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Novel Adaptive UKF for Tightly-coupled INS/GPS Integration with Experimental Validation on an UAV

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Abstract—A Novel Adaptive Unscented Kalman Filter namely (NAUKF) has been developed and applied to fuse the outputs of strap-down IMU, the measurements of GPS satellites (pseudo-range and Doppler), strap-down magnetometer and a barometric altimeter, using tight coupling architecture. The proposed filter NAUKF considers the residual unmodeled noises of process and measurement as *non-zero* mean Gaussian white noises, estimates and compensates for the mean (bias) and covariance of the noise online, based on the principles of adaptive filtering, even if they are time-varying quantities. Employing the adaptive filtering principle into UKF, the nonlinearity of system can be restrained and the NAUKF is obtained. The noise statistic estimators designed in the proposed algorithm are built on basis of forgetting factors which are usually calculated empirically, in this research a new calculation concept based on "Genetic Algorithm" as an optimization tool is utilized to determine the optimal values of forgetting factors. The performance and convergence of noise statistic estimators used in NAUKF are checked by Monte-Carlo simulation. The utilization of NAUKF for Tightly-Coupled INS/GPS Integration (TCI) showed a superiority against the UKF for TCI especially during the GPS outages. The comparison is done experimentally by means of real flight trip of a*n UAV.*

Keywords: Unscented Kalman Filter (UKF), Adaptive Filtering, Tightly-Coupled INS/GPS Integration (TCI)

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INTRODUCTION

MEMS-based INS/GPS integrated navigation system exhibits large errors because of its nonlinear model and uncertain noise statistic characteristics ($\mathbf{Q}$,$\mathbf{R}$) [16]; especially talking about tightly-coupled INS/GPS integration (TCI), the problem becomes more complicated where the system faces three challenges: 1-The measurement model is variable because it depends on the number of used satellites. 2-The measurement model is highly nonlinear, but on the other hand the measurement model of loosely-coupled INS/GPS integration (LCI) is completely linear. 3-The errors' statistics of satellites raw measurements (Pseudo-range, Doppler) are variable and strongly environment-dependent, such as the multi-path error which depends on the terrains surrounding the antenna of GPS receiver.

Typically Unscented Kalman Filter (UKF) is utilized to realize TCI systems because of its derivative free calculation process and superior performance in highly non-linear systems, [13, 5, 20]. But UKF needs the stochastic models of state and measurements noises in addition to their statistic characteristics which are not known precisely, to work properly; thus many INS/GPS integrated systems use the adaptive filtering to estimate the unknown parameters in the run time [21, 18, 6, 1].

In this paper a Novel Adaptive Unscented Kalman Filter (NAUKF) is developed on basis of unscented Kalman filtering and adaptive filtering which gives the opportunity to estimate the states without any prior knowledge about the first moment (mean) and second moment (covariance) of either process noise $\mathcal{W}$ or measurement noise $\mathcal{V}$, because the noise's moments are on-line estimated to adaptively compensate the time varying noise characteristics [11]. The resultant filter (NAUKF) is utilized to realize the TCI; this new combination (TCI-NAUKF) showed more superiority against the typical (TCI-UKF) as will be seen next.

The main contributions of (TCI-NAUKF) can be summarized in the following points:

1. The NAUKF supposes nonzero-mean noises, where other adaptive filters focus only on the noise covariance and discard the mean [21, 18, 6, 1].

2. The GPS receiver clock offset t_u and drift t_{ru} are omitted from the state vector because the noise mean of each in-view satellite's raw measurements is esti-

mated in the run time; that reduces the mathematical complexity of the filter.

3. The proposed filter estimates the noise characteristics of each in-view satellite's raw measurements separately, where typical (TCI) considers them of same characteristics.

4. The proposed noise estimators are based on the "forgetting factors" which were introduced by Deng and Guo [19]; subsequently, the estimators have the ability to estimate the unknown time-varying noise. The problem is that, the initial values of forgetting factors affect the estimator's performance; and usually they are determined empirically according to the problem's nature. The "Genetic Algorithm" is proposed to achieve their optimal values.

The development of this work will pass through successive phases; where firstly "Unscented Kalman Filtering Algorithm for Nonzero Mean Noise" will be introduced, so the resultant filter can deal with noise of known bias. Secondly the "Statistic Estimator of Noise's Mean & Covariance" will be developed. Thirdly the development of "Noise Real-time Tracker (Statistic Estimator)" based on forgetting factors concept will be introduced in detail; then fourthly the utilization of the "Genetic Algorithm" for "Getting Optimal Forgetting Factors" also will be discussed; then the novel adaptive UKF (NAUKF) will be the result of the integration of previous phases. Fifthly the employment of the NAUKF into the TCI will be expounded in a separate section. Finally, a "Comparative Study" will be done to evaluate the performance of the proposed integration (TCI-NAUKF) compared to the typical one (TCI-UKF).

UKF ALGORITHM FOR NONZERO MEAN NOISE

The integrated-navigation systems are almost nonlinear, and can be modeled as the following general nonlinear discrete system:

$$\text{(a) } \underline{\text{Process}}\text{: } \boxed{\mathbf{x}_{k+1} = f(t_k, \mathbf{x}_k) + \mathcal{W}_k}, \quad \text{(b) } \underline{\text{Measurments}}\text{: } \boxed{z_{k+1} = h(t_{k+1}, \mathbf{x}_{k+1}) + \mathcal{V}_{k+1}}, \tag{1}$$

where equation (a) is the process (or state) equation, $f(\cdot)$ is the state's nonlinear vector function, $\mathbf{x}_{k+1}$ is $n \times 1$ state vector at time instant t_{k+1}, $\mathcal{W}_k$ is $n \times 1$ process additive noise and is supposed to be nonzero-mean Gaussian white noise of unknown mean $\mathbf{q}$ and unknown diagonal covariance matrix $\mathbf{Q}_k$; equation (b) is the measurement equation, $h(\cdot)$ is the measurements' nonlinear vector function, z_{k+1} is $m \times 1$ measurement vector at time instant t_{k+1}, and finally $\mathcal{V}_{k+1}$ is $m \times 1$ measurement additive noise and is supposed to be nonzero-mean Gaussian white noise of unknown mean $\mathbf{r}$ and unknown diagonal covariance matrix $\mathbf{R}_{k+1}$.

$\{x_0, \mathcal{W}_k, \mathcal{V}_k\}$ are assumed uncorrelated random variables with means and covariances.

Assuming that $\mu_k = \mathcal{W}_k - \mathbf{q}$ and $\eta_k = \mathcal{V}_k - \mathbf{r}$; by substituting them into (1) yield

$$\text{(a) } \underline{\text{Process}}: \boxed{\mathbf{x}_{k+1} = f(t_k, \mathbf{x}_k) + \mathbf{q} + \mu_k}, \quad \text{(b) } \underline{\text{Measurments}}\text{: } \boxed{z_{k+1} = h(t_{k+1}, \mathbf{x}_{k+1}) + \mathbf{r} + \eta_{k+1}}. \tag{2}$$

It is clear that μ_k and η_{k+1} are zero-mean Gaussian white noise of unknown diagonal covariance matrices $\mathbf{Q}_k$ and $\mathbf{R}_{k+1}$ respectively.

Sigma Points Selection: we form a matrix $\chi_{k/k}$ of $(2n+1)$ sigma points $\chi_{i,k/k}$ (with corresponding weights W_i), according to the following equations [12]:

$$\begin{cases} \chi_{0,k/k} = \hat{x}_{k/k}, \ \ \chi_{i,k/k} = \hat{x}_{k/k} \pm \left(\sqrt{(n+\lambda)\mathbf{P}_{k/k}}\right)_i, \\ (+) \text{ for: } 1 \le i \le n, \ (-) \text{ for: } n+1 \le i \le 2n \\ W_0^{(m)} = \dfrac{\lambda}{(n+\lambda)}, \ \ W_0^{(c)} = W_0^{(m)} + \left(1 - \alpha^2 + \beta\right), \\ W_i^{(m)} = W_i^{(c)} = \dfrac{\lambda}{2(n+\lambda)} \ \ \text{for } 1 \le i \le 2n, \end{cases} \tag{3}$$

where $\lambda = \alpha^2(n+\kappa) - n$ is a scaling parameter. α determines the spread of the sigma points around $\hat{x}_{k/k}$ and is usually set to a small positive value (e.g., 1e-3). κ is a secondary scaling parameter which is usually set to 0, and β is used to incorporate prior knowledge of the distribution of $\mathbf{x}$ (for Gaussian distributions, $\beta = 2$ is optimal). $\left(\sqrt{(n+\lambda)\mathbf{P}_{k/k}}\right)_i$ is the i-th row of the matrix square root.

A priori Estimation: These sigma points $\boldsymbol{\chi}_{k/k}$ are propagated through the nonlinear state function $(f(\cdot) + \mathbf{q})$ to obtain the a priori estimate of sigma points $\boldsymbol{\chi}_{k+1/k}$ at t_{k+1} [17]:

$$\chi_{i,k+1/k} = f(\chi_{i,k/k}) + \mathbf{q} \quad \text{with: } \ i = 0, \ldots, 2n. \tag{4}$$

The a priori estimate $\hat{x}_{k+1/k}$ and covariance matrix $\mathbf{P}_{k+1/k}$ of the state vector at t_{k+1} are as:

$$\begin{cases} \hat{x}_{k+1/k} = \sum_{i=0}^{2n} W_i^{(m)} \chi_{i,k+1/k} + \mathbf{q}, \\ \mathbf{P}_{k+1/k} = \sum_{i=0}^{2n} W_i^{(c)} (\delta\chi_{i,k+1})(\delta\chi_{i,k+1})^T + \mathbf{Q}_k \\ \text{with: } \delta\chi_{i,k+1} = \chi_{i,k+1/k} - \hat{x}_{k+1/k}. \end{cases} \tag{5}$$

Posteriori Estimation: Propagating the sigma points $\chi_{k+1/k}$ through the nonlinear measurement function $(h(\cdot) + \mathbf{r})$ leads the measurement sigma points $\mathbf{Z}_{k+1/k}$ as [17]:

$$Z_{i,k+1/k} = h(\chi_{i,k+1/k}) + \mathbf{r} \text{ with: } i = 0,\ldots,2n. \tag{6}$$

Computing the predicted measurement vector $\hat{z}_{k+1/k}$, the covariance of the measurement $\mathbf{P}_{zz,k+1}$ and the cross-covariance of the state and measurement $\mathbf{P}_{xz,k+1}$ are as follows:

$$\begin{cases} \hat{z}_{k+1/k} = \sum_{i=0}^{2n} W_i^{(m)} h(\chi_{i,k+1/k}) + \mathbf{r} \text{ Define: } \delta Z_{i,k+1} \\ = Z_{i,k+1/k} - \hat{z}_{k+1/k} \\ \mathbf{P}_{zz,k+1} = \sum_{i=0}^{2n} W_i^{(c)} (\delta Z_{i,k+1})(\delta Z_{i,k+1})^T + \mathbf{R}_{k+1}, \\ \mathbf{P}_{xz,k+1} = \sum_{i=0}^{2n} W_i^{(c)} (\delta\chi_{i,k+1})(\delta Z_{i,k+1})^T. \end{cases} \tag{7}$$

The posteriori estimate $\hat{z}_{k+1/k+1}$ and covariance $\mathbf{P}_{k+1/k+1}$ of the state vector at t_{k+1} are calculated depending on the measurement vector z_{k+1} and the filter gain $\mathbf{K}_{k+1}$ as follows:

$$\begin{cases} \hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + \mathbf{K}_{k+1}\left(z_{k+1} - \hat{z}_{k+1/k}\right) \\ \text{with: } \mathbf{K}_{k+1} = \mathbf{P}_{xz,k+1}\mathbf{P}_{zz,k+1}^{-1} \\ \mathbf{P}_{k+1/k+1} = \mathbf{P}_{k+1/k} - \mathbf{K}_{k+1}\mathbf{P}_{zz,k+1}\mathbf{K}_{k+1}^T. \end{cases} \tag{8}$$

The obtained filter has been built to deal easily with *known* noises' bias which are almost unknown or time varying; that leads to develop noise estimators in the next section.

STATISTIC ESTIMATOR OF NOISE'S MEAN & COVARIANCE

The expected value (mean) and the covariance of state noise $\mathcal{W}_k$ and measurement noise $\mathcal{V}_{k+1}$ by definition are as follows:

$$\begin{cases} \mathbf{q} = \mathrm{E}[\mathcal{W}_k] = \mathrm{E}[x_{k+1} - f(x_k)], \\ \mathbf{Q}_k = \mathrm{Var}(\mathcal{W}_k) = \mathrm{Var}(x_{k+1} - f(x_k)), \\ \mathbf{r} = \mathrm{E}[\mathcal{V}_{k+1}] = \mathrm{E}[z_{k+1} - h(x_{k+1})], \\ \mathbf{R}_{k+1} = \mathrm{Var}(\mathcal{V}_{k+1}) = \mathrm{Var}(z_{k+1} - h(x_{k+1})). \end{cases} \tag{9}$$

By taking into consideration the ergodicity of the two processes $\mathcal{W}_k$ and $\mathcal{V}_{k+1}$, the temporal average and variance become equal to the probabilistic mean and variance respectively, [2]; so the following approximations are valid:

$$\begin{cases} \mathbf{q} \approx \mathbf{q}_k^{(\mathrm{ar})} = \frac{1}{k}\sum_{j=1}^{k}\left(x_{j+1} - f(x_j)\right), \\ \mathbf{Q}_k \approx \mathbf{Q}_k^{(\mathrm{ar})} = M_{\mathbf{q},k}^{(\mathrm{ar2})} - \left(\mathbf{q}_k^{(\mathrm{ar})}\right)\left(\mathbf{q}_k^{(\mathrm{ar})}\right)^T, \\ \mathbf{r} \approx \mathbf{r}_{k+1}^{(\mathrm{ar})} = \frac{1}{k}\sum_{j=1}^{k}\left(z_{j+1} - h(x_{j+1})\right), \\ \mathbf{R}_{k+1} \approx \mathbf{R}_{k+1}^{(\mathrm{ar})} = M_{\mathbf{r},k+1}^{(\mathrm{ar2})} - \left(\mathbf{r}_{k+1}^{(\mathrm{ar})}\right)\left(\mathbf{r}_{k+1}^{(\mathrm{ar})}\right)^T, \end{cases} \tag{10}$$

where $\mathbf{q}_k^{(\mathrm{ar})}$ and $\mathbf{r}_{k+1}^{(\mathrm{ar})}$ are the arithmetic or temporal mean, $\mathbf{Q}_k^{(\mathrm{ar})}$ and $\mathbf{R}_{k+1}^{(\mathrm{ar})}$ are the arithmetic or temporal covariance, and finally $M_{\mathbf{q},k}^{(\mathrm{ar2})}$ and $M_{\mathbf{r},k+1}^{(\mathrm{ar2})}$ are the arithmetic or temporal second moment of $\mathcal{W}_k$ and $\mathcal{V}_{k+1}$ respectively and defined as follows:

$$\begin{aligned} M_{\mathbf{q},k}^{(\mathrm{ar2})} &= \frac{1}{k}\sum_{j=1}^{k}\left(x_{j+1} - f(x_n)\right)\left(x_{j+1} - f(x_n)\right)^T, \\ M_{\mathbf{r},k+1}^{(\mathrm{ar2})} &= \frac{1}{k}\sum_{j=1}^{k}\left(z_{j+1} - h(x_{j+1})\right)\left(z_{j+1} - h(x_{j+1})\right)^T. \end{aligned} \tag{11}$$

It's clear that the previous arithmetic quantities depend directly on the true state vector which is not available; hence it will be replaced by its UKF's estimate of state vector to predict the means $(\hat{\mathbf{q}}_k, \hat{\mathbf{r}}_{k+1})$ and covariances $(\hat{\mathbf{Q}}_k, \hat{\mathbf{R}}_{k+1})$, as follows:

$$\begin{cases} \hat{\mathbf{q}}_k = \frac{1}{k}\sum_{j=1}^{k}\delta\hat{x}_j, \quad \hat{\mathbf{Q}}_k = \hat{M}_{\mathbf{q},k}^{(2)} - (\hat{\mathbf{q}}_k)(\hat{\mathbf{q}}_k)^T \\ \text{with: } \hat{M}_{q,k}^{(2)} = \frac{1}{k}\sum_{j=0}^{k-1}(\delta\hat{x}_j)(\delta\hat{x}_j)^T \\ \text{and } \delta\hat{x}_j = \hat{x}_{j+1/j+1} - f(\hat{x}_{j/j}) \\ \hat{\mathbf{r}}_{k+1} = \frac{1}{k}\sum_{j=1}^{k}\delta z_{j+1}, \quad \hat{\mathbf{R}}_{k+1} = \hat{M}_{\mathbf{r},k+1}^{(2)} - (\hat{\mathbf{r}}_{k+1})(\hat{\mathbf{r}}_{k+1})^T \\ \text{with: } \hat{M}_{\mathbf{r},k+1}^{(2)} = \frac{1}{k}\sum_{j=1}^{k}(\delta z_{j+1})(\delta z_{j+1})^T \\ \text{and } \delta z_{j+1} = z_{j+1} - h(\hat{x}_{j+1/j}). \end{cases} \tag{12}$$

Assuming the components of measurement vector being independent leads to diagonal covariance matrix $\mathbf{R}_{k+1}$; so the calculation of covariance matrix estimate $\hat{\mathbf{R}}_{k+1}$ becomes scalar equations as follows:

$$\begin{cases} \hat{\mathbf{r}}_{k+1}^{(i)} = \dfrac{1}{k}\sum_{j=1}^{k}\delta z_{j+1}^{(i)} \Rightarrow \boxed{\hat{\mathbf{r}}_{k+1} = \left[\hat{\mathbf{r}}_{k+1}^{(1)},\dots,\hat{\mathbf{r}}_{k+1}^{(m)}\right]^T} \\ \hat{\mathbf{R}}_{k+1}^{(i)} = \hat{M}_{\mathbf{r},k+1}^{(2,i)} - \left(\hat{\mathbf{r}}_{k+1}^{(i)}\right)^2 \\ \text{with: } \hat{M}_{\mathbf{r},k+1}^{(2,i)} = \dfrac{1}{k}\sum_{j=1}^{k}\left(\delta z_{j+1}^{(i)}\right)^2 \\ \Rightarrow \boxed{\hat{\mathbf{R}}_{k+1} = \operatorname{diag}\left[\hat{\mathbf{R}}_{k+1}^{(1)},\dots,\hat{\mathbf{R}}_{k+1}^{(m)}\right]}, \end{cases} \tag{13}$$

where $\delta z_{j+1}^{(i)}$ are the i-th component in the measurement estimation error vector δz_{j+1}.

NOISE REAL-TIME TRACKER (DYNAMIC STATISTIC ESTIMATOR)

In the previous section we have assumed that the statistic properties of noise are not variant with time which doesn't consist with real world especially the noise's mean where it depends severely on the environment's conditions in various cases. This section concentrates on employing the statistic noise estimators into the TCI, this kind of integration uses directly the raw measurements (Pseudo-range and Doppler) of each satellite in view whose noise's properties depend strongly on the nearby terrains and the weather conditions [15, 14, 9]. For these reasons the equations of noise estimator (12) and (13) will be modified by using the forgetting factor concept that gives the estimator the power of tracking the changes of mean and covariance online [21]; in the reference [21], the matrices $\hat{\mathbf{Q}}_k, \hat{\mathbf{R}}_{k+1}$ are estimated on basis of the adaptive filtering algorithm using the maximum a posteriori estimation which was developed by Sage and Husa [22]. The algorithm in [21] proposed a unique time-varying forgetting factor for updating the two matrices $\hat{\mathbf{Q}}_k, \hat{\mathbf{R}}_{k+1}$; in this paper different updating weights are used for estimating the covariance and mean of each state and measurement as follows:

$$\begin{aligned} \hat{\mathbf{q}}_k &= \left(\mathbf{I}_n - \operatorname{diag}(d_{\mathbf{q},k})\right)\hat{\mathbf{q}}_{k-1} + \operatorname{diag}(d_{\mathbf{q},k})\delta\hat{x}_{k+1}, \\ \hat{\mathbf{Q}}_k &= \hat{M}_{\mathbf{q},k}^{(2)} - (\hat{\mathbf{q}}_k)(\hat{\mathbf{q}}_k)^T. \end{aligned} \tag{14}$$

$$\begin{aligned} \text{With: } \hat{M}_{\mathbf{q},k}^{(2)} &= \left(\mathbf{I}_n - \operatorname{diag}(d_{\mathbf{q},k})\right)\hat{M}_{\mathbf{q},k-1}^{(2)} + \operatorname{diag}(d_{\mathbf{q},k}) \\ &\times\left[(\delta\hat{x}_k)(\delta\hat{x}_k)^T + \mathbf{P}_{k+1/k+1} - \mathbf{\Phi}_k\mathbf{P}_{k/k}\mathbf{\Phi}_k^T\right], \end{aligned} \tag{15}$$

$$\text{and } \begin{cases} \hat{\mathbf{r}}_{k+1}^{(i)} = \left(1 - d_{\mathbf{r},k}^{(i)}\right)\hat{\mathbf{r}}_k^{(i)} + d_{\mathbf{r},k}^{(i)}\delta z_{k+1}^{(i)}, \\ \hat{\mathbf{R}}_{k+1}^{(i)} = \hat{M}_{\mathbf{r},k+1}^{(2,i)} - \left(\hat{\mathbf{r}}_{k+1}^{(i)}\right)^2 \\ \text{with: } \hat{M}_{\mathbf{r},k+1}^{(2,i)} = \left(1 - d_{\mathbf{r},k}^{(i)}\right)\hat{M}_{\mathbf{r},k}^{(2,i)} \\ + d_{\mathbf{r},k}^{(i)}\left[\left(\delta z_{k+1}^{(i)}\right)^2 - \mathbf{HPH}_{k+1}^{(i)}\right]. \end{cases} \tag{16}$$

$\mathbf{\Phi}_k$ is the process matrix and defined by $\mathbf{\Phi}_k = (\partial f/\partial x)\big|_{x=\hat{x}_{k/k}}$. The term $\mathbf{HPH}_{k+1}^{(i)}$ is the i-th element of the diagonal of the matrix $\mathbf{H}_{k+1}\mathbf{P}_{k+1/k}\mathbf{H}_{k+1}$, where $\mathbf{H}_{k+1}$ is the measurement matrix and defined by $\mathbf{H}_{k+1} = (\partial h/\partial x)\big|_{x=\hat{x}_{k+1/k}}$. The coefficients $d_{\mathbf{q},k} = \left[d_{\mathbf{q}1,k},\dots,d_{\mathbf{q}n,k}\right]$ and $d_{\mathbf{r},k}^{(i)}$ are the updating weights of the states and the i-th measurement respectively at time t_{k+1}. An updating weight d_k is defined as follows:

$$d_k = \frac{1-b_0}{1-b_0^{k+1}} \text{ with: } 0 < b_0 < 1. \tag{17}$$

The parameter b_0 is called "*forgetting factor*"; its value depends on the problem's nature and is determined empirically. The utilization of "Noise Real-time Trackers" with the unscented Kalman filtering algorithm for nonzero mean noise will form the proposed filter which will be called namely *Novel Adaptive Unscented Kalman Filter* (NAUKF).

(NAUKF) FOR INS/GPS/MAG/BARO INTEGRATED SYSTEMS

Typically, the state vector, used by TCI, consists of the following three groups:

1. Navigational parameters: they are Attitude (represented for example by quaternion $\overline{\mathbf{q}}$), Position (represented by LLA, $\mathbf{p}^n = $ [Latitude L, Longitude l, Altitude h] coordinates), and Velocity $V^n = [V_N, V_E, V_D]$.

2. The inertial measurements model: in this paper the model mentioned in [5] will be taken into consideration, where the gyroscopes' measurement model is given by:

$$\tilde{\omega}_{ib}^b = (\mathbf{I}_3 + \mathcal{K}_g)\,\omega_{ib}^b + b_g + \mathcal{W}_g, \quad \dot{b}_g = \mathcal{W}_{bg}, \tag{18}$$

where b_g is the gyro "bias", $\mathcal{K}_g$ is a diagonal matrix of gyro scale factors $k_g = [k_{gx}, k_{gy}, k_{gz}]$ and $\mathcal{W}_g$ and $\mathcal{W}_{bg}$ are 3×1 zero-mean Gaussian white-noise processes with spectral densities given by $\sigma_g^2\mathbf{I}_3$ and $\sigma_{bg}^2\mathbf{I}_3$, respectively. The accelerometers' measurement model is as:

$$\tilde{f}^b = (\mathbf{I}_3 + \mathcal{K}_a)\,f^b + b_a + \mathcal{W}_a, \quad \dot{b}_a = \mathcal{W}_{ba}, \tag{19}$$

where b_a is the accelerometer "bias", K_a is a diagonal matrix of accelerometer scale factors $k_a = [k_{ax}, k_{ay}, k_{az}]$, and $\mathcal{W}_a$ and $\mathcal{W}_{ba}$ are 3×1 zero-mean Gaussian white-noise processes with spectral densities given by $\sigma_a^2\mathbf{I}_3$ and $\sigma_{ba}^2\mathbf{I}_3$, respectively.

3. Errors of GPS receiver clock: they are the receiver clock bias Δt_u, and the clock drift Δt_{ru}, both of which are modeled as a random walk process, [13, 13].

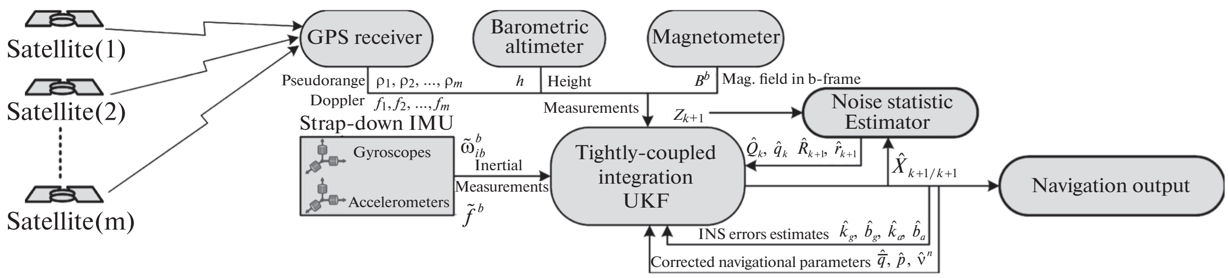

Fig. 1. The architecture of INS/GPS/Mag/Baro integrated system with NAUKF.

The clock error states are defined in the units of range and range rates, by multiplying by the speed of light (c), for compatibility with position and velocity states.

Hence the proposed state vector of TCI is as: $x = [\bar{\mathbf{q}}, \mathbf{p}^n, V^n, ƙ_g, b_g, ƙ_a, b_a, \Delta t_u, \Delta t_{ru}]$. The final integration will actually use a MEMS-base strap-down IMU, a GPS receiver, a strap-down three-axial magnetometer and a barometric altimeter, where the noise specifications of these sensors are not supposed to be known precisely; the proposed architecture of TCI with NAUKF is computed as clarified in Fig. 1. The major benefits of using NAUKF with TCI are:

1. Estimation of the offset and noise level of pseudo-range and Doppler of each satellite in view independently; that eliminates the need of estimating the GPS receiver clock offset and drift.

2. Estimation of the residual magnetometer hard iron, which had not been removed by calibration; this will enhance the heading estimation especially during GPS outages.

3. Estimation of the uncalibrated barometer offset which depends directly on uncontrollable issues; like the ambient temperature and the stability of its power supply.

The minimum required forgetting factors to be calculated for tightly-coupled INS/GPS/Magnetometer/Barometer integration with NAUKF, are as follows:

1. Theoretically the state noise statistic estimator needs a forgetting factor for each state, but the determination of their values is very difficult and time-consuming; subsequently we will divide these factors logically to two groups, each group has unique value as follows: firstly $b_{\mathrm{qG},0}$ for the gyro-dependent states $[\bar{\mathbf{q}}, ƙ_g, b_g]$, and secondly $b_{\mathrm{qA},0}$ for the accelerometer-dependent states $[\mathrm{LLA}, V^n, ƙ_a, b_a]$.

2. The barometer needs one factor $b_{\mathrm{bar},0}$.

3. The magnetometer needs three factors $b_{mx,0}, b_{my,0}, b_{mz,0}$, by the same logic they will be considered equal $b_{\mathrm{mag},0}$ because the magnetic sensor of each axis has the same nature.

4. Namely each satellite needs two factors, one for the pseudo-range measurement and another for the Doppler measurement, but it's logical to consider that all satellites have equal values of these factors; hence there are only two factors ($b_{\rho,0}$ for pseudo-range, $b_{\mathrm{d},0}$ for Doppler).

Forgetting factors are usually determined empirically; this work is time consuming and does not lead to optimal solution, but in the next section a new technique is applied to achieve the optimal values of the forgetting factors, where the problem of "tight coupling (TCI) with (NAUKF)" will be treated as an optimization problem with forgetting factors as the parameters to be optimized.

OPTIMAL FORGETTING FACTORS OF (TCI-NAUKF)

As mentioned in the previous section the forgetting factors cannot be calculated analytically, but unfortunately the performance of the INS/GPS/Mag/Baro integration with NAUKF and tight coupling is affected strongly by the choice of these factors. We propose this problem as an optimization problem which can be realized by "simulation" as follows:

1. The cost function: Function's inputs are the forgetting factors, besides a simulation of the output of strap-down IMU, satellites raw data viewed by GPS receiver, a strap-down magnetometer and finally a barometric altimeter, during a simulated flight trip; the sensors specs are taken from the datasheet of the real sensors which will be used in the validation of employing the NAUKF into the TCI later. Function's output actually is the RMS value of horizontal error.

2. Optimization tool: There are many methods that can be used to find out the values of forgetting factors and minimize the RMS of "Horizontal Error", such as *Pattern Search*, *Multi Start*, *Global Search* and *Genetic Algorithm*; but the random nature of a part of the cost function's inputs which are the sensors' outputs, makes "*Genetic Algorithm*" is the best candidate, where for every optimization loop the sensors' outputs are regenerated according to their stochastic model [8, 4, 3].

Table 1. Forgetting factors calculated by genetic algorithm

$b_{qG,0}$	$b_{qA,0}$	$b_{bar,0}$	$b_{mag,0}$	$b_{\rho,0}$	$b_{d,0}$
0.26650	0.67590	0.99999	0.99994	0.97500	0.75130

Fig. 1 clarifies exactly how the optimization loop is done by means of genetic algorithm to achieve the optimal forgetting factors which are used by the noise estimator of the NAUKF.

This algorithm is used to calculate the forgetting factors corresponding to the real hardware which is used in the comparative study (see Table 3); so all the hardware's stochastic specs mentioned by the manufacturer are taken into consideration to generate sensors' noise during the simulation. The utilization of this algorithm leads up to the following values of forgetting factors corresponding the real used hardware.

TESTING THE CONVERGENCE OF NOISE REAL-TIME TRACKER BY MONTE-CARLO SIMULATION

The convergence of the proposed filter and performance of the noise statistic estimator will be checked according to estimation accuracy and stability of parameters of state and measurement noises by means of "Monte-Carlo simulation" based on the following:

• Noise of the different sensors is regenerated in each occurrence of "Monte-Carlo simulation" from the stochastic specifications of the real hardware which is used in the comparative study (see Table 3).

• The initial noise level of either state or measurements are given as **ten times** of their nominal values.

• The generated noise will be added to the nominal sensors output to obtain the simulated measurements to form the input of the proposed integration combination "TCI-NAUKF".

• The nominal outputs of sensors (IMU, GPS, barometer and magnetometer) are generated in MATLAB by means of a known-model aircraft tracking a predefined trajectory (as shown in Fig. 1).

• The start GMT time of simulated trip is set as 07:30AM, on 01-JAN-2015; thus the Receiver Independent Exchange "RINEX" file of NavStar GPS associated to this date is "brdc0010.15n", and the satellites in-view for the proposed start time are: [SV04, SV11, SV19, SV22, SV28, and SV32].

• Output rate of the sensors IMU, barometer and magnetometer is (10 [ms]), and GPS output rate is (1 [s]).

The following results are actually the average of all executed simulations' results.

Fig. 4 shows the estimated noise level (σ_ρ, σ_d) of the pseudo-range and Doppler measurements respectively for each satellite.

Fig. 1 shows the estimated noise level σ_{bar} and offset h_0 of barometric altimeter's output.

Fig. 6 shows the estimated noise level σ_{mag} and offset B_0 of each axis measurements of the strap-down three-axial magnetometer.

Table 2. Results on noise estimators used in NAUKF for TCI

Sensor	Noise	Symbol	Nominal value	Unit	Average accuracy	Settling time
Gyroscope	Angle random walk	σ_g	0.450	$\mathrm{deg}/\sqrt{\mathrm{h}}$	97.3%	<1 s
Accelerometer	Velocity random walk	σ_a	0.125	$(\mathrm{m/s})/\sqrt{\mathrm{h}}$	97.9%	<1 s
Magnetometer	White noise (noise level)	σ_{mag}	0.2	$(\%)/\sqrt{\mathrm{Hz}}$	99.9%	<1 s
	Offset (bias)	B_0	5.0	%	98.3%	<1 s
Barometer	White noise (noise level)	σ_{bar}	1/3	$\mathrm{m}/\sqrt{\mathrm{Hz}}$	99.7%	<1 s
	Offset (bias)	h_0	50	m	98.8%	<1 s
GPS (Pseudo-range)	White noise (noise level)	σ_ρ	25/3	$\mathrm{m}/\sqrt{\mathrm{Hz}}$	98.1%	<10 s
GPS (Doppler)	White noise (noise level)	σ_d	2.5/3	$\mathrm{Hz}/\sqrt{\mathrm{Hz}}$	97.6%	<10 s

Table 3. Inertial Sensors Specs

Gyros' noise	Value	Unit	Accelerometers' noise	Value	Unit
Fixed drift range	±250	deg/h			
Angle random walk	0.45	$\mathrm{deg}/\sqrt{\mathrm{h}}$	Velocity random walk	0.125	$(\mathrm{m/s})/\sqrt{\mathrm{h}}$
Bias instability	3.50	deg/h	Bias instability	0.1	mg
Rate random walk	9.40	$\mathrm{deg}/\mathrm{h}^{3/2}$	Acceleration random walk	21.15	$(\mathrm{m/s})/\mathrm{h}^{3/2}$

Fig. 7 shows the estimated noise level of each axis measurements of the strap-down IMU (Gyroscopes + Accelerometers).

Table 2 summarizes the estimation results obtained from utilizing the NAUKF for the TCI. It's shown that all noise's parameters of either states or measurements are well estimated with more than 97.3% average accuracy; and the noise estimators are convergent and stable with settling time approximately 1 second for the high-rate sensors (IMU, barometer and magnetometer) and approximately 10 second for the low-rate sensors (GPS).

COMPARATIVE STUDY

A strap-down MEMS-based IMU, GPS receiver, magnetometer and barometric altimeter were implemented on an Unmanned Aerial Vehicle (UAV) which performed a flight trip of trajectory shown in Fig. 1a; the sensors output were acquired during the flight, then the TCI with either UKF or NAUKF was executed off-line under MATLAB environment, where four software GPS outages of length 150 sec were performed in different places of the trajectory as shown in Fig. 1b. The comparison was done between the proposed combination (TCI-NAUKF) and the typical combination (TCI-UKF) by means of the "RMS of horizontal error (RMS-HE)" which is taken into consideration as performance index whose value at time t_k is equal to the RMS value of horizontal error signal between the end time of last GPS outage to the current timet_k. Actually horizontal error describes the distance between the projection of estimated position on the local horizontal plane and the projection of the reference (GPS) position on the same plane.

On the other hand, the errors of vertical channel (Height, Down-velocity) are ignored because the height is measured by the "barometric altimeter"; thus this channel is not affected by the GPS outages.

SENSORS' SPECIFICATIONS

Noise level on magnetometers ($0.2\%/\sqrt{\mathrm{Hz}}$) of the magnitude of Earth's magnetic field in the initial position. Noise level on barometric altimeter ($1/3\ \mathrm{m}/\sqrt{\mathrm{Hz}}$).

Specifications of inertial sensors: Table 3 shows the noise components most-commonly appearing in the inertial sensors' output; for more details on noise classification see [10].

EVALUATION OF (TCI-NAUKF)'S PERFORMANCE

The proposed combination (TCI-NAUKF), and the typical (TCI-UKF) will use the data of the real flight trip; then by computing four successive GPS outages of length 150 s as shown in Fig. 1; the comparison will be done by means of the RMS of horizontal error.

Table 4 gives the values of horizontal error, its (RMS-HE), and the relative improvement of the combination (TCI-NAUKF) compared to the (TCI-UKF) in certain chosen time lengths of GPS outage during each outage period of the four software outages; in addition to the averaged improvement achieved by the proposed combination (TCI-NAUKF). Results show that (TCI-NAUKF) achieved a superiority against the (TCI-UKF), where the averaged (RMS-HE) related to (TCI-NAUKF) is decreased by 37% relatively to the (RMS-HE) related to (TCI-UKF) at the end of GPS outage.

CONCLUSION

In this paper a novel adaptive filter NAUKF has been developed based on adaptive filtering and unscented Kalman filtering and utilized for tightly-coupled INS/GPS integration; the developed filter can investigate the state and measurement noises of unknown and time-varying mean and covariance, this is the case of tight coupling where the raw measure-

Table 4. (RMS-HE) and relative improvement achieved by (TCI-NAUKF)

Outage length (t), s	Outage nb. 1			Outage nb. 2			Outage nb. 3			Outage nb. 4			Outages' average		
	RMS-HE UKF, meter	RMS-HE NAUKF, meter	Improvement, %	RMS-HE UKF, meter	RMS-HE NAUKF, meter	Improvement, %	RMS-HE UKF, meter	RMS-HE NAUKF, meter	Improvement, %	RMS-HE UKF, meter	RMS-HE NAUKF, meter	Improvement, %	RMS-HE UKF, meter	RMS-HE NAUKF, meter	Improvement, %
30	25	18	28	18	18	−1	25	11	55	18	18	3	22	16	24
60	45	43	3	25	31	−22	32	22	32	34	25	25	34	30	11
90	75	78	−3	51	48	5	45	33	26	55	30	45	56	48	16
120	122	95	22	103	78	25	63	45	27	80	37	54	92	64	31
150	177	108	39	180	119	34	84	56	33	108	62	43	137	86	37

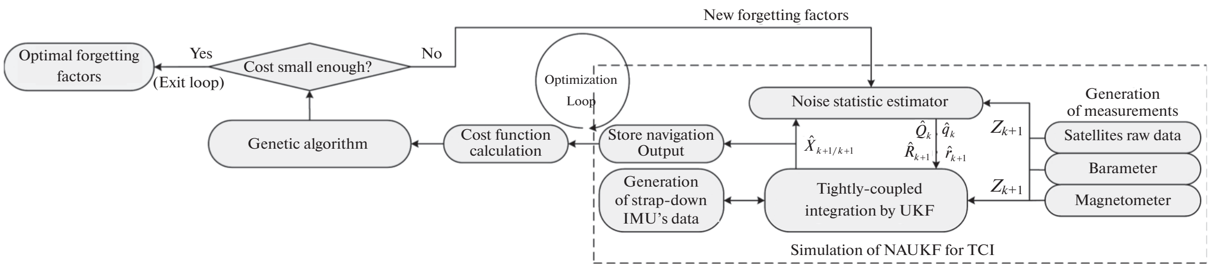

Fig. 2. Using genetic algorithm to calculate the optimal forgetting factors of the NAUKF.

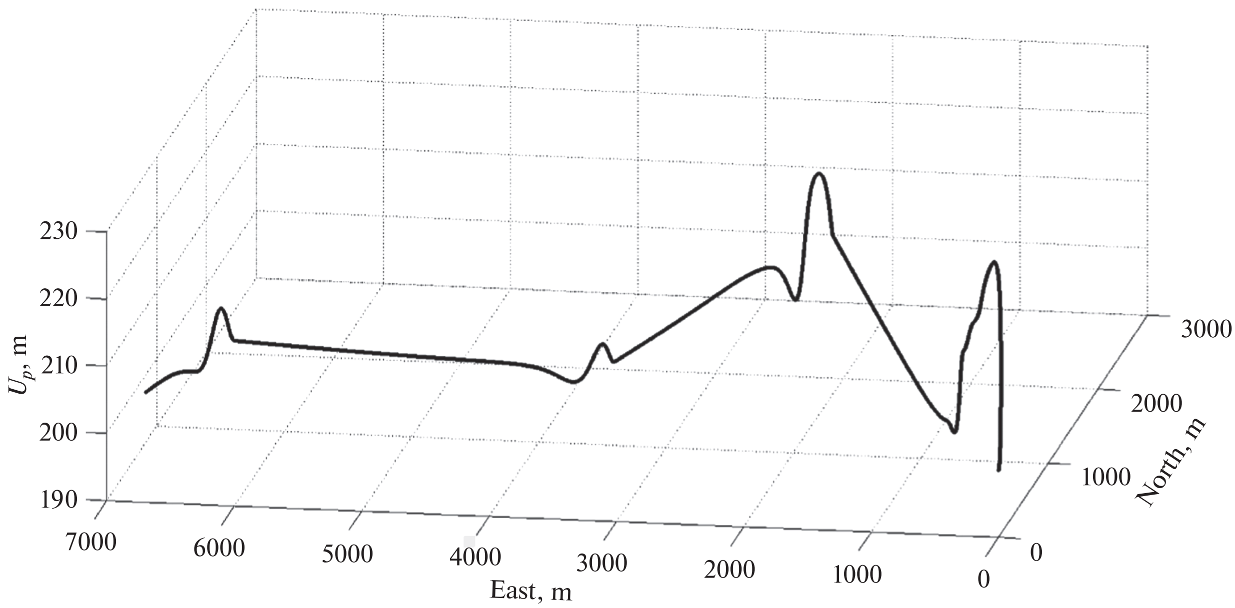

Fig. 3. Reference trajectory of simulated trip.

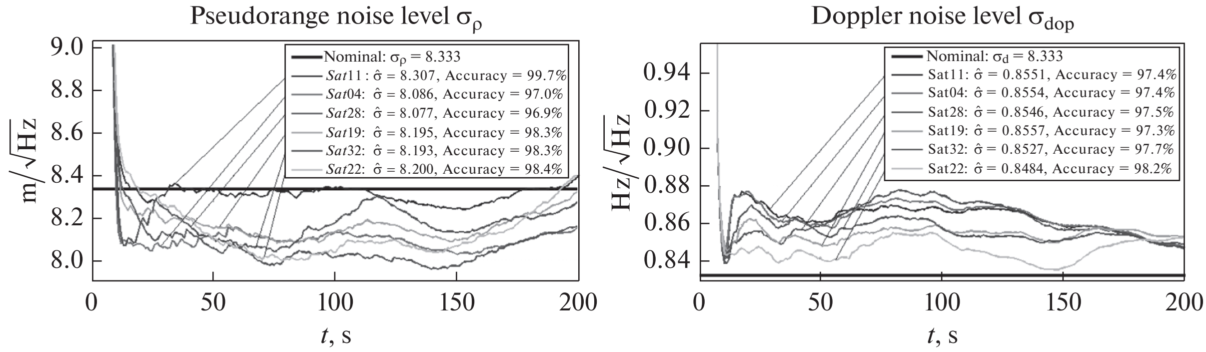

Fig. 4. Estimated noise level of the measurements of GPS satellites.

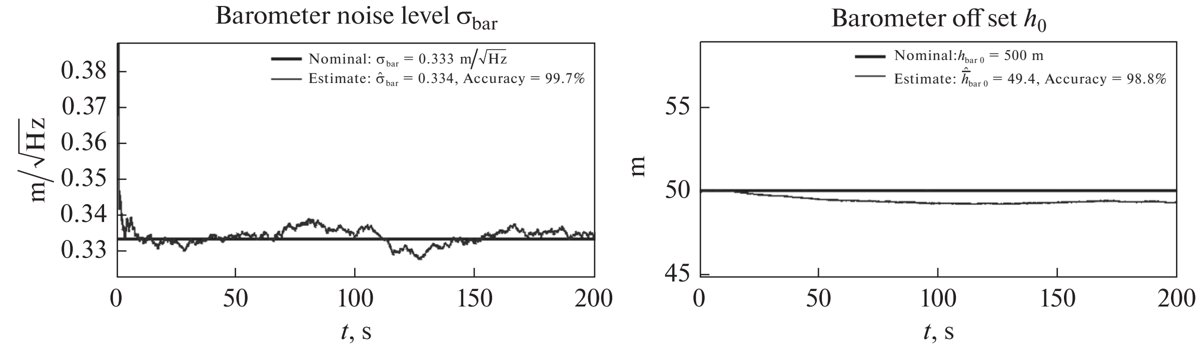

Fig. 5. Estimated noise level & offset of barometric altimeter.

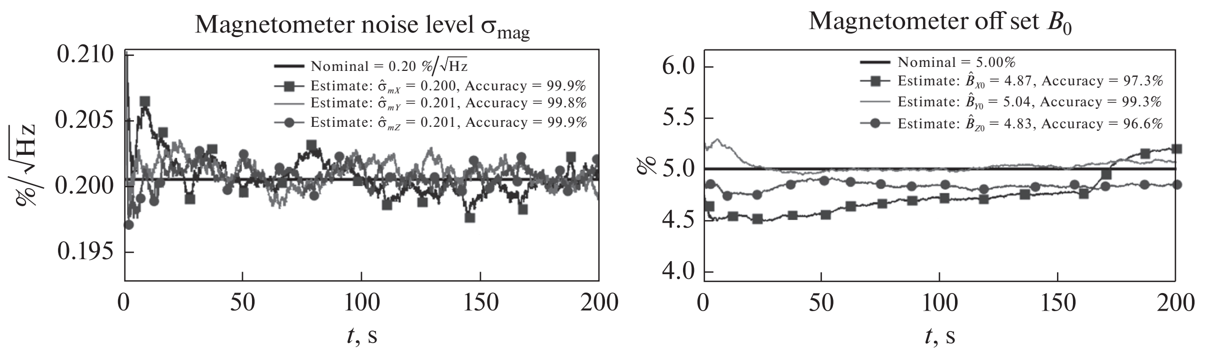

Fig. 6. Estimated noise level & offset of magnetometer.

ments noise of satellite vehicles (SV) depends directly on environment's terrains and weather's conditions, also the state noise consists basically of gyroscopes and accelerometers noises where a non-negligible part of them is associated to the dynamics of the vehicle carrying the strap-down inertial system. The core of the novel filter is the real-time statistic noise tracker which is based on the forgetting factor concept, hence it can estimate the first and second moment of noise and track their variations in the real time. Monte-Carlo simulation showed acceptable convergence of noise statistic estimators used in NAUKF for TCI.

The experimental results confirmed the superiority of the new combination (TCI-NAUKF) against the combination (TCI-UKF).

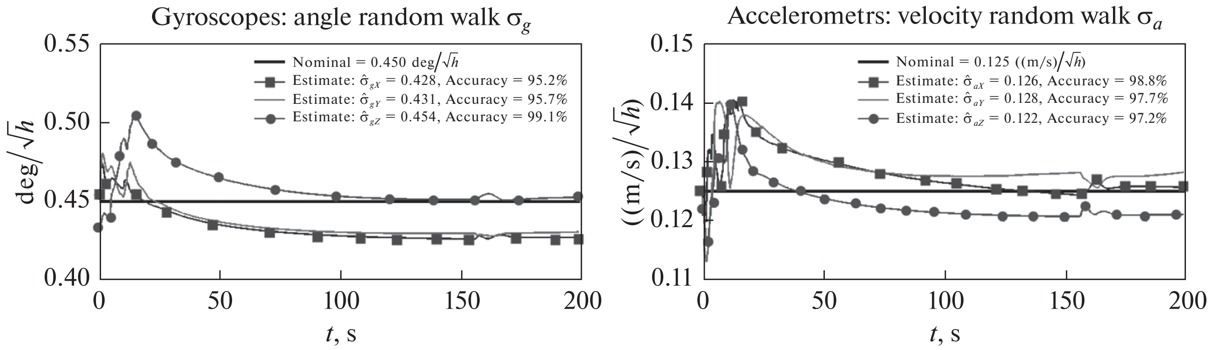

Fig. 7. Estimated noise level of inertial sensors.

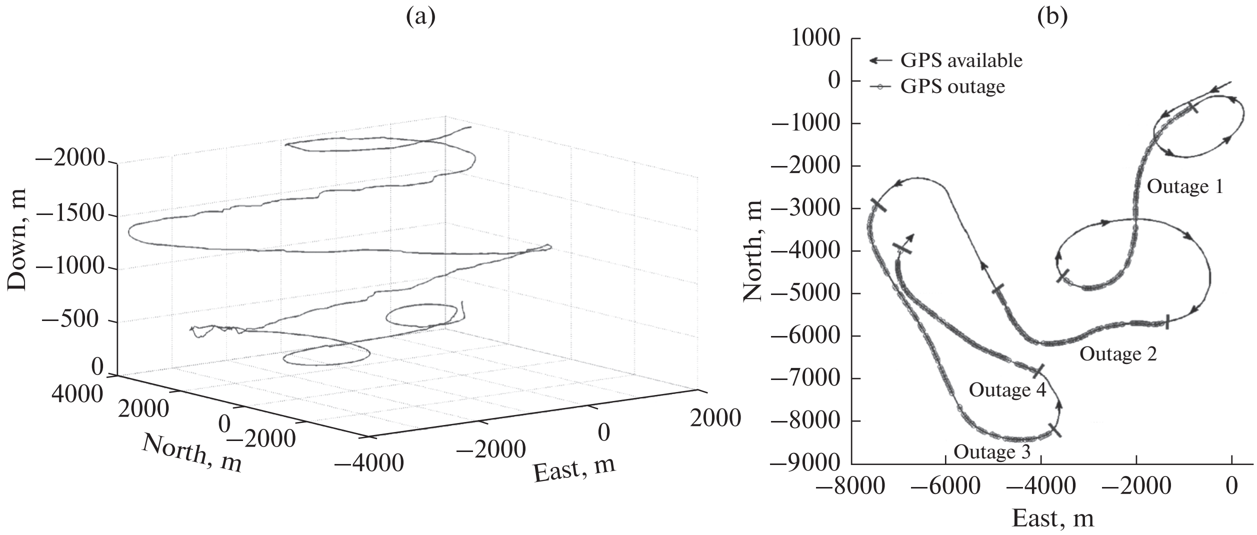

Fig. 8. (a) 3D Trajectory of flight trip. (b) GPS outages on the horizontal trajectory.

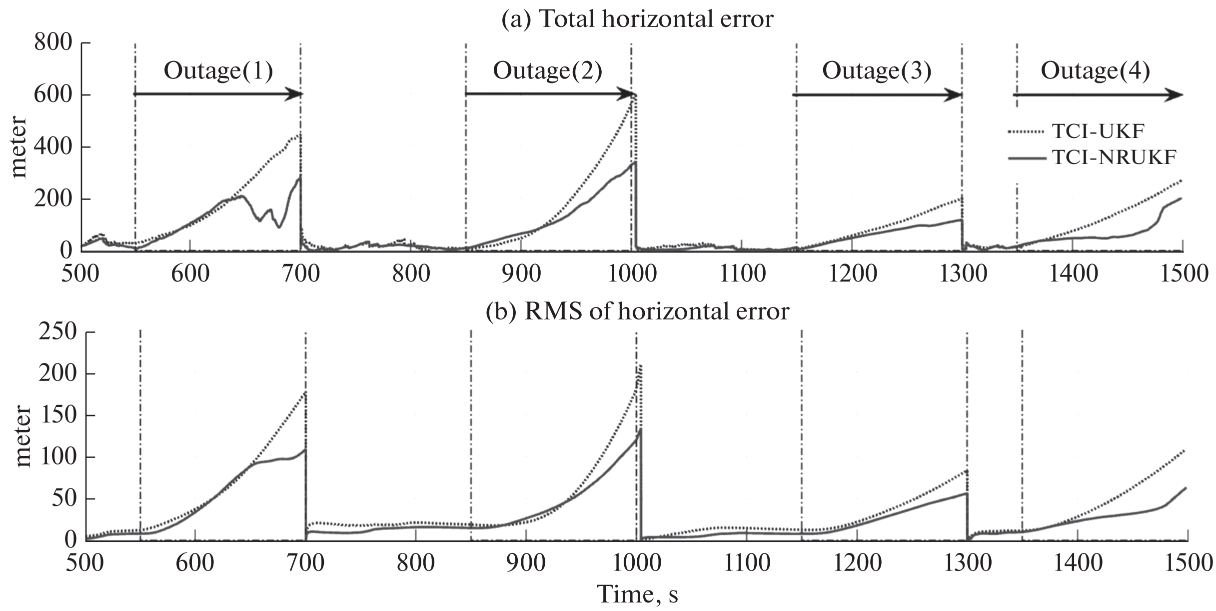

Fig. 9. (a) Horizontal error. (b) RMS of Horizontal error.

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