Multi-Leader Multi-follower Stackelberg Game in Mobile Blockchain Mining

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01

问题背景

Paper Background

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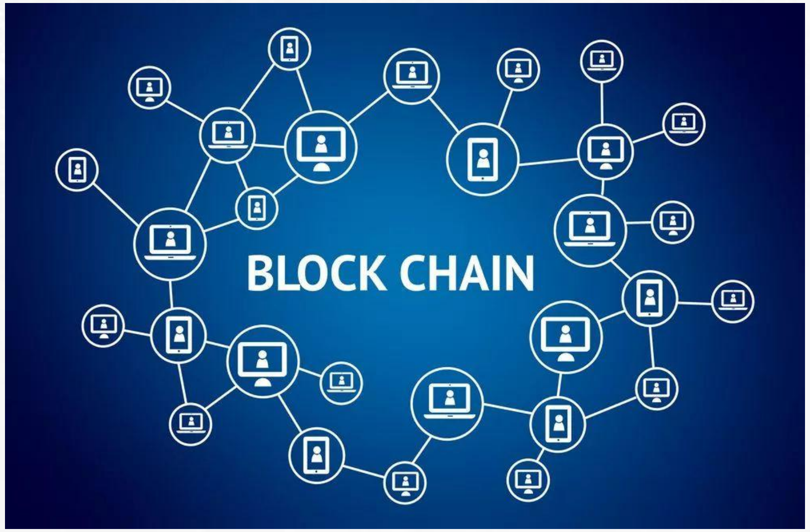
- Block Chain



区块链技术与比特币

MINER

Southeast University

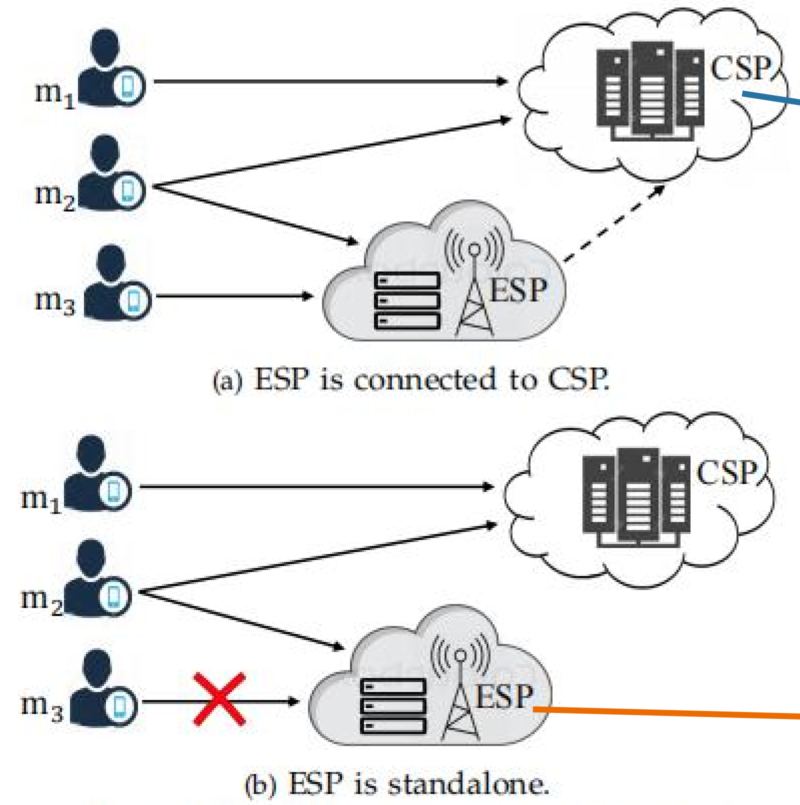


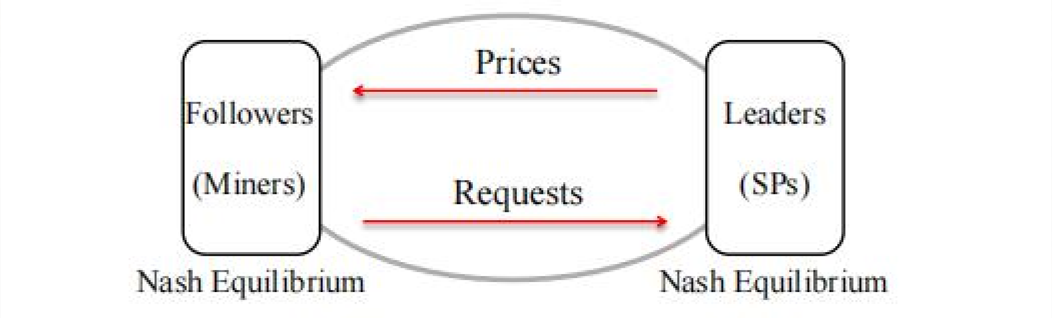
Fig. 2: Different operation modes of the ESP.

计算能力无限，有传输时延

计算能力有限，无传输时延

两种服务模型

Stackelberg Equilibrium



斯塔克伯格模型

Motivation and contributions

- 我们提出了来一个斯塔克伯格模型来解决移动区块链挖掘网络中一个基于价格的计算资源管理问题
- 我们基于其提供的不同质量和价格将ESP和CSP分开，实现资源的最优配置
- 分析了两种边缘操作模式的斯塔克尔伯格平衡(SE)的存在唯一性，在此基础上提出了得到SE解的算法
- 分析了在特殊条件下(矿工同质),SE解的算法
- 我们研究了矿工人数不确定性的影响

02

模型设计

模型设计

Problem 2 (SP SUBGAME: $\mathrm{OP_{SP}}$).

$$\text{maximize} \quad V_e = (P_e - C_e) \cdot E \ \ \text{where } E = \sum_{i=1}^{N} e_i \quad \text{(2a)}$$

$$\text{maximize} \quad V_c = (P_c - C_c) \cdot C \ \ \text{where } C = \sum_{i=1}^{N} c_i \quad \text{(2b)}$$

SPs的效益函数

Problem 1 (MINER SUBGAME: $\mathrm{OP_{MINER}}$).

$$\text{maximize} \quad U_i = R \cdot W_i - (P_e \cdot e_i + P_c \cdot c_i), \quad \text{(1a)}$$

$$\text{subject to} \quad P_e \cdot e_i + P_c \cdot c_i \le B_i, \quad e_i \ge 0, \quad c_i \ge 0. \quad \text{(1b)}$$

Miners的问题规划

$$V_e(P_e^*, \boldsymbol{E}^*) \ge V_e(P_e, \boldsymbol{E}^*), \forall P_e, \quad \text{(3a)}$$

$$V_c(P_c^*, \boldsymbol{C}^*) \ge V_c(P_c, \boldsymbol{C}^*), \forall P_c, \quad \text{(3b)}$$

$$U_i(e_i^*, c_i^*, P_e^*, P_c^*) \ge U_i(e_i, c_i, P_e^*, P_c^*), \forall i. \quad \text{(3c)}$$

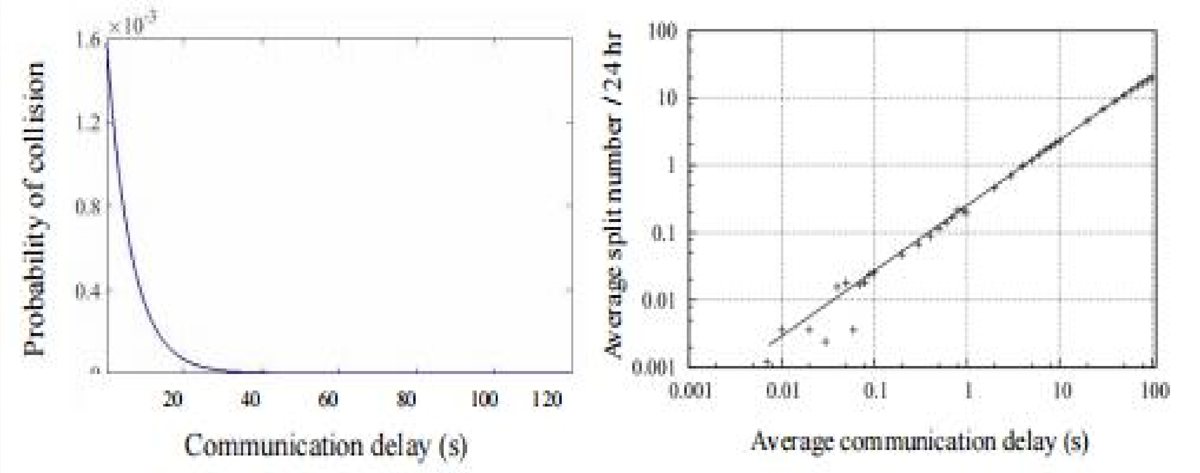
均衡的定义

获胜概率Wi

$$W_i^e(r_i, \boldsymbol{r}_{-i}) = e_i/S + e_i \sum_{j\neq i} \beta c_j/ES, \quad (4)$$

$$W_i^c(r_i, \boldsymbol{r}_{-i}) = c_i/S - c_i \sum_{j\neq i} \beta e_j/ES. \quad (5)$$

Wi的定义



传输时延对Wi的影响

$$\begin{aligned}\sum_{i=1}^{N} W_i &= \sum_{i=1}^{N} (W_i^e + W_i^c) \\ &= \sum_{i=1}^{N} [e_i/S + c_i/S] \\ &\quad + \beta \sum_{i=1}^{N} [e_i(C-c_i)/ES + c_i(E-e_i)/ES] \\ &= 1 + \beta \sum_{i=1}^{N} (e_i C - c_i E)/ES = 1. \qquad \square\end{aligned}$$

获胜概率Wi

1.在Connected Mode中，假定计算请求被传输到CSP的概率为h

如果计算请求被ESP接纳的获胜概率(r=[ei,ci])： $W_i^h = (e_i + c_i)/S + \beta(e_i C - c_i E)/ES.$

如果计算请求被ESP传输给CSP的获胜概率(r=[0,ei+ci])： $W_i^{1-h} = (1-\beta)(e_i + c_i)/S.$

总体而言，获胜概率为: $W_i = h \cdot W_i^h + (1-h) \cdot W_i^{1-h} = (1-\beta)(e_i + c_i)/S + \beta h e_i/E,$

2.在Standalone Mode中，假定计算请求被接纳的概率为h

如果计算请求被ESP接纳的获胜概率(r=[ei,ci])： $W_i^h = (e_i + c_i)/S + \beta(e_i C - c_i E)/ES.$

如果计算请求被ESP传输给CSP的获胜概率(r=[0,ci])，S=S-ei: $W_i^{1-h} = (1-\beta)c_i/(S - e_i).$

模型求解-存在性证明

Proof. Denote the equivalent variational inequality (VI) problem $\mathcal{VI}(\mathcal{K},\mathrm{F}) \equiv \mathcal{NEP}(\mathcal{X},U)$, where

$$\begin{aligned}
&\mathrm{F} := (\nabla_i U_i)_{i=1}^n, \quad \mathcal{X} = ([e_i,c_i]^\intercal)_{i=1}^n, \quad \mathcal{U} = (U_i)_{i=1}^n, \\
&\mathcal{K} := \mathcal{K}_1 \times \mathcal{K}_2 \times \cdots \times \mathcal{K}_n, \qquad (11)\\
&\mathcal{K}_i := \{(e_i,c_i) | P_e \cdot e_i + P_c \cdot c_i \le B_i, e_i \ge 0, c_i \ge 0\}.
\end{aligned}$$

转换为变分不等式问题

$$\begin{aligned}
U_{ee}^i = &-(R(1-\beta)/S^2 + \beta h/E^2)\cdot(R(1-\beta)/S^2 + \beta h/E^2)\\
&+(R(1-\beta)/S + 2\beta h/E - P_e)\cdot(\beta h/E^3 - R(1-\beta)/S^3),\\
U_{ec}^i = &R(1-\beta)/S \cdot (R(1-\beta)/S^2 + \beta h/E^2)\\
&+2(R(1-\beta)/S + \beta h/E - P_e)R(1-\beta)/S^3\\
&-(R(1-\beta)/S^2 - 2R(1-\beta)/S^3 \cdot c_i),\\
U_{ce}^i = &(-R(1-\beta)/S^2 - R(1-\beta)/S^3 \cdot e_i)\\
&+(-R(1-\beta)/S^3 + 2R(1-\beta)/S^3 \cdot c_i),\\
U_{cc}^i = &2R(1-\beta)/S^3 \cdot e_i - 2(R(1-\beta)/S^2 - R(1-\beta)/S^3 \cdot c_i).
\end{aligned}$$

Since $det(\mathcal{H}) = U_{ee}^i \cdot U_{cc}^i - U_{ec}^i \cdot U_{ce}^i > 0, \forall [e_i,c_i]^\intercal \in \mathcal{K}_i$,

Denote $\mathcal{H}^i$ for the Hessian matrix of U_i as below.

$$\mathcal{H} := \begin{bmatrix} U_{ee}^i & U_{ec}^i \\ U_{ce}^i & U_{cc}^i \end{bmatrix}, \qquad (18)$$

函数的Hession矩阵

Since $det(\mathcal{H}) = U_{ee}^i \cdot U_{cc}^i - U_{ec}^i \cdot U_{ce}^i > 0, \forall [e_i,c_i]^\intercal \in \mathcal{K}_i$,

Hession矩阵正定，转换是有效的

模型求解-唯一性证明

Claim 2: There is at most one NE for Problem 1e.

To show the uniqueness of the NE point, we first introduce the matrices $\mathcal{J}_{low}$, defined as

$$[\mathcal{J}_{low}]_{ij} := \inf_{x\in\mathcal{K}} \begin{cases} |\nabla^2_{ii}U_i|, & \text{if } i=j, \\ -\frac{1}{2}(|\nabla^2_{ij}U_i| + |\nabla^2_{ji}U_j|), & \text{else.} \end{cases} \quad (21)$$

$$\nabla^2_{ii}U_i = U^i_{ee} + U^i_{cc} \quad (22a)$$
$$= R[-8(1-\beta)(S-e_i-c_i)/S^3] - 2\beta(E-e_i)/E^3,$$
$$\nabla^2_{ij}U_i = \nabla^2_{ji}U_j \quad (22b)$$
$$= R(1-\beta)[1-2(S-e_i-c_i)]/S^2 + h\beta(2e_i - E)/E^3.$$

$$\mathcal{J}_{low} = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}, \quad (23)$$

$$a_{11} = \inf_{(e_1,c_1)\in\mathcal{K}} |\nabla^2_{11}U_1|,\ a_{22} = \inf_{(e_2,c_2)\in\mathcal{K}} |\nabla^2_{22}U_2|, \quad (24)$$

$$a_{12} = (-\frac{1}{2}) \inf_{(e_1,c_1)\in\mathcal{K}}^{(e_2,c_2)\in\mathcal{K}} (|\nabla^2_{12}U_1| + |\nabla^2_{21}U_2|). \quad (25)$$

$$a_{12} + \sqrt{a_{11}a_{22}} = \inf_{(e_1,c_1)\in\mathcal{K}}^{(e_2,c_2)\in\mathcal{K}} R(1-\beta)[1-2(S-e_i-c_i)]/S^2$$
$$+ h\beta(2e_i - E)/E^3$$
$$- 8(1-\beta)\sqrt{\prod_{i=1,2}(S-e_i-c_i)/S^3} > 0.$$

解是唯一的

模型求解

$$L_i = R\cdot[(1-\beta)(e_i+c_i)/S+\beta h e_i/E]-(P_e\cdot e_i+P_c\cdot c_i) - \lambda_1(P_e\cdot e_i+P_c\cdot c_i-B_i)+\lambda_2 e_i+\lambda_3 c_i, \quad (12)$$

and the complementary slackness conditions are

$$\begin{aligned}&\lambda_1(P_e\cdot e_i+P_c\cdot c_i-B_i)=0,\\ &\lambda_2 e_i=0,\quad \lambda_3 c_i=0,\quad \lambda_1>0,\lambda_2,\lambda_3,e_i,c_i\geq 0.\end{aligned} \quad (13)$$

KKT条件 ↓

$$e_i=\sqrt{\frac{h\beta E_{-i}R}{(1+\lambda_1)(P_e-P_c)}}-E_{-i}, \quad (14)$$

$$c_i=\sqrt{\frac{R(1-\beta)(E_{-i}+C_{-i})}{(1+\lambda_1)P_c}}-\sqrt{\frac{h\beta E_{-i}R}{(1+\lambda_1)(P_e-P_c)}}-C_{-i},$$

$B_i=P_e e_i+P_c c_i$, where $E_{-i}=\sum_{j\neq i}e_j, C_{-i}=\sum_{j\neq i}c_j$.

Solving Eq. (14) yields that

$$1+\lambda_1=\left[\frac{(P_e-P_c)\sigma_1\sqrt{E_{-i}}+P_c\sigma_2\sqrt{E_{-i}+C_{-i}}}{B_i+P_cC_{-i}+P_eE_{-i}}\right]^2, \quad (15)$$

where: $\sigma_1^2 = h\beta R/(P_e-P_c)$ and $\sigma_2^2=(1-\beta)R/P_c$.

下层解的close-form形式

Algorithm 1 Asynchronous Best-Response Algorithm

Output: $j, j\in\{e,c\}$

Input: Initialize k as 1 and randomly pick a feasible $P_j^{(0)}$

1: **for** iteration k **do**
2: Receive the miners' request vectors $\mathbf{r}^{(k-1)}$
3: Predict the strategy of the other SP
4: Decide $P_j^{(k)}=P_j^{(k-1)}+\Delta\frac{\partial V_j\left(P_j,P_{-j}^{(k-1)},\mathbf{r}^{(k-1)}\right)}{\partial P_j}$
5: **if** $P_j^{(k)}=P_j^{(k-1)}$ **then** Stop
6: **else** send $P_j^{(k)}$ to miners and set $k\leftarrow k+1$

上层解法：Asynchronous Best-response Algorithm

同质化矿工的情形

Theorem 3. *The unique Nash equilibrium for miner m_i in the* NEP$_{\text{HOMOMINER}}$ *problem is given below*

$$\begin{cases} e_i^* = \frac{B\beta h}{(1-\beta+h\beta)(P_e-P_c)}, \\ c_i^* = \frac{B[(1-\beta)(P_e-P_c)-P_c\beta h]}{P_c(1-\beta+h\beta)(P_e-P_c)}, \end{cases} \tag{28}$$

provided that the prices set by the ESP and the CSP satisfy $P_c < \frac{1-\beta}{1-\beta+h\beta}P_e$.

同质矿工的情况下，可以轻易得到下层解

Problem 2b (SP SUBGAME: NEP$_{\text{SPHOMOMINER}}$).

$$\text{maximize} \quad V_e=(P_e-C_e)\cdot e_i^*,\quad V_c=(P_c-C_c)\cdot c_i^*, \tag{30a}$$

$$\text{subject to} \quad P_c < \frac{1-\beta}{1-(1-h)\beta}P_e, \tag{30b}$$

where $e_i^*=\frac{B\beta h}{(1-\beta+h\beta)(P_e-P_c)}$, $c_i^*=\frac{B[(1-\beta)(P_e-P_c)-P_c\beta h]}{P_c(1-\beta+h\beta)(P_e-P_c)}$.

Theorem 4. *The unique Nash equilibrium for the SPs in the* NEP$_{\text{SPHOMOMINER}}$ *problem is given below:*

$$\begin{cases} P_e^* = \bar{p}, \\ P_c^* = \frac{C_c\bar{p}(1-\beta)-\bar{p}\sqrt{C_c h\beta(\bar{p}-C_c)(1-\beta)}}{[1-\beta(1-h)]C_c-\beta h P_e}, \end{cases} \tag{31}$$

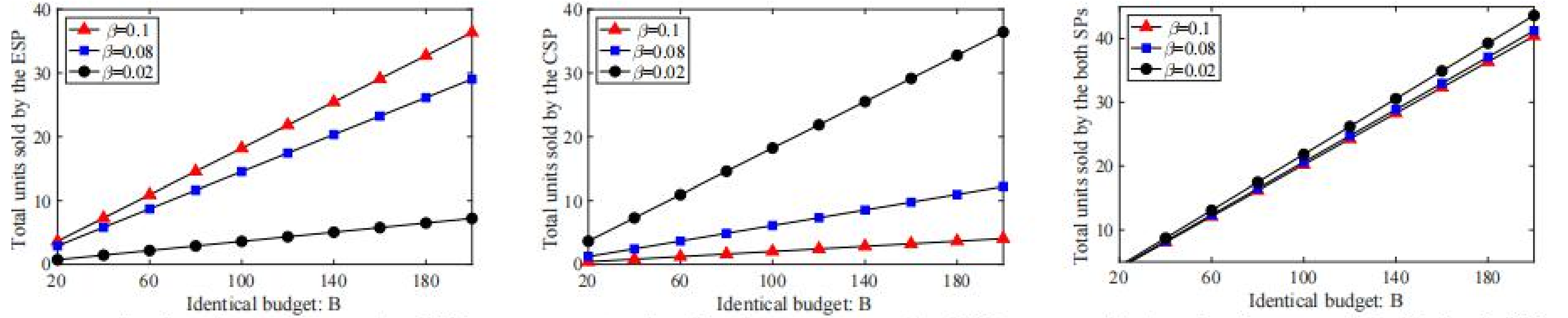
where $\bar{p}$ *is the solution to* $\partial V_e/\partial P_e = 0$.

上层解也可以求得

03

实验步骤

预算变化实验

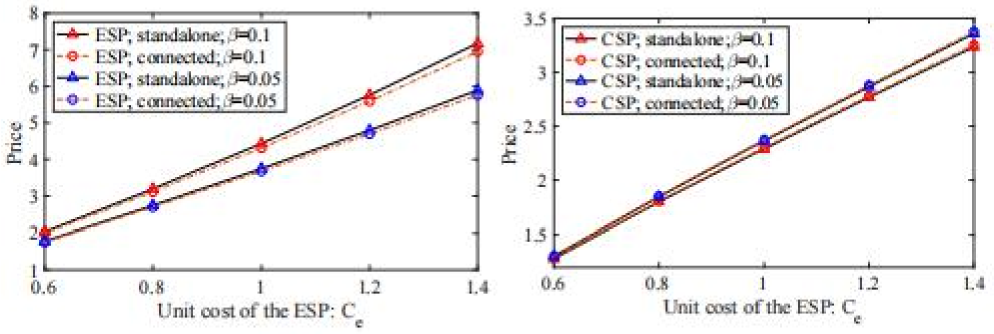


(a) A miner's request to the ESP. (b) A miner's request to the CSP. (c) A miner's request to the both SPs.

Fig. 8: m_i's budget B_1 varies from 20 to 200, with 5 miners in total.

不同预算下SP的收益

价格随成本的变化

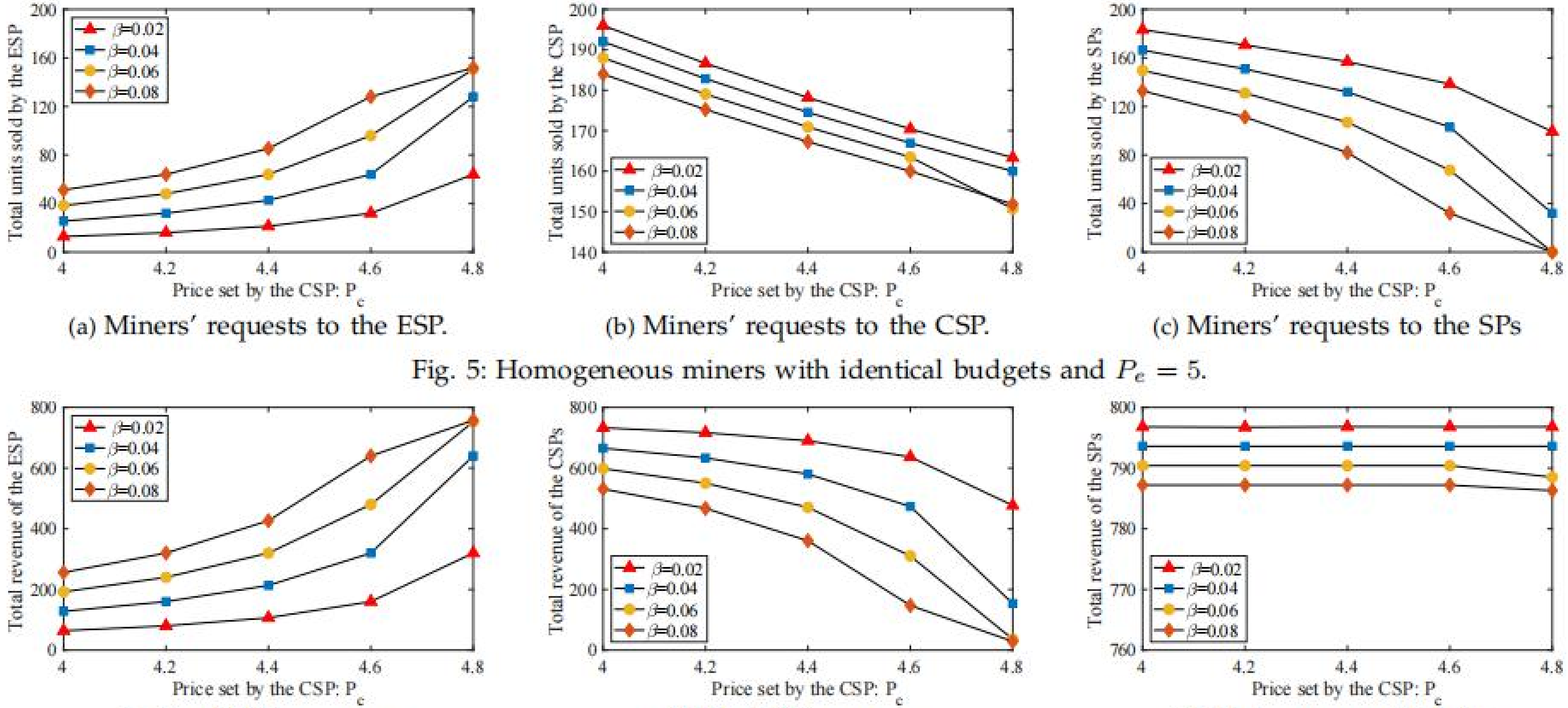


(a) $\beta = 0.02$.　　(b) $\beta = 0.06$.

Fig. 9: The CSP's unit cost is 0.5, while the ESP's unit cost changes.

随ESP的cost的变化，其价格的变化

同质矿工实验



(a) Miners' requests to the ESP. (b) Miners' requests to the CSP. (c) Miners' requests to the SPs

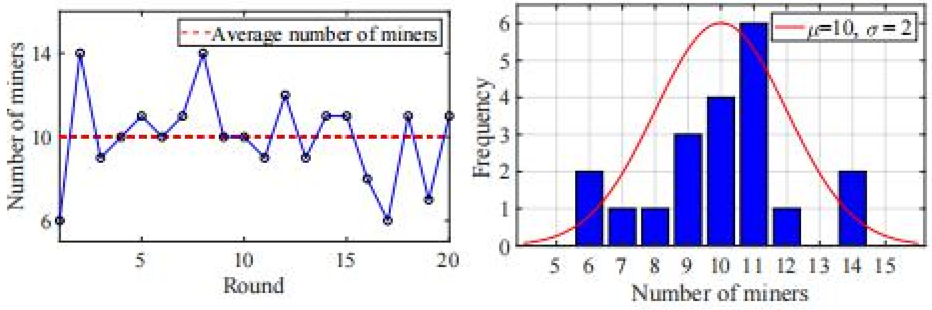
Fig. 5: Homogeneous miners with identical budgets and $P_e = 5$.

(a) The ESP's revenue. (b) The CSP's revenue. (c) Total revenue at SP side.

Fig. 6: Homogeneous miners with identical budgets and $P_e = 5$.

同质矿工的实验结果图

动态矿工数

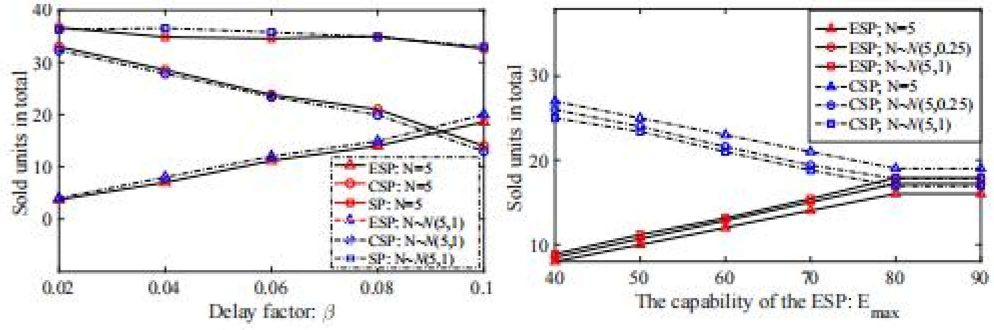


(a) Statistics on the miner number among 20 mining rounds.

(b) Corresponding histogram and distribution $N(\mu, \sigma^2)$.

Fig. 4: A toy example for population dynamics of moblie miners.

TABLE 2: Optimal requests of homogeneous miners with sufficiently large budgets where $\gamma = (N-1)R/N$.



(a) $\beta = 0.02$.

(b) $\beta = 0.06$.

Fig. 10: Miner number: fixed vs dynamic.

矿工数服从N(10,4)分布

动态静态对比图

04

结论

Research Methods And Processes

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总结与展望

- **本文总结**
  - 我们提出了一个最优价格策略和最优计算卸载请求的移动矿工之间的斯塔克尔伯格博弈
  - 讨论了该对策中斯塔克尔伯格平衡的存在性和唯一性，并提出了实现SE点的算法
  - 不确定性使矿工更积极地从ESP购买计算资源

- **留下的思考**
  - 模型设计还可以继续优化
  - 上层的均衡证明比较模糊
  - 依旧是通过迭代进行数值求解

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感谢大家的观看

主讲人：赵逸新

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