Confidence Intervals

Ken Mwai

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Confidence intervals

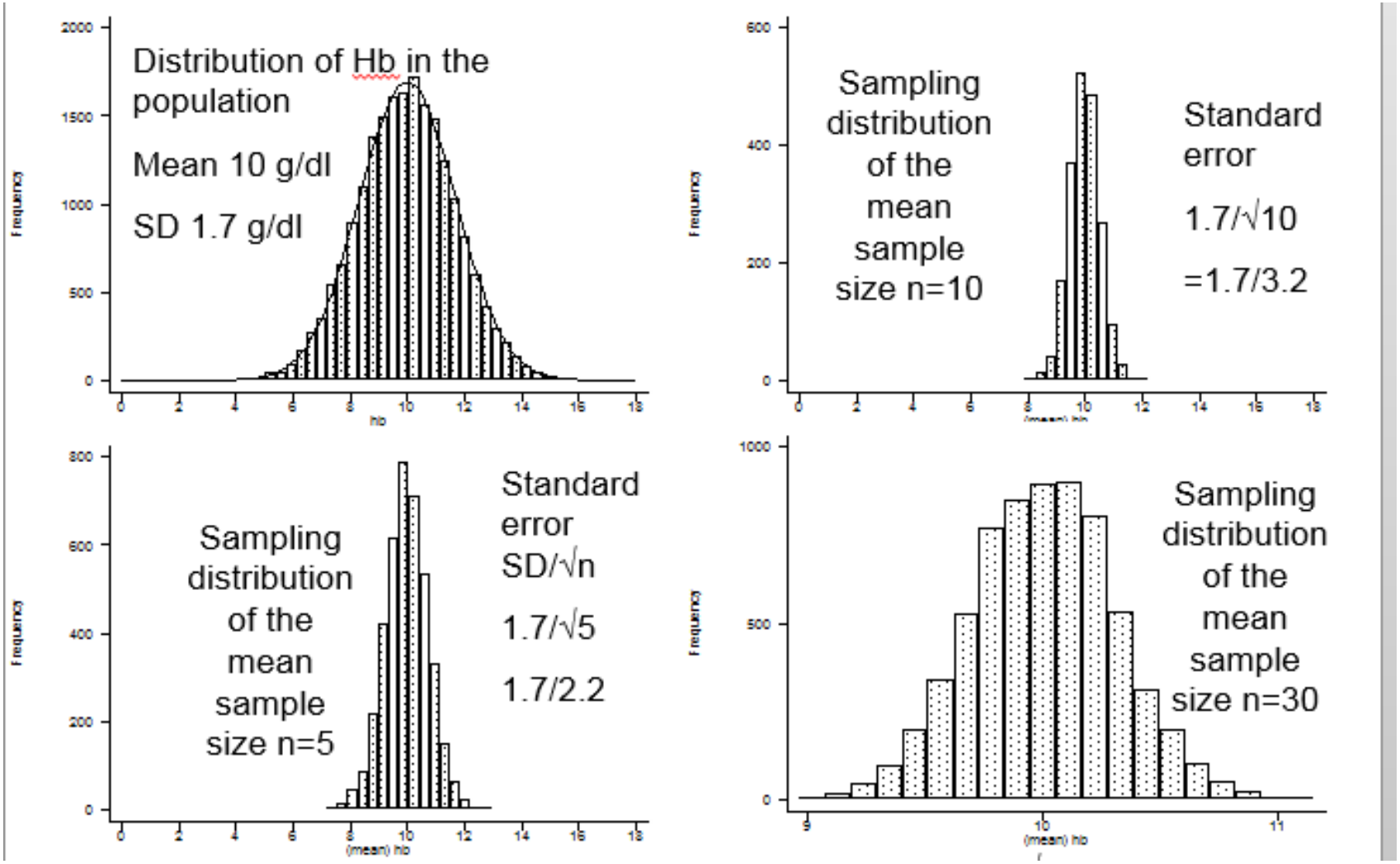
Point Estimation:

1. Provides a single value -Based on observations from one sample
2. Gives no information about how close the value is to the unknown population parameter
3. Example: Sample mean $x = 3$ is point estimate of unknown population mean

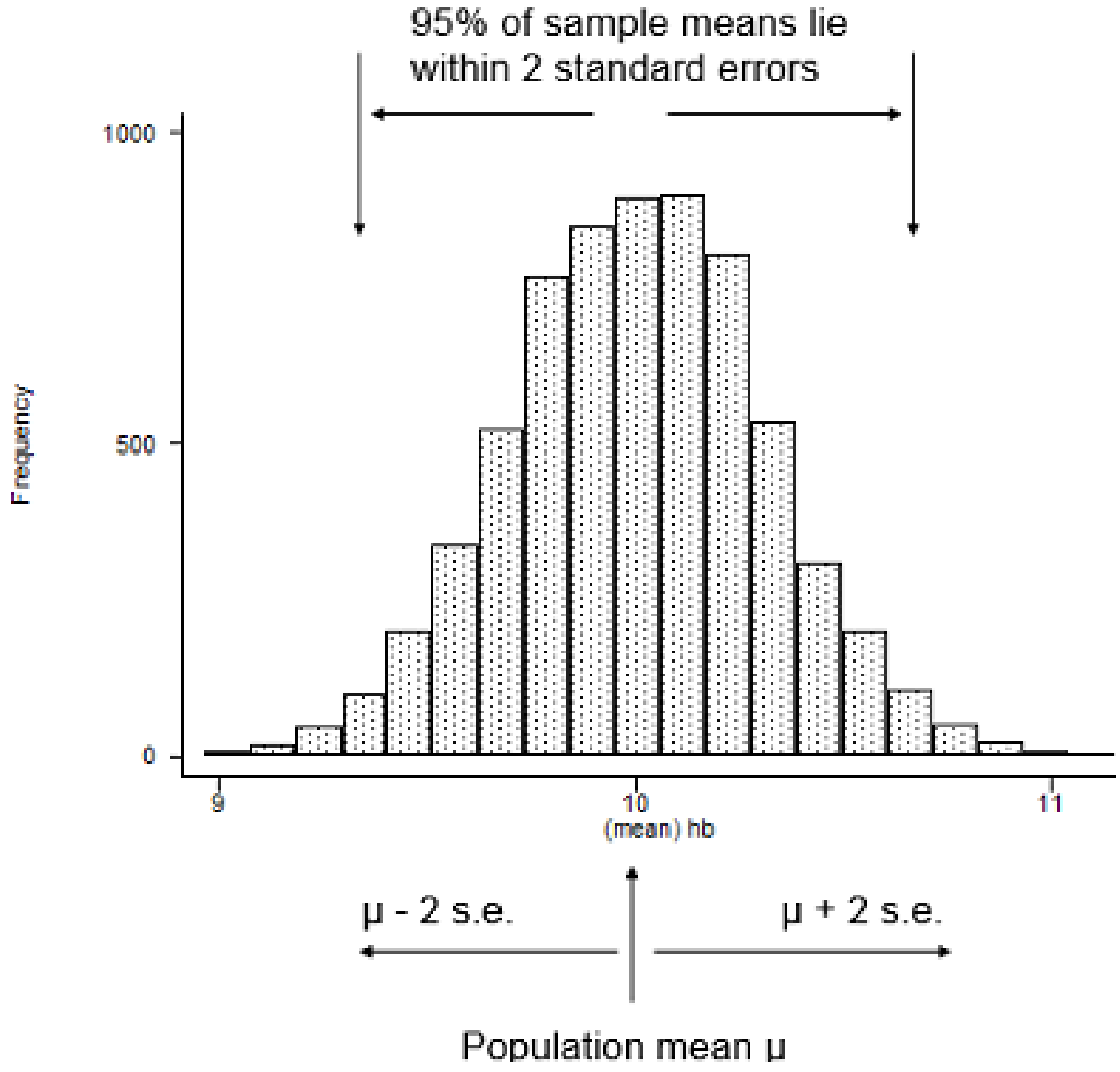
Confidence intervals

Interval estimation:

1. Provides a range of values -Based on observations from one sample
2. Gives information about closeness to unknown population parameter
   - Stated in terms of probability -Knowing exact closeness requires knowing unknown population parameter
3. Example: The range between 50 and 70 contains the true unknown parameter value with 95% confidence

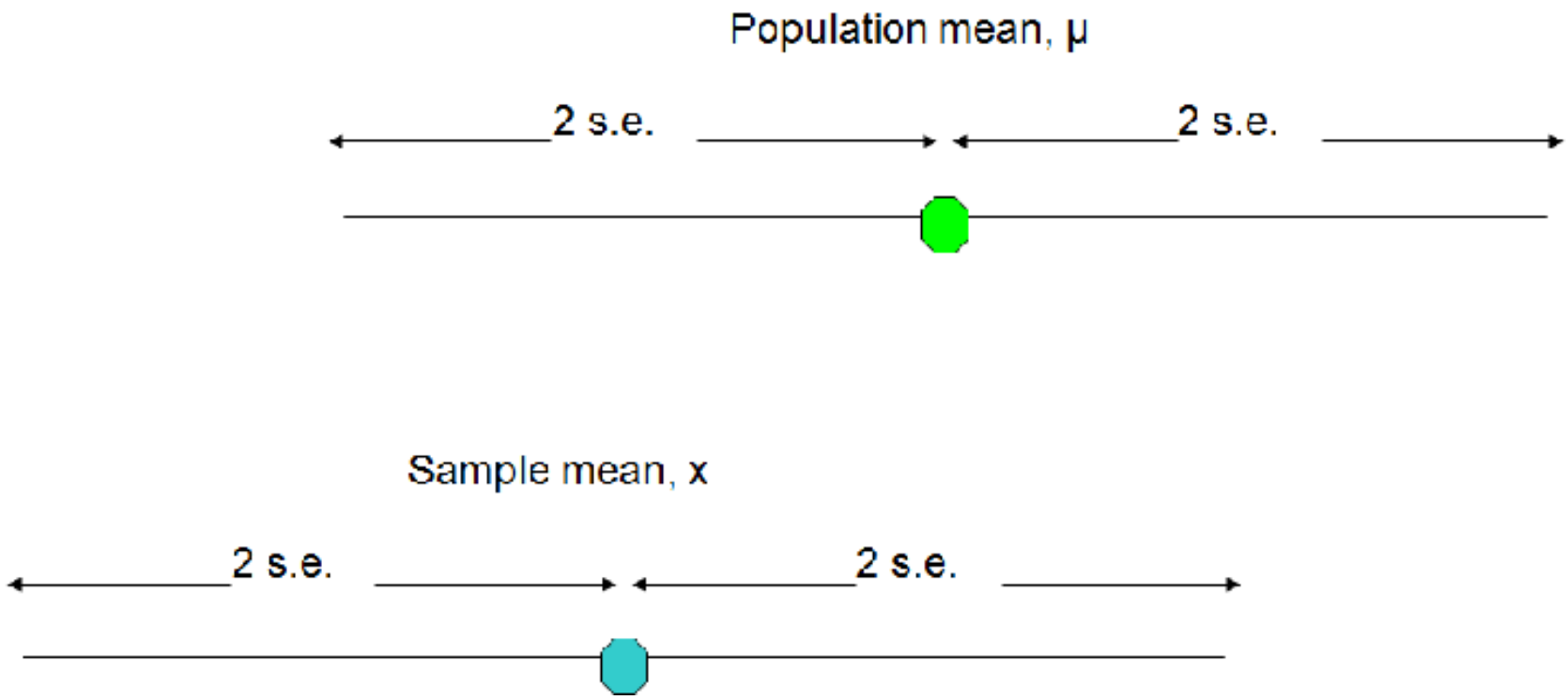


- Bigger sample size, less variability in sample means: $s.e. = SD\sqrt{n}$
- Average value of sample means is equal to the population mean
- Sample means follow normal distribution if the distribution in the population is normal


95% of sample means lie
within 2 standard errors
1000
500
0
Frequency
9
10
11
(mean) hb
μ - 2 s.e.
μ + 2 s.e.
Population mean μ

- If we repeated our sampling many times, 95% of sample means would be within 2 standard errors of the population mean.
- In practice we usually have only one sample but we can use the sample data to quantify the uncertainty in our single estimate

95% of sample means lie within 2 s.e. of the population mean so we can also say: 95% of the time the population mean will lie within 2 s.e. of the our sample mean



Confidence interval on μ (α known)

$$P(-1.96 <= \frac{P(\bar{X} - \mu)}{\frac{\alpha}{\sqrt{n}}}) = 0.95$$

$$P(\bar{X} - 1.96\frac{\alpha}{\sqrt{n}} <= \mu <= \bar{X} + 1.96\frac{\alpha}{\sqrt{n}}) = 0.95$$

$$(\bar{X} - 1.96\frac{\alpha}{\sqrt{n}}, \bar{X} + 1.96\frac{\alpha}{\sqrt{n}})$$

is a 95% confidence interval for μ

A survey of haemoglobin status in children $<$ 5yrs in Kilifi district. 30 children gave a finger prick blood sample and the mean Hb was 9.6g/dl with a standard deviation of 1.5 g/dl.

- Sample estimate of the SD: 1.5 g/dl
- Standard error $s.e. = SD\sqrt{n}$

```
se <- 1.55/sqrt(30)
lower_ci <- 9.6-(2*se)
upper_ci <- 9.6+(2*se)
```

- Interpretation: we can say with 95% confidence that the mean Haemoglobin concentration in children in the population could be as small as 9.03402 or as big as 10.16598 g/dl

Slide with R Code and Output

```
summary(cars)
```

```
##      speed           dist       
##  Min.   : 4.0   Min.   :  2.00  
##  1st Qu.:12.0   1st Qu.: 26.00  
##  Median :15.0   Median : 36.00  
##  Mean   :15.4   Mean   : 42.98  
##  3rd Qu.:19.0   3rd Qu.: 56.00  
##  Max.   :25.0   Max.   :120.00
```

Slide with Plot