

Newton's laws of motion

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Newton's First and Second laws, in Latin, from the original 1687 edition of the *Principia Mathematica*.

Newton's Laws of Motion are three physical laws which provide relationships between the forces acting on a body and the motion of the body, first formulated by Sir Isaac Newton. Newton's laws were first published in his work [*Philosophiæ Naturalis Principia Mathematica*](#) (1687). The laws form the basis for classical mechanics. Newton used them to explain many results concerning the motion of physical objects. In the third volume of the text, he showed that the laws of motion, combined with his law of universal gravitation, explained Kepler's laws of planetary motion.

The Three Laws of Motion

Newton's Laws of Motion describe only the motion of a body as a whole and are valid only for motions relative to a reference frame. The following are brief modern formulations of Newton's three laws of motion:

First Law

Objects in motion tend to stay in motion, and objects at rest tend to stay at rest unless an outside force acts upon them.

Second law

The rate of change of the momentum of a body is directly proportional to the net force acting on it, and the direction of the change in momentum takes place in the direction of the net force.

Third law

To every action (force applied) there is an equal but opposite reaction (equal force applied in the opposite direction).

It is important to note that these three laws together with his law of gravitation provide a satisfactory basis for the explanation of motion of everyday macroscopic objects under everyday conditions. However, when applied to extremely high speeds or extremely small objects, Newton's laws break down; this was remedied by Albert Einstein's Special Theory of Relativity for high speeds and by quantum mechanics for small objects.



Newton's first law: law of inertia

Lex I: Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

- *"An object at rest will remain at rest unless acted upon by an external and unbalanced force . An object in motion will remain in motion unless acted upon by an external and unbalanced force"*

This law is also called the **law of inertia** or **Galileo's principle**.

The net force on an object is the vector sum of all the forces acting on the object. Newton's first law says that if this sum is zero, the state of motion of the object does not change. Essentially, it makes the following two points:

- An object that is not moving will not move until a force acts upon it.
- An object that is in motion will not change velocity (including stopping) until a force acts upon it.

The first point seems relatively obvious to most people, but the second may take some thinking through, because everyone knows that things don't keep moving forever. If one slides a hockey puck along a table, it doesn't move forever, it slows and eventually comes to a stop. But according to Newton's laws, this is because a force is acting on the hockey puck and, sure enough, there is frictional force between the table and the puck, and that frictional force is in the direction opposite the movement. It's this force which causes the object to slow to a stop. In the absence (or virtual absence) of such a force, as on an air hockey table or ice rink, the puck's motion isn't hindered.

Although the 'Law of Inertia' is commonly attributed to Galileo, Aristotle wrote the first known description of it:

[N]o one could say why a thing once set in motion should stop anywhere; for why should it stop *here* rather than *here*? So that a thing will either be at rest or must be moved *ad infinitum*, unless something more powerful get in its way.

However, a key difference between Galileo's idea from Aristotle's is that Galileo realised that force acting on a body determines *acceleration*, not velocity. This insight leads to Newton's First Law - no force means no acceleration, and hence the body will continue to maintain its velocity.

The 'Law of Inertia' apparently occurred to many different natural philosophers independently, for example in China the inertia of motion appears in the 3rd century BC *Mo Tzu* and René Descartes also formulated the law, although he did not perform any experiments to confirm it.

There are no perfect demonstrations of the law, as friction usually causes a force to act on a moving body, and even in outer space relativistic effects or gravitational forces act, but the law serves to emphasize the elementary causes of changes in an object's state of motion: *forces*.

Newton's second law - historical development

In an exact original 1792 translation (from Latin) Newton's Second Law of Motion reads:

"LAW II: The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed. — If a force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and successively. And this motion (being always directed the same way with the generating force), if the body moved before, is added to or subtracted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both."

Newton here is basically saying that the rate of change in the momentum of an object is directly proportional to the amount of force exerted upon the object. He also states that the change in direction of momentum is determined by the angle from which the force is applied. Interestingly, Newton is restating in his further explanation another prior idea of Galileo, what we call today the Galilean transformation or the addition of velocities.

An interesting fact when studying Newton's Laws of Motion from the Principia is that Newton himself does not explicitly write formulae for his laws which was common in scientific writings of that time period. In fact, it is today commonly added when stating Newton's second law that Newton has said, "and inversely proportional to the mass of the object." This however is not found in Newton's second law as directly translated above. In fact, the idea of mass is not introduced until the third law.

In mathematical terms, the differential equation can be written as:

$$F = k \frac{d(m \cdot v)}{dt}$$

where F is force, m is mass, v is velocity, t is time and k is the constant of proportionality. The product of the mass and velocity is the momentum of the object.

If mass of an object in question is known to be constant and using the definition of acceleration, this differential equation can be rewritten as:

$$F = k \cdot m \cdot a$$

where a is the acceleration.

Using only SI Units for the definition of Newton, the constant of proportionality is unity (1). Hence:

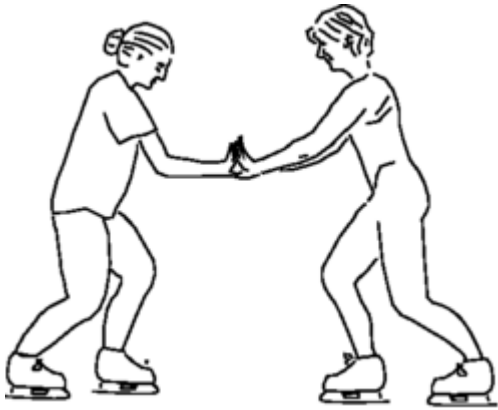
$$F = m a$$

However, it has been a common convention to describe Newton's second law in the mathematical formula $F = ma$ where F is Force, a is acceleration and m is mass. This is actually a combination of laws two and three of Newton expressed in a very useful form. This formula in this form did not even begin to be used until the 18th century, after Newton's death, but it is implicit in his laws.

Newton's Third Law of Motion states: "LAW III: To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts. -- Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tied to a rope, the horse (if I may so say) will be equally drawn back towards the stone: for the distended rope, by the same endeavour to relax or unbend itself, will draw the horse as much towards the stone, as it does the stone towards the horse, and will obstruct the progress of the one as much as it advances that of the other. If a body impinge upon another, and by its force change the motion of the other, that body also (because of the equality of the mutual pressure) will undergo an equal change, in its own motion, toward the contrary part. The changes made by these actions are equal, not in the velocities but in the motions of the bodies; that is to say, if the bodies are not hindered by any other impediments. For, because the motions are equally changed, the changes of the velocities made toward contrary parts are reciprocally proportional to the bodies. This law takes place also in attractions, as will be proved in the next scholium."

The explanation of mass is expressed here for the first time in the words "reciprocally proportional to the bodies" which have now been traditionally added to Law 2 as "inversely proportional to the mass of the object." This is because Newton in his definition 1 had already stated that when he said "body" he meant "mass". Thus we arrive at $F=ma$. When the formula $F=ma$ is taken into account, Law II can be also interpreted as a quantitative restatement of Law I, where mass also acts as a measurement of inertia.

Newton's third law: law of reciprocal actions



Newton's third law. The skaters' forces on each other are equal in magnitude, and in opposite directions

Lex III: Actioni contrariam semper et æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales et in partes contrarias dirigi.

- All forces occur in pairs, and these two forces are equal in magnitude and opposite in direction.

The third law follows mathematically from the law of conservation of momentum.

As shown in the diagram opposite, the skaters' forces on each other are equal in magnitude, and opposite in direction. Although the forces are equal, the accelerations are not: the less massive skater will have a greater acceleration due to Newton's second law. If a basketball hits the ground, the basketball's force on the Earth is the same as Earth's force on the basketball. However, due to the ball's much smaller mass,

Newton's second law predicts that its acceleration will be much greater than that of the Earth. Not only do planets accelerate toward stars, but stars also accelerate toward planets.

The two forces in Newton's third law are of the same type, e.g., if the road exerts a forward frictional force on an accelerating car's tires, then it is also a frictional force that Newton's third law predicts for the tires pushing backward on the road.

Importance and range of validity

Newton's laws were verified by experiment and observation for over 200 years, and they are excellent approximations at the scales and speeds of everyday life. Newton's laws of motion, together with his law of universal gravitation and the mathematical techniques of calculus, provided for the first time a unified quantitative explanation for a wide range of physical phenomena.

In quantum mechanics concepts such as force, momentum, and position are defined by linear operators that operate on the quantum state. At speeds that are much lower than the speed of light, Newton's laws are just as exact for these operators as they are for classical objects. At speeds comparable to the speed of light, the second law holds in the original form $F = dp / dt$, which says that the force is the derivative of the momentum of the object with respect to time, but some of the newer versions of the second law (such as the constant mass approximation above) do not hold at relativistic velocities.

References

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