Non Linear Models

Bayesian approach

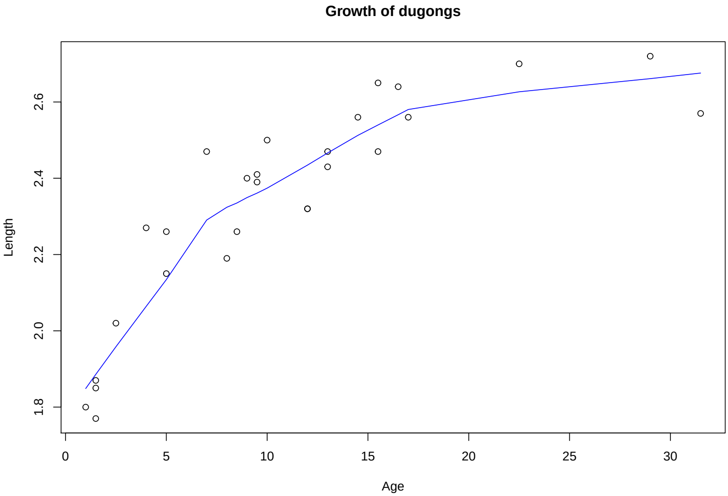
Friday, April 7, 2017

Non linear vs Linear approach

- the linear mean structure is: $Y_i = x_i' \beta + \epsilon_i$
  - the generic form: $Y_i = g(x_i, \beta) + \epsilon_i$ for a known function g
- **Lets consider a non linear mean structure**
  - The idea is to model non transformed data

Non transformed data

- The data are length and age measurements for 27 captured dugongs (sea cows).
    - **Carlin and Gelfand (1991)** model this data using a nonlinear growth


Growth of dugongs
Length
Age
1.8
2.0
2.2
2.4
2.6
0
5
10
15
20
25
30

Non-linear Dugong growth model

$$Y_i = \alpha - \beta * \gamma^{x_i} + \epsilon_i, i = 1, 2, ..., n$$

- Where $\alpha > 0$, $\beta > 0$, $0 \leq \gamma \leq 1$ and as usual $\epsilon_i \sim N(0, \sigma^2)$
- α corresponds to the average length of a fully grown dugong
- $\alpha - \beta$ length of a dugong at birth and γ determines the growth rate

Sampling approach: why?

- The nonlinearity of the model eliminates any hope for a closed form full conditional for γ
- **Sampling** is the best approach - types of sampling?

get data and code here?

https://goo.gl/d5pbBo

OpenBugs Model ..

- We run three parallel Gibbs sampling chains of 20,000 iterations each following a 1000-iteration burn-in
- Obtain posterior density estimates and autocorrelation plots for $\alpha > 0$, $\beta > 0$, γ and σ
- **Investigate the bivariate posterior of (α, γ) using the Correlation tool on the inference menu**

Winbugs Code

```
model
    {
        for( i in 1 : N ) {
            Y[i] ~ dnorm(mu[i], tau)
            mu[i] <- alpha - beta * pow(gamma,x[i])
        }
        alpha ~ dflat()
        beta ~ dflat()
        gamma ~ dunif(0.5, 1.0)
  U1 <- log(alpha);
  U2 <- log(beta);
  U3 <- logit(gamma);
  tau <- 1/(sigma*sigma)
  sigma ~ dunif(0.01, 100)
    }
```

```
#data
list(
x = c( 1.0,  1.5,  1.5,  1.5, 2.5,   4.0,  5.0,  5.0,  7.0,
    8.0,  8.5,  9.0,  9.5, 9.5,  10.0, 12.0, 12.0, 13.0,
13.0, 14.5, 15.5, 15.5, 16.5, 17.0, 22.5, 29.0, 31.5),
Y = c(1.80, 1.85, 1.87, 1.77, 2.02, 2.27, 2.15, 2.26, 2.47,
    2.19, 2.26, 2.40, 2.39, 2.41, 2.50, 2.32, 2.32, 2.43,
2.47, 2.56, 2.65, 2.47, 2.64, 2.56, 2.70, 2.72, 2.57), N = 27)
#initial values
list(alpha = 1, beta = 1, sigma = 1, gamma = 0.9)
list(alpha = 10, beta = 10, sigma = 10, gamma = 0.7)
list(alpha = 100, beta = 100, sigma = 100, gamma = 0.5)
```

Stata Implementation

```
insheet using "Dugongs.csv" , clear
*****model1
bayesmh length ,dots(1000) ///
rseed(12345) saving(nl_growth_model1, replace) ///
mcmcsize(25000) burnin(5000) thinning(9) ///
likelihood(normal({var})) ///
prior({length:_cons}, density({alpha}-{beta}*{gamma}^age)) ///
prior({alpha}, flat)  ///
prior({beta}, flat) ///
prior({gamma}, uniform(0.5,1.0)) ///
block({beta} {alpha} {gamma} ) blocksummary ///
prior({var},igamma(0.001, 0.001))
***diagnostics graph
bayesstats ess _all
bayesgraph diagnostics _all
bayesgraph matrix _all
```

```
****model2 - with more ierations compare with the bugs output
bayesmh length ,dots(1000) ///
rseed(2468) saving(nl_growth_model2, replace) ///
mcmcsize(100000) burnin(5000) thinning(9) ///
likelihood(normal({var})) ///
prior({length:_cons}, density({alpha}-{beta}*{gamma}^age)) //
prior({alpha}, normal(0,1000))  ///
prior({beta},igamma(0.001,0.001)) ///
prior({gamma}, beta(0.5,1.0)) ///
block({beta} {alpha} {gamma} ) blocksummary ///
prior({var},igamma(0.001, 0.001))
**daignostics graphs
bayesstats ess _all
bayesgraph diagnostics _all
bayesgraph matrix _all
```

Some intro

https://youtu.be/3OJEae7Qb_o?list=PLTn3eOV1DiQi8OT3K7vrB_7cXYaLNb-Y-