Regression Analysis Final Project Code Apendix

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2019-2020

Loading data, neccesary libraries, and partitioning data into training/test sets

```
rm(list=ls())
library(ggplot2)
library(MASS)
library(car)
library(lmtest)
library(olsrr)
library(mctest)
library(ppcor)
library(GGally)
library(dplyr)
library(Metrics)
library(ggeffects)
```

Code for tasks 1-5

Creating training and test data sets

```
data=read.table("C:/Users/kebro/OneDrive/KU Leuven/Regression/invertebrate.txt",header=T)
rnum=0773111
set.seed(rnum)
d.test <- sample(1:dim(data)[1], 200)
data.test <- data[d.test, ]
data.training <- data[-d.test, ]
n <- dim(data.training)[1]
p <- dim(data.training)[2]
```

Descriptive Analysis of training data

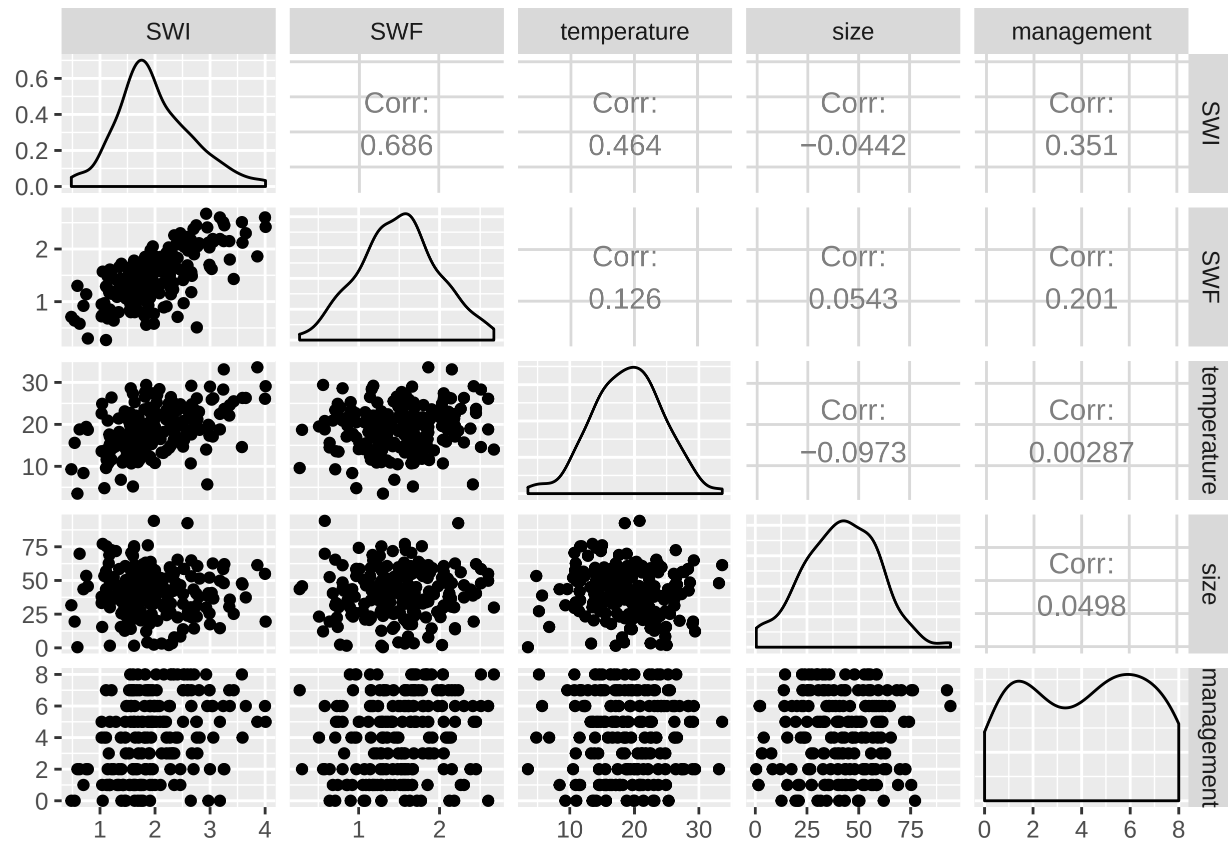
```
summary(data.training)
```

```
##       SWI             SWF          temperature         size      
##  Min.   :0.480   Min.   :0.270   Min.   : 3.50   Min.   : 0.50  
##  1st Qu.:1.567   1st Qu.:1.177   1st Qu.:15.38   1st Qu.:29.23  
##  Median :1.895   Median :1.525   Median :19.05   Median :42.10  
##  Mean   :1.994   Mean   :1.498   Mean   :18.95   Mean   :41.32  
##  3rd Qu.:2.357   3rd Qu.:1.802   3rd Qu.:22.52   3rd Qu.:54.70  
##  Max.   :4.010   Max.   :2.670   Max.   :33.60   Max.   :94.20  
##    management       duration    
##  Min.   :0.000   Min.   :19.00  
##  1st Qu.:2.000   1st Qu.:30.00  
##  Median :4.000   Median :33.00  
##  Mean   :4.115   Mean   :32.75  
##  3rd Qu.:6.000   3rd Qu.:37.00  
##  Max.   :8.000   Max.   :44.00  
```

```r
str(data.training)
```

```
## 'data.frame':    200 obs. of  6 variables:
##  $ SWI        : num  0.59 1.08 1.6 2.95 1.38 0.7 0.48 1.11 1.14 1.57 ...
##  $ SWF        : num  1.3 0.97 1.67 2.41 1.44 0.92 0.71 0.27 1.48 1.65 ...
##  $ temperature: num  3.5 4.8 5.2 5.7 6.8 8.4 9.3 9.6 10.5 10.7 ...
##  $ size       : num  0.5 53.3 27.2 38.8 15.5 43.5 31.6 43.6 52 70.5 ...
##  $ management : int  2 4 8 6 4 1 0 7 2 7 ...
##  $ duration   : int  20 22 20 22 19 24 22 26 25 25 ...
```

```r
ggpairs(data.training[,-ncol(data.training)])
```



From the ggpairs plot all data seems normally distributed with the exception of management, generally weak corelations between variables

Fitting a first order linear model predicting SWI as a function of all variables -duration

```r
fit.basic=lm(SWI~.-duration,data=data.training)
fit.basic.sum=summary(fit.basic)
fit.basic.sum
```

```
##
## Call:
## lm(formula = SWI ~ . - duration, data = data.training)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.02005 -0.27531 -0.02608  0.24343  1.53643
##
```

```
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.357059   0.147639  -2.418   0.0165 *
## SWF          0.834942   0.059745  13.975  < 2e-16 ***
## temperature  0.048518   0.005275   9.198  < 2e-16 ***
## size        -0.001927   0.001576  -1.222   0.2231
## management   0.063350   0.011409   5.553 9.13e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3961 on 195 degrees of freedom
## Multiple R-squared:  0.6694, Adjusted R-squared:  0.6626
## F-statistic:  98.7 on 4 and 195 DF,  p-value: < 2.2e-16
```

```r
#model coefficients and their 95% confidence intervals
coefficients(fit.basic)
```

```
##  (Intercept)          SWF  temperature         size   management
## -0.357058626  0.834941694  0.048517959 -0.001926504  0.063350391
```

```r
confint(fit.basic)
```

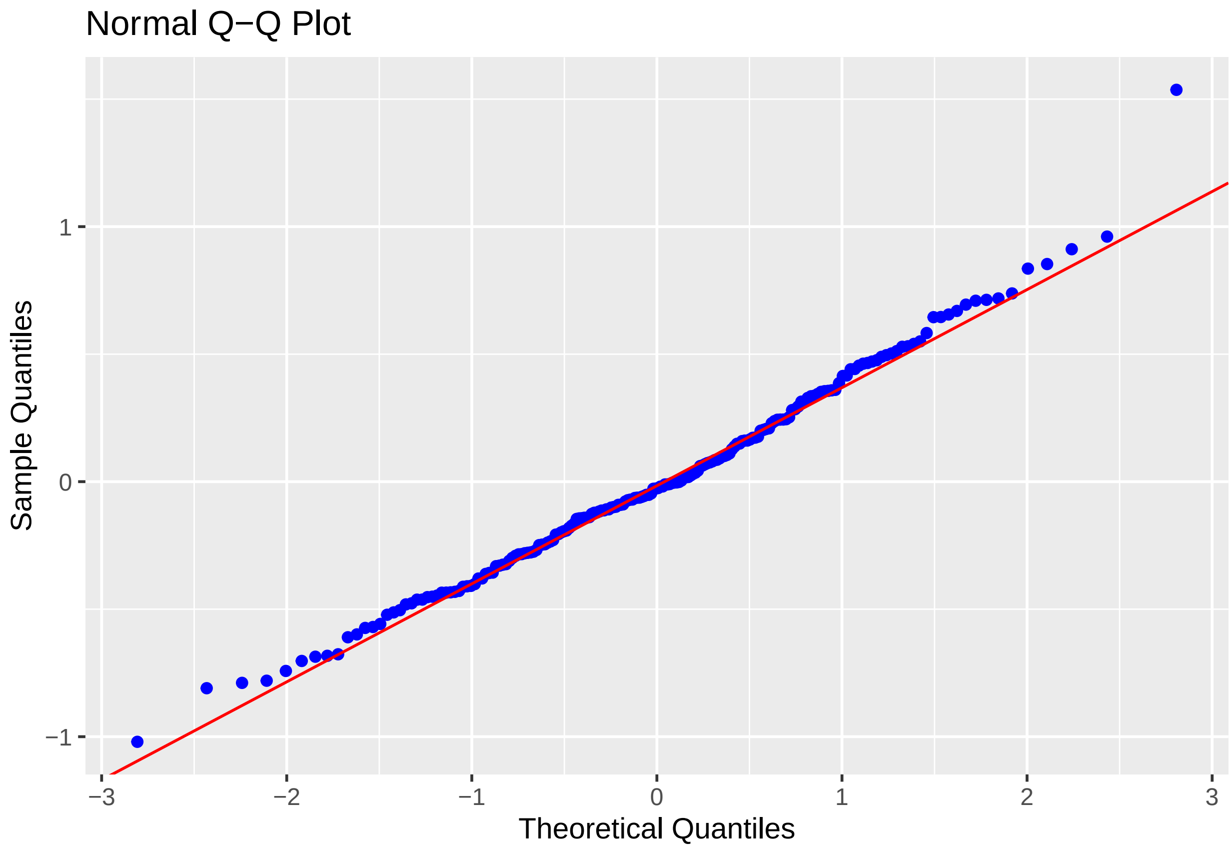
```
##                    2.5 %       97.5 %
## (Intercept) -0.648232700 -0.065884552
## SWF          0.717111387  0.952772002
## temperature  0.038114533  0.058921385
## size        -0.005035407  0.001182398
## management   0.040849086  0.085851697
```

```r
#ANOVA analysis of model
anova(fit.basic)
```

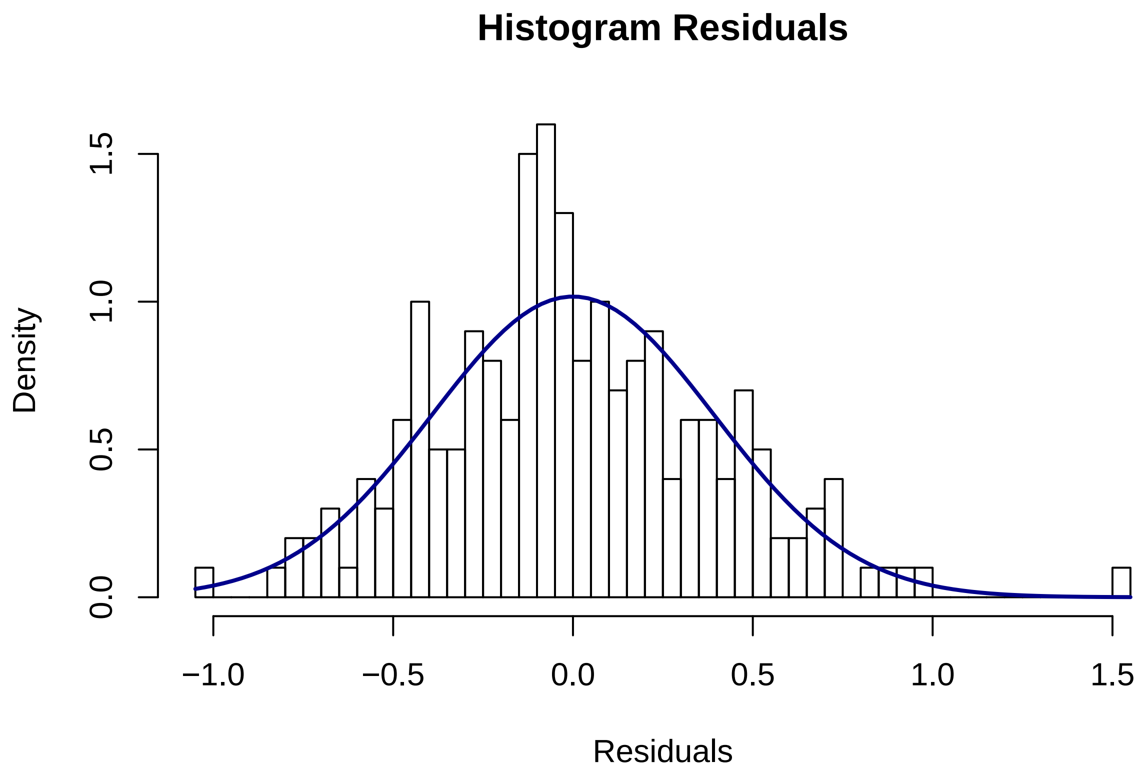
```
## Analysis of Variance Table
##
## Response: SWI
##              Df Sum Sq Mean Sq  F value    Pr(>F)
## SWF           1 43.515  43.515 277.3264 < 2.2e-16 ***
## temperature   1 13.431  13.431  85.5981 < 2.2e-16 ***
## size          1  0.161   0.161   1.0291    0.3116
## management    1  4.838   4.838  30.8309 9.126e-08 ***
## Residuals   195 30.597   0.157
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Testing model assumptions

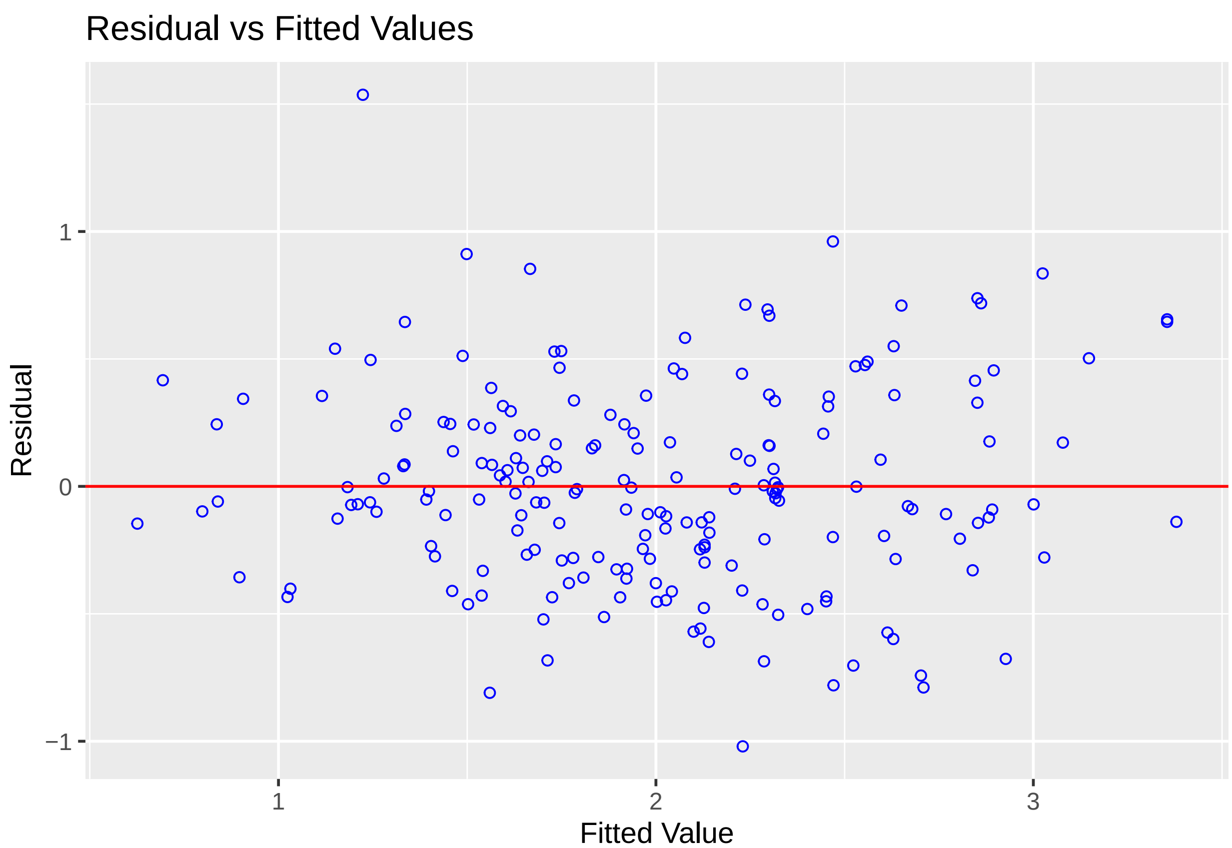
```r
#QQ-Plot and Histogram to visualize normality of errors
ols_plot_resid_qq(fit.basic)
```




```
hist(fit.basic$residuals,breaks = 50,
     xlab="Residuals", main="Histogram Residuals",
     probability = T)
curve(dnorm(x, mean=mean(fit.basic$residuals), sd=sd(fit.basic$residuals)),
      col="darkblue", lwd=2, add=TRUE, yaxt="n")
```




```r
##Visualiztion of Residuals vs. Fitted Values to evaluate homoscedasticity
ols_plot_resid_fit(fit.basic)
```



First order model shows possible non-constant variance, normality appears to hold true, linearity is met.

Formal Tests of assumptions

```
#Breusch-Pagan Test for homoscedasticity
ncvTest(fit.basic)
```

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 3.496839, Df = 1, p = 0.061486
```

```
#Normality of errors tests
ols_test_normality(fit.basic)
```

```
## -----------------------------------------------
##        Test             Statistic       pvalue
## -----------------------------------------------
## Shapiro-Wilk              0.9897         0.1589
## Kolmogorov-Smirnov        0.0513         0.6690
## Cramer-von Mises          29.964         0.0000
## Anderson-Darling          0.4314         0.3030
## -----------------------------------------------
```

```
ols_test_correlation(fit.basic)
```

```
## [1] 0.9939204
```

Tests for multi-colinearity

```r
#Covariance and correlation matracies of each predictor variable
cov(data.training[,c(-1,-6)])
```

```
##                   SWF temperature        size management
## SWF         0.2347857  0.32936558   0.4723405 0.24435930
## temperature 0.3293656 29.12542688  -9.4289621 0.03900251
## size        0.4723405 -9.42896206 322.2585023 2.24644975
## management  0.2443593  0.03900251   2.2464497 6.32339196
```

```r
cor(data.training[,c(-1,-6)])
```

```
##                    SWF  temperature        size  management
## SWF         1.00000000  0.125952345  0.05430217 0.200547852
## temperature 0.12595235  1.000000000 -0.09732525 0.002873964
## size        0.05430217 -0.097325254  1.00000000 0.049764479
## management  0.20054785  0.002873964  0.04976448 1.000000000
```

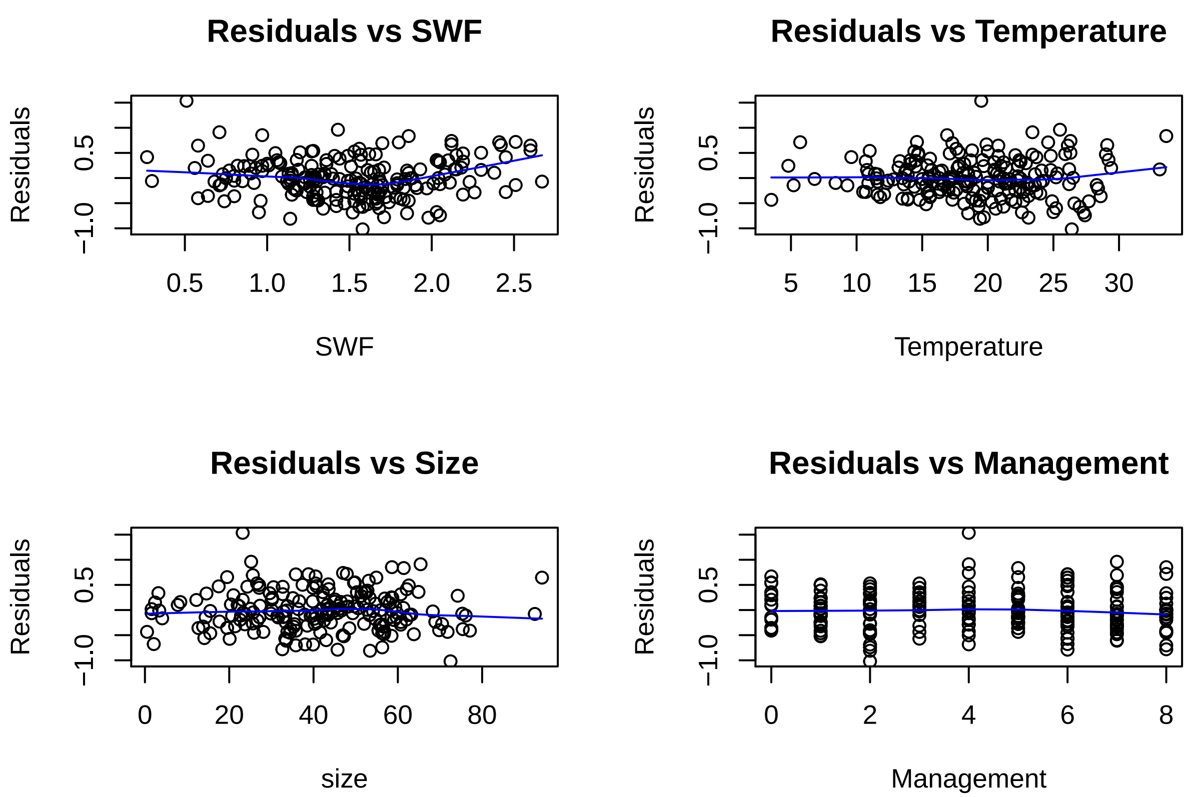
```r
#formal multicolinearity tests
omcdiag(data.training[,c(-1,-6)],data.training[,1])
```

```
## 
## Call:
## omcdiag(x = data.training[, c(-1, -6)], y = data.training[, 1])
## 
## 
## Overall Multicollinearity Diagnostics
## 
##                        MC Results detection
## Determinant |X'X|:         0.9295         0
## Farrar Chi-Square:        14.3796         1
## Red Indicator:             0.1088         0
## Sum of Lambda Inverse:     4.1503         0
## Theil's Method:           -1.8644         0
## Condition Number:         13.0682         0
## 
## 1 --> COLLINEARITY is detected by the test
## 0 --> COLLINEARITY is not detected by the test
```

```r
#no severe multicolinearity discovered
```

```r
par(mfrow=c(2,2))
#Checking linearity of SWF
plot(data.training$SWF,fit.basic$residuals,
     main="Residuals vs SWF",xlab="SWF",ylab="Residuals")
lines(lowess(fit.basic$residuals~data.training$SWF),col="blue")

#Checking linearity of Temperature
plot(data.training$temperature,fit.basic$residuals,
     main="Residuals vs Temperature",xlab="Temperature",ylab="Residuals")
lines(lowess(fit.basic$residuals~data.training$temperature),col="blue")

#Checking linearity of Size
plot(data.training$size,fit.basic$residuals,
     main="Residuals vs Size",xlab="size",ylab="Residuals")
lines(lowess(fit.basic$residuals~data.training$size),col="blue")
```

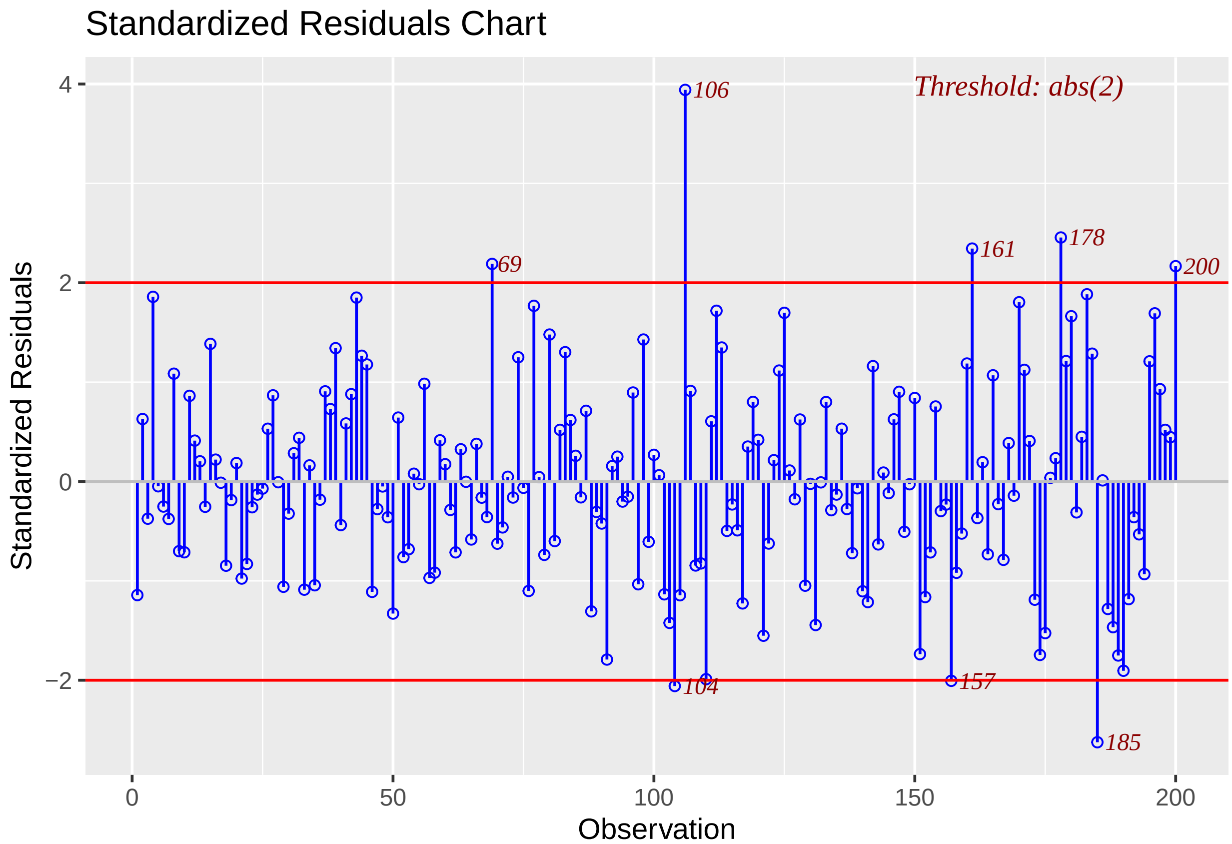
```r
#Checking linearity of Management
plot(data.training$management,fit.basic$residuals,
     main="Residuals vs Management",xlab="Management",ylab="Residuals")

lines(lowess(fit.basic$residuals~data.training$management),col="blue")
```



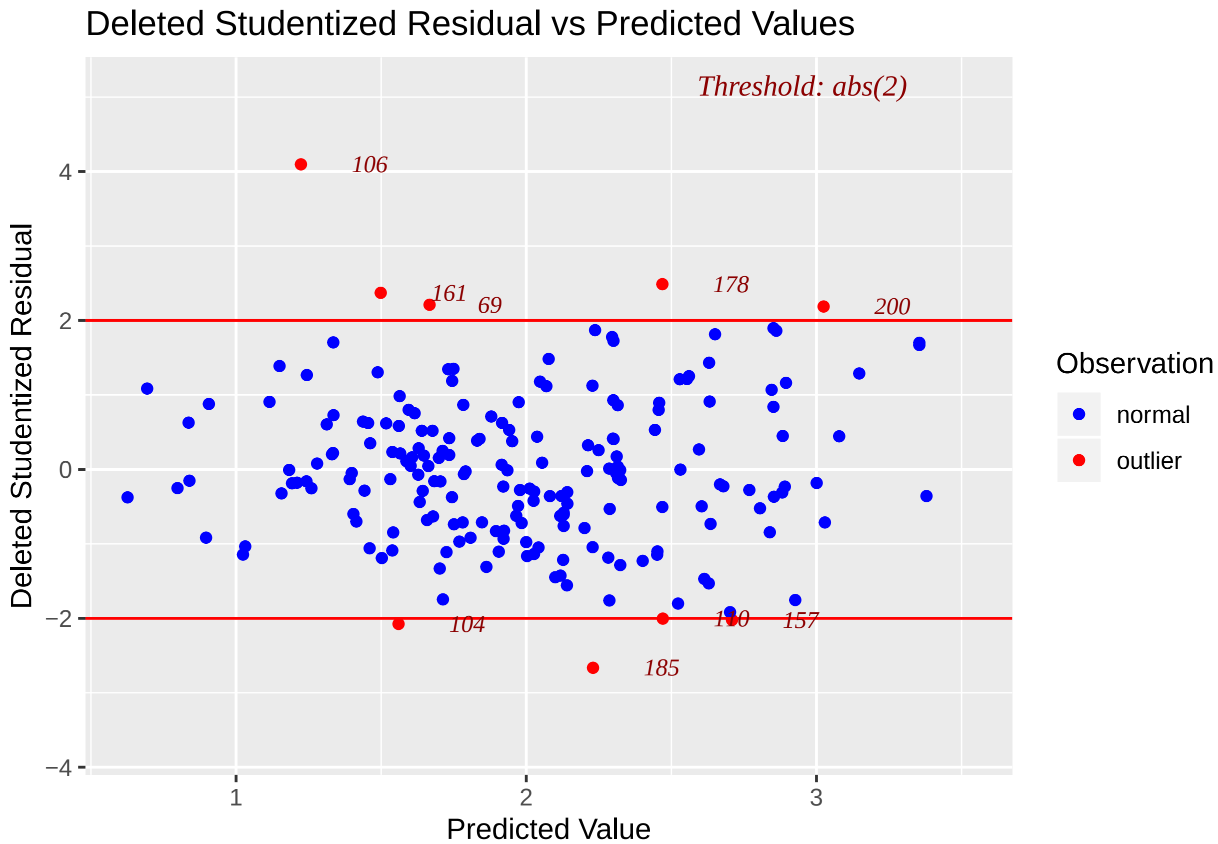
SWF appears to be nonlinear. Temperature and size may be non-linear as well. Managment is seemingly linear.

Checking for outliers and heavily influential observations

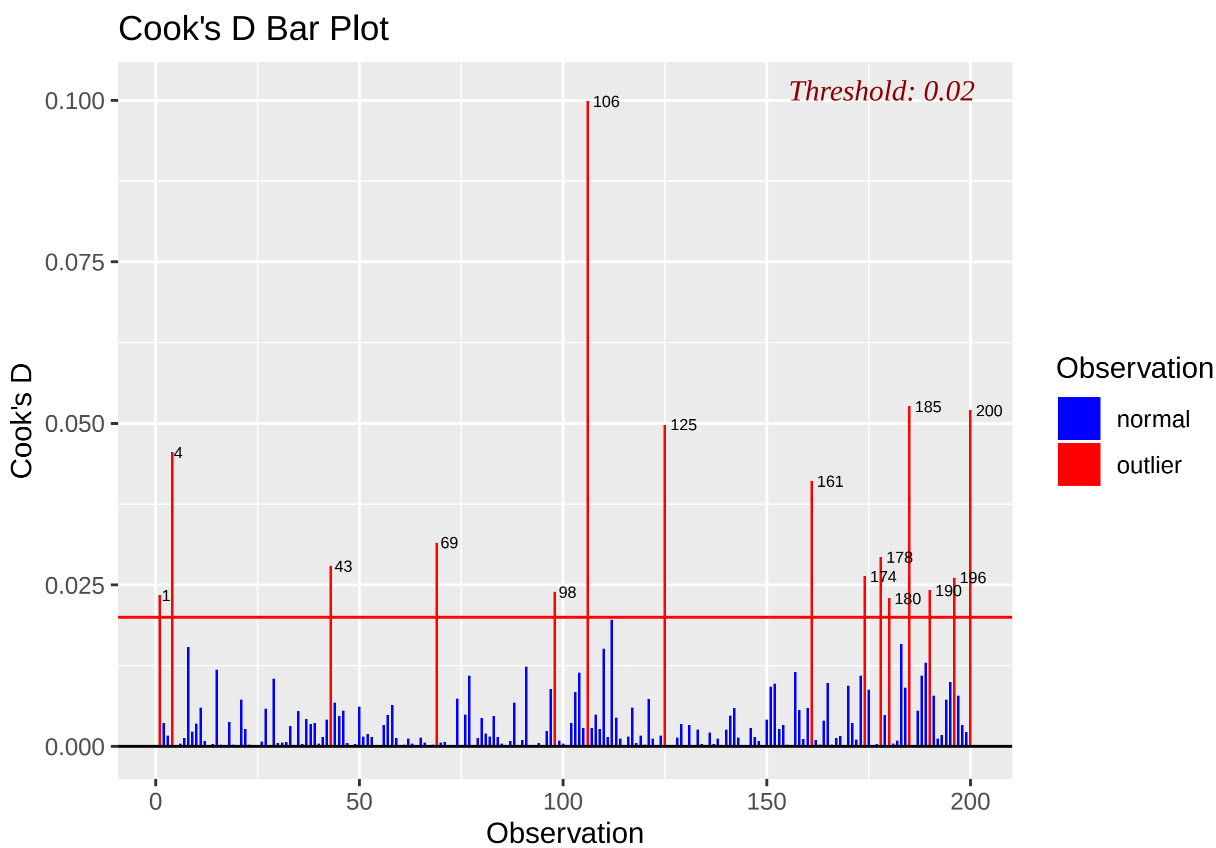
```r
mod=fit.basic
#Standardized Residual plot
ols_plot_resid_stand(mod)
```




```r
#studentized residual plot
ols_plot_resid_stud_fit(mod)
```




```
#cooks distance plot
ols_plot_cooksd_bar(mod)
```




```r
#DFBetas for each variable
ols_plot_dfbetas(mod)
```

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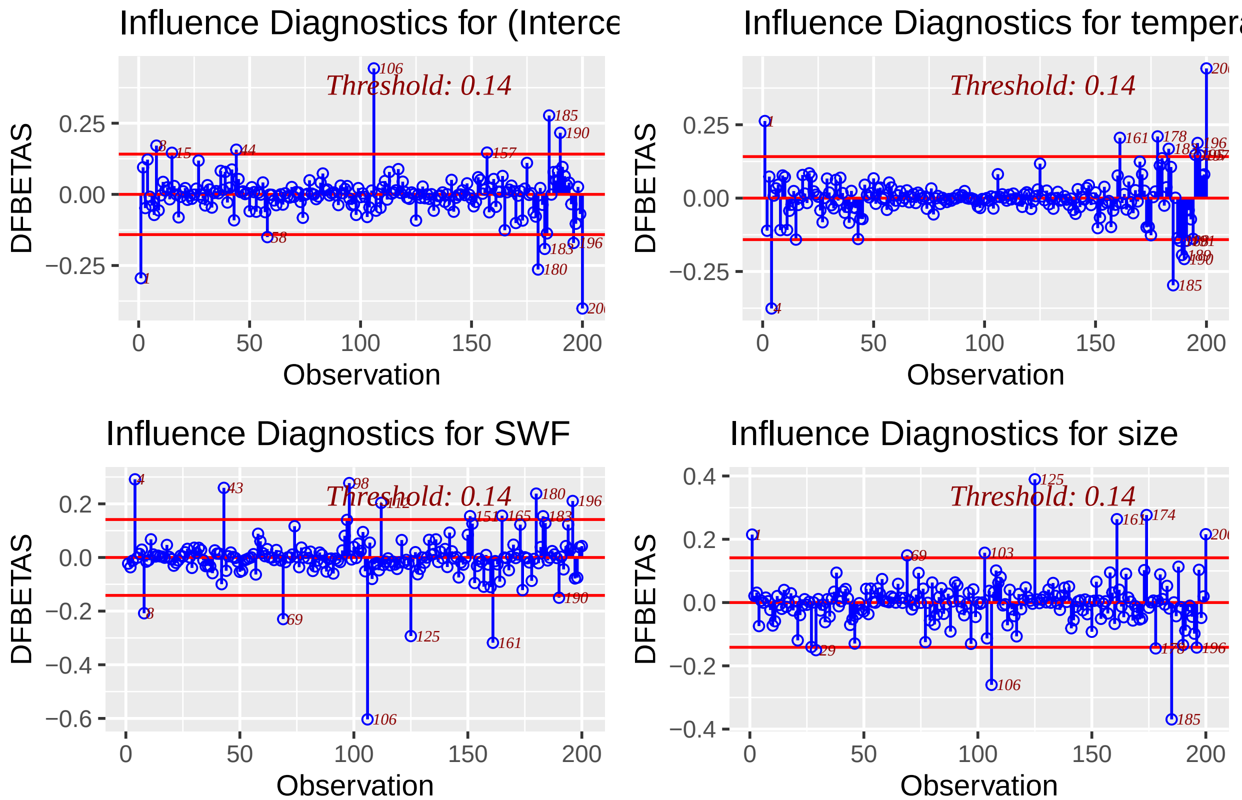
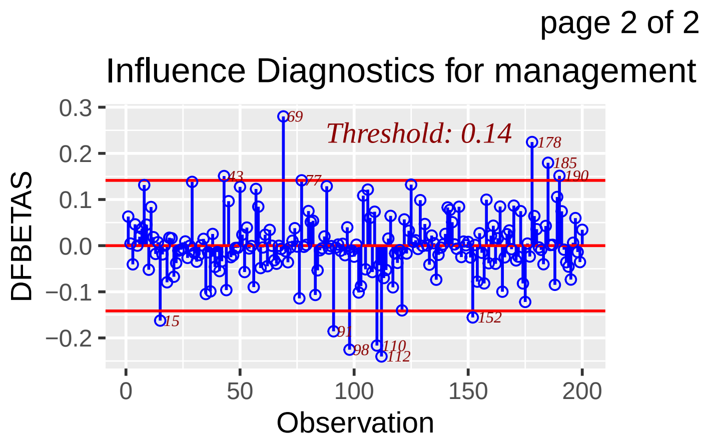
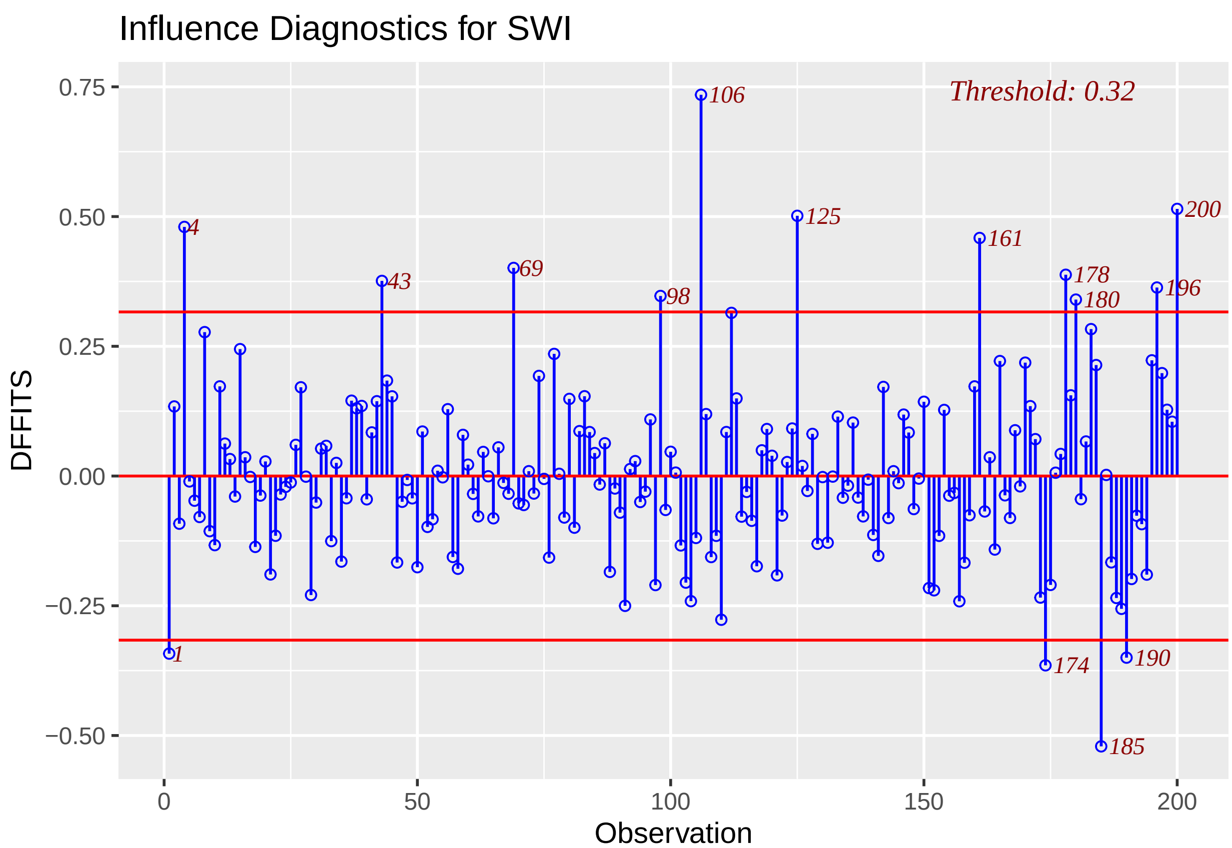
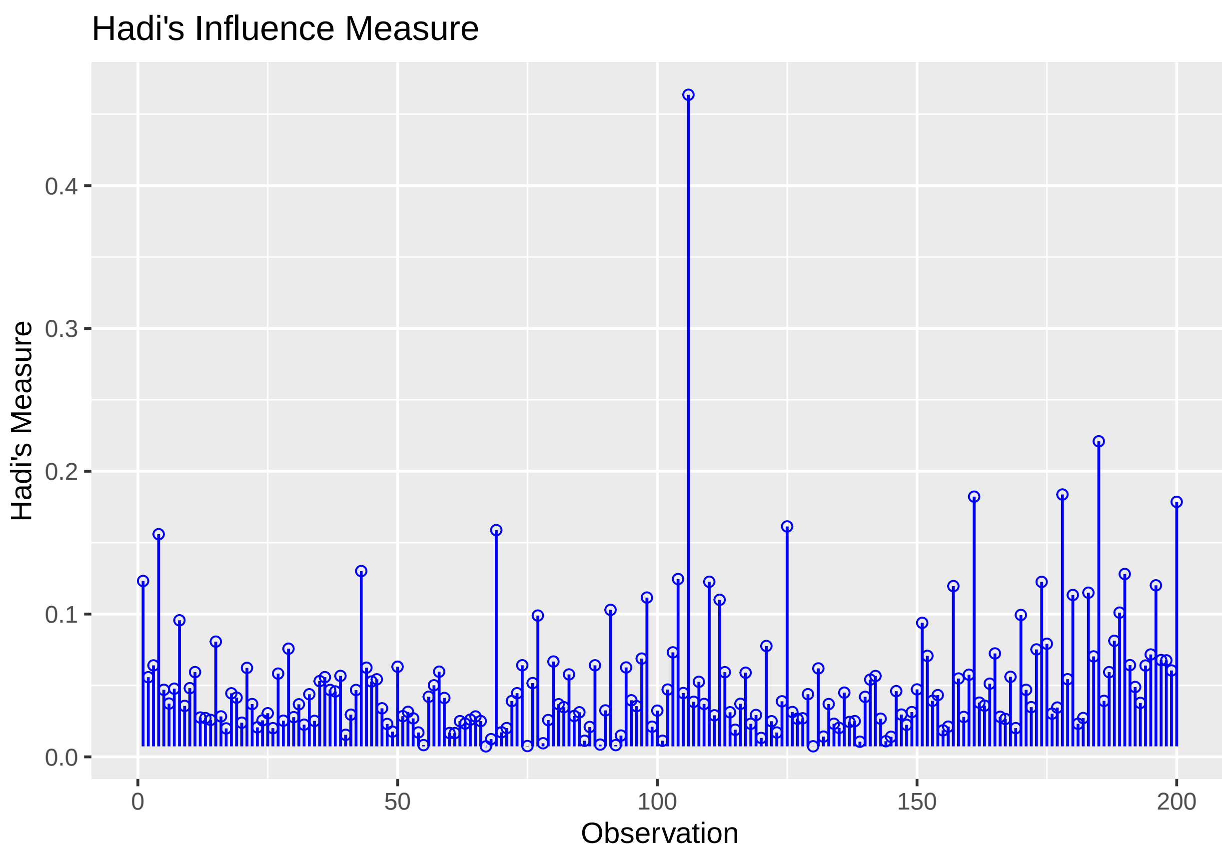

Influence Diagnostics for (Interce
Influence Diagnostics for tempera
Influence Diagnostics for SWF
Influence Diagnostics for size
Threshold: 0.14
DFBETAS
Observation




```
#Difference in fit chart for each sample
ols_plot_dffits(mod)
```




```r
#Plot for observation influence using hadi's distance
ols_plot_hadi(mod)
```



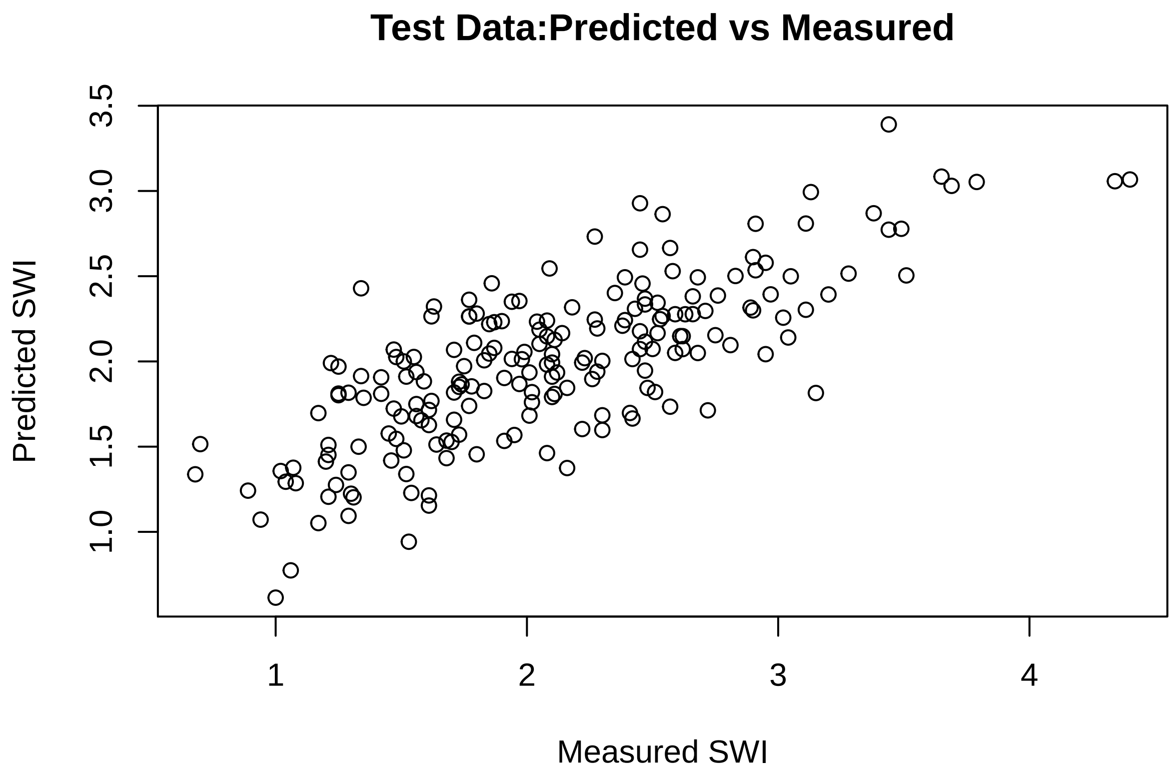
A several outliers are detected in sample index 69, 106, 161, 178, 185, and 200. Samples 104, 110, and 157 are potential outliers. Sample 169 proves to be highly influential

Evaluating test data performance with RMSE and a plot.

```
results.basic=predict(fit.basic,data.test)
rmse(results.basic,data.test$SWI)
```

```
## [1] 0.4481482
```

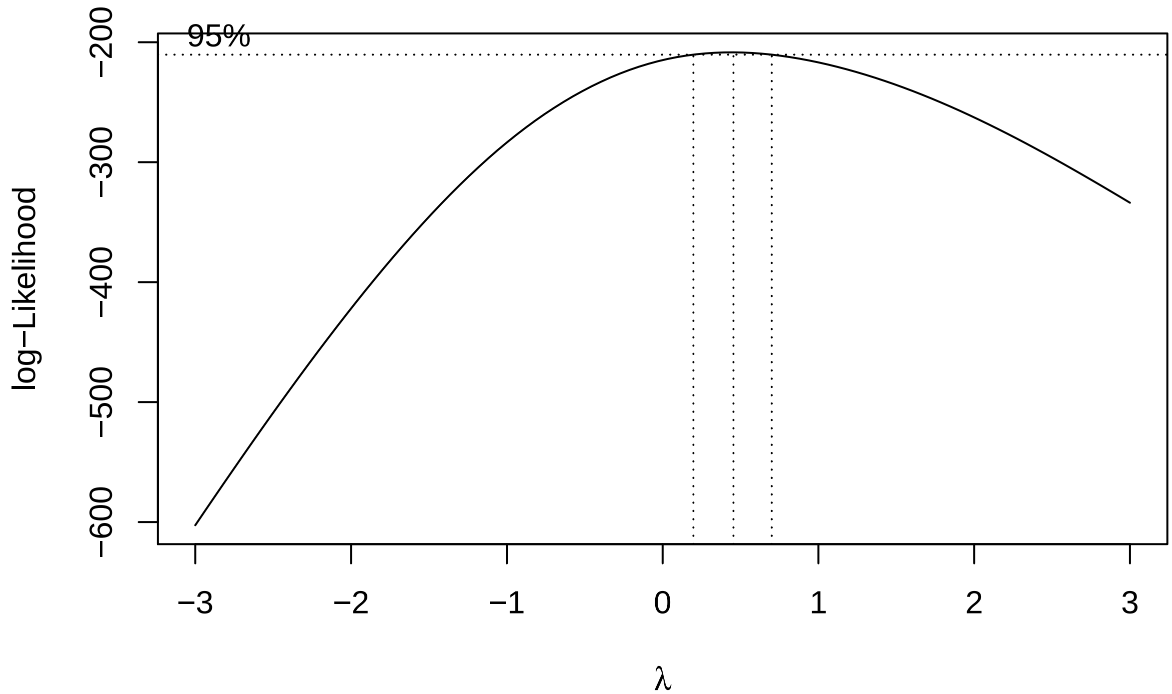
```
plot(data.test$SWI,results.basic,
     xlab="Measured SWI",ylab="Predicted SWI",main="Test Data:Predicted vs Measured")
```



The basic model has a poor mapping to the actual data. Problems with assumptions need to be adressed.

Box-Cox Transform to address homoscedasticity.Size removed because its insignificant, full model with interaction effects considered for ox cox transform

```
fullmodel=lm(SWI~SWF*temperature*management,data=data.training)
bc=boxcox(fullmodel,lambda = seq(-3,3))
```




```r
lambda=bc$x[which(bc$y==max(bc$y))]
#measured lambda
lambda
```

```
## [1] 0.4545455
```

```r
fit.bc=lm(SWI^lambda~SWF*temperature*management,data=data.training)
summary(fit.bc)
```

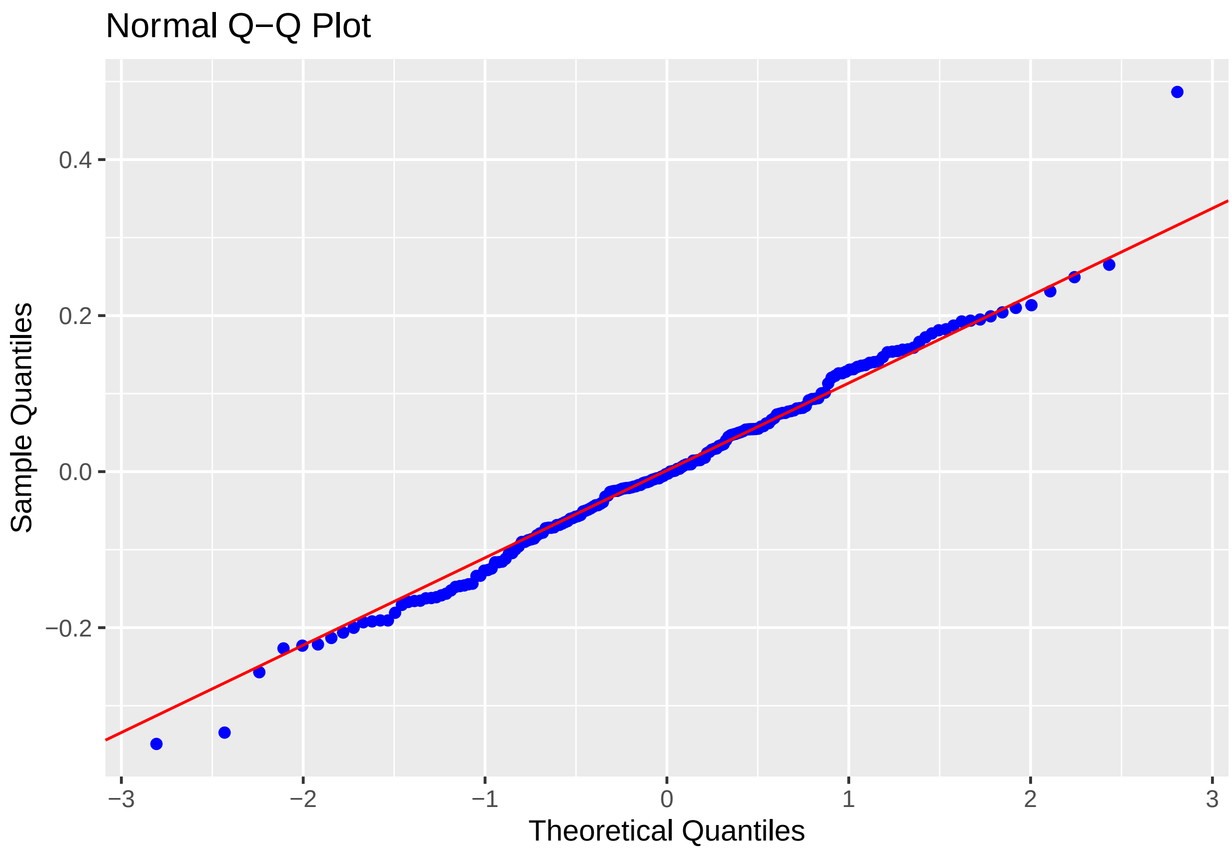
```
##
## Call:
## lm(formula = SWI^lambda ~ SWF * temperature * management, data = data.training)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.34903 -0.07397 -0.00286  0.07710  0.48664
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)                 0.260388   0.210552   1.237   0.2177
## SWF                         0.486445   0.150045   3.242   0.0014 **
## temperature                 0.027846   0.010756   2.589   0.0104 *
## management                  0.066891   0.041767   1.602   0.1109
## SWF:temperature            -0.008817   0.007499  -1.176   0.2411
## SWF:management             -0.032088   0.027864  -1.152   0.2509
## temperature:management     -0.001350   0.002187  -0.617   0.5379
## SWF:temperature:management  0.000984   0.001446   0.681   0.4970
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1238 on 192 degrees of freedom
## Multiple R-squared:  0.6804, Adjusted R-squared:  0.6688
## F-statistic:  58.4 on 7 and 192 DF,  p-value: < 2.2e-16
```

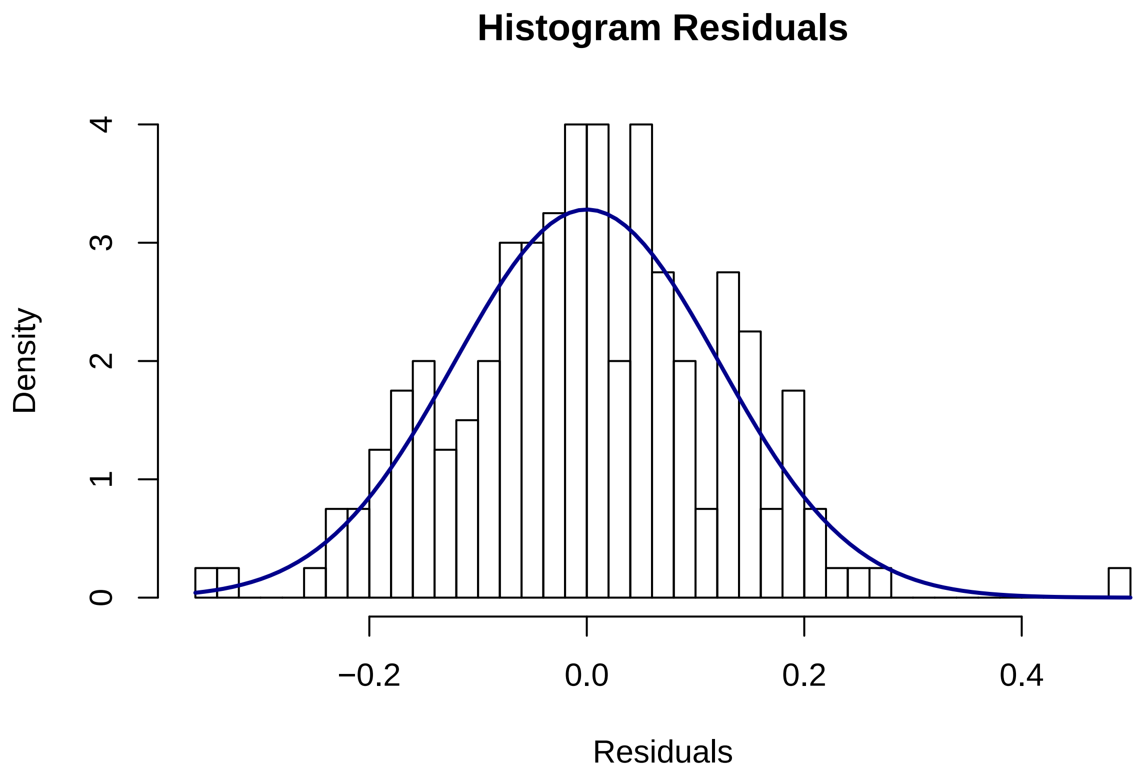
lambda of .45455

model assumptions for full interaction model

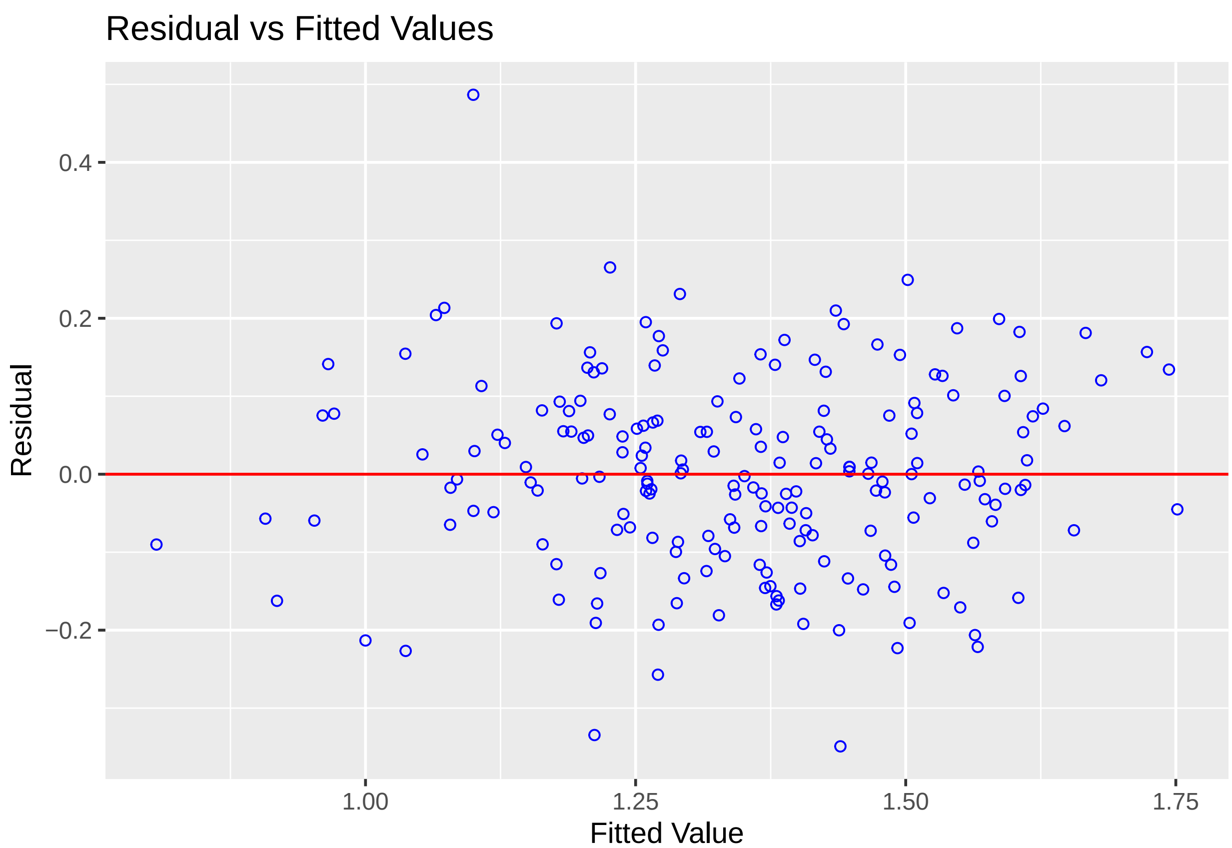
```r
#QQ-Plot and Histogram to visualize normality of errors
ols_plot_resid_qq(fit.bc)
```




```r
hist(fit.bc$residuals,breaks = 50,
     xlab="Residuals", main="Histogram Residuals",
     probability = T)
curve(dnorm(x, mean=mean(fit.bc$residuals), sd=sd(fit.bc$residuals)),
      col="darkblue", lwd=2, add=TRUE, yaxt="n")
```




```
##Visualiztion of Residuals vs. Fitted Values to evaluate homoscedasticity
ols_plot_resid_fit(fit.bc)
```




```
#Breusch-Pagan Test for homoscedasticity
ncvTest(fit.bc)
```

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 1.128714, Df = 1, p = 0.28805
```

```
#Normality of errors tests
ols_test_normality(fit.bc)
```

```
## -----------------------------------------------
##        Test             Statistic       pvalue
## -----------------------------------------------
## Shapiro-Wilk              0.9903         0.1960
## Kolmogorov-Smirnov        0.0402         0.9023
## Cramer-von Mises         51.8664         0.0000
## Anderson-Darling          0.2579         0.7154
## -----------------------------------------------
```

```
ols_test_correlation(fit.bc)
```

```
## [1] 0.993773
```

Variance is now constant. All assumptions are met.

Performing model trimming to only th most influential variables.

```
fit.bc.full=lm(SWI^lambda~SWF*temperature*management,data=data.training)
fit.bc.null=lm(SWI^lambda~1,data=data.training)
```

Stepwise variable selection

Fitting the forward selection. backwards elimination, and both-ways selection model

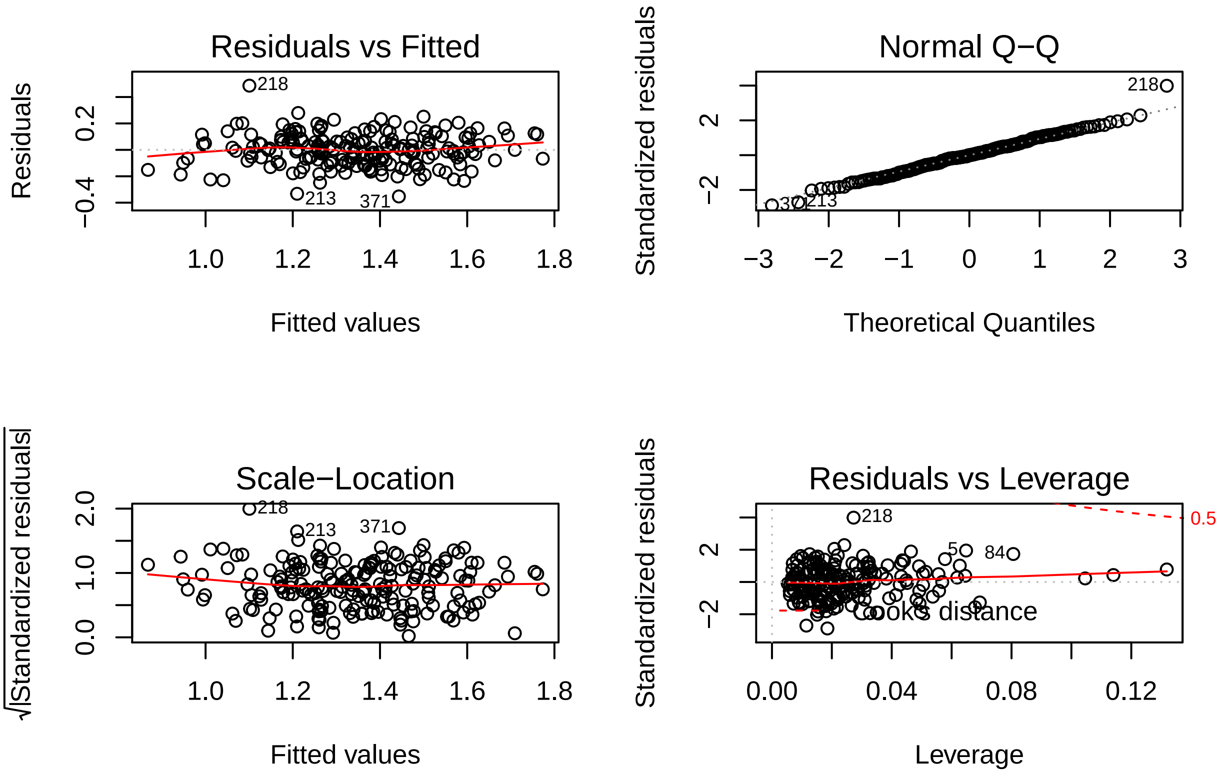
```
fit.bc.backwards=
    lm(formula = SWI^lambda ~ SWF + temperature + management + SWF:management,
    data = data.training)

fit.bc.forward=
  lm(formula = SWI^lambda ~ SWF + temperature + management + SWF:management,
    data = data.training)

fit.bc.both=
    lm(formula = SWI^lambda ~ SWF + temperature + management + SWF:management,
    data = data.training)
```

Box-cox+variable elimination model diagnostics

```
summary(fit.bc.backwards)
```

```
##
## Call:
## lm(formula = SWI^lambda ~ SWF + temperature + management + SWF:management,
##     data = data.training)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35281 -0.07840 -0.00021  0.07708  0.48590
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)     0.503743   0.057804   8.715 1.26e-15 ***
## SWF             0.311480   0.035253   8.836 5.82e-16 ***
## temperature     0.015378   0.001642   9.368  < 2e-16 ***
## management      0.041077   0.011447   3.589 0.000421 ***
## SWF:management -0.012876   0.007191  -1.791 0.074918 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1235 on 195 degrees of freedom
## Multiple R-squared:  0.6769, Adjusted R-squared:  0.6702
## F-statistic: 102.1 on 4 and 195 DF,  p-value: < 2.2e-16
```

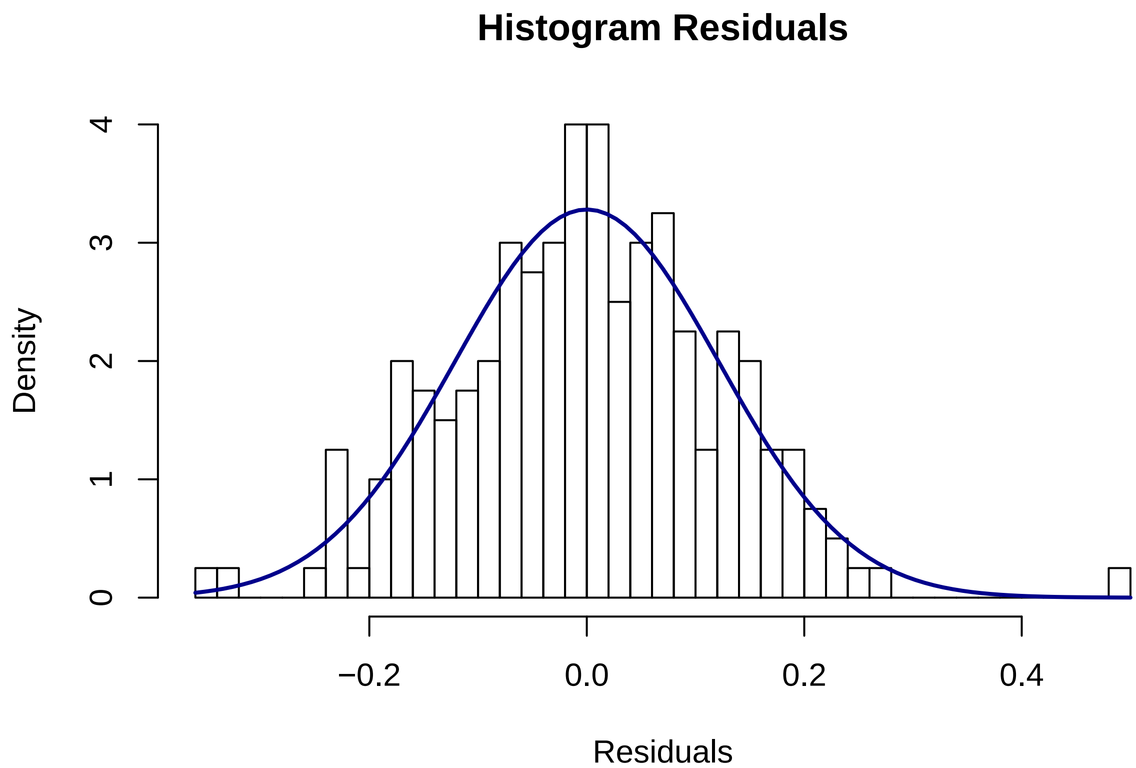
```
par(mfrow = c(2,2))
plot(fit.bc.backwards)
```




```r
#normality test
ols_test_normality(fit.bc.backwards)
```

```
## -----------------------------------------------
##        Test             Statistic       pvalue
## -----------------------------------------------
## Shapiro-Wilk              0.9919         0.3289
## Kolmogorov-Smirnov        0.0375         0.9415
## Cramer-von Mises         51.7743         0.0000
## Anderson-Darling          0.1785         0.9176
## -----------------------------------------------
```

```r
par(mfrow = c(1,1))
hist(fit.bc.backwards$residuals,breaks = 50,
     xlab="Residuals", main="Histogram Residuals",
     probability = T)
curve(dnorm(x, mean=mean(fit.bc$residuals), sd=sd(fit.bc$residuals)),
      col="darkblue", lwd=2, add=TRUE, yaxt="n")
```

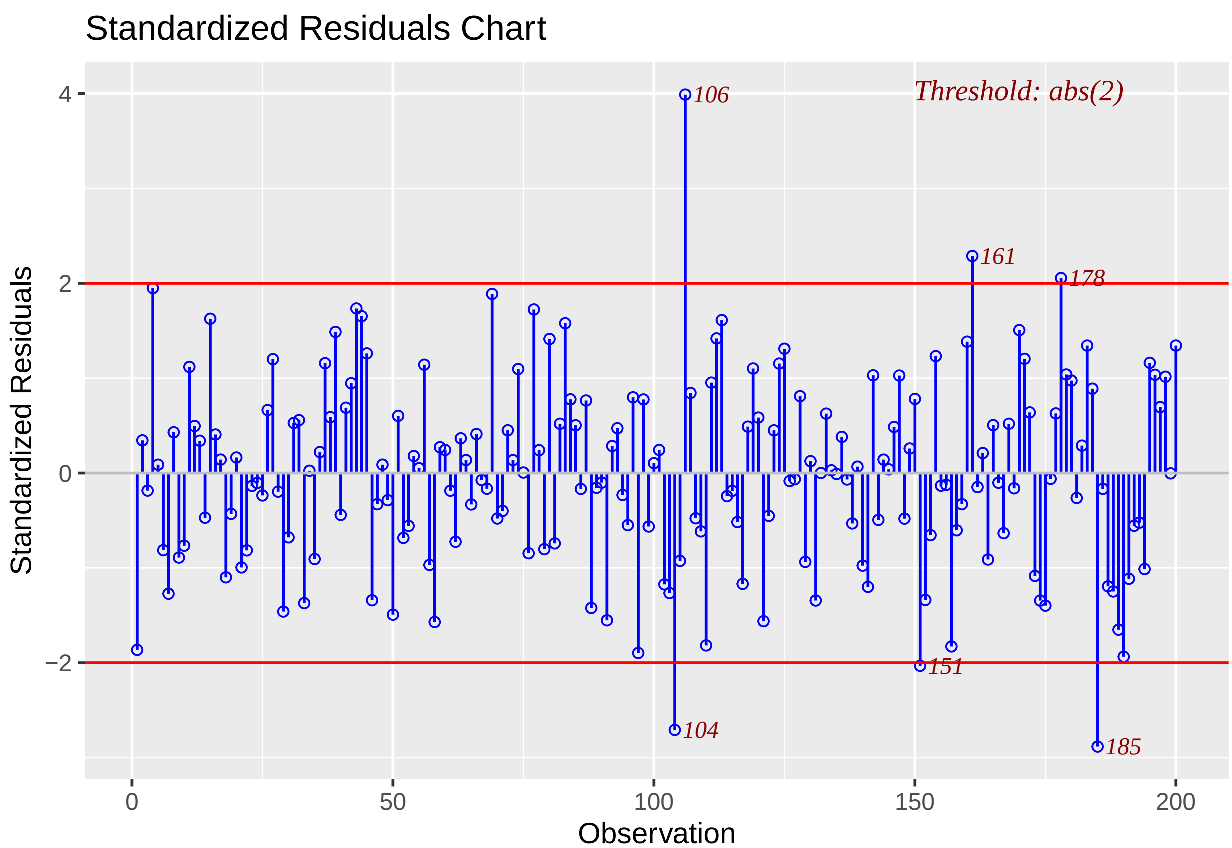



```r
#Breusch-Pagan Test for homoscedasticity
ncvTest(fit.bc.backwards)
```

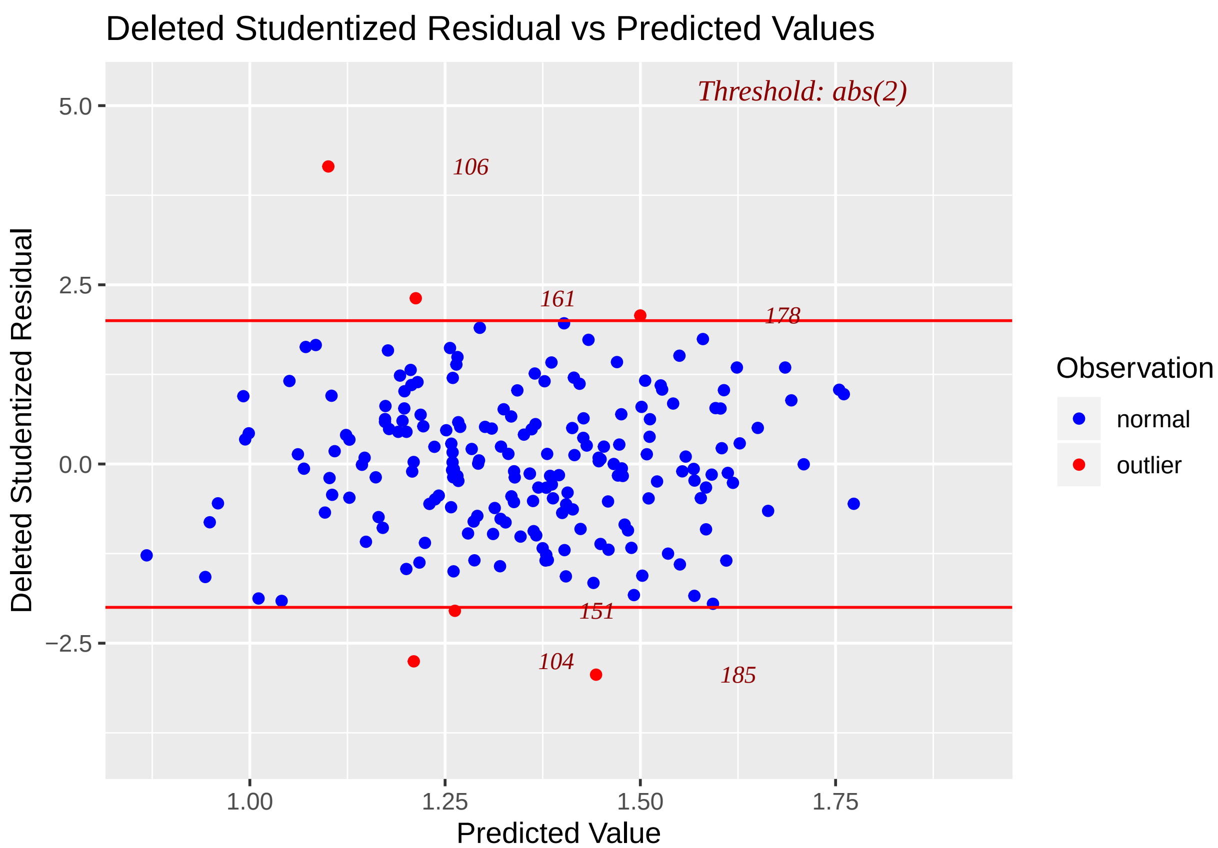
```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 1.708554, Df = 1, p = 0.19117
```

```r
fit.bc=fit.bc.backwards
```

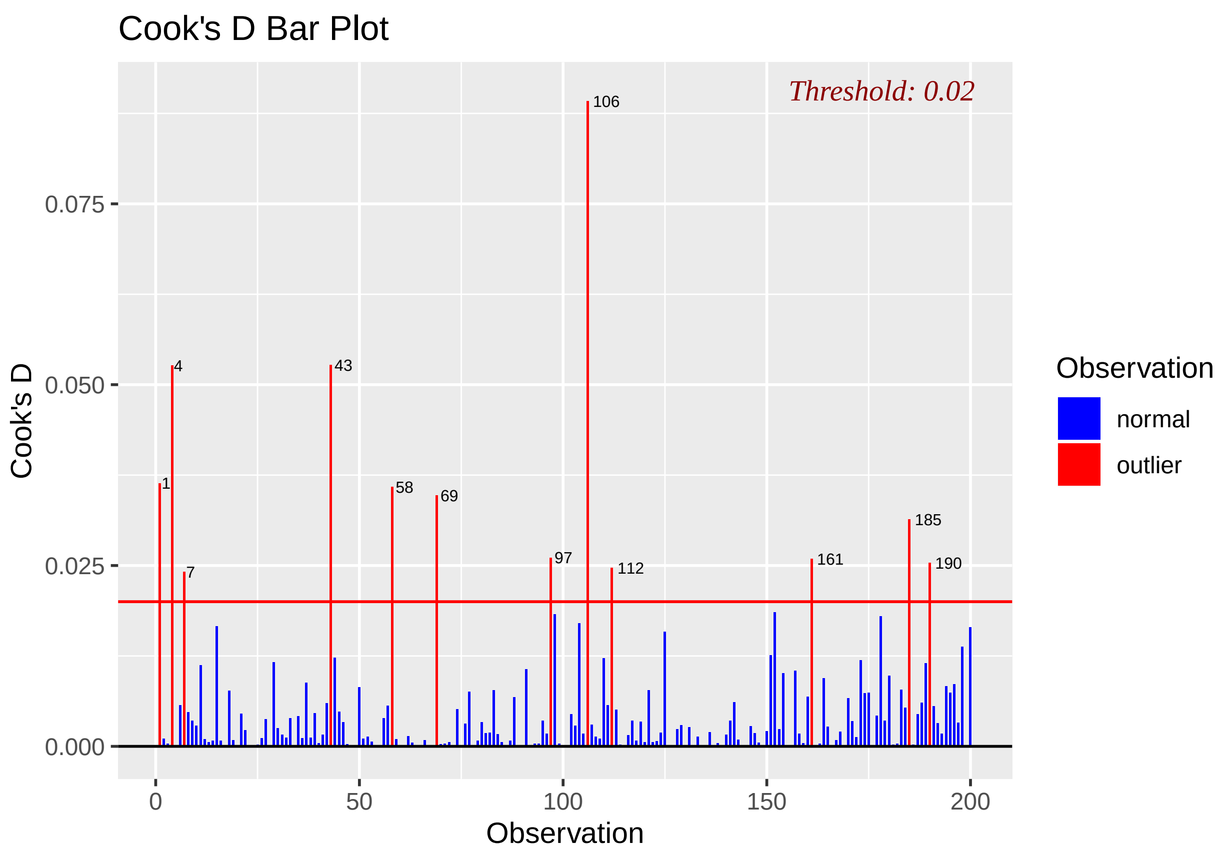
```r
mod=fit.bc
#Standardized Residual plot
ols_plot_resid_stand(mod)
```



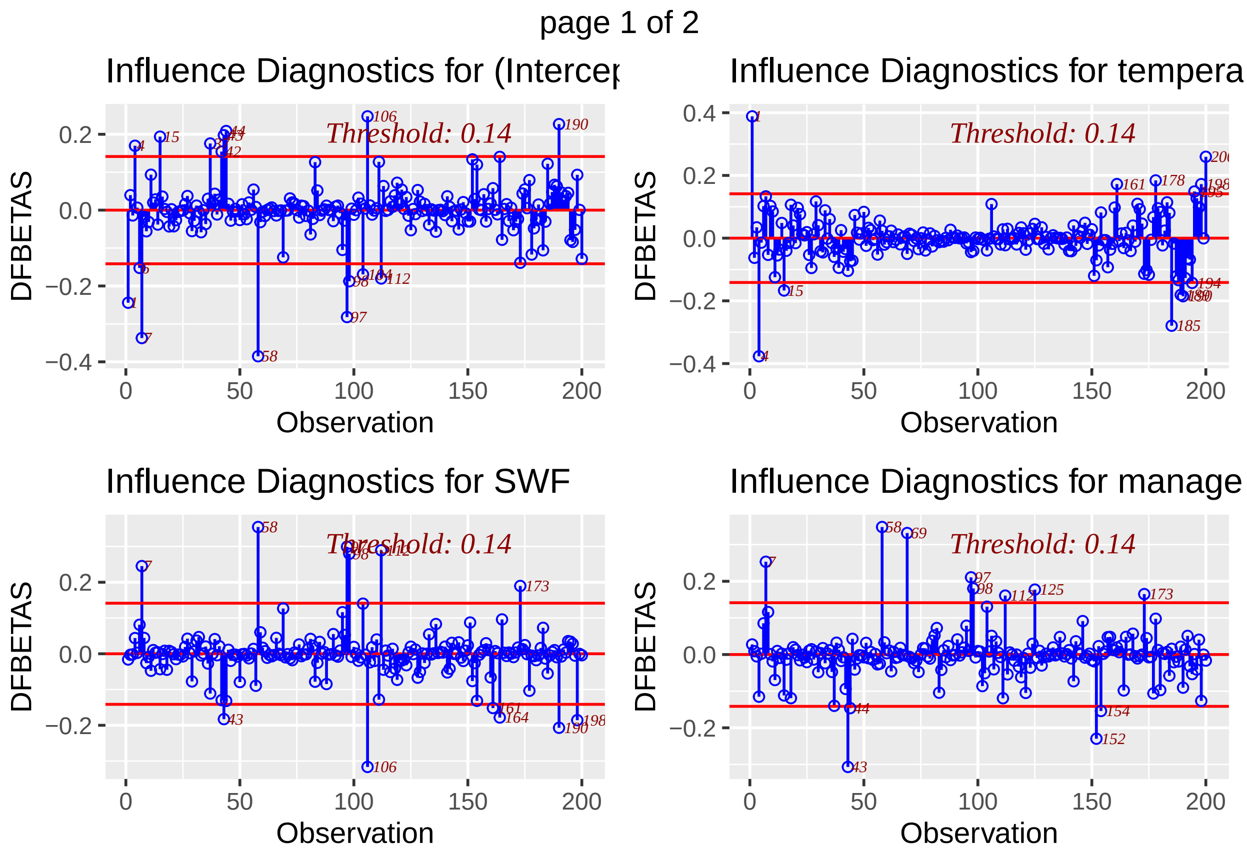

```
#studentized residual plot
ols_plot_resid_stud_fit(mod)
```

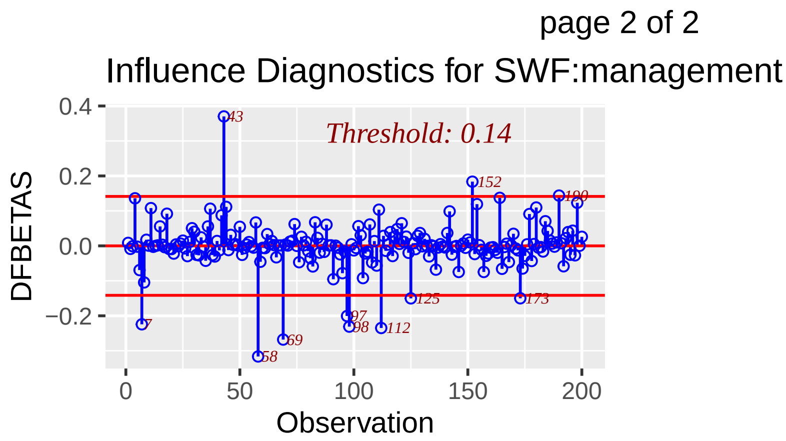



```r
#cooks distance plot
ols_plot_cooksd_bar(mod)
```

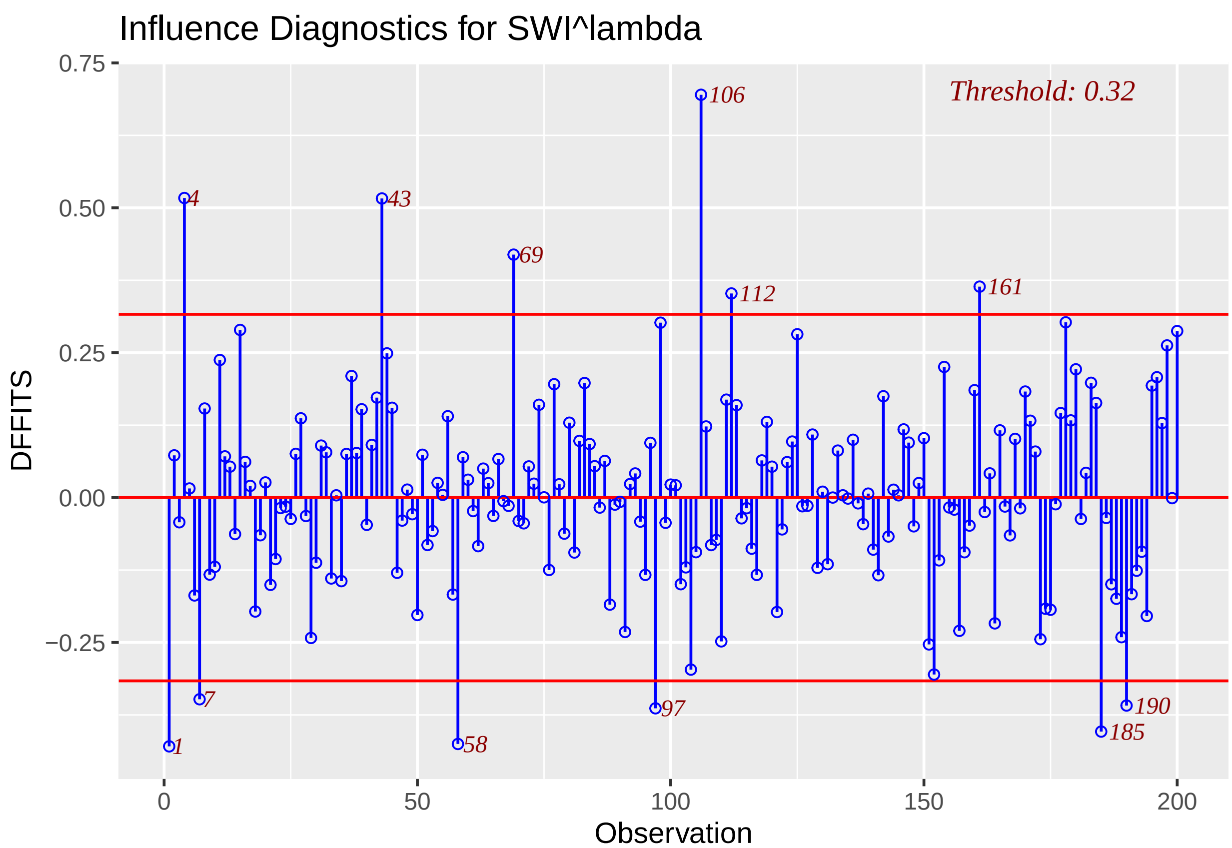



```r
#DFBetas for each variable
ols_plot_dfbetas(mod)
```

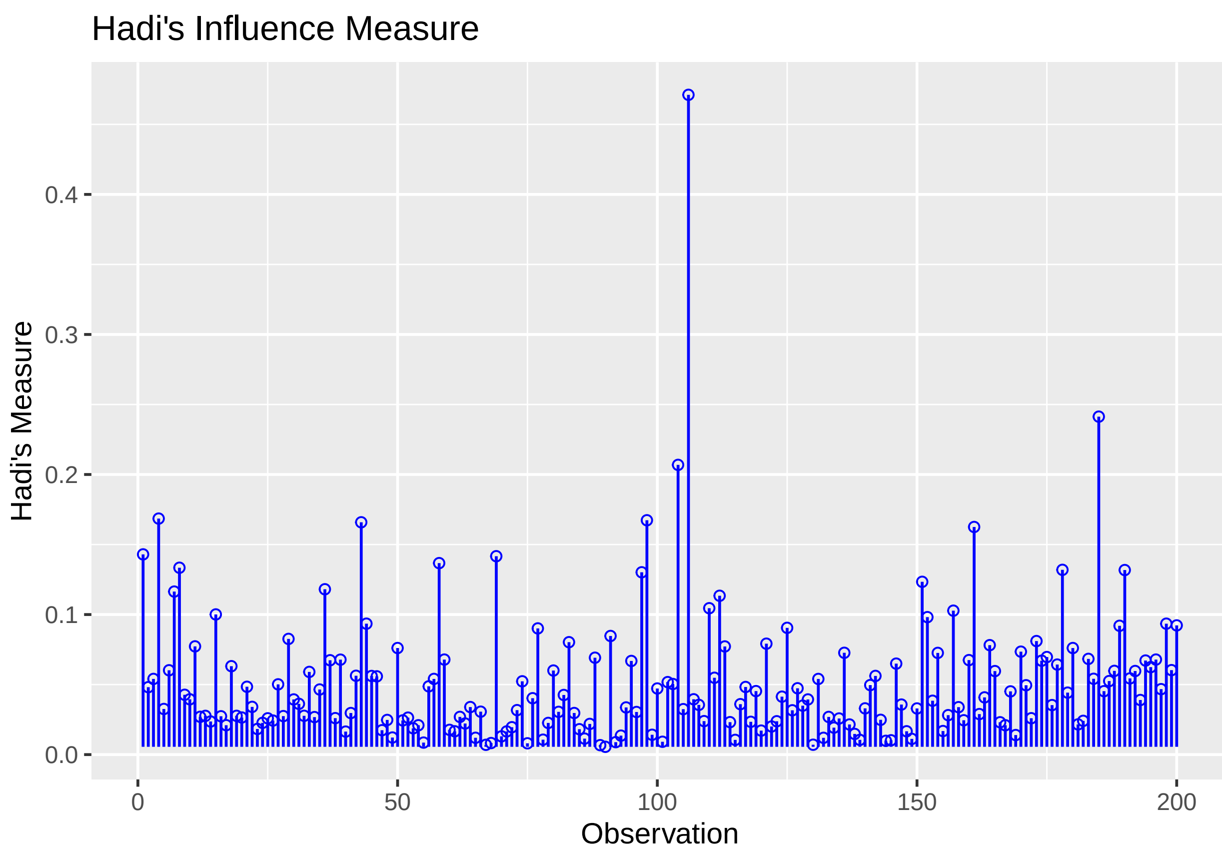

page 1 of 2
Influence Diagnostics for (Intercep
Influence Diagnostics for tempera
Influence Diagnostics for SWF
Influence Diagnostics for manage
Threshold: 0.14
DFBETAS
Observation




```r
#Difference in fit chart for each sample
ols_plot_dffits(mod)
```




```r
#Plot for observation influence using hadi's distance
ols_plot_hadi(mod)
```

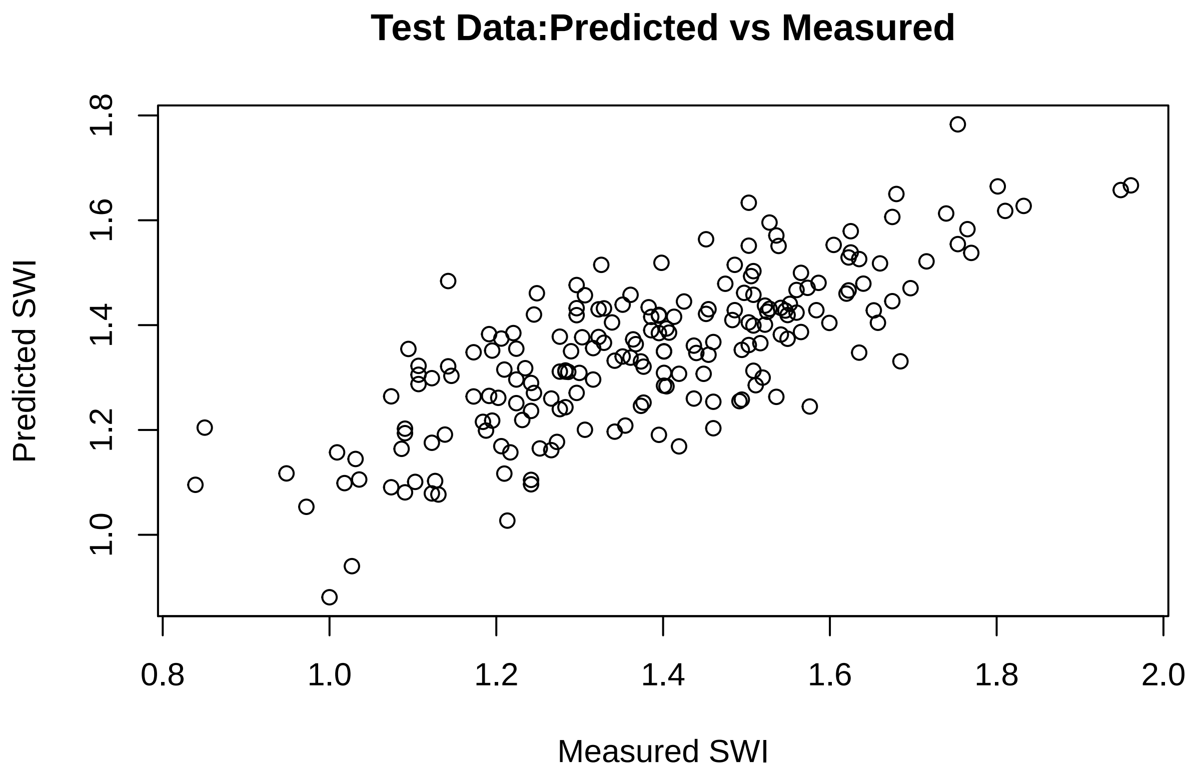


All assumptions still met, outliers still present.

```
results.bc=predict(fit.bc,data.test)
rmse(results.bc,data.test$SWI^lambda)
```

```
## [1] 0.1362177
```

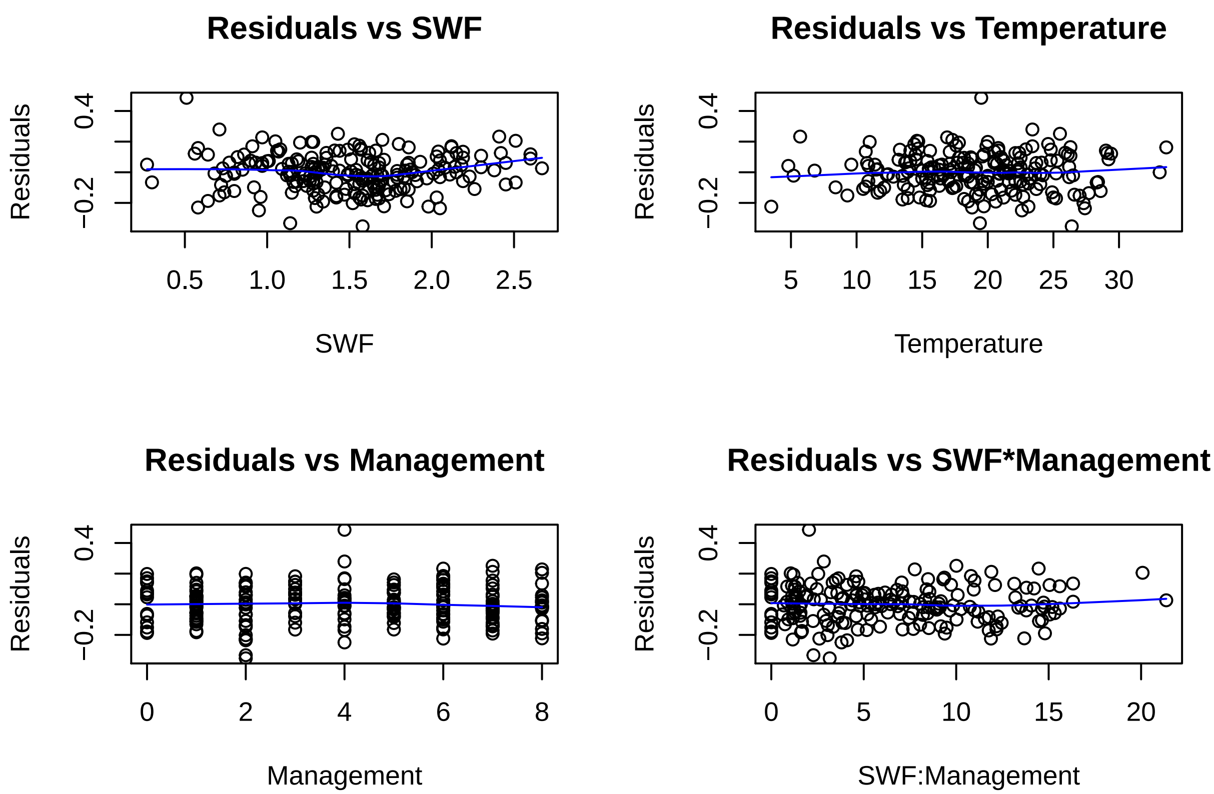
```
plot(data.test$SWI^lambda,results.bc,xlab="Measured SWI",ylab="Predicted SWI",main="Test Data:Predicted
```



Model sees drastic improvemt in RMSE after variable selection and box cox transfrom.

Checking variable linearity

```
par(mfrow=c(2,2))
#Checking linearity of SWF
plot(data.training$SWF,fit.bc$residuals,main="Residuals vs SWF",xlab="SWF",ylab="Residuals")
lines(lowess(fit.bc$residuals~data.training$SWF),col="blue")

#Checking linearity of Temperature
plot(data.training$temperature,fit.bc$residuals,main="Residuals vs Temperature",xlab="Temperature",ylab=
lines(lowess(fit.bc$residuals~data.training$temperature),col="blue")

#Checking linearity of Management
plot(data.training$management,fit.bc$residuals,main="Residuals vs Management",xlab="Management",ylab="Re
lines(lowess(fit.bc$residuals~data.training$management),col="blue")

plot(data.training$management*data.training$SWF,fit.bc$residuals,main="Residuals vs SWF*Management",xlab
lines(lowess(fit.bc$residuals~data.training$management*data.training$SWF),col="blue")
```



SWF Appears to be non-linear. Polynomial effects will be tested to see if they bring significance

Fitting polynomial model with interaction terms and SWF^2.

```
fit.poly=
  lm(formula = SWI ~ SWF * temperature * management * I(SWF^2), data = data.training)
summary(fit.poly)
```

```
##
## Call:
## lm(formula = SWI ~ SWF * temperature * management * I(SWF^2),
##     data = data.training)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.93183 -0.22112 -0.00805  0.22828  1.21197
##
## Coefficients:
##                                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)                            -3.9238033  4.4174404  -0.888    0.376
## SWF                                     9.1872558 10.6882098   0.860    0.391
## temperature                             0.1782457  0.2337267   0.763    0.447
## management                              0.4765277  0.7137731   0.668    0.505
## I(SWF^2)                               -5.8771375  7.9605384  -0.738    0.461
## SWF:temperature                        -0.3037308  0.5644875  -0.538    0.591
## SWF:management                         -0.8985644  1.7250861  -0.521    0.603
## temperature:management                  0.0017881  0.0417186   0.043    0.966
## SWF:I(SWF^2)                            1.2919949  1.8415574   0.702    0.484
```

```
## temperature:I(SWF^2)               0.2006896  0.4194884   0.478    0.633
## management:I(SWF^2)                0.5541672  1.2807618   0.433    0.666
## SWF:temperature:management         0.0016545  0.0983606   0.017    0.987
## SWF:temperature:I(SWF^2)          -0.0411284  0.0969055  -0.424    0.672
## SWF:management:I(SWF^2)           -0.1076724  0.2942715  -0.366    0.715
## temperature:management:I(SWF^2)   -0.0010898  0.0719051  -0.015    0.988
## SWF:temperature:management:I(SWF^2)  0.0001218  0.0163892   0.007    0.994
##
## Residual standard error: 0.3812 on 184 degrees of freedom
## Multiple R-squared:  0.7111, Adjusted R-squared:  0.6876
## F-statistic:  30.2 on 15 and 184 DF,  p-value: < 2.2e-16
```

```
anova(fit.poly)
```

```
## Analysis of Variance Table
##
## Response: SWI
##                                      Df Sum Sq Mean Sq  F value    Pr(>F)
## SWF                                   1 43.515  43.515 299.5144 < 2.2e-16
## temperature                           1 13.431  13.431  92.4465 < 2.2e-16
## management                            1  4.765   4.765  32.7959 4.105e-08
## I(SWF^2)                              1  2.831   2.831  19.4888 1.722e-05
## SWF:temperature                       1  0.006   0.006   0.0427    0.8365
## SWF:management                        1  0.282   0.282   1.9376    0.1656
## temperature:management                1  0.183   0.183   1.2600    0.2631
## SWF:I(SWF^2)                          1  0.110   0.110   0.7600    0.3844
## temperature:I(SWF^2)                  1  0.001   0.001   0.0079    0.9295
## management:I(SWF^2)                   1  0.125   0.125   0.8626    0.3542
## SWF:temperature:management            1  0.000   0.000   0.0012    0.9726
## SWF:temperature:I(SWF^2)              1  0.115   0.115   0.7907    0.3750
## SWF:management:I(SWF^2)               1  0.444   0.444   3.0581    0.0820
## temperature:management:I(SWF^2)       1  0.001   0.001   0.0047    0.9457
## SWF:temperature:management:I(SWF^2)   1  0.000   0.000   0.0001    0.9941
## Residuals                           184 26.732   0.145
##
## SWF                                 ***
## temperature                         ***
## management                          ***
## I(SWF^2)                            ***
## SWF:temperature
## SWF:management
## temperature:management
## SWF:I(SWF^2)
## temperature:I(SWF^2)
## management:I(SWF^2)
## SWF:temperature:management
## SWF:temperature:I(SWF^2)
## SWF:management:I(SWF^2)             .
## temperature:management:I(SWF^2)
## SWF:temperature:management:I(SWF^2)
## Residuals
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Stepwise variable section for polynomial model both ways model = forward selection model

fitting each model and comparing them against each other.

```
fit.poly.backward=
  lm(formula = SWI ~ SWF + temperature + management + I(SWF^2) +
    SWF:temperature + SWF:management + SWF:I(SWF^2) + temperature:I(SWF^2) +
    management:I(SWF^2) + SWF:temperature:I(SWF^2) + SWF:management:I(SWF^2),
    data = data.training)
summary(fit.poly.backward)
```

```
##
## Call:
## lm(formula = SWI ~ SWF + temperature + management + I(SWF^2) +
##     SWF:temperature + SWF:management + SWF:I(SWF^2) + temperature:I(SWF^2) +
##     management:I(SWF^2) + SWF:temperature:I(SWF^2) + SWF:management:I(SWF^2),
##     data = data.training)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.97111 -0.22625 -0.01605  0.22938  1.21856
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -4.10952    1.98707  -2.068  0.04000 *
## SWF                       9.32558    4.42890   2.106  0.03657 *
## temperature               0.18700    0.08345   2.241  0.02621 *
## management                0.50321    0.18992   2.650  0.00875 **
## I(SWF^2)                 -6.02560    3.07492  -1.960  0.05152 .
## SWF:temperature          -0.30731    0.18822  -1.633  0.10420
## SWF:management           -0.85401    0.41003  -2.083  0.03862 *
## SWF:I(SWF^2)              1.33376    0.66455   2.007  0.04618 *
## temperature:I(SWF^2)      0.20590    0.13117   1.570  0.11817
## management:I(SWF^2)       0.52431    0.27703   1.893  0.05995 .
## SWF:temperature:I(SWF^2) -0.04284    0.02835  -1.511  0.13238
## SWF:management:I(SWF^2)  -0.10325    0.05857  -1.763  0.07954 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3781 on 188 degrees of freedom
## Multiple R-squared:  0.7096, Adjusted R-squared:  0.6926
## F-statistic: 41.77 on 11 and 188 DF,  p-value: < 2.2e-16
```

```
fit.poly.forward=
 lm(formula = SWI ~ I(SWF^2) + temperature + management,
    data = data.training)
summary(fit.poly.forward)
```

```
##
## Call:
## lm(formula = SWI ~ I(SWF^2) + temperature + management, data = data.training)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.01224 -0.25139 -0.01333  0.21775  1.37081
##
```

```
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.142086   0.110001   1.292    0.198
## I(SWF^2)    0.281397   0.018583  15.142  < 2e-16 ***
## temperature 0.047491   0.005039   9.425  < 2e-16 ***
## management  0.061961   0.010918   5.675 4.93e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3795 on 196 degrees of freedom
## Multiple R-squared:  0.6949, Adjusted R-squared:  0.6902
## F-statistic: 148.8 on 3 and 196 DF,  p-value: < 2.2e-16
```

```r
anova(fit.poly.backward,fit.poly.forward)
```

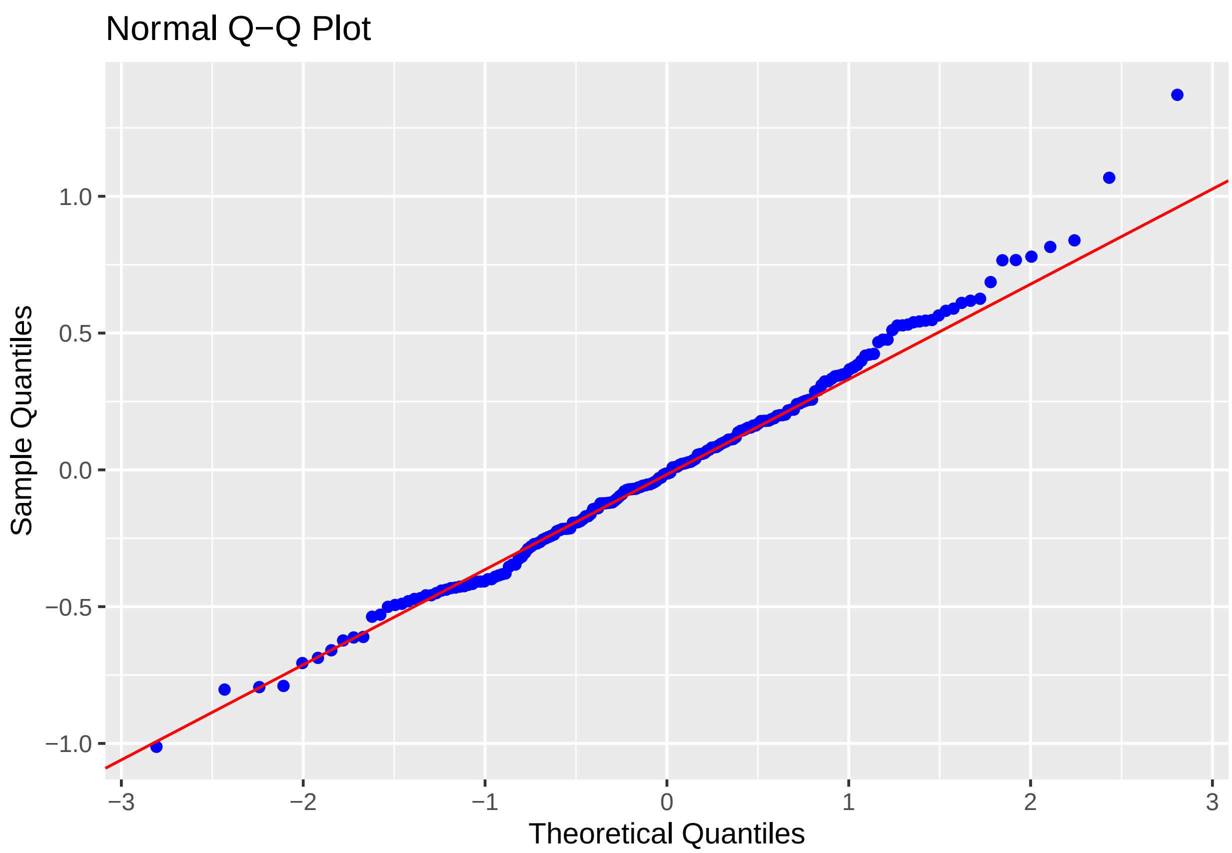
```
## Analysis of Variance Table
##
## Model 1: SWI ~ SWF + temperature + management + I(SWF^2) + SWF:temperature +
##     SWF:management + SWF:I(SWF^2) + temperature:I(SWF^2) + management:I(SWF^2) +
##     SWF:temperature:I(SWF^2) + SWF:management:I(SWF^2)
## Model 2: SWI ~ I(SWF^2) + temperature + management
##   Res.Df    RSS Df Sum of Sq      F Pr(>F)
## 1    188 26.871
## 2    196 28.235 -8   -1.3645 1.1934  0.305
```

```r
#no real difference found between the two models
fit.poly=fit.poly.forward
```

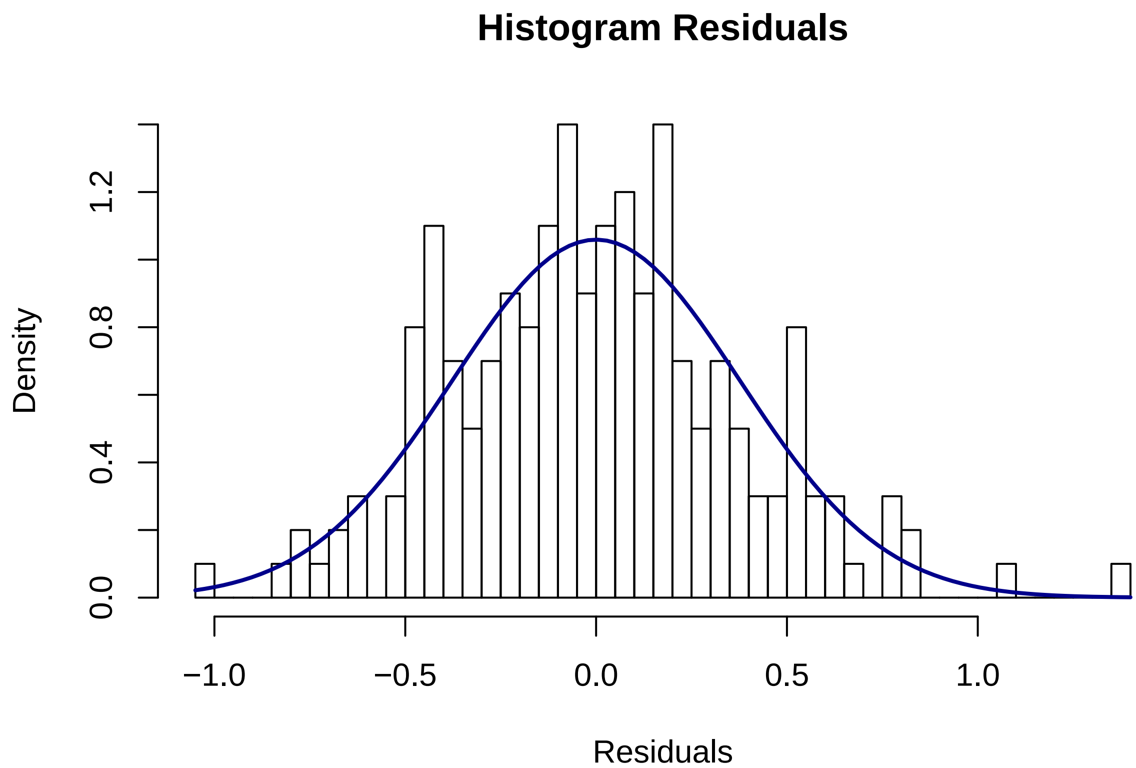
Anova model comparison shows no significant defference between the two models. Forward selection shall be used as it is substantially smaller than the backward elimination model. Interaction effects lose significane in the pressence of polynomial terms.

Checking model assumptions of the polynomial model.

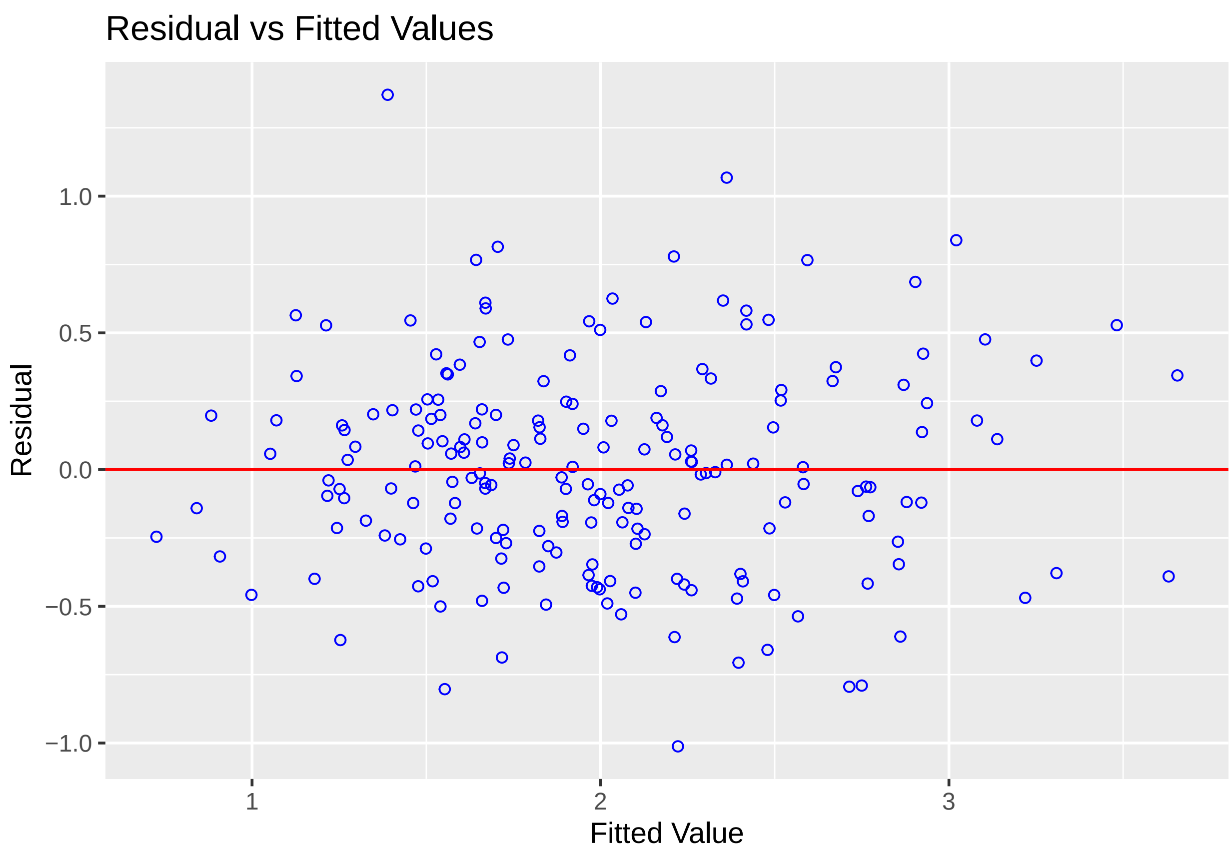
```r
#QQ-Plot and Histogram to visualize normality of errors
ols_plot_resid_qq(fit.poly)
```




```
hist(fit.poly$residuals,breaks = 50,
     xlab="Residuals", main="Histogram Residuals",
     probability = T)
curve(dnorm(x, mean=mean(fit.poly$residuals), sd=sd(fit.poly$residuals)),
      col="darkblue", lwd=2, add=TRUE, yaxt="n")
```




```
##Visualiztion of Residuals vs. Fitted Values to evaluate homoscedasticity
ols_plot_resid_fit(fit.poly)
```




```
#Breusch-Pagan Test for homoscedasticity
ncvTest(fit.poly)
```

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 3.096221, Df = 1, p = 0.078474
```

```
#Normality of errors tests
ols_test_normality(fit.poly)
```

```
## -----------------------------------------------
##        Test             Statistic       pvalue
## -----------------------------------------------
## Shapiro-Wilk              0.9919         0.3364
## Kolmogorov-Smirnov        0.0406         0.8958
## Cramer-von Mises          30.616         0.0000
## Anderson-Darling          0.3461         0.4794
## -----------------------------------------------
```

```
ols_test_correlation(fit.poly)
```

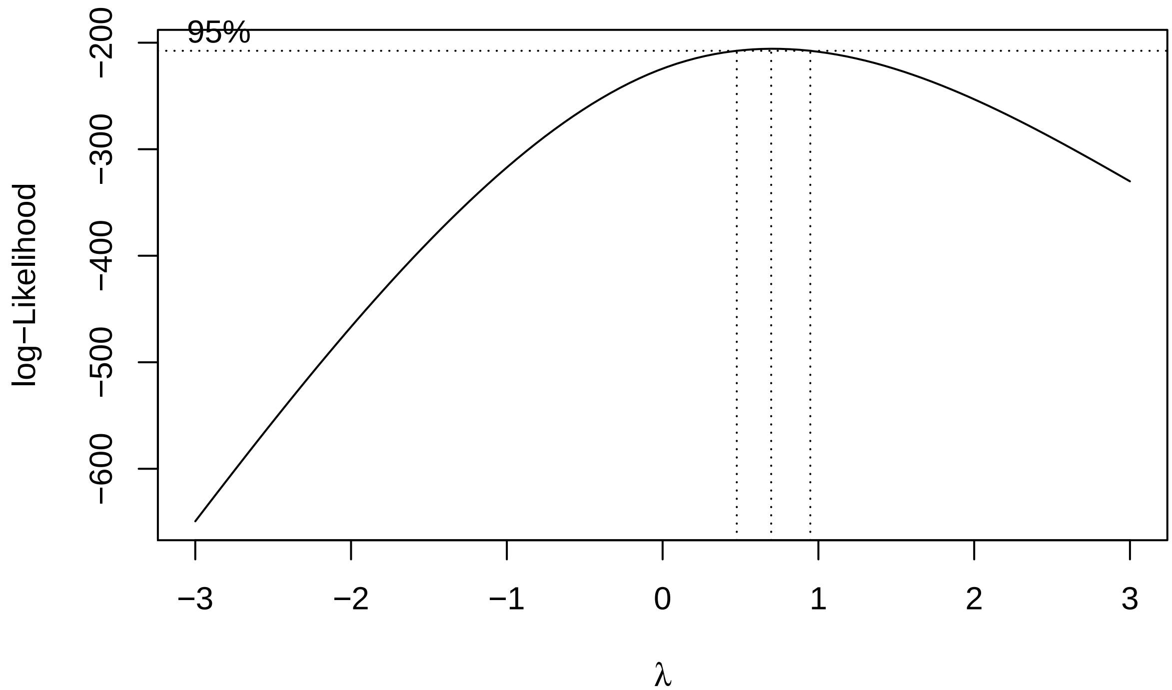
```
## [1] 0.9952231
```

Non-constant variance is an issue, will be addressed with a box cox transform. Other assumptions met.

Box-cox transform for polynomial model.

```
fullmodelpoly=
  lm(formula = SWI ~ I(SWF^2) + temperature + management, data = data.training)
```

```r
bcpoly=boxcox(fullmodelpoly,lambda = seq(-3,3))
```




```r
lambdapoly=bcpoly$x[which(bcpoly$y==max(bcpoly$y))]
lambdapoly
```

```
## [1] 0.6969697
```

```r
fit.bc.poly=
  lm(SWI^lambdapoly ~ I(SWF^2) + temperature + management, data = data.training)
summary(fit.bc.poly)
```

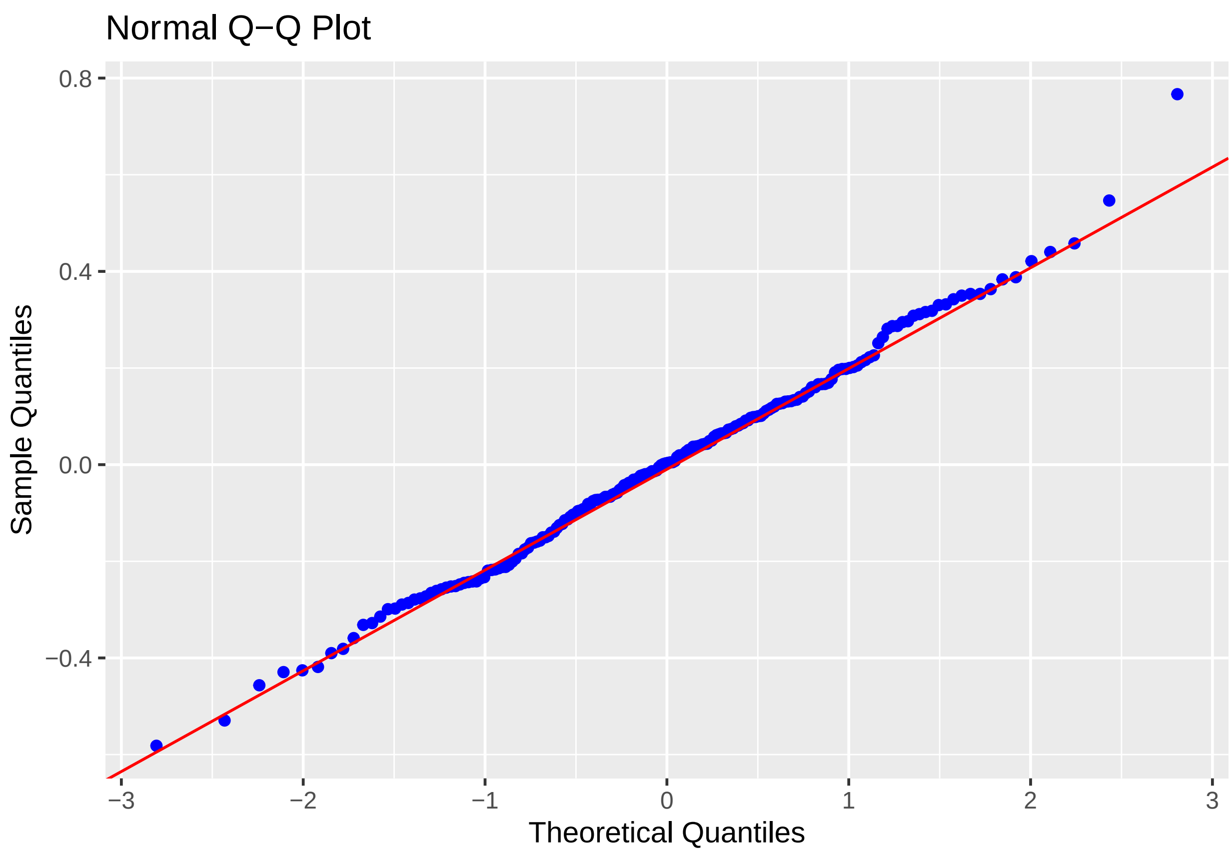
```
##
## Call:
## lm(formula = SWI^lambdapoly ~ I(SWF^2) + temperature + management,
##     data = data.training)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.58175 -0.15022  0.00329  0.13104  0.76671
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.546052   0.062436   8.746 1.01e-15 ***
## I(SWF^2)    0.156010   0.010548  14.791  < 2e-16 ***
## temperature 0.027054   0.002860   9.460  < 2e-16 ***
## management  0.037048   0.006197   5.978 1.05e-08 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2154 on 196 degrees of freedom
## Multiple R-squared:  0.6917, Adjusted R-squared:  0.687
## F-statistic: 146.6 on 3 and 196 DF,  p-value: < 2.2e-16
```

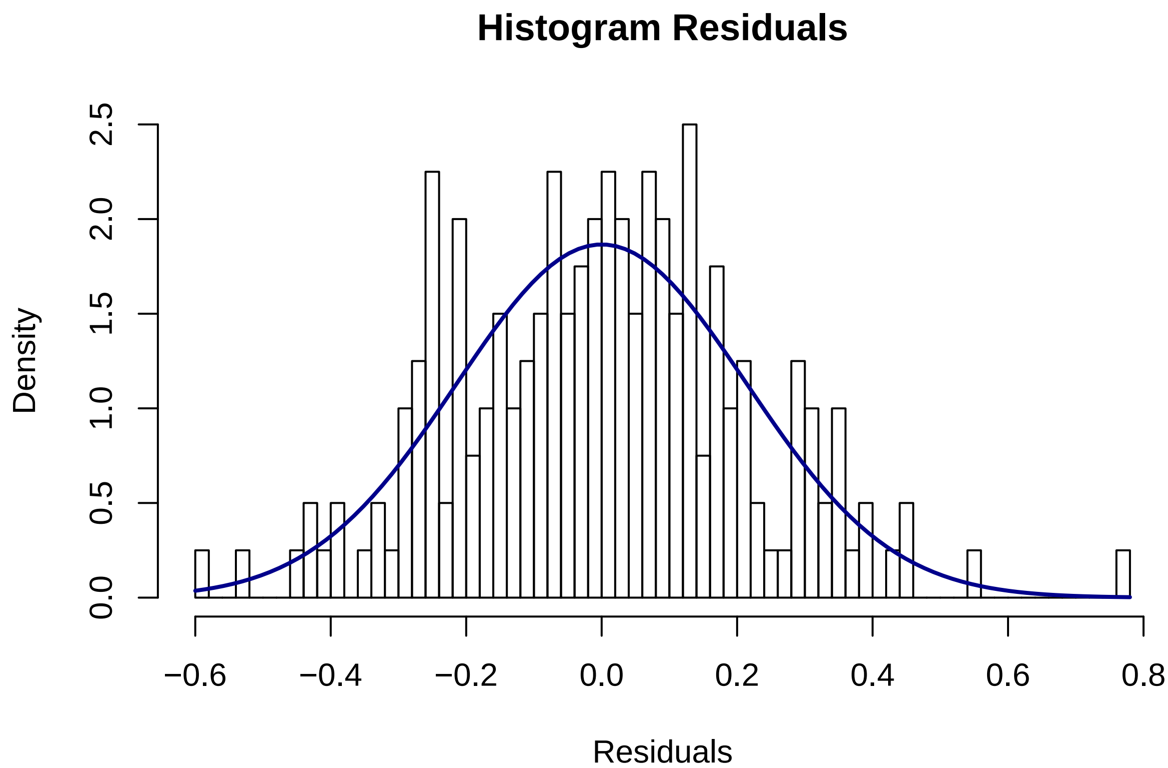
lambda of .69697

Model Diagnostics of box-cox transformed polynomial model

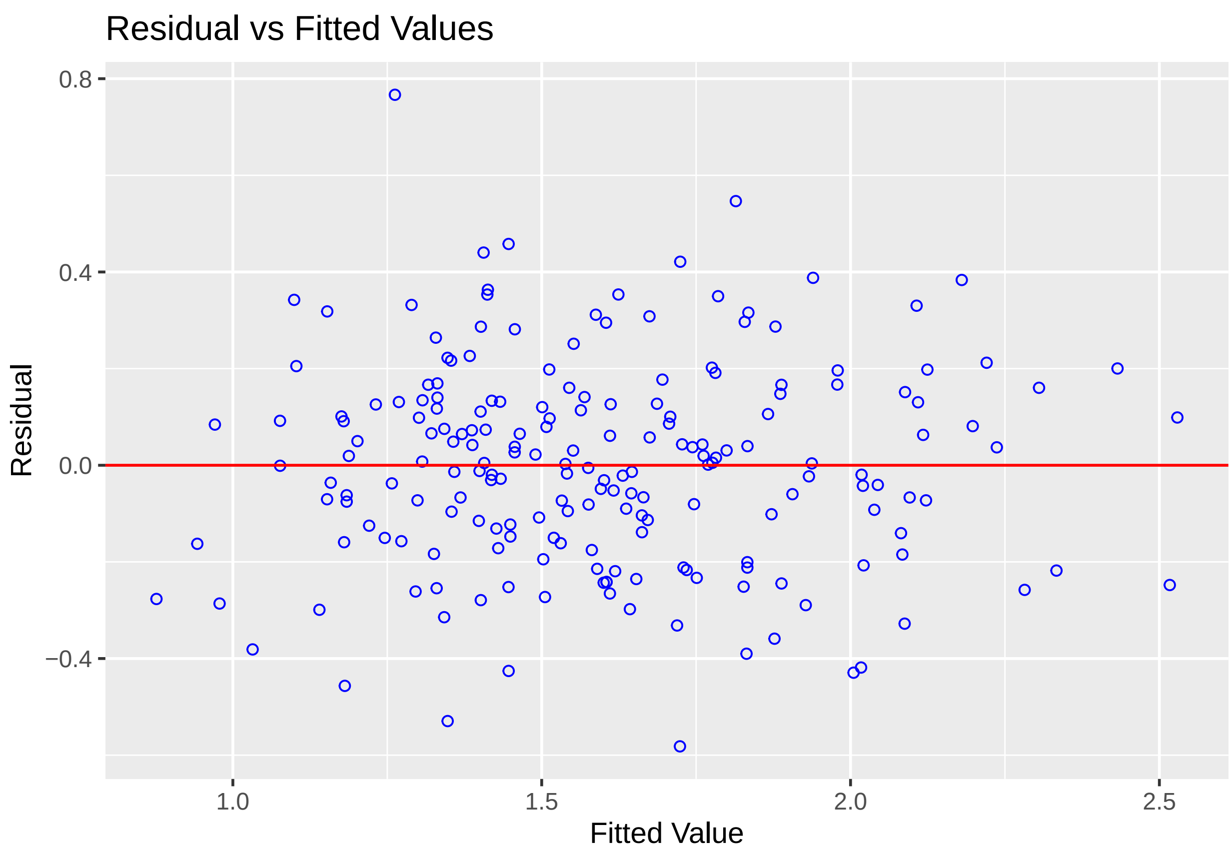
```
#QQ-Plot and Histogram to visualize normality of errors
ols_plot_resid_qq(fit.bc.poly)
```




```
hist(fit.bc.poly$residuals,breaks = 50,
     xlab="Residuals", main="Histogram Residuals",
     probability = T)
curve(dnorm(x, mean=mean(fit.bc.poly$residuals), sd=sd(fit.bc.poly$residuals)),
      col="darkblue", lwd=2, add=TRUE, yaxt="n")
```




```
##Visualiztion of Residuals vs. Fitted Values to evaluate homoscedasticity
ols_plot_resid_fit(fit.bc.poly)
```




```r
#Breusch-Pagan Test for homoscedasticity
ncvTest(fit.bc.poly)
```

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 0.00738277, Df = 1, p = 0.93153
```

```r
#Normality of errors tests
ols_test_normality(fit.bc.poly)
```

```
## -----------------------------------------------
##        Test             Statistic       pvalue
## -----------------------------------------------
## Shapiro-Wilk              0.9952         0.7809
## Kolmogorov-Smirnov        0.0297         0.9946
## Cramer-von Mises         42.4756         0.0000
## Anderson-Darling          0.2129         0.8518
## -----------------------------------------------
```

```r
ols_test_correlation(fit.bc.poly)
```

```
## [1] 0.9968848
```

All assumptions met.

Final model to be used for further analysis

```r
fit.fin=fit.bc.poly
summary(fit.fin)
```

```
##
## Call:
## lm(formula = SWI^lambdapoly ~ I(SWF^2) + temperature + management,
##     data = data.training)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.58175 -0.15022  0.00329  0.13104  0.76671
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.546052   0.062436   8.746 1.01e-15 ***
## I(SWF^2)    0.156010   0.010548  14.791  < 2e-16 ***
## temperature 0.027054   0.002860   9.460  < 2e-16 ***
## management  0.037048   0.006197   5.978 1.05e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2154 on 196 degrees of freedom
## Multiple R-squared:  0.6917, Adjusted R-squared:  0.687
## F-statistic: 146.6 on 3 and 196 DF,  p-value: < 2.2e-16
```

As an alternative model, an iteratively reweighted least squares model without the box cox transformation and only the main effects minus size and where SWF is taken to the 2nd power shall be considered.

```r
#Base Model
fit.alt.primary=
  lm(formula = SWI ~ I(SWF^2) + temperature + management,
     data = data.training)
summary(fit.alt.primary)
```

```
##
## Call:
## lm(formula = SWI ~ I(SWF^2) + temperature + management, data = data.training)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.01224 -0.25139 -0.01333  0.21775  1.37081
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.142086   0.110001   1.292    0.198
## I(SWF^2)    0.281397   0.018583  15.142  < 2e-16 ***
## temperature 0.047491   0.005039   9.425  < 2e-16 ***
## management  0.061961   0.010918   5.675 4.93e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3795 on 196 degrees of freedom
## Multiple R-squared:  0.6949, Adjusted R-squared:  0.6902
## F-statistic: 148.8 on 3 and 196 DF,  p-value: < 2.2e-16
```

```r
#First itteration
resid=residuals(fit.alt.primary)
fit.alt.std=lm(abs(resid)~ I(SWF^2) + temperature + management, data = data.training)
summary(fit.alt.std)
```

```
##
## Call:
## lm(formula = abs(resid) ~ I(SWF^2) + temperature + management,
##     data = data.training)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.34608 -0.18305 -0.03833  0.12883  1.07880
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.088615   0.065975   1.343  0.18077
## I(SWF^2)    0.003972   0.011146   0.356  0.72193
## temperature 0.009990   0.003022   3.306  0.00113 **
## management  0.001890   0.006548   0.289  0.77314
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2276 on 196 degrees of freedom
## Multiple R-squared:  0.0567, Adjusted R-squared:  0.04226
## F-statistic: 3.927 on 3 and 196 DF,  p-value: 0.009437
```

```r
w=1/fit.alt.std$fitted^2
fit.alt1=lm(formula = SWI ~ I(SWF^2) + temperature + management,
           data = data.training, weights = w)
summary(fit.alt1)
```

```
##
## Call:
## lm(formula = SWI ~ I(SWF^2) + temperature + management, data = data.training,
##     weights = w)
##
## Weighted Residuals:
##     Min      1Q  Median      3Q     Max
## -2.7562 -0.9340 -0.0447  0.7948  4.7449
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.142925   0.088352   1.618    0.107
## I(SWF^2)    0.288042   0.018296  15.744  < 2e-16 ***
## temperature 0.046763   0.004367  10.709  < 2e-16 ***
## management  0.061181   0.010080   6.070 6.52e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.271 on 196 degrees of freedom
## Multiple R-squared:  0.7268, Adjusted R-squared:  0.7226
## F-statistic: 173.8 on 3 and 196 DF,  p-value: < 2.2e-16
```

```r
#second iteration
resid1=residuals(fit.alt1)
fit.alt1.std=lm(abs(resid1)~ I(SWF^2) + temperature + management, data = data.training)
summary(fit.alt1.std)
```

```
##
```

```
## Call:
## lm(formula = abs(resid1) ~ I(SWF^2) + temperature + management,
##     data = data.training)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -0.3455 -0.1795 -0.0414  0.1433  1.0900
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.086517   0.065965   1.312 0.191200
## I(SWF^2)    0.002407   0.011144   0.216 0.829198
## temperature 0.010142   0.003022   3.357 0.000948 ***
## management  0.002665   0.006547   0.407 0.684379
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2276 on 196 degrees of freedom
## Multiple R-squared:  0.05765,    Adjusted R-squared:  0.04323
## F-statistic: 3.997 on 3 and 196 DF,  p-value: 0.008612
```

```
w1=1/fit.alt1.std$fitted^2
fit.alt2=lm(formula = SWI ~ I(SWF^2) + temperature + management,
           data = data.training, weights = w1)
summary(fit.alt2)
```

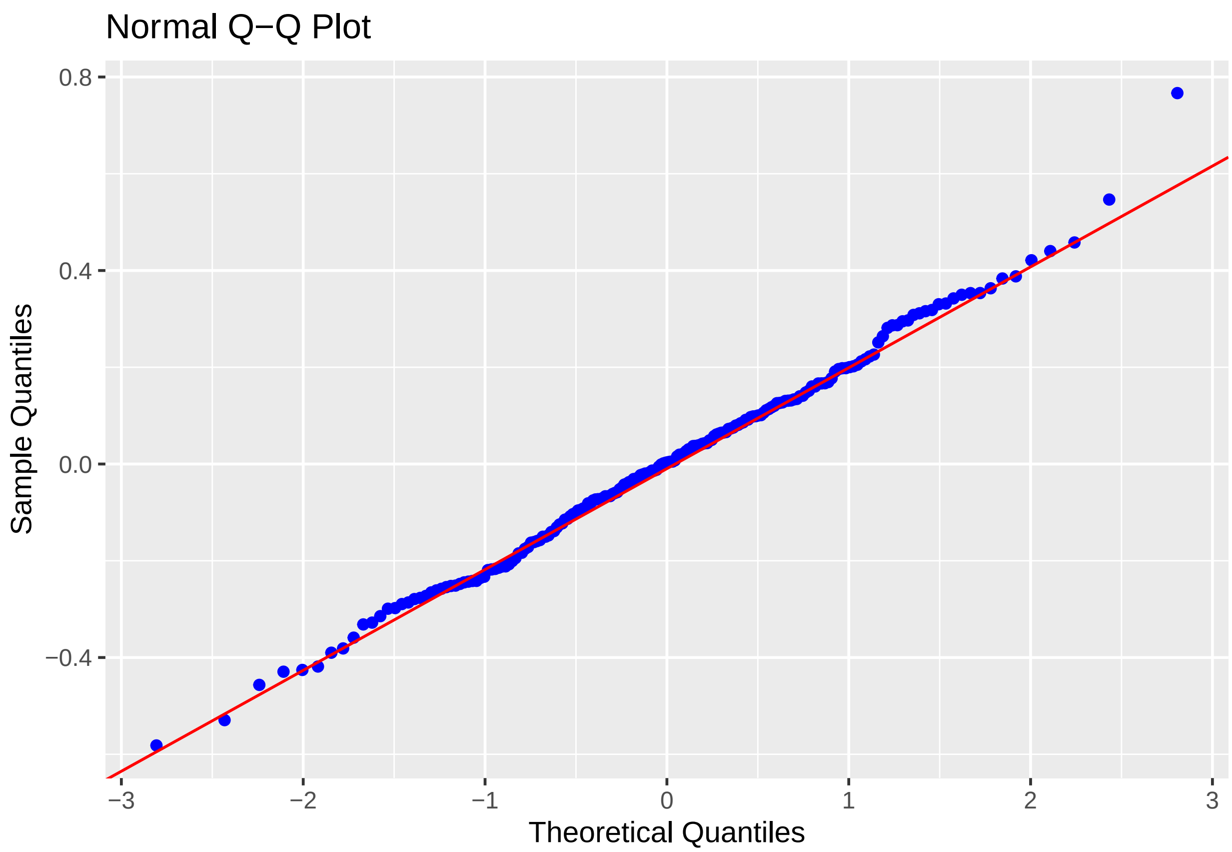
```
##
## Call:
## lm(formula = SWI ~ I(SWF^2) + temperature + management, data = data.training,
##     weights = w1)
##
## Weighted Residuals:
##     Min      1Q  Median      3Q     Max
## -2.7578 -0.9322 -0.0453  0.8008  4.6982
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.142132   0.087878   1.617    0.107
## I(SWF^2)    0.289410   0.018172  15.926  < 2e-16 ***
## temperature 0.046648   0.004352  10.718  < 2e-16 ***
## management  0.061068   0.010063   6.069 6.55e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.271 on 196 degrees of freedom
## Multiple R-squared:  0.7299, Adjusted R-squared:  0.7258
## F-statistic: 176.6 on 3 and 196 DF,  p-value: < 2.2e-16
```

```
#nothing really changed setting alternative model to the second iteration.
fit.alt=fit.alt2
```

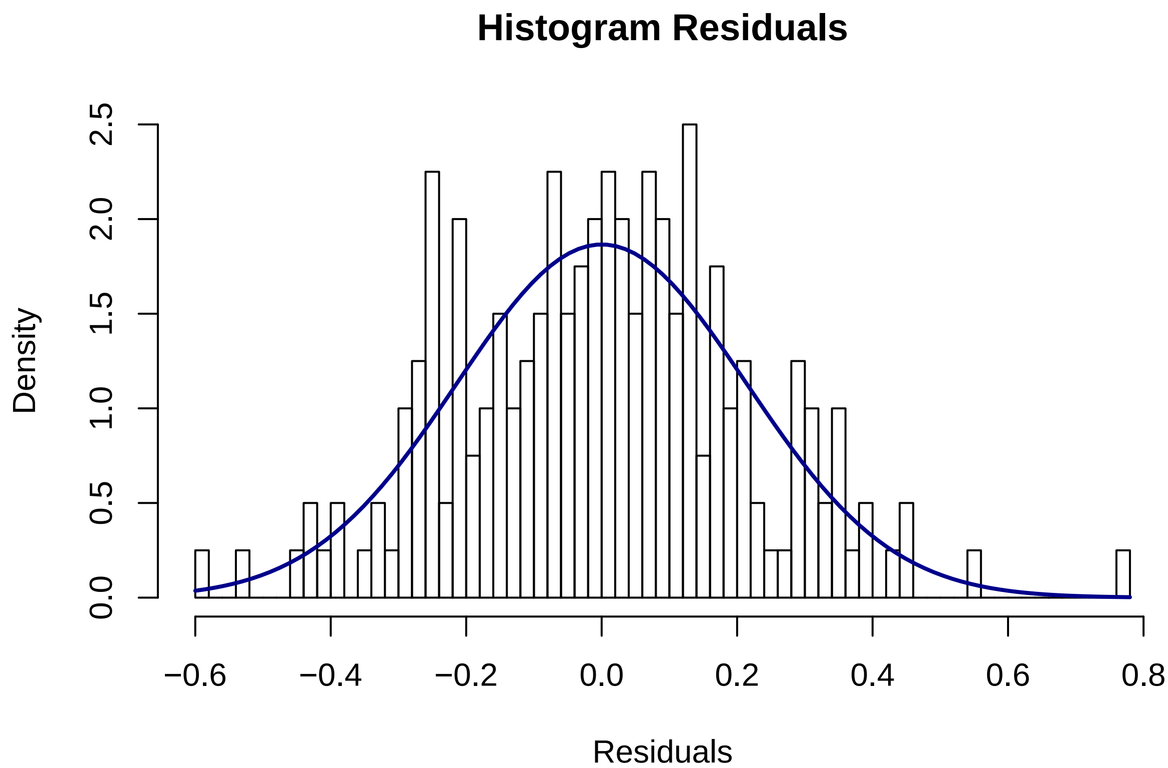
2 Ttterations of rewieghting seems appropriate

Model Diagnostics for final model.

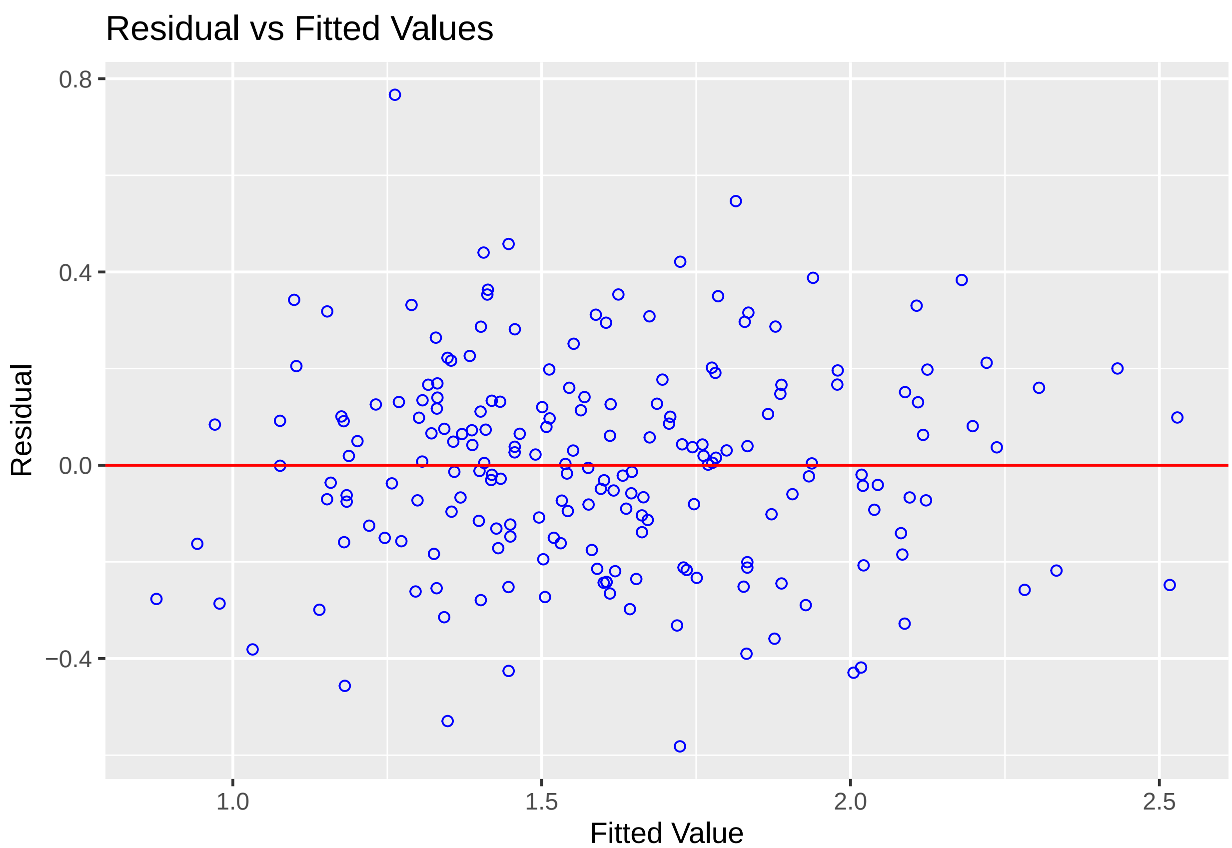
```r
#QQ-Plot and Histogram to visualize normality of errors
ols_plot_resid_qq(fit.fin)
```




```r
hist(fit.fin$residuals,breaks = 50,
     xlab="Residuals", main="Histogram Residuals",
     probability = T)
curve(dnorm(x, mean=mean(fit.fin$residuals), sd=sd(fit.fin$residuals)),
      col="darkblue", lwd=2, add=TRUE, yaxt="n")
```




```
##Visualiztion of Residuals vs. Fitted Values to evaluate homoscedasticity
ols_plot_resid_fit(fit.fin)
```




```
#Breusch-Pagan Test for homoscedasticity
ncvTest(fit.fin)
```

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 0.00738277, Df = 1, p = 0.93153
```

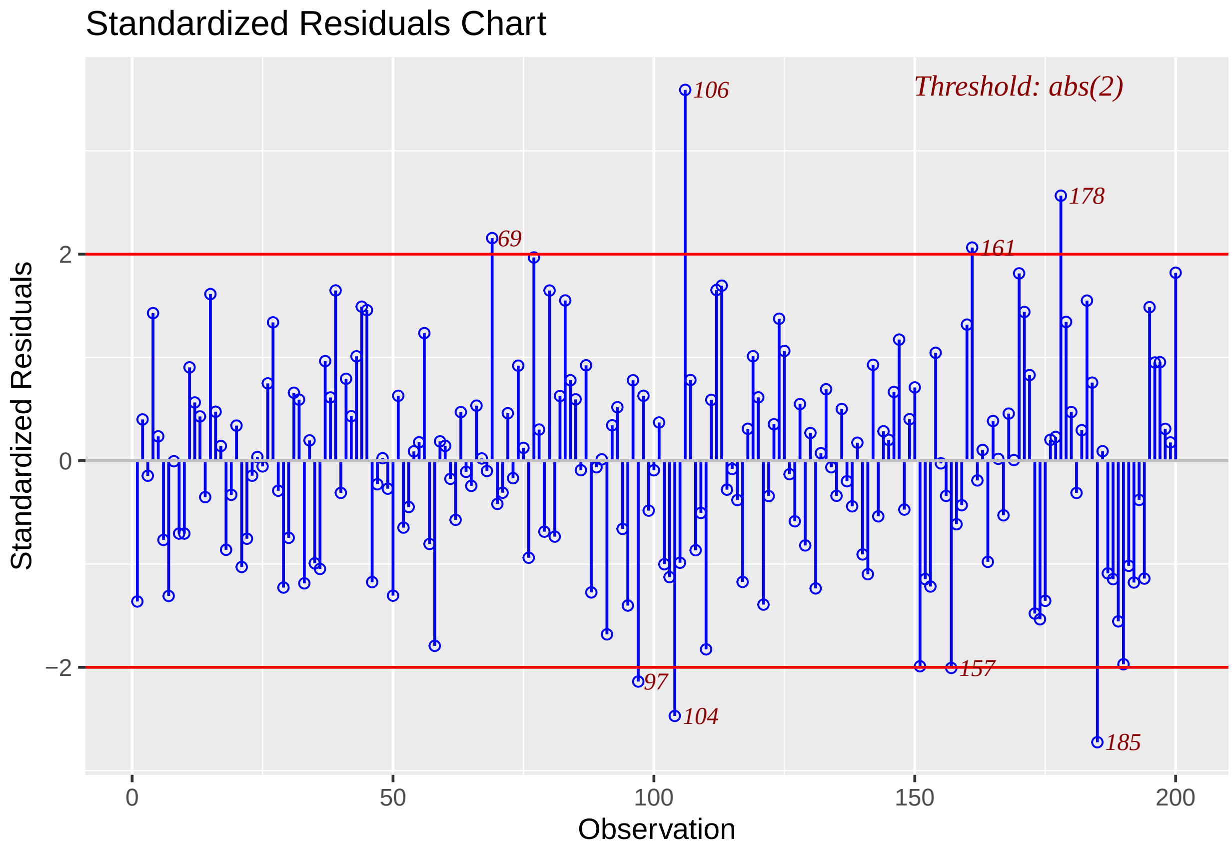
```
#Normality of errors tests
ols_test_normality(fit.fin)
```

```
## -----------------------------------------------
##        Test             Statistic       pvalue
## -----------------------------------------------
## Shapiro-Wilk              0.9952         0.7809
## Kolmogorov-Smirnov        0.0297         0.9946
## Cramer-von Mises         42.4756         0.0000
## Anderson-Darling          0.2129         0.8518
## -----------------------------------------------
```

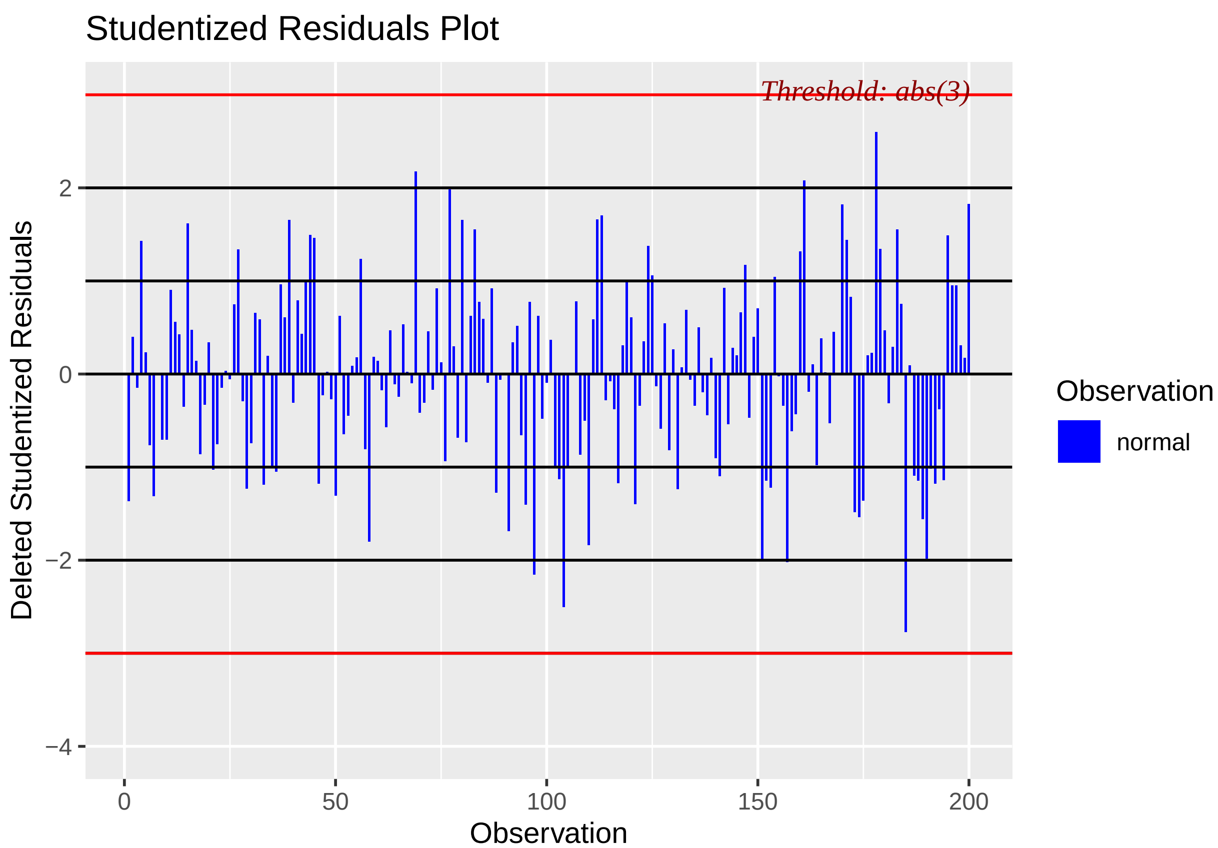
```
ols_test_correlation(fit.fin)
```

```
## [1] 0.9968848
```

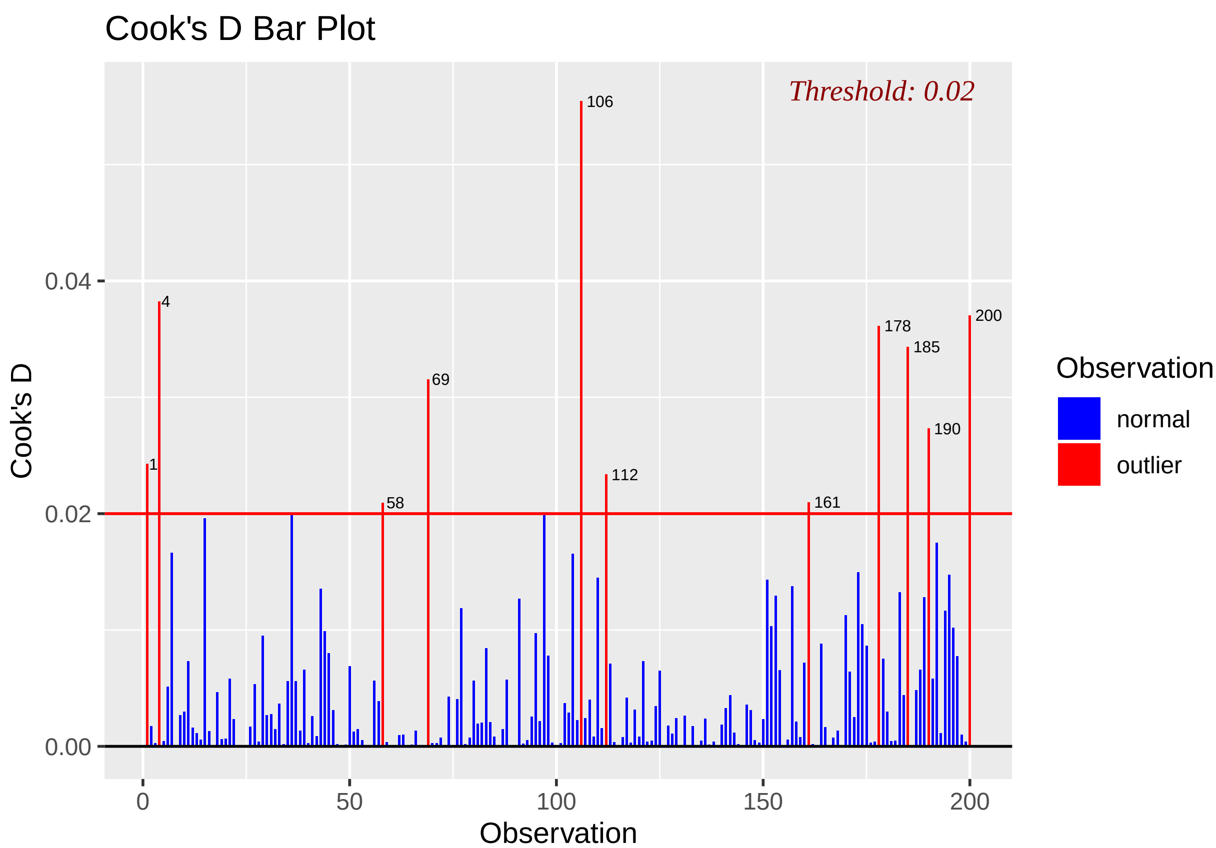
```
mod=fit.fin
#Standardized Residual plot
ols_plot_resid_stand(mod)
```



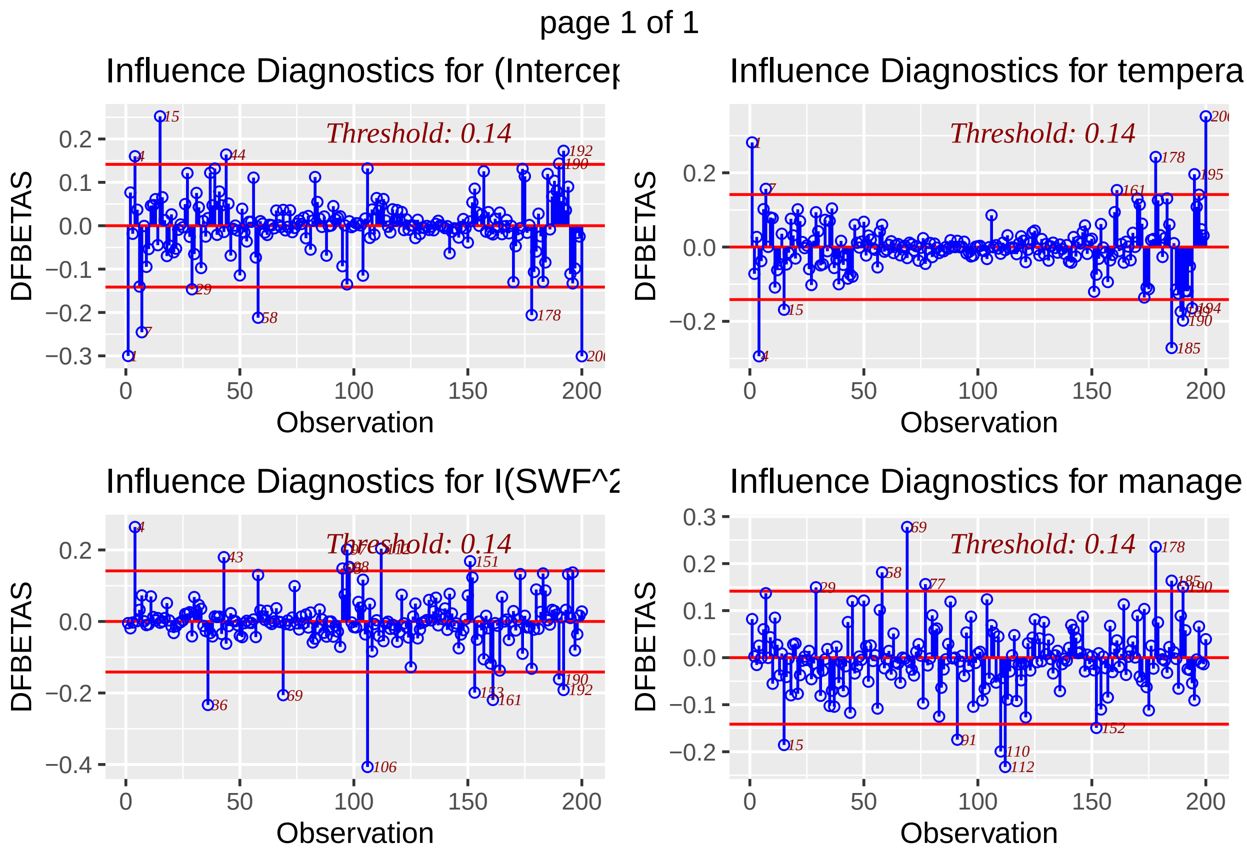

```r
#studentized residual plot
ols_plot_resid_stud(mod)
```



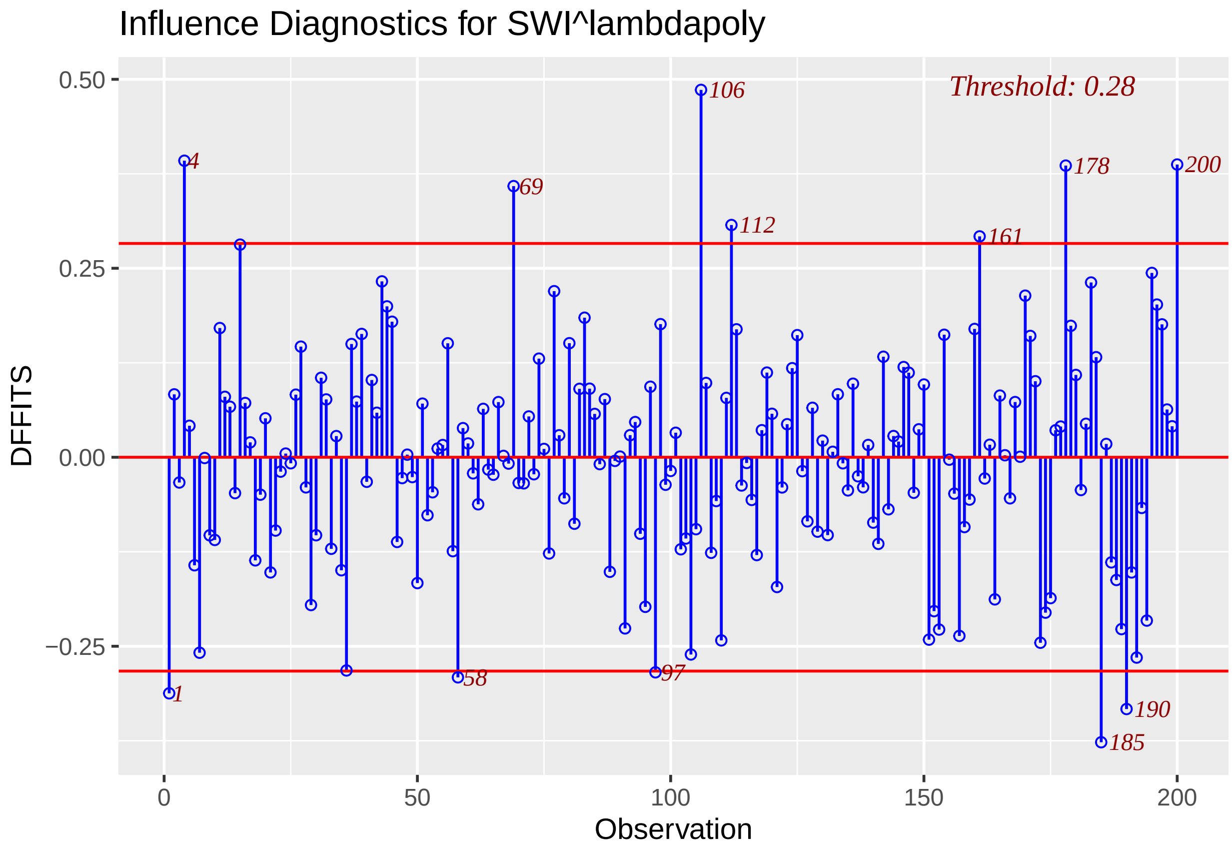

```r
#cooks distance plot
ols_plot_cooksd_bar(mod)
```



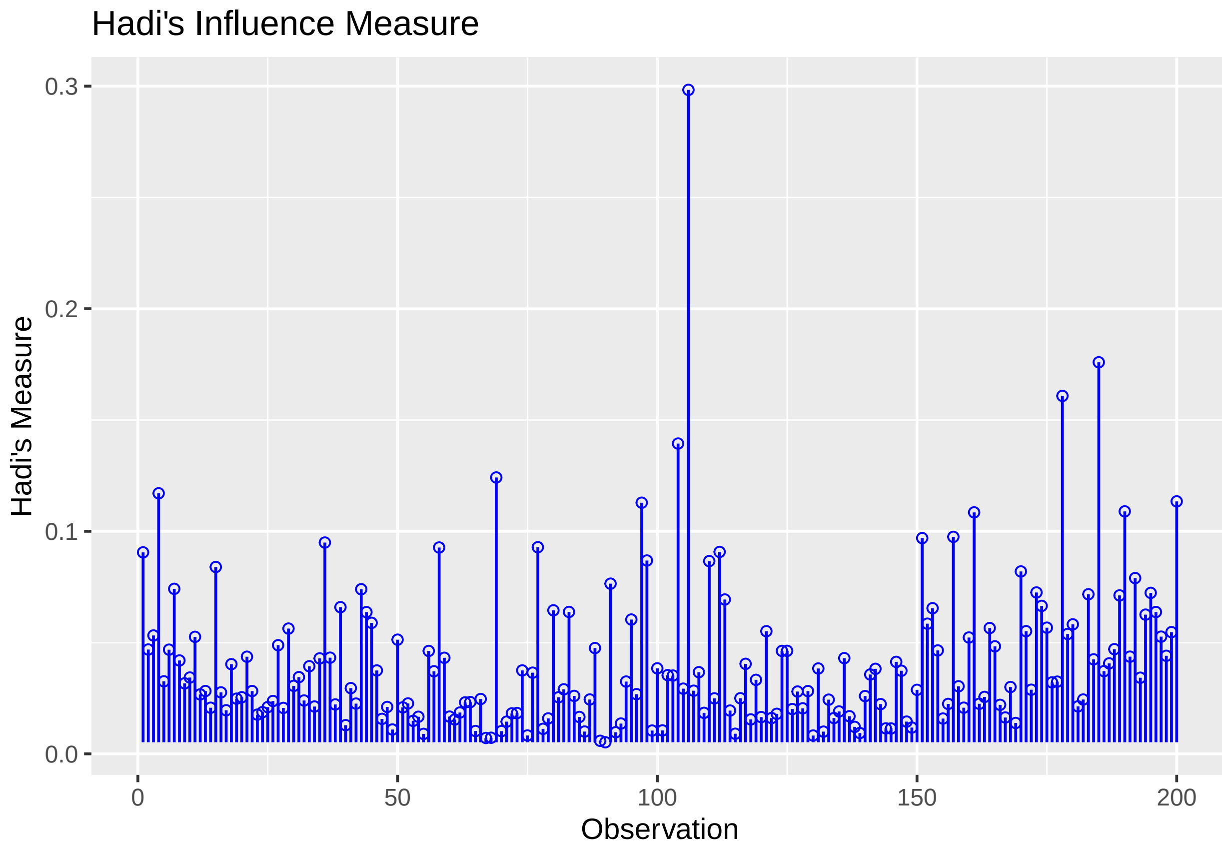

```r
#DFBetas for each variable
ols_plot_dfbetas(mod)
```




```r
#Difference in fit chart for each sample
ols_plot_dffits(mod)
```

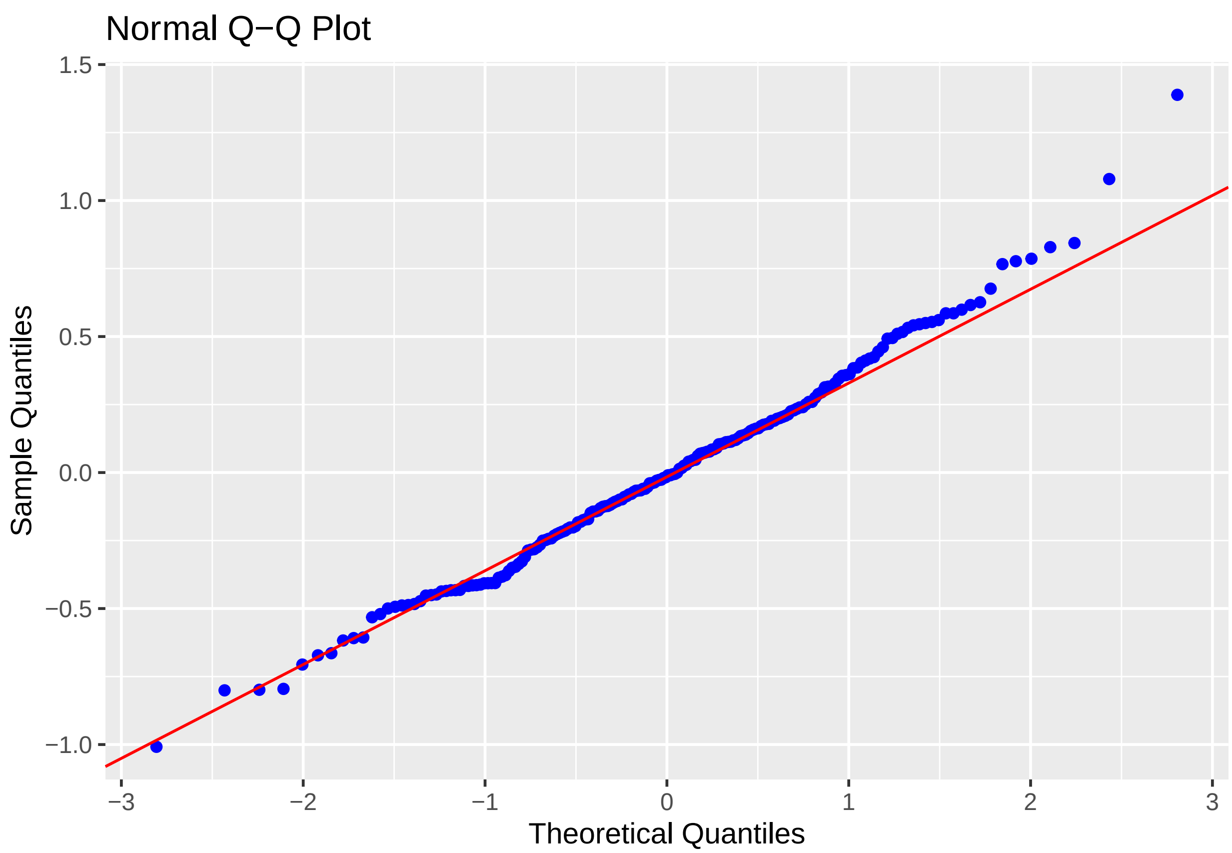



```r
#Plot for observation influence using hadi's distance
ols_plot_hadi(mod)
```

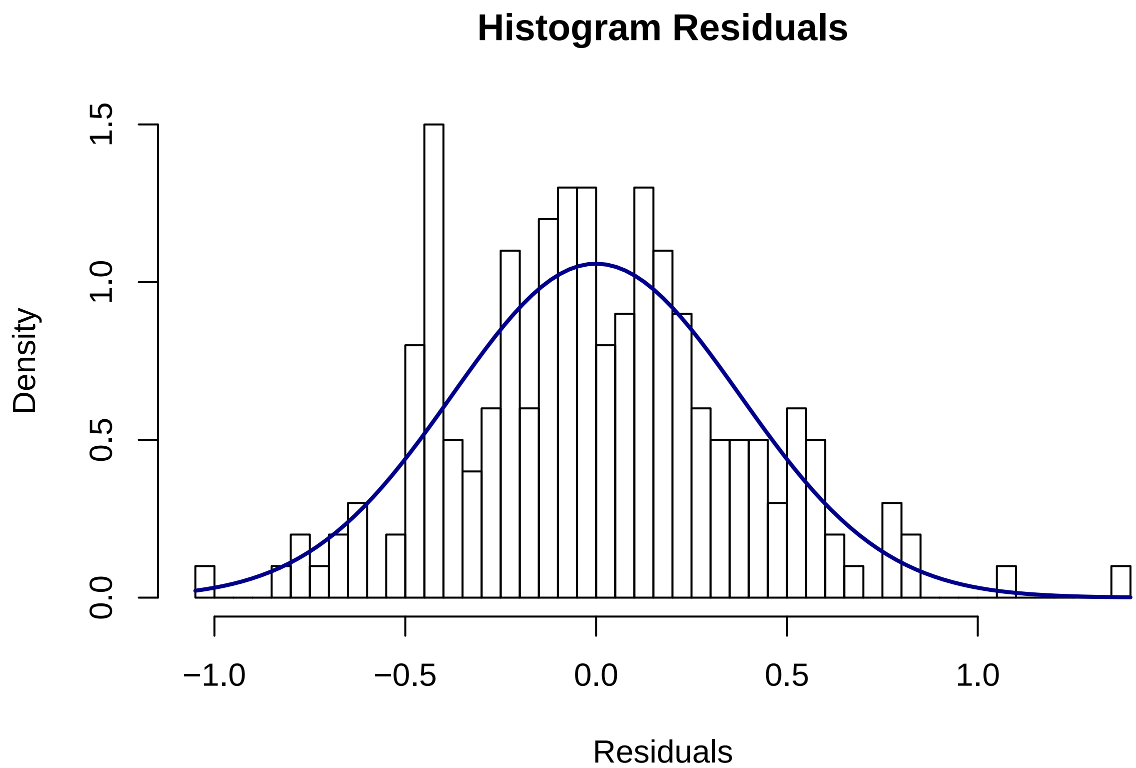


Alternative model diagnostics

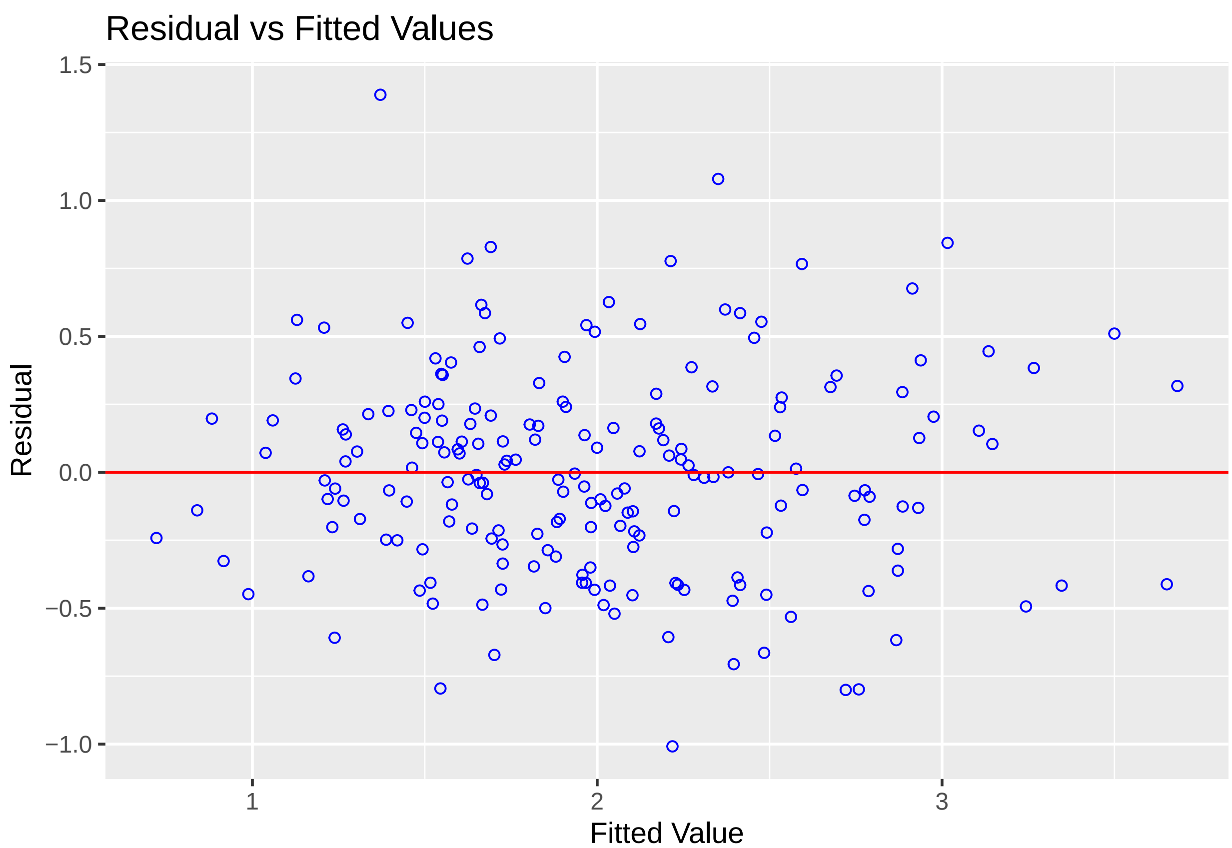
```
#QQ-Plot and Histogram to visualize normality of errors
ols_plot_resid_qq(fit.alt)
```




```r
hist(fit.alt$residuals,breaks = 50,
     xlab="Residuals", main="Histogram Residuals",
     probability = T)
curve(dnorm(x, mean=mean(fit.alt$residuals), sd=sd(fit.alt$residuals)),
      col="darkblue", lwd=2, add=TRUE, yaxt="n")
```




```
##Visualiztion of Residuals vs. Fitted Values to evaluate homoscedasticity
ols_plot_resid_fit(fit.alt)
```




```
#Breusch-Pagan Test for homoscedasticity
ncvTest(fit.alt)
```

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 0.5727483, Df = 1, p = 0.44917
```

```
#Normality of errors tests
ols_test_normality(fit.alt)
```

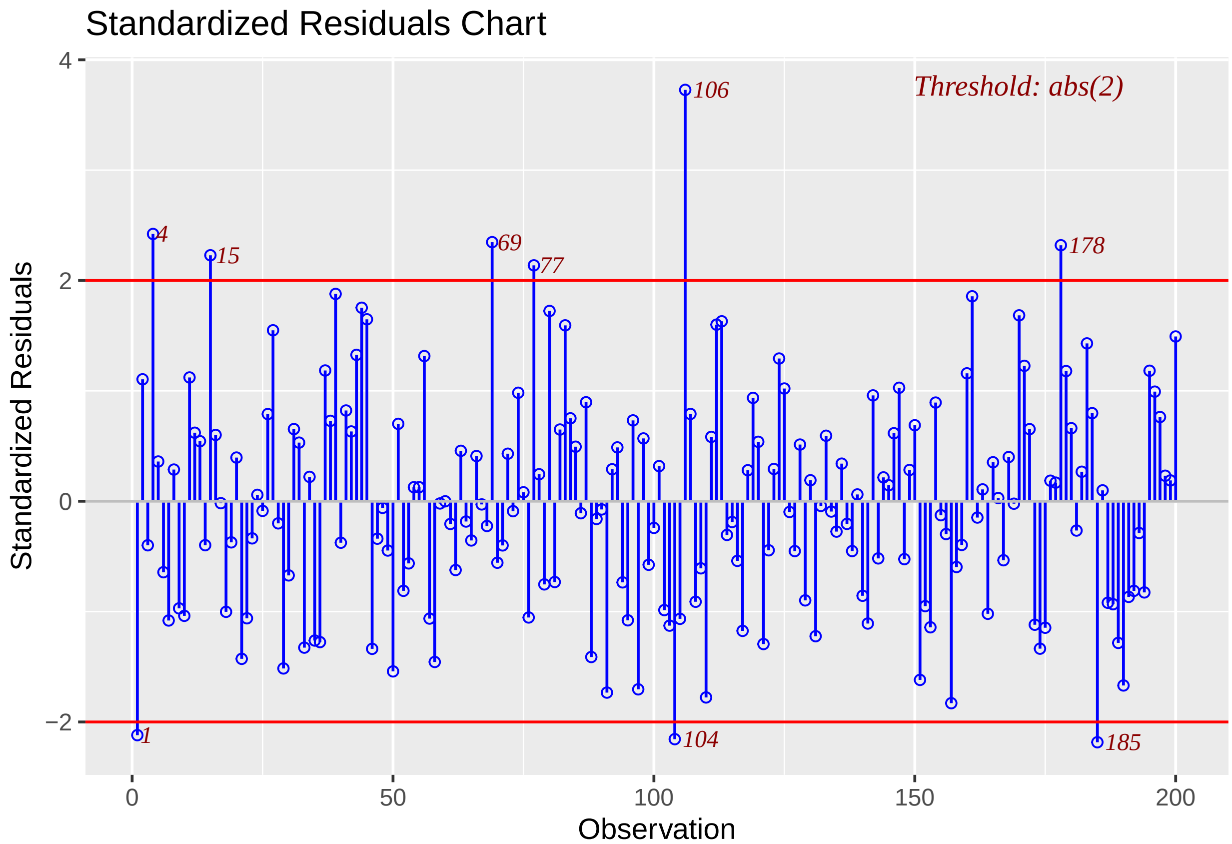
```
## -----------------------------------------------
##        Test             Statistic       pvalue
## -----------------------------------------------
## Shapiro-Wilk              0.9912         0.2689
## Kolmogorov-Smirnov        0.0365         0.9530
## Cramer-von Mises         30.6541         0.0000
## Anderson-Darling          0.3552         0.4570
## -----------------------------------------------
```

```
ols_test_correlation(fit.alt)
```

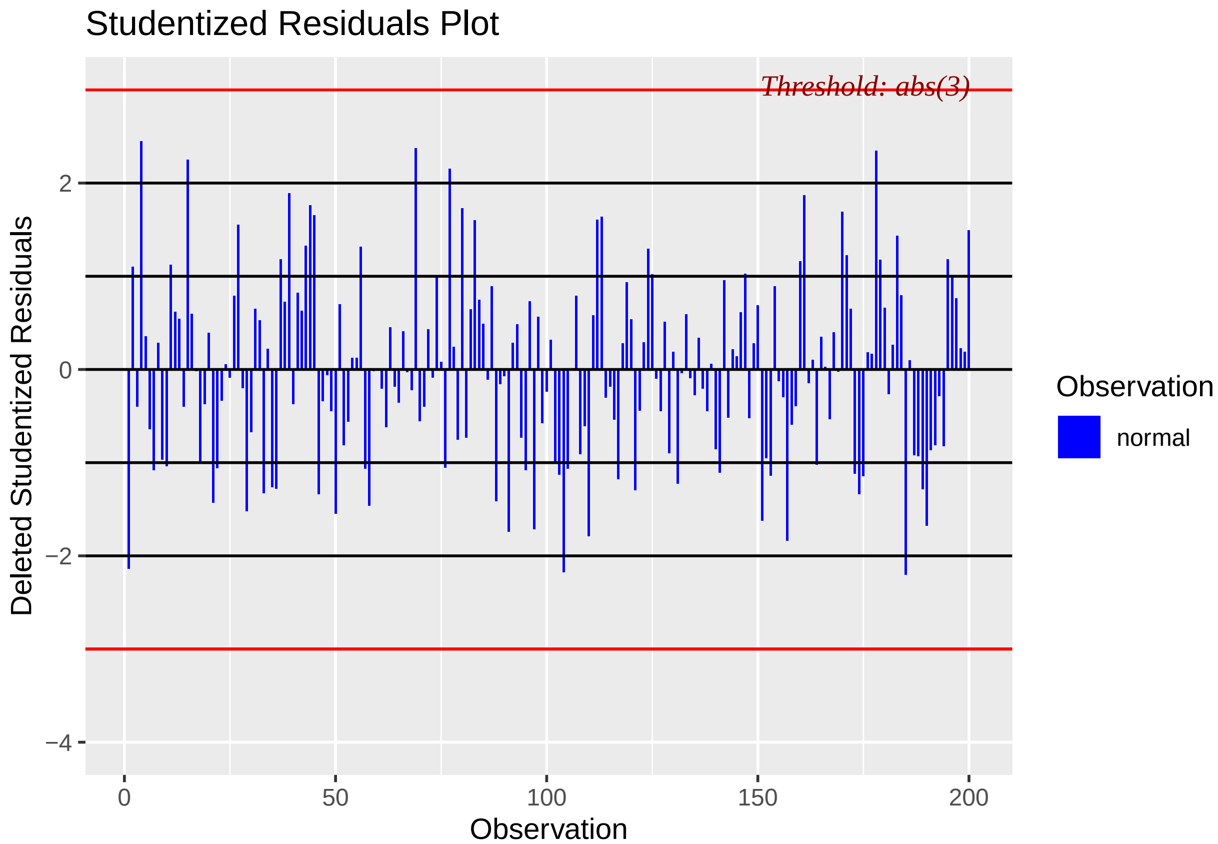
```
## Warning in cor(h, out): the standard deviation is zero
```

```
## [1] NA
```

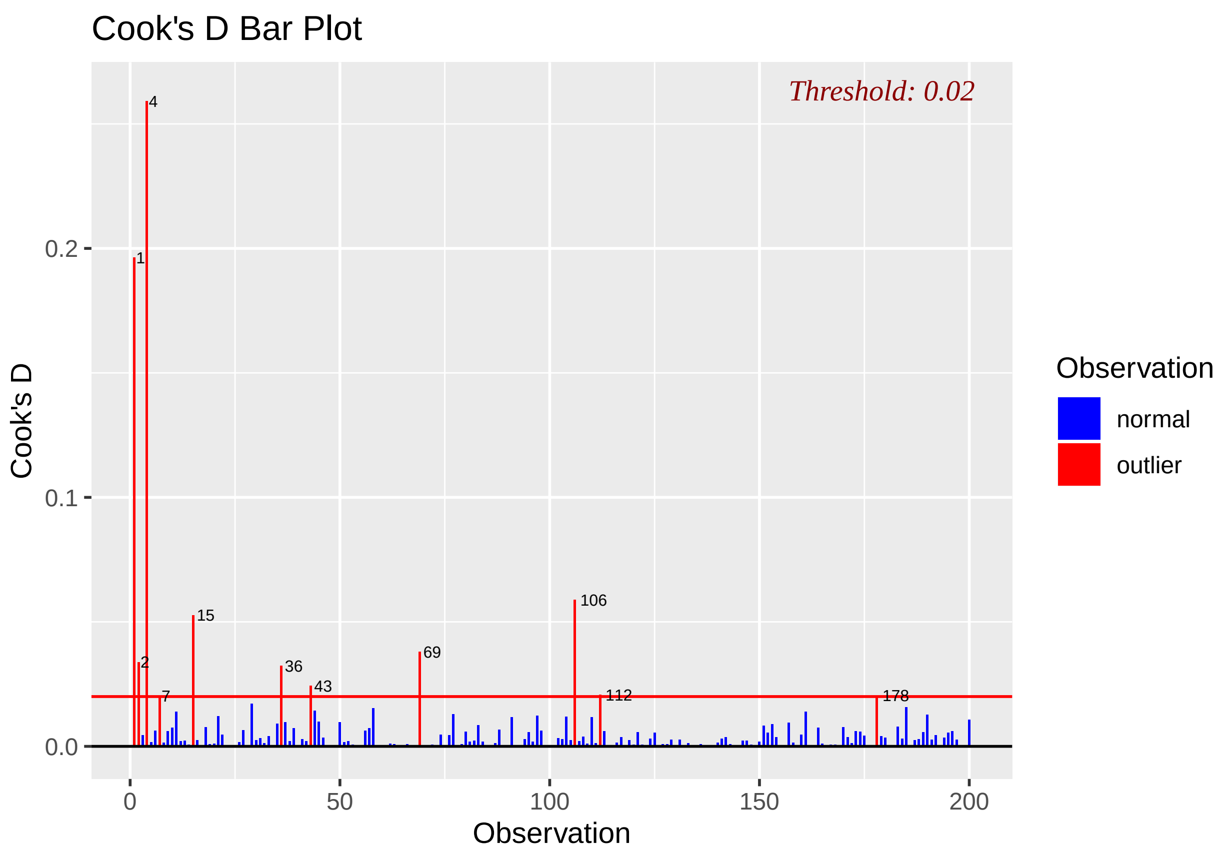
```
mod=fit.alt
#Standardized Residual plot
ols_plot_resid_stand(mod)
```



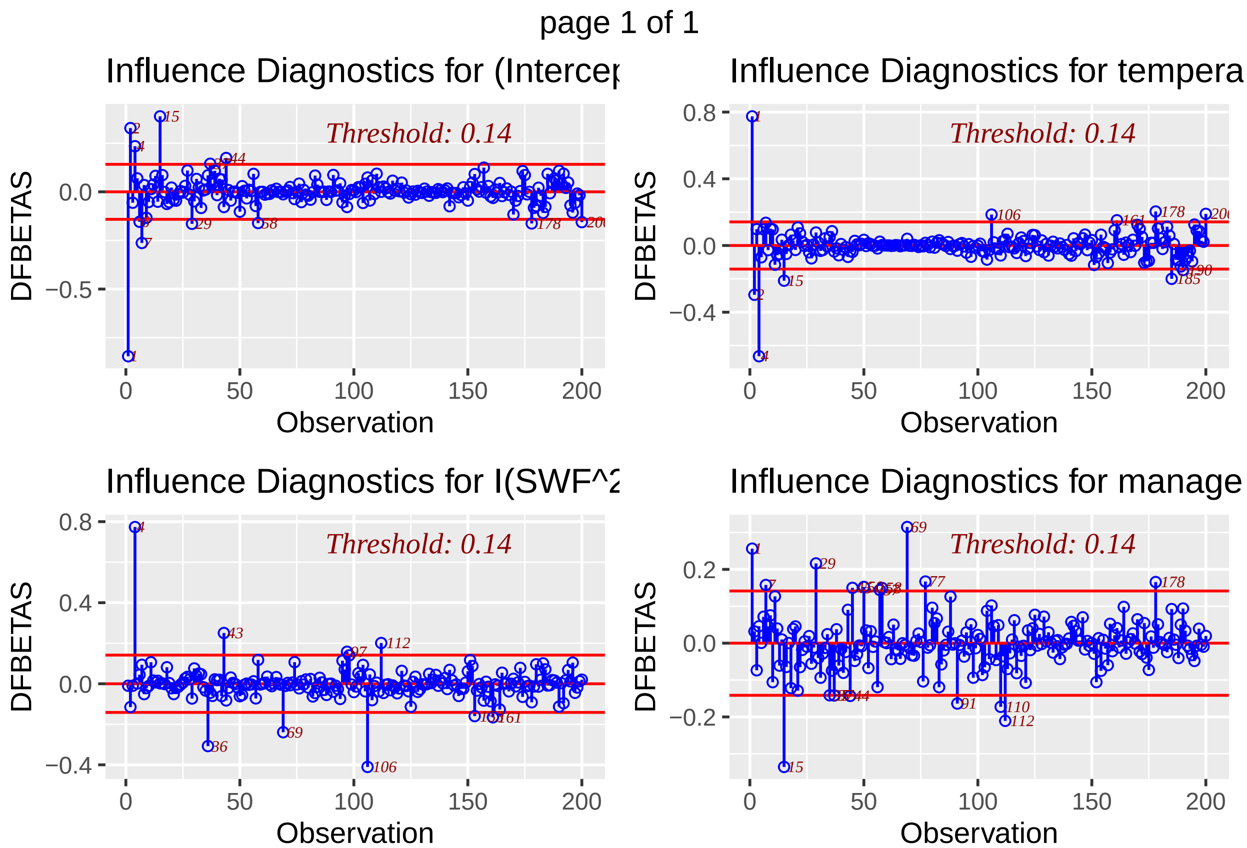

```r
#studentized residual plot
ols_plot_resid_stud(mod)
```



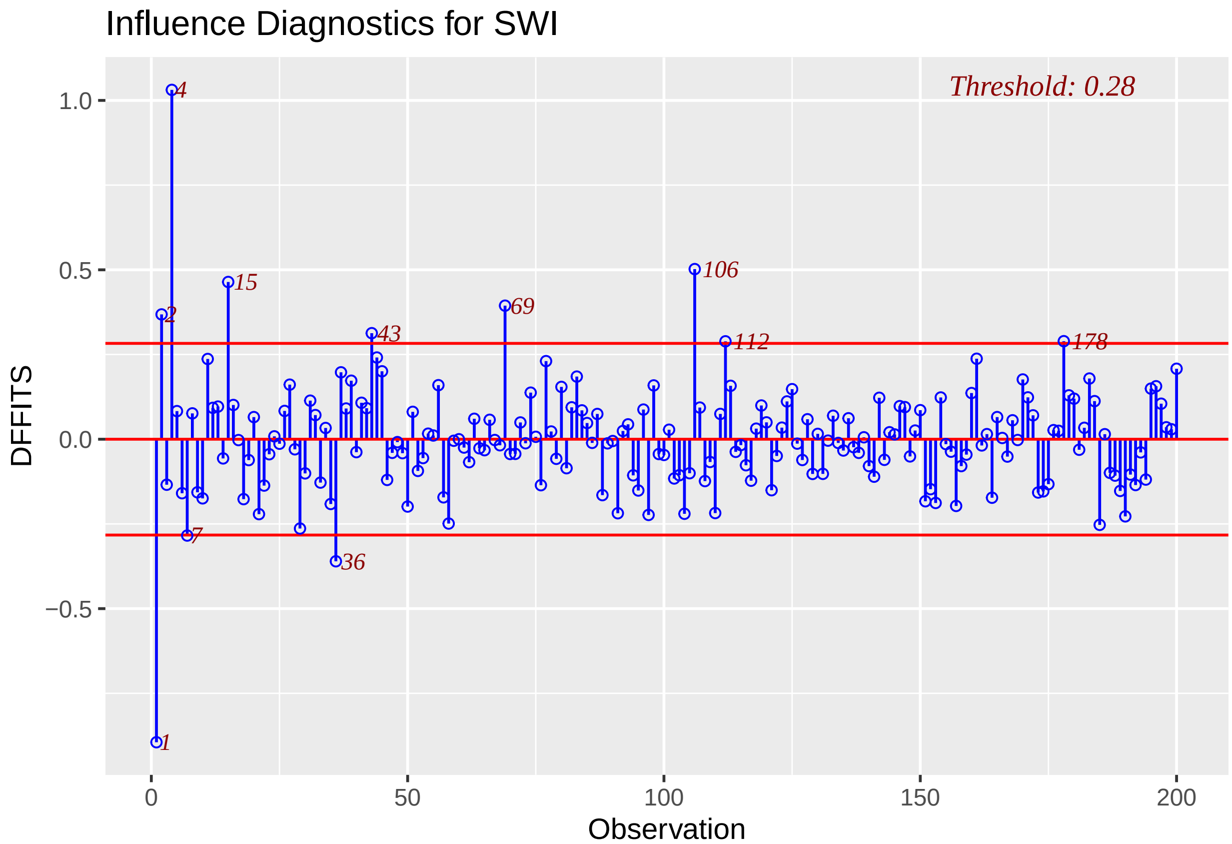

```r
#cooks distance plot
ols_plot_cooksd_bar(mod)
```



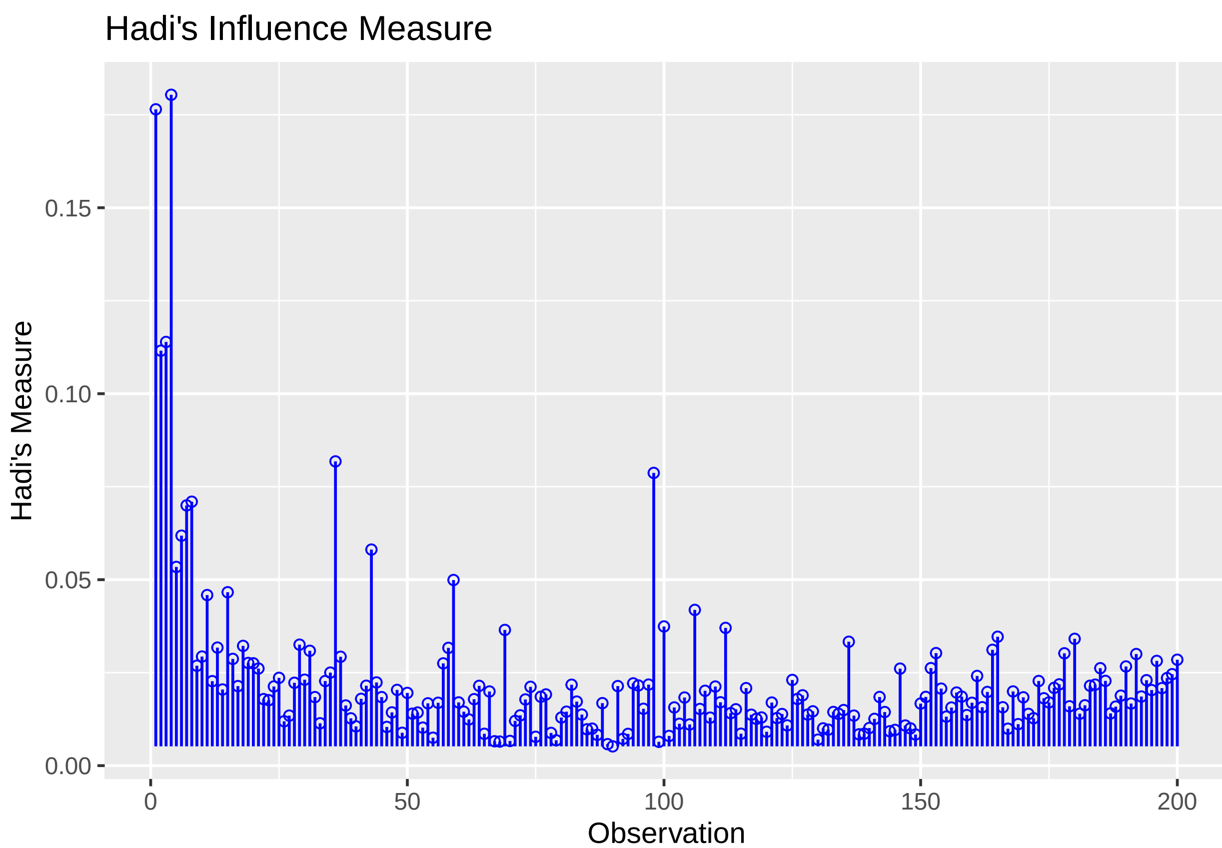

```r
#DFBetas for each variable
ols_plot_dfbetas(mod)
```




```r
#Difference in fit chart for each sample
ols_plot_dffits(mod)
```




```r
#Plot for observation influence using hadi's distance
ols_plot_hadi(mod)
```



All assumptions met for both models. Outliers are still influencing the models but remain unredacted as per assignment instructions.

RMSE of models against the test data.

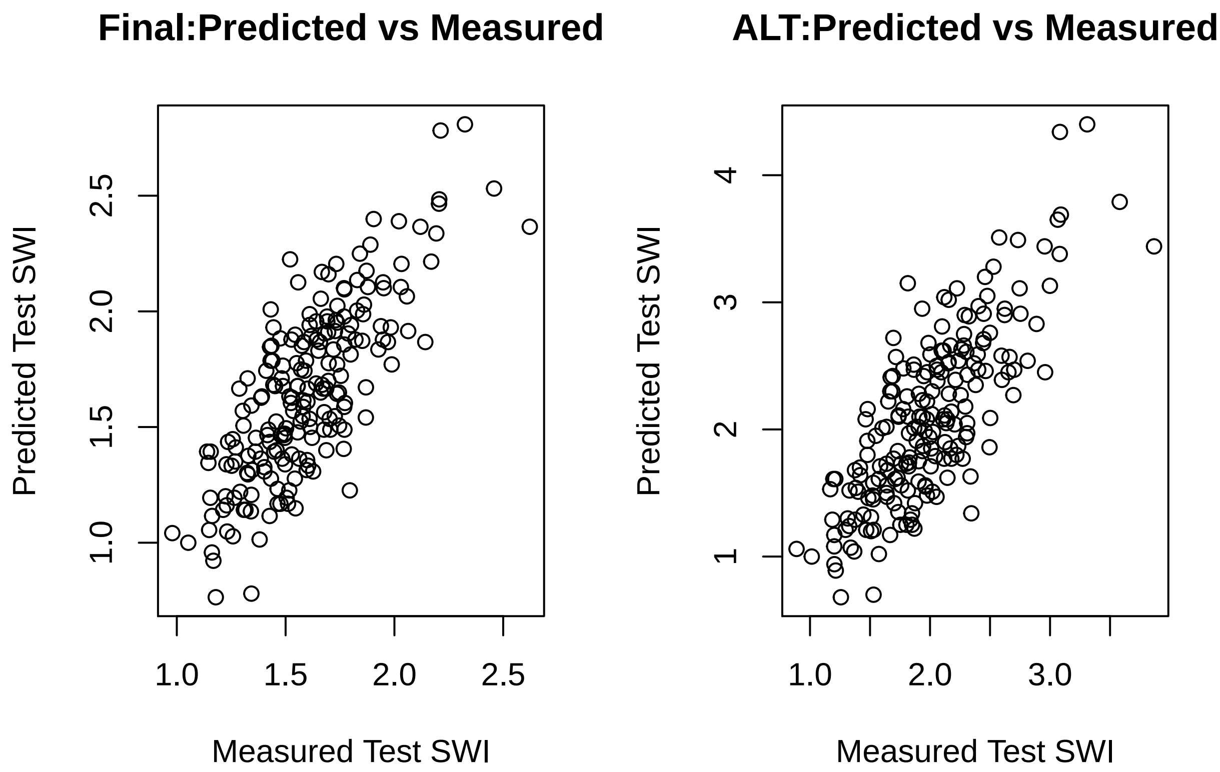
```r
#RMSE of final and alternative models when test data is applied
results.fin=predict(fit.fin,data.test)
results.alt=predict(fit.alt,data.test)
rmse(results.fin,data.test$SWI^lambdapoly)
```

```
## [1] 0.2426738
```

```r
rmse(results.alt,data.test$SWI)
```

```
## [1] 0.434435
```

```r
#Plots of final and alternative models predicting test data results
par(mfrow=(c(1,2)))
plot(results.fin,data.test$SWI^lambdapoly,
     xlab="Measured Test SWI",ylab="Predicted Test SWI",main="Final:Predicted vs Measured")
plot(results.alt,data.test$SWI,
     xlab="Measured Test SWI",ylab="Predicted Test SWI",main="ALT:Predicted vs Measured")
```



Final model with box-cox transform is out performing the WLS model.

Fitting the test data to the models and evaluating the results.

```r
fit.fin.test=lm(formula = SWI^lambdapoly ~ I(SWF^2) + temperature + management,
           data = data.test)
fit.alt.test=lm(formula = SWI ~ I(SWF^2) + temperature + management,
           data = data.test, weights = w1)
summary(fit.fin.test)
```

```
##
## Call:
## lm(formula = SWI^lambdapoly ~ I(SWF^2) + temperature + management,
##     data = data.test)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.67351 -0.17479 -0.00278  0.17206  0.66883
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.495702   0.080482   6.159 4.07e-09 ***
## I(SWF^2)    0.170526   0.011368  15.000  < 2e-16 ***
## temperature 0.030994   0.003425   9.048  < 2e-16 ***
## management  0.036173   0.006678   5.417 1.77e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.2368 on 196 degrees of freedom
## Multiple R-squared:  0.6104, Adjusted R-squared:  0.6045
## F-statistic: 102.4 on 3 and 196 DF,  p-value: < 2.2e-16
```

```r
summary(fit.alt.test)
```

```
##
## Call:
## lm(formula = SWI ~ I(SWF^2) + temperature + management, data = data.test,
##     weights = w1)
##
## Weighted Residuals:
##     Min      1Q  Median      3Q     Max
## -3.8662 -1.0027 -0.1475  0.9436  4.1657
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.05709    0.14166  -0.403    0.687
## I(SWF^2)     0.30982    0.02146  14.440  < 2e-16 ***
## temperature  0.05863    0.00599   9.788  < 2e-16 ***
## management   0.07078    0.01192   5.940 1.28e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.514 on 196 degrees of freedom
## Multiple R-squared:  0.6165, Adjusted R-squared:  0.6106
## F-statistic:   105 on 3 and 196 DF,  p-value: < 2.2e-16
```

```r
#comparing to base model to examine potential overfitting
fit.basic.test=lm(formula = SWI ~ SWF + temperature + management + size,
           data = data.test)


#investigating if the generalization problem resides
#within the choice of training/test data.
summary(fit.basic)
```

```
##
## Call:
## lm(formula = SWI ~ . - duration, data = data.training)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.02005 -0.27531 -0.02608  0.24343  1.53643
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.357059   0.147639  -2.418   0.0165 *
## SWF          0.834942   0.059745  13.975  < 2e-16 ***
## temperature  0.048518   0.005275   9.198  < 2e-16 ***
## size        -0.001927   0.001576  -1.222   0.2231
## management   0.063350   0.011409   5.553 9.13e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.3961 on 195 degrees of freedom
## Multiple R-squared:  0.6694, Adjusted R-squared:  0.6626
## F-statistic:  98.7 on 4 and 195 DF,  p-value: < 2.2e-16
```

```r
summary(fit.basic.test)
```

```
##
## Call:
## lm(formula = SWI ~ SWF + temperature + management + size, data = data.test)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.23253 -0.28300 -0.00478  0.28897  1.29020
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.638053   0.179246  -3.560 0.000466 ***
## SWF          0.919424   0.063734  14.426  < 2e-16 ***
## temperature  0.054235   0.006331   8.567  3.2e-15 ***
## management   0.068697   0.012296   5.587  7.7e-08 ***
## size         0.001090   0.001561   0.698 0.485735
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4356 on 195 degrees of freedom
## Multiple R-squared:  0.5974, Adjusted R-squared:  0.5892
## F-statistic: 72.35 on 4 and 195 DF,  p-value: < 2.2e-16
```

```r
#generalization problem appears to be a result of the test data set
#as perfromance is worse even without any transforms or reweighting
```

Merging the training and test data together and assigning class labels. (this is only used for making one of the following plots easier to code)

```r
data.training$class=rep(2,200)
data.test$class=rep(3,200)
data.withclass=rbind(data.training,data.test)
```

two sided t-tests to compare distributions of each variable across training and test data

```r
t.test(data.training$SWI,data.test$SWI,"two.sided")
```

```
##
##  Welch Two Sample t-test
##
## data:  data.training$SWI and data.test$SWI
## t = -1.4506, df = 398, p-value = 0.1477
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.23258612  0.03508612
## sample estimates:
## mean of x mean of y
##   1.99375   2.09250
```

```r
t.test(data.training$SWF,data.test$SWF,"two.sided")
```

```
##
##  Welch Two Sample t-test
```

```
##
## data:  data.training$SWF and data.test$SWF
## t = 0.011292, df = 397.96, p-value = 0.991
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.09520335  0.09630335
## sample estimates:
## mean of x mean of y
##   1.49750   1.49695
```

```
t.test(data.training$temperature,data.test$temperature,"two.sided")
```

```
##
##  Welch Two Sample t-test
##
## data:  data.training$temperature and data.test$temperature
## t = 0.0077209, df = 395.14, p-value = 0.9938
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.014527  1.022527
## sample estimates:
## mean of x mean of y
##   18.9495   18.9455
```

```
t.test(data.training$management,data.test$management,"two.sided")
```

```
##
##  Welch Two Sample t-test
##
## data:  data.training$management and data.test$management
## t = 0, df = 398, p-value = 1
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.4943628  0.4943628
## sample estimates:
## mean of x mean of y
##     4.115     4.115
```

```
t.test(data.training$size,data.test$size,"two.sided")
```

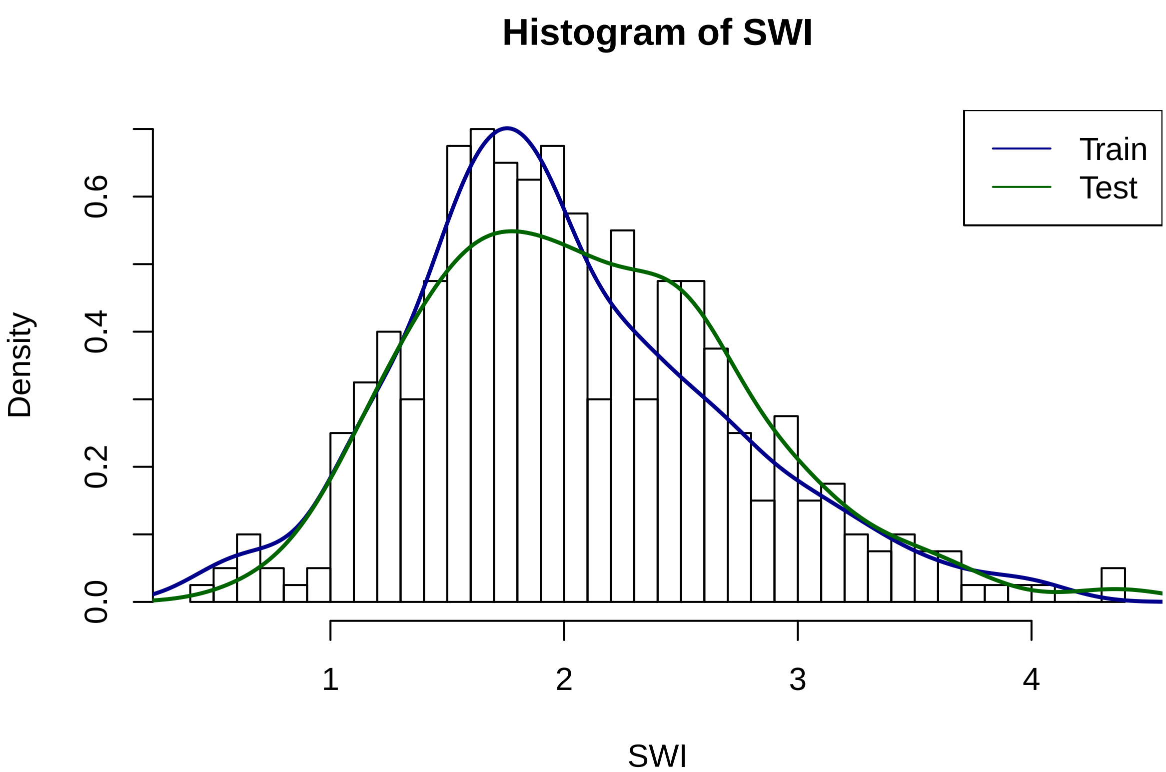
```
##
##  Welch Two Sample t-test
##
## data:  data.training$size and data.test$size
## t = 0.49369, df = 393.78, p-value = 0.6218
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -2.791402  4.663402
## sample estimates:
## mean of x mean of y
##   41.3155   40.3795
```

```
#SWI shows a loose relation between the two data sets
#relative to the tests performed on other variables.

#This implies that either the training or test data set
#may have an unusually high number of outliers.
```

P-value of t-test for SWI is only .14 compared to .9+ for each other variable excluding size.

Plot of Histogram of all SWI values and the density plots of the training and test data

```
hist(data.withclass$SWI,breaks=50,prob=T,main="Histogram of SWI",xlab="SWI")
lines(density(data.training$SWI), col="darkblue", lwd=2, yaxt="n")
lines(density(data.test$SWI), col="darkgreen", lwd=2, yaxt="n")
legend("topright",legend=c("Train","Test"),col=c("darkblue","darkgreen"),lty=c(1,1))
```



This plot shows quite the substantial difference in the two distributions leading to poor generalization with model building.

Final and alternative model 95% regression coeficients interval

```
confint(fit.fin)
```

```
##                  2.5 %     97.5 %
## (Intercept) 0.42291995 0.66918447
## I(SWF^2)    0.13520836 0.17681183
## temperature 0.02141378 0.03269399
## management  0.02482665 0.04926958
```

```
confint(fit.fin.test)
```

```
##                  2.5 %     97.5 %
## (Intercept) 0.33697952 0.65442507
## I(SWF^2)    0.14810604 0.19294625
## temperature 0.02423844 0.03774877
## management  0.02300331 0.04934291
```

```r
confint(fit.alt)
```

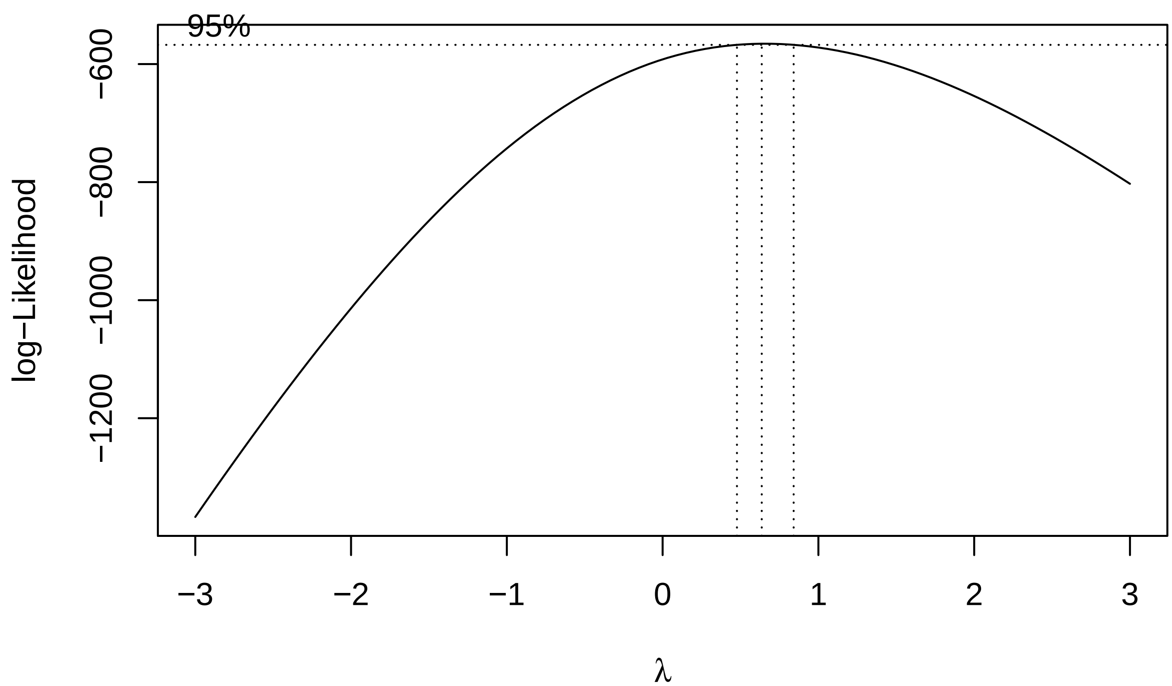
```
##                    2.5 %     97.5 %
## (Intercept) -0.03117542 0.31543985
## I(SWF^2)     0.25357277 0.32524800
## temperature  0.03806482 0.05523197
## management   0.04122228 0.08091336
```

```r
confint(fit.alt.test)
```

```
##                    2.5 %     97.5 %
## (Intercept) -0.33646650 0.22229391
## I(SWF^2)     0.26750405 0.35213102
## temperature  0.04682043 0.07044803
## management   0.04728440 0.09428369
```

Fitting ultimate model to full data set.

```r
fullmodel.ult=lm(SWI~I(SWF^2)+temperature+management,data=data)
bc.ult=boxcox(fullmodel.ult,lambda = seq(-3,3))
```




```r
lambdault=bc$x[which(bc$y==max(bc$y))]
lambdault
```

```
## [1] 0.4545455
```

```r
fit.bc.ult=lm(SWI^lambdault~I(SWF^2)+temperature+management,data=data)
summary(fit.bc.ult)
```

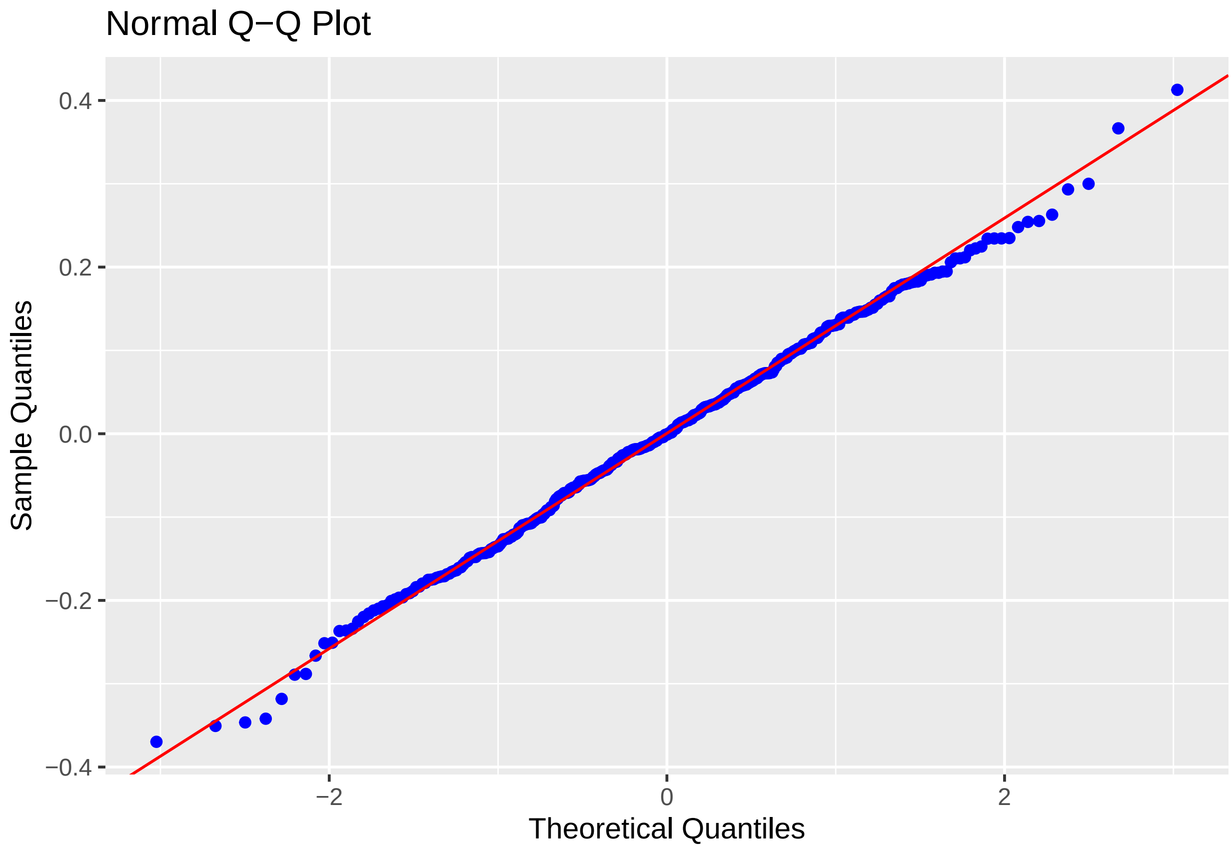
```
##
## Call:
## lm(formula = SWI^lambdault ~ I(SWF^2) + temperature + management,
##     data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.36971 -0.08664 -0.00097  0.08763  0.41273
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.760397   0.027906  27.249  < 2e-16 ***
## I(SWF^2)    0.088873   0.004299  20.673  < 2e-16 ***
## temperature 0.015846   0.001232  12.867  < 2e-16 ***
## management  0.020289   0.002552   7.949 1.98e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1273 on 396 degrees of freedom
## Multiple R-squared:  0.6393, Adjusted R-squared:  0.6366
## F-statistic:   234 on 3 and 396 DF,  p-value: < 2.2e-16
```

```r
confint(fit.bc.ult)
```

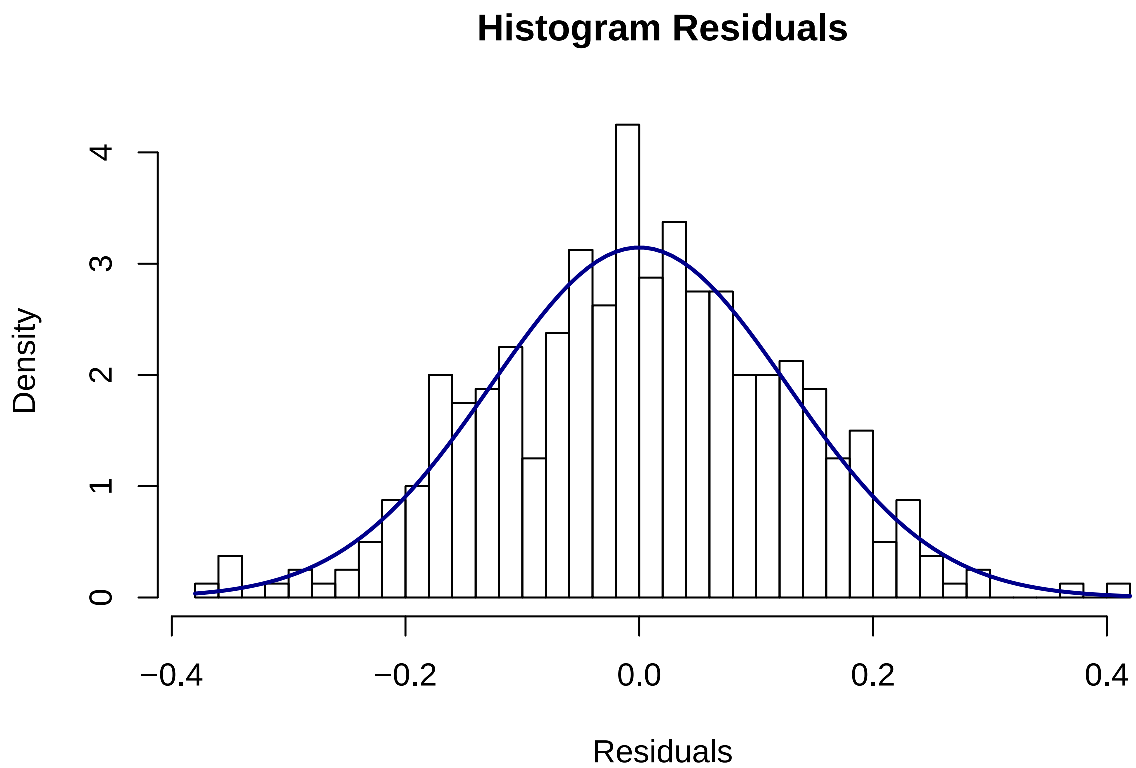
```
##                  2.5 %     97.5 %
## (Intercept) 0.70553500 0.81525939
## I(SWF^2)    0.08042145 0.09732497
## temperature 0.01342487 0.01826734
## management  0.01527115 0.02530666
```

Model diagnostics for ultimate model

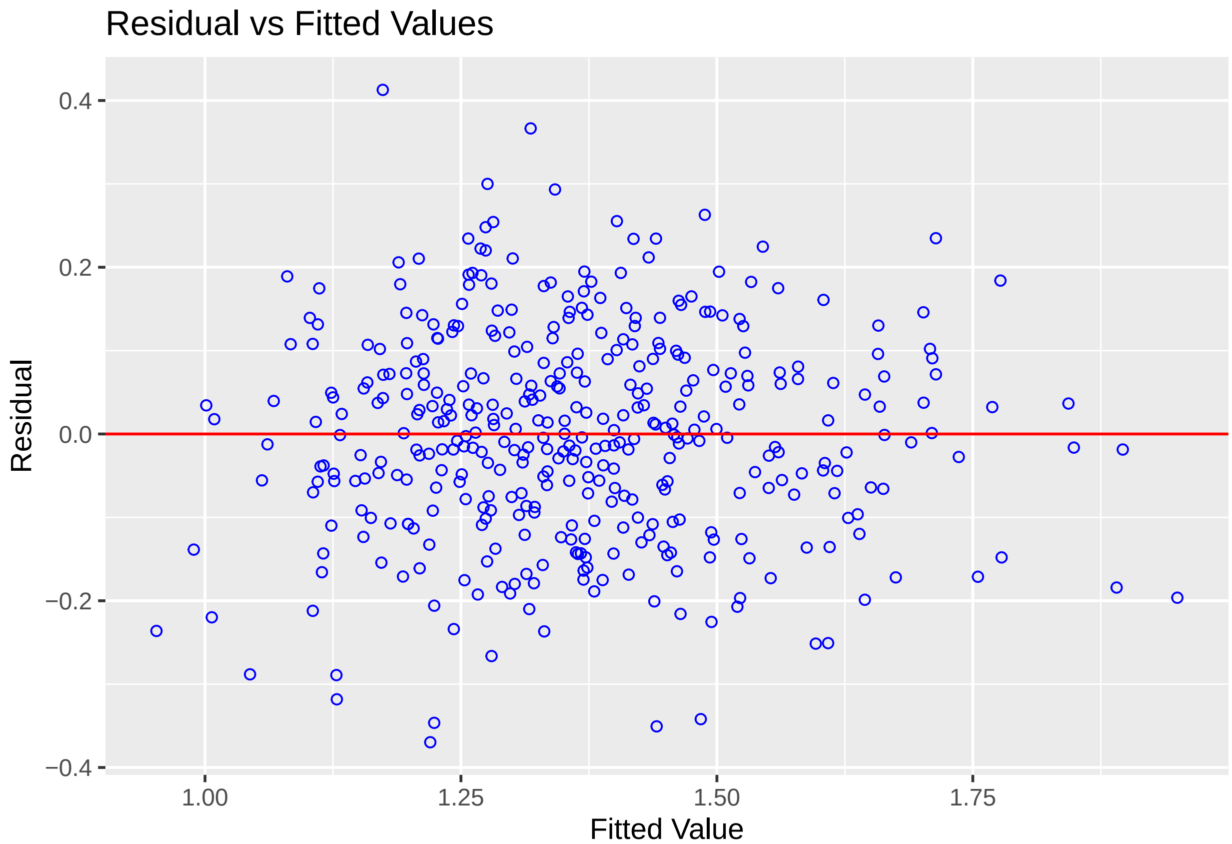
```r
mod=fit.bc.ult
#QQ-Plot and Histogram to visualize normality of errors
ols_plot_resid_qq(mod)
```




```
hist(mod$residuals,breaks = 50,
     xlab="Residuals", main="Histogram Residuals",
     probability = T)
curve(dnorm(x, mean=mean(mod$residuals), sd=sd(mod$residuals)),
      col="darkblue", lwd=2, add=TRUE, yaxt="n")
```




```
##Visualiztion of Residuals vs. Fitted Values to evaluate homoscedasticity
ols_plot_resid_fit(mod)
```




```r
#Breusch-Pagan Test for homoscedasticity
ncvTest(mod)
```

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 1.258875, Df = 1, p = 0.26186
```

```r
#Normality of errors tests
ols_test_normality(mod)
```

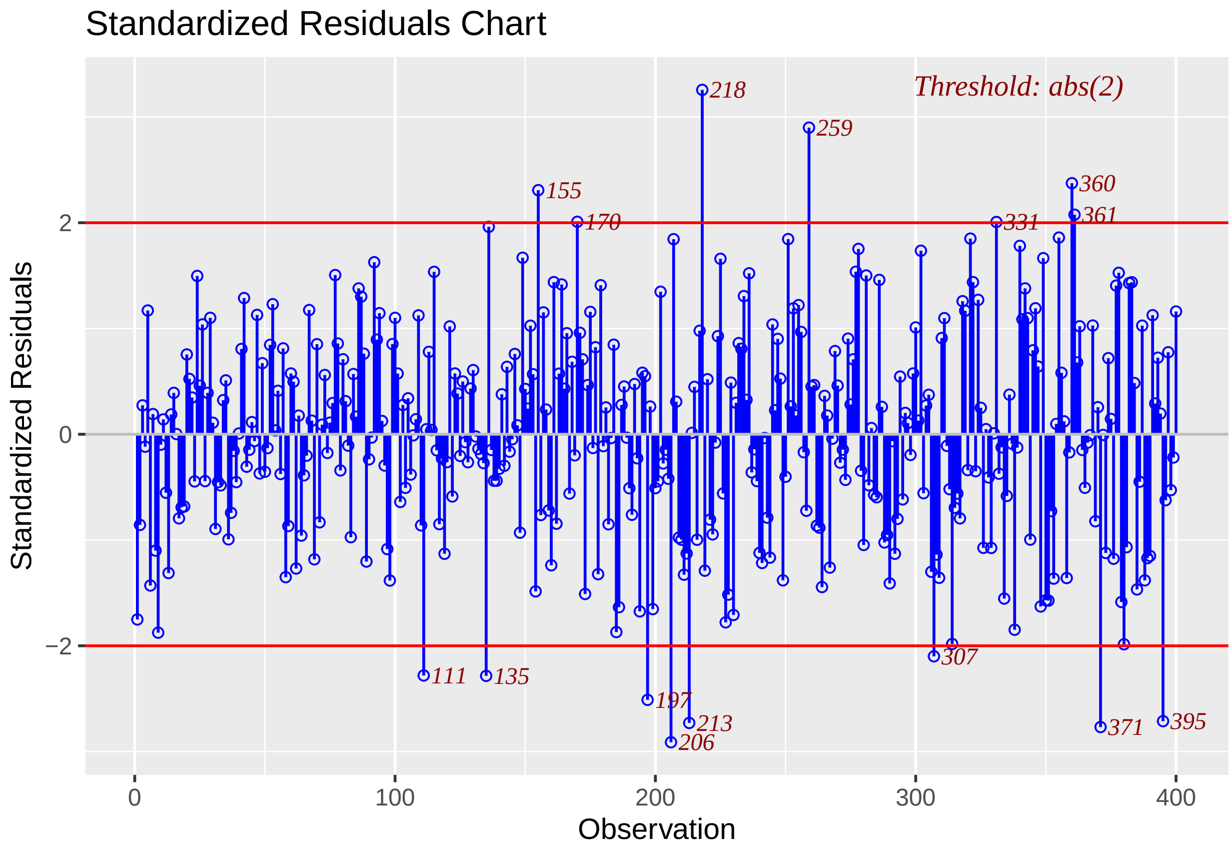
```
## -----------------------------------------------
##        Test             Statistic       pvalue
## -----------------------------------------------
## Shapiro-Wilk              0.9976         0.8368
## Kolmogorov-Smirnov        0.0239         0.9767
## Cramer-von Mises        102.138          0.0000
## Anderson-Darling          0.2144         0.8492
## -----------------------------------------------
```

```r
ols_test_correlation(mod)
```

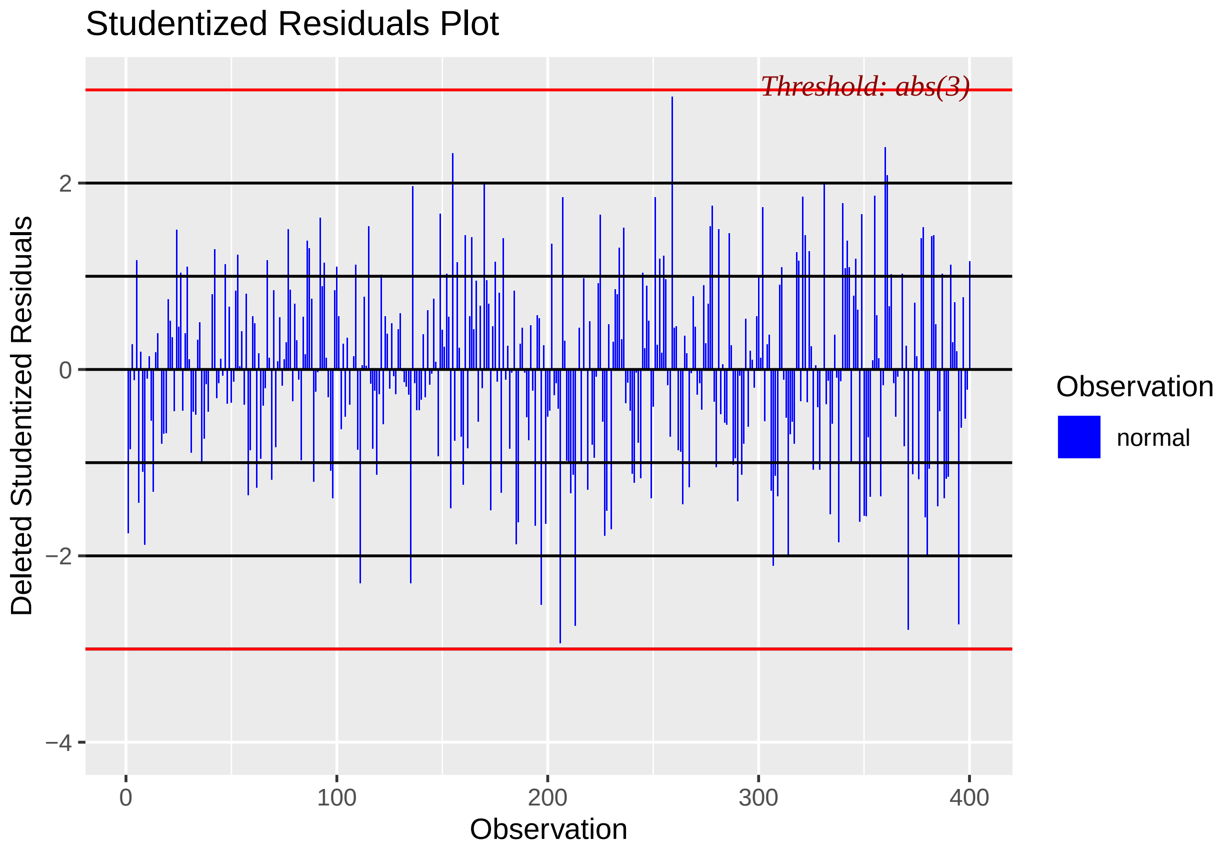
```
## [1] 0.9987477
```

More ultimate model diagnostics

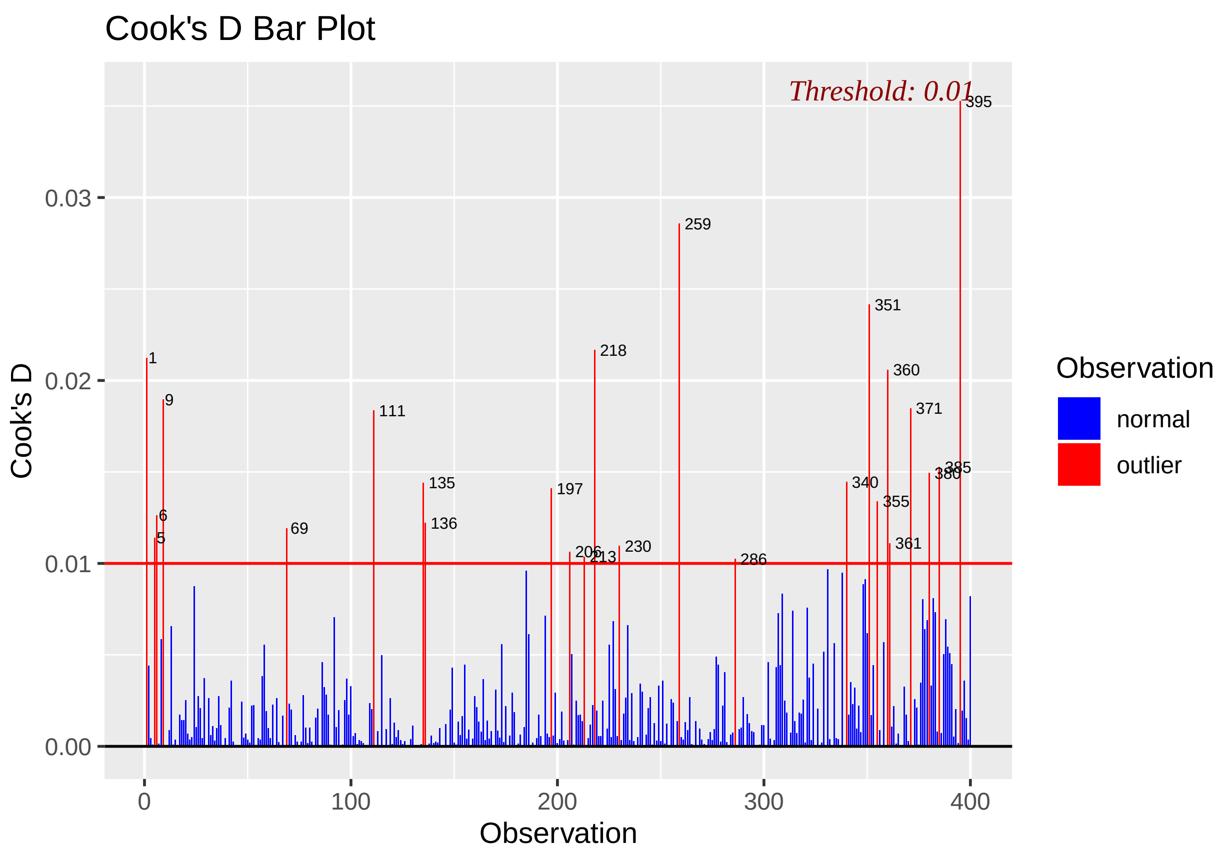
```r
#Standardized Residual plot
ols_plot_resid_stand(mod)
```



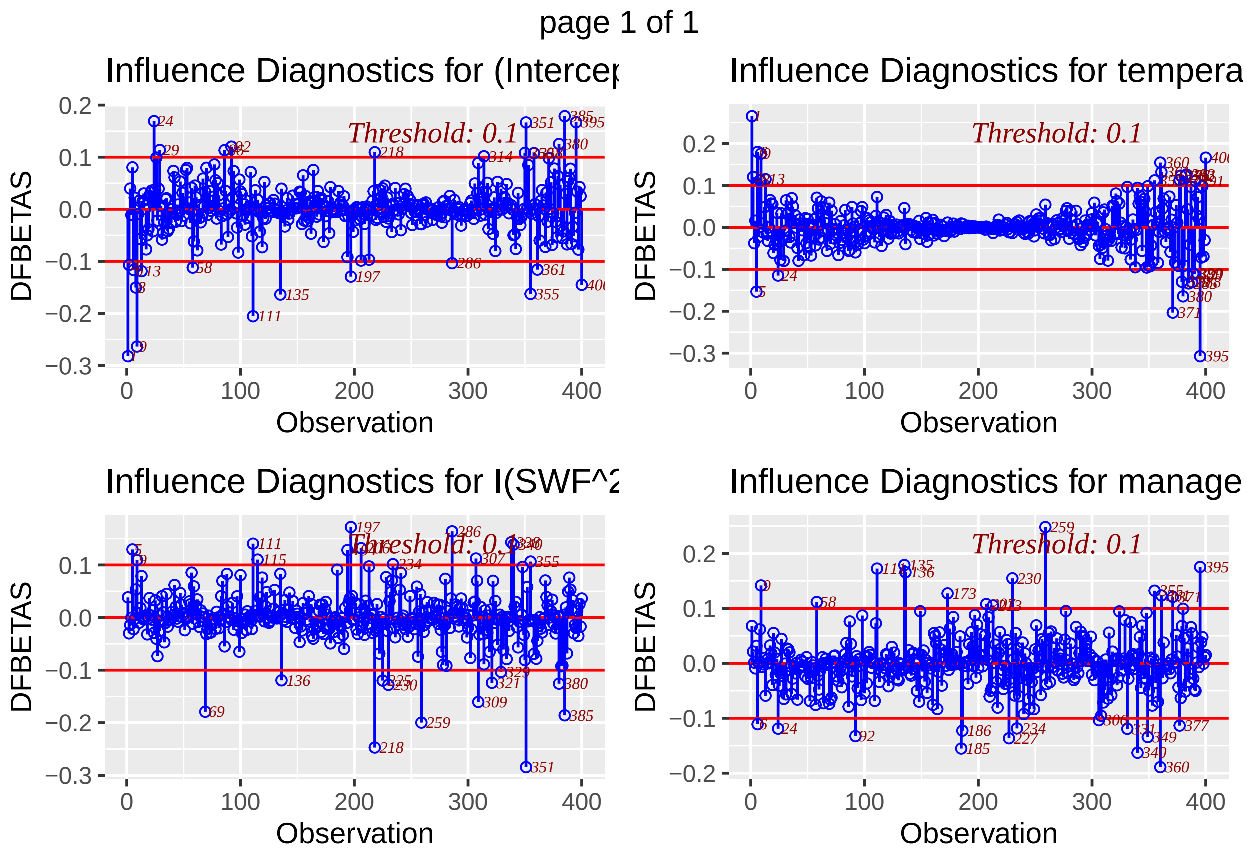

```r
#studentized residual plot
ols_plot_resid_stud(mod)
```



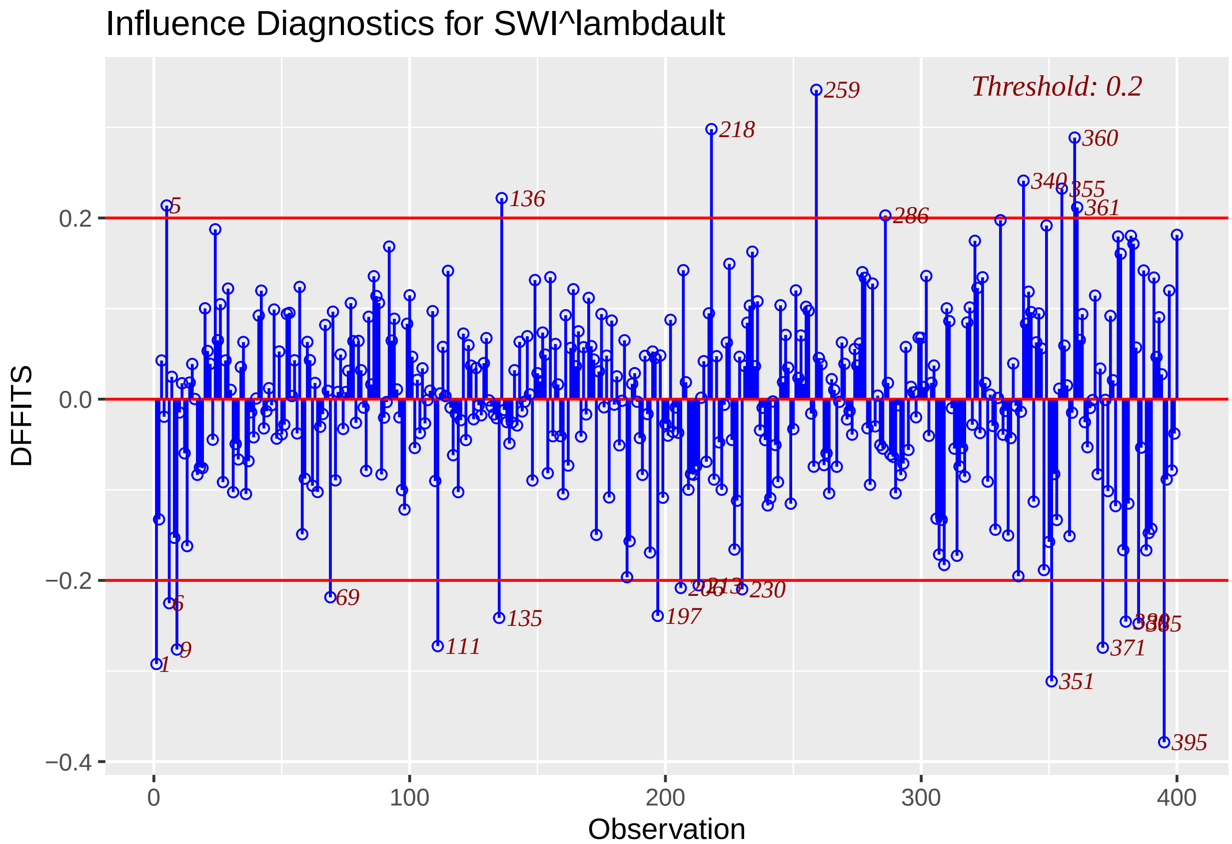

```
#cooks distance plot
ols_plot_cooksd_bar(mod)
```



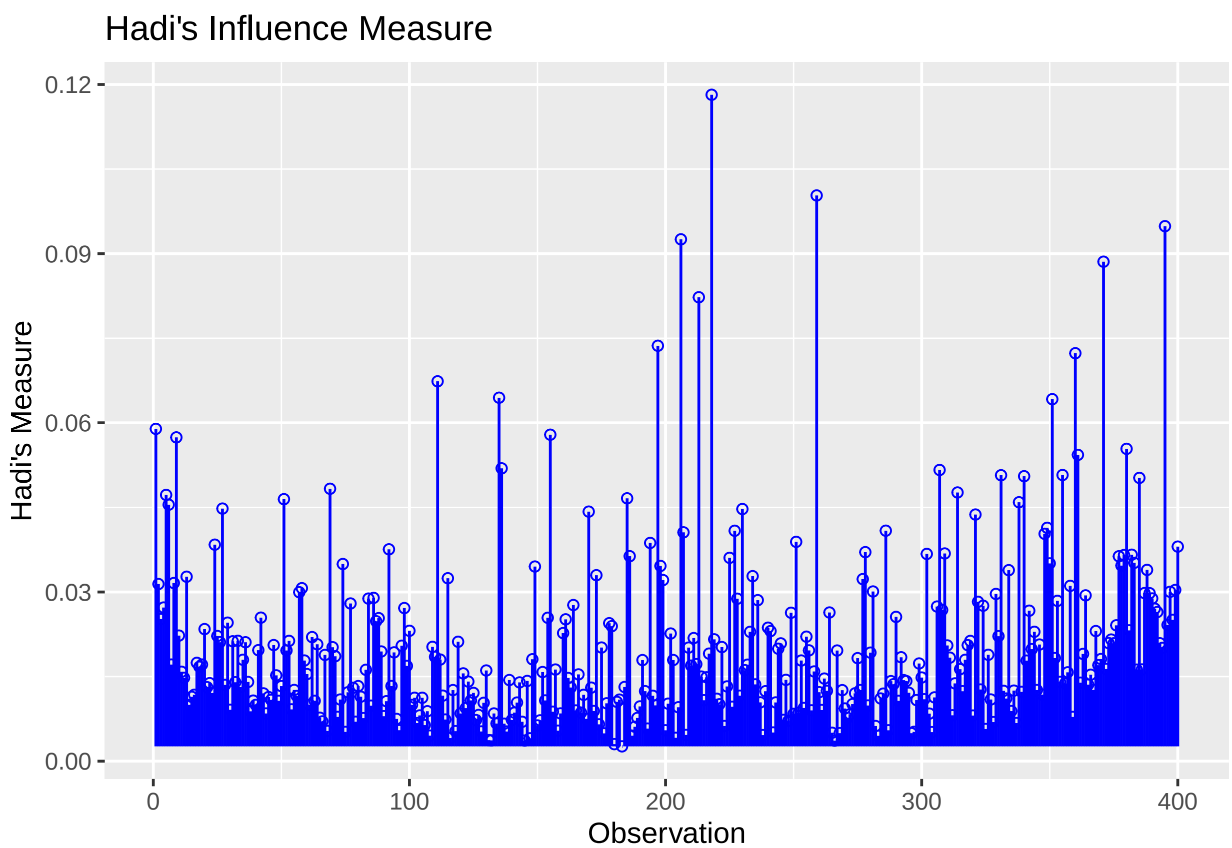

```
#DFBetas for each variable
ols_plot_dfbetas(mod)
```




```r
#Difference in fit chart for each sample
ols_plot_dffits(mod)
```




```r
#Plot for observation influence using hadi's distance
ols_plot_hadi(mod)
```

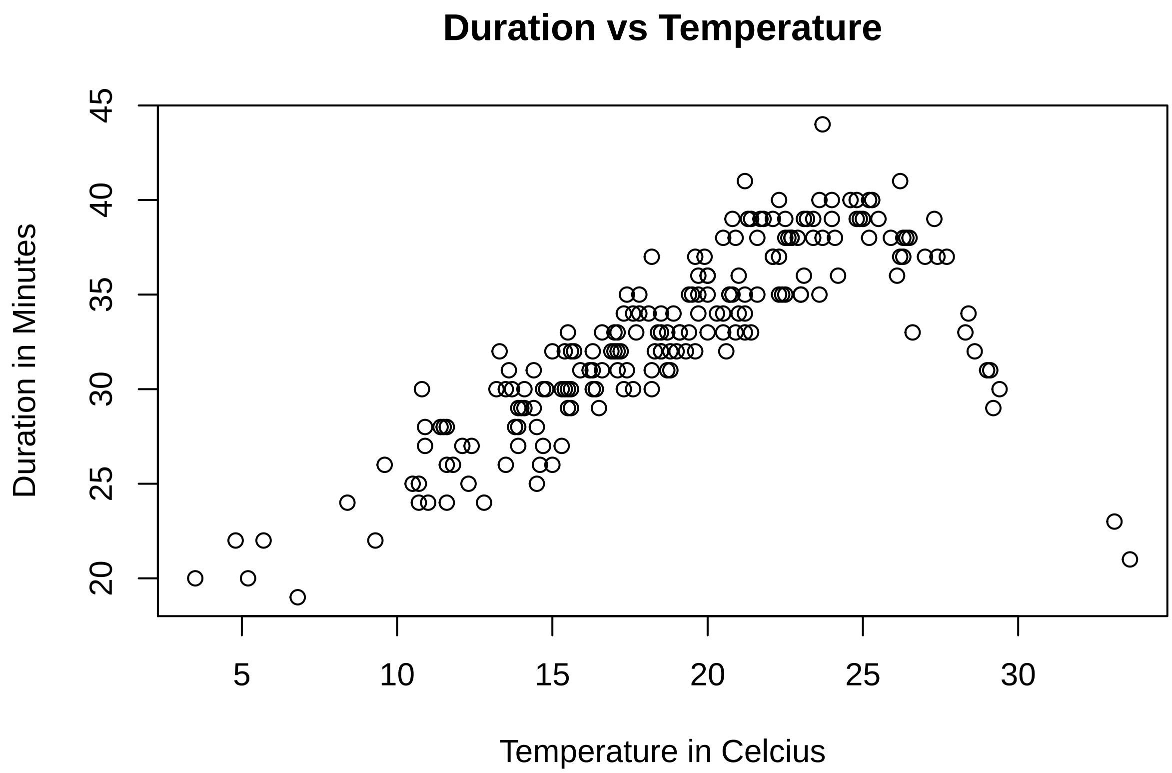


Model assumptions are met, the presence of outliers proves to be impactful to the model.

Task 6 Non-parametric Regression for Duration~Temperature

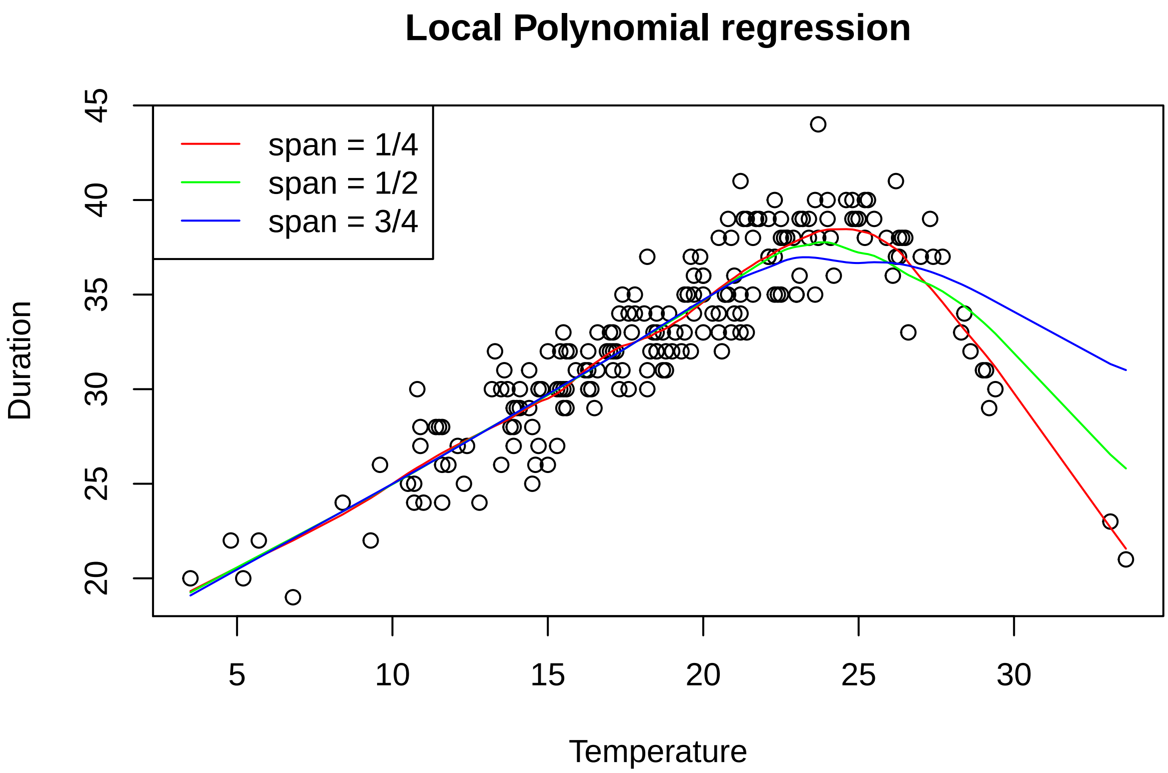
Data exploration

```
plot(data.training$temperature,data.training$duration,
     main="Duration vs Temperature",
     xlab="Temperature in Celcius",ylab="Duration in Minutes")
```



local linear model degree=1

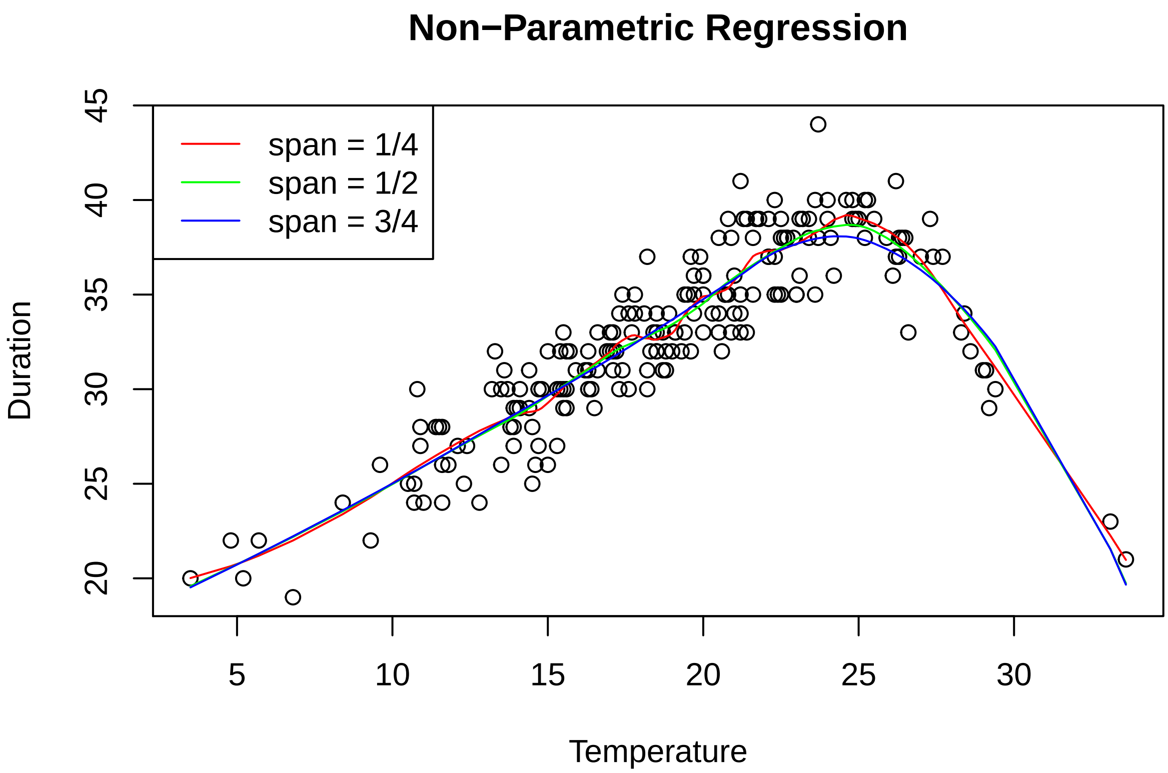
```
plot(data.training$temperature, data.training$duration,
      main = "Local Polynomial regression",xlab = "Temperature",ylab="Duration")
s <- c(1/4, 2/4, 3/4)
colors <- c("red", "green", "blue")
for (i in 1:length(s)) {
  lines(data.training$temperature,
  predict(loess(data.training$duration ~ data.training$temperature,
  span = s[i], degree = 1), data = mcycle), col = colors[i])}
legend("topleft", c("span = 1/4", "span = 1/2", "span = 3/4"), lty = 1, col = colors)
```



Fit could be better, struggles at the tail for temp>25.

local polynomial model degree=2

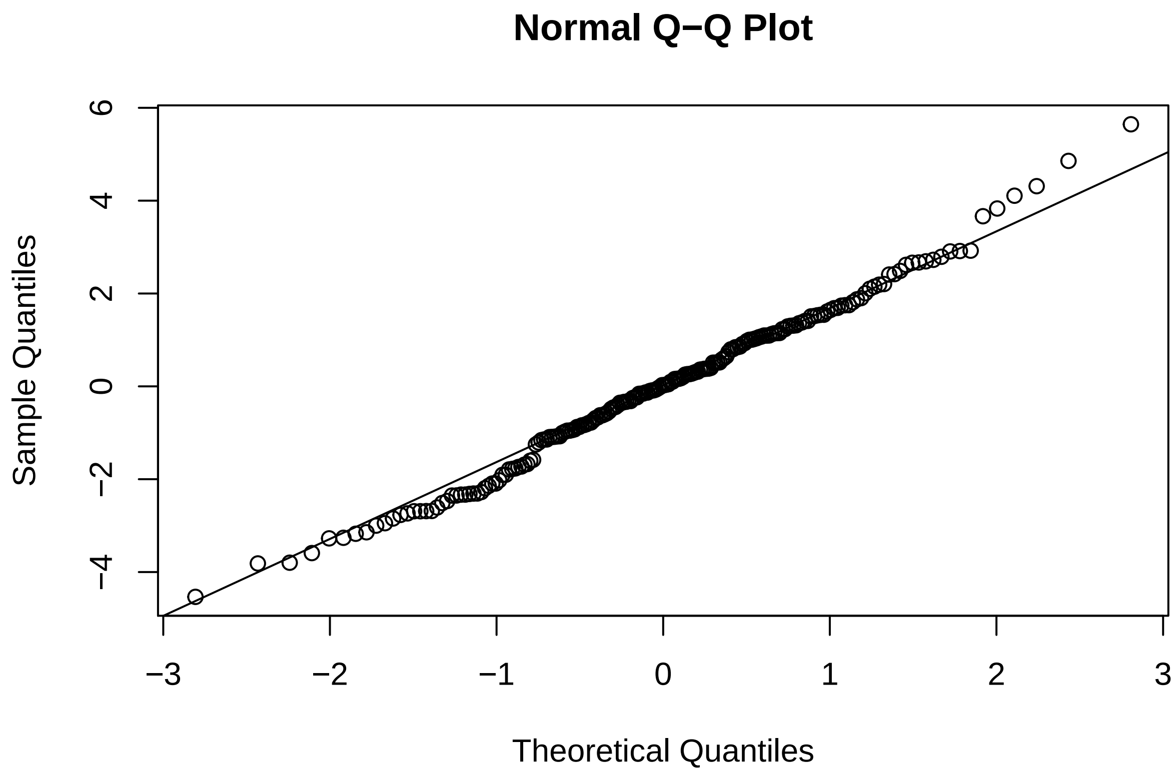
```
plot(data.training$temperature, data.training$duration,
     main = "Non-Parametric Regression",xlab = "Temperature",ylab="Duration")
s <- c(1/4, 2/4, 3/4)
colors <- c("red", "green", "blue")
for (i in 1:length(s)){
  lines(data.training$temperature,
        predict(loess(data.training$duration ~ data.training$temperature,
                      span = s[i], degree = 2), data = mcycle), col = colors[i])}
legend("topleft", c("span = 1/4", "span = 1/2", "span = 3/4"), lty = 1, col = colors)
```



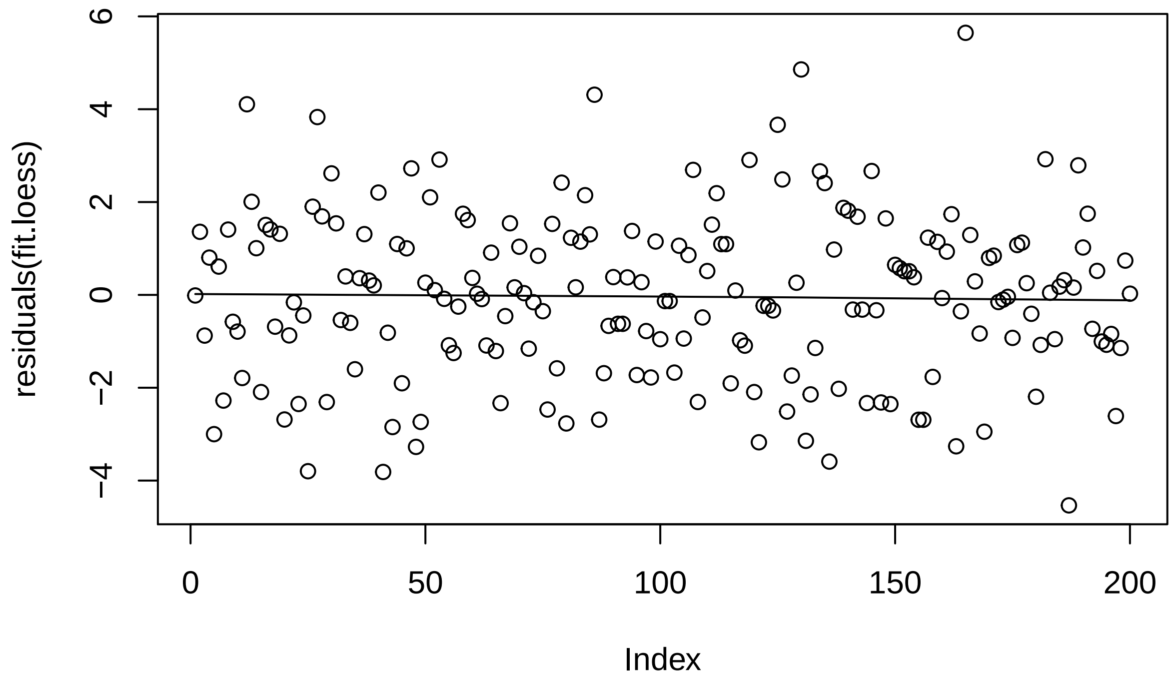
2nd degree fit proves to be quite sufficient.

Model Diagnosticsfor non-parametric model

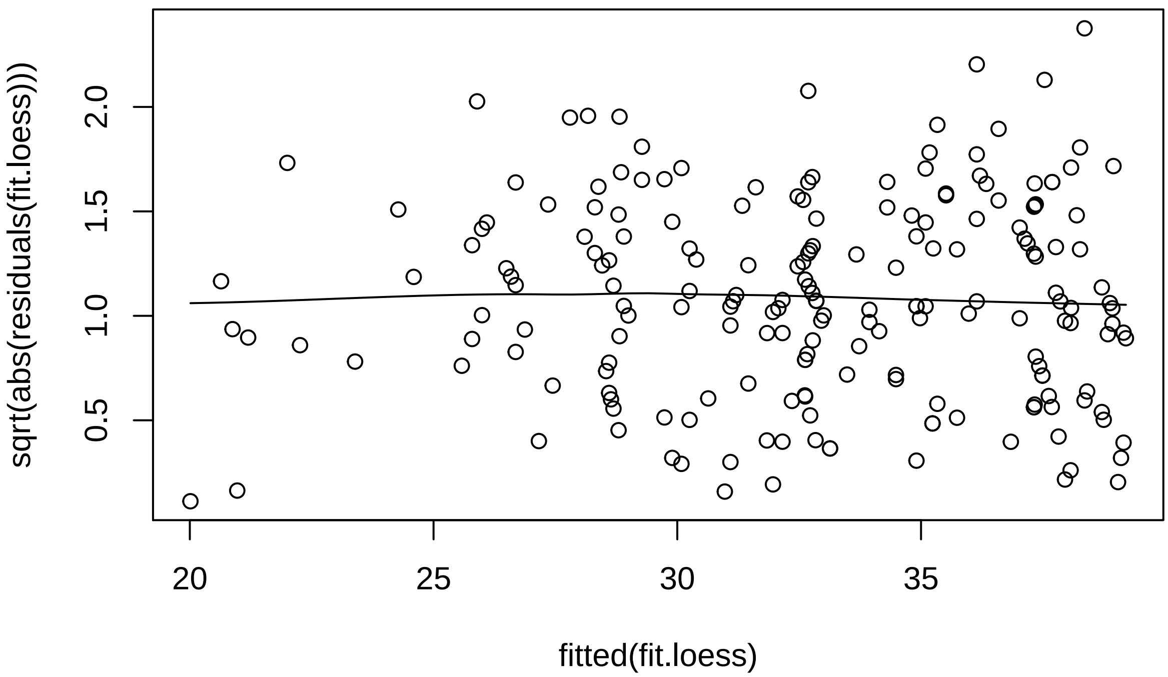
```
fit.loess <- loess(duration ~ temperature, span = 1/4, degree = 2,data=data.training)
qqnorm(residuals(fit.loess))
qqline(residuals(fit.loess))
```




```
scatter.smooth(residuals(fit.loess), span = 1, degree = 1)
```




```
scatter.smooth(fitted(fit.loess), sqrt(abs(residuals(fit.loess))), span = 1, degree = 1)
```




```r
shapiro.test(residuals(fit.loess))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  residuals(fit.loess)
## W = 0.99311, p-value = 0.4752
```

Heavy tails in QQ plot show potential deviation from normality, Shapiro-Wilks test fails to reject normaility. Varaince appears to be constant. Linearity appears to hold as well.

Confidence interval for several temperature measurement

```r
t <- c(seq(6, 33, by = 3))
t.pred <- predict(fit.loess, t, se = TRUE)
t.upper <- t.pred$fit + qnorm(0.975) * t.pred$se.fit
t.lower <- t.pred$fit - qnorm(0.975) * t.pred$se.fit
data.frame("pred" = t.pred$fit, "lower" = t.lower, "upper" = t.upper)
```

```
##       pred    lower    upper
## 1  21.40291 20.12482 22.68100
## 2  23.97338 22.89848 25.04828
## 3  27.06465 26.16349 27.96581
## 4  29.27451 28.45160 30.09742
## 5  32.74872 31.92041 33.57704
## 6  35.73759 34.92458 36.55059
## 7  38.70893 37.84195 39.57590
## 8  36.84233 35.96330 37.72136
```

```
## 9  29.74875 28.55294 30.94455
## 10 22.51513 20.26060 24.76966
```

Fitting a polynomial model to predict Duration~Temperature. Using Forward selection to reduce model to most influential predictors.

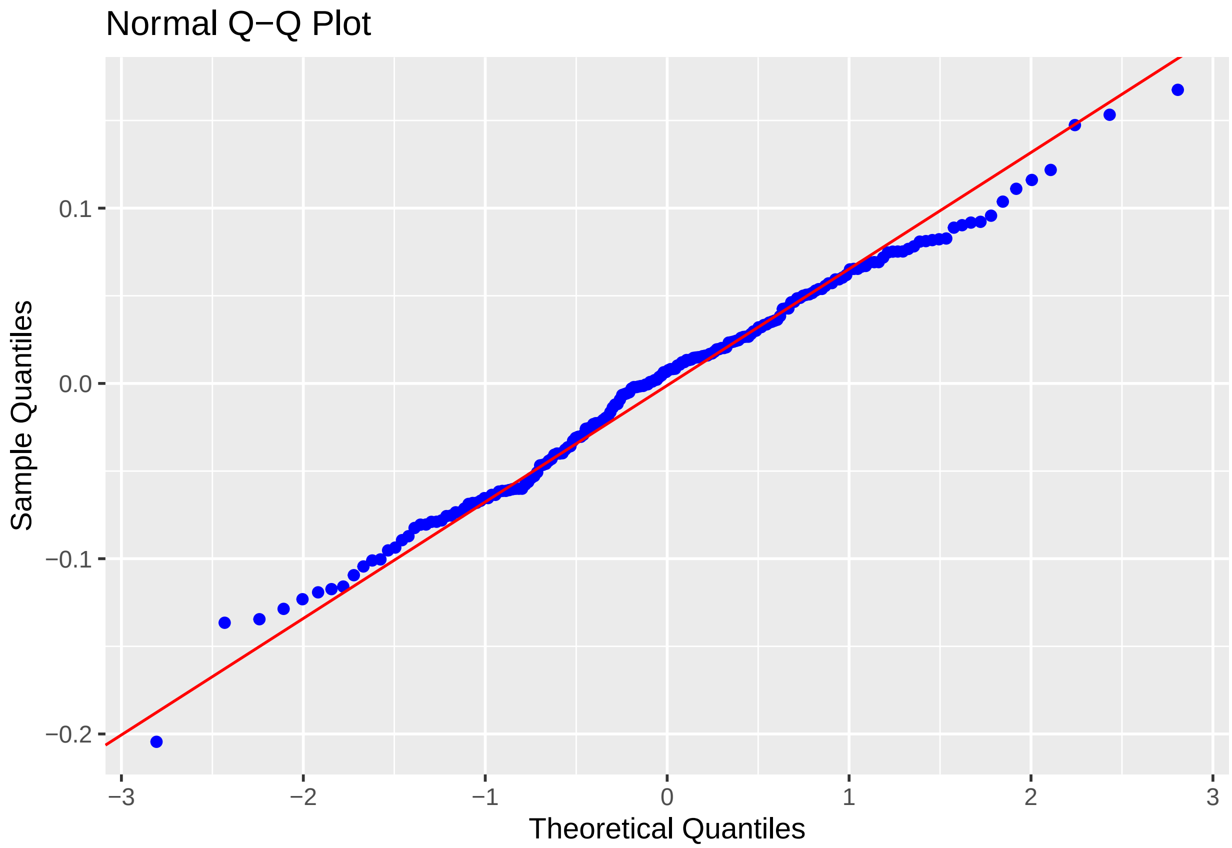
Fitting final polynomial model of log(duration) ~ I(temperature^3) + I(temperature^4). (Normality test failed so a log transform was taken to adress this.)

```
fit.dur.poly3=lm(formula = log(duration) ~ I(temperature^3) + I(temperature^4),
    data = data.training)
summary(fit.dur.poly3)
```

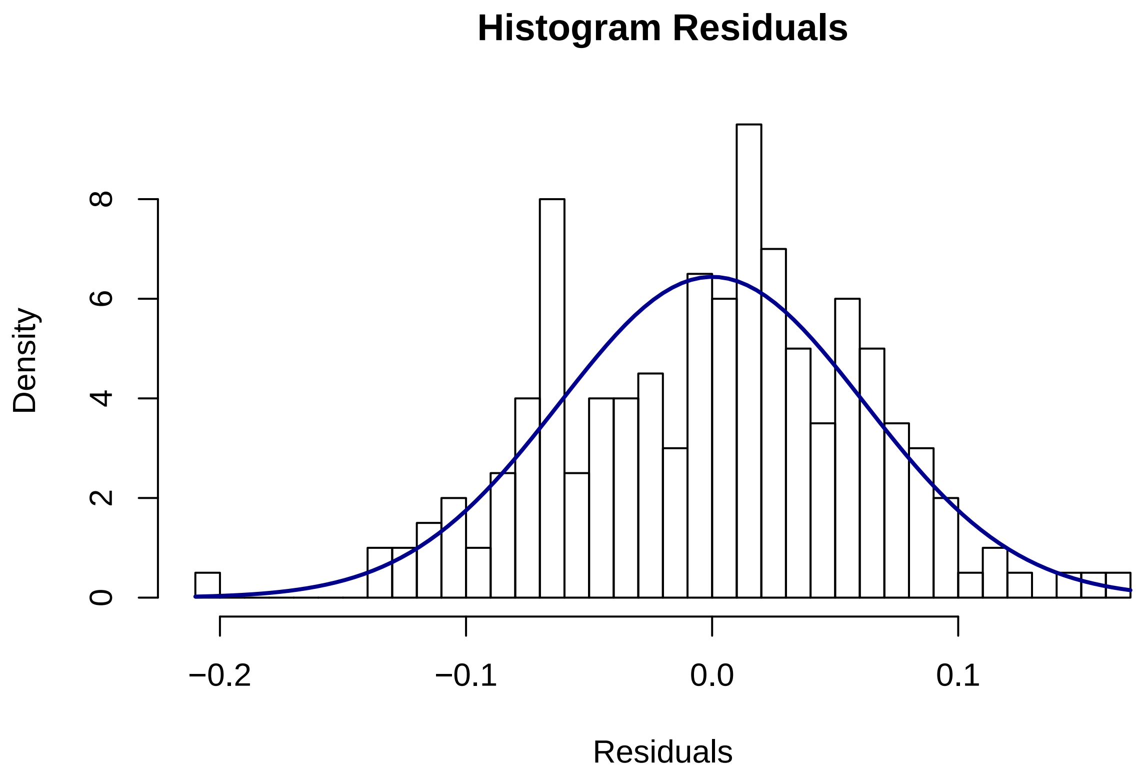
```
##
## Call:
## lm(formula = log(duration) ~ I(temperature^3) + I(temperature^4),
##     data = data.training)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.204459 -0.045988  0.007033  0.043664  0.167501
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)       3.113e+00  1.189e-02  261.95   <2e-16 ***
## I(temperature^3)  1.425e-04  4.671e-06   30.52   <2e-16 ***
## I(temperature^4) -4.355e-06  1.534e-07  -28.39   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06226 on 197 degrees of freedom
## Multiple R-squared:  0.8463, Adjusted R-squared:  0.8448
## F-statistic: 542.5 on 2 and 197 DF,  p-value: < 2.2e-16
```

Model Diagnostics

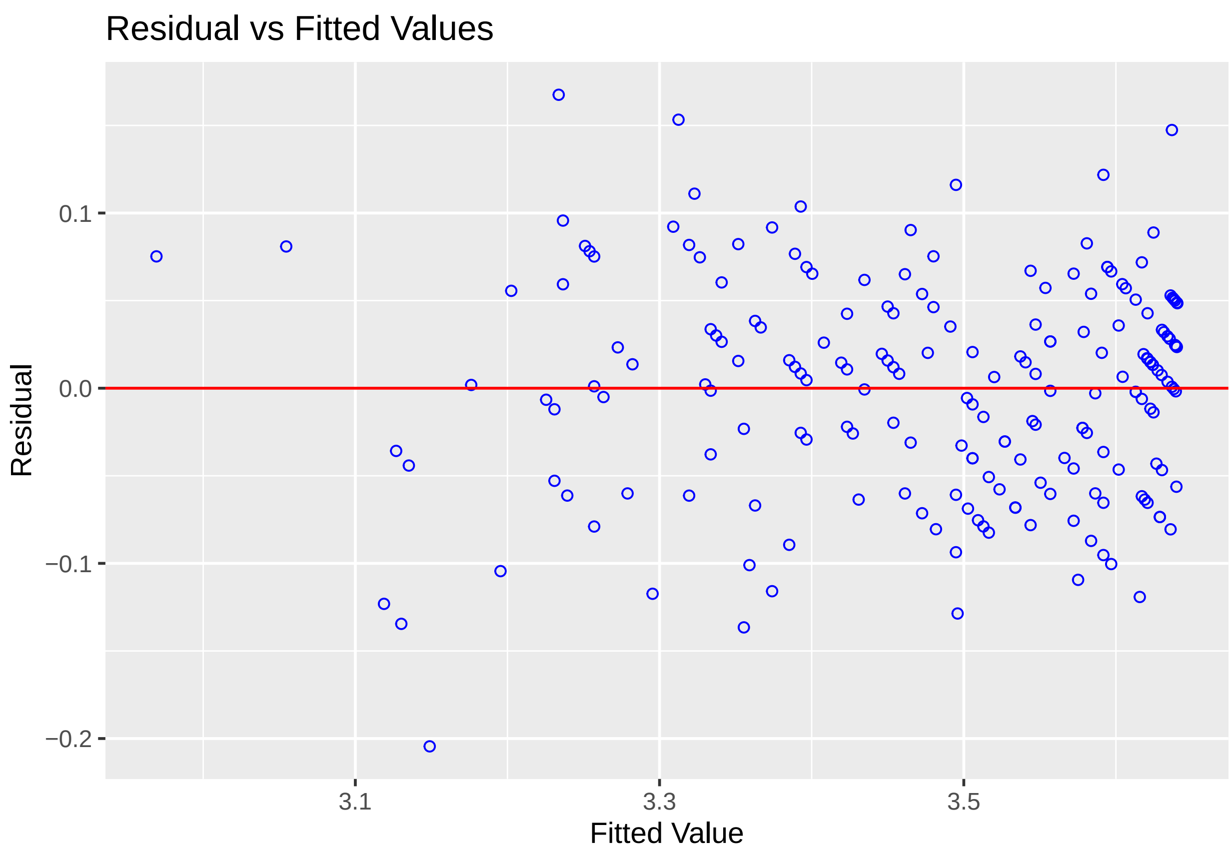
```
mod=fit.dur.poly3
#QQ-Plot and Histogram to visualize normality of errors
ols_plot_resid_qq(mod)
```




```r
hist(mod$residuals,breaks = 50,
     xlab="Residuals", main="Histogram Residuals",
     probability = T)
curve(dnorm(x, mean=mean(mod$residuals), sd=sd(mod$residuals)),
      col="darkblue", lwd=2, add=TRUE, yaxt="n")
```




```
##Visualiztion of Residuals vs. Fitted Values to evaluate homoscedasticity
ols_plot_resid_fit(mod)
```




```r
#Breusch-Pagan Test for homoscedasticity
ncvTest(mod)
```

```
## Non-constant Variance Score Test
## Variance formula: ~ fitted.values
## Chisquare = 16.04692, Df = 1, p = 6.1792e-05
```

```r
#Normality of errors tests
ols_test_normality(mod)
```

```
## Warning in ks.test(y, "pnorm", mean(y), sd(y)): ties should not be present
## for the Kolmogorov-Smirnov test
```

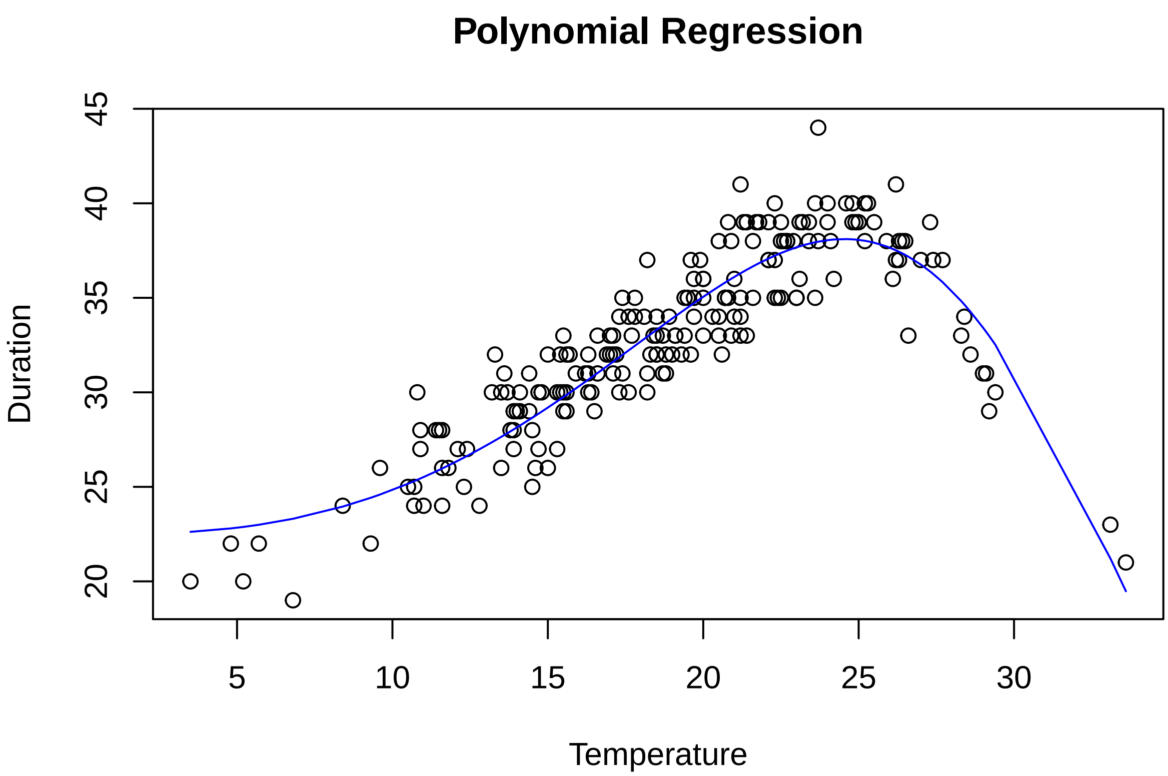
```
## -----------------------------------------------
##        Test             Statistic       pvalue
## -----------------------------------------------
## Shapiro-Wilk              0.9926         0.4057
## Kolmogorov-Smirnov        0.0616         0.4347
## Cramer-von Mises         58.6631         0.0000
## Anderson-Darling          0.582          0.1283
## -----------------------------------------------
```

```r
ols_test_correlation(mod)
```

```
## [1] 0.9959428
```

Plot of Quadratic Regression model along with regression line fit.

```r
fit.dur.poly.coef=coefficients(fit.dur.poly3)
temp=data.training$temperature
y=fit.dur.poly.coef[1]+fit.dur.poly.coef[2]*temp^3+fit.dur.poly.coef[3]*temp^4
plot(data.training$temperature, data.training$duration,
     main = "Polynomial Regression",xlab = "Temperature",ylab="Duration")
lines(x=data.training$temperature,y=exp(y),col="blue")
```



Comparing models with RMSE(fitted,actual)

```r
#Loess model RMSE
rmse(fit.loess$fitted,data.training$duration)
```

```
## [1] 1.780385
```

```r
#polynomial model RMSE
rmse(exp(fitted.values(fit.dur.poly3)),data.test$duration)
```

```
## [1] 6.440638
```

Non-parametric model proves to be superior than the polynomial model.