Basics of Machine Learning

1.3 Optimization in Relation to Problem Solving

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Original Loss Profiles

Weather Station	Year	Theta0	Theta1	Iteration	Step Size	Result (Loss Path)
Basel	1960	3	-10	500	0.01	Large visual loss path (black line)
Basel	1990	-1	1	500	0.05	Small loss path
Basel	2020	0	0	500	0.05	Very large loss path
Madrid	1960	0	0	500	0.05	Small loss path
Madrid	1990	0	0	300	0.001	Small loss path
Madrid	2020	0	0	500	0.01	Miniscule, a couple of dots
Stockholm	1960	1	0	300	0.1	Small loss path
Stockholm	1990	0	0	100	0.01	Very small loss path, basically a dot
Stockholm	2020	0	0	150	0.1	Small loss path

Rerun optimizations (X,Y variables closer to objective based on loss path)

Weather Station	Year	Theta0	Theta1	Iteration	Step Size	Result (Result Path)
Basel	1960	-0.82	1.89	500	0.01	Smaller, closer to goal loss path

Basel	1990	0.07	-0.07	500	0.05	Incredibly small loss path
Basel	2020	0.09	0.06	500	0.05	Reduced loss path
Madrid	1960	0.09	-0.3	500	0.05	Small loss path
Madrid	1990	0.09	-0.09	300	0.001	Small loss path
Madrid	2020	0.1	-0.03	500	0.01	Small loss path
Stockholm	1960	0.32	-0.68	300	0.1	Smaller loss path closer to goal
Stockholm	1990	0.052	0.029	100	0.01	Loss path resembles a dot, guess is close to minimum and gradient descent has converged on answer quickly
Stockholm	2020	0.18	-0.05	150	0.1	Small loss path

General Statistics and Theta Values over Time

	Basel	Madrid	Stockholm
Min Temp			
1960	-3.19	-2.32	-2.35
1990	-1.18	-1.74	-1.52
2020	-1.68	-1.63	-1.07
Change	+1.51	+0.96	+1.28
Max temp			
1960	1.82	1.68	1.68
1990	1.99	2.16	1.98
2020	2.30	2.32	2.15
Change	+ 0.48	+ 0.64	+ 0.47
Difference between min and max			
1960	5.01	4	4.03
1990	3.17	3.9	3.5

2020	3.98	3.95	3.22
Theta 1 Rerun (X)			
1960	-0.82	0.09	0.31
1990	0.07	0.09	0.052
2020	0.09	0.1	0.18
Theta 0 Rerun (Y)			
1960	1.89	-0.3	-0.67
1990	-0.07	-0.09	0.029
2020	0.06	-0.03	-0.05

Observations about the data:

1. The mean temperature of each station increased over the time periods looked at (approximately 60 years)
2. Madrid had the largest increase in maximum temperature (+0.64) between the 3 stations looked at
3. Basel had the largest increase in minimum temperature (+1.51) between the 3 stations
4. The difference between the minimum and maximum temperature has decreased over the span of years looked at

With this data set, the starting points for gradient descent were already very close to the discovered minimum. The range of the data was very low with minimums in the -2s and maximums in the positive 2s. While looking at this data, I determined a good starting guess for my theta values would be around 0. This led to the loss paths being relatively small as they were close to their goal (the minimum). A couple of initial Theta 1 and theta 0 "guesses" were changed to demonstrate what larger loss paths may look like (see Basel 1960). With retrying the optimization with the respective X,Y coordinates found in the original graphs of each weather station and year looked at, the loss function did improve. Since the loss function decreases, we can say that the model, gradient descent is suitable to determine patterns in the temperature data over time.

Additionally, to further solidify the suitability of gradient descent for this data, we can look at parameter changes over integrations (Theta values). As demonstrated in the line graphs below, most of the parameters for the initial guesses converge and stabilize on a specific value as the algorithm approaches a solution (loss stabilizes/approaches 0)

Graphs

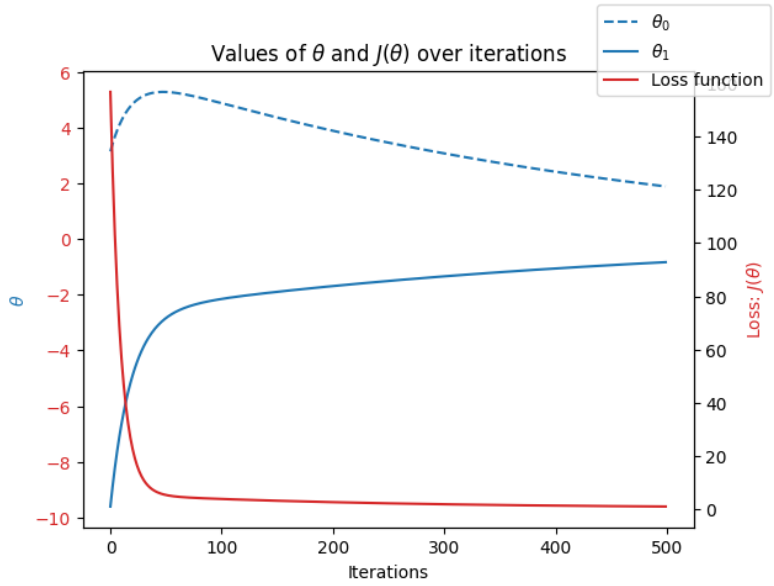
The top graph demonstrates the changes in parameters (Theta0/Theta1) over iterations. Convergence and stabilization on a specific value as the loss stabilizes indicates the algorithm has/is determining a solution. (See first graph in each subsection)

The left-hand graph depicts the loss function with the initial theta guess. Some theta guesses were close to the minimums that we were looking for with gradient descent.

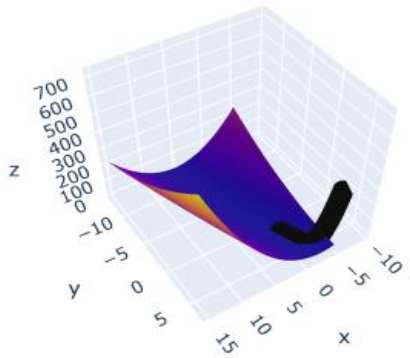
The right-hand graph depicts the loss function with the revised theta values based on the lower most point of the black line on the left hand graph. These loss functions are typically smaller as we are minimizing loss and more accurately predicting data.

Due to initial theta0 and theta1 "guesses" being close to the goal, loss paths are small however there are slight differences that can be noted between the left and right graphs below.

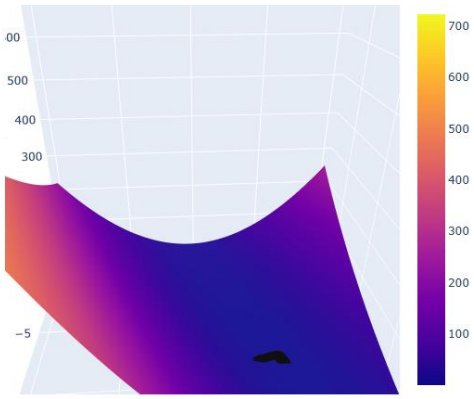
Basel 1960



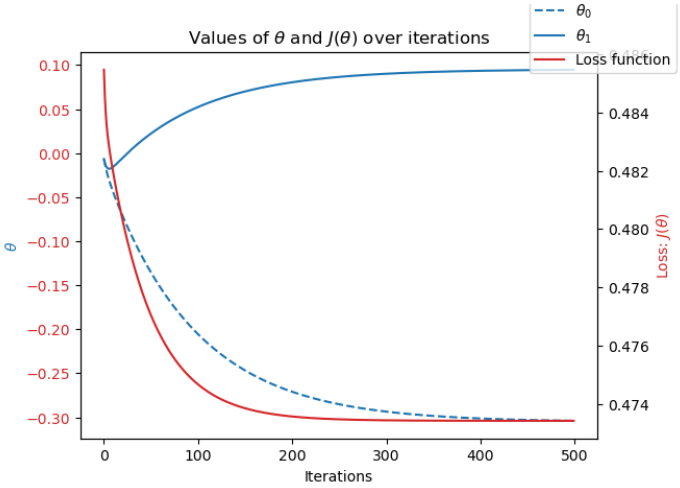
Original Loss profile with line towards minimum



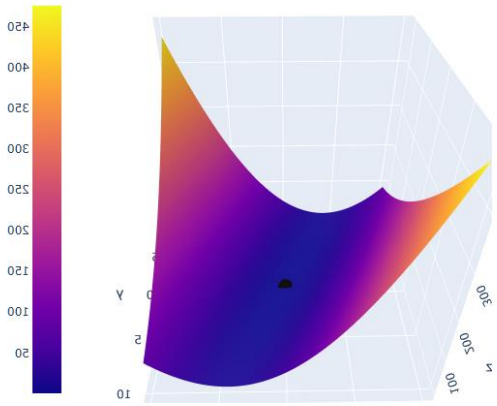
After optimization closer to objective



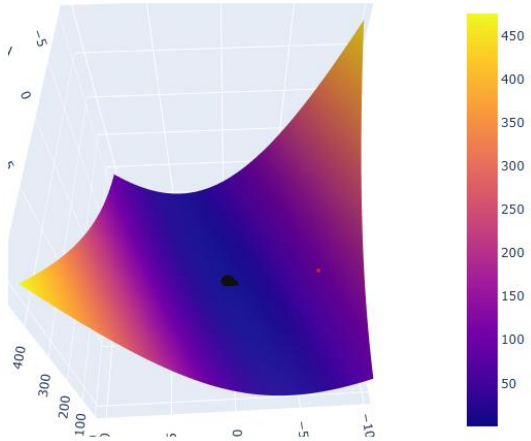
Madrid 1960



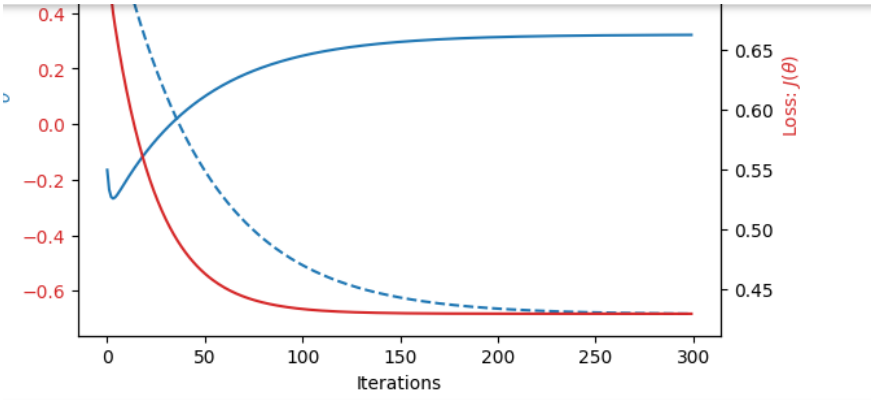
Original Loss profile with line towards minimum



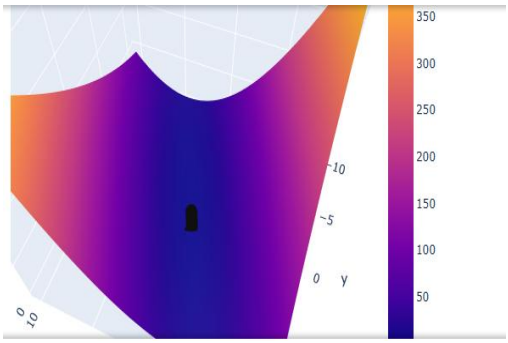
After optimization closer to objective



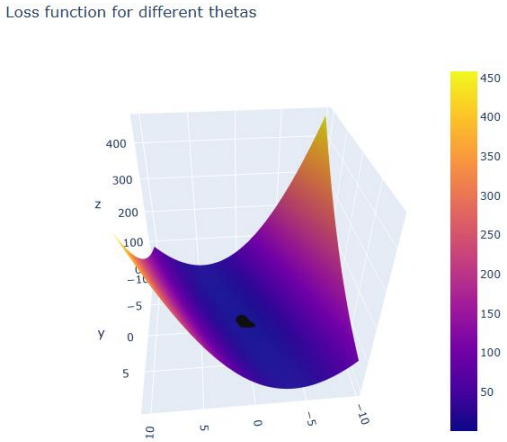
Stockholm 1960



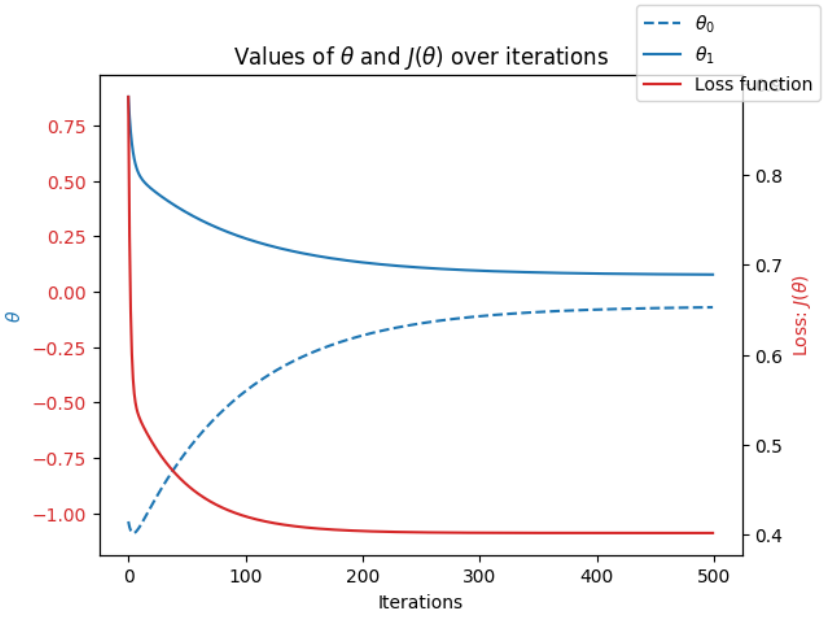
Original Loss profile with line towards minimum



After optimization closer to objective



Basel 1990



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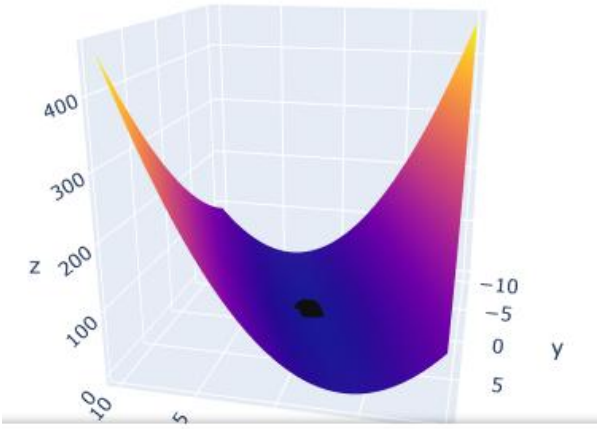
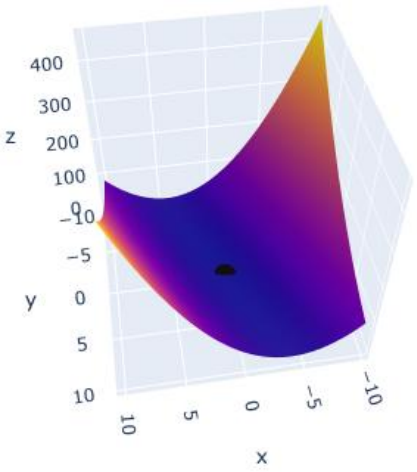
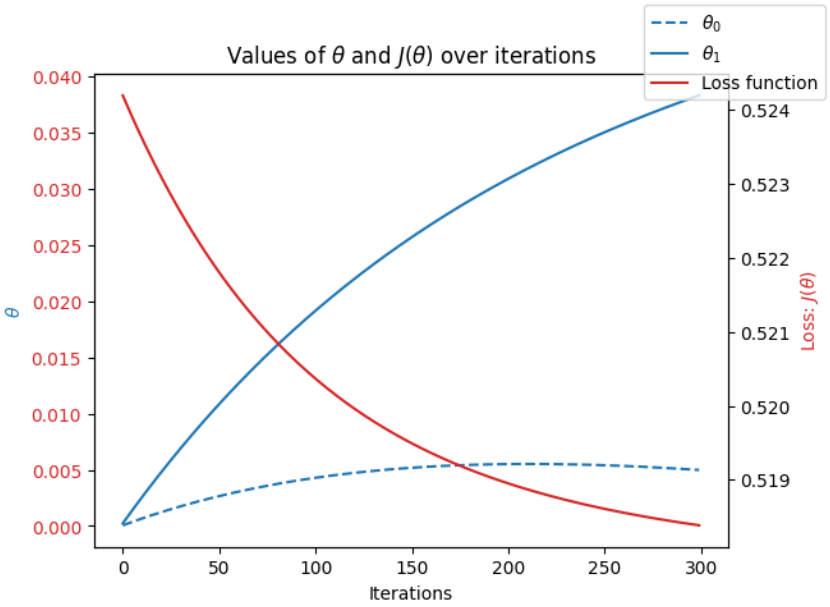




Madrid 1990



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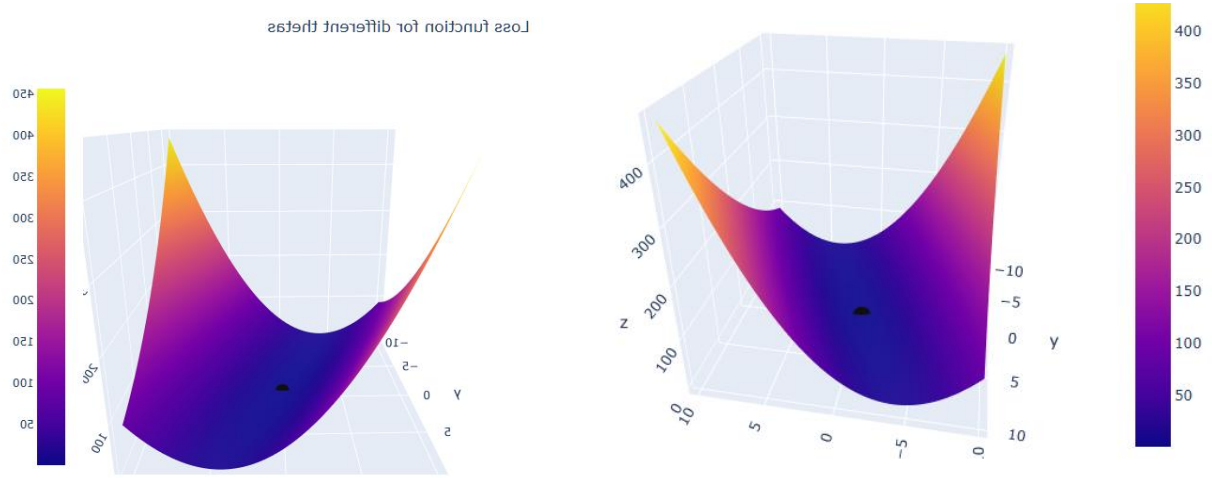

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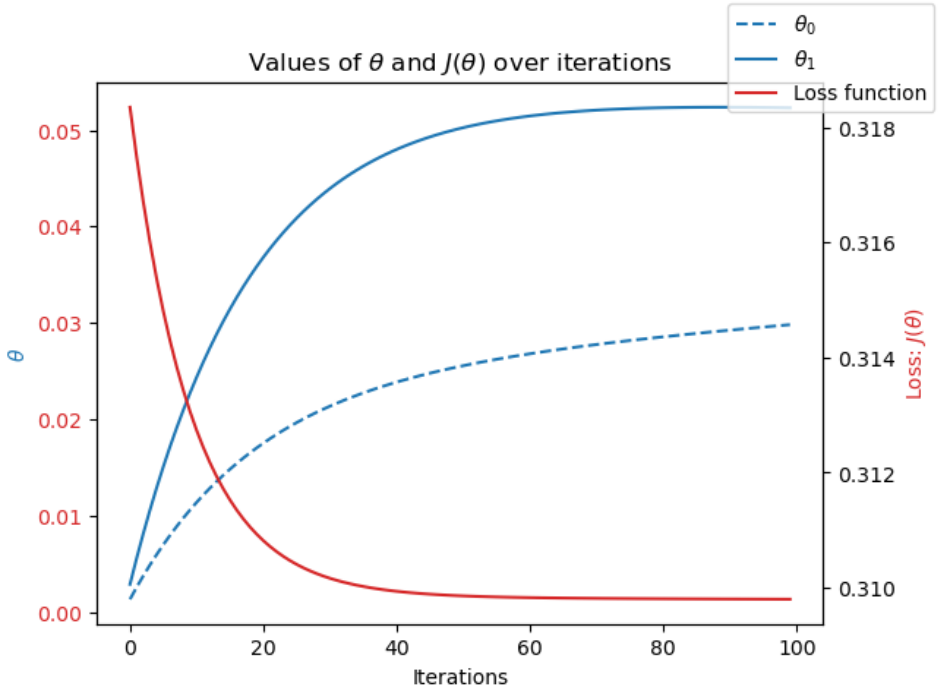
Stockholm 1990



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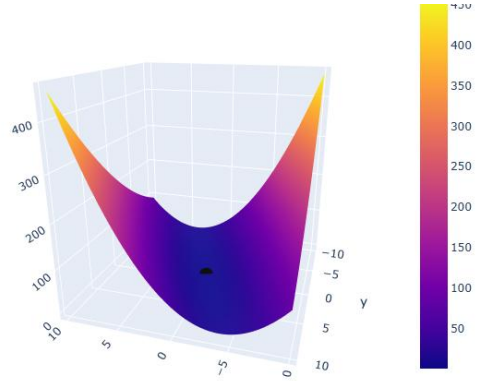

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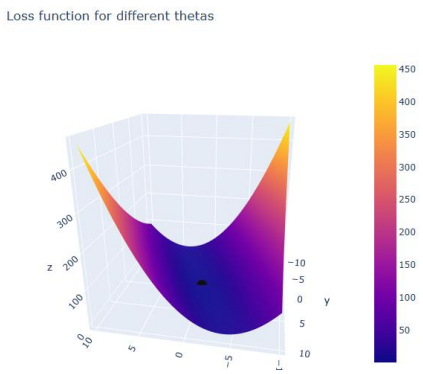
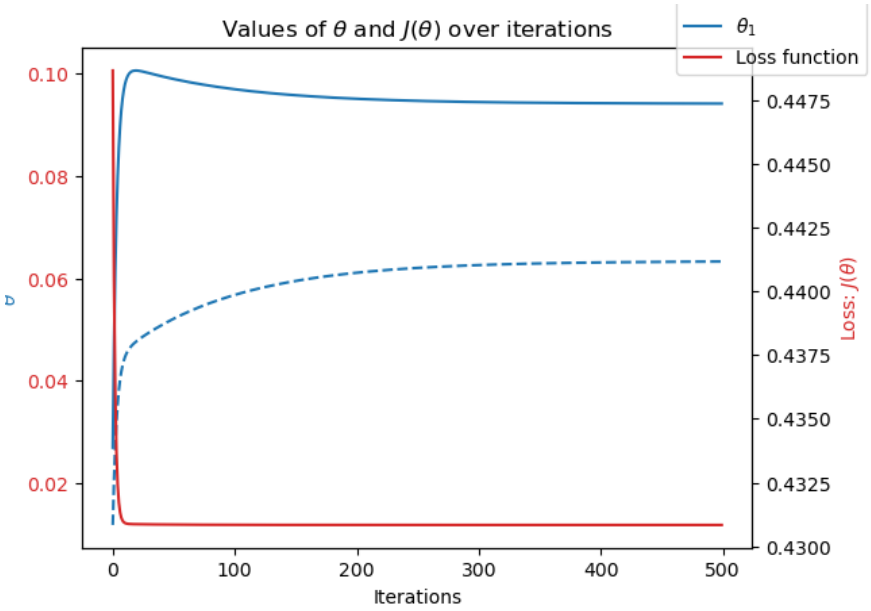


Basel 2020



Original Loss profile with line towards minimum

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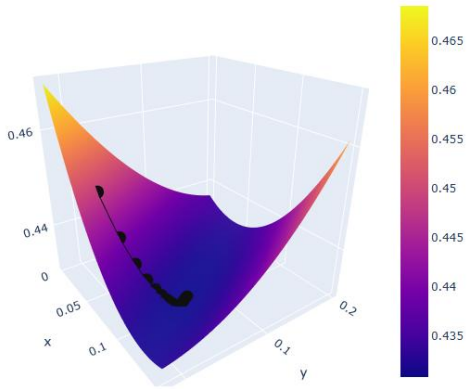
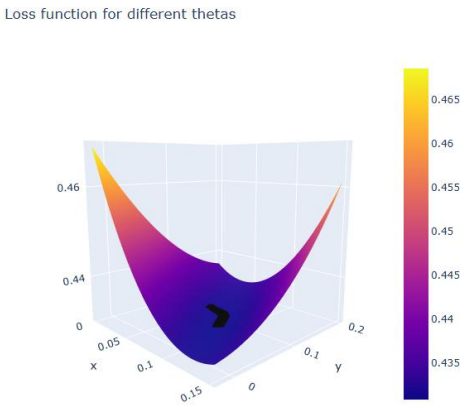
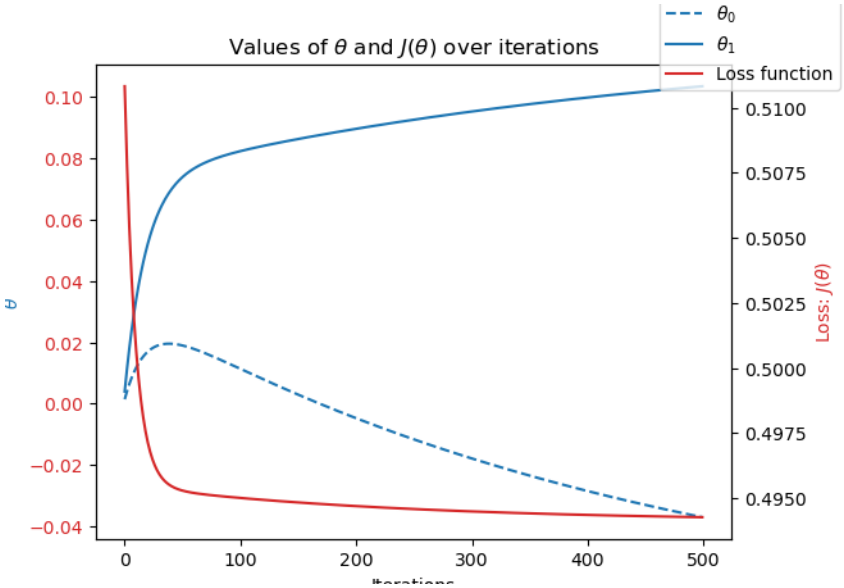




Madrid 2020



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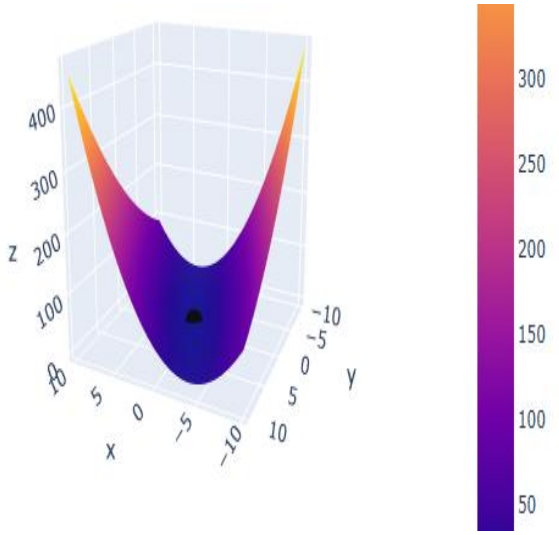

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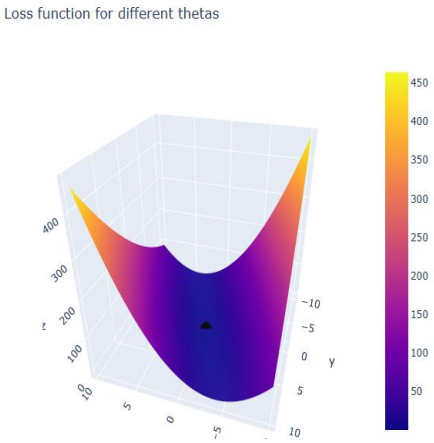
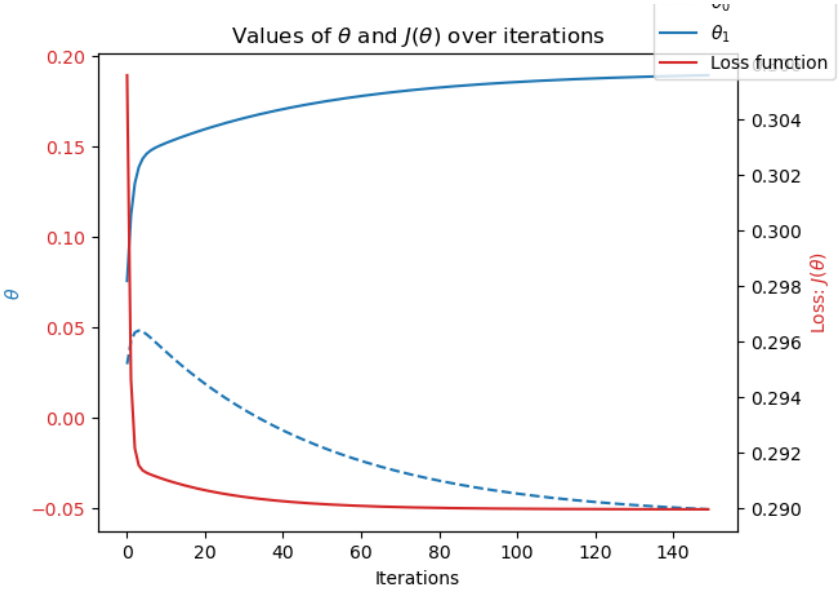


Stockholm 2020



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