

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Registration matrix (24 trials)

= GROUND TRUTH

```
-------REG ARUCO to KUKA ----------
[[ 9.70307596e-01  4.76785513e-02  2.37128500e-01 -7.14842213e-01]
 [-2.36887572e-01 -1.07235591e-02  9.71477886e-01 -6.55058557e-01]
 [ 4.88615197e-02 -9.98805167e-01  8.89305382e-04  4.65641889e-01]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00]]
```

** Original in meters: analysis later converted to millimeters

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Repeatability of Registration

- Take 5 Trials (4 positions each)
- Calculate their individual registration matrix T
- Get mean and std of each T component (for Rotation and translation matrices)
- Calculate their respective bias
- Calculate their respective TAE: total analytical error

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$$T^{\cancel{aruco}}_{\cancel{kuka}} \begin{bmatrix} R_{00} & R_{01} & R_{02} & T_{03} \\ R_{10} & R_{11} & R_{11} & T_{13} \\ R_{20} & R_{21} & R_{21} & T_{23} \\ R_{30} & R_{31} & R_{31} & T_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \longrightarrow$$

Kuka

Aruco

Registration matrix obtained from 24 trials = "Ground Truth"

$$T^{\cancel{aruco}}_{\cancel{kuka},trial=i} \begin{bmatrix} R_{00} & R_{01} & R_{02} & T_{03} \\ R_{10} & R_{11} & R_{11} & T_{13} \\ R_{20} & R_{21} & R_{21} & T_{23} \\ R_{30} & R_{31} & R_{31} & T_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Kuka

Aruco

Get registration matrix for each of the 5 trials used for repeatability.

➔ Get their combined component means and std.

➔ Compare with ground truth

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Mean matri

Not that intuitive!

- G   ment mean:

$$T^{aruco}_{kuka} mean = \frac{1}{n} \begin{bmatrix} \sum_i^n R_{00} & \sum_i^n R_{01} & \sum_i^n R_{02} & T_{03} \\ R_{10} & \sum_i^n R_{11} & \sum_i^n R_{11} & \sum 13 \\ \sum & \sum_i^n R_{21} & \sum_i^n R_{21} & \sum 23 \\ \sum_i & \sum_i^n R_{31} & \sum_i^n R_{31} & \sum 23 \\ 0 & 0 & 0 & \end{bmatrix}$$

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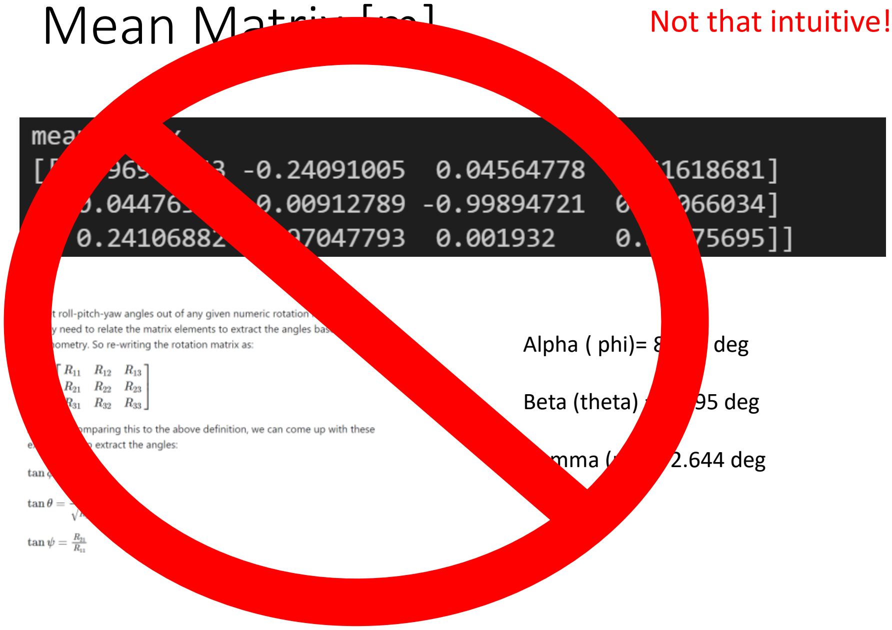
Mean Matri

Not that intuitive!



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Change of Approach

- Rotation matrix elements are not intuitive➔ **get equivalent roll,pitch, yaw angles for analysis**
- Get row, pitch, yaw for every single trial
- Get mean and standard deviation
- Mean, and Standard deviation of tx,ty,tz

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Rotation R convention

R= product of "3 elemental rotation matrices":

ZYX Euler angles (roll, pitch, yaw)

Tait–Bryan angles

$$X_1 Z_2 Y_3 = \begin{bmatrix} c_2 c_3 & -s_2 & c_2 s_3 \\ s_1 s_3 + c_1 c_3 s_2 & c_1 c_2 & c_1 s_2 s_3 - c_3 s_1 \\ c_3 s_1 s_2 - c_1 s_3 & c_2 s_1 & c_1 c_3 + s_1 s_2 s_3 \end{bmatrix}$$

$$X_1 Y_2 Z_3 = \begin{bmatrix} c_2 c_3 & -c_2 s_3 & s_2 \\ c_1 s_3 + c_3 s_1 s_2 & c_1 c_3 - s_1 s_2 s_3 & -c_2 s_1 \\ s_1 s_3 - c_1 c_3 s_2 & c_3 s_1 + c_1 s_2 s_3 & c_1 c_2 \end{bmatrix}$$

$$Y_1 X_2 Z_3 = \begin{bmatrix} c_1 c_3 + s_1 s_2 s_3 & c_3 s_1 s_2 - c_1 s_3 & c_2 s_1 \\ c_2 s_3 & c_2 c_3 & -s_2 \\ c_1 s_2 s_3 - c_3 s_1 & c_1 c_3 s_2 + s_1 s_3 & c_1 c_2 \end{bmatrix}$$

$$Y_1 Z_2 X_3 = \begin{bmatrix} c_1 c_2 & s_1 s_3 - c_1 c_3 s_2 & c_3 s_1 + c_1 s_2 s_3 \\ s_2 & c_2 c_3 & -c_2 s_3 \\ -c_2 s_1 & c_1 s_3 + c_3 s_1 s_2 & c_1 c_3 - s_1 s_2 s_3 \end{bmatrix}$$

$$\boxed{Z_1 Y_2 X_3 = \begin{bmatrix} c_1 c_2 & c_1 s_2 s_3 - c_3 s_1 & s_1 s_3 + c_1 c_3 s_2 \\ c_2 s_1 & c_1 c_3 + s_1 s_2 s_3 & c_3 s_1 s_2 - c_1 s_3 \\ -s_2 & c_2 s_3 & c_2 c_3 \end{bmatrix}}$$

1. 1, 2, 3 represent the angles α, β and γ, i.e. the angles corresponding to the first, second and third elemental rotations respectively.
2. X, Y, Z are the matrices representing the elemental rotations about the axes x, y, z of the fixed frame (e.g., X_1 represents a rotation about x by an angle α).
3. s and c represent sine and cosine (e.g., s_1 represents the sine of α).

https://en.wikipedia.org/wiki/Euler_angles

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Roll, pitch, yaw Stats in deg

```
----- Mean Roll, pitch, yaw in Degree---------
[-89.889168     2.61635045 -13.95553427]
----- STD Roll, pitch, yaw in Degree---------
[0.12450735 0.20097041 0.4041064 ]
```

- From each trial's registration matrix, get the roll, pitch, yaw angles (degrees) from its rotation matrix

➔ Calculate mean and std over the 5 trials

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Translation Stats in mm

```
----- Mean Translations [mm]---------
[-716.64789188 -653.13726675  465.03366962]
----- STD Translations [mm]---------
[4.52526638 2.21553389 1.77274879]
```

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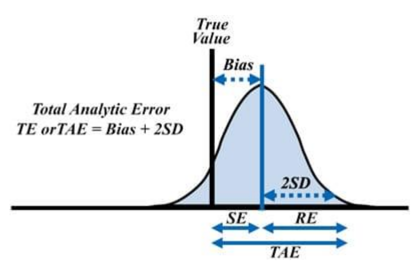
Bias and TAE

1) Calculate Bias for translation and angles
2) Calculated TAE

*** Ground Truth is the Calibration Registration Matrix, and its corresponding angles, derived from 24 trials!



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TAE

- Includes imprecision and trueness (bias) [1]
- TE (total error, a.k.a. total analytical error) – The sum of random error (imprecision) and systematic error (bias or inaccuracy). [2]

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Ground Truth (aruco→ kuka)

```
-----  Calibration Registration Roll, Pitch, Yaw in Degrees-------
(-89.94898561467923, 2.8006740273117106, -13.719622040602603)
-----  Calibration Registration Translation in mm-------
[-714.84221293 -655.0585569   465.64188876]
```

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TAE

Angles [deg]

```
------ANGLE BIAS Measurement in DEG---------
[-0.05981761  0.18432358  0.23591223]
------Total Analytical Error Angles in DEG---------
[0.18919709 0.5862644  1.04412503]
```

Translation [mm]

```
-------Translation BIAS Measurement in mm---------
[ 1.80567895 -1.92129015  0.60821914]
-------Total Analytical Error Translation in mm---------
[10.85621171  2.50977763  4.15371673]
```

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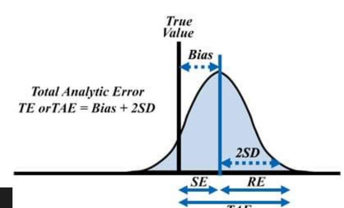
GT and TAE all together

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```

Takeaways: Comparing the single trial to 24 trials derived registration matrix, gives the corresponding bias and TAE.
BIAS & Error of rotational angles (roll, pitch, yaw) <1 deg
BIAS of translation <2mm. TEA: shows that translation is more imprecise with single trials (only 4 point coordinates)

➔ Realsense to kuka registration calibration should use more than 4 points to obtain the registration matrix, to reduce the TAE. TAE depends on the repeatability of the robot, as well as the noise of the aruco marker

➔ Evaluation of position and tracking will thus be done using the 24 trials-based registration matrix. Future recommendations: optimize the registration precision and accuracy .

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