MIE1622 Assignment 2 Report

Implement investment strategies in Python

1. Equal risk contributions

```
def strat_equal_risk_contr(x_init, cash_init, mu, Q, cur_prices):
  port_value = np.dot(cur_prices,x_init) + cash_init
  n = 20

  class erc(object):
    def __init__(self):
        pass

    def objective(self, x):
        # The callback for calculating the objective
        y = x * np.dot(Q, x)
        fval = 0
        for i in range(n):
            for j in range(i,n):
                xij = y[i] - y[j]
                fval = fval + xij*xij
        fval = 2*fval
        return fval

    def gradient(self, x):
        # The callback for calculating the gradient
        grad = np.zeros(n)
        # Insert your gradient computations here
        # You can use finite differences to check the gradient
        y = x * np.dot(Q, x)
        for i in range(n):
          for j in range(n):
            diff1 = np.dot(Q[i],x) + np.dot(Q[i][i],x[i])
            diff2 = np.dot(Q[i][j], x[i])
            g = (y[i]-y[j]) * (diff1 - diff2)
            grad[i] += g
          grad[i] *= 4
        return grad

    def constraints(self, x):
        # The callback for calculating the constraints
        return [1.0] * n

    def jacobian(self, x):
        # The callback for calculating the Jacobian
        return np.array([[1.0] * n])

  w0 = (x_init*cur_prices)/port_value
  lb = [0.0] * n  # lower bounds on variables
  ub = [1.0] * n  # upper bounds on variables
  cl = [1]        # lower bounds on constraints
  cu = [1]        # upper bounds on constraints

  # Define IPOPT problem
  nlp = ipopt.Problem(n=len(w0), m=len(cl), problem_obj=erc(), lb=lb, ub=ub, cl=cl, cu=cu)

  # Set the IPOPT options
  nlp.add_option('jac_c_constant'.encode('utf-8'), 'yes'.encode('utf-8'))
  nlp.add_option('hessian_approximation'.encode('utf-8'), 'limited-memory'.encode('utf-8'))
  nlp.add_option('mu_strategy'.encode('utf-8'), 'adaptive'.encode('utf-8'))
  nlp.add_option('tol'.encode('utf-8'), 1e-10)

  # Solve the problem
  w_erc, info = nlp.solve(w0)

  x_optimal = np.floor(w_erc*port_value / cur_prices)
  tran_cost = np.dot(cur_prices , abs(x_optimal-x_init)) * 0.005
  cash_optimal = port_value - np.dot(cur_prices,x_optimal) - tran_cost

  return x_optimal, cash_optimal
```

2. Leveraged equal risk contributions

```
def strat_lever_equal_risk_contr(x_init, cash_init, mu, Q, cur_prices):
  port_value = np.dot(cur_prices,x_init) + cash_init
  n = 20

  class erc(object):
    def __init__(self):
        pass

    def objective(self, x):
        # The callback for calculating the objective
        y = x * np.dot(Q, x)
        fval = 0
        for i in range(n):
            for j in range(i,n):
                xij = y[i] - y[j]
                fval = fval + xij*xij
        fval = 2*fval
        return fval

    def gradient(self, x):
        # The callback for calculating the gradient
        grad = np.zeros(n)
        # Insert your gradient computations here
        # You can use finite differences to check the gradient
        y = x * np.dot(Q, x)
        for i in range(n):
          for j in range(n):
            diff1 = np.dot(Q[i],x) + np.dot(Q[i][i],x[i])
            diff2 = np.dot(Q[i][j], x[i])
            g = (y[i]-y[j]) * (diff1 - diff2)
            grad[i] += g
          grad[i] *= 4
        return grad

    def constraints(self, x):
        # The callback for calculating the constraints
        return [1.0] * n

    def jacobian(self, x):
        # The callback for calculating the Jacobian
        return np.array([[1.0] * n])

  bor_val = init_value

  interest = bor_val*r_rf/6

  if period == 1:
    port_value = port_value + bor_val

  w0 = (x_init*cur_prices)/port_value
  lb = [0.0] * n  # lower bounds on variables
  ub = [1.0] * n  # upper bounds on variables
  cl = [1]        # lower bounds on constraints
  cu = [1]        # upper bounds on constraints

  # Define IPOPT problem
  nlp = ipopt.Problem(n=len(w0), m=len(cl), problem_obj=erc(), lb=lb, ub=ub, cl=cl, cu=cu)

  # Set the IPOPT options
  nlp.add_option('jac_c_constant'.encode('utf-8'), 'yes'.encode('utf-8'))
  nlp.add_option('hessian_approximation'.encode('utf-8'), 'limited-memory'.encode('utf-8'))
  nlp.add_option('mu_strategy'.encode('utf-8'), 'adaptive'.encode('utf-8'))
  nlp.add_option('tol'.encode('utf-8'), 1e-10)

  # Solve the problem
  w_erc, info = nlp.solve(w0)

  x_optimal = np.floor(w_erc*port_value / cur_prices)
  tran_cost = np.dot(cur_prices , abs(x_optimal-x_init)) * 0.005
  cash_optimal = port_value - np.dot(cur_prices,x_optimal) - tran_cost - interest

  return x_optimal, cash_optimal
```

When calculating new cash, not like other strategies, interest is also being deducted in the leveraged equal risk contributions.

3. Robust mean-variance optimization

```
def strat_robust_optim(x_init, cash_init, mu, Q, cur_prices):
  port_value = np.dot(cur_prices,x_init) + cash_init
  n = 20

  cpx = cplex.Cplex()
  cpx.objective.set_sense(cpx.objective.sense.minimize)
  c  = [0.0] * n
  lb = [0.0] * n
  ub = [1.0] * n

  A = []
  for k in range(n):
    A.append([[0,1],[1.0,mu[k]]])

  var_names = ["w_%s" % i for i in range(1,n+1)]

  x_minVar, cash_minVar = strat_min_variance(x_init, cash_init, mu, Q, cur_prices)
  w_minVar = (x_minVar*cur_prices)/port_value
  ret_minVar = np.dot(mu, w_minVar)
  Portf_Retn = ret_minVar # target return
  var_matr = np.diag(np.diag(Q))
  Qq_rMV = var_matr
  w0 = (x_init*cur_prices)/port_value
  rob_init = np.dot(w0, np.dot(var_matr, w0)) # return estimation error of initial portfolio
  rob_bnd  = rob_init # target return estimation error

  cpx.linear_constraints.add(rhs=[1.0,Portf_Retn], senses="EG")
  cpx.variables.add(obj=c, lb=lb, ub=ub, columns=A, names=var_names)
  Qmat = [[list(range(n)), list(2*Q[k,:])] for k in range(n)]
  cpx.objective.set_quadratic(Qmat)
  Qcon = cplex.SparseTriple(ind1=var_names, ind2=range(n), val=np.diag(var_matr))
  cpx.quadratic_constraints.add(rhs=rob_bnd, quad_expr=Qcon, name="Qc")

  cpx.parameters.threads.set(4)
  cpx.parameters.timelimit.set(60)
  cpx.parameters.barrier.qcpconvergetol.set(1e-12)
  cpx.set_results_stream(None)
  cpx.set_warning_stream(None)
  cpx.solve()
  w_rMV = np.array(cpx.solution.get_values())
  w_rMV[w_rMV<1e-6] = 0 #round near-zero
  w_rMV = w_rMV / np.sum(w_rMV)

  x_optimal = np.floor(w_rMV*port_value / cur_prices)
  tran_cost = np.dot(cur_prices , abs(x_optimal-x_init)) * 0.005
  cash_optimal = port_value - np.dot(cur_prices,x_optimal) - tran_cost


  return x_optimal, cash_optimal
```

The target return is the predicted return of minimum variance portfolio, and the target risk estimation error is calculated by the diagonal matrix of variance and the weights from the previous robust mean-variance portfolio.

```
# Compute portfolio value
p_values = np.dot(data_prices[day_ind_start:day_ind_end+1,:], x[strategy, period-1]) + cash[strategy, period-1]
# We need to subtract borrow value (= init_value) when we calculate the portfolio value from applying leveraged equal risk contribbutions
if strategy == 5:
  portf_value[strategy][day_ind_start:day_ind_end+1] = np.reshape(p_values, (p_values.size,1)) - init_value
else:
  portf_value[strategy][day_ind_start:day_ind_end+1] = np.reshape(p_values, (p_values.size,1))
print('  Strategy "{0}", value begin = $ {1:.2f}, value end = $ {2:.2f}'.format( strategy_names[strategy],
      portf_value[strategy][day_ind_start][0], portf_value[strategy][day_ind_end][0]))
```

Also, also set that the new portfolio value needs to reduce the initial value for this strategy only.

Rounding procedure:

```
x_optimal = np.floor(w_rMV*port_value / cur_prices)
```

I use the np.floor method in every strategy to make sure I always trade integer number of shares.

Validation procedure:

```
if cash[strategy, period-1] < 0:
  port_value = np.dot(cur_prices, curr_positions) + curr_cash
  ratio = x[strategy, period-1]/sum(x[strategy, period-1])
  exce_cash = abs(cash[strategy, period-1])*ratio
  exce_x = np.ceil(exce_cash/cur_prices)
  x[strategy, period-1] = x[strategy, period-1] - exce_x
  new_tran_cost = np.dot(cur_prices , abs(x[strategy, period-1]-curr_positions)) * 0.005
  cash[strategy, period-1] = port_value - np.dot(cur_prices,x[strategy, period-1]) - new_tran_cost
```

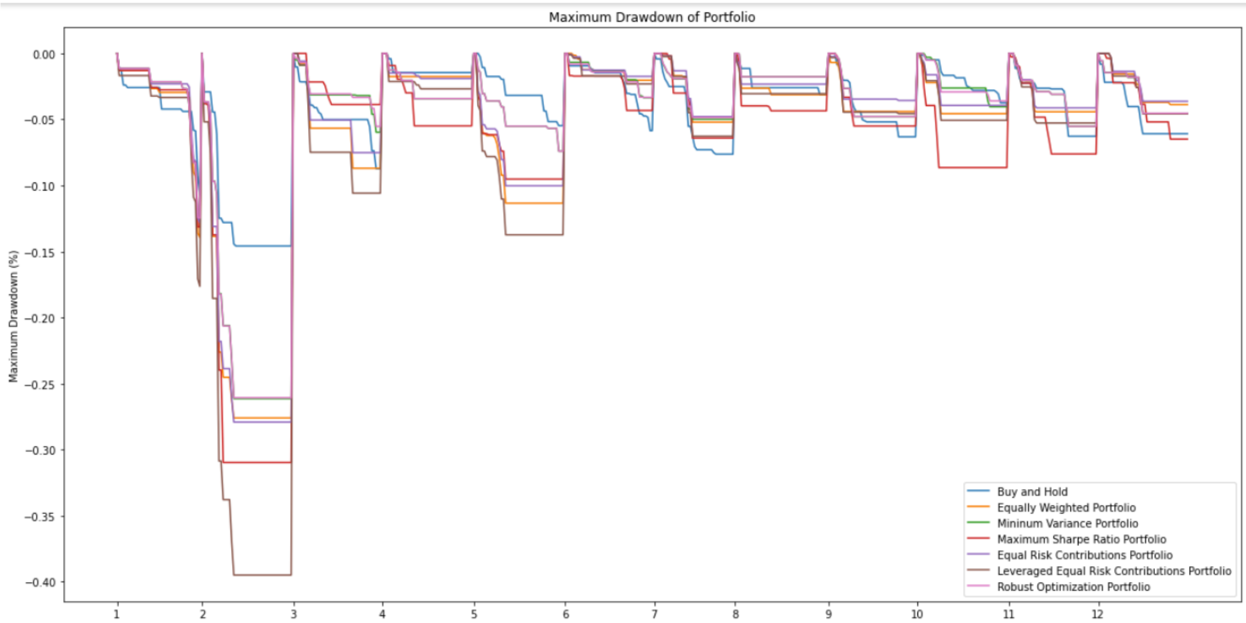
I use the code above to calculate that how much cash is being negative and deduct corresponding amount of shares in the portfolio to make cash become non-negative again.

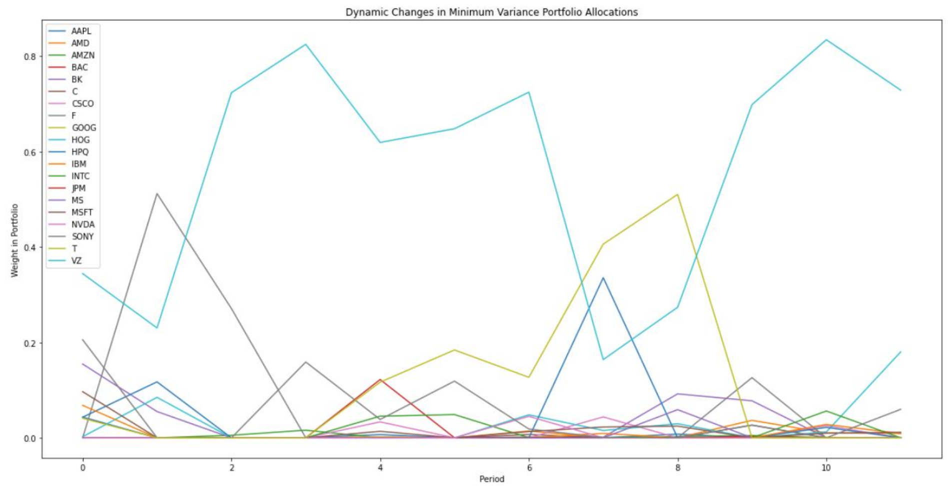
Analyze your results

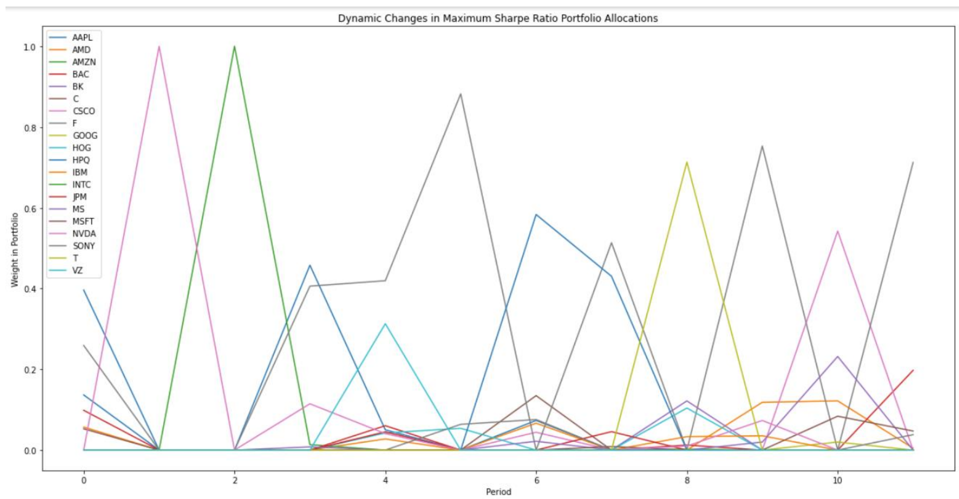
```
Initial portfolio value = $ 1000013.0


Period 1: start date 01/02/2020, end date 02/28/2020
  Strategy "Buy and Hold", value begin = $ 1000013.00, value end = $ 893956.82
  Strategy "Equally Weighted Portfolio", value begin = $ 990898.24, value end = $ 893208.59
  Strategy "Mininum Variance Portfolio", value begin = $ 992758.41, value end = $ 916240.12
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990064.37, value end = $ 922095.98
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 991764.84, value end = $ 904269.40
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 985914.90, value end = $ 857709.01
  Strategy "Robust Optimization Portfolio", value begin = $ 992742.31, value end = $ 916349.19

Period 2: start date 03/02/2020, end date 04/30/2020
  Strategy "Buy and Hold", value begin = $ 945076.08, value end = $ 949228.39
  Strategy "Equally Weighted Portfolio", value begin = $ 931395.29, value end = $ 862353.83
  Strategy "Mininum Variance Portfolio", value begin = $ 955988.34, value end = $ 851552.27
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 962082.00, value end = $ 1017240.71
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 943554.83, value end = $ 857984.71
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 907350.70, value end = $ 791157.68
  Strategy "Robust Optimization Portfolio", value begin = $ 956225.07, value end = $ 853066.34

Period 3: start date 05/01/2020, end date 06/30/2020
  Strategy "Buy and Hold", value begin = $ 937916.75, value end = $ 913415.30
  Strategy "Equally Weighted Portfolio", value begin = $ 831101.45, value end = $ 934159.54
  Strategy "Mininum Variance Portfolio", value begin = $ 827264.88, value end = $ 854237.62
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 974390.72, value end = $ 1175795.01
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 827544.07, value end = $ 923533.33
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 745522.53, value end = $ 875972.00
  Strategy "Robust Optimization Portfolio", value begin = $ 828663.74, value end = $ 869417.83


Period 4: start date 07/01/2020, end date 08/31/2020
  Strategy "Buy and Hold", value begin = $ 905419.70, value end = $ 994693.42
  Strategy "Equally Weighted Portfolio", value begin = $ 927755.25, value end = $ 1060727.96
  Strategy "Mininum Variance Portfolio", value begin = $ 856556.48, value end = $ 981500.25
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1219685.93, value end = $ 1606785.15
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 919911.78, value end = $ 1060207.85
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 866799.35, value end = $ 1058106.00
  Strategy "Robust Optimization Portfolio", value begin = $ 874641.80, value end = $ 1002483.84

Period 5: start date 09/01/2020, end date 10/30/2020
  Strategy "Buy and Hold", value begin = $ 993194.54, value end = $ 971914.18
  Strategy "Equally Weighted Portfolio", value begin = $ 1068338.47, value end = $ 999243.10
  Strategy "Mininum Variance Portfolio", value begin = $ 983247.54, value end = $ 942756.98
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1640764.45, value end = $ 1553142.55
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1068126.51, value end = $ 1002768.01
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1064771.08, value end = $ 975347.11
  Strategy "Robust Optimization Portfolio", value begin = $ 1004277.62, value end = $ 962896.08

Period 6: start date 11/02/2020, end date 12/31/2020
  Strategy "Buy and Hold", value begin = $ 983801.02, value end = $ 1004435.74
  Strategy "Equally Weighted Portfolio", value begin = $ 1008081.57, value end = $ 1194326.17
  Strategy "Mininum Variance Portfolio", value begin = $ 951192.00, value end = $ 1005965.27
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1551740.49, value end = $ 1789134.34
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1010990.80, value end = $ 1186792.03
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 982403.61, value end = $ 1222387.15
  Strategy "Robust Optimization Portfolio", value begin = $ 970844.27, value end = $ 1043592.46
```

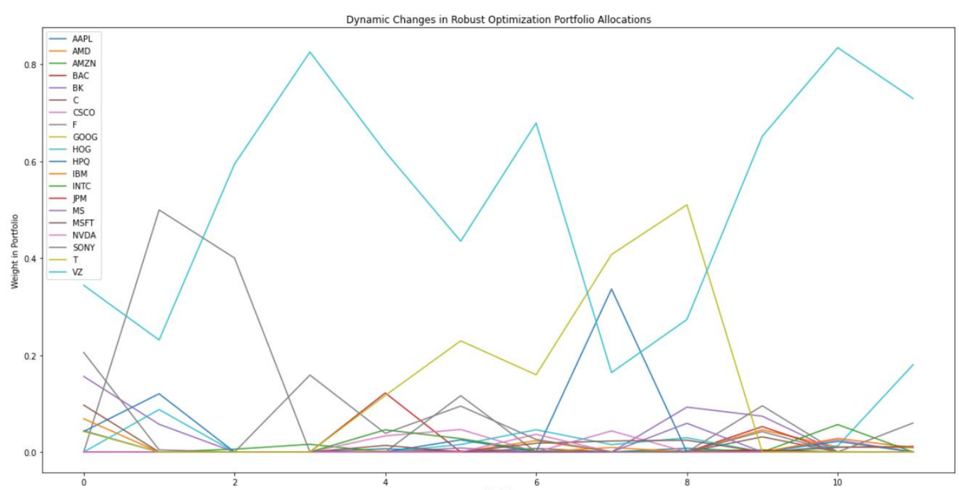
```
Period 7: start date 01/04/2021, end date 02/26/2021
  Strategy "Buy and Hold", value begin = $ 1005601.39, value end = $ 956244.08
  Strategy "Equally Weighted Portfolio", value begin = $ 1180783.14, value end = $ 1267218.28
  Strategy "Mininum Variance Portfolio", value begin = $ 1003981.40, value end = $ 975148.49
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1737569.08, value end = $ 1851392.00
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1173763.69, value end = $ 1226772.08
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1200463.75, value end = $ 1273745.57
  Strategy "Robust Optimization Portfolio", value begin = $ 1040054.32, value end = $ 1012251.07

Period 8: start date 03/01/2021, end date 04/30/2021
  Strategy "Buy and Hold", value begin = $ 957791.35, value end = $ 1019731.32
  Strategy "Equally Weighted Portfolio", value begin = $ 1297587.56, value end = $ 1398874.11
  Strategy "Mininum Variance Portfolio", value begin = $ 975468.82, value end = $ 1088054.39
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1899769.93, value end = $ 2059602.42
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1250751.78, value end = $ 1362050.22
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1302438.47, value end = $ 1454273.19
  Strategy "Robust Optimization Portfolio", value begin = $ 1013689.22, value end = $ 1130748.49

Period 9: start date 05/03/2021, end date 06/30/2021
  Strategy "Buy and Hold", value begin = $ 1022204.61, value end = $ 987842.85
  Strategy "Equally Weighted Portfolio", value begin = $ 1397748.67, value end = $ 1459313.30
  Strategy "Mininum Variance Portfolio", value begin = $ 1087868.73, value end = $ 1076783.13
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2051066.72, value end = $ 2014754.84
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1361002.02, value end = $ 1390889.63
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1448665.91, value end = $ 1490399.32
  Strategy "Robust Optimization Portfolio", value begin = $ 1130519.33, value end = $ 1118952.59

Period 10: start date 07/01/2021, end date 08/31/2021
  Strategy "Buy and Hold", value begin = $ 993283.49, value end = $ 975250.19
  Strategy "Equally Weighted Portfolio", value begin = $ 1466719.25, value end = $ 1517805.45
  Strategy "Mininum Variance Portfolio", value begin = $ 1076813.56, value end = $ 1086661.37
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2013665.31, value end = $ 2121761.08
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1397128.55, value end = $ 1448789.01
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1494817.92, value end = $ 1565959.25
  Strategy "Robust Optimization Portfolio", value begin = $ 1118949.97, value end = $ 1134380.38

Period 11: start date 09/01/2021, end date 10/29/2021
  Strategy "Buy and Hold", value begin = $ 974520.08, value end = $ 949068.41
  Strategy "Equally Weighted Portfolio", value begin = $ 1513571.60, value end = $ 1563474.39
  Strategy "Mininum Variance Portfolio", value begin = $ 1081129.14, value end = $ 1057285.32
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2102424.44, value end = $ 2144868.51
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1442868.67, value end = $ 1457697.42
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1553715.95, value end = $ 1575539.60
  Strategy "Robust Optimization Portfolio", value begin = $ 1127863.54, value end = $ 1102995.18

Period 12: start date 11/01/2021, end date 12/31/2021
  Strategy "Buy and Hold", value begin = $ 951350.41, value end = $ 932471.35
  Strategy "Equally Weighted Portfolio", value begin = $ 1584847.25, value end = $ 1646671.96
  Strategy "Mininum Variance Portfolio", value begin = $ 1054688.50, value end = $ 1048743.13
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2113787.35, value end = $ 2217141.21
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1467876.00, value end = $ 1520269.68
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1585804.55, value end = $ 1657667.19
  Strategy "Robust Optimization Portfolio", value begin = $ 1100262.88, value end = $ 1094050.27
```


Daily Values of Portfolio
1e6
Buy and Hold Portfolio
Equally Weighted Portfolio
Minimum Variance Portfolio
Maximum Sharpe Ratio Portfolio
Equal Risk Contributions Portfolio
Leveraged Equal Risk Contributions Portfolio
Robust Optimization Portfolio
Portfolio Value
Day


Maximum Drawdown of Portfolio
Maximum Drawdown (%)
Period
Buy and Hold
Equally Weighted Portfolio
Mininum Variance Portfolio
Maximum Sharpe Ratio Portfolio
Equal Risk Contributions Portfolio
Leveraged Equal Risk Contributions Portfolio
Robust Optimization Portfolio







The trading amount of maximum Sharpe Ratio portfolio is much more volatile than the other two portfolios. And this volatile trading pattern appears in many stocks in the maximum Sharpe Ratio portfolio. On the other hand, the trading amount doesn't really reduce in robust optimization portfolio while comparing to the minimum variance portfolio, since the target return for the robust optimization portfolio is set up to be the predicted return of minimum variance portfolio.

According to the graph of the daily value of portfolios, the returns of the portfolios can be separated into 3 classes. The lowest class has buy and hold portfolio, minimum variance portfolio and the robust optimization portfolio, and their final values are approximately around $1,000,000. The middle class has the equally weighted portfolio, the leveraged and the normal equal risk contributions portfolio, and their final values are approximately around $1,500,000. The highest class only has the maximum Sharpe Ratio portfolio with the final values that is around $2,200,000. And the leveraged equal risk contributions portfolio has the highest maximum drawdown among all the portfolios in all the periods. Due to its high performance in values, I would choose the maximum Sharpe Ratio portfolio.

Test your trading strategies for years 2008 and 2009

```
Initial portfolio value = $ 385097.15


Period 1: start date 01/02/2008, end date 02/29/2008
  Strategy "Buy and Hold", value begin = $ 385097.15, value end = $ 325918.34
  Strategy "Equally Weighted Portfolio", value begin = $ 381649.89, value end = $ 326929.57
  Strategy "Mininum Variance Portfolio", value begin = $ 383262.49, value end = $ 327144.09
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 381265.54, value end = $ 332652.59
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 381854.88, value end = $ 328109.33
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 378354.15, value end = $ 306048.14
  Strategy "Robust Optimization Portfolio", value begin = $ 383262.39, value end = $ 327133.70

Period 2: start date 03/03/2008, end date 04/30/2008
  Strategy "Buy and Hold", value begin = $ 325807.08, value end = $ 349997.20
  Strategy "Equally Weighted Portfolio", value begin = $ 322097.69, value end = $ 354821.22
  Strategy "Mininum Variance Portfolio", value begin = $ 322718.41, value end = $ 365547.71
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 325785.77, value end = $ 344234.61
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 324033.55, value end = $ 360849.19
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 296525.49, value end = $ 347146.59
  Strategy "Robust Optimization Portfolio", value begin = $ 322708.60, value end = $ 365544.16

Period 3: start date 05/01/2008, end date 06/30/2008
  Strategy "Buy and Hold", value begin = $ 357929.49, value end = $ 322881.56
  Strategy "Equally Weighted Portfolio", value begin = $ 366424.15, value end = $ 308970.75
  Strategy "Mininum Variance Portfolio", value begin = $ 372947.72, value end = $ 351399.81
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 349025.01, value end = $ 312518.46
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 371619.76, value end = $ 322250.26
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 359066.31, value end = $ 291214.08
  Strategy "Robust Optimization Portfolio", value begin = $ 372944.29, value end = $ 351396.38


Period 4: start date 07/01/2008, end date 08/29/2008
  Strategy "Buy and Hold", value begin = $ 324349.75, value end = $ 326489.53
  Strategy "Equally Weighted Portfolio", value begin = $ 309425.79, value end = $ 315897.27
  Strategy "Mininum Variance Portfolio", value begin = $ 351751.30, value end = $ 356195.63
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 324814.45, value end = $ 314800.05
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 321897.25, value end = $ 325995.56
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 287838.92, value end = $ 293491.23
  Strategy "Robust Optimization Portfolio", value begin = $ 351747.90, value end = $ 356192.75

Period 5: start date 09/02/2008, end date 10/31/2008
  Strategy "Buy and Hold", value begin = $ 333252.73, value end = $ 274022.75
  Strategy "Equally Weighted Portfolio", value begin = $ 316675.00, value end = $ 231420.37
  Strategy "Mininum Variance Portfolio", value begin = $ 348289.18, value end = $ 269010.37
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 306067.46, value end = $ 229119.53
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 325754.45, value end = $ 241694.02
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 290277.41, value end = $ 174617.09
  Strategy "Robust Optimization Portfolio", value begin = $ 348286.12, value end = $ 269017.67

Period 6: start date 11/03/2008, end date 12/31/2008
  Strategy "Buy and Hold", value begin = $ 282342.11, value end = $ 305967.56
  Strategy "Equally Weighted Portfolio", value begin = $ 230011.81, value end = $ 198885.85
  Strategy "Mininum Variance Portfolio", value begin = $ 269394.11, value end = $ 248173.94
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 226378.28, value end = $ 175247.44
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 240851.03, value end = $ 211878.79
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 170565.19, value end = $ 130489.26
  Strategy "Robust Optimization Portfolio", value begin = $ 269856.95, value end = $ 251501.28
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Period 7: start date 01/02/2009, end date 02/27/2009
  Strategy "Buy and Hold", value begin = $ 313366.90, value end = $ 258275.19
  Strategy "Equally Weighted Portfolio", value begin = $ 207366.91, value end = $ 169935.27
  Strategy "Mininum Variance Portfolio", value begin = $ 256222.08, value end = $ 244193.85
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 174617.71, value end = $ 145398.83
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 220753.36, value end = $ 188482.39
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 139872.30, value end = $ 95257.85
  Strategy "Robust Optimization Portfolio", value begin = $ 259917.71, value end = $ 247662.05

Period 8: start date 03/02/2009, end date 04/30/2009
  Strategy "Buy and Hold", value begin = $ 248688.22, value end = $ 286368.72
  Strategy "Equally Weighted Portfolio", value begin = $ 161713.12, value end = $ 260066.85
  Strategy "Mininum Variance Portfolio", value begin = $ 234569.46, value end = $ 319274.90
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 138400.02, value end = $ 180376.00
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 180603.30, value end = $ 270304.74
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 81478.69, value end = $ 209466.97
  Strategy "Robust Optimization Portfolio", value begin = $ 237900.24, value end = $ 323828.76

Period 9: start date 05/01/2009, end date 06/30/2009
  Strategy "Buy and Hold", value begin = $ 287805.37, value end = $ 285824.08
  Strategy "Equally Weighted Portfolio", value begin = $ 259634.59, value end = $ 273277.43
  Strategy "Mininum Variance Portfolio", value begin = $ 316797.53, value end = $ 320211.72
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 179218.47, value end = $ 184969.52
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 269856.63, value end = $ 280347.35
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 205895.31, value end = $ 220790.76
  Strategy "Robust Optimization Portfolio", value begin = $ 321314.78, value end = $ 324775.24

Period 10: start date 07/01/2009, end date 08/31/2009
  Strategy "Buy and Hold", value begin = $ 286766.63, value end = $ 298338.27
  Strategy "Equally Weighted Portfolio", value begin = $ 272967.79, value end = $ 321758.02
  Strategy "Mininum Variance Portfolio", value begin = $ 319907.28, value end = $ 341210.60
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 180480.59, value end = $ 195141.25
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 280343.01, value end = $ 318898.80
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 217904.85, value end = $ 272788.95
  Strategy "Robust Optimization Portfolio", value begin = $ 324466.92, value end = $ 346073.01

Period 11: start date 09/01/2009, end date 10/30/2009
  Strategy "Buy and Hold", value begin = $ 291703.36, value end = $ 290193.57
  Strategy "Equally Weighted Portfolio", value begin = $ 310182.86, value end = $ 328338.22
  Strategy "Mininum Variance Portfolio", value begin = $ 333055.45, value end = $ 349798.81
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 186718.97, value end = $ 186532.29
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 309206.36, value end = $ 327941.74
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 256110.31, value end = $ 282777.17
  Strategy "Robust Optimization Portfolio", value begin = $ 337803.27, value end = $ 354854.71

Period 12: start date 11/02/2009, end date 12/31/2009
  Strategy "Buy and Hold", value begin = $ 288596.05, value end = $ 323101.02
  Strategy "Equally Weighted Portfolio", value begin = $ 329690.48, value end = $ 375806.51
  Strategy "Mininum Variance Portfolio", value begin = $ 347213.39, value end = $ 389876.96
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 185544.74, value end = $ 210082.30
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 328584.85, value end = $ 367596.90
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 280816.16, value end = $ 336565.41
  Strategy "Robust Optimization Portfolio", value begin = $ 352228.24, value end = $ 395513.04
```
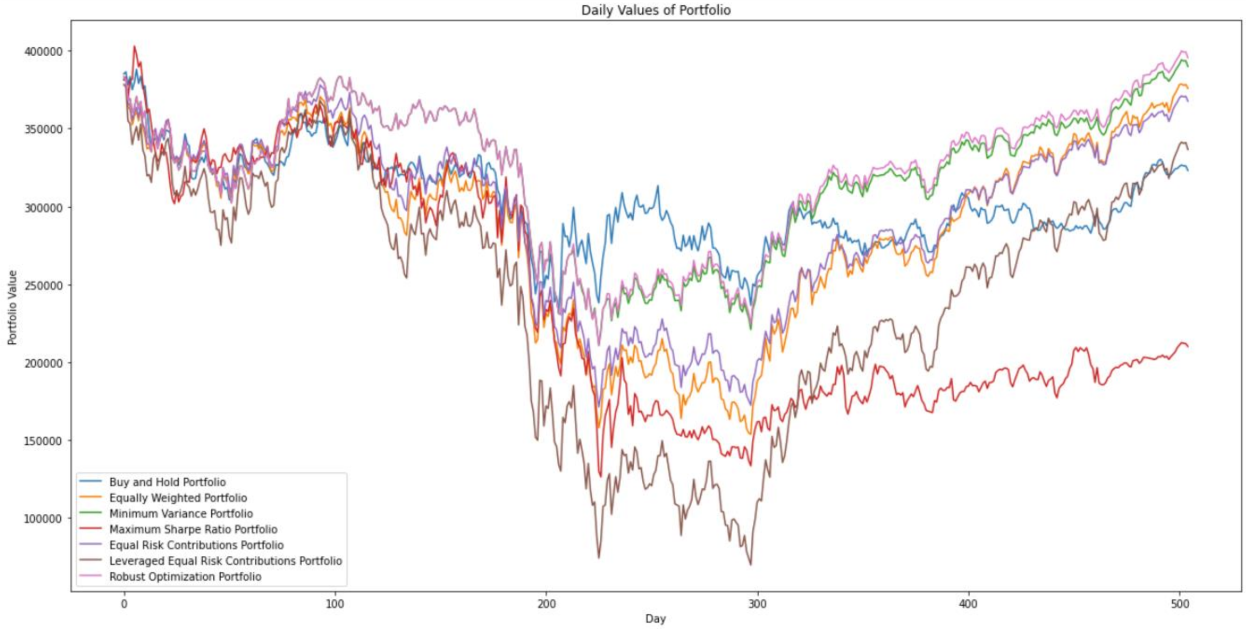
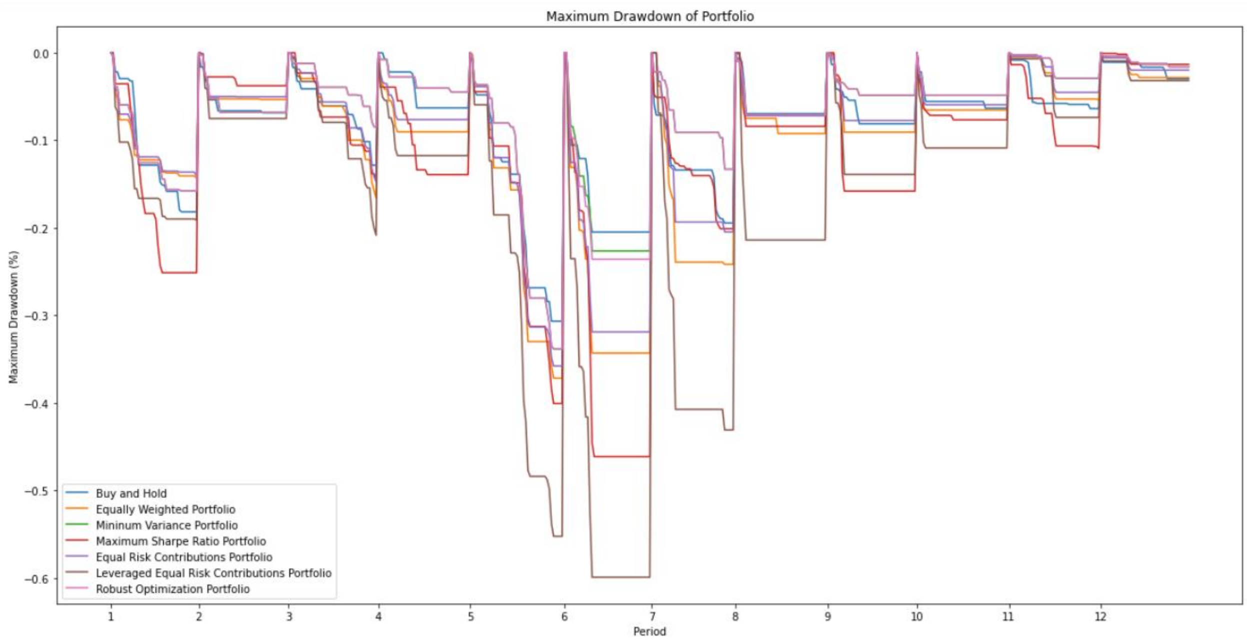
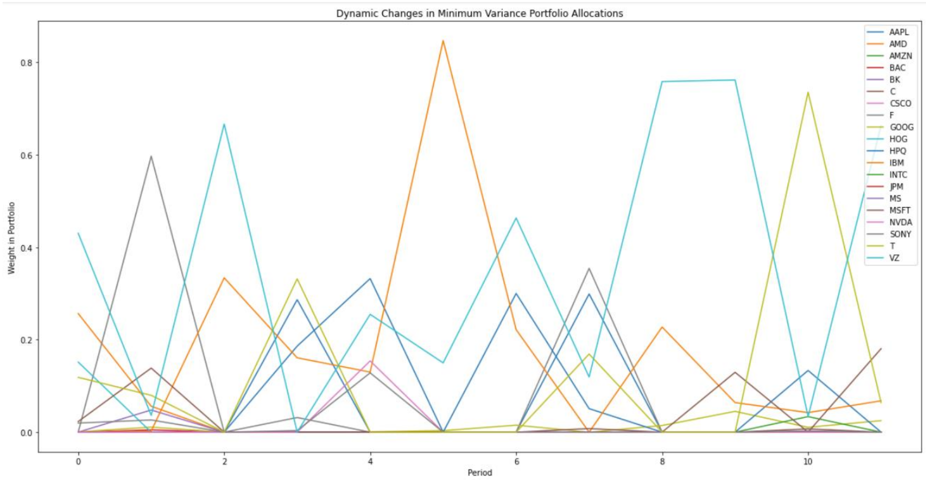
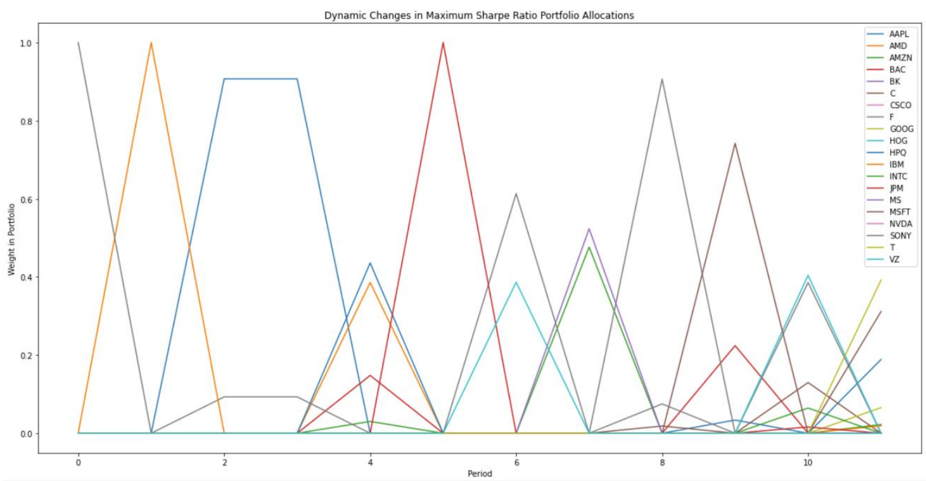
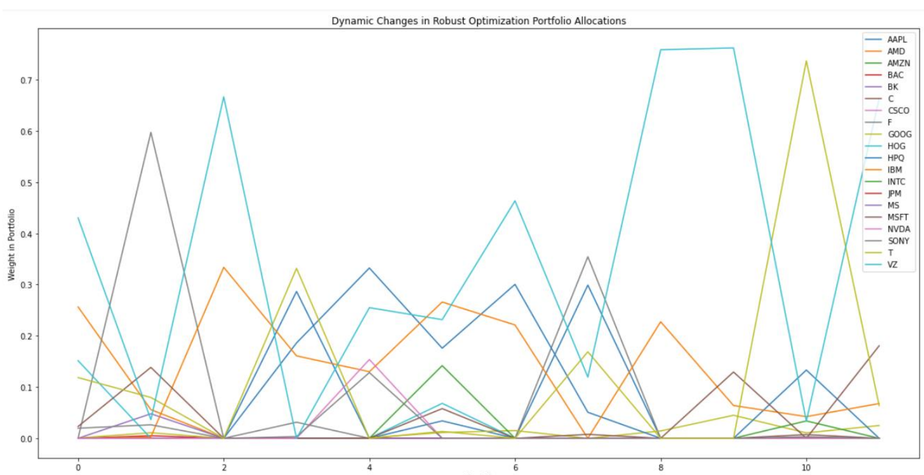

In certain period, the maximum Sharpe Ratio is infeasible due to unable to find a point where capital market line is tangent to the efficient frontier while also keeping the constraints. In these situations, I keep the positions and cash to be same as the last period by adding new code in the function of maximum Sharpe Ratio strategy.


Daily Values of Portfolio
Portfolio Value
Day
Buy and Hold Portfolio
Equally Weighted Portfolio
Minimum Variance Portfolio
Maximum Sharpe Ratio Portfolio
Equal Risk Contributions Portfolio
Leveraged Equal Risk Contributions Portfolio
Robust Optimization Portfolio


Maximum Drawdown of Portfolio
Maximum Drawdown (%)
Period
Buy and Hold
Equally Weighted Portfolio
Mininum Variance Portfolio
Maximum Sharpe Ratio Portfolio
Equal Risk Contributions Portfolio
Leveraged Equal Risk Contributions Portfolio
Robust Optimization Portfolio







The trading amount of maximum Sharpe Ratio portfolio is much more volatile than the other two portfolios. And this volatile trading pattern appears in many stocks in the maximum Sharpe Ratio portfolio. On the other hand, though the target return for the robust optimization portfolio is set up to be the predicted return of minimum variance portfolio, the trading amount does reduce in certain stocks in robust optimization portfolio while comparing to the minimum variance portfolio, such as AMD.

According to the graph of the daily value of portfolios, the returns of the portfolios can be separated into 2 classes. The lower class only has the maximum Sharpe Ratio portfolio with the final values that is around $200,000. The other class has all other portfolios, and their final values are approximately around $350,000. And the leveraged equal risk contributions portfolio has the highest maximum drawdown among all the portfolios in all the periods. Due to its high and stable performance on each period's maximum drawdown and its highest final values, I would like to choose the robust optimization portfolio.