

Calibration Quality

Comparing single trial registration matrix to "ground truth" registration matrix obtained from a combined 24 trials

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Kuka → Aruco Registration

- 29 calibration trials captures → **5 for repeatability** +24 for registration matrix
- JSON Calibration
- Get registration matrix (source=JSON, target=aruco X24 trials)

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Registration matrix (24 trials)

= GROUND TRUTH

```
----- LOAD Calibration Registration Matrix-----
----- Calibration Registration Matrix-----
[[ [ 9.70307596e-01 -2.36887572e-01 4.88615197e-02 5.15689627e-01]
   [ 4.76785513e-02 -1.07235591e-02 -9.98805167e-01 4.92143606e-01]
   [ 2.37128500e-01 9.71477886e-01 8.89305382e-04 8.05470266e-01]
   [ 0.00000000e+00 0.00000000e+00 0.00000000e+00 1.00000000e+00]]
```

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Repeatability of Registration

- Take 5 Trials (4 positions each)
- Calculate their individual registration matrix T
- Get mean and std of each T component (for Rotation and translation matrices)
- Calculate their respective bias
- Calculate their respective TAE: total analytical error

***NOTE ALL IN standard units (Meters)

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$$T_{kuka} = \begin{bmatrix} R_{00} & R_{01} & R_{02} & T_{03} \\ R_{10} & R_{11} & R_{12} & T_{13} \\ R_{20} & R_{21} & R_{22} & T_{23} \\ R_{30} & R_{31} & R_{32} & T_{33} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Registration matrix obtained from 24 trials = "Ground Truth"

$$T_{kuka, trial=i} = \begin{bmatrix} R_{00} & R_{01} & R_{02} & T_{03} \\ R_{10} & R_{11} & R_{12} & T_{13} \\ R_{20} & R_{21} & R_{22} & T_{23} \\ R_{30} & R_{31} & R_{32} & T_{33} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Get registration matrix for each of the 5 trials used for repeatability.

- Get their combined component means and std.
- Compare with ground truth

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Mean matrix

Not that intuitive!

- Get component mean:

$$T_{kuka, mean} = \frac{1}{n} \begin{bmatrix} \sum_{i=1}^n R_{00} & \sum_{i=1}^n R_{01} & \sum_{i=1}^n R_{02} & \sum_{i=1}^n T_{03} \\ \sum_{i=1}^n R_{10} & \sum_{i=1}^n R_{11} & \sum_{i=1}^n R_{12} & \sum_{i=1}^n T_{13} \\ \sum_{i=1}^n R_{20} & \sum_{i=1}^n R_{21} & \sum_{i=1}^n R_{22} & \sum_{i=1}^n T_{23} \\ \sum_{i=1}^n R_{30} & \sum_{i=1}^n R_{31} & \sum_{i=1}^n R_{32} & \sum_{i=1}^n T_{33} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Mean Matrix [rad]

Not that intuitive!

mean =

```

[[ 0.956233 -0.24091005  0.04564778  1.618681]
 [ 0.044768  0.00912789 -0.99894721  0.0466034]
 [ 0.24106882  0.7047793  0.001932  0.75695]]

```

Roll (phi) = 5.0 deg
Pitch (theta) = 25.0 deg
Yaw (gamma) = 2.644 deg

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- Rotation matrix elements are not intuitive → get equivalent roll, pitch, yaw angles for analysis
- Get roll, pitch, yaw for every single trial
- Get mean and standard deviation
- Mean, and Standard deviation of tx, ty, tz

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Rotation R convention

R= product of "3 elemental rotation matrices":

ZYX Euler angles (roll, pitch, yaw)

Tait-Bryan angles

$$X_1 Z_0 Y_1 = \begin{bmatrix} c_1 c_2 & -s_1 c_2 & s_1 s_2 \\ s_1 c_2 + c_1 s_2 & c_1 c_2 & c_1 s_2 - s_1 s_2 \\ c_1 s_2 - c_1 s_2 & c_1 s_2 & c_1 s_2 + s_1 s_2 \end{bmatrix}$$

$$X_0 Y_1 Z_1 = \begin{bmatrix} c_2 c_3 & -s_2 c_3 & s_2 s_3 \\ c_1 c_3 + c_2 s_3 & c_1 s_3 - c_2 s_3 & -s_1 s_3 \\ c_1 s_3 - c_2 s_3 & c_1 c_3 + c_2 s_3 & c_1 s_3 \end{bmatrix}$$

$$Y_1 X_1 Z_1 = \begin{bmatrix} c_1 c_3 + c_2 s_3 & c_1 s_3 - c_2 s_3 & -s_1 s_3 \\ c_1 s_3 - c_2 s_3 & c_1 c_3 + c_2 s_3 & c_1 s_3 \\ c_2 c_3 & -s_2 c_3 & s_2 s_3 \end{bmatrix}$$

$$Y_1 Z_0 X_1 = \begin{bmatrix} c_1 c_2 & s_1 c_2 & s_2 \\ c_1 s_2 & s_1 s_2 & c_2 \\ -s_2 & c_2 & c_1 \end{bmatrix}$$

$$Z_0 Y_1 X_1 = \begin{bmatrix} c_1 c_2 & c_1 s_2 & -c_2 \\ c_2 c_3 & c_2 s_3 & -s_1 \\ -s_2 & c_2 & c_1 \end{bmatrix}$$

- 1, 2, 3 represent the angles α , β and γ , i.e. the angles corresponding to the first, second and third elemental rotations respectively.
- X, Y, Z are the matrices representing the elemental rotations about the axes x, y, z of the fixed frame (e.g., X_1 represents a rotation about x by an angle α).
- a and b represent yaw and course (e.g., a_1 represents the yaw of a).

https://en.wikipedia.org/wiki/Euler_angles

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Roll, pitch, yaw Stats in deg

```
----- Mean Roll, pitch, yaw in Degree-----
89.88598465 13.94998535 2.64366285
----- STD Roll, pitch, yaw in Degree-----
0.12825733 0.40497782 0.23282181
```

- From each trial's registration matrix, get the roll, pitch, yaw angles (degrees) from its rotation matrix

→ Calculate mean and std over the 5 trials

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Translation Stats in mm

```
----- Mean Translations [mm]-----
[516.18681283 490.66033575 805.75694641]
----- STD Translations [mm]-----
[1.85007297 1.77898862 3.17776836]
```

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Not that intuitive!

Rotation matrix

```
0.00168304 0.001684352 0.00350377 0.00185
0.00393204 0.0016917 0.00016885 0.00177
0.00686226 0.0016917 0.0021703 0.003177
```

which yaw angles out of any given matrix rotation matrix, you calculate the matrix elements to extract the angles based on the following definitions:

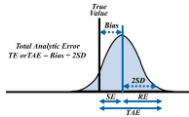
Alpha (phi) = 66.83 deg
 Beta (theta) = 68.14 deg
 Gamma (psi) = 66.83 deg

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TAE

- 1) Calculate Bias for translation and angles
- 2) Calculated TAE

*** Ground Truth is the Calibration Registration Matrix, and its corresponding angles, derived from 24 trials!



TAE

- Includes imprecision and trueness (bias) [1]
- TE (total error, a.k.a. total analytical error) – The sum of random error (imprecision) and systematic error (bias or inaccuracy). [2]

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Ground Truth

```
----- Calibration Registration Roll, Pitch, Yaw in Degrees-----
(89.9475506014213, 13.717123869291916, 2.813112386178329)
----- Calibration Registration Translation in mm-----
[515.6896274 492.14360628 805.47026631]
```

TAE

Angles [deg] Translation [mm]

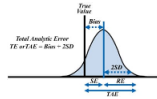
```
----- Angle Bias Measurement in DEG-----
[ 0.00150995 0.11286348 0.1094995]
----- Total Analytical Error Angles in DEG-----
[0.1180802 0.5770627 0.61362015]

----- Translation Bias Measurement in mm-----
[-0.49719544 1.48327093 -0.2866801 ]
----- Total Analytical Error Translation in mm-----
[3.2809605 5.04124778 6.0686963]
```

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GT and TAE all together



```
----- Calibration Registration Roll, Pitch, Yaw in Degrees-----
(89.9475506014213, 13.717123869291916, 2.813112386178329)
----- Calibration Registration Translation in mm-----
[515.6896274 492.14360628 805.47026631]
```

```
----- ANGLES BIAS Measurement in DEG-----
[ 0.00150995  0.2028848  0.10040951]
-----Total Analytical Error Angles in DEG-----
[0.21000061 0.57709257 0.63340315]

-----Translation BIAS Measurement in mm-----
[ 0.4072354  1.4822703  -0.2860062 ]
-----Total Analytical Error Translation in mm-----
[1.2026005  5.04114778  6.06805063]
```

Takeaways:

Both translation and rotation obtained from single trial (only 4-point 3D coordinates) have a low bias
TAE of Translation is however higher

- Improved kuka to realsense calibration should use more than 4 points to obtain the registration matrix.
- Evaluation of position and tracking will thus be done using the 24 trials based registration matrix. Future recommendations: optimize the registration precision and accuracy .