Regression Analysis

Chapter 0. Introduction

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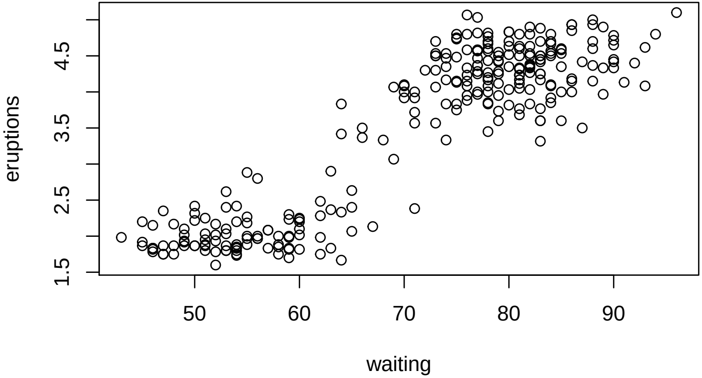
Practical information

- Classes and computer sessions (3, possibly a 4th will be added)
- PC sessions will focus on the use of R in regression analysis
- Additional exercises will be provided to do at home (not graded)
- Material will be provided on Toledo
- Suggested reference: Applied Linear Statistical Models, 5th Edition, Kutner et al. (2005)

Practical information

- Evaluation
  - Open book exam: course notes + slides
  - Focus on correct application of regression analysis
  - Individual written project at the end of semester involving data analysis tasks
  - Oral exam with written preparation + possibly questions related to the project
  - Final grade: project grade * 0.35 + exam result * 0.65
  - Second chance exam: opportunity to write a new report (new data and questions)

How can one best capture the relationship between variables?



Simple regression

- A method to model the **relation** between an input variable X and an output or *response* variable Y
  - **Asymmetric**
  - To what extent does the outcome Y **change** due to a change in the value of X?
  - **Predict** Y from X
  - X is the *independent* variable, or *regressor*, Y is the *dependent* variable.

Regression: a more general definition

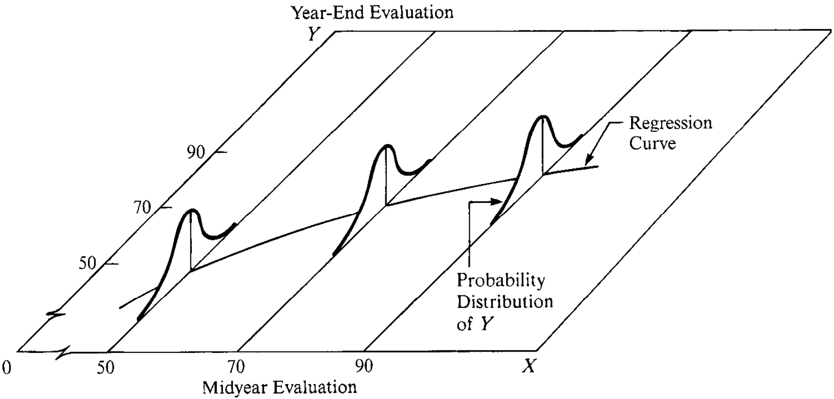
- Regression analysis models the relationship between a set of predictor variables $X_1, X_2, \ldots, X_{l-1}$ and a response variable Y that are measured on n observations.
- Find a **relation** between the X_j ($j = 1, \ldots, l-1$) and Y, which reveals the joint influence of the X-variables on Y.
- Predict the dependent variable Y from the independent variables $X_1, \ldots, X_{l-1}$
- In a very general form, we seek real functions g, f and a parameter vector $\boldsymbol{\beta} = (\beta_0, \ldots, \beta_{p-1})^t$ such that $g(Y)$ can be well described by $f(X_1, \ldots, X_{l-1}, \boldsymbol{\beta})$.
- Unless otherwise stated, we will assume that the response variable is *continuous*.

Regression: a more general definition

- The observations will in general not satisfy this functional relation exactly
- A **stochastic component** ϵ **expresses the variation of the data points around the regression curve**.
- A regression model thus postulates that:

1. There is a probability distribution of Y for each level of $X = (X_1, \dots, X_{I-1})$.
2. The means of these probability distributions vary in some systematic fashion with X.

FIGURE 1.4 **Pictorial Representation of Regression Model.**



General linear regression Model:

$$g(y_i) = \beta_0 + \beta_1 f_1(x_{i1}, \ldots, x_{i,l-1}) + \ldots + \beta_{p-1} f_{p-1}(x_{i1}, \ldots, x_{i,l-1}) + \epsilon_i$$

for $i = 1, \ldots, n$ and for certain choices of $g, f_1, \ldots, f_{p-1}$. The error terms ϵ_i represent the random variation of the data points around the regression curve. We assume that the standard Gauss-Markov conditions are satisfied:

$$\begin{aligned} \mathrm{E}[\epsilon_i] &= 0 \\ \mathrm{Var}[\epsilon_i] &= \sigma^2 \\ \mathrm{E}[\epsilon_i \epsilon_j] &= 0 \text{ for all } i \neq j. \end{aligned}$$

This general linear model includes:

1. The first-order regression model:
$l = p, g(y_i) = y_i, f_j(x_{i1}, \ldots, x_{i,p-1}) = x_{ij} \;\; (j = 1, \ldots, p-1)$
$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_{p-1} x_{i,p-1} + \epsilon_i.$$
2. Simple regression: the first-order regression model with $p = 2$.
3. Polynomial regression: $g, f_1, \ldots, f_{l-1}$ as in the first-order regression model, and e.g. additionally
$f_l = X_1^2, f_{l+1} = X_3^2, f_{l+2} = X_1 X_2$.
4. Variable selection: $g, f_1, \ldots, f_{l-3}$ as in the first-order regression model, all other $f_j = 0$.
5. Transformations in X or Y:
$g(Y) = \log(Y), g(Y) = \frac{y^\lambda - 1}{\lambda}, f_j = \log(X_j)$.

- Linear models are linear in $\boldsymbol{\beta}$ and not necessarily in the independent variables X_j!
- An example of a nonlinear model is

$$y_i = \beta_0 + \beta_1 e^{\beta_2 x_i} + \epsilon_i.$$

Course contents

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2. The general linear model
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4. Polynomial regression
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8. Multicollinearity
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10. Nonlinear regression
11. Nonparametric regression