MEC6602 : Homework 1,
20% of final grade
Due date as per course Syllabus

You will be judged on both content (75%) and form (25%). Content refers to the exercises requested, the precision of the analytical and numerical calculations (25 points), and the quality of the analysis of your results. Form refers to style, clarity of presentation and quality of figures/tables. The detailed evaluation grid is:

Evaluation table

	points
exercises	5
precision	5
analysis	5
form	5

IMPORTANT: no points deducted for 'handwritten' or 'software editor' submissions, as the many equations are difficult and time-consuming to write on a computer! Put your efforts elsewhere!

Wave equation

The 1D wave equation is :

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

1. Perform a theoretical analysis of the accuracy, stability and consistency of the following algorithms:

a. Explicit ‘backward’, and ‘forward’
   Backward :$u_j^{n+1} = u_j^n - \frac{c\Delta t}{\Delta x}(u_j^n - u_{j-1}^n)$
   Forward : $u_j^{n+1} = u_j^n - \frac{c\Delta t}{\Delta x}(u_{j+1}^n - u_j^n)$
b. Forward Time-Centered Space : $u_j^{n+1} = u_j^n - \frac{c\Delta t}{2\Delta x}(u_{j+1}^n - u_{j-1}^n)$
c. Leap-Frog : $u_j^{n+1} = u_j^{n-1} - \frac{c\Delta t}{\Delta x}(u_{j+1}^n - u_{j-1}^n)$
d. Lax-Wendroff: $u_j^{n+1} = u_j^n - \frac{c\Delta t}{2\Delta x}\left(u_{j+1}^n - u_{j-1}^n\right) + \frac{1}{2}\left(\frac{c\Delta t}{\Delta x}\right)^2 (u_{j+1}^n - 2u_j^n + u_{j-1}^n)$
e. Lax: $u_j^{n+1} = (u_{j+1}^n + u_{j-1}^n)/2 - \frac{c\Delta t}{2\Delta x}(u_{j+1}^n - u_{j-1}^n)$
f. Hybride explicit-implicit with $\theta = 0{,}0.5\ (Crank - Nicolson),1.0$

$$u_j^{n+1} + \theta\left(\frac{CFL}{2}\right)(u_{j+1}^{n+1} - u_{j-1}^{n+1}) = u_j^n - (1-\theta)\frac{CFL}{2}(u_{j+1}^n - u_{j-1}^n)$$

g. Your own algorithm with 2^{nd} order space, 4th order time.

2. Write a program in the programming language of your choice to solve the above wave equation on the domain $[0, \pi]$ with these initial conditions :

$$u = 0; x < 0.5 \ and \ x > 1.0 \ ;$$
$$u = 1; 0.5 \leq x \leq 1.0.$$

For each algorithm, run numerical simulations varying the CFL number and report your observations when the mid-wave is at x=2.5. Comment on accuracy and stability.

Note that in a later assignment you will need to invert a block-tridiagonal matrix and solve similar equations. You are therefore advised to choose your programming language carefully and to make your code modular, so you can re-use it!

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