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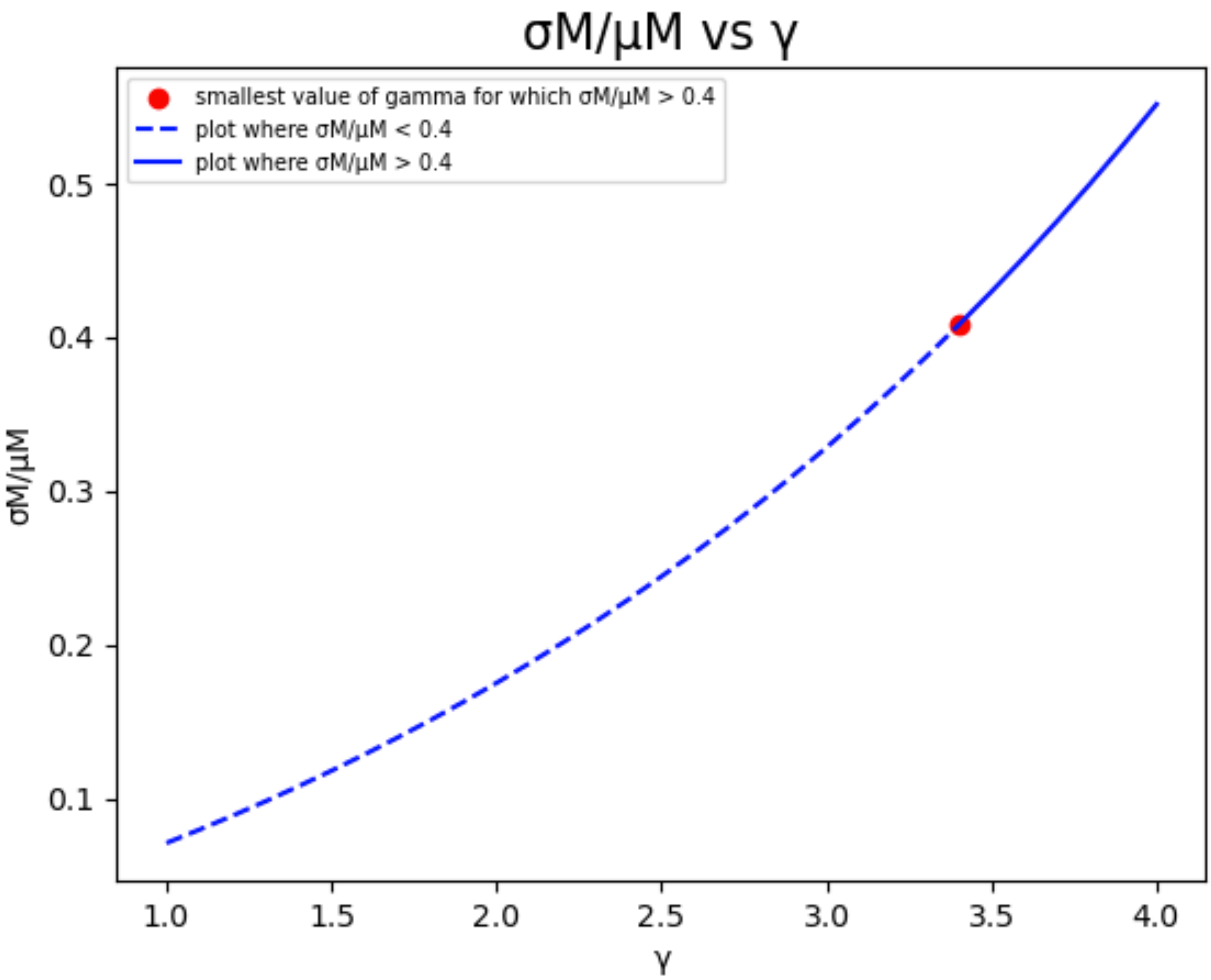
QF600 – Asset Pricing

Homework 5

Calculate μ_M and σ_M for each value of γ, and plot σ_M/μ_M (on the vertical axis) vs γ (on the horizontal axis)

Find the smallest value of γ (in your data) for which $\sigma_M/\mu_M > 0.4$.



γ for which $\sigma_M/\mu_M > 0.4$ = **3.4**

Explain (in words, without using any mathematical equations or formulas) the economic significance of this result.

The smallest value of γ in the data for which $\sigma M/\mu M > 0.4$, is 3.4. If we assume that the assignment is analysing the U.S. stock market, then the Hansen-Jagannathan bound is satisfied for $\gamma = 3.4$, based on the Sharpe ratio of around 0.4 for the U.S. stock market. This result eliminates the equity premium puzzle, since $\gamma = 3.4$ represents a reasonable degree of (relative) risk aversion. This is because when optimal consumption growth also contains a random variable that represents the effect of rare disasters, the possibility of rare disasters greatly increases volatility and negative (left) skewness of consumption growth. So we need only a small amount of magnification (via γ) to match volatility and negative (left) skewness of the pricing kernel, and we arrive at a reasonable degree of (relative) risk aversion.

In the assignment, we assume that the investor has time-separable utility of consumption, meaning that the investor's utility of consumption at a given point in time is not affected by past or future consumption. We consider an investor with power utility of consumption, and we suppose that optimal consumption growth has a lognormal distribution with a mean of μM and a variance of σM.

Under these conditions (assuming we do not take into account the effect of rare disasters) if we consider the U.S. stock market which has a Sharpe ratio of around 0.4, and σ_C of approximately 2% based on real annual per capita consumption for post-war U.S. economy (i.e., after World War 2), investor must have γ greater than or approximately equal to 20, which is widely considered to be an unreasonably high degree of (relative) risk aversion. This leads to the equity premium puzzle, which states that an investor with a time-separable power utility of consumption and lognormal consumption growth must have an unreasonably high degree of risk aversion. Risk aversion magnifies volatility of consumption growth, so investors must have a high degree of risk aversion since consumption growth is very stable but pricing kernel is very volatile.

Since investor's optimal consumption growth has lognormal distribution, it has a small amount of negative (left) skewness. So, for investor with power utility of consumption, distribution for pricing kernel will have positive (right) skewness that increases with investor's (relative) risk aversion. Empirical evidence suggests that probability distribution for pricing kernel should have a large amount of positive skewness. Hence, investor must also have high degree of (relative) risk aversion to satisfy 'skewness bound' for pricing kernel. However, when taking into account the effect of rare disasters, it can be seen that empirical data on post-war consumption understates volatility and skewness of consumption growth.

Historical data usually covers time periods without disasters, which makes consumption growth appear less volatile. Moreover, excluding disasters severely understates the amount of negative (left) skewness in consumption growth. Disasters are events that result in great economic disruption, such as the Great Depression, World War 1, and World War 2.

To solve this equity premium puzzle, we account for the existence of rare disasters. In this context, a rare disaster is defined as ν, an independent random variable, which take the value of zero with a probability of 98.3% or the value of ln(0.65) with a probability of 1.7%. The interpretation of this probability is that, in most cases, consumption growth occurs gradually. However, there is a 1.7% chance that rare disaster ν affects consumption growth, resulting in $\nu = \ln(0.65)$ which approximates to $\ln(2/3)$, and $C1/C0 = 2/3$. This implies that a rare disaster has a probability of occurring once every 17 years, and this rare disaster will result in a fall in consumption by 1/3. When taking these rare disasters into account, we can eliminate the equity premium puzzle (as explained in the first paragraph) and obtain a more reasonable degree of (relative) risk aversion.