

Prologue

Project goal: re-affirm the atomic nature of matter by tracking the motion of particles undergoing Brownian motion, fitting this data to Einstein's model, and estimating Avogadro's number

Relevant lecture material

- → Recursion 🗷
- → Using Data Types ♂
- → Creating Data Types &

Files

- → project5.pdf

 (project description)
- → projects.zip C (starter files for the exercises/problems, report.txt file for the project report, and run_tests.py file to test your solutions)

Exercise 1. (Sum of Integers) Implement the functions $sum_iter()$ and $sum_irec()$ in $sum_of_ints.py$ that take an integer n as argument and return the sum $S(n) = 1 + 2 + 3 + \cdots + n$, computed iteratively (using a loop) and recursively. The recurrence equation for the latter implementation is

$$S(n) = \begin{cases} 1 & \text{if } n = 1, \\ n + S(n-1) & \text{if } n > 1. \end{cases}$$

```
$ python3 sum_of_ints.py 100
5050
```

```
sum_of_ints.py
import stdio
import sys
# Returns the sum S(n) = 1 + 2 + ... + n, computed iteratively.
def sum_iter(n):
# Returns the sum S(n) = 1 + 2 + ... + n, computed recursively.
def sum_rec(n):
# Test client [DO NOT EDIT]. Reads an integer n from command line and
# writes the sum S(n) = 1 + 2 + ... + n, computed both iteratively and
# recursively.
def main():
   n = int(sys.argv[1])
    stdio.writeln(sum iter(n))
    stdio.writeln(sum_rec(n))
if __name__ == '__main__':
    main()
```

Exercise 2. (Bit Counts) Implement the functions zeros() and ones() in bits.py that takes a bit string (ie, a string of zeros and ones) s as argument and returns the number of zeros and ones in s, each computed recursively. The number of zeros in a bit string is 1 or 0 (if the first character is '0' or '1') plus the number of zeros in the rest of the string; number of zeros of an empty string is 0 (base case). The number of ones in a bit string can be defined analogously.

\$ python3 bits.py 101001001001111100010111111
zeros = 11, ones = 14, total = 25

```
bits.py
import stdio
import sys
# Return the number of zeros in s, computed recursively.
def zeros(s):
# Return the number of ones in s, computed recursively.
def ones(s):
# Test client [DO NOT EDIT]. Reads a string s from command line and writes the
# the number of zeros and ones in s, both computed recursively.
def main():
    s = sys.argv[1]
    stdio.writef('zeros = %d, ones = %d, total = %d\n',
                 zeros(s), ones(s), len(s))
if __name__ == '__main__':
   main()
```

Exercise 3. (String Reversal) Implement the function reverse() in reverse.py that takes a string s as argument and returns the reverse of the string, constructed recursively. The reverse of a string is the last character concatenated with the reverse of the string up to the last character; the reverse of an empty string is an empty string (base case).

```
$ python3 reverse.py bolton
notlob
$ python3 reverse.py amanaplanacanalpanama
amanaplanacanalpanama
```

```
import stdio
import sys

# Returns the reverse of the string s, computed recursively.
def reverse(s):
...

# Test client [DO NOT EDIT]. Read a string s from command line and writes its
# reverse, computed recursively.
def _main():
    s = sys.argv[i]
    stdio.writeln(reverse(s))

if __name__ == '__main__':
    _main()
```

Exercise 4. (Palindrome) Implement the function is_palindrome() in palindrome.py, using recursion, such that it returns true if the argument s is a palindrome (ie, reads the same forwards and backwards), and False otherwise. You may assume that s is all lower case and doesn't any whitespace characters. A string is a palindrome if the first character is the same as the last and the rest of the string is a palindrome; an empty string is a palindrome (base case).

```
$ python3 palindrome.py bolton
False
$ python3 palindrome.py amanaplanacanalpanama
True
```

```
palindrome.py
import stdio
import sys
# A recursive function that returns True if s is a palindrome, and False
# otherwise.
def is_palindrome(s):
# Test client [DO NOT EDIT]. Read a string s from command line and writes
# whether or not s is a palindrome.
def _main():
    s = sys.argv[1]
    stdio.writeln(is_palindrome(s))
if __name__ == '__main__':
   main()
```

Exercise 5. (Rational Number) Define a data type rational in rational.py that represents a rational number, ie, a number of the form a/b where a and $b \neq 0$ are integers. The data type must support the following API:

method	description
Rational(x, y)	a new rational r from the numerator x and denominator y
r + s	sum of r and s
r - s	difference of r and s
r * s	product of r and s
abs(r)	absolute value of r
str(r)	the string representation of r as $_{^{'}x/y'}$

Use the private function $_{\tt gcd()}$ to ensure that the numerator and denominator never have any common factors. For example, the rational number 2/4 must be represented as 1/2.

\$ python3 rational.py 100 3.1315929035585515

```
rational.py
import stdio
import sys
# Returns the GCD of p and q, computed using Euclid's algorithm.
def _gcd(p, q):
   return p if q == 0 else _gcd(q, p % q)
class Rational:
    Represents a rational number.
    ....
    def __init__(self, x, y=1):
        Constructs a new rational given its numerator and denominator.
        d = gcd(x, y)
        self._x = ...
        self._y = ...
    def __add__(self, other):
        Returns the sum of self and other.
        ....
    def __sub__(self, other):
        Returns the difference of self and other.
```

```
rational.py
    def __mul__(self, other):
        Returns the product of self and other.
    def __abs__(self):
        Return the absolute value of self.
    def __str__(self):
        Returns a string representation of self.
        a, b = self._x, self._y
        if a == 0 or b == 1:
            return str(a)
        if h < 0.
            a *= -1
            b *= -1
        return str(a) + '/' + str(b)
# Test client [DO NOT EDIT]. Reads an integer n as command-line argument and
# writes the value of PI computed using Leibniz series:
# PI/4 = 1 - 1/3 + 1/5 - 1/7 + ... + (-1)^n/(2n-1).
def _main():
    n = int(sys.argv[1])
    total = Rational(0)
    sign = Rational(1)
    for i in range(1, n + 1):
        total += sign * Rational(1, 2 * i - 1)
        sign *= Rational(-1)
```

```
stdio.writeln(4.0 * total._x / total._y)
if __name__ == '__main__':
   _main()
```



Student

The guidelines for the project problems that follow will be of help only if you have read the description $\ensuremath{\mathcal{C}}$ of the project and have a general understanding of the problems involved. It is assumed that you have done the reading.

Instructor

Please summarize the project description \mathcal{C} for the students before you walk them through the rest of this checklist document.

Problem 1. ($Particle\ Identification$) Define a data type Blob that has the following API:

method	description
Blob()	an empty blob b
b.add(i, j)	add a pixel (i, j) to the b
b.mass()	the number of pixels in b , ie, its mass
b.distanceTo(c)	the distance between the centers of b and c
str(b)	string representation of b 's mass and center of mass

Next, define a data type ${\mbox{\scriptsize BlobFinder}}$ that has the following API:

method	description
BlobFinder(pic, tau)	a blob finder bf to find blobs in the picture pic using a luminance threshold tau
bf.getBeads(P)	list of all beads with $\geq P$ pixels

```
Hints
```

 \leadsto Blob

→ Instance variables

- → Number of pixels, _P (int)
- \rightsquigarrow x-coordinate of center of mass, _x (float)
- → y-coordinate of center of mass, _y (float)
- → Blob()
 - → Initialize the instance variables appropriately
- \rightsquigarrow b.add(i, j)
 - \rightsquigarrow Use the idea of running average¹ to update the x- and y-coordinates of the center of mass of blob b to include the new point (i, j)
 - → Increment the number of pixels in blob b by 1
- → b.mass()
 - → Return the number of pixels in the blob b
- → h distanceTo(c)
 - → Return the Euclidean distance between the center of mass of blob b and the center of mass of blob c

¹If \bar{x}_{n-1} is the average value of n-1 points $x_1, x_2, \ldots, x_{n-1}$, then the average value \bar{x}_n of n points $x_1, x_2, \ldots, x_{n-1}, x_n$ is $\frac{\bar{x}_{n-1} \cdot (n-1) + x_n}{n}$

- → BlobFinder
 - → Instance variable
 - → Blobs identified by this blob finder, _blobs (list of Blob objects)
 - → BlobFinder()
 - \rightsquigarrow Initialize _blobs to an empty list

 - Enumerate the pixels of pic, and for each pixel (i, j): 1. Create a Blob object called blob; 2. Call _findBlob() with the appropriate arguments; and 3. Add blob to _blobs if it has a non-zero mass
 - \rightsquigarrow bf._findBlob()
 - → Base case: return if pixel (i, j) is out of bounds, or if it is marked, or if its luminance (use the function luminance(luminance()) for this) is less than tau
 - → Mark the pixel (i, j)
 - → Add the pixel (i, j) to the blob blob
 - → Recursively call _findBlob() on the N, E, W, and S pixels
 - \rightsquigarrow bf.getBeads(P)
 - → Return a list of blobs from _blobs that have a mass of at least P

Problem 2. (Particle Tracking) Implement a client program bead_tracker.py that takes an integer P, a float tau, a float delta, and a sequence of JPEG filenames as command-line arguments, identifies the beads in each JPEG image using blobFinder, and prints out (one per line, formatted with 4 decimal places to the right of decimal point) the radial distance that each bead moves from one frame to the next (assuming it is no more than delta).

Hints

- → Read command-line arguments P, tau, and delta
- → Construct a BlobFinder object for the frame sys.argv[4] and from it get a list of beads prevBeads that have at least P pixels
- \rightsquigarrow For each frame starting at sys.argv[5]
 - \leadsto Construct a BlobFinder object and from it get a list of beads currBeads that have at least P pixels
 - → For each bead currBead in currBeads, find a bead prevBead from prevBeads that is no further than delta and is closest to currBead, and if such a bead is found, write its distance (using format string '%.4f\n') to currBead
 - → Write a newline character
 - \leadsto Set prevBeads to currBeads

Problem 3. (*Data Analysis*) Implement a client program avogadro.py that reads in the displacements from standard input and computes an estimate of Boltzmann's constant and Avogadro's number using the formulae described above.

Hints

- → Calculate var as the sum of the squares of the n displacements (each converted from pixels to meters) read from standard input
- → Divide var by 2 * n
- → Initialize eta, rho, T, and R to appropriate values
- → Estimate Boltzman constant k as 6 * math.pi * var * eta * rho / T
- → Estimate Avogadro's number N_A as R / k
- → Write k and N_A using format string '%e' (for scientific notation)

Be sure to test your programs thoroughly using ten datasets (they are under the $_{\tt data}$ directory), obtained by William Ryu (Princeton University) using fluorescent imaging

Each run contains a sequence of two hundred 640-by-480 color JPEG images, framecocoo.jpg through framecologs.jpg and is stored in a subdirectory run_1 through run_10, and the directory also contains some reference solutions

Epilogue

Use the template file report.txt to write your report for the project

Your report must include

- → Time (in hours) spent on the project
- → Difficulty level (1: very easy; 5: very difficult) of the project
- → A short description of how you approached each problem, issues you encountered, and how you resolved those issues
- --- Acknowledgement of any help you received
- → Other comments (what you learned from the project, whether or not you enjoyed working on it, etc.)

Epilogue

Before you submit your files

→ Make sure your programs meet the style requirements by running the following command on the terminal

\$ pycodestyle program >

where cprogram> is the .py file whose style you want to check

→ Make sure your programs meet the input and output specifications by running the following command on the terminal

\$ python3 run_tests.py -v [<items>]

where the optional argument <irems> lists the exercises/problems (Exercise1, Problem2, etc.)
you want to test, separated by spaces; all the exercises/problems are tested if no
argument is given

- → Make sure your code is adequately commented, is not sloppy, and meets any project-specific requirements, such as corner cases and running time
- → Make sure your report uses the given template, isn't too verbose, doesn't contain lines that exceed 80 characters, and doesn't contain spelling mistakes

Epilogue

Files to submit

- 1. sum_of_ints.py
- $2.\ \mathrm{bits.py}$
- 3. reverse.py
- $4. \ {\tt palindrome.py}$
- 5. rational.py
- 6. blob.py
- 7. blob_finder.py
- 8. bead_tracker.py
- 9. avogadro.py
- 10. report.txt