Proceedings of the 34th Chinese Control Conference
July 28-30, 2015, Hangzhou, China

Autonomous Landing of Quadrotor based on Ground Effect Modelling

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Abstract: Due to the ground effect, it is difficult for quadrotor to take off or land precisely. This paper presents a new ground effect model of quadrotor based on the Cheeseman's model for helicopter, and a ground effect compensator is introduced into the control system. Meanwhile by combining the proportional-integral-derivative (PID) control method and the robust compensation technique, a robust altitude controller is designed to keep the height of quadrotor and land it at an accurate time. Experimental results demonstrate the effectiveness of our ground effect model and the control method.

Key Words: Quadrotor, Ground Effect, Landing, Robust Control

1 Introduction

Autonomous landing is an important problem for Unmanned Aerial Vehicles (UAV) to achieve autonomous flight. Different landing strategies have been applied in recent years. And vision-based landing schemes of miniature vertical takeoff and landing (VTOL) aircrafts on fixed or moving targets have attracted many researchers' attention [1–6]. However most work mainly focused on are the accuracy of landing point and whether the aircraft can land on the target safely or not. The landing time and landing process have not been concerned. Generally, at the end of landing when aircrafts approach the surface the thrust is turned off and the aircrafts drop to the surface directly, so the landing time is not guaranteed. When an aircraft is required to land on a moving target, the accuracy of the landing time is one of the keys to the successful landing. Actually what affects the landing time significantly is ground effect.

Ground effect is an aerodynamic phenomena which reduces the induced drag of aircraft and thereby increases its lift-to-drag ratio. It is the lift increase generated by rotors when an aircraft is close to a surface. The ground effect has been studied for years. An effective ground effect model for helicopter rotor was put forward by Cheeseman and Bennett [7] based on the method of image. It is shown that the ratio of the thrust in ground effect (IGE) to that outside of ground effect (OGE) for a single rotor operating at constant power, is a function of the rotor radius and the vertical distance from propeller disk to the ground. Nonaka [8] represented the ground effect as a 2nd-order polynomial function of altitude for a small helicopter with coaxial rotors. Sharf et al. [9] focused on the experimental evaluation of the ground effect of rotors in proximity to the ground, using a Draganflyer X8 rotorcraft which was actuated by 8 propellers arranged in 4 coaxial pairs in a quadrotor configuration. And they pointed out that the ground effect on the X8 vehicle was stronger than predicated by theory in [7]. However, the proposed models in [7–9] are based on single rotor or coaxial dual-rotor arrangement, it is worthwhile to question their application to a quadrotor. Ryan and Kim [10] modeled ground effect forces experienced by a quadrotor using visual feedback. The data presented in [10] were for a specific setup in a laboratory environment and the mathematical relationship were too complex to give.

In practice, many control methods have been applied for aircraft's altitude control, such as PID [11], LQI (Linear Quadratic with Integral) [12], backstepping [13], adaptive control [16], sliding mode control [15, 16], H_∞ control [17], robust control [18] and so on. However those altitude controllers were designed to control aircraft's altitude high above the ground without ground effect. And few literature discusses the robust control of aircraft in ground effect. In [8] and [10], sliding mode controllers were designed to deal with the unmodeled dynamics and external disturbances for altitude control with ground effect compensation.

In this paper we present a ground effect model for quadrotor based on Cheeseman's [7] and introduce a ground effect compensator in the altitude controller. A PID controller combined with a robust compensator is designed in the altitude channel. Experimental results show that the quadrotor could track the predefined trajectory very well and land at the exact time with our ground effect compensator and the robust controller. It is a foundation for landing an aircraft on a ship which demands strict landing time and good tracking performance.

The rest of the paper is organized as follows: Section 2 describes the quadrotor system and its dynamical model. An improved ground effect model for quadrotor is presented in Section 3.The robust altitude controller is designed in Section 4 and experimental results on the quadrotor are given in Section 5. Finally, Section 6 concludes the paper and points out our future work.

2 The Quadrotor System

The experimental quadrotor developed by our UAV laboratory, is depicted in Fig. 1. It is based on the mechanical frame of the Flycker MH750, and the diagonal wheelbase of rotors is shorten to 600mm because of the modified arms. Four MH-4115 rotors and four 1505 propellers combine to give the quadrotor a maximum take-off weight of 5.5 Kg. The control system of the quadrotor contains an onboard flight control processor, a downward facing camera

This work was supported by the National Natural Science Foundation of China under Grants 61374034 , 61210012 and 61203071.

with a 2-axis gimbal mounted on the bottom of the vehicle, an onboard minicomputer and a sensor system. The sensor system consists of an inertial measurement unit (IMU) module (which includes a 3-axis digital accelerometer, three gyroscopes and a compass), an ultrasonic sensor, and a Novatel GPS module. The flight data are sent to the ground station via ZigBee, and the station is used for monitoring quadrotor's current status and modifying controller parameters. The quadrotor is capable of flying autonomously without the ground computer. An Ubuntu operating system is installed in the minicomputer, and the vision processing program runs in a Robot Operating System (ROS) environment. The total weight of our quadrotor is approximately 4.3 kg including a 5300mAh battery.

Fig. 1: Quadrotor used in this paper

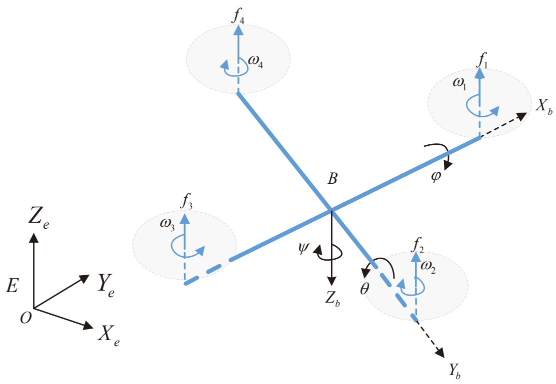

Fig. 2: Quadrotor coordinate system

Let $E = \{X_e, Y_e, Z_e\}$ denote a right-hand inertial frame such that Z_e denotes the vertical direction upwards. Let $B = \{X_b, Y_b, Z_b\}$ be the body fixed frame linked to the quadrotor, which is considered as a rigid body. The origin of the body fixed frame is the mass center of the quadrotor. The coordinate frames E and B are depicted in Fig. 2.The Euler angles $\eta = [\varphi, \theta, \psi]^T$ represent the roll, pitch, yaw angles respectively. The orientation of the rigid body is given by a rotation matrix $R : B \to E$, and $R \in SO(3)$ as follows:

$$R = \begin{bmatrix} C_\theta C_\psi & S_\varphi S_\theta C_\psi - C_\varphi S_\psi & S_\varphi S_\psi + C_\varphi S_\theta C_\psi \\ C_\theta S_\psi & C_\varphi C_\psi + S_\varphi S_\theta S_\psi & C_\varphi S_\theta S_\psi - S_\varphi C_\psi \\ -S_\theta & C_\theta S_\varphi & C_\varphi C_\theta \end{bmatrix}$$

here $C_\bullet = \cos(\bullet)$ and $S_\bullet = \sin(\bullet)$.

The full dynamics of the quadrotor based on Newton-Euler equations of motion can be described as

$$F = m\ddot{\xi} \tag{1}$$

$$J\dot{\Omega} = -\Omega \times J\Omega + \Gamma \tag{2}$$

where F is the total force, m the mass of the quadrotor, $\xi = [x, y, z]^T$ denotes the position of the mass center of the quadrotor relative to the origin $O \in E$ expressed in the frame E, $J = diag(I_{xx}, I_{yy}, I_{zz}) \in {}^{3\times3}$ the simplified inertia matrix, Ω the angular velocity of the quadrotor in the body frame, Γ the total external torque. When we neglect the air resistances and propeller gyroscopic effects, Eqs (1) and (2) can be rewritten as follows (see, also [19]):

$$\begin{aligned} m\ddot{x} &= (C_\varphi S_\theta C_\psi + S_\varphi S_\psi)\sum_{i=1}^{4} f_i \\ m\ddot{y} &= (C_\varphi S_\theta S_\psi - S_\varphi C_\psi)\sum_{i=1}^{4} f_i \\ m\ddot{z} &= C_\varphi C_\theta \sum_{i=1}^{4} f_i \text{ - } mg \\ I_{xx}\ddot{\varphi} &= \dot{\theta}\dot{\psi}(I_{yy} - I_{zz}) + lb(\omega_4^2 - \omega_2^2) \\ I_{yy}\ddot{\theta} &= \dot{\varphi}\dot{\psi}(I_{zz} - I_{xx}) + lb(\omega_1^2 - \omega_3^2) \\ I_{zz}\ddot{\psi} &= \dot{\varphi}\dot{\theta}(I_{xx} - I_{yy}) + d(\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2) \end{aligned} \tag{3}$$

where $f_i = b\omega_i^2$ $(i = 1, 2, 3, 4)$ are the thrusts produced by the four rotors respectively, ω_i the angular velocity of rotor i, l the displacement of the rotors with respect to the center of mass of the quadrotor, b the thrust coefficient and d the drag coefficient. Define the control inputs U_i $(i = 1, 2, 3, 4)$ as

$$\begin{aligned} U_1 &= b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ U_2 &= b(\omega_4^2 - \omega_2^2) \\ U_3 &= b(\omega_1^2 - \omega_3^2) \\ U_4 &= d(\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2) \end{aligned} \tag{4}$$

Combining (3) and (4) results in that

$$\begin{aligned} \ddot{x} &= \frac{1}{m}(C_\varphi S_\theta C_\psi + S_\varphi S_\psi)U_1 \\ \ddot{y} &= \frac{1}{m}(C_\varphi S_\theta S_\psi - S_\varphi C_\psi)U_1 \\ \ddot{z} &= \frac{1}{m}C_\varphi C_\theta U_1 - g \\ \ddot{\varphi} &= \dot{\theta}\dot{\psi}\left(\frac{I_{yy} - I_{zz}}{I_{xx}}\right) + \frac{l}{m}U_2 \\ \ddot{\theta} &= \dot{\varphi}\dot{\psi}\left(\frac{I_{zz} - I_{xx}}{I_{yy}}\right) + \frac{l}{m}U_3 \\ \ddot{\psi} &= \dot{\varphi}\dot{\theta}\left(\frac{I_{xx} - I_{yy}}{I_{zz}}\right) + \frac{1}{m}U_4 \end{aligned} \tag{5}$$

3 Ground Effect Model

In this section, an improved ground effect model is developed for our quadrotor through experimentations. It is well known that the aircraft operating near the ground experiences "ground effect" which produces more thrust than flying at a large distance from the ground. A commonly-used model of ground effect of helicopter was developed by Cheeseman and Bennett [7] as

$$\frac{T_{IGE}}{T_{OGE}} = \frac{1}{1 - \left(\frac{r}{4z_r}\right)^2} \tag{6}$$

where r is the radius of the rotor, z_r is the vertical distance from rotor to the ground, T_{OGE} is the thrust generated by the rotor outside of ground effect , while T_{IGE} is the thrust generated by the same rotor in ground effect which includes the normal thrust of rotor and the additional thrust because of ground effect. Note that T_{IGE} and T_{OGE} are in the case of constant power, and T_{IGE} is always not less than T_{IGE}. However the empirical formula (4) is obtained from a single rotor. It may be unsuitable for quadrotor. The four rotors stand close to each other for quadrotor, and there may exist some unpredictable airflow influences between them. So we introduce a correction coefficient into Eq (4) to model the ground effect of our quadrotor as follows

$$\frac{T_{input}}{T_{output}} = 1 - \rho\left(\frac{r}{4z_r}\right)^2 \tag{7}$$

where T_{input} is the input thrust command we give, T_{output} is the actual thrust generated by quadrotor, ρ is a coefficient to be determined.

Remark 1: Ideally, T_{output} is equal to T_{input} outside of ground effect. But in condition of ground effect, more thrust will be produced than expected, so T_{output} is greater than T_{input} in ground effect.

Remark 2: T_{input} can be easily gotten by recording the input thrust commands. In general, the actual thrust generated by rotors, T_{output}, is equal to quadrotor's weight plus its vertical acceleration (i.e., $T_{output} = m(g + \ddot{z})$), and it is hard to be measured. But quadrotor's vertical acceleration is almost zero in hover, so T_{output} can be regarded as a constant value.

To identify the coefficient ρ, a few hover experiments at different altitudes have been performed with our quadrotor. Fig. 3 depicts the average input thrust T_{output} which maintains the quadrotor hovering at each altitude z_r for several seconds (red stars). Note that the altitude z_r is the vertical distance from the rotor to the ground, so the minimum of z_r is 0.43 m (the height of landing gear). From Eq (7), the function representing the input thrust for different altitude is given by

$$T_{input} = T_{output}\left(1 - \rho\left(\frac{r}{4z_r}\right)^2\right) \tag{8}$$

where r = 0.1905 m is the radius of 15$''$ propeller. Using least square approximation, we get ρ = 8.6 when T_{output} = 41.73 N . The fitted curve is shown in Fig. 3 (green line).

Fig. 4 depicts the ratio of T_{input} to T_{output} against z_r/r. The smaller T_{input}/T_{output} is, the more significant the ground effect is, and T_{input}/T_{output} equals 1 without ground effect. From Fig. 4 we can see that the influence of the ground is still apparent at height corresponding to z_r/r = 4, much higher than that predicted by the models in [7] and

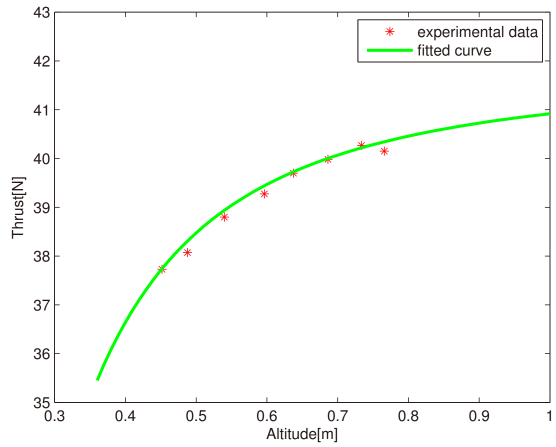

Fig. 3: Input thrust vs. Altitude

[9]. Cheeseman and Bennett [7] indicated the ground effect is significant in the range $0.5 < z_r/r < 2$, while in Sharf's [9] $0.5 < z_r/r < 3$. The different arrangement of rotors may be the major cause leading to the different results in [7], [9] and this paper. Cheeseman and Bennett's model is based on a single isolated propeller, and Sharf's model relates to multiple coaxial pairs, while we use a common quadrotor with four rotors. The rotor clearance (tip-to-tip) is 0.045 m in our quadrotor while 0.064 m for top rotor and 0.085 m for bottom rotor in Sharf's [9].

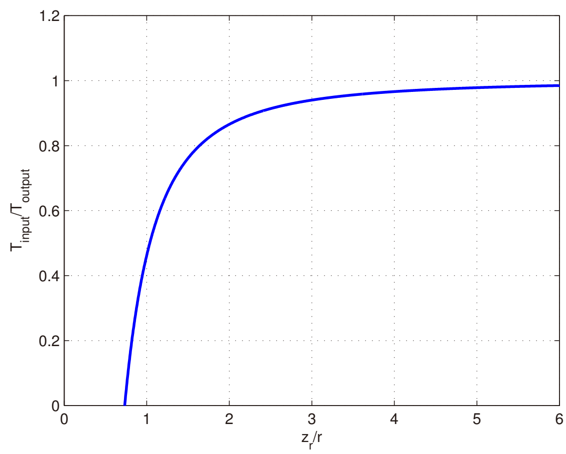

Fig. 4: Ground effect vs. z_r/r

4 Altitude Controller Design

Many uncertainties including parameter perturbations, coupling, nonlinear dynamics, and external disturbances are ignored in above quadrotor model and ground effect model. To cope with the model errors and the uncertainties, a robust altitude controller is proposed in this section, which includes two parts: a nominal PID controller and a robust compensator. The robust compensator is easy to be realized in practical applications and the parameters can be tuned on-line.

Combining the ground effect model in (8), air resistances, and other uncertainties into (5), the dynamics of altitude can be given by:

$$\ddot{z} = \frac{1}{m} C_{\varphi} C_{\theta} U_z - g - k_z \dot{z} + d_z \tag{9}$$

$$U_1 = U_z \left(1 - \rho \left(\frac{r}{4z_r} \right)^2 \right) \tag{10}$$

where k_z is the drag coefficient, d_z the external disturbances. U_z is the virtual control input while U_1 is the actual control input for altitude.

Remark 3: z_r is the vertical distance from rotor to the ground, but z is the vertical distance from the mass center of quadrotor to the ground, and there is a constant bias of 0.26 m between them, i.e. $z_r = z + 0.26$.

Let $a_z = 1/m$, then (9) can be expressed as

$$\ddot{z} = a_z^N (U_z + q_z) - g \tag{11}$$

where a_z^N is the nominal parameter, q_z is the named equivalent disturbances and takes the following form

$$q_z = (-k_z \dot{z} + \cos\theta \cos\varphi a_z U_z - a_z^N U_z + d_z)/a_z^N \tag{12}$$

The virtual control input U_z consists of two parts: the nominal control input U_z^N and the robust compensating input U_z^{RC}:

$$U_z = U_z^N + U_z^{RC} \tag{13}$$

The nominal input U_z^N is designed for the nominal system without the equivalent disturbances q_z, while the robust compensating input U_z^{RC} is introduced to restrain the influence of q_z.

A. Nominal controller design

From Eq (11), the nominal model of altitude is as follows

$$\ddot{z} = a_z^N U_z - g \tag{14}$$

The nominal controller is designed as a PID controller

$$\begin{aligned} U_z^N = \Big(& k_p (z_d - z) + k_i \int (z_d - z) dt \\ & + k_d (\dot{z}_d - \dot{z}) + \ddot{z}_d + g \Big) / a_z^N \end{aligned} \tag{15}$$

where k_p, k_i, k_d are positive controller parameters, z is the quadrotor's altitude and z_d is the desired altitude. The nominal controller input U_z^N makes the altitude z track the desired one z_d.

B. Robust compensator design

The robust compensator is constructed based on the robust filter

$$F_z(s) = f_z g_z / \big((s + f_z)(s + g_z) \big)$$

where s is the Laplace operator, f_z and g_z are positive constants to be determined. The robust compensating input U_z^{RC} is construct as

$$U_z^{RC}(s) = -F_z(s) q_z(s) \tag{16}$$

If the parameters f_z and g_z are sufficiently large, one can expect that U_z^{RC} would approximate $-q_z$ and thereby the effect of the disturbances in the altitude channel can be attenuated. However, the equivalent disturbances q_z cannot be measured directly. Then from (11), one can obtain an expression of q_z as

$$q_z = (\ddot{z} + g)/a_z^N - U_z \tag{17}$$

Therefore U_z^{RC} can be realized with states z_{z1} and z_{z2} as

$$\begin{aligned} \dot{z}_{z1} &= -g_z z_{z1} - g_z^2 z + a_z^N U_z - g \\ \dot{z}_{z2} &= -f_z z_{z2} + (f_z + g_z) z + z_{z1} \\ U_z^{RC} &= f_z g_z (z_{z2} - z)/a_z^N \end{aligned} \tag{18}$$

The expected virtual control input U_z is obtained above, while the actual control input U_1 is smaller than expected in ground effect. So U_1 should be compensated by the ground effect compensator (10) based on U_z.

The block diagram of the whole altitude controller is depicted in Fig. 5.

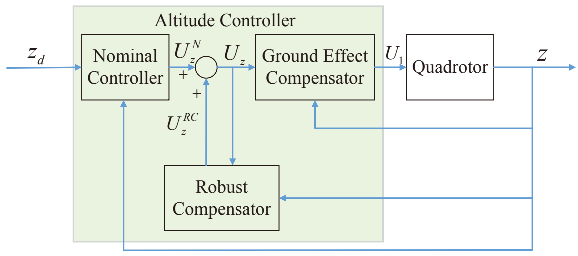

Fig. 5: The block diagram of the altitude controller

5 Experimental Results

To verify the ground effect model and the altitude controller, four outdoor experiments have been performed. The first three experiments were conducted without robust compensator to verify the effectiveness of our ground effect model. There was no ground effect compensator in the first experiment, while a ground effect compensator based on Cheeseman's model in Eq (6) (i.e., ρ = 1) was introduced into the second experiment and the third one was performed with the ground effect compensator based on our quadrotor's model in Eq (7) (i.e., ρ = 8.6). Different from the third experiment, a robust compensator was also added in the fourth experiment. The horizontal position of the quadrotor is provided by a vision system, and camera and ultrasonic sensor are used to measure the height. Since the ultrasonic sensor is fixed with 0.18 m ground clearance, the minimum of quadrotor's actual altitude is 0.18 m. In the experiments we made the quadrotor hover in the beginning, then trigged the landing command for a descent flight, and the landing time was set as 6 s. A nonzero landing speed is designed to ensrue the quadrotor land to the ground safely, so the reference altitude is smaller than 0.18 m in the end.

Fig. 6 and Fig. 7 show the response and tracking error of altitude in the first experiment respectively. The landing command was trigged at t = 2.89 s and the quadrotor was designed to touch the ground at t = 8.89 s. Without the ground effect compensator, one can see that the quadrotor could not track well when the altitude is below 0.5 m where ground effect is significant (see the enlarged part). And it could not land on the ground as expected because of the ground effect. The actual altitude at t = 8.89 s is 0.31 m

which much higher than expected, 0.18 m, and the root mean square error (RMSE) is 0.1394 m.

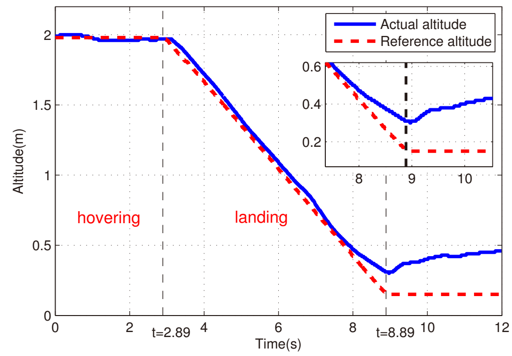

Fig. 6: Altitude response in the first experiment (without ground effect and robust compensators)

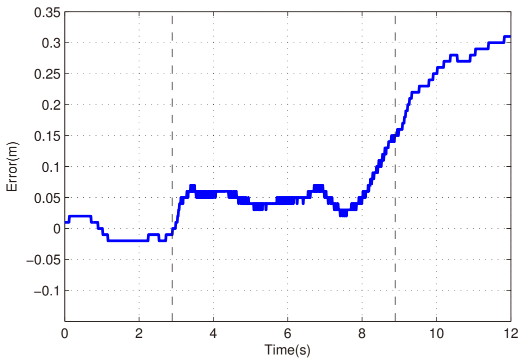

Fig. 7: Altitude tracking error in the first experiment

The experimental results in the second experiment are shown in Fig. 8 and Fig. 9. Though a ground effect compensator based on Cheeseman's model was introduced into the altitude controller, there were not much differences between the first and the second experiment. The quadrotor still descended slower in the end and could not land on the ground as desired. The actual altitude at $t = 8.89$ s is 0.305 m and the RMSE is 0.1265 m.

Fig. 10 and Fig. 11 give the landing results in the third experiment. With the ground effect compensator based on our quadrotor's ground effect model in Eq (7), the quadrotor could land on the ground in spite of the adverse ground effect. However the track error is still significant, the maximum error is 0.10 m and the RMSE is 0.0536 m. And owing to the nonzero landing speed, the quadrotor jumped slightly after touching the ground.

The experimental results with ground effect and robust compensators in the fourth experiment are depicted in Fig. 12 and Fig. 13. The landing command was trigged at $t = 3.19$ s and the quadrotor was designed to touch the ground at $t = 9.19$ s. In fact, the actual altitude is 0.20 m at $t = 9.19$ s which is almost the same as the ground clearance, 0.18 m. The maximum error is 0.07 m and the RMSE is 0.0314 m, smaller than those in the third experiment.

From the first three experiments, one can see that the ground effect compensator plays an important role in the altitude controller. Without the ground effect compensator, the nominal PID controller could not deal with the ground effect near the ground. The second experiment shows that the helicopter's ground effect model is not suitable for quadrotor, and the results in the third experiment demonstrate the effectiveness of the ground effect model we proposed. Comparing the results in the third and fourth experiment, we can see that errors are made smaller and a better tracking performance can be achieved because of the robust compensator. Moreover, a precise landing with accurate time can be realized successfully with the two compensators.

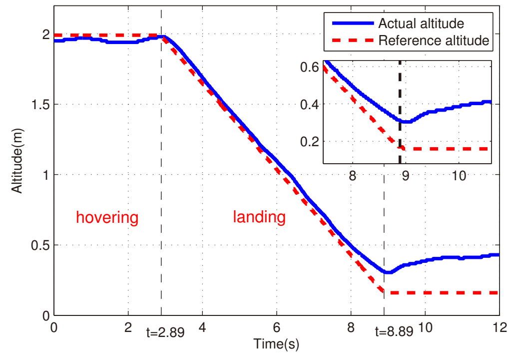

Fig. 8: Altitude response in the second experiment (with ground effect compensator when ρ = 1 and without robust compensator)

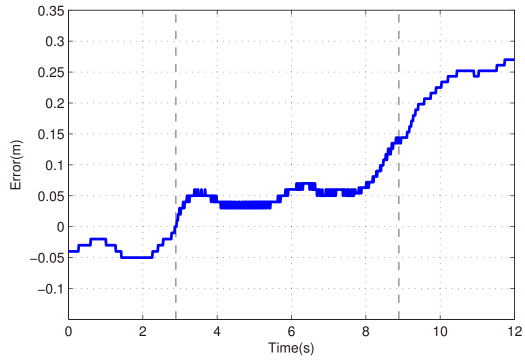

Fig. 9: Altitude tracking error in the second experiment

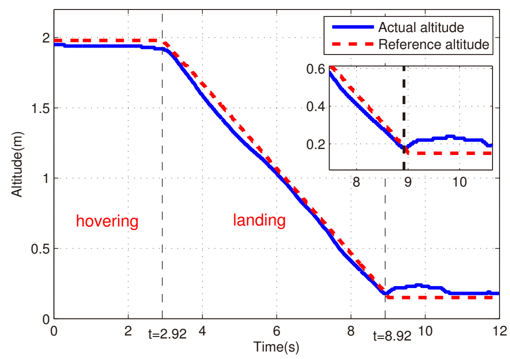

Fig. 10: Altitude response in the third experiment (with ground effect compensator when ρ = 8.6 and without robust compensator)

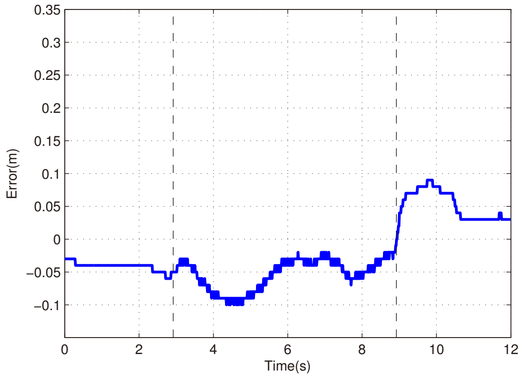

Fig. 11: Altitude tracking error in the third experiment

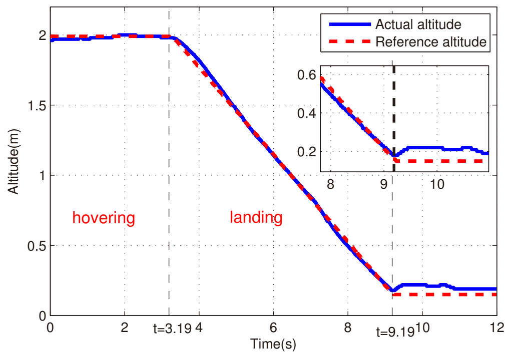

Fig. 12: Altitude response in the fourth experiment (with ground effect compensator when ρ = 8.6 and with robust compensator)

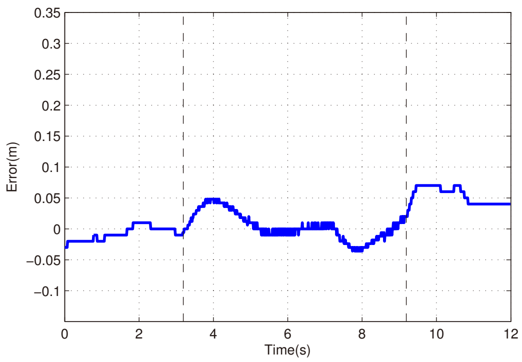

Fig. 13: Altitude tracking error in the fourth experiment

6 Conclusion and Future Work

In this paper, an experiment based ground effect model for quadrotor was built and a robust altitude controller was proposed. The effectiveness of the model and controller have been demonstrated in some flight experiments. Moreover, we have successfully landed the quadrotor in strict time that we set in advance, and a good tracking performance was achieved.

In the future, we plan to land the quadrotor on a moving platform or a ship which requires higher performance.

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