A study of parameters used in Ant Colony Optimization to solve Vehicle Routing Problem with Time Window

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I. INTRODUCTION

Vehicle Routing Problem (VRP) is a combinational optimization problem of which the objective is to find the least cost route (minimum number of vehicles and total distances) under some constraints. A typical VRP can be stated as follows: Each vehicle must depart at and return to depot. Each customer is to be serviced exactly once by only one vehicle and each vehicle has a limited capacity. The customers are placed specific coordinates. The Vehicle Routing Problem with Time Windows (VRPTW) is an extension of the VRP. Each customer has time window individually: each customer is to be serviced between ready time and duetime. If the vehicle reaches a customer before ready time, this vehicle must wait until ready time. Visiting a customer after due time is treated as a infeasible solution.

The VRPTW is NP-Complete and instances with 100 customers or more are very hard to solve optimally. The VRPTW was represented as a multi-objective problem and implemented Ant Colony Optimization (ACO) to solve. A method to evaluate solutions was applied Pareto Ranking which use three informations which are discussed later to evaluate each other [1]. I use a direct interpretation of the VRPTW as a multi-objective problem, in which the three objective dimensions are number of vehicles, total distance (without considering waiting time) and total time (considering waiting time). My experimental results use the standard Solomon's VRPTW benchmark problem instances available at [2]. Solomon's data is clustered into six classes; C1, C2, R1, R2, RC1 and RC2.The purpose of this paper is to consider the effects of each function part of ACO and different parameters.

The rest of this paper is organized as follows. Section II presents a formal definition of the VRPTW. In Section III, gives description on procedure and each formula used in ACO and how the ACO is applied to VRPTW and Pareto Ranking is proposed and described. Numerical results are presented in Section IV. Conclusion is given in Section V.

II. VRPTW

The Vehicle Routing Problem with Time Windows (VRPTW) is an extension of the VRP. There is customer set $C = \{c_0, c_1, c_2, ..., c_n\}$ on the two-dimensional surface represented as x and y coordinates, where c_0 represents the depot and $c_i(i = 1, 2, 3, ..., n)$ represents customer i. Each vehivle has limited capacity Q and customer c_i has own demand q_i, where $q_i < Q$ and $q_0 = 0$. t_{ij} represents distnce between c_i and c_j, where $t_{ij} = t_{ji}$. Acceptable time window $[b_i, e_i]$ is assined customer i, where b_i represents ready time to ride vehicle and e_i represents the latest time to be picked up. Customer i must be picked up by e_i and the vehicle need to wait until b_i if the vehicle arrives at customer i before b_i. Moreover, customer c_i has service time s_i. When a vehicle visit customer c_i, this vehicle must wait there for it's service time. In this paper, two objecttives were picked to be minimize as follows:

- the number of vehicles
- the total distance of all vehicles.

III. MOACO_VRPTW

The principle of ACO algorithms (Dorigo and Gambardella, 1997) is based on the way ants search for foodEach ant consider pheromone trails left by all other ant colony members which preceded its route, the pheromone trail being a signal, a smell left by every ant on its way. This pheromone evaporates with time, and therefore the probabilistic value of each route for each ant changes with time. When ants construct their routes, the path to the food will be characterized by higher pheromone traces and thus all ants will follow the same path. So, ACO can be used to find a solution to the shortest path problem. In VRPTW, a solution is described in terms of paths through depots to customers in accordance with the problems' constraints.

In this algorithm, There are m ants to find better solutions. Each ant $k \in \{1, ..., m\}$ create one feasible solution, starting at the depot and choose next node c_j stochastically from the set N_i^k which contains nodes which do not violate any constrains from node i, where i represents that ant k is in node c_i. Note that N_i^k does not contain the depot. N_i^k is calculated at each node c_i. When N_i^k is empty set, which means that there are not any feasible node c_j from c_i, ank k will go back to the depot and construct new route as next vehicle in same way. This process is repeated until all nodes have been visited and a feasible solution ψ^k is found.

An ant k moves from node c_i to node c_j using heuristic as well as pheromone information. To choose the nexr node c_j from node c_i, the probability between two nodes $p_k(i, j)$ is culculated at each node c_i. In this process, ant k chooses next node c_j using probability q_0 as follows:

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procedure CHOOSE-NEXT-NODE
  if $random <= q_0$ **then**
    Choose the city with larger $p_k(i,j)$
  else $random > q_0$
    Randomly choose c_j using probabilities $p_k(i,j)$
  end if
end procedure

where:

$$\mathrm{p}_k(i,j) = \begin{cases} \dfrac{[\tau(i,j)] \cdot [\eta_L(i,j)]^\beta}{\sum_{u \in N_i^k} [\tau(i,u)] \cdot [\eta_L(i,u)]^\beta} & \text{if } j \in N_i^k \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

β weight the relative importance of the objectives with respect to the pheromone trail, given by τ. The delivery time information, denoted as η_L, which is considered waiting time is calculated as follows:

$$\begin{aligned} ct_j = & \max(now + t_{ij}, b_j) \\ \Delta t_{ij} = & ct_j - now \\ d_{ij} = & \Delta t_{ij}(e_j - now) \\ d_{ij} = & \max(1, d_{ij}) \\ \eta_L(i,j) = & 1/d_{ij} \end{aligned} \quad (2)$$

where now represents the current time at node c_i. Each vehicle begins its trip at a depot c_0 with $now = 0$.

In this paper, Pareto Ranking was picked as the way to evaluate solutions. A solution which was not dominated by other solutions is supposed to be assigned to P which is the Pareto optimal set. Every time where a new feasible solution ψ^k is created by ant k, ψ^k is compared to Pareto optimal solutions in P one after another. If ψ^k dominates some Pareto optimal solutions, ψ^k is included P and dominated solutions in P are erased. Hereinafter the Pareto optimal solution is referred to as ψ^P.

In this ACO, two methods of updating τ were conducted. The first method, which is called local update of pheromone, is calculated as follows:

$$\tau(i,j) = (1-\rho)\tau(i,j) + \rho\tau_0 \qquad ((i,j) \in \psi^k) \quad (3)$$

where ρ determins how much the pheromone put on the edge (i,j) will evaporate and ant k put on the edge. τ_0 is initially calculated as follows:

$$\tau_0 = 1/(n \cdot L_\psi^h) \quad (4)$$

where n and L_ψ^h represents the number of nodes (depots + customers) and total distance of all vehicles which are gained through Nearest Neighbor (NN) heuristic. This value is fixed not to construct the same route as the route constructed before by other ants. The local update of pheromone is conducted every time ant k decides next node j. While another method, which is called global update of pheromone, is calculated as follows:

$$\tau(i,j) = \begin{cases} (1-\alpha)\tau(i,j) + \alpha/\overline{L^P} & \text{if } (i,j) \in \psi^P \\ (1-\alpha)\tau(i,j) & \text{otherwise} \end{cases} \quad (5)$$

where $\overline{L^P}$ represents the average distance of Pareto optimal solutions and α works in same fanction as ρ in formula (3). The global update of pheromone is conducted to put strong pheromone on the best known route so far after all ants finish constructing the route.

The pseudocode for the new proposal denoted as MOACO-VRPTW (Multi-Objective Ant Colony Optimization for the Vehicle Routing Problem with Time Windows) follows:

procedure MOACO-VRPTW
  /* Initialization */
  $\psi^h \leftarrow$ feasible initial solution using NN
  $\tau_0 \leftarrow 1/(n \cdot L_\psi^h)$
  repeat /* Main Loop */
    for each ant $k \in \{1, \cdots, m\}$ **do**
      /* Construct a solution (ψ^k) */
      $\psi^k \leftarrow build\_tour(k)$
      if $\psi^k \in P$ **then**
        Save ψ^k and
        erase dominated solution from P
      end if
    end for
    if New Pareto optimal solutions aren't found **then**
      Perform global updating using equation (5)
    end if
  until A stopping criterion is met
end procedure

procedure $build\_tour(k)$
  /* Initialization */
  Put ant k at depot c_0
  $\psi^k = \phi$
  $\psi^k \leftarrow \psi^k \cup c_0,\ now \leftarrow 0,\ load_k \leftarrow 0$
  /* The ant builds its tour. Tour is stored in ψ^k */
  repeat
    /* Starting from node i compute the set N_i^k
    of feasible nodes*/
    Compute N_i^k
    if $N_i^k = \{\}$ /* no feasible nodes */ **then**
      $now \leftarrow 0$
      $load_k \leftarrow 0$
      $\psi^k \leftarrow \psi^k \cup c_0$
    else
      /* $\forall j \in N_i^k$ compute visibility */
      for every $j \in N_i^k$ **do**
        Compute $\eta_L(i,j)$ using equation (2)
      end for
      $c_j \leftarrow$ Choose-Next-Node
      $\psi^k \leftarrow \psi^k \cup c_j$
      $now \leftarrow ct_j + s_j$
      $load_k \leftarrow load_k + q_j$
      Perform local updating using equation (3)
      $i \leftarrow j$
    end if
  until a stopping criterion is met

end procedure

IV. RESULT

A. Comparison with previous studies

In this paper, to compare the quality of MOACO_VRPTW, solved six types of benchmards using same parameters used in [3]. According to [3], the best values of parameters were found as follows: $m = 100,\ \beta = 1.0,\ \rho = 0.15,\ \alpha = 0.15,\ q_0 = 0.85$ with the number of $\text{cycles(Iterations)} = 300$. The values in all of tables I - VI were studied on each of the six data sets by running 10 computer simulations. Tours and Length represent the number of vehicles and the total distance at one solution. BestKnown and Precedent in TABLE I represent the value known as the best value so far in [2] and the average values of 10 runs using [3]'s method. The difference between Precedent's method and MOACO_VRPTW is the way of evaluation, particularly Pareto ranking method used in MOACO_VRPTW. Therefor, if Pareto optimal set P has some solutions, the solution which has smaller number of vehicle than the other was picked as values of MOACO_VRPTW in TABLE I. As can be seen from TABLE I, most of values in MOACO_VRPTW are superior to values in Precedent, which can be considered that Pareto ranking method makes solutions to converge on better values.

TABLE I
EACH SOLUTIONS VALUE

	BestKnown		Precedent		MOACO_VRPTW	
	Tours	Length	Tours	Length	Tours	Length
C109	10.0	828.94	10.0	953.11	10.0	965.34
C208	3.0	588.32	3.0	727.63	3.0	601.69
R112	9.0	982.14	13.17	1434.8	11.0	1238.57
R211	2.0	885.71	3.18	1333.5	3.0	1199.78
RC108	10.0	1139.82	13.00	1593.0	11.50	1421.54
RC208	3.0	828.14	3.38	1560.8	3.0	1303.57

B. Study about parameters

In this study, those parameters which were used in IV-A was used as default setting. In order to find better parameter set than default setting, each value was changed one after another. Firstly, the number of ant was changed 10 to 100 with every 10 interval, which is shown in TABLE , and $m = 40$ was decided as better value by the rule as follows: the number of vehicles and total distance of each parameter set was renked each other and the parameter set has the minimum value which is sum of each ranking value is determined as a better parameter set. TABLE III - VI show results and $\beta = 2.5,\ q_0 = 0.85,\ \alpha = 0.2$ and $\rho = 0.2$ were decided as better parameters in the same way mentioned above.

V. CONCLUSION

This paper represents a multi-objective ant colony optimization approach to the vehicle routing problem with time windows. In this MOACO, the Pareto ranking method was applied. Therefor, the pheromone trail could be able to take not only global best solution but Pareto optimal set into

TABLE II
THE AVERAGE VALUES OF 10 SOLUTIONS OF EACH PARAMETER SET, PARTICULARLY m WAS CHANGED

	$m = 10$		$m = 20$		$m = 30$		$m = 40$	
	Tours	Length	Tours	Length	Tours	Length	Tours	Length
C109	10.0	1088.66	10.0	994.06	10.0	906.44	10.0	940.5
C208	3.0	663.05	3.0	618.33	3.0	603.93	3.0	601.92
R112	11.3	1305.37	11.0	1238.73	11.0	1231.19	11.0	1208.67
R211	3.0	1276.46	3.0	1145.97	3.0	1140.5	3.0	1161.22
RC108	12.3	1544.68	11.8	1466.69	11.6	1436.86	11.5	1435.65
RC208	3.0	1377.53	3.0	1242.71	3.0	1231.21	3.0	1263.34
	$m = 50$		$m = 60$		$m = 70$		$m = 80$	
	Tours	Length	Tours	Length	Tours	Length	Tours	Length
C109	10.0	946.97	10.0	947.9	10.0	959.96	10.0	954.4
C208	3.0	602.01	3.0	602.39	3.0	602.22	3.0	603.46
R112	11.0	1231.19	11.0	1251.4	10.9	1234.09	11.0	1239.9
R211	3.0	1184.2	3.0	1186.47	3.0	1185.36	3.0	1191.52
RC108	11.7	1422.08	11.9	1436.37	11.6	1420.77	11.7	1424.7
RC208	3.0	1289.09	3.0	1281.55	3.0	1281.95	3.0	1288.59
	$m = 90$		$m = 100$					
	Tours	Length	Tours	Length				
C109	10.0	955.24	10.0	956.79				
C208	3.0	602.46	3.0	604.21				
R112	11.0	1238.62	11.0	1244.34				
R211	3.0	1189.39	3.0	1193.58				
RC108	11.5	1432.04	11.3	1383.53				
RC208	3.0	1287.99	3.0	1309.38				

TABLE III
THE AVERAGE VALUES OF 10 SOLUTIONS OF EACH PARAMETER SET, PARTICULARLY β WAS CHANGED

	$\beta = 1.5$		$\beta = 2.0$		$\beta = 2.5$		$\beta = 3.0$	
	Tours	Length	Tours	Length	Tours	Length	Tours	Length
C109	10.0	935.3	10.0	902.95	10.0	905.5	10.0	895.64
C208	3.0	601.8	3.0	601.88	3.0	603.08	3.0	601.92
R112	11.0	1253.84	10.4	1166.27	10.4	1143.96	10.2	1148.23
R211	3.0	1145.92	3.0	1088.6	3.0	1074.38	3.0	1057.18
RC108	11.8	1426.1	11.1	1345.87	11.1	1331.19	11.0	1308.71
RC208	3.0	1259.32	3.0	1176.02	3.0	1164.44	3.0	1173.58

TABLE IV
THE AVERAGE VALUES OF 10 SOLUTIONS OF EACH PARAMETER SET, PARTICULARLY q_0 WAS CHANGED

	$q_0 = 0.8$		$q_0 = 0.85$		$q_0 = 0.9$		$q_0 = 0.95$	
	Tours	Length	Tours	Length	Tours	Length	Tours	Length
C109	10.0	913.02	10.0	895.64	10.0	937.1	10.0	967.15
C208	3.0	602.26	3.0	601.92	3.0	603.49	3.0	604.52
R112	10.0	1128.24	10.2	1148.23	10.1	1136.76	10.4	1154.14
R211	3.0	1070.04	3.0	1057.18	3.0	1057.79	3.0	1072.83
RC108	11.0	1294.43	11.0	1308.71	11.0	1292.06	11.0	1293.75
RC208	3.0	1177.71	3.0	1173.58	3.0	1174.05	3.0	1186.81

TABLE V
THE AVERAGE VALUES OF 10 SOLUTIONS OF EACH PARAMETER SET, PARTICULARLY α WAS CHANGED

	$\alpha = 0.1$		$\alpha = 0.15$		$\alpha = 0.2$		$\alpha = 0.25$	
	Tours	Length	Tours	Length	Tours	Length	Tours	Length
C109	10.0	917.78	10.0	895.64	10.0	908.39	10.0	911.66
C208	3.0	601.9	3.0	601.92	3.0	604.34	3.0	604.44
R112	10.6	1175.72	10.2	1148.23	10.0	1136.81	10.3	1146.8
R211	3.0	1076.21	3.0	1057.18	3.0	1046.88	3.0	1049.85
RC108	11.0	1302.57	11.0	1308.71	11.0	1288.9	11.0	1314.96
RC208	3.0	1180.11	3.0	1173.58	3.0	1154.45	3.0	1154.19

TABLE VI
THE AVERAGE VALUES OF 10 SOLUTIONS OF EACH PARAMETER SET, PARTICULARLY ρ WAS CHANGED

	$\rho = 0.1$		$\rho = 0.15$		$\rho = 0.2$		$\rho = 0.25$	
	Tours	Length	Tours	Length	Tours	Length	Tours	Length
C109	10.0	912.8	10.0	933.27	10.0	897.89	10.0	911.05
C208	3.0	604.69	3.0	603.46	3.0	602.37	3.0	601.85
R112	10.2	1131.64	10.2	1155.49	10.2	1138.79	10.3	1145.34
R211	3.0	1074.5	3.0	1039.64	3.0	1042.12	3.0	1063.54
RC108	11.0	1311.82	11.0	1315.53	11.0	1295.55	11.0	1298.17
RC208	3.0	1171.38	3.0	1150.61	3.0	1148.07	3.0	1150.16

account. The solution quality of this MOACO_VRPTW was superior to simple MOACO. Moreover, the better parameter set was decided in IV-B under ranking rule menthioned earlier. This parameter tuning was note conducted with continuous value. Hence, This MOACO_VRPTW did not perform the best quality. Further study about parameter tuning is needed in order to find the best parameter set in this study.

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