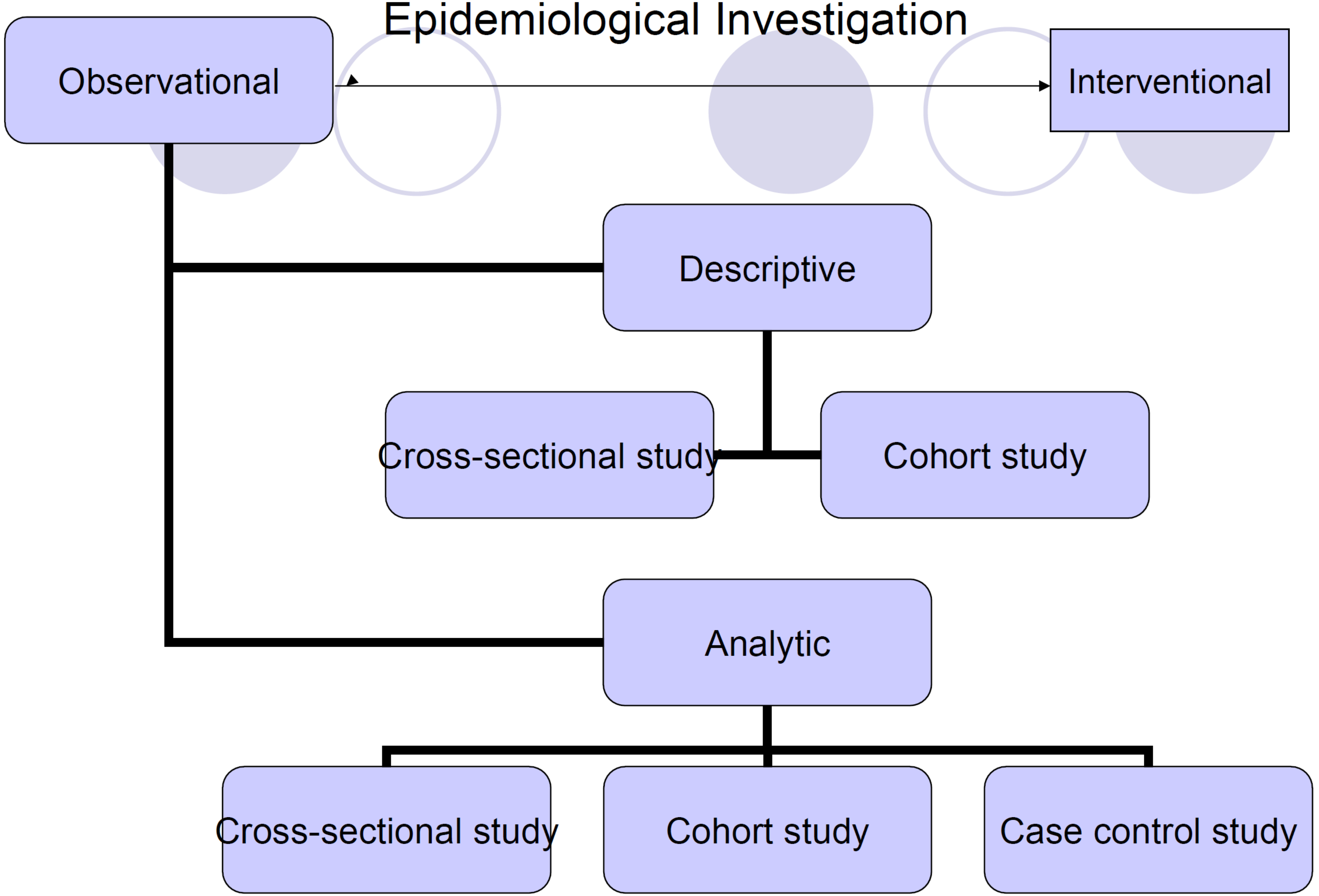
Measures of association

MWAI

Epidemiological Investigation



2X2 or contingency table

Exposure	**Outcome**		**Total**
	Yes	No	
Yes	a	b	a+b
No	c	d	c+d
Total	a+c	b+d	a+b+c+d

Presentation of data in Case control or Cohort studies with count denominators

2x2 table

- Contains two rows and two columns
- Each represents the presence or absence of exposure or disease.
  - ***a= number of individuals who are exposed and have outcome***
  - ***b= number who are exposed and do not have outcome***
  - ***c=number who are not exposed and have outcome***
  - ***d= number who are not exposed and do not have outcome***
- The margin represent total number in each row and column calculated by adding relevant cells

2x2 table

a+b	the total number of individuals exposed
c+d	the total number of individuals not exposed
a+c	the total number of individuals with outcome
b+d	the total number of individuals without outcome

The sum of all four cells a+b+c+d is the total sample size, represent T or N

Measures of Association

- Quantifies the relationship between exposure and outcome among two groups
  - ODDS Ratio
  - Risk Ratio
- Ratio - ***how much more one group is to develop a disease than the another***

Important Jargon

- **Exposure (E)** ≡ an explanatory factor; the independent variable i.e. any potential health determinant;
- **Outcome (D)** ≡ the response; the dependent variable i.e. any health-related outcome;
- **Measure of association (syn. measure of effect)** ≡ a statistic that quantifies the relationship between an exposure and a disease
- **Measure of potential impact** ≡ a statistic that quantifies the potential impact of removing a hazardous exposure

Relative Risk/Risk Ratio

- Relative Risk/Risk Ratio (RR) →relative effect associated with exposure or the "risk"
  - Cohort studies

$$RR = \frac{R_1}{R_0}$$

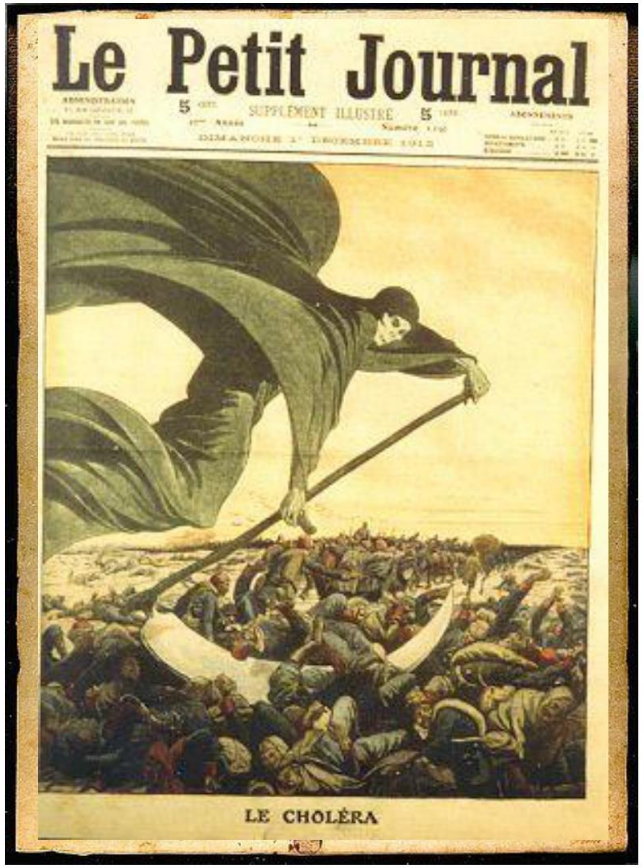
where

$R_1 \equiv$ risk in the exposed group
$R_0 \equiv$ risk in the non-exposed group

Example

Fitness & Mortality (Blair et al., 1995)

- Is improved fitness associated with decreased mortality?
- Exposure ≡ improved fitness (1 = yes, 0 = no)
- Disease ≡ death (1 = yes, 0 = no)
- Mortality rate, group 1: $R_1 = 67.7$
- Mortality rate, group 0: $R_0 = 122.0$



RR?

$$RR = \frac{R_1}{R_0}$$

where

$R_1 \equiv$ risk in the exposed group
$R_0 \equiv$ risk in the non-exposed group

Example
Relative Risk

What is the effect of improved fitness on mortality in relative terms?

$$RR = \frac{R_1}{R_0} = \frac{67.7}{122.0} = 0.55$$

The effect of the exposure is to cut the risk *almost* in half.

2-by-2 Table Format

	Outcome +	Outcome −	Total
Exposure +	A_1	B_1	N_1
Exposure –	A_0	B_0	N_0
Total	M_1	M_0	N

$$\boxed{R_1 = \frac{A_1}{N_1}}$$

$$\boxed{R_0 = \frac{A_0}{N_0}}$$

Food borne Outbreak Example

Exposure ≡ eating a particular dish

Outcome ≡ gastroenteritis

	Outcome +	Outcome −	Total	
Exposure +	63	25	88	R_E
Exposure –	1	6	7	
Total	64	31	95	$R_{N\bar{E}}$

Food borne Outbreak Example

Exposure ≡ eating a particular dish

Outcome ≡ gastroenteritis

	Outcome +	Outcome −	Total
Exposure +	63	25	88
Exposure –	1	6	7
Total	64	31	95

$$R_1 = \frac{A_1}{N_1} = \frac{63}{88} = 0.7159$$

$$R_0 = \frac{A_0}{N_0} = \frac{1}{7} = 0.1429$$

Food borne Outbreak Data

	Outcome+	Outcome −	Total
Exposure +	63	25	88
Exposure –	1	6	7
Total	64	31	95

$$RR = \frac{R_1}{R_0} = \frac{63/88}{1/7} = \frac{0.7159}{0.1429} = 5.01$$

Exposed group had 5 times the risk

ODD Ratio

- An *odds ratio* (OR) → is a measure of association between an exposure and an outcome.
- The OR represents the *odds* that an outcome will occur given a particular exposure, compared to the *odds* of the outcome occurring in the absence of that exposure.
  - Case control studies

$$\text{ODDS} = \frac{\text{Number with Outcome (D+)}}{\text{NumberwithoutOutcome(D-)}}$$

$$\text{ODDS RATIO} = \frac{\text{odds in exposed} \left(\dfrac{A_1}{B_1}\right)}{\text{odds in non exposed} \left(\dfrac{A_0}{B_0}\right)}$$

	D+	D−	Total
E+	A_1	B_1	N_1
E−	A_0	B_0	N_0
Total	M_1	M_0	N

$$OR = \frac{A_1 / B_1}{A_0 / B_0} = \frac{A_1 B_0}{B_1 A_0}$$

“Cross-product ratio”

Odds Ratio, Example

Milunsky et al, 1989, Table 4

NTD = Neural Tube Defect

	NTD+	NTD−
Folic Acid+	10	10,703
Folic Acid−	39	11,905

$$OR = \frac{A_1 B_0}{B_1 A_0} = \frac{10 \cdot 11{,}905}{10{,}703 \cdot 39} = 0.29$$

~~Exposed group had 0.29 times (*about* a quarter) the risk of the non-exposed group~~

What if?

- Want to deal with confounding

What if?

- Want to deal with confounding during analysis
  - Regression (only works if you can identify and measure the confounders)
  - Stratify
  - adjustment (usually distorted by choice of standard)

Logistic regression

- **Logistic regression** - a regression modelling technique for producing Odds Ratios (ORs); models the log odds of a binary "outcome"
  - *Effect of T.B infection on death in HIV positive patients crude(unadjusted) OR; 95% CI and hypothesis tests*
  - *Effect of mothers education on childs' measles immunisation status*
  - *Effect of ethnicity on risk of death from breast cancer*
  - *Effect of gender on being a high wage earner*

Why model log odds?

- The reason for modelling the log odds rather than risk or odds is that the log odds can take any value, positive or negative, whereas risks are constrained to lie between 0 and 1.

- When using statistical models it is easier to model a quantity which is unconstrained than one which is constrained.

- This avoids the possibility of predicting impossible values (like risks which are negative or greater than 1) from the model.

- Modelling log odds is referred to as **logistic regression**, and the models arereferred to as **logistic models**.

Example

Calculate the prevalence, odds and log odds of Microfilariae infection in the forest and savannah areas

Microfilariae Infection	Savannah	Forest	Total
Positive	267	213	480
Negative	281	541	822
Total	548	754	1302

	savannah	forest	overall
Risk/prevalence			63.1
Odds			1.712
Log odds			0.538

What are the odds ratio and log odds ratio?

	savannah	forest	overall
Risk/prevalence(%)	51.3	71.8	63.1
Odds	1.052	2.540	1.712
Log odds	0.052	0.932	0.538

The Odds ratio $= 2.41$ Whilst the log odds ratio? $= 0.881$

area	**odds**	**log odds of disease**
0=savannah	1.052	0.051
1=forest	$1.052 \times 2.41 = 2.536$	$0.051 + 0.881 = 0.932$

Summarise results in a model

log odds = Baseline + Area

where Baseline = log (odds in savannah) = $(0.051 + 0.881 \times 0) = 0.051$

Area = log odds for individuals in the forest and 0 individuals in the savannah

= $(0.051 + 0.881 \times 1) = 0.932$

Example in R - https://bit.ly/2OTXImY

- **Calculate the prevalence, odds and log odds of Microfilariae infection in the forest and savannah areas**

- onch <-**read.csv**("onchall.csv") *# Read in CSV data*
- **m1 <- glm(mf~area, data=onch,family=binomial)*# Run model***
- **summary**(m1)