



LECTURE 2:

OPTIMIZATION IN DEEP LEARNING

University of Washington, Seattle

Fall 2025



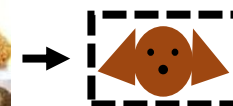
Previously in EEP 596...

Fixed model

$$x_n \rightarrow \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn} \rightarrow X_k$$

Learned model

$$x_n \rightarrow \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{i2\pi}{N}kn} \rightarrow X_k$$



Model



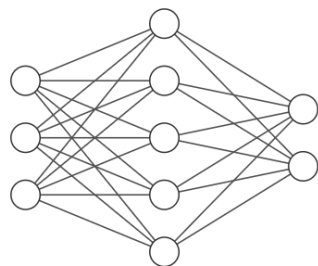
1: Café Poodle
0: Not Café Poodle

Fixed vs Learned model

Classical ML methods

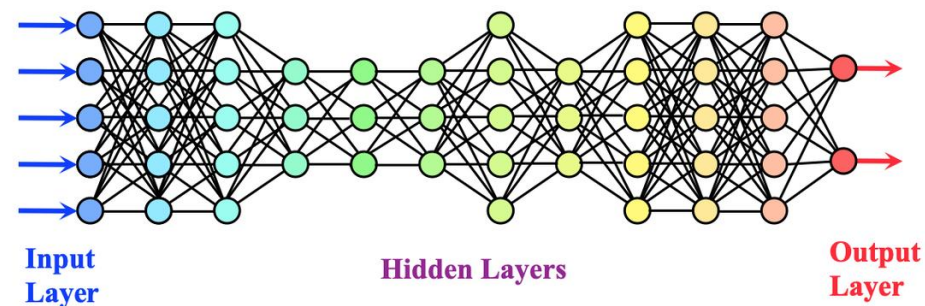
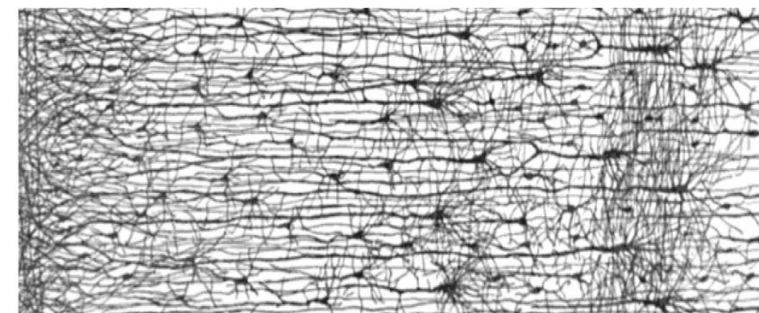


Previously in EEP 596...



1: Café Poodle
0: Not Café Poodle

ANN as a model

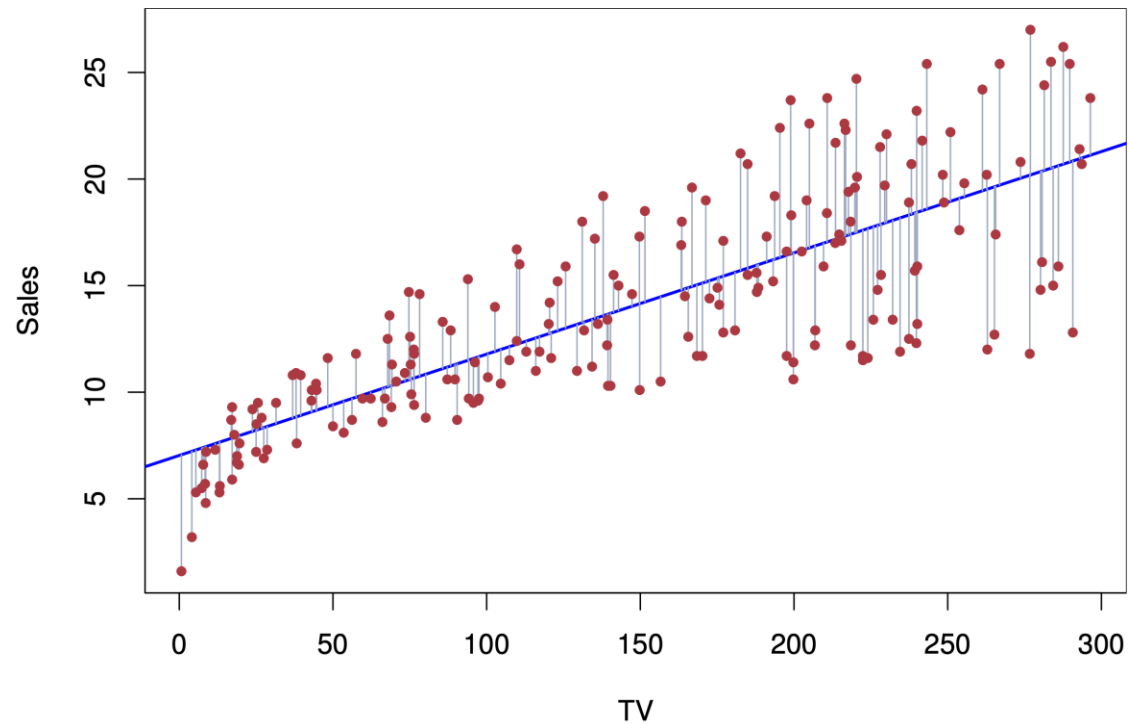


Deep ANN as brain analogue



Previously in EEP 596...

$$Y = f(X, W)$$



Linear Regression



OUTLINE

Part 1: Binary Classification

- Binary Logistic Regression
- Linear vs Logistic regression
- Logistic regression and Neuron

Part 2: Training and Optimization of a Neuron

- Binary Cross Entropy Loss function
- Training Logistic Regression
- Gradient descent
- Back-propagation algorithm

Part 3: Stochastic Gradient Descent

- GD vs SGD
- Convergence of SGD
- Learning rate and convergence
- Comparing GD variants

Part 4: Optimization Techniques in DL

- Variable learning rate
- Advanced methods
- Cross validation
- Regularization/Normalization/Initialization
- Hyperparameter tunings



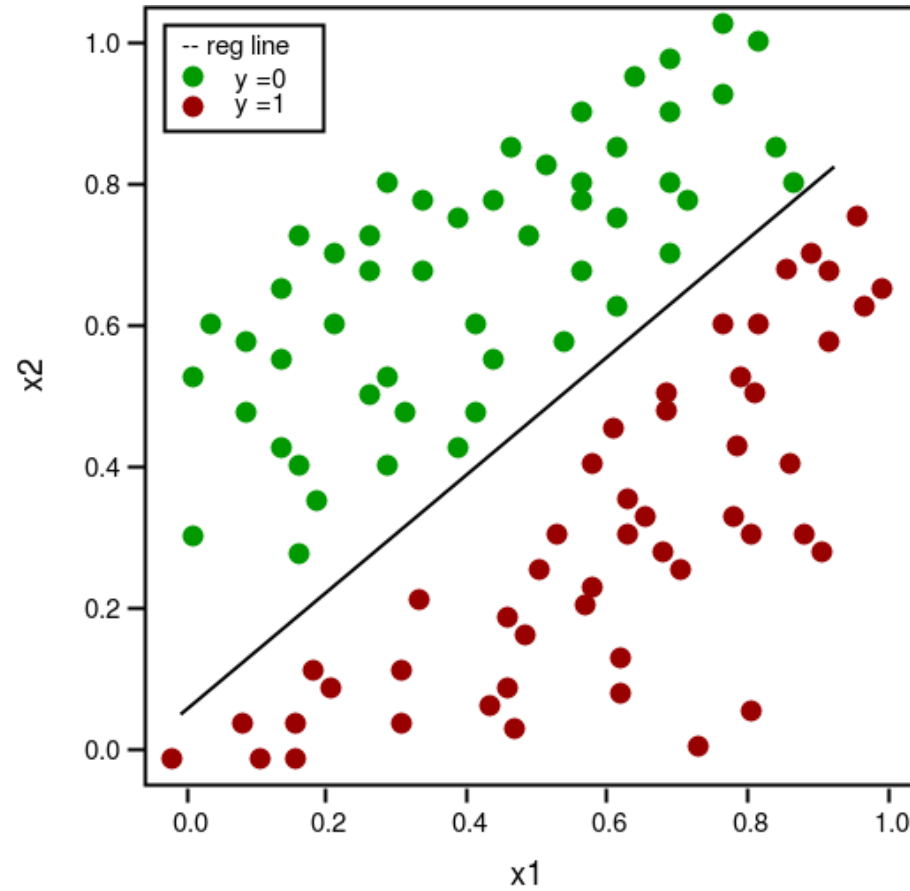
PART 1:

Binary Classification



Binary classification problem

$$y = \sigma(\vec{w}^T \vec{x} + b)$$





Iris Flower Data

iris setosa



petal sepal

iris versicolor



petal sepal

iris virginica



petal sepal

	<code>Id</code>	<code>SepalLengthCm</code>	<code>SepalWidthCm</code>	<code>PetalLengthCm</code>	<code>PetalWidthCm</code>	<code>Species</code>
0	1	5.1	3.5	1.4	0.2	Iris-setosa
1	2	4.9	3.0	1.4	0.2	Iris-setosa
2	3	4.7	3.2	1.3	0.2	Iris-setosa
3	4	4.6	3.1	1.5	0.2	Iris-setosa
4	5	5.0	3.6	1.4	0.2	Iris-setosa
5	6	5.4	3.9	1.7	0.4	Iris-setosa
6	7	4.6	3.4	1.4	0.3	Iris-setosa
7	8	5.0	3.4	1.5	0.2	Iris-setosa
8	9	4.4	2.9	1.4	0.2	Iris-setosa
9	10	4.9	3.1	1.5	0.1	Iris-setosa

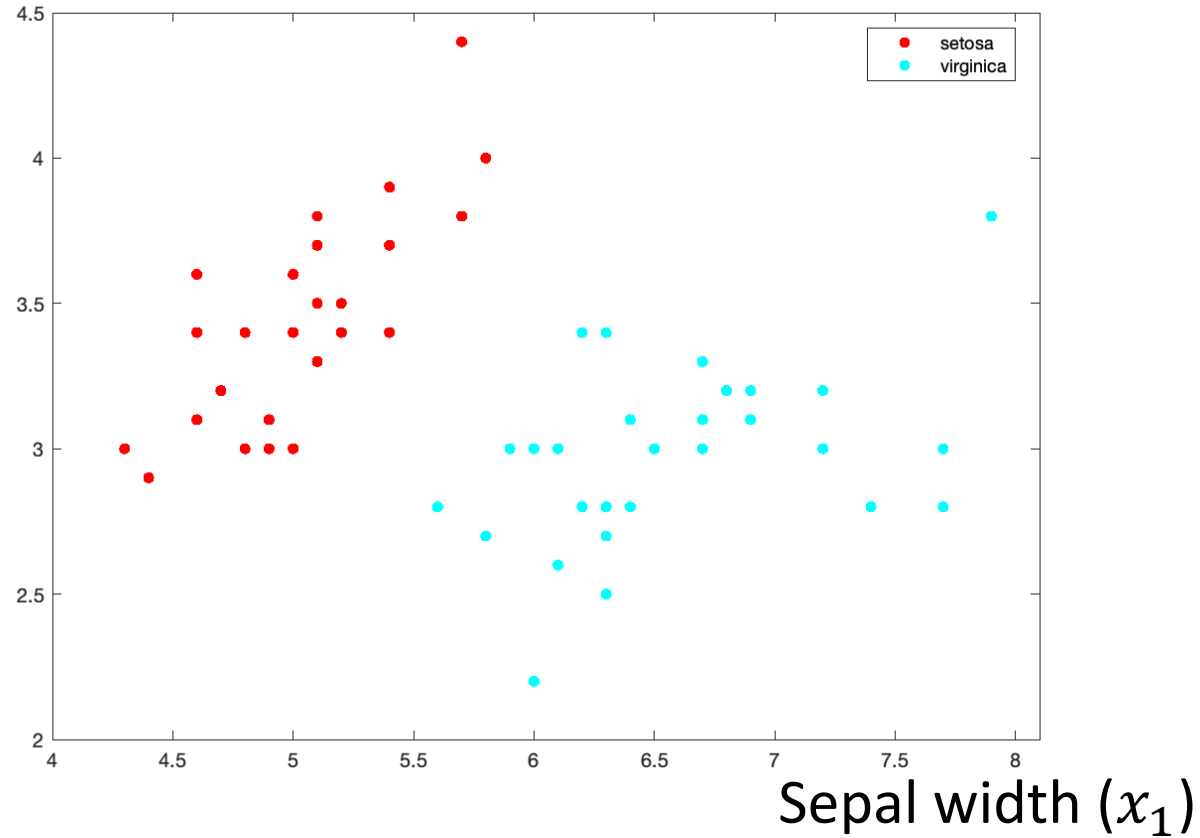
Features

Labels (Targets)

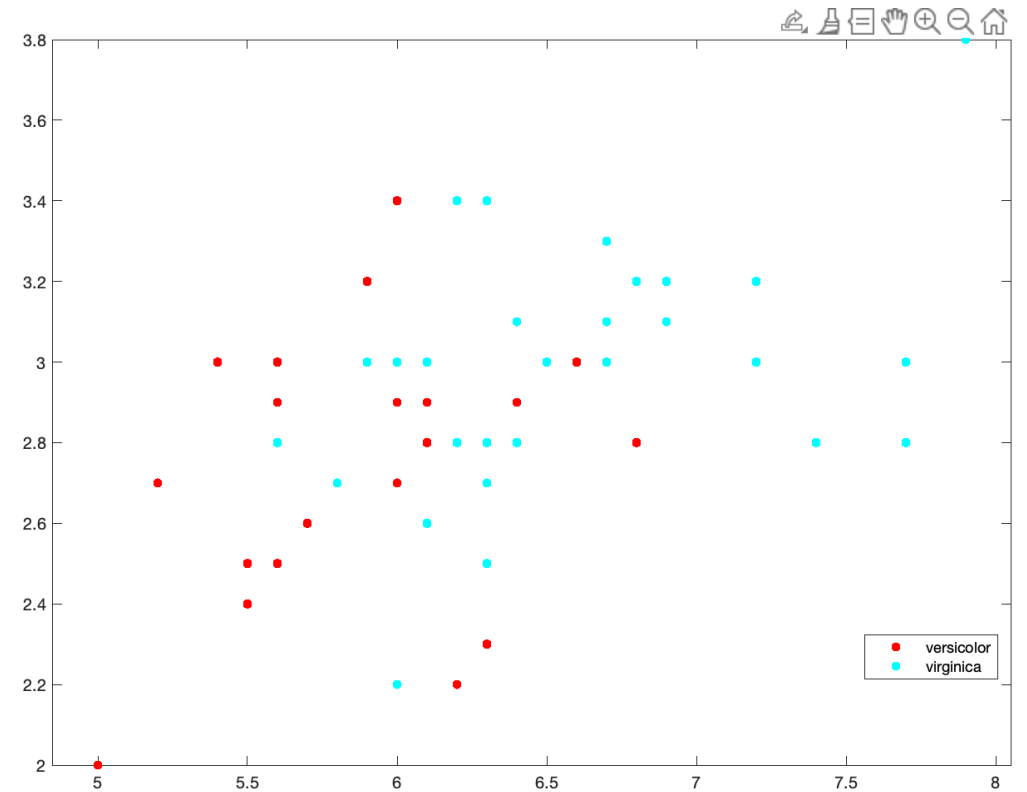


Linear vs Logistic regression

Sepal length (x_2)



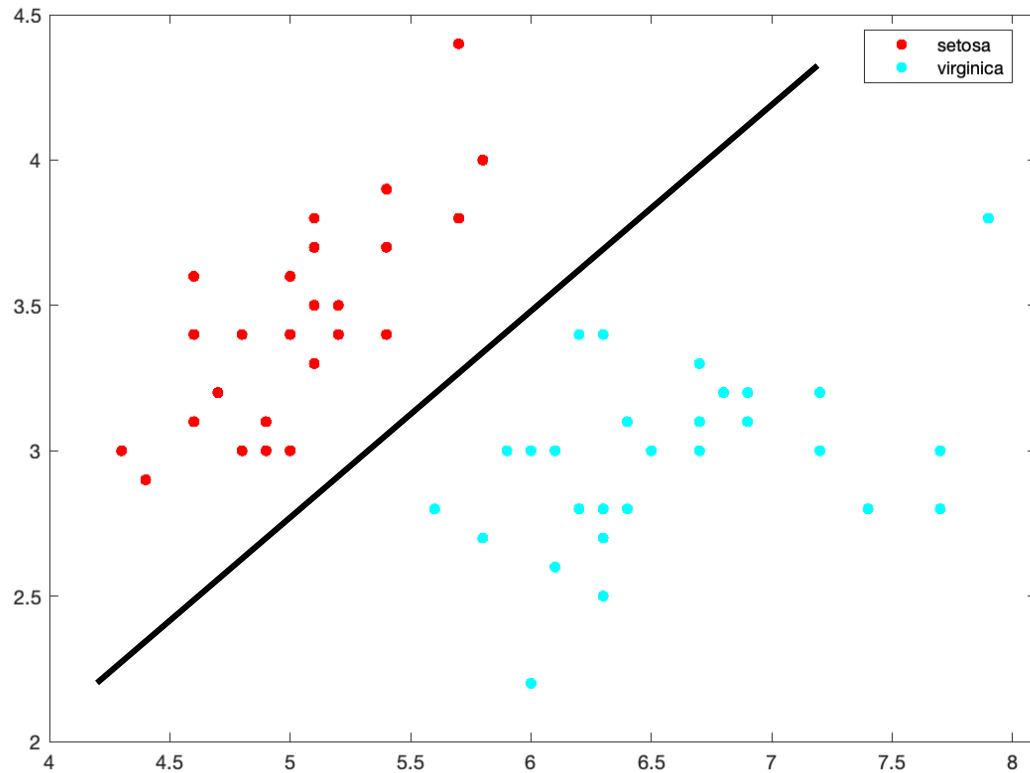
Setosa vs Virginica



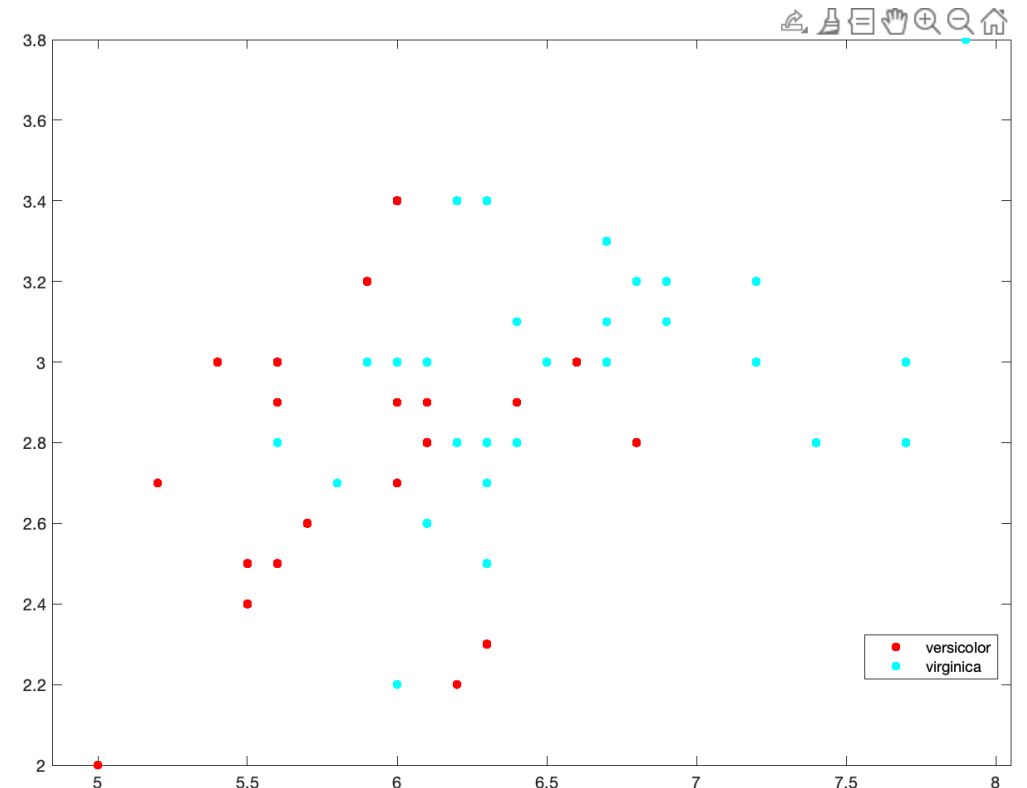
Virsicolor vs Virginica



Linear vs Logistic regression



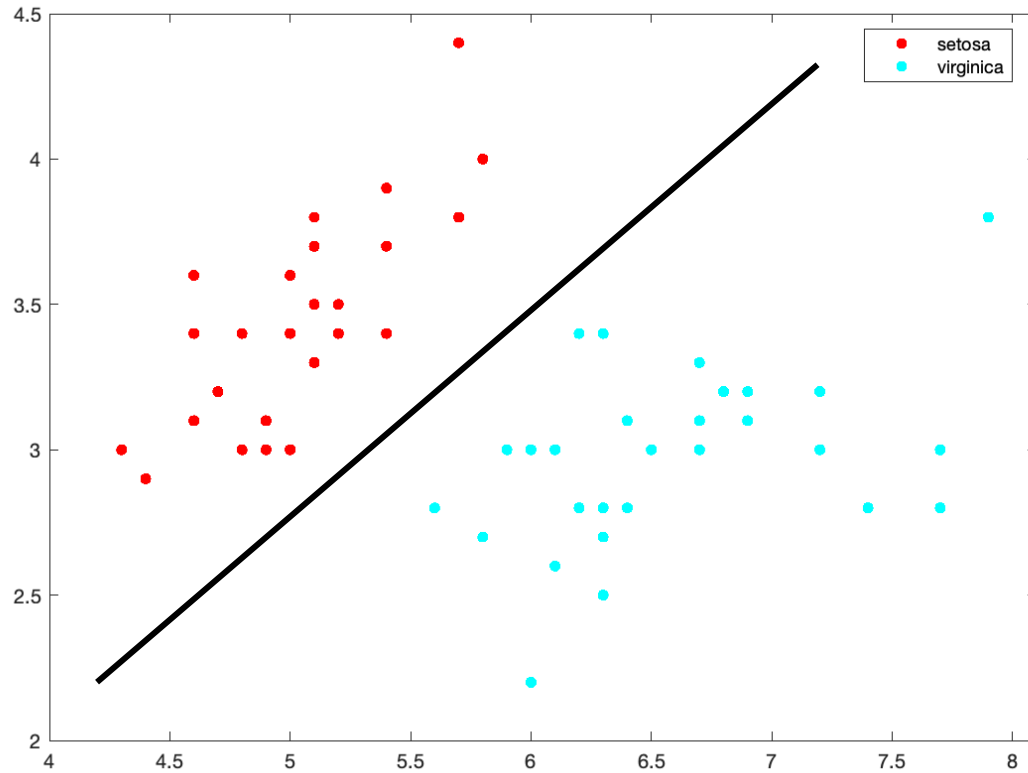
Setosa vs Virginica



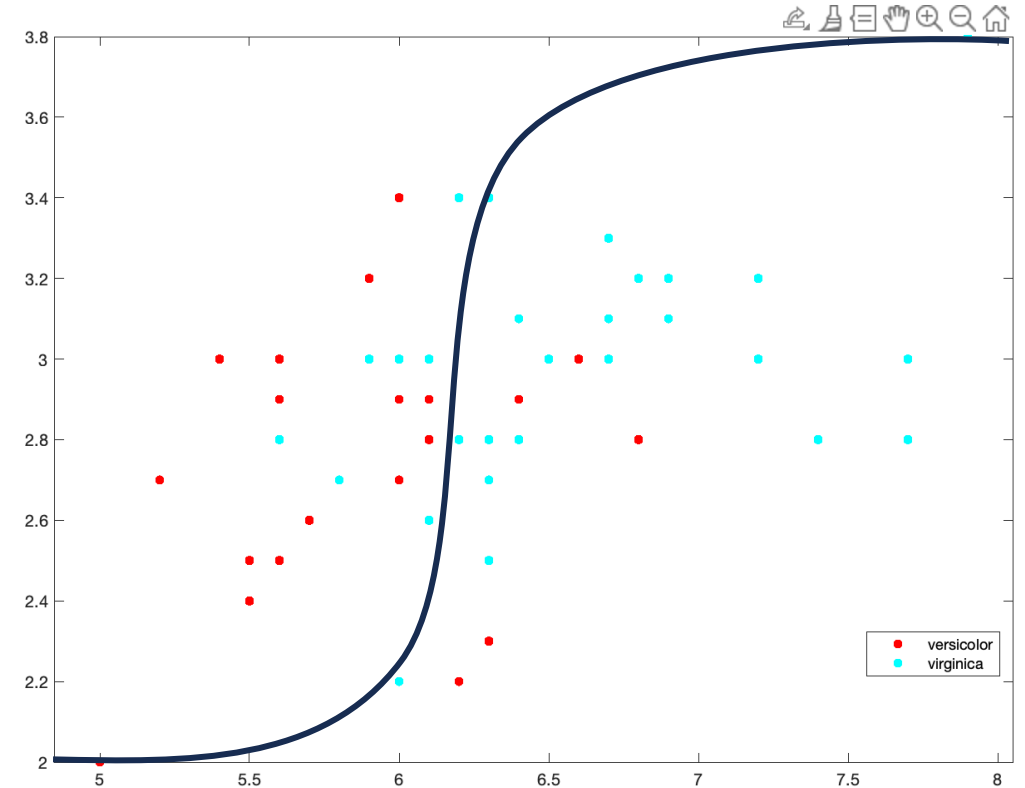
Virsicolor vs Virginica



Linear vs Logistic regression



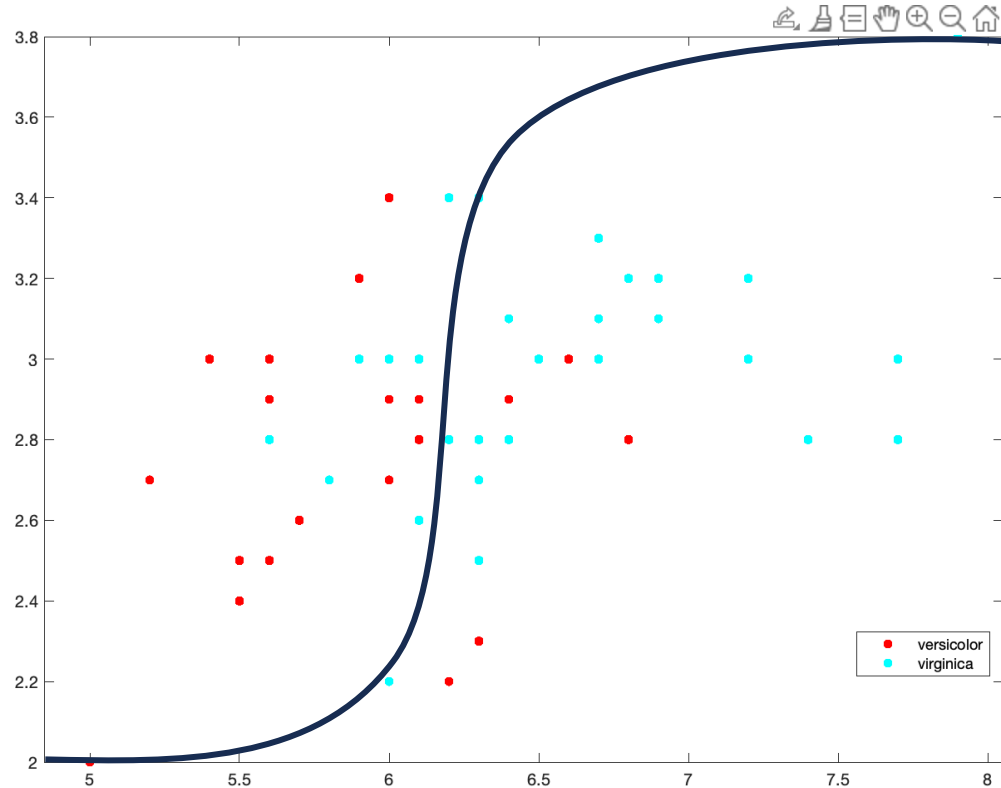
Setosa vs Virginica



Virsicolor vs Virginica



Linear vs Logistic regression



$$p = \frac{1}{1 + e^{-z}}$$

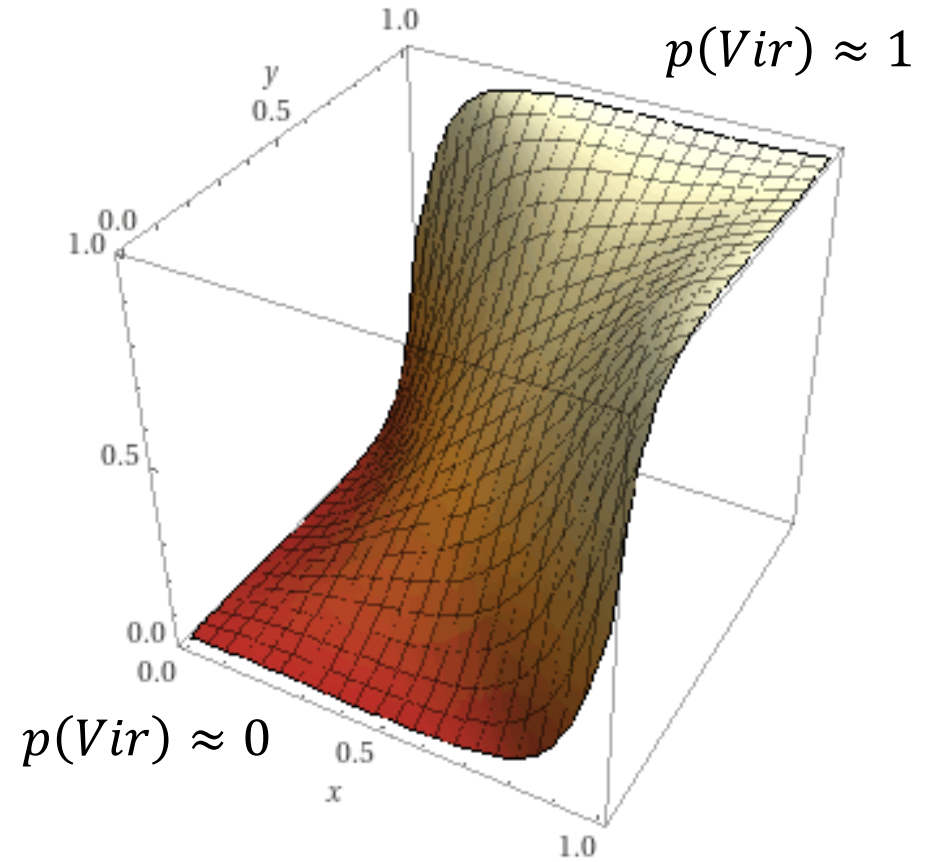
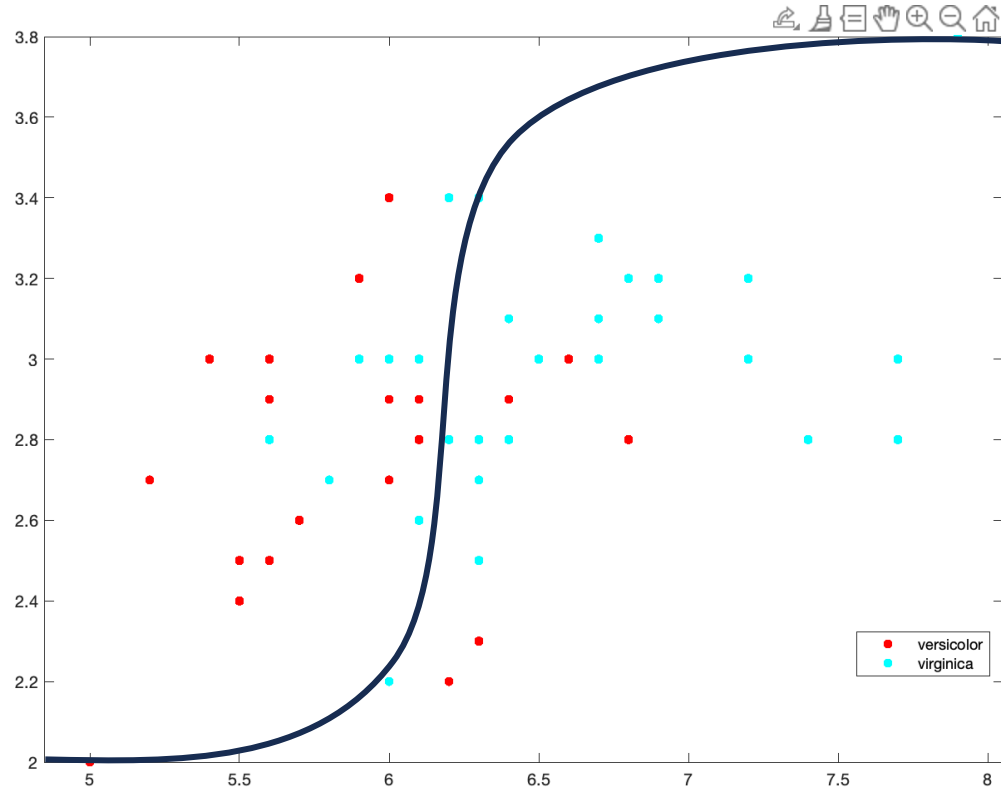
$$0 \leq p \leq 1$$

$$z = \vec{\beta}^T \vec{x} + \beta_0$$

$$\vec{x} = [x_1, x_2] \quad \vec{\beta} = [\beta_1, \beta_2]$$

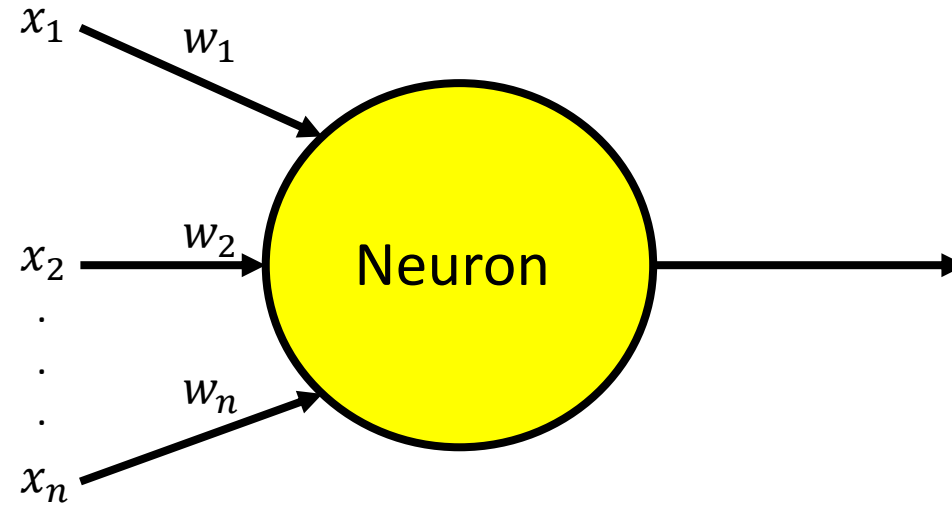


Linear vs Logistic regression



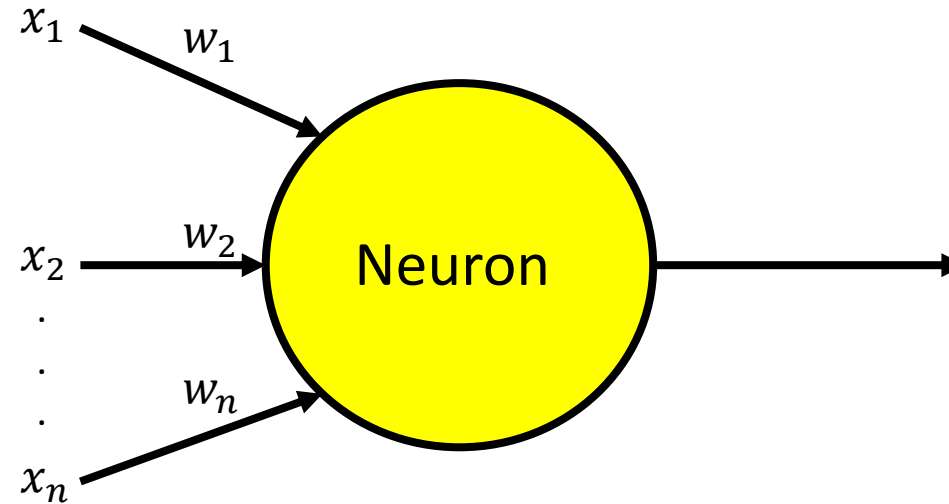


Artificial Neuron as Logistic Regression Model





Artificial Neuron as Logistic Regression Model



Input

$$z = \vec{w}^T \vec{x} + b$$

$$\vec{x} \in \mathbb{R}^n$$

Activation

$$\sigma = \frac{1}{1 + e^{-z}}$$

Output

$$y = \sigma(\vec{w}^T \vec{x} + b)$$

$$y = P(y = 1 | \vec{x}), 0 \leq y \leq 1$$

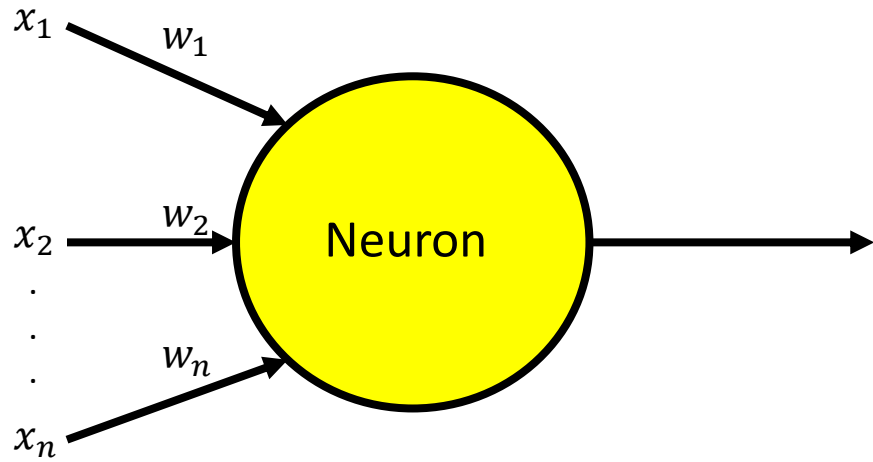


PART 2:

Training and Optimization of a Neuron



Cross Entropy Loss function (Binary)



Input

Activation

Output

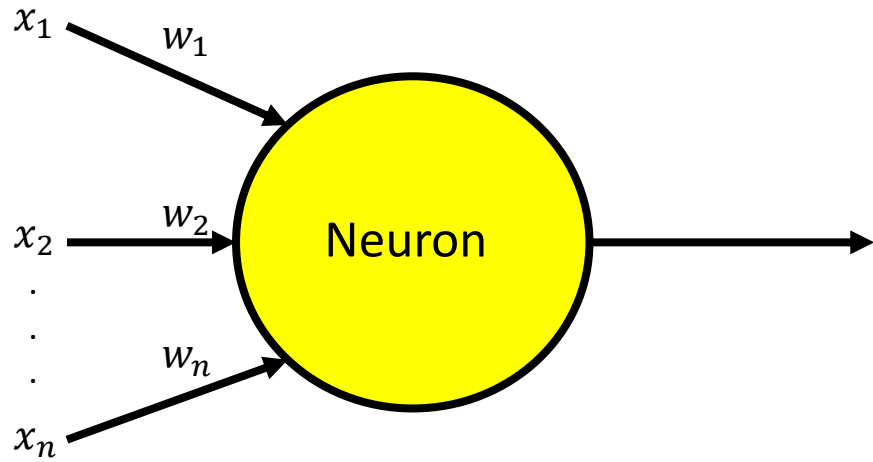
$$z = \vec{w}^T \vec{x}$$

$$\sigma = \frac{1}{1 + e^{-z}}$$

$$y = \sigma(\vec{w}^T \vec{x} + b)$$



Cross Entropy Loss function (Binary)



Input

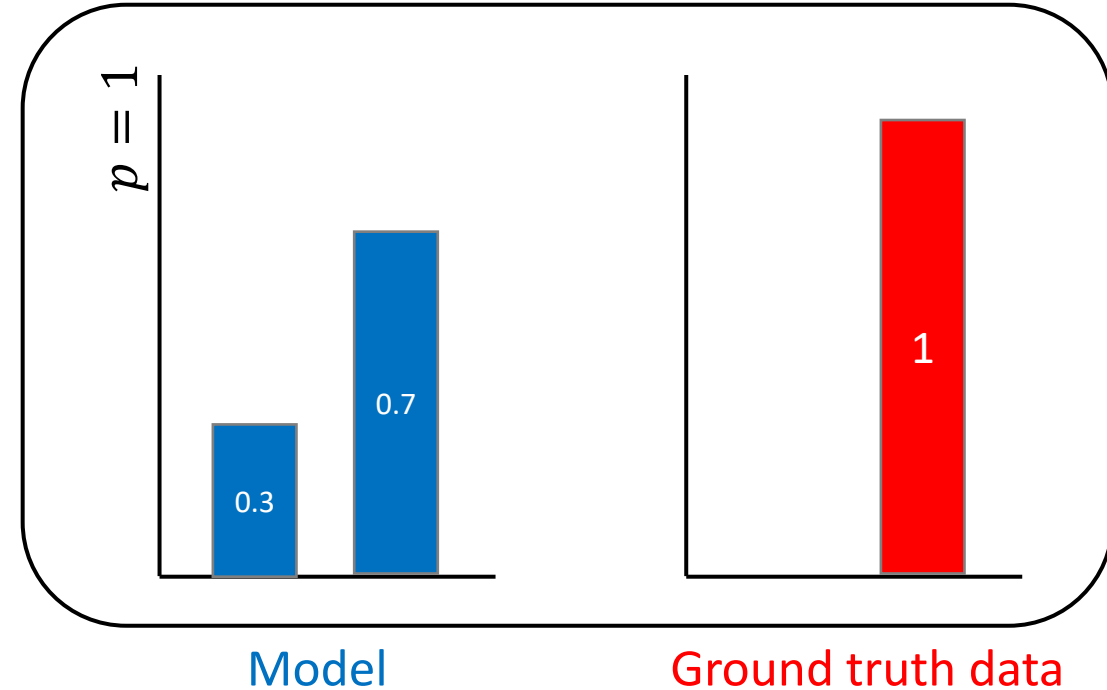
Activation

Output

$$z = \vec{w}^T \vec{x}$$

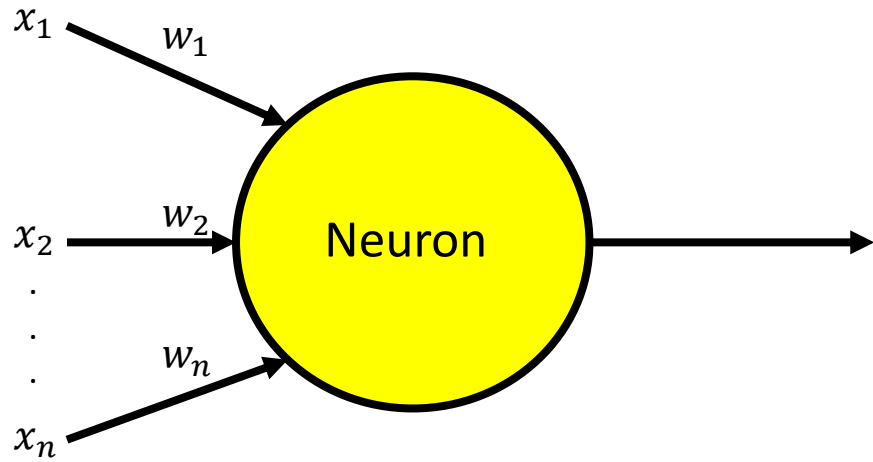
$$\sigma = \frac{1}{1 + e^{-z}}$$

$$y = \sigma(\vec{w}^T \vec{x} + b)$$





Cross Entropy Loss function (Binary)



Input

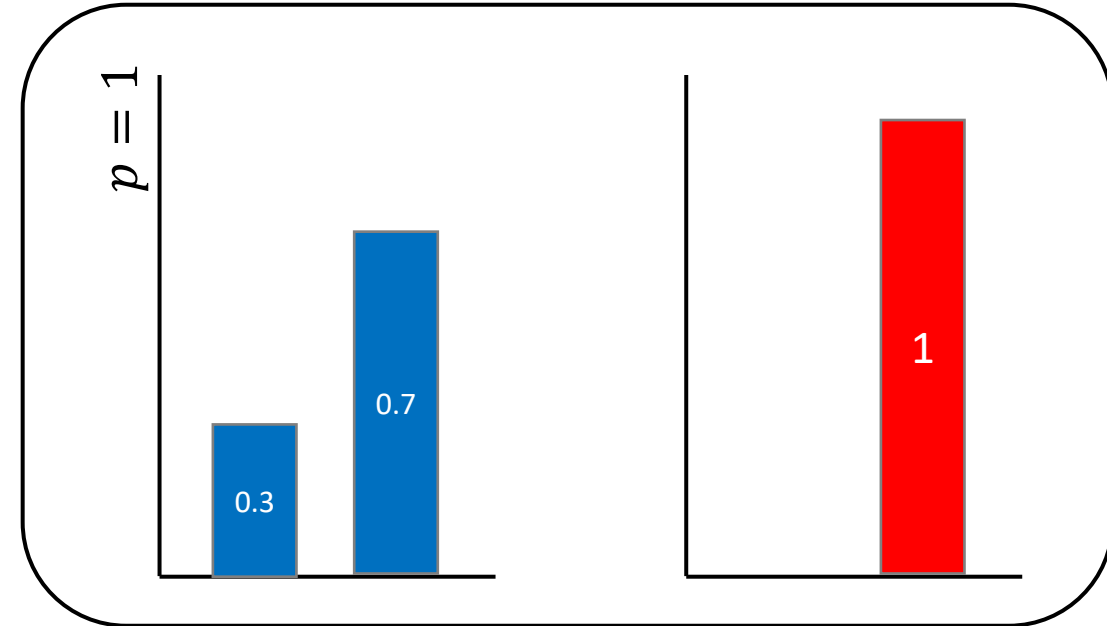
Activation

Output

$$z = \vec{w}^T \vec{x}$$

$$\sigma = \frac{1}{1 + e^{-z}}$$

$$y = \sigma(\vec{w}^T \vec{x} + b)$$



Model

Ground truth data

$J \approx$ Measure of difference in distributions



Cross Entropy Loss function (Binary)

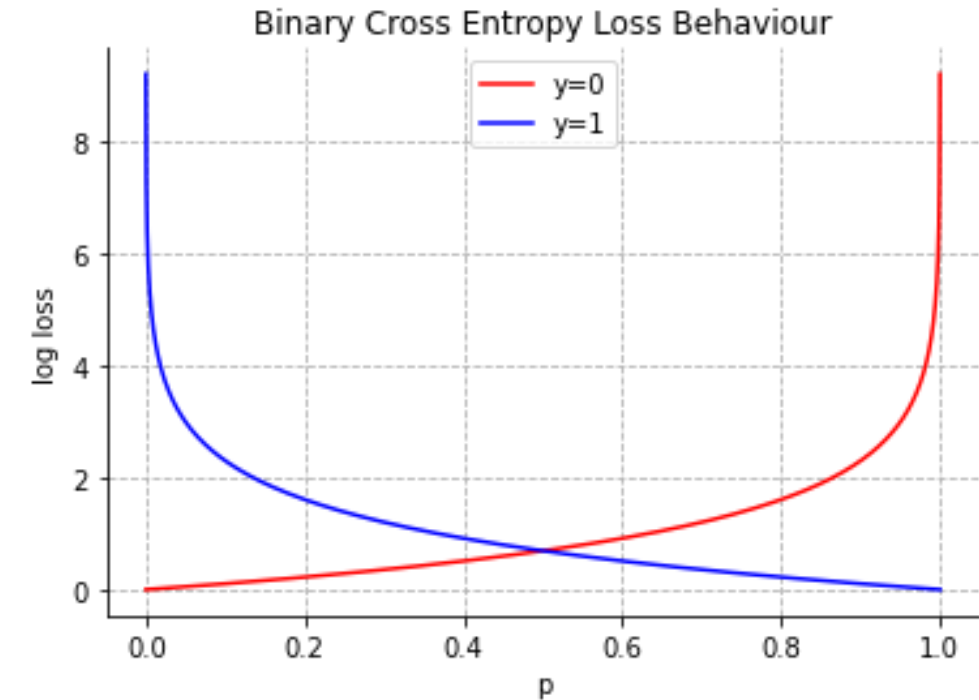
$$L(\hat{y}, y) = f(x) = \begin{cases} -\log(\hat{y}), & y = 1 \\ -\log(1 - \hat{y}), & y = 0 \end{cases}$$



Cross Entropy Loss function (Binary)

$$L(\hat{y}, y) = f(x) = \begin{cases} -\log(\hat{y}), & y = 1 \\ -\log(1 - \hat{y}), & y = 0 \end{cases}$$

$$L(\hat{y}, y) = -(y \log \hat{y} + (1 - y) \log (1 - \hat{y}))$$





Cross Entropy Loss function (Binary)

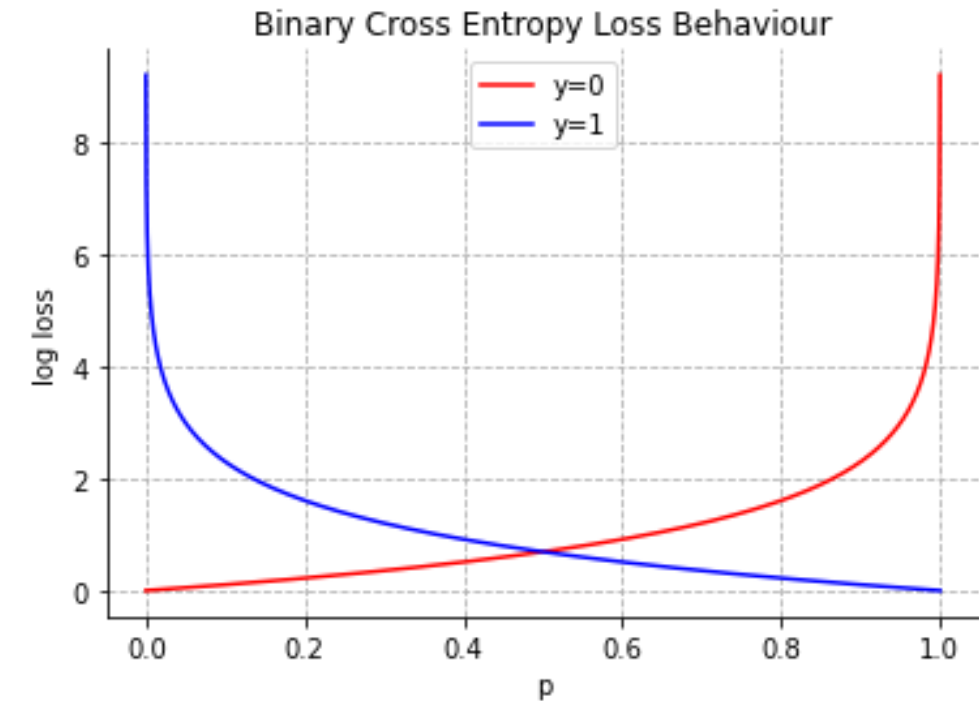
$$L(\hat{y}, y) = f(x) = \begin{cases} -\log(\hat{y}), & y = 1 \\ -\log(1 - \hat{y}), & y = 0 \end{cases}$$

$$L(\hat{y}, y) = -(y \log \hat{y} + (1 - y) \log (1 - \hat{y}))$$

Cross Entropy Loss is **Convex**

$$\Leftrightarrow L(\hat{y}, y)'' \geq 0$$

The line segment between any two points does not lie below the graph





Logistic Regression Training

Training Set D

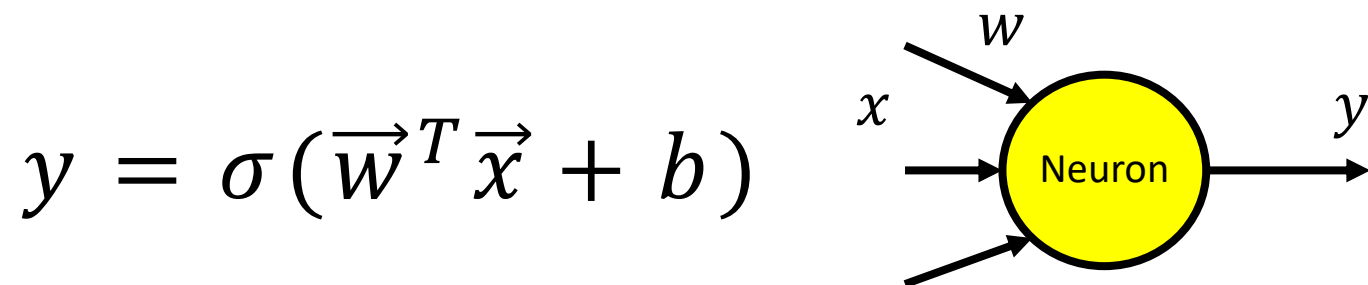
$$D : \{(\vec{x}^{(1)}, y^{(1)}), \dots, (\vec{x}^{(i)}, y^{(i)}), \dots, (\vec{x}^{(m)}, y^{(m)})\}$$

Cost J

$$\begin{aligned} J(\{\hat{y}\}^m, \{y\}^m; \{\vec{x}\}^m) &= \frac{1}{m} \sum_{i=1}^m L(\hat{y}^{(i)}, y^{(i)}) \\ &= -\frac{1}{m} \sum_{i=1}^m (y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log (1 - \hat{y}^{(i)})) \end{aligned}$$



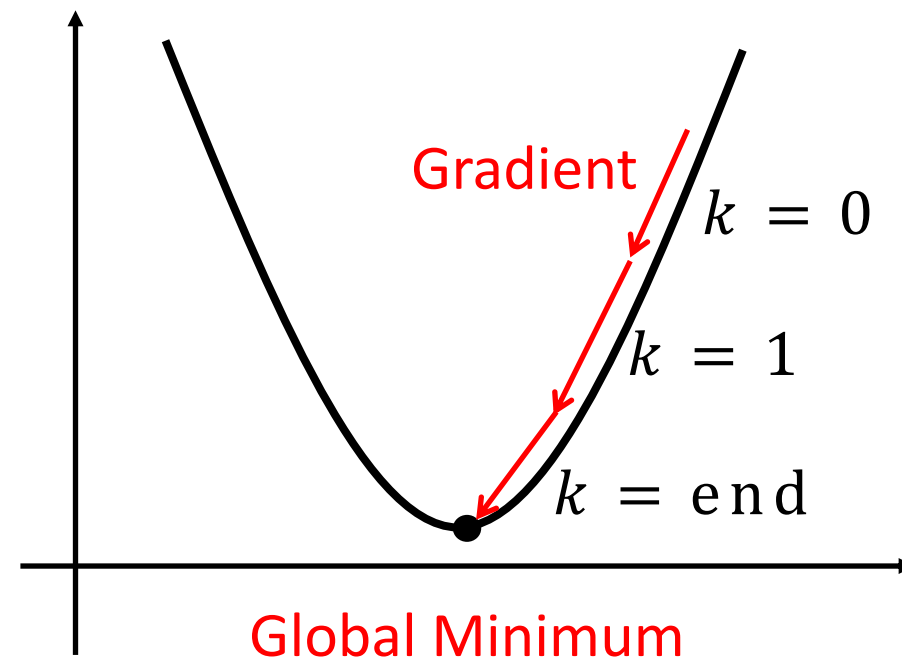
Minimizing Loss using Gradient Descent



$$J = L(\vec{w}, b, y)$$

$$\vec{w}_{k+1} = \vec{w}_k - \alpha \nabla_{\vec{w}} J(\vec{w}_k; b)$$

$$b_{k+1} = b_k - \alpha \frac{\partial}{\partial b} J(\vec{w}; b_k)$$





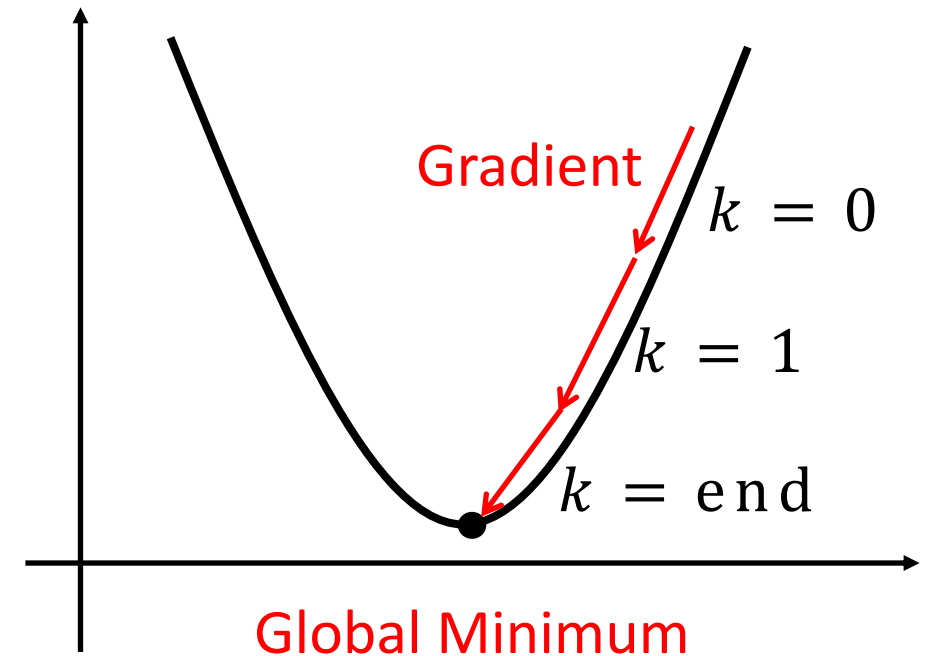
Minimizing Loss using Gradient Descent

$$y = \sigma(\vec{w}^T \vec{x} + b)$$

$$\vec{w}_{k+1} = \vec{w}_k - \underset{\substack{\text{Learning rate} \\ \downarrow}}{\alpha} \nabla_{\vec{w}} J(\vec{w}_k; b)$$

$$b_{k+1} = b_k - \alpha \frac{\partial}{\partial b} J(\vec{w}; b_k)$$

$$J = L((\vec{w}, b), y)$$



Iteratively adjust (\vec{w}, b) until we reach the global minimum



Finding Gradient w.r.t. weights and biases

$$J = L(\hat{y}, y)$$



Finding Gradient w.r.t. weights and biases

$$J = L(\hat{y}, y)$$

$$\underbrace{\hat{y}}_{\hat{y} = \sigma(z)} \text{ Activation function}$$



Finding Gradient w.r.t. weights and biases

$$J = L(\hat{y}, y)$$

$$\underbrace{\hat{y} = \sigma(z)}_{\text{Activation function}}$$

$$\underbrace{z = \vec{w}^T \vec{x} + b}_{\text{Integrate Inputs}}$$



Finding Gradient w.r.t. weights and biases

$$J = L(\hat{y}, y)$$
$$\underbrace{\hat{y} = \sigma(z)}$$
$$\underbrace{z = \vec{w}^T \vec{x} + b}$$

$$J = L(\sigma(\vec{w}^T \vec{x} + b), y)$$



Finding Gradient w.r.t. weights and biases

$$J = L(\hat{y}, y)$$

$$\hat{y} = \sigma(z)$$

$$z = \vec{w}^T \vec{x} + b$$

$$J = L(\sigma(\boxed{\vec{w}}^T \vec{x} + b), y)$$



Using Chain Rule to Compute Gradients

$$J = L(\hat{y}, y)$$

$\underbrace{\hspace{1.5cm}}$

$$\hat{y} = \sigma(z)$$

$\underbrace{\hspace{1.5cm}}$

$$z = \vec{w}^T \vec{x} + b$$

$$J = L(\sigma(\boxed{\vec{w}}^T \vec{x} + b), y)$$

$$P = f(g(h(x)))$$

$$\frac{dP}{dx} = \frac{df}{dg} * \frac{dg}{dh} * \frac{dh}{dx}$$



Using Chain Rule to Compute Gradients

$$J = L(\overbrace{\sigma(\underbrace{\vec{w}^T \vec{x} + b}_{z})}^{\hat{y}}, y)$$

$$P = f(g(h(x)))$$

$$\frac{dP}{dx} = \frac{df}{dg} * \frac{dg}{dh} * \frac{dh}{dx}$$



Using Chain Rule to Compute Gradients

$$J = L(\underbrace{\sigma(\underbrace{\vec{w}^T \vec{x} + b}_{z})}_{\hat{y}}, y)$$

$$P = f(g(h(x)))$$

$$\begin{aligned} \frac{\partial J}{\partial \vec{w}} &= \frac{\partial L}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z} * \frac{\partial z}{\partial \vec{w}} \\ &= \frac{\partial L}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z} * \nabla_{\vec{w}} z \end{aligned}$$

$$\frac{dP}{dx} = \frac{df}{dg} * \frac{dg}{dh} * \frac{dh}{dx}$$

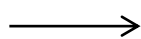


Using Chain Rule to Compute Gradients

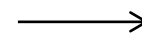
$$\nabla_{\vec{w}} L(\hat{y}, y) = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z} \nabla_{\vec{w}} z$$

FWD

$$z = \vec{w}^T \vec{x} + b$$



$$\hat{y} = \sigma(z)$$



$$L(\hat{y}, y)$$

BWD

$$\nabla_{\vec{w}} z$$



$$\frac{\partial \hat{y}}{\partial z}(z)$$



$$\frac{\partial L}{\partial \hat{y}}(\hat{y}, y)$$



Using Chain Rule to Compute Gradients

BWD

$$\nabla_{\vec{w}} z$$

$$\frac{\partial \hat{y}}{\partial z}(z)$$

$$\frac{\partial L}{\partial \hat{y}}(\hat{y}, y)$$

$$\nabla_{\vec{w}} L$$

$$\vec{x}$$

$$\sigma(z)(1 - \sigma(z))$$

$$-\frac{y}{\hat{y}} + \frac{1 - y}{1 - \hat{y}}$$

$$\frac{\partial L}{\partial b}$$

$$1$$

$$\sigma(z)(1 - \sigma(z))$$

$$-\frac{y}{\hat{y}} + \frac{1 - y}{1 - \hat{y}}$$



Training Terminologies

- **Forward Propagation:**
Computing the **loss through forward pass** for a single training example
- **Backward Propagation:**
Computing **gradients of parameters** through backward pass for a single training example
- **Batch:**
Training set could be divided into **smaller sets** called batches
- **Iteration:**
When an **entire batch** is passed both **forward and backward**
- **Epoch:**
When an **entire dataset** is passed both **forward and backward** through the NN once



Batch Gradient Descent

Training Set D

$$D : \{(\vec{x}^{(1)}, y^{(1)}), \dots, (\vec{x}^{(i)}, y^{(i)}), \dots, (\vec{x}^{(m)}, y^{(m)})\}$$

Cost J

$$\begin{aligned} J(\{\hat{y}\}^m, \{y\}^m; \{\vec{x}\}^m) &= \frac{1}{m} \sum_{i=1}^m L(\hat{y}^{(i)}, y^{(i)}) \\ &= -\frac{1}{m} \sum_{i=1}^m (y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log (1 - \hat{y}^{(i)})) \end{aligned}$$

Sum the gradients for all m -examples where m = total number of samples in training set (Single epoch)

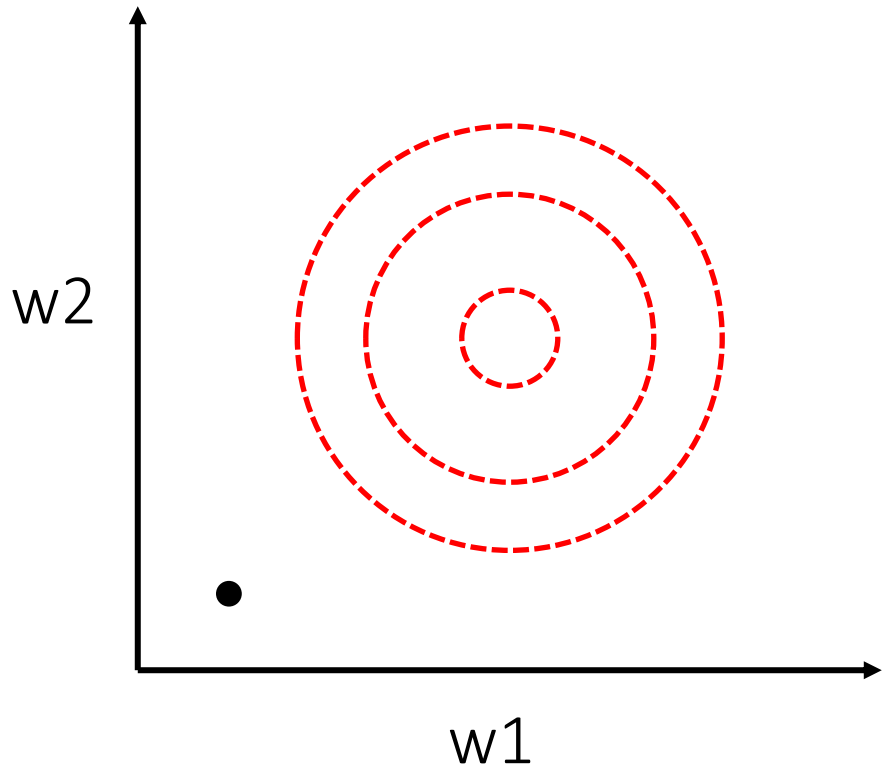


PART 3:

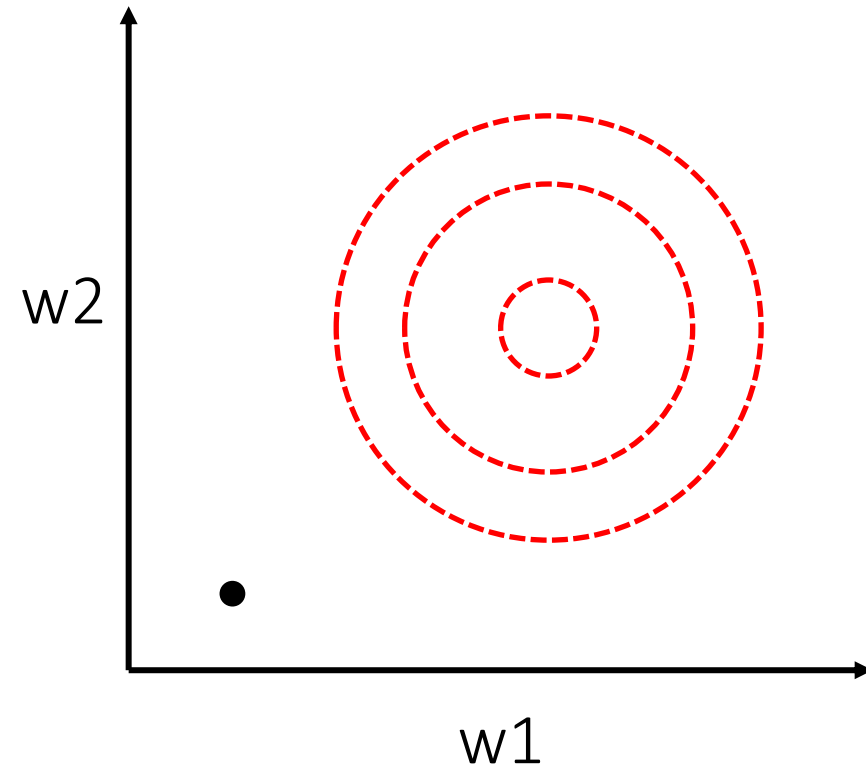
Stochastic Gradient Descent (SGD)



Gradient descent (GD) vs Stochastic Gradient Descent (SGD)



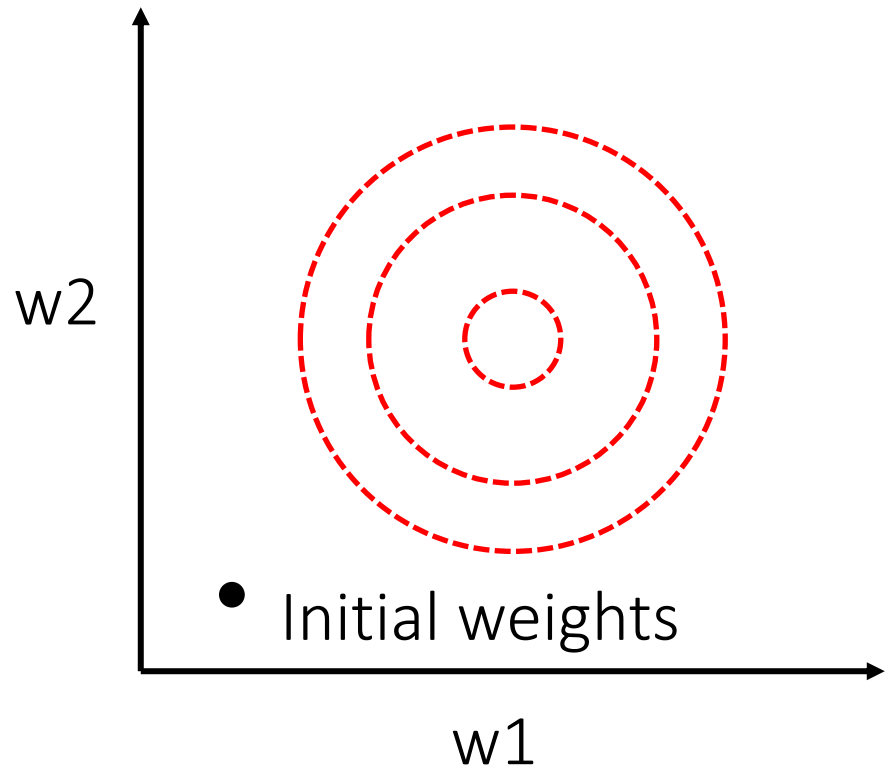
Batch Gradient Descent
1 iteration: FWD pass and BWD pass on
whole training set



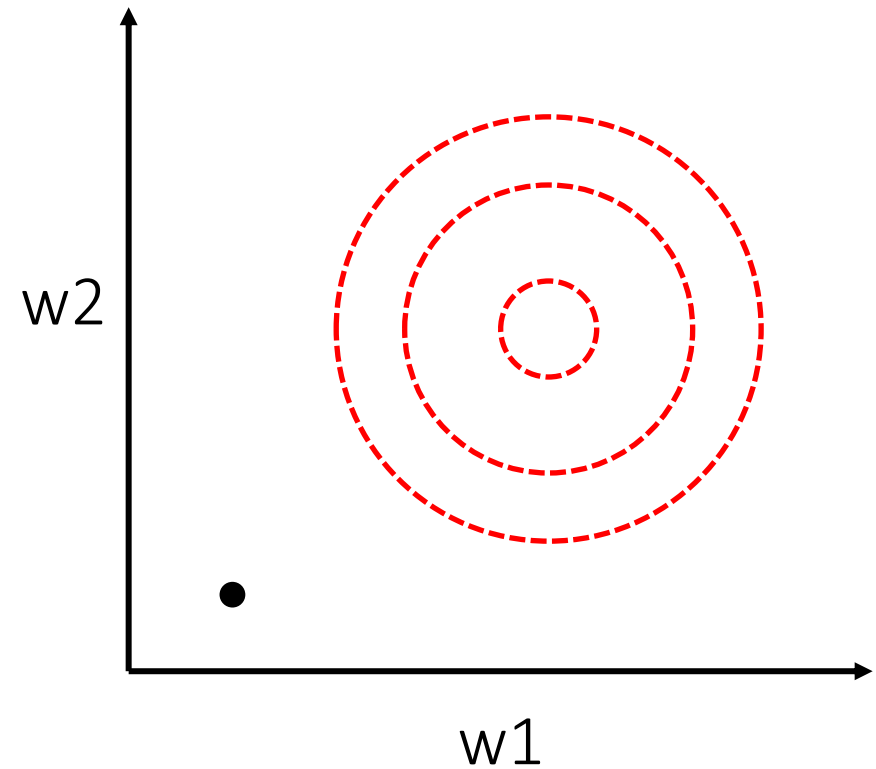
Stochastic Gradient Descent
1 iteration: FWD pass and BWD pass on
subset of training set



GD vs SGD



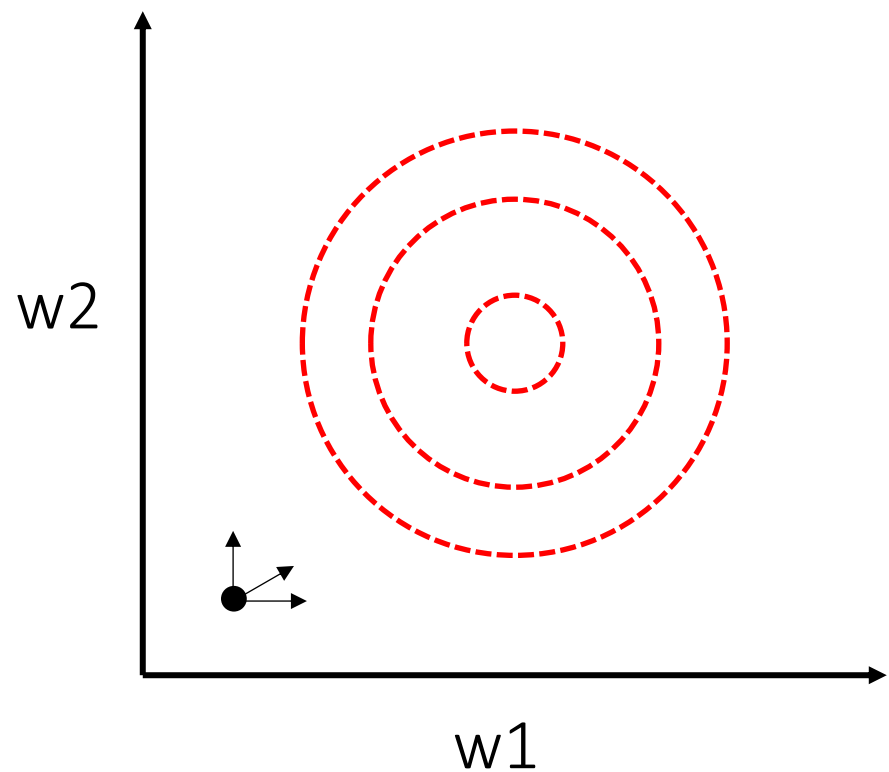
Batch Gradient Descent



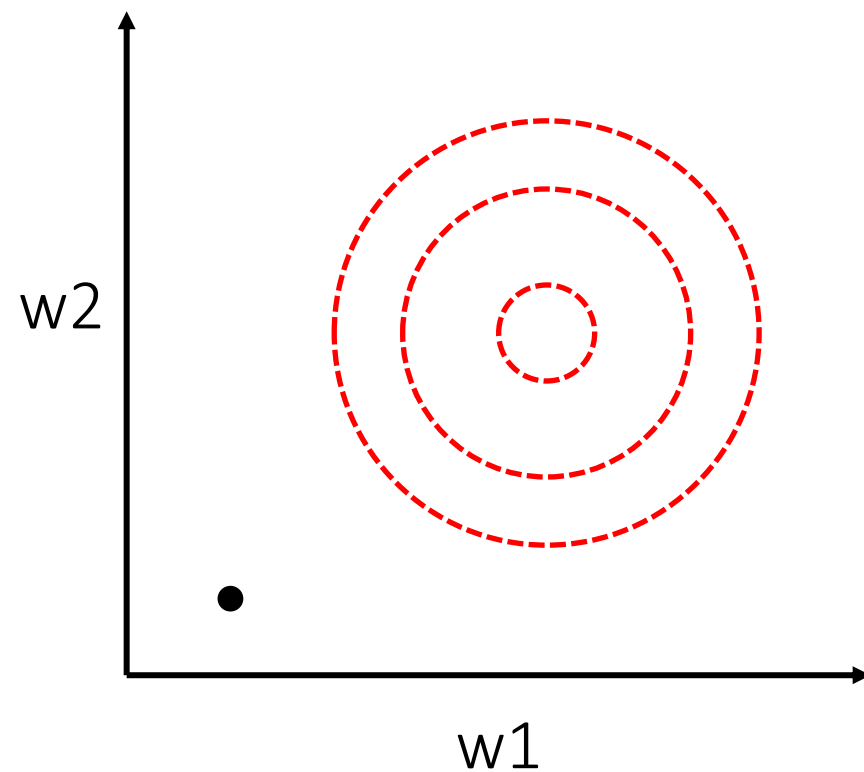
Stochastic Gradient Descent



GD vs SGD



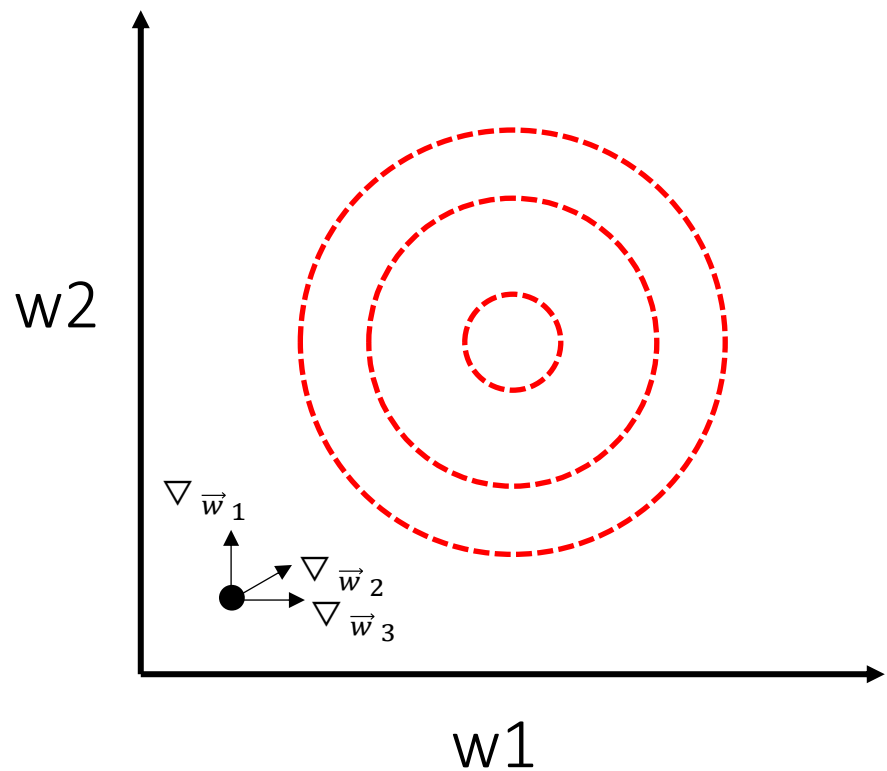
Batch Gradient Descent



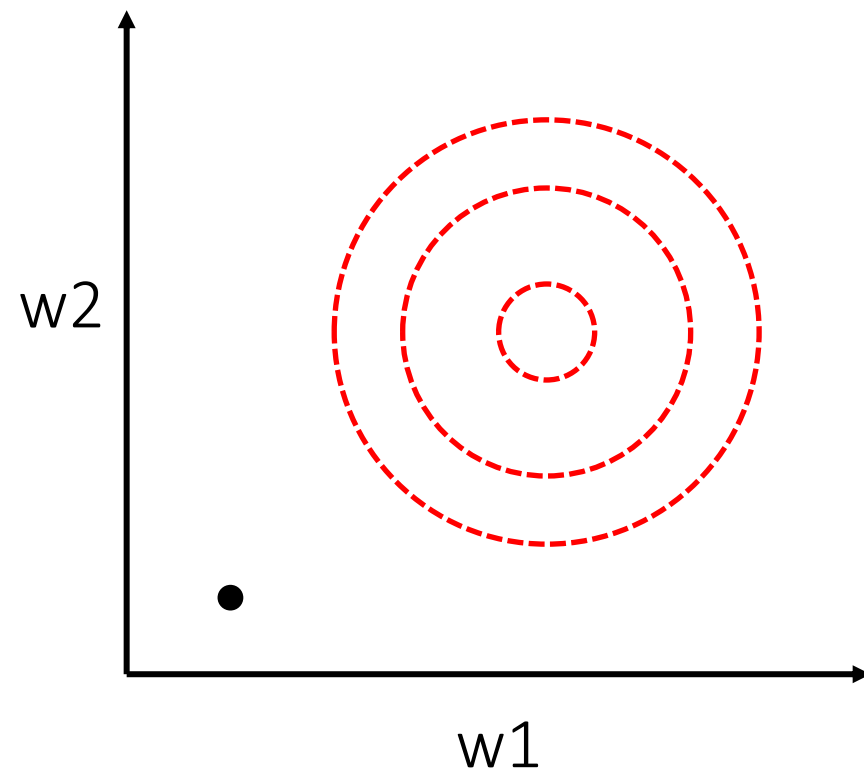
Stochastic Gradient Descent



GD vs SGD



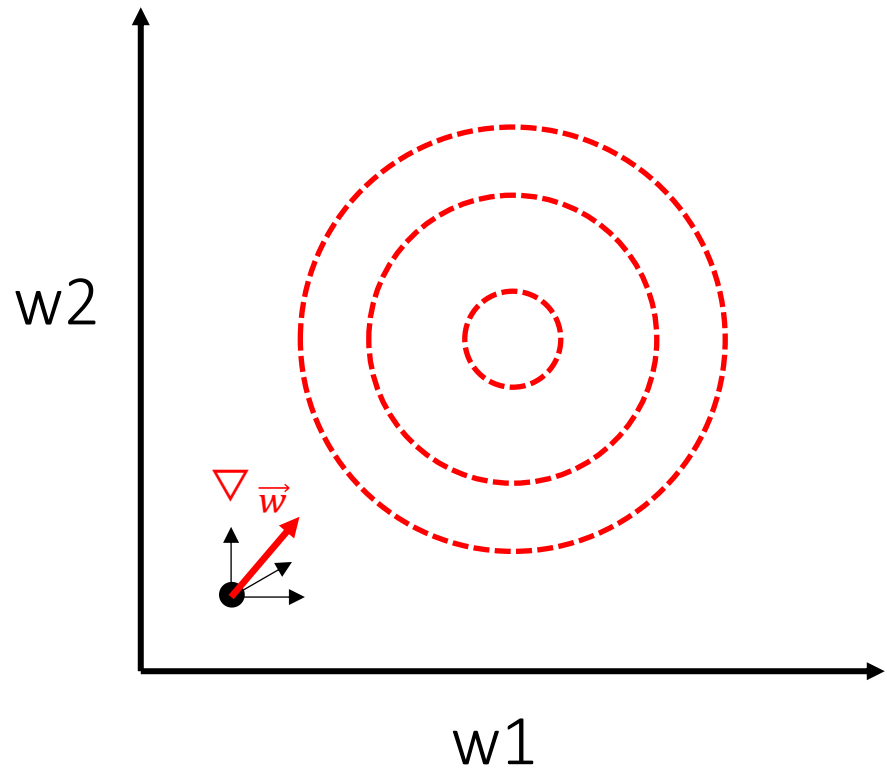
Batch Gradient Descent



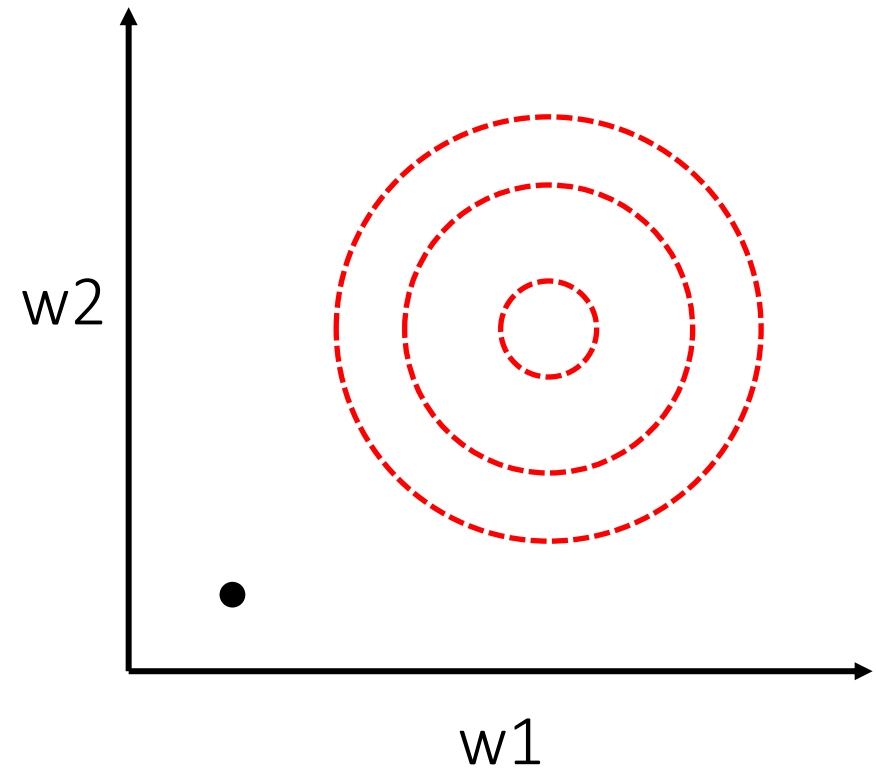
Stochastic Gradient Descent



GD vs SGD



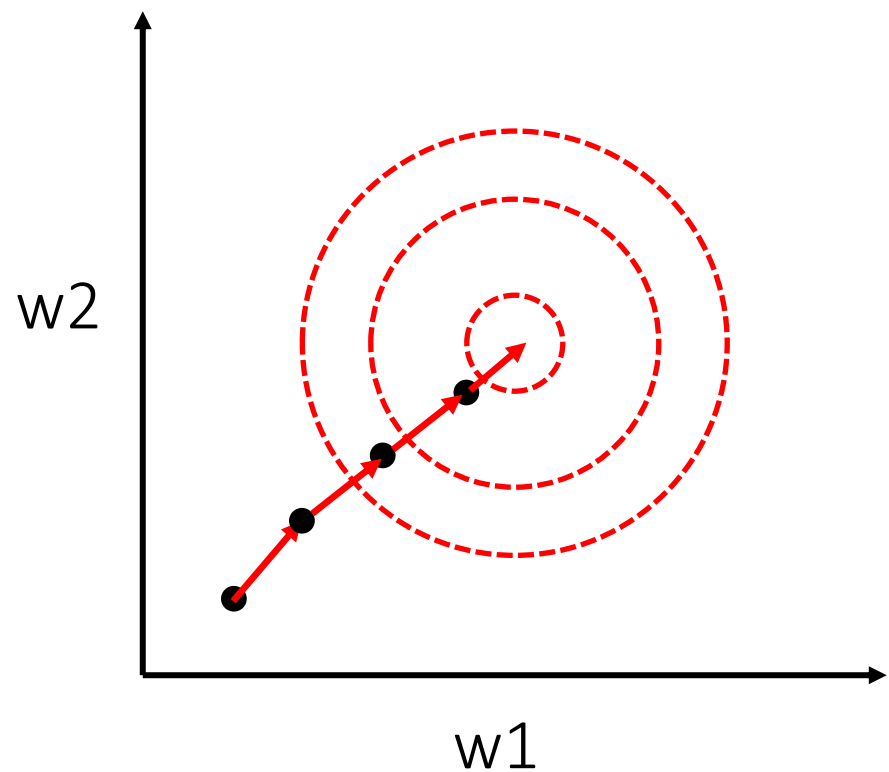
Batch Gradient Descent



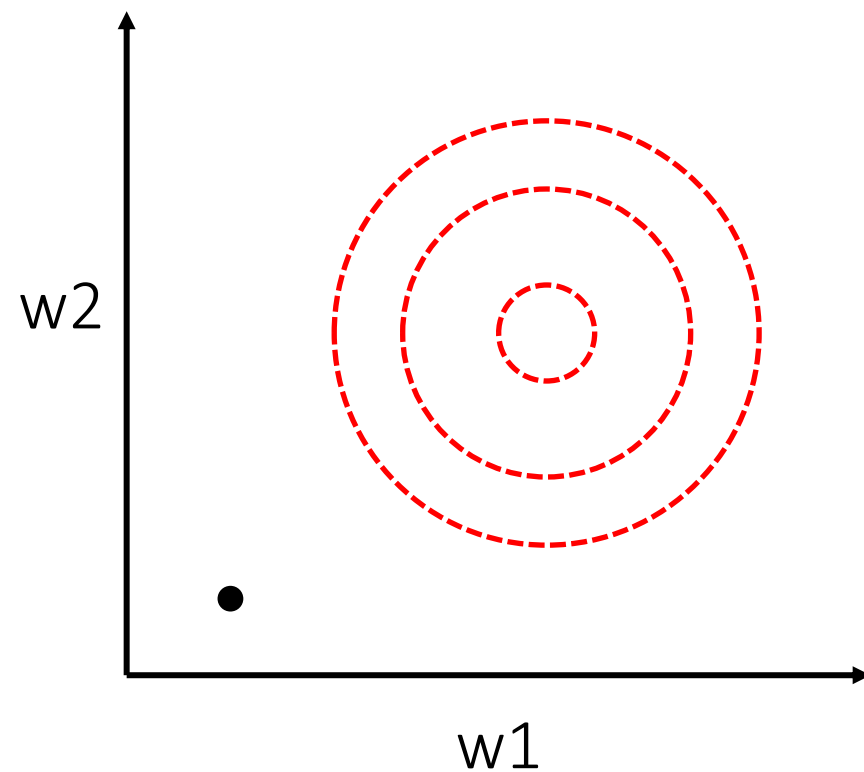
Stochastic Gradient Descent



GD vs SGD



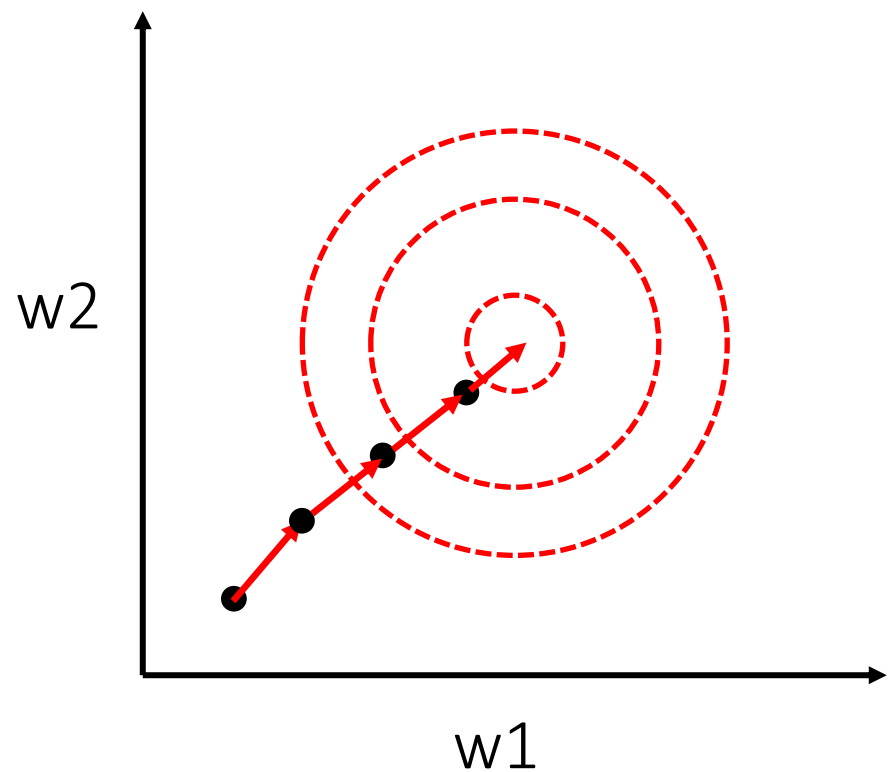
Batch Gradient Descent



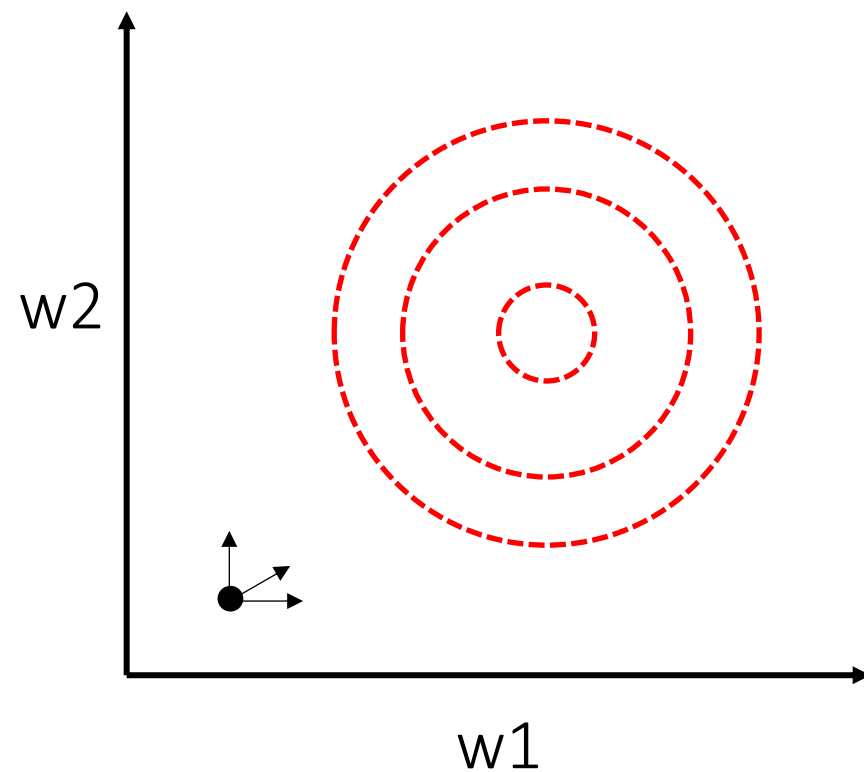
Stochastic Gradient Descent



GD vs SGD



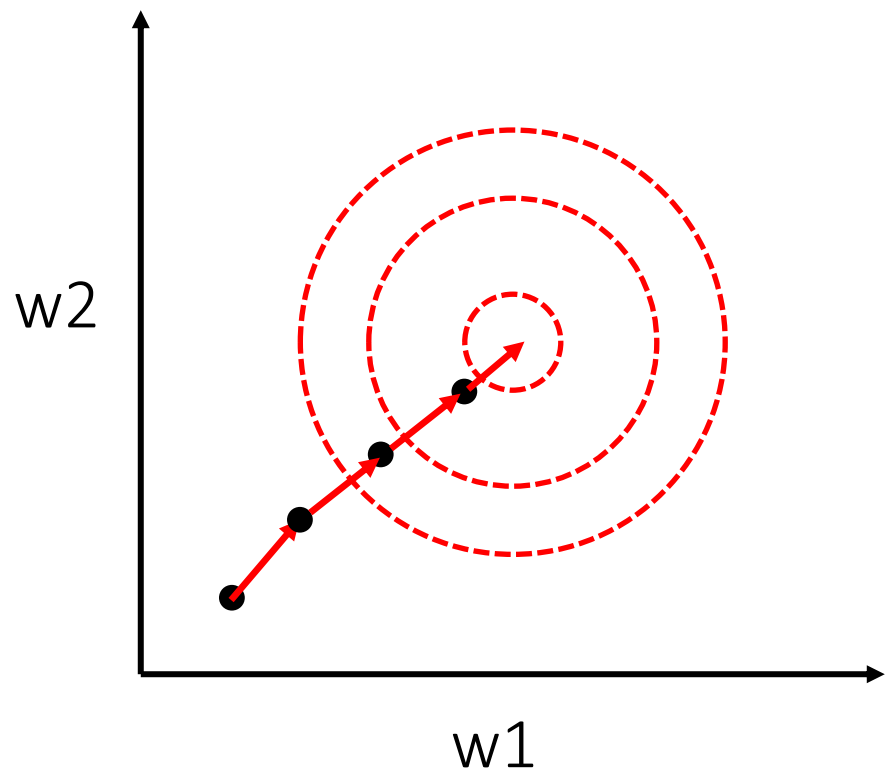
Batch Gradient Descent



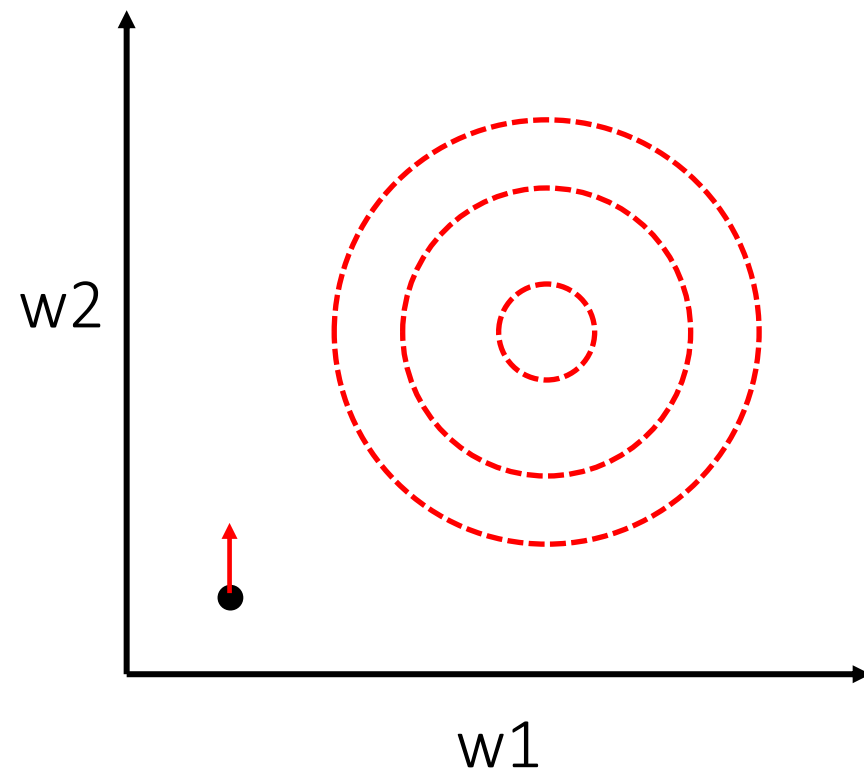
Stochastic Gradient Descent



GD vs SGD



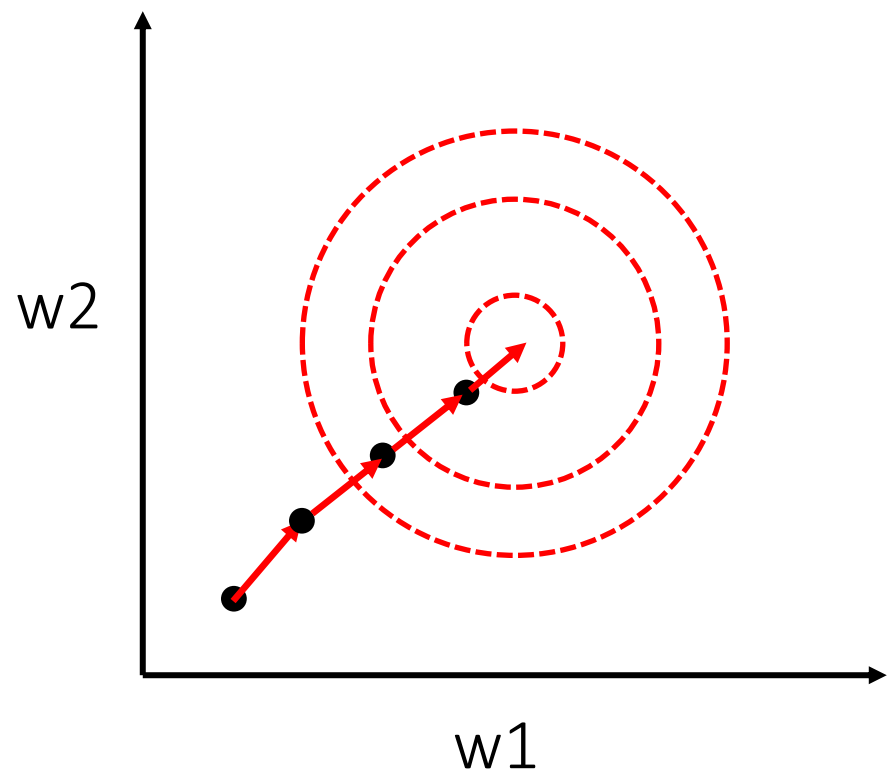
Batch Gradient Descent



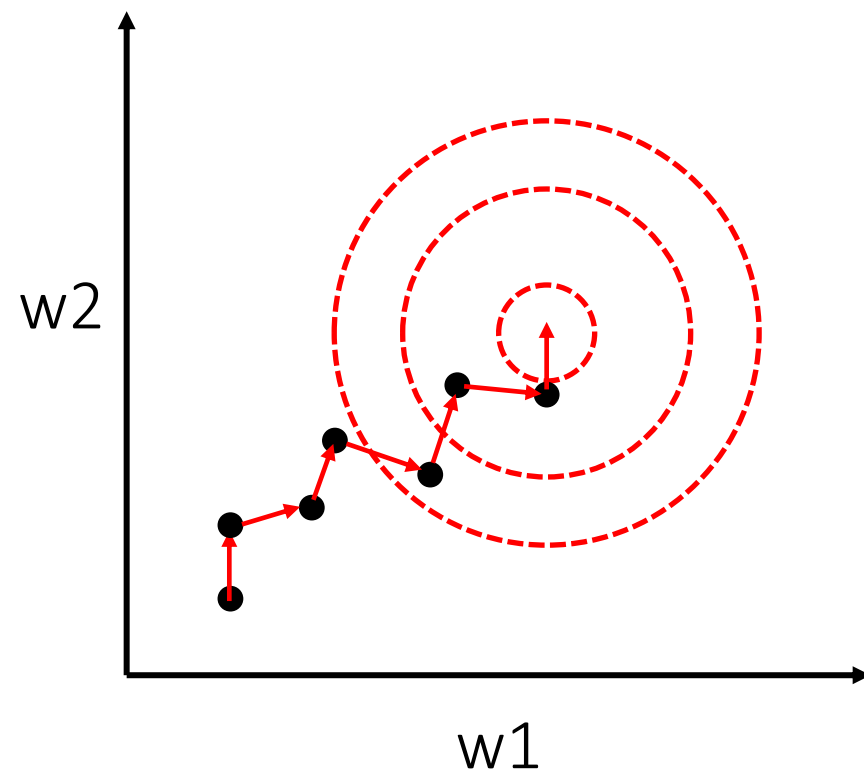
Stochastic Gradient Descent



GD vs SGD



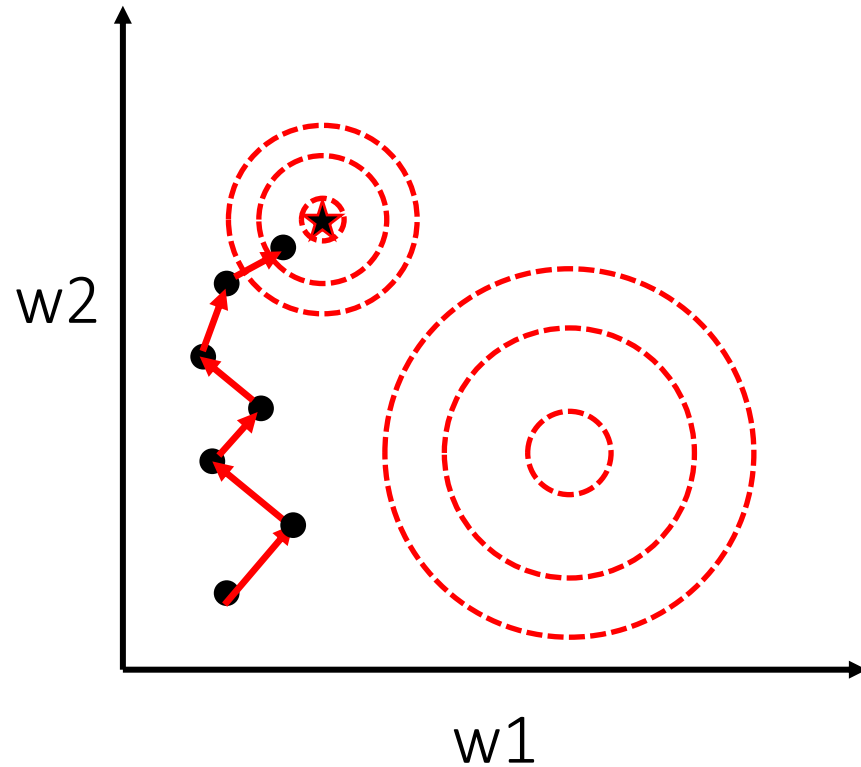
Batch Gradient Descent



Stochastic Gradient Descent



Pros and Cons of SGD

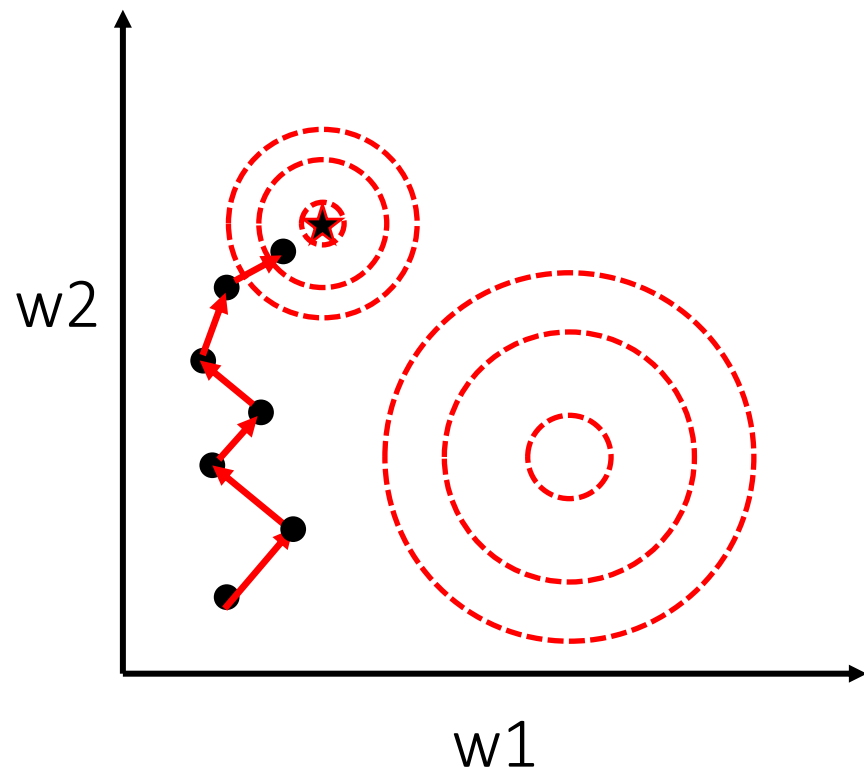


Pros:

Still consistently converges to minimum
May take shortcut to minimum

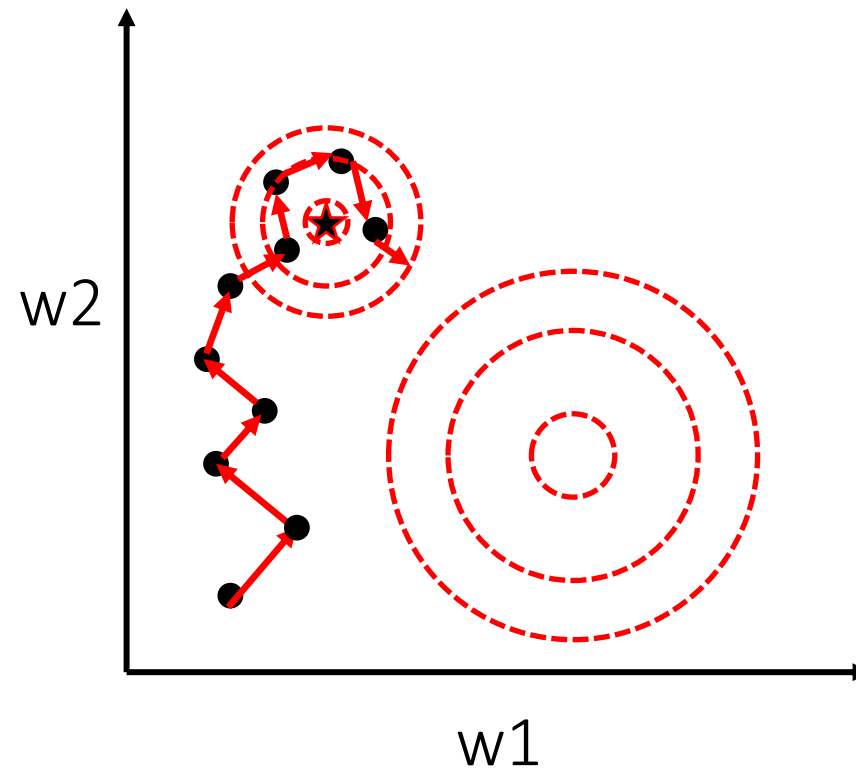


Pros and Cons of SGD



Pros:

Still consistently converges to minimum
May take shortcut to minimum



Cons:

Not useful when we are already close to minimum
Hard to parallelize



GD vs SGD



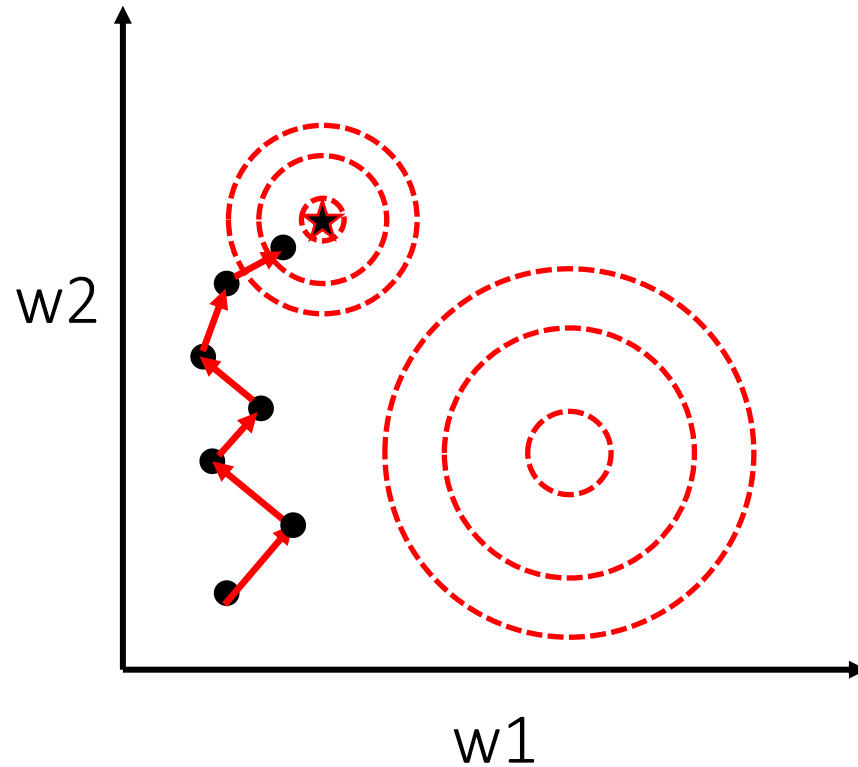
Journal of Mathematical Analysis and Applications
Volume 114, Issue 2, March 1986, Pages 512-527



On convergence of the stochastic subgradient
method with on-line stepsize rules

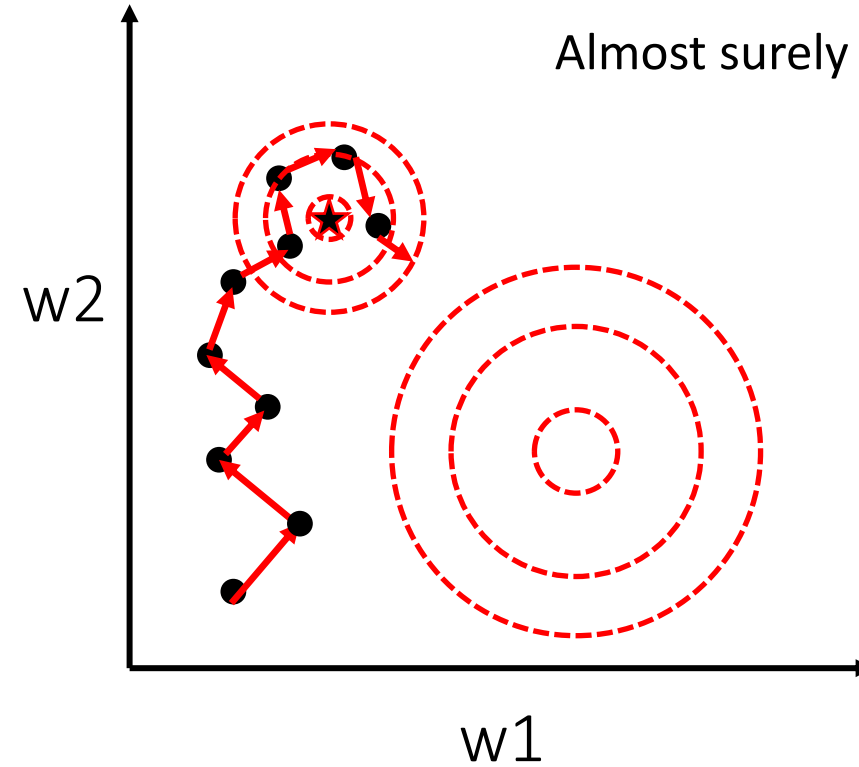
Andrzej Ruszczyński *, Wojciech Syski

Almost surely convergence



Pros:

Still consistently converges to minimum
May take shortcut to minimum

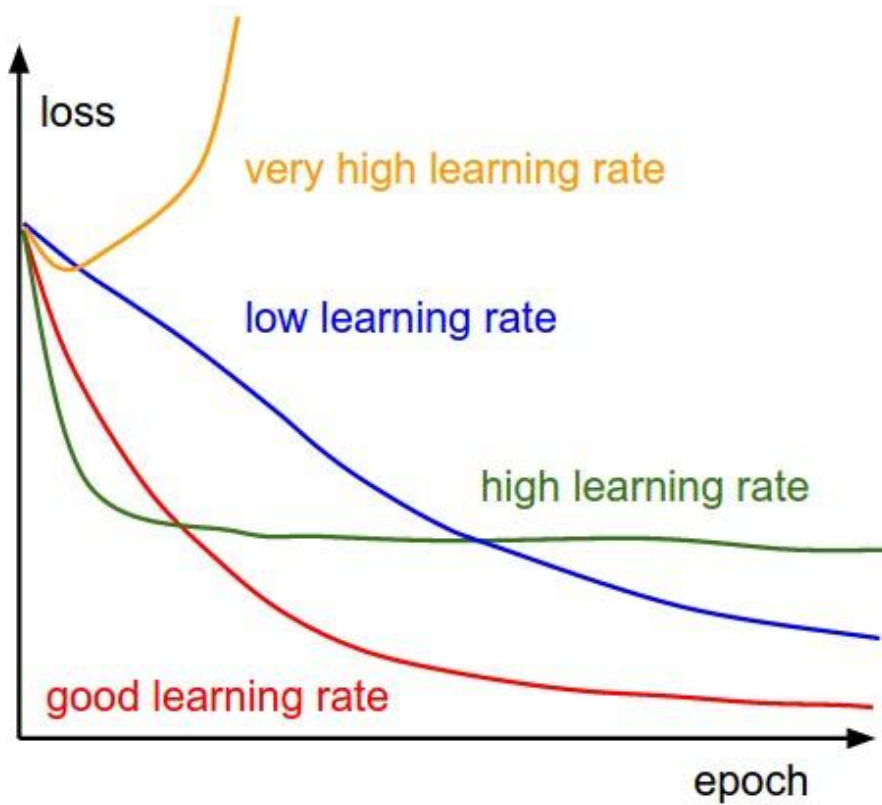


Cons:

Not useful when we are already close to minimum
Hard to parallelize



Effects of learning rate (α) on SGD

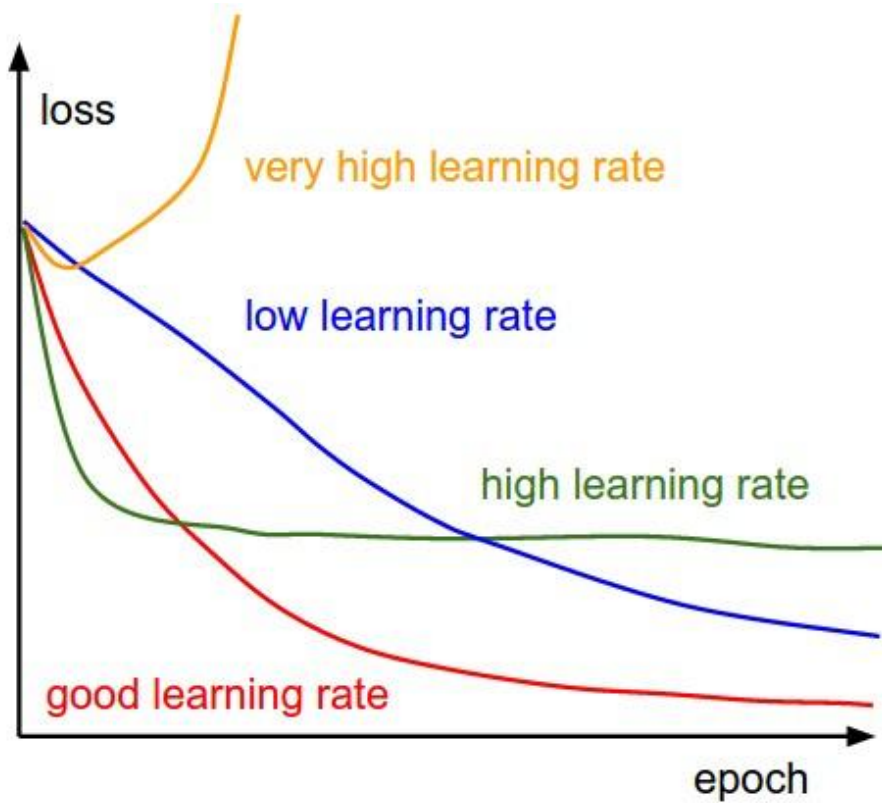


$$\vec{w}_{k+1} = \vec{w}_k - \boxed{\alpha} \nabla_{\vec{w}} J(\vec{w}_k; b)$$

$$b_{k+1} = b_k - \boxed{\alpha} \frac{\partial}{\partial b} J(\vec{w}; b_k)$$

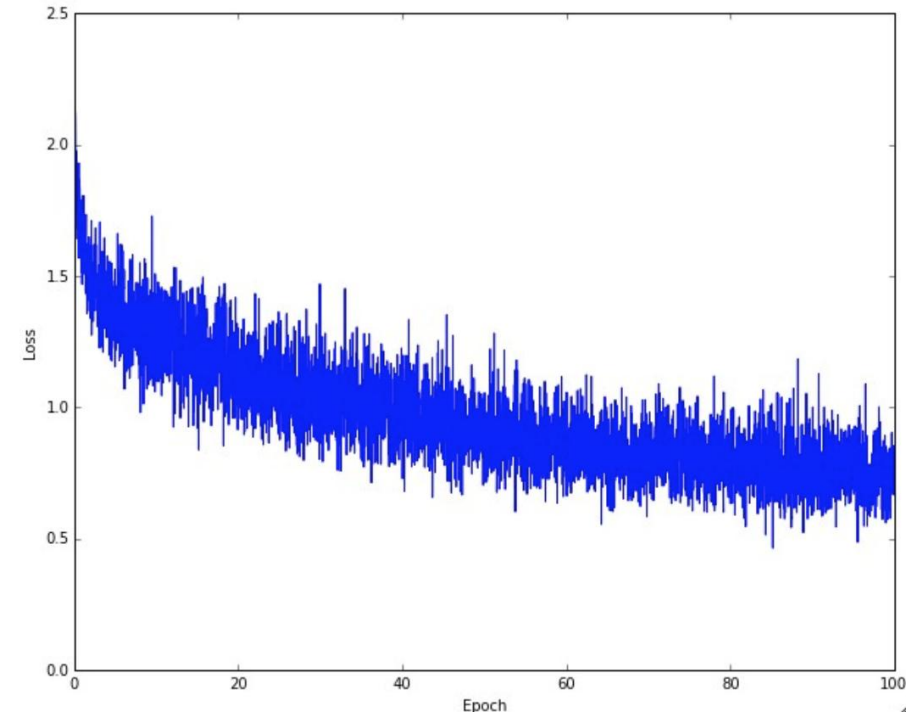


Effects of learning rate (α) on SGD



$$\vec{w}_{k+1} = \vec{w}_k - \boxed{\alpha} \nabla_{\vec{w}} J(\vec{w}_k; b)$$

$$b_{k+1} = b_k - \boxed{\alpha} \frac{\partial}{\partial b} J(\vec{w}; b_k)$$



Loss curve is typically noisy with SGD



Effects of learning rate on SGD

	SGD	Mini-batch GD	Batch GD
Data batch size per iteration	1		
	<div>(-) Can loose speedup from oscillations</div> <div>(-) hard to parallelize</div>		

N = Total # of datapoints in training set

m = Number of mini-batches for training set



Effects of learning rate on SGD

	SGD	Mini-batch GD	Batch GD
Data batch size per iteration	1	N/m	
	<div>(-) Can loose speedup from oscillations</div> <div>(-) hard to parallelize</div>	<div>(+) The whole mini-batch is evaluated in parallel</div> <div>(+) Mostly consistent convergence</div>	

N = Total # of datapoints in training set

m = Number of mini-batches for training set



Effects of learning rate on SGD

	SGD	Mini-batch GD	Batch GD
Data batch size per iteration	1	N/m	n
	<ul style="list-style-type: none">(-) Can loose speedup from oscillations(-) hard to parallelize	<ul style="list-style-type: none">(+) The whole mini-batch is evaluated in parallel(+) Mostly consistent convergence	<ul style="list-style-type: none">(+) Consistent convergence(+) Maximum parallelization(-) Too long per iteration(-) Hardware memory limit (RAM, VRAM)

N = Total # of datapoints in training set

m = Number of mini-batches for training set



PART 4:

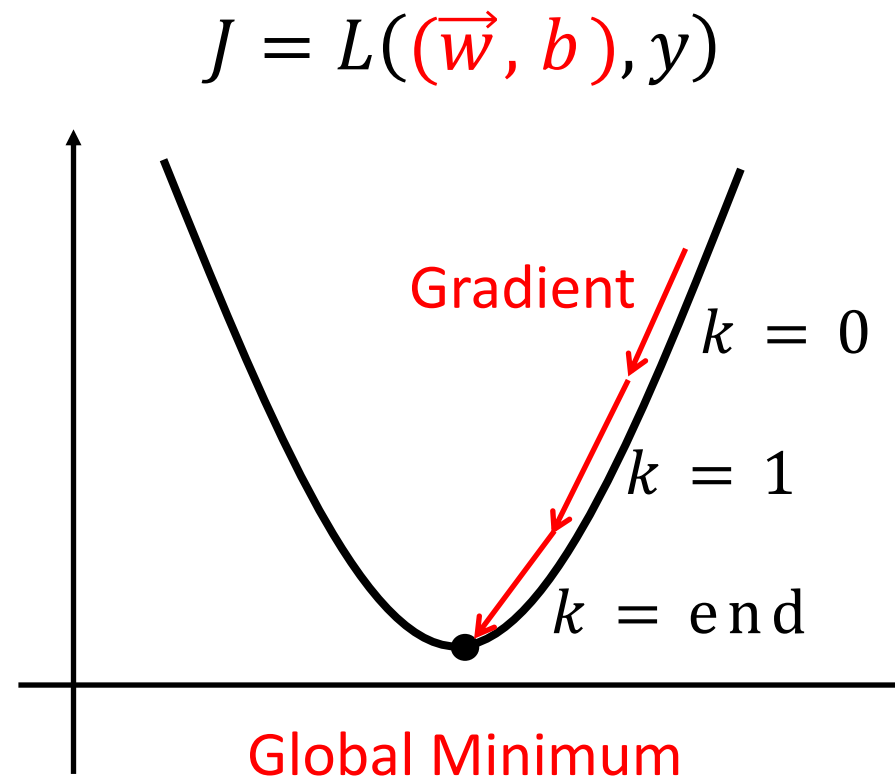
Optimization Techniques in Deep Learning



Variable Learning Rates

$$\vec{w}_{k+1} = \vec{w}_k - \alpha \nabla_{\vec{w}} J(\vec{w}_k; b)$$

$$b_{k+1} = b_k - \alpha \frac{\partial}{\partial b} J(\vec{w}; b_k)$$



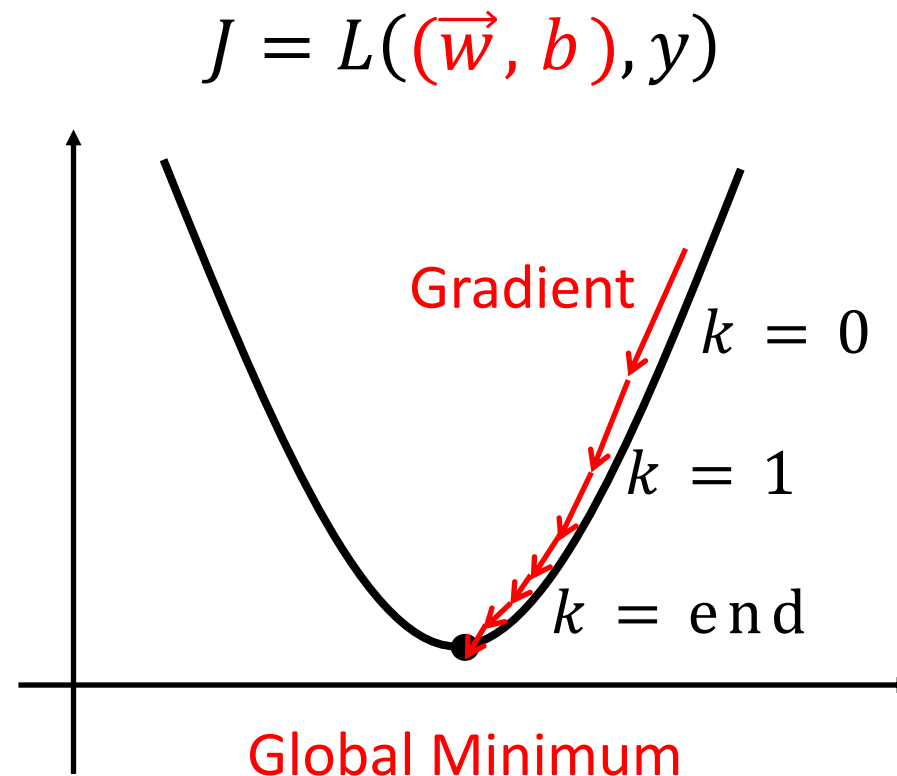


Variable Learning Rates

$$\vec{w}_{k+1} = \vec{w}_k - \alpha \nabla_{\vec{w}} J(\vec{w}_k; b)$$

$$b_{k+1} = b_k - \alpha \frac{\partial}{\partial b} J(\vec{w}; b_k)$$

$$\alpha = f(hp_1, hp_2, \dots)$$





Variable Learning Rates

$$\vec{w}_{k+1} = \vec{w}_k - \alpha \nabla_{\vec{w}} J(\vec{w}_k; b)$$

$$\alpha = \frac{1}{1 + \text{decr} \cdot \text{epnum}} \alpha_0$$

$$b_{k+1} = b_k - \alpha \frac{\partial}{\partial b} J(\vec{w}; b_k)$$

$$\alpha = d^{\text{epnum}} \cdot \alpha_0$$

$$\alpha = f(\text{hp}_1, \text{hp}_2, \dots)$$

$$\alpha = \frac{d}{\sqrt{\text{epnum}}} \cdot \alpha_0$$



Advanced Methods

Momentum

“Accelerate” gradients vectors in the right directions, to lead to faster converging.

AdaGrad

Adagrad uses a different learning rate for every parameter w_j at every step k . It eliminates the need to manually tune the learning rate.

RMSProp

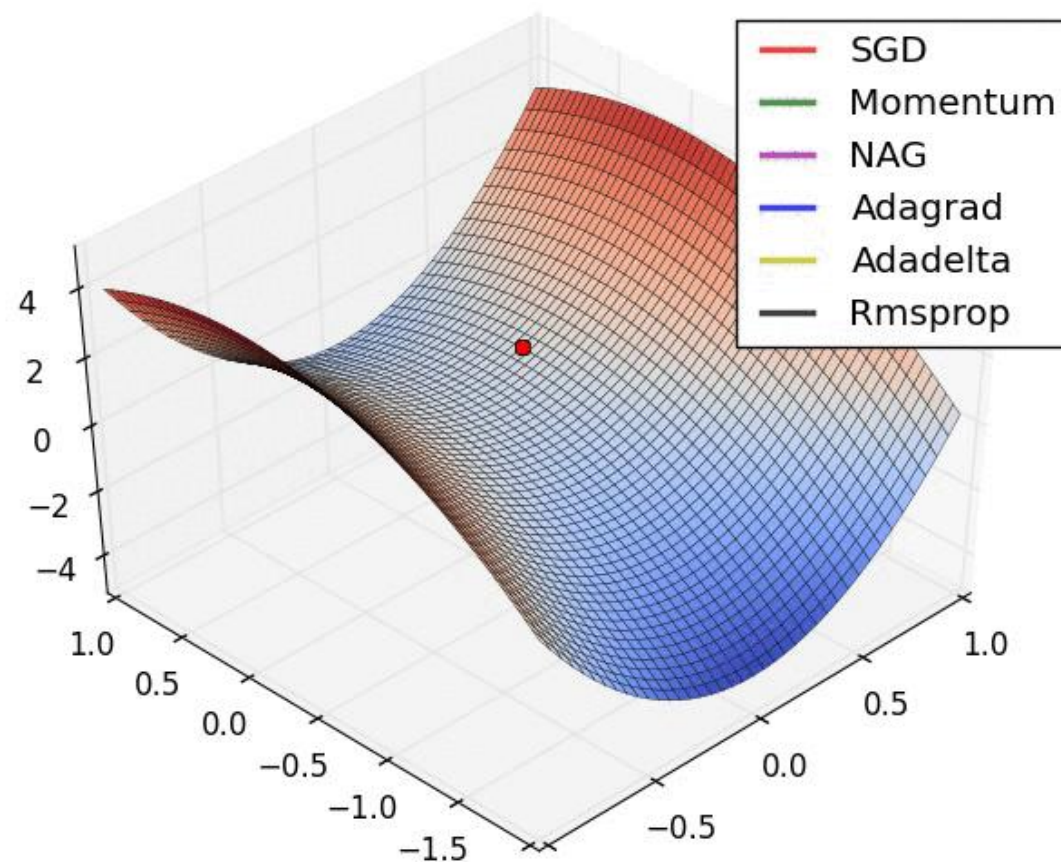
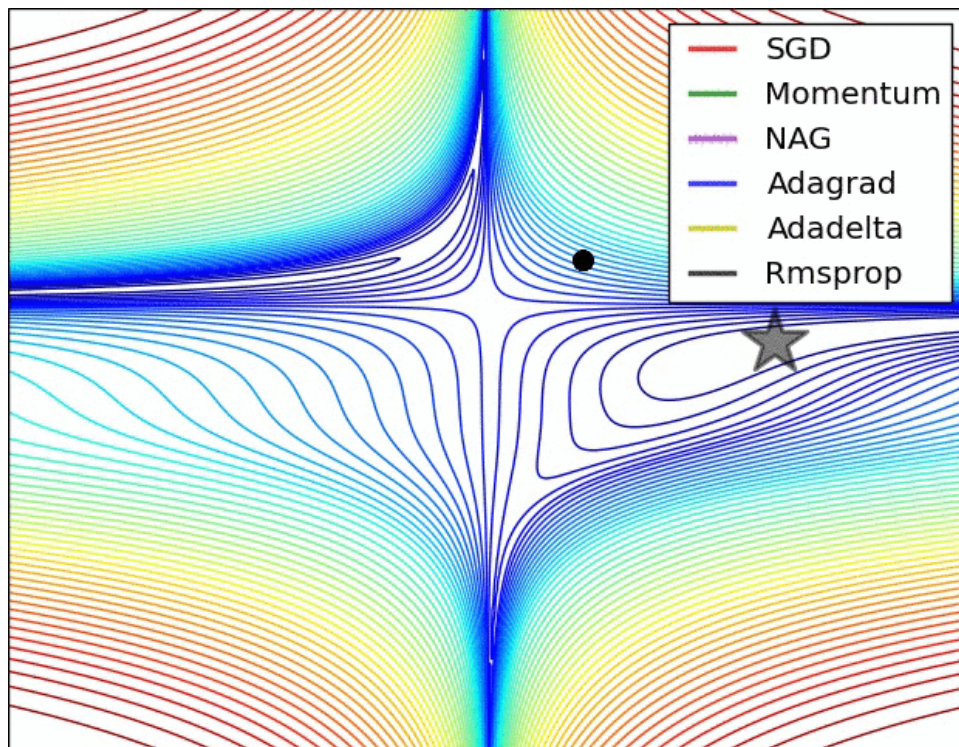
“Extended” and weighted version of AdaGrad via moving average of squared gradients

AdaM

Adaptive learning rate + Momentum



Advanced Methods



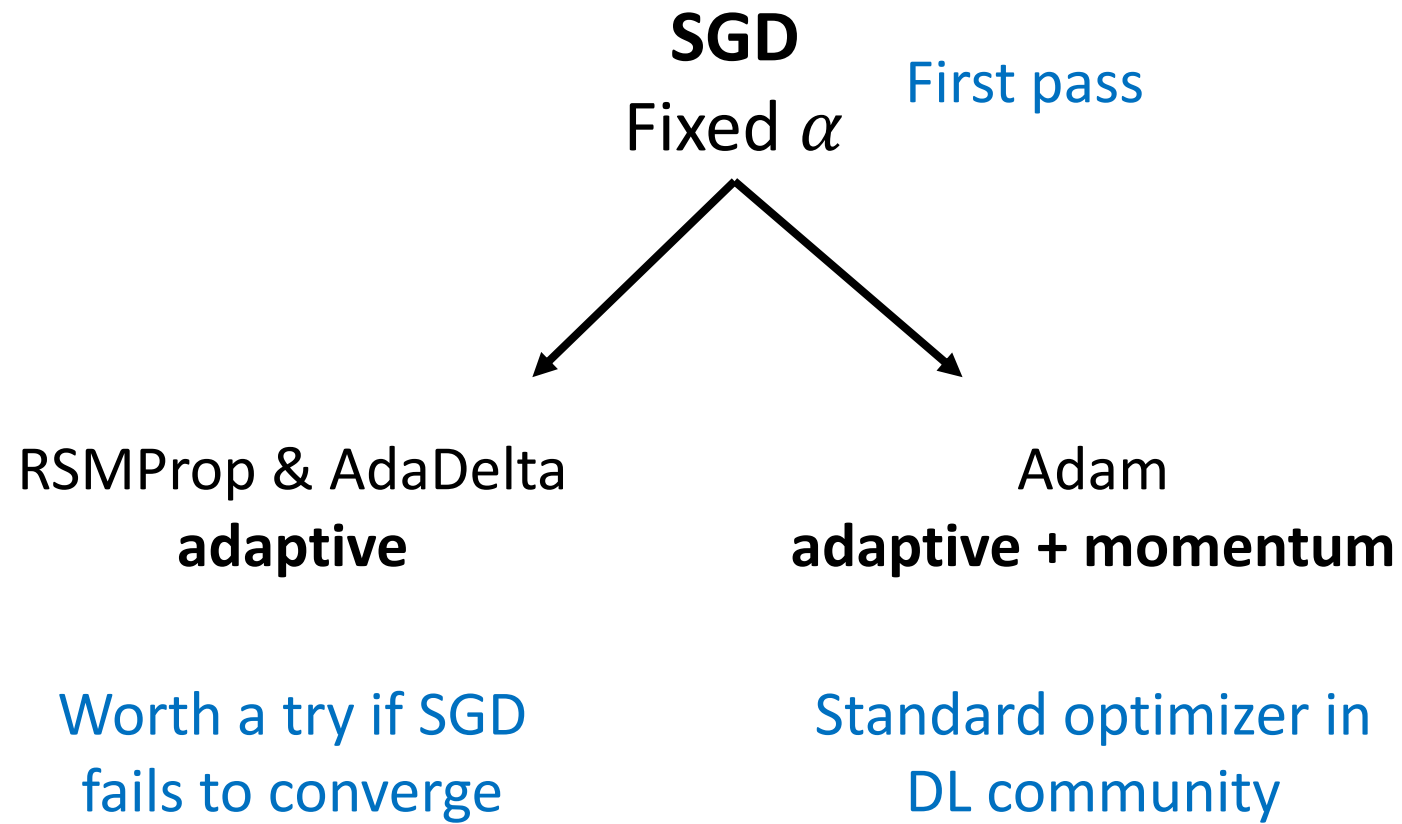


Advanced Methods

SGD
Fixed α First pass



Advanced Methods





Optimizer vs Optimization Techniques

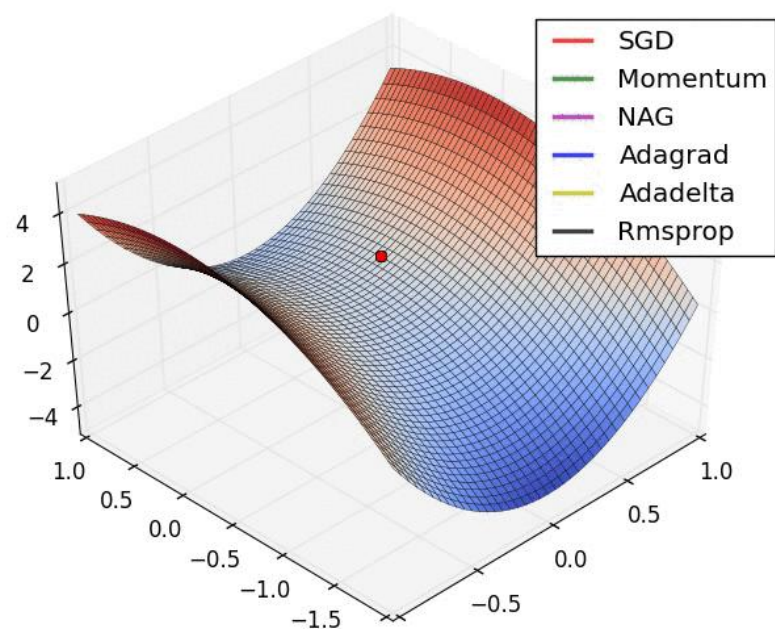
Optimizers

Optimization Techniques



Optimizer vs Optimization Techniques

Optimizers



Optimization Techniques



Optimizer vs Optimization Techniques

Optimizers

- Vanilla SGD
- Momentum
- AdaGrad
- RMSProp
- Adam

Optimization Techniques



Optimizer vs Optimization Techniques

Optimizers

- Vanilla SGD
- Momentum
- AdaGrad
- RMSProp
- Adam

Optimization Techniques

Everything else that contributes to optimization



Optimizer vs Optimization Techniques

Optimizers

- Vanilla SGD
- Momentum
- AdaGrad
- RMSProp
- Adam

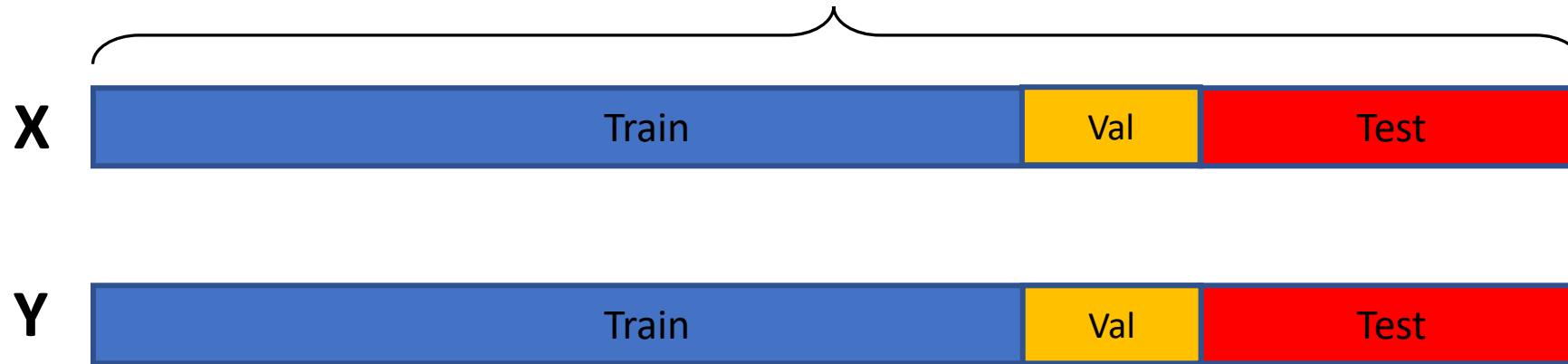
Optimization Techniques

- Data splitting (Train/Val/Test)
- Regularization
- Data normalization
- Batch-normalization
- Network initialization
- Hyperparameter tunings



Cross Validation in Supervised Learning

Dataset (X inputs, Y targets)



During training

Training loss ↓

Validation performance ↑

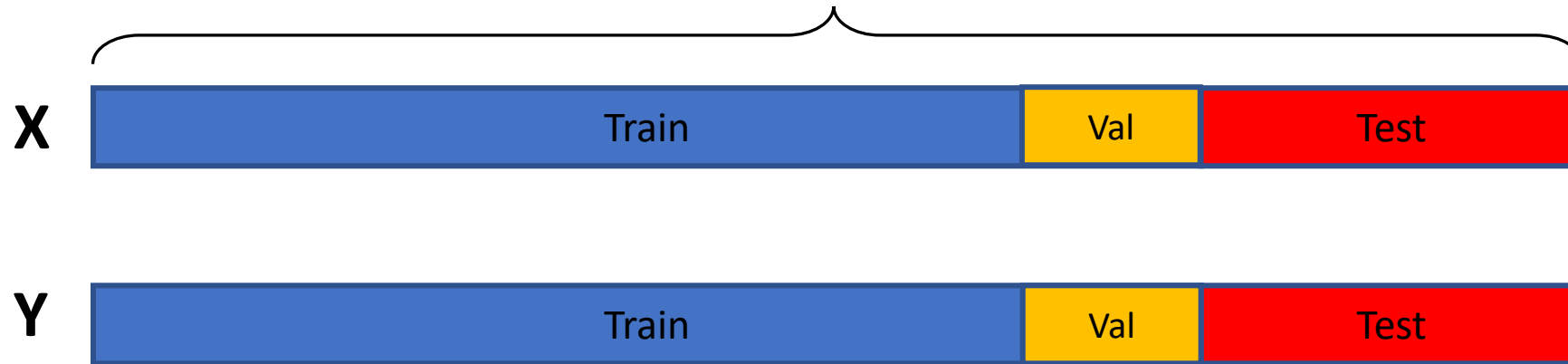
After training

Testing performance ↑



Cross Validation in Supervised Learning

Dataset (X inputs, Y targets)



Common ratios

70, 15, 15

80, 10, 10

60, 20, 20

⋮

During training

Training loss ↓

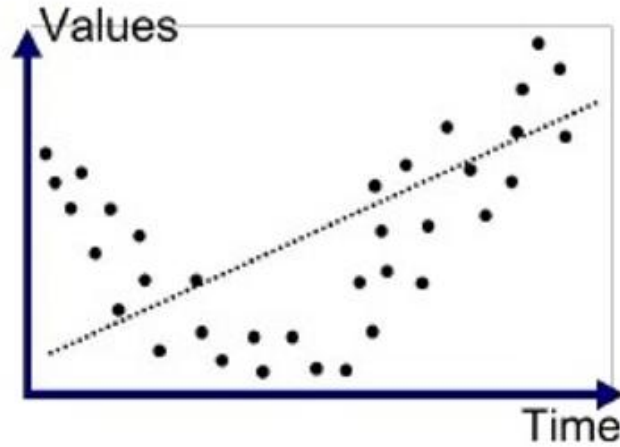
Validation performance ↑

After training

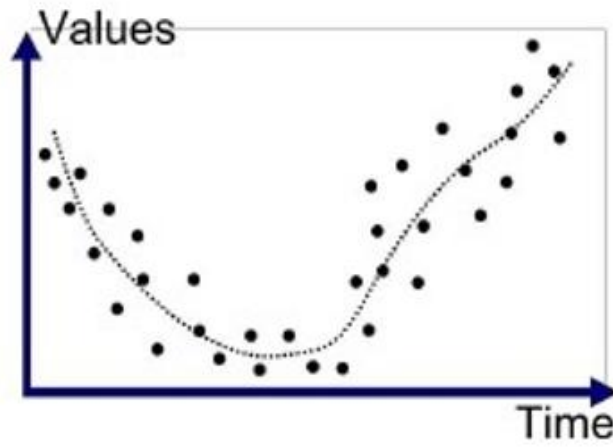
Testing performance ↑



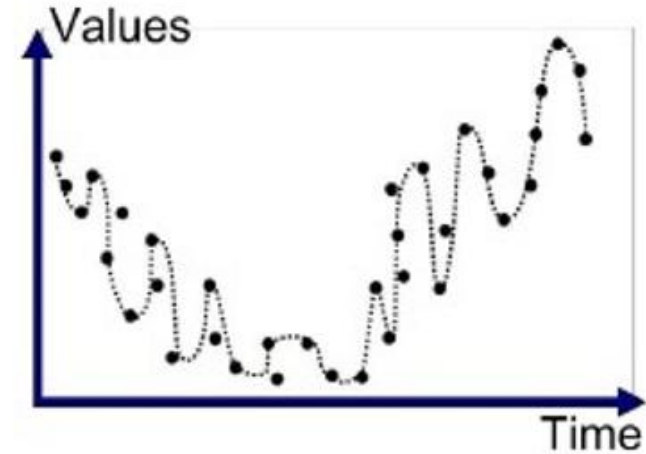
Overfitting vs Underfitting



Underfitted



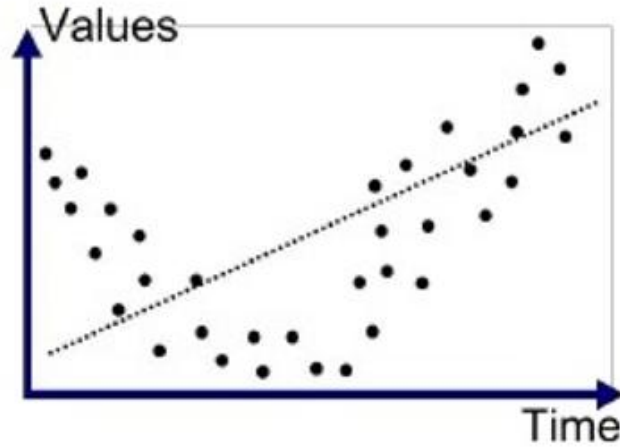
Good Fit/Robust



Overfitted

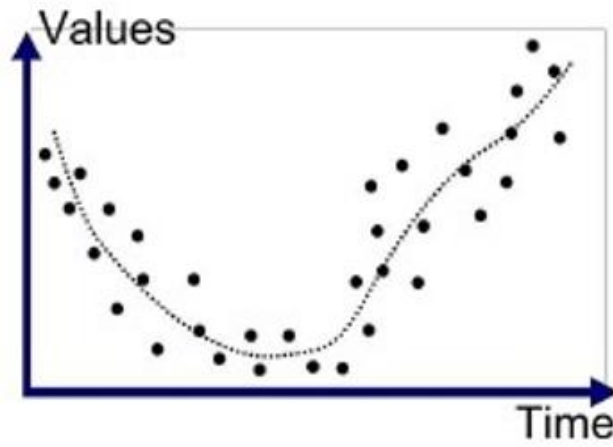


Overfitting vs Underfitting



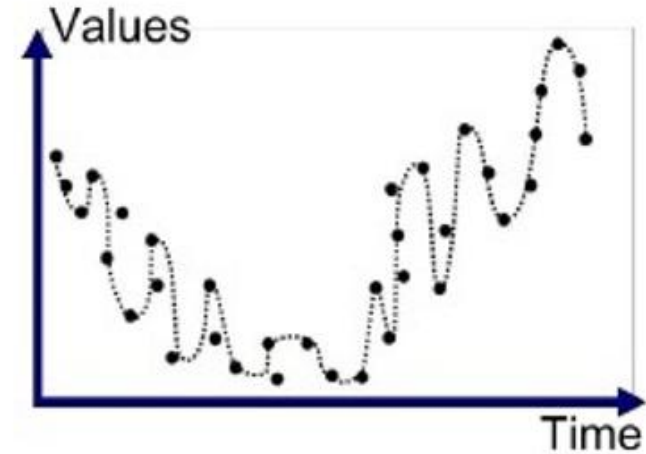
Underfitted

Bad training accuracy
Bad testing accuracy



Good Fit/Robust

Good training accuracy
Good testing accuracy

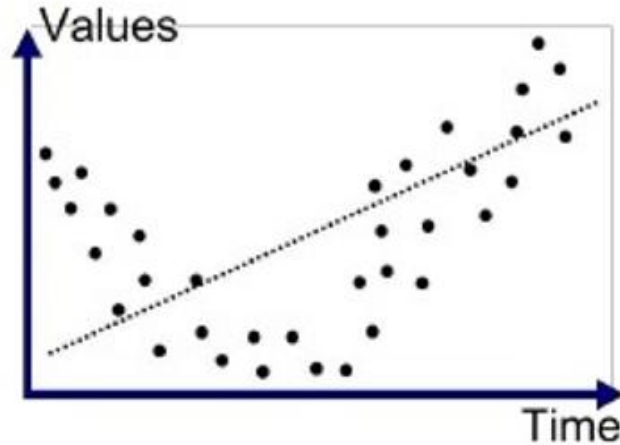


Overfitted

Great training accuracy
Bad testing accuracy



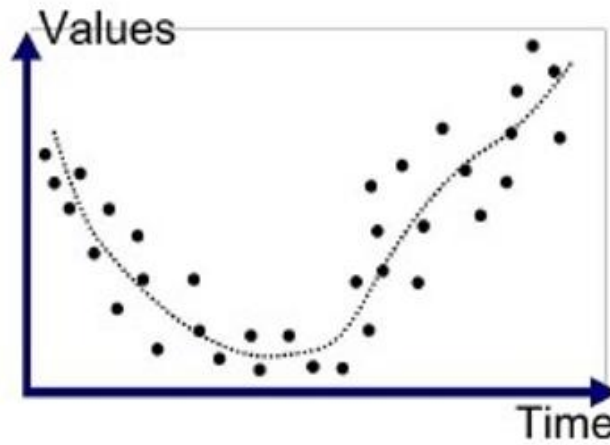
Overfitting vs Underfitting



Underfitted

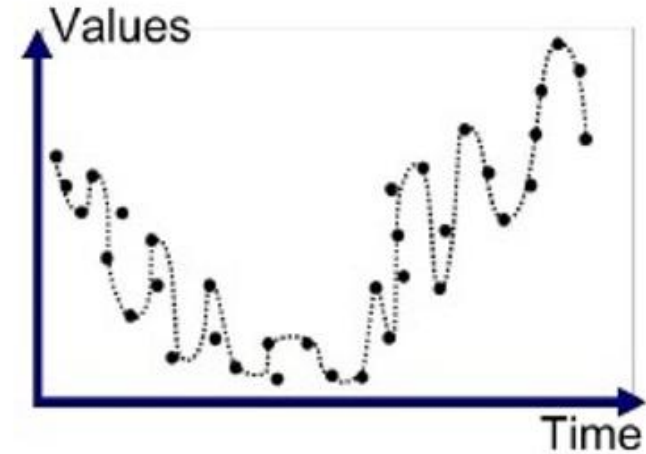
Bad training accuracy
Bad testing accuracy

High Bias



Good Fit/Robust

Good training accuracy
Good testing accuracy



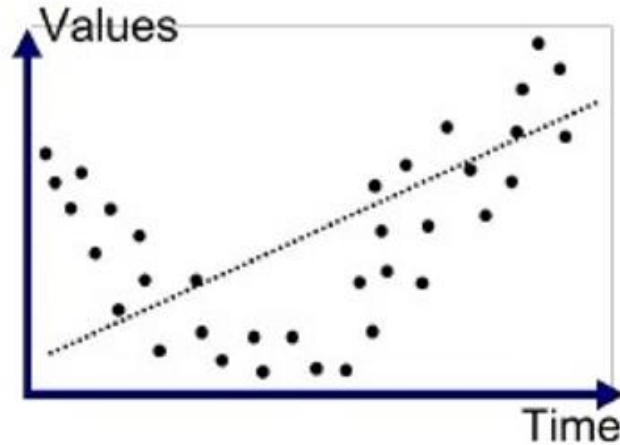
Overfitted

Great training accuracy
Bad testing accuracy

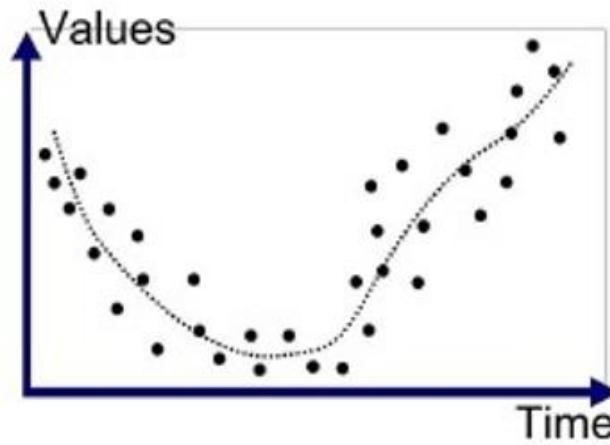
High Variance



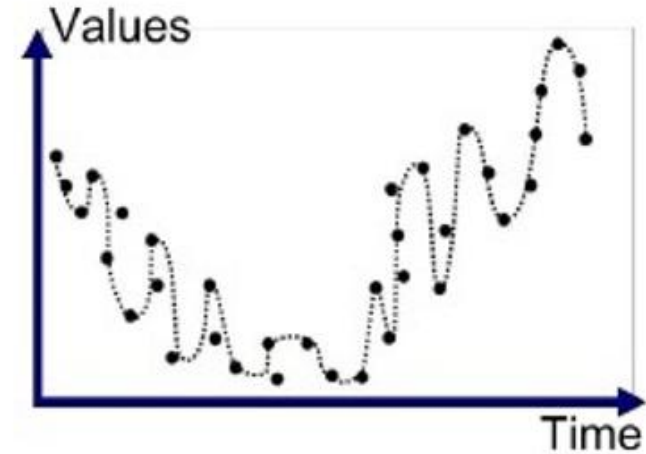
Remedies for Overfitting/Underfitting



Underfitted



Good Fit/Robust

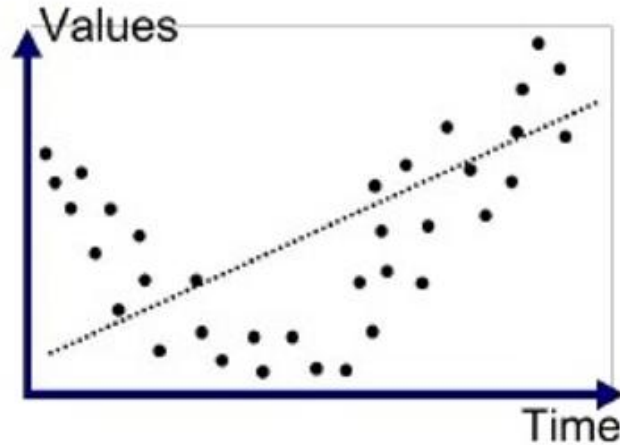


Overfitted

- More Layers/Neurons
- Longer Training
- Architecture
- Hyperparameter tunings

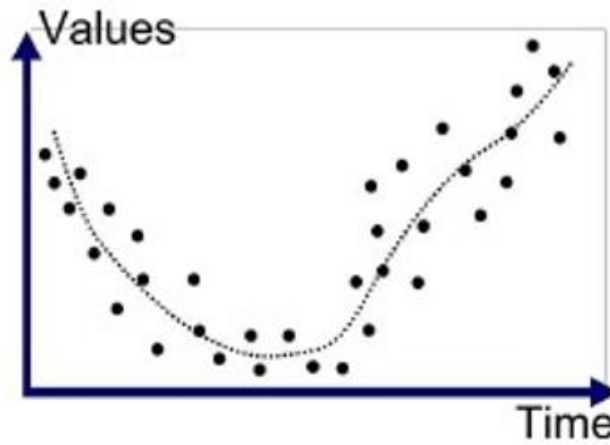


Remedies for Overfitting/Underfitting

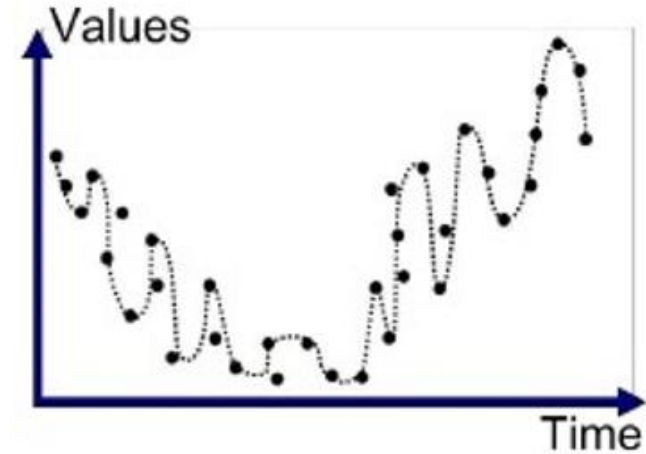


Underfitted

- More Layers/Neurons
- Longer Training
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Good Fit/Robust

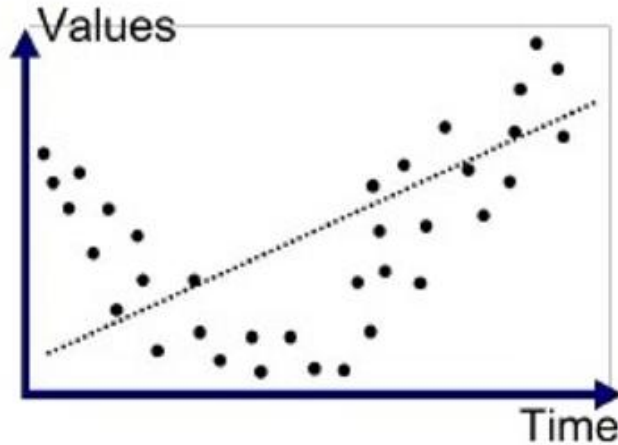


Overfitted

- More training data
- Regularization
- Dropout
- Initialization

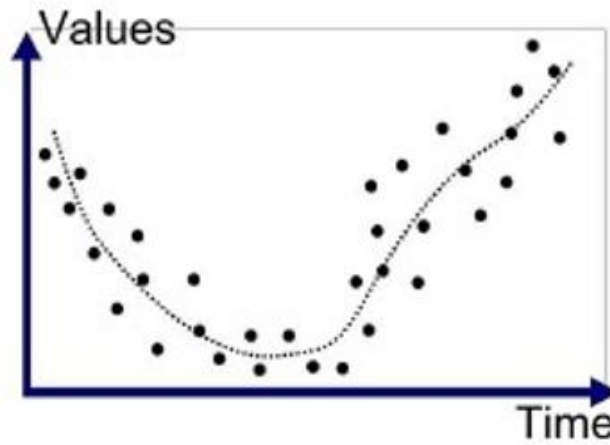


Remedies for Overfitting/Underfitting

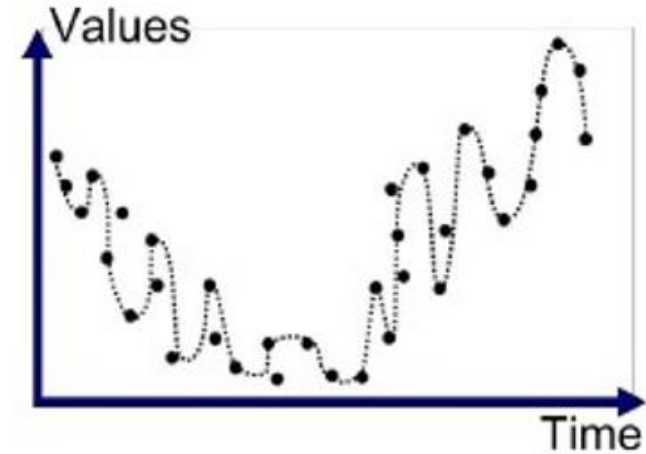


Underfitted

- More Layers/Neurons
- Longer Training
- Architecture
- Hyperparameter tunings



Good Fit/Robust



Overfitted

- More training data
- Regularization
- Dropout
- Initialization



L1, L2 Regularizations

L1 Regularization

$$Loss = Error(y, \hat{y}) + \lambda \sum_{i=1}^N |w_i|$$

L2 Regularization

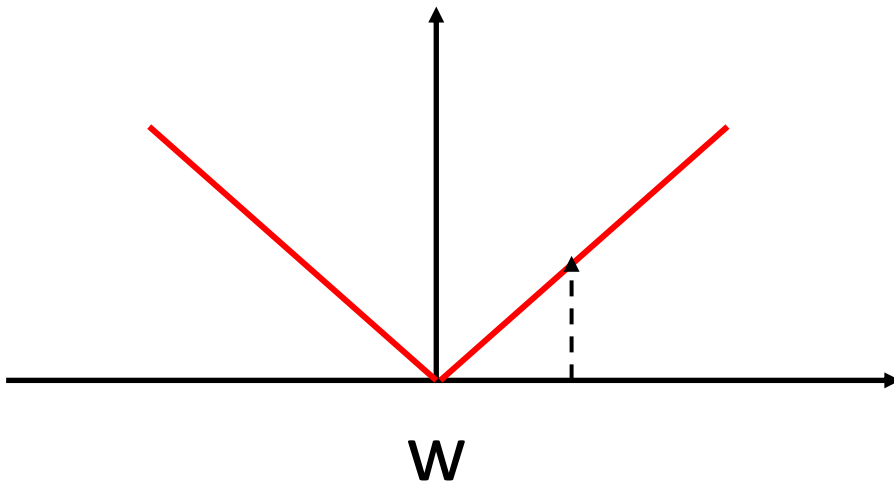
$$Loss = Error(y, \hat{y}) + \lambda \sum_{i=1}^N w_i^2$$



L1, L2 Regularizations

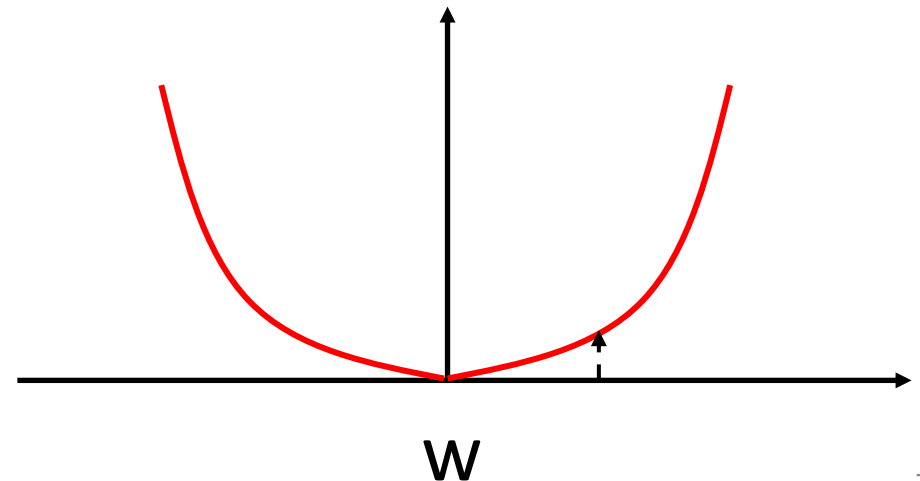
L1 Regularization

$$Loss = Error(y, \hat{y}) + \lambda \sum_{i=1}^N |w_i|$$



L2 Regularization

$$Loss = Error(y, \hat{y}) + \lambda \sum_{i=1}^N w_i^2$$





L1, L2 Regularizations

L1 Regularization

$$Loss = Error(y, \hat{y}) + \lambda \sum_{i=1}^N |w_i|$$

Penalizes **sum of absolute values of weights**

Results in a sparse model

Not suitable for learning complex patterns

Robust to outliers

L2 Regularization

$$Loss = Error(y, \hat{y}) + \lambda \sum_{i=1}^N w_i^2$$

Penalizes **sum of squared values of weights**

Results in a dense model

Learns complex patterns

Sensitive to outliers



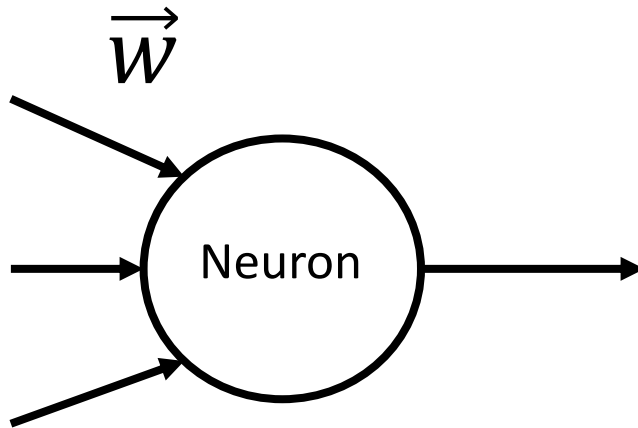
Single-neuron Regularization

$$J(\vec{w}, b) = \frac{1}{m} \sum_{i=1}^m L(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2m} ||\vec{w}||_2^2$$

Cost function

$$+ \frac{\lambda}{m} ||\vec{w}||_1$$

Weight regularization terms

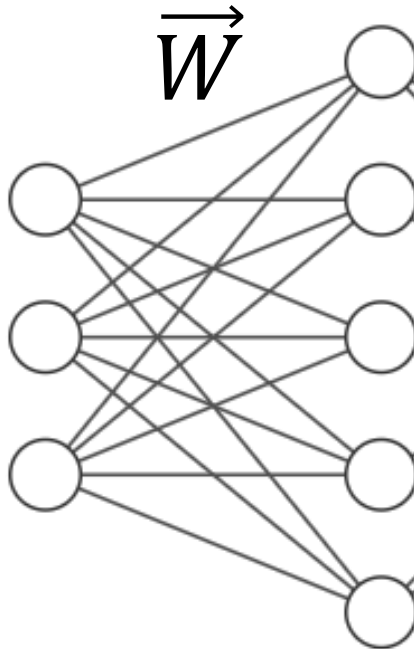




Multi-neuron Regularization

$$J(W^{[1]}, b^{[1]}, \dots, W^{[L]}, b^{[L]}) = \frac{1}{m} \sum_{i=1}^m L(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2m} \sum_{l=1}^L ||W^{[l]}||_F^2$$

Cost function

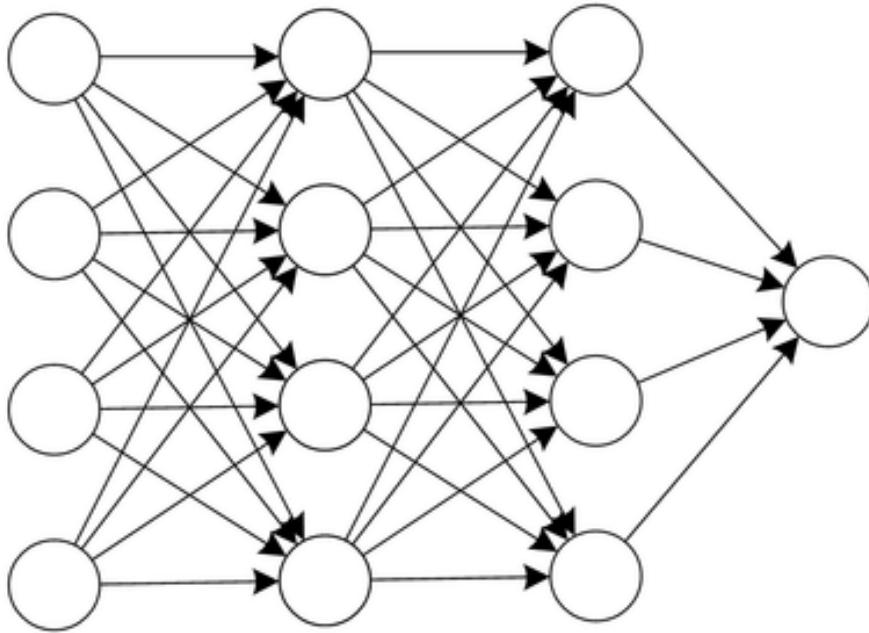


$$||W^{[l]}||_F^2 = \sum_{i=1}^{n^{[l]}} \sum_{j=1}^{n^{[l-1]}} (w_{ij}^{[l]})^2$$

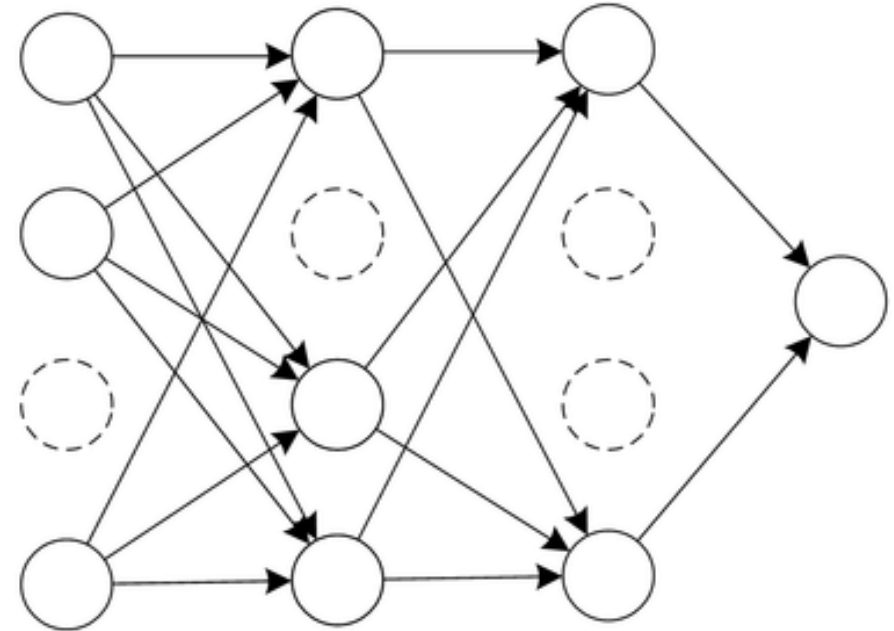
Weight regularization term over multiple layer (Frobenius norm)



Dropout Regularization



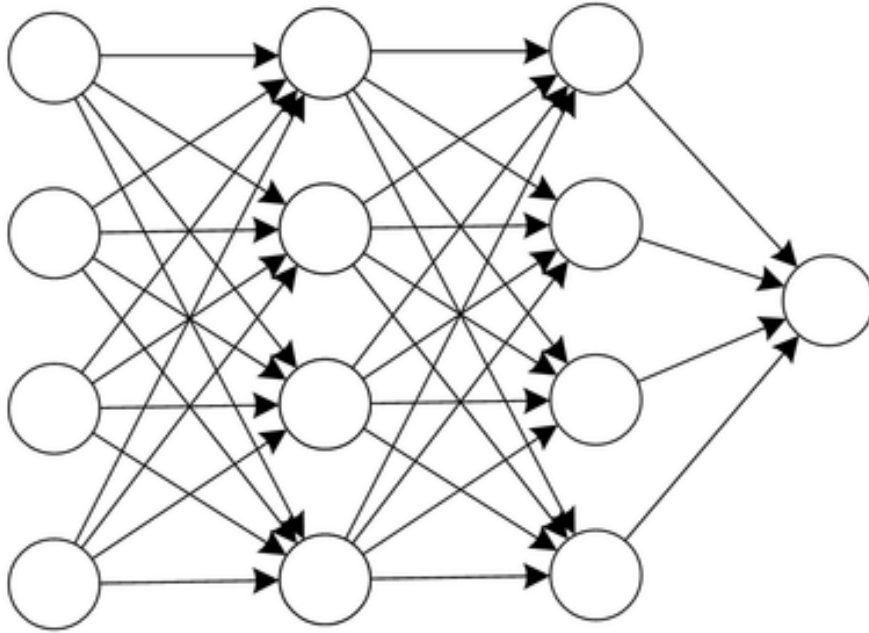
Standard Neural Network



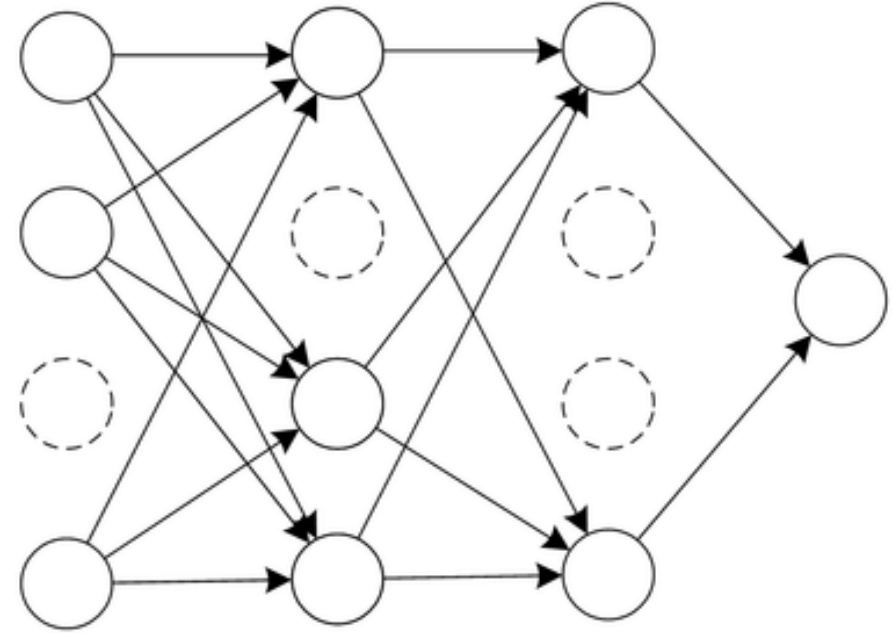
Network with Dropout



Dropout Regularization



Standard Neural Network

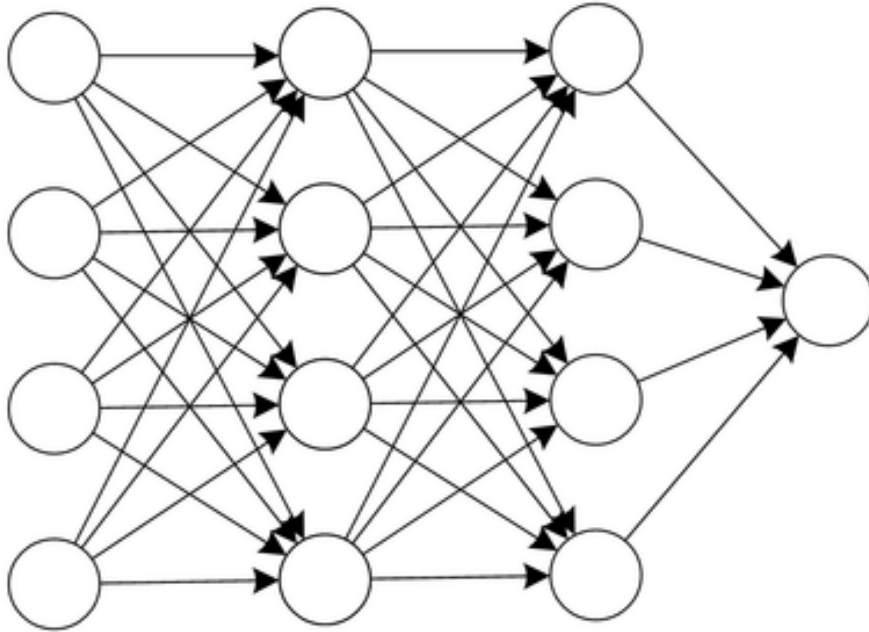


Network with Dropout

Dropout forces the network to learn **more robust features** + **different random subsets** of other neurons

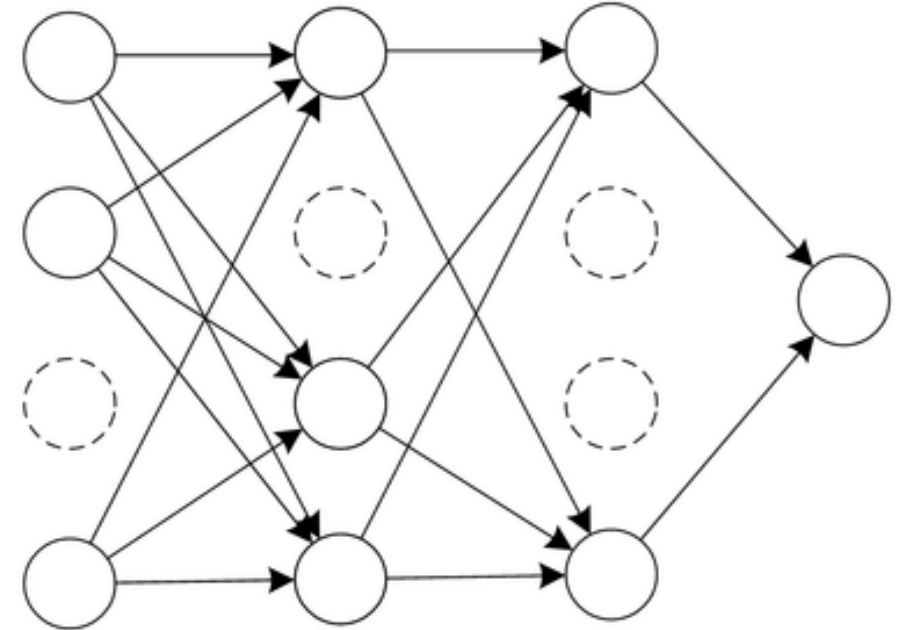


Dropout Regularization



Standard Neural Network

- Effectively spreading the weights
- Similar to L2 reg
- Testing with dropout $p_d=0$



Network with Dropout

- Can depend on weights (W)
- J could not be well defined in each pass



Data Augmentation

(a)



(b)



(c)



(d)



(e)



(f)



(g)



(h)





Data Augmentation

(a)



(b)



(c)



(d)



- Add noise
- Distortions
- Synthetic images
- Resize resolutions
- Rotation
- Add symmetries

(e)



(f)



(g)

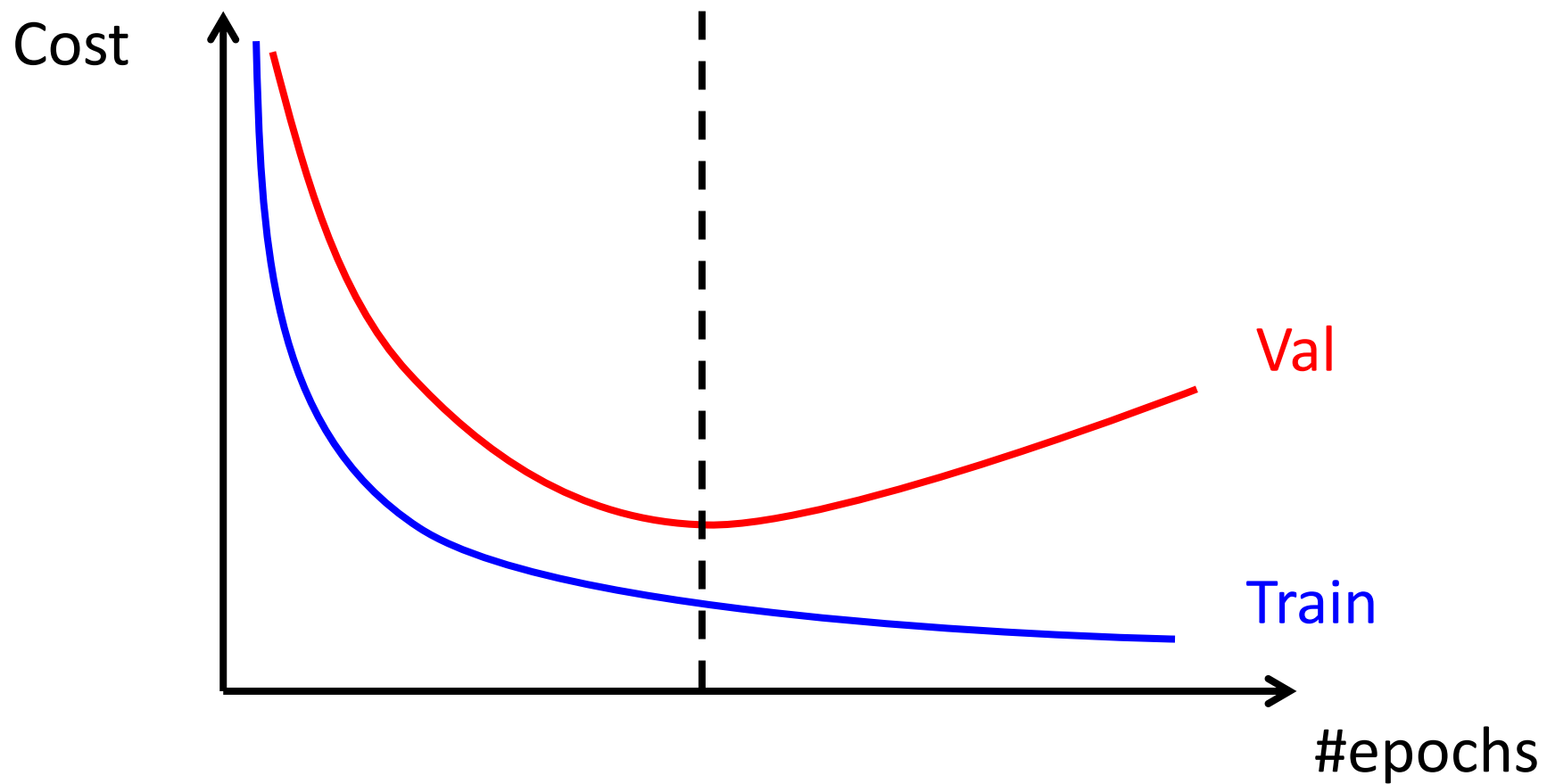


(h)





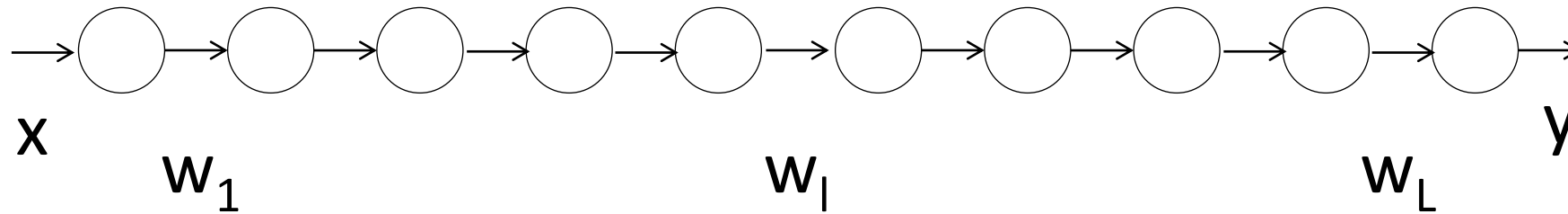
Early Stopping





Exploding/Vanishing Gradients

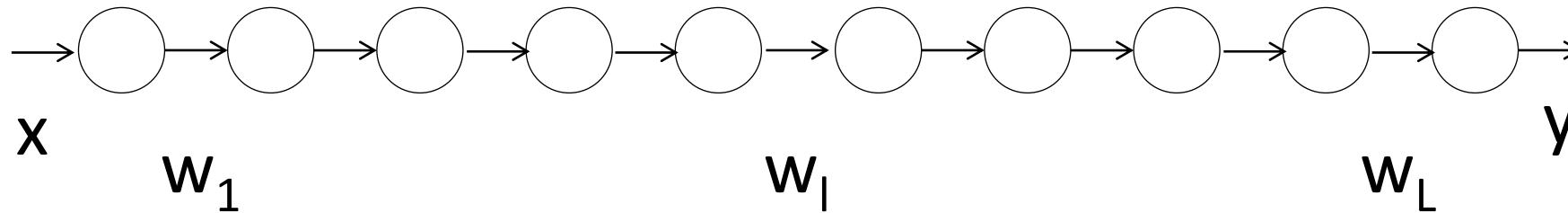
Very deep neural network





Exploding/Vanishing Gradients

Very deep neural network



$$\hat{y} = w_L \cdot \dots \cdot w_l \cdot \dots \cdot w_2 \cdot w_1 \cdot x$$

$$w_l = w > 1; \quad \hat{y} = w^L x \rightarrow \infty$$

$$w_l = w < 1; \quad \hat{y} = w^L x \rightarrow 0$$



Exploding/Vanishing Gradients

With **activation**:

$$\dots w_3 \sigma_3(w_2 \sigma_2(\sigma'_1(w_1 x)))$$

For **gradients**:

$$\frac{\partial J}{\partial w_1} = \sigma'_3(z_3) w_3 \sigma'_2(z_2) w_2 \sigma'_1(z_1) x$$



Remedies for exploding/vanishing gradients: Data Normalization

Zero mean:

$$\mu = \frac{1}{m} \sum_{i=1}^m x^{(i)}$$
$$x^{(i)\mu} = x^{(i)} - \mu$$

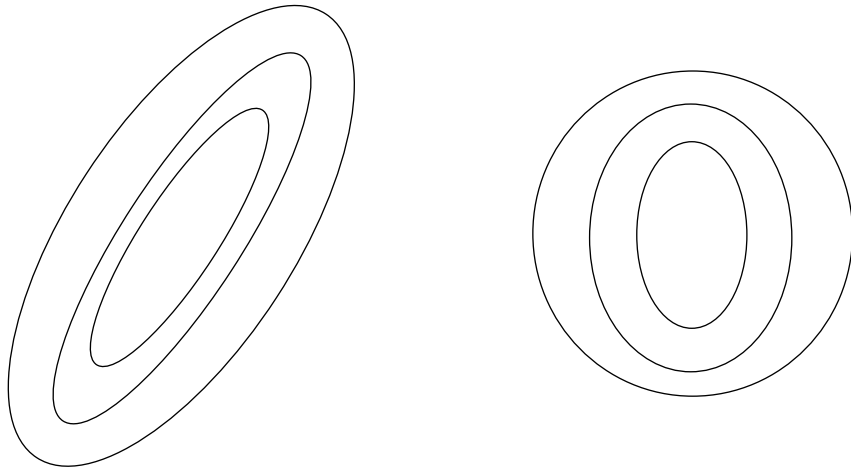
Normalized Variances

$$\sigma^2 = \frac{1}{m} \sum_{i=1}^m x^{(i)2}$$
$$x^{(i)\mu, \sigma^2} = x^{(i)\mu} / \sigma$$



Intuition for data normalization

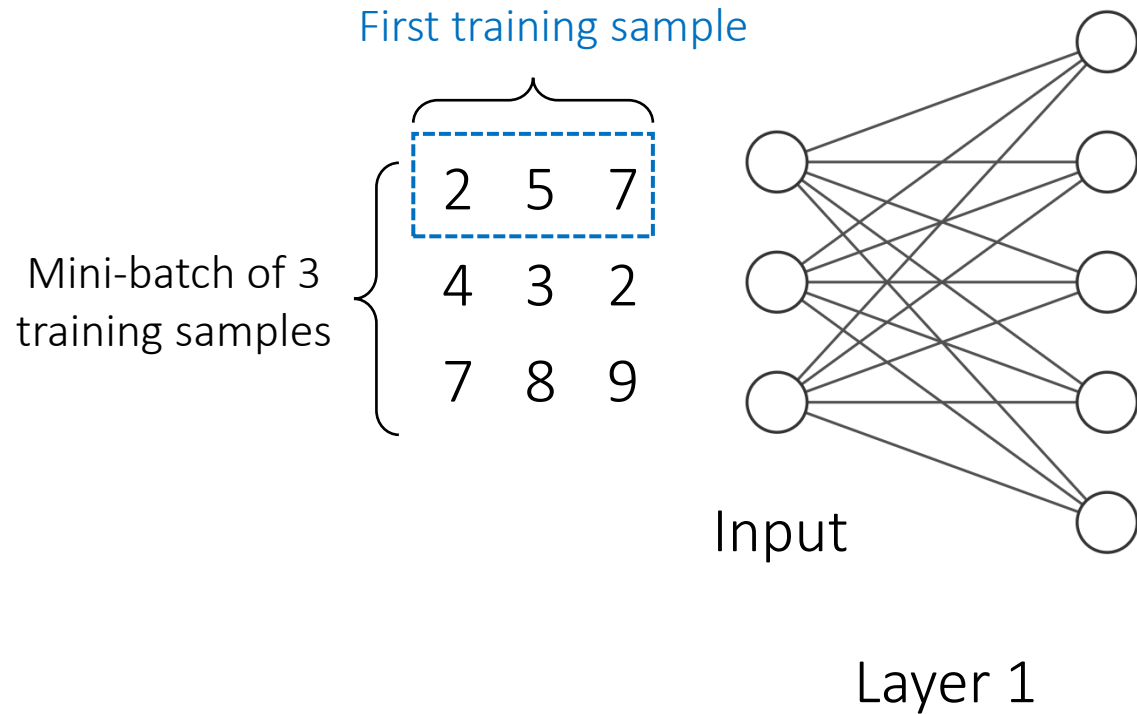
If **inputs have different scales**, the **cost function** will also have to include different scales → increased likelihood of instability



Remember to **normalize all sets**:
training, validation, testing



Remedies for Vanishing/Exploding Gradients: Batch Normalization

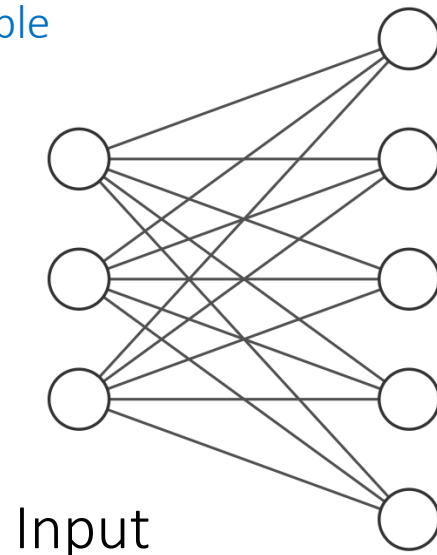




First training sample

Mini-batch of 3
training samples

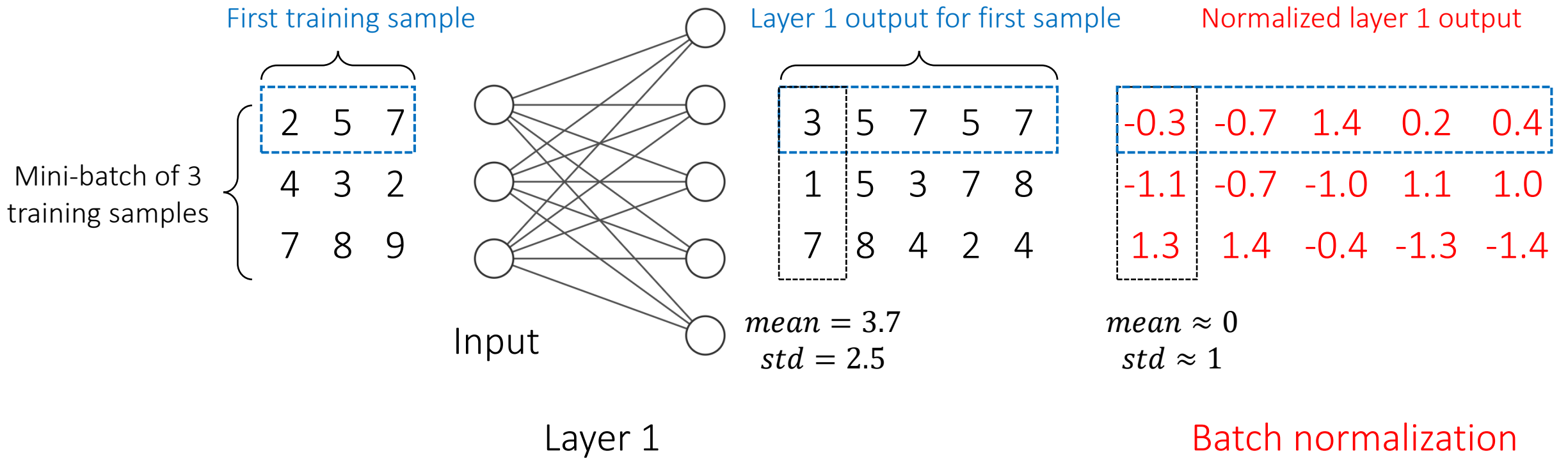
2 5 7		
4	3	2
7	8	9



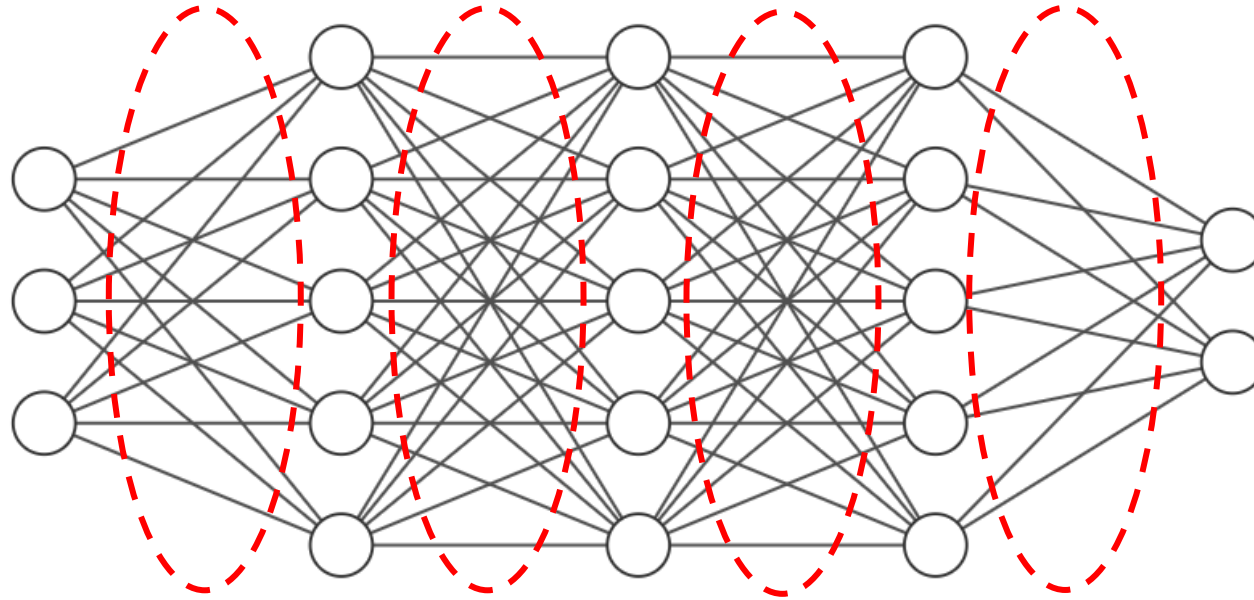
Layer 1 output for first sample

3	5	7	5	7
1	5	3	7	8
7	8	4	2	4

$mean = 3.7$
 $std = 2.5$

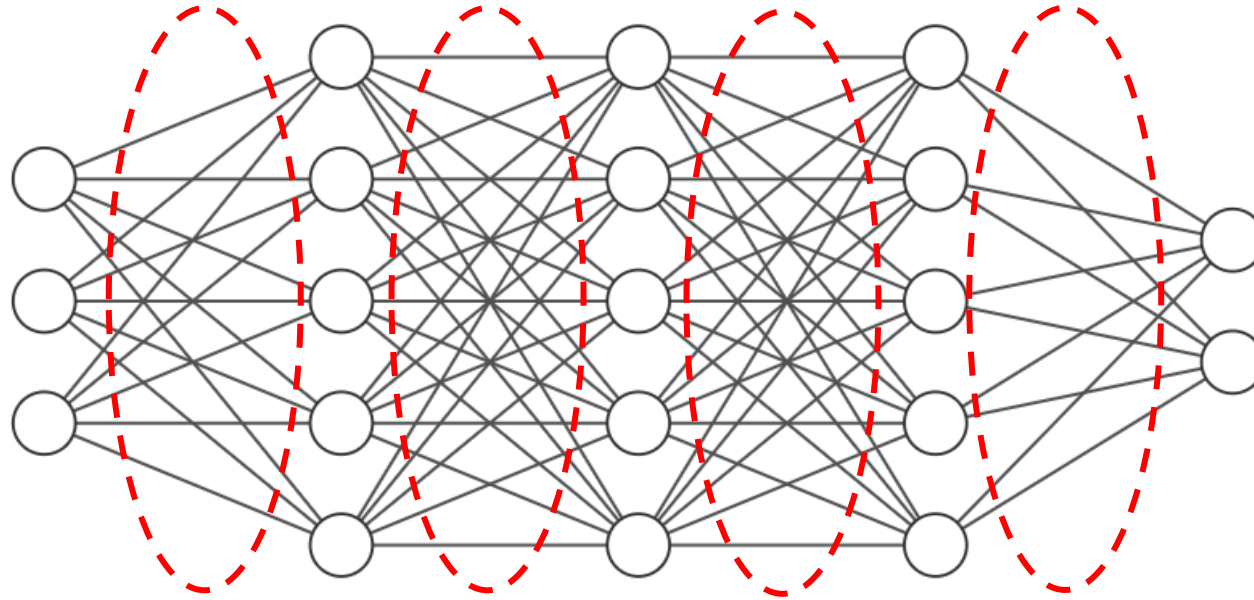


Remedies for Vanishing/Exploding Gradients: Weight Initialization



Proper weight initialization plays essential roles in preventing exploding/vanishing gradients

Remedies for Vanishing/Exploding Gradients: Weight Initialization



Proper weight initialization plays essential roles in preventing
exploding/vanishing gradients



Faster convergence



Network Initialization

- Zero \rightarrow Problematic
- Random Normal (0,1) \rightarrow Problematic
- Xavier (tanh):

$$Var(w^{[l]}) : 1/n^{[l-1]}$$

$$w^{[l]} = N(0, 1) \cdot \sqrt{\frac{1}{n^{[l-1]}}}$$



Network Initialization

- He (ReLU):

$$Var(w^{[l]}) : 2/n^{[l-1]}$$

- Other: $w^{[l]} = N(0, 1) \cdot \sqrt{\frac{2}{n^{[l-1]}}}$

$$Var(w^{[l]}) : \frac{2}{n^{[l-1]} + n^{[l]}}$$



Hyperparameters

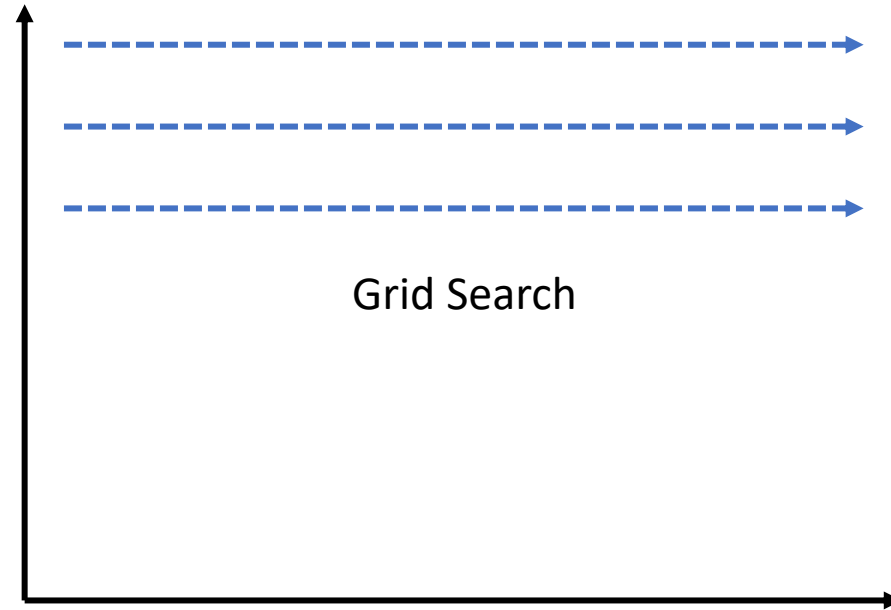
- Learning rate
- Number of layers
- Neurons in each layer
- Activation function
(ReLU, Tanh, sigmoid)
- Training batch size
(SGD, Mini-batch, Batch Gradient)
- Optimizer
(SGD, Adam, RMS Prop etc)
- Number of training epochs



Hyperparameters

- Learning rate
- Number of layers
- Neurons in each layer
- Activation function (ReLU, Tanh, sigmoid)
- Training batch size (SGD, Mini-batch, Batch Gradient)
- Optimizer (SGD, Adam, RMS Prop etc)
- Number of training epochs

Number of layers



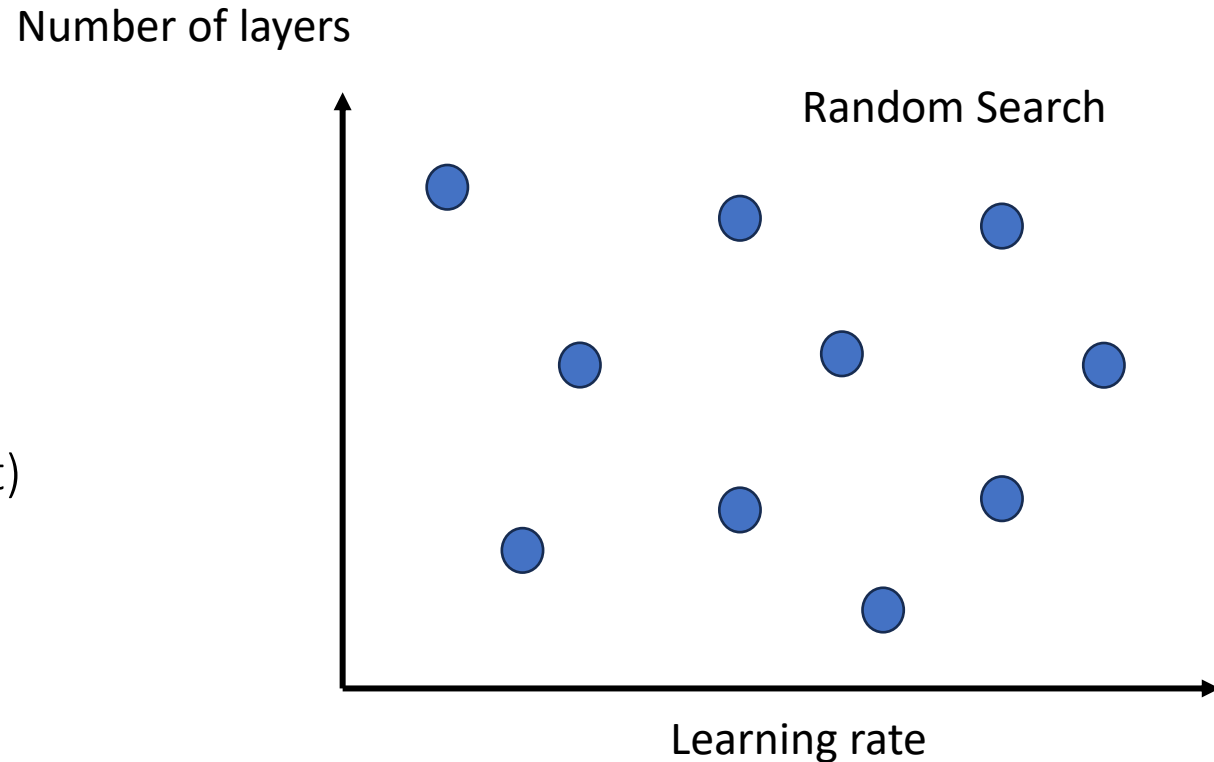
Grid Search

Learning rate



Hyperparameters

- Learning rate
- Number of layers
- Neurons in each layer
- Activation function (ReLU, Tanh, sigmoid)
- Training batch size (SGD, Mini-batch, Batch Gradient)
- Optimizer (SGD, Adam, RMS Prop etc)
- Number of training epochs





Summary

Optimizers

- Vanilla SGD
- Momentum
- AdaGrad
- RMSProp
- Adam

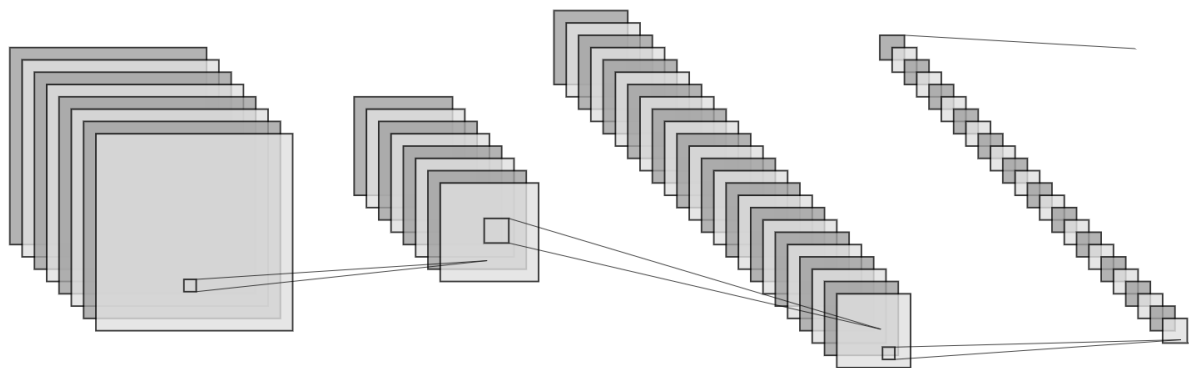
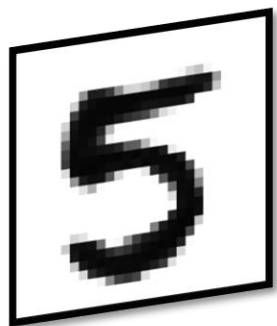


Optimization Techniques

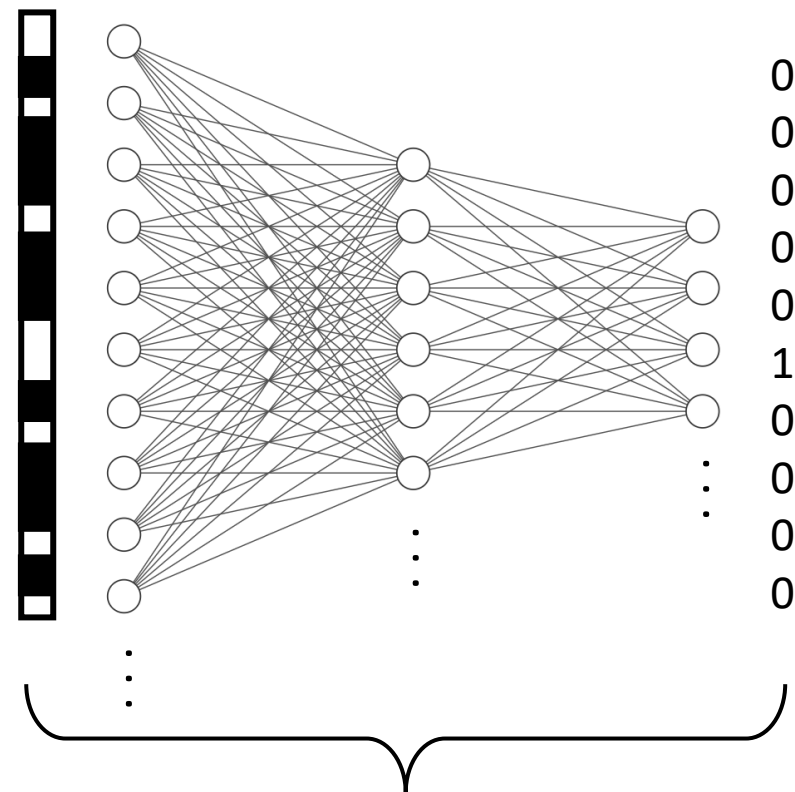
- Data splitting (Train/Val/Test)
- Regularization
- Data normalization
- Batch-normalization
- Network initialization
- Hyperparameter tunings



Next episode in EEP 596



Convolution Layers + Pooling Layers
(Image feature extraction)



Fully connected layers
(Classifier)