Entanglement branching operator and its applications

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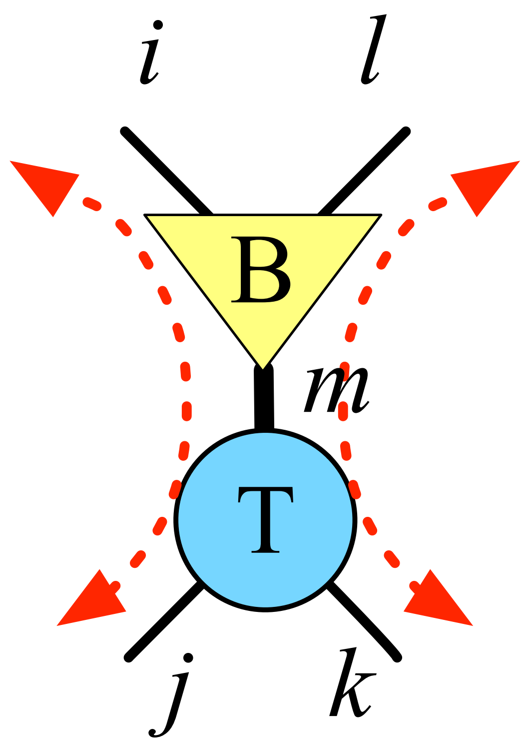
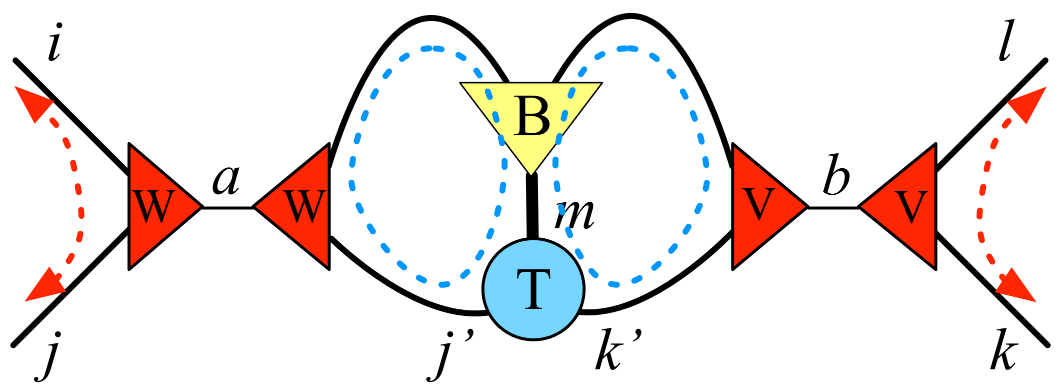
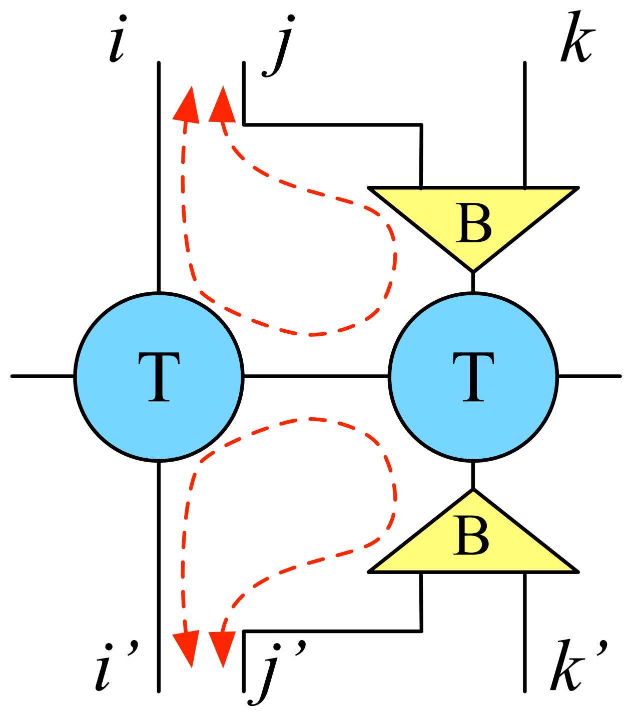
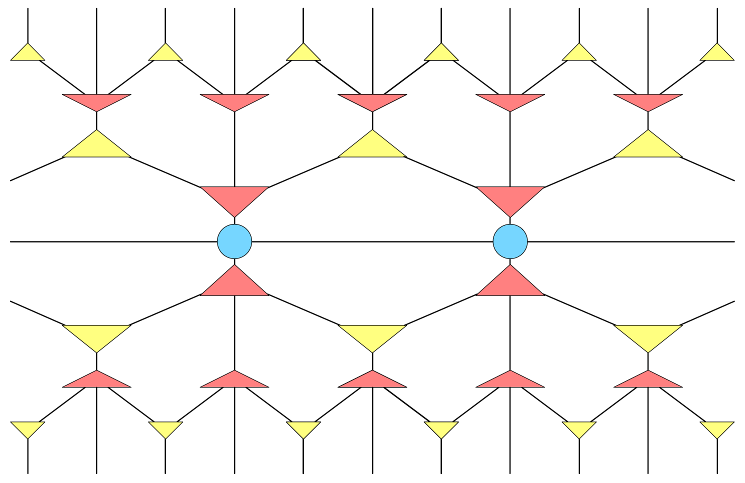
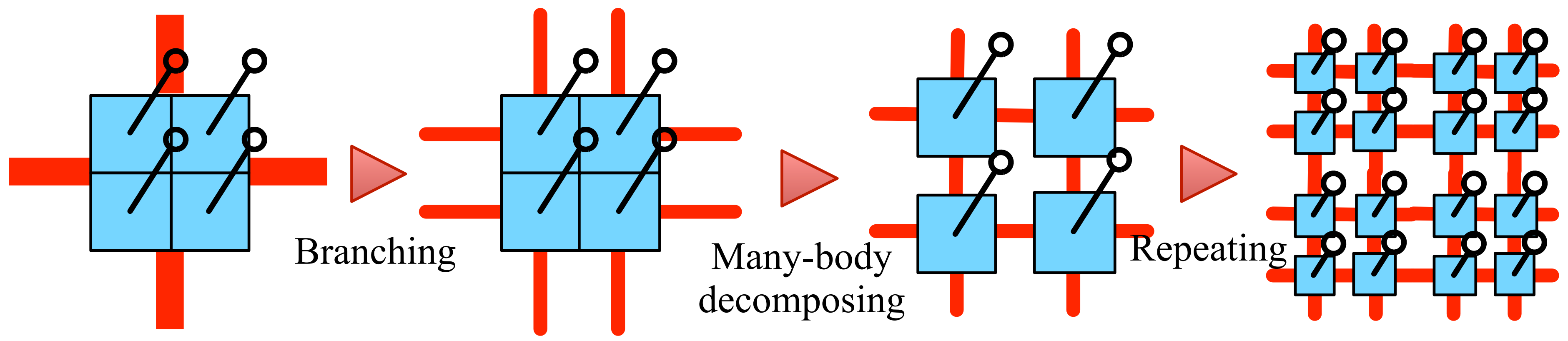






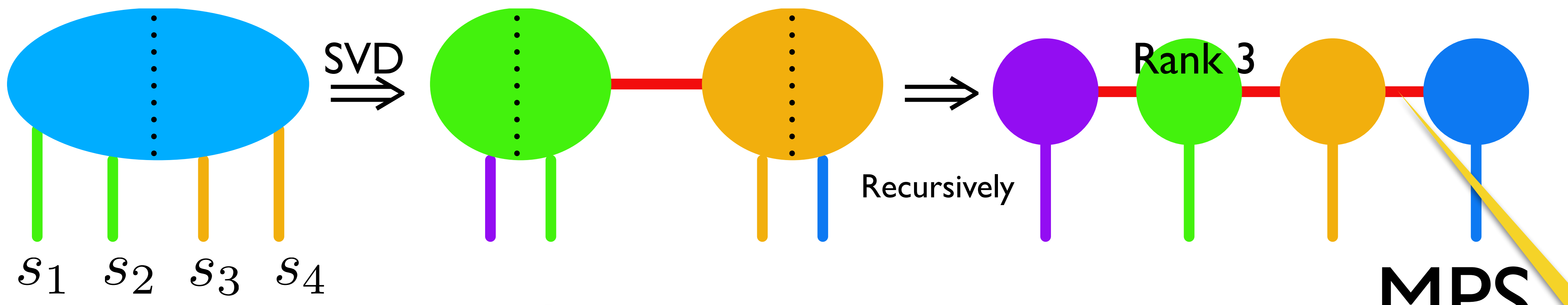




Reference: “Entanglement branching operator”, Phys. Rev. B **97**, 045124 (2018)

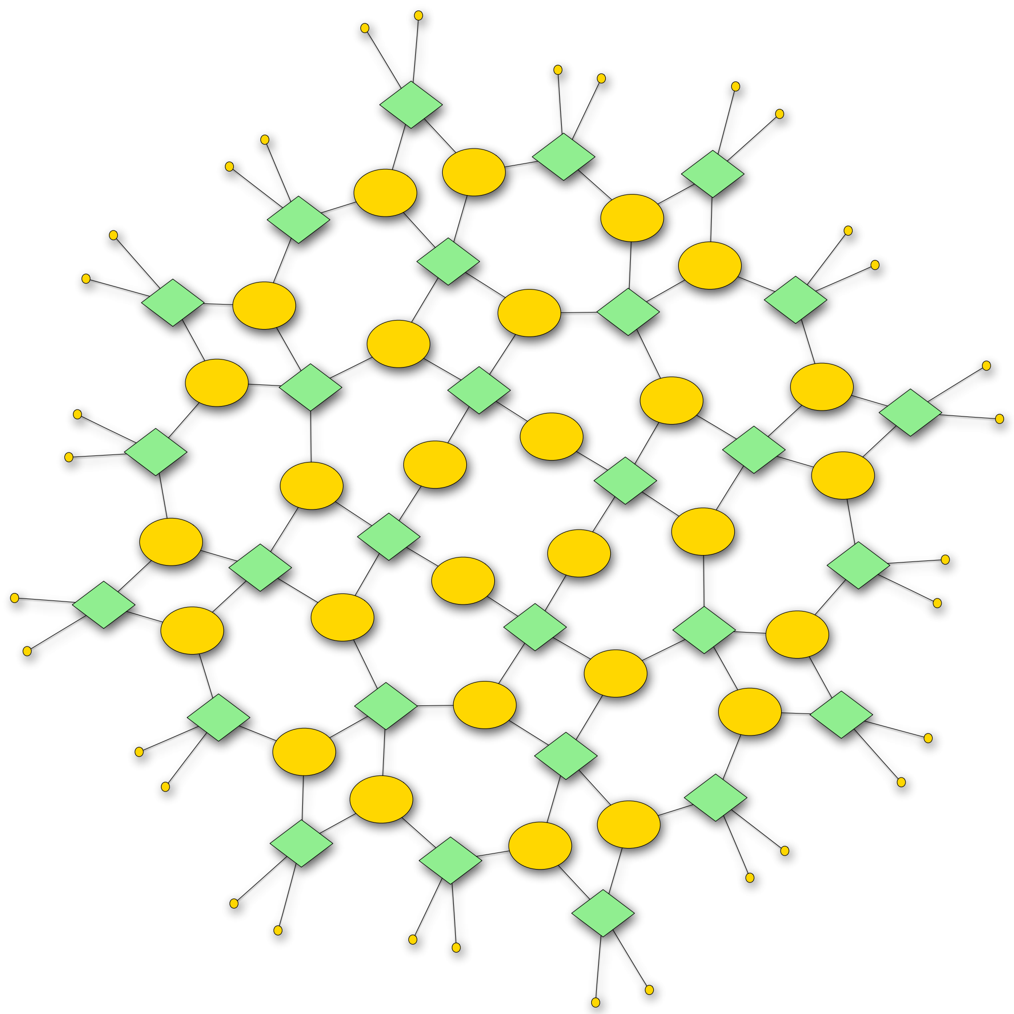
Tensor network, tensor network algorithm, and entanglement flow

1D quantum state

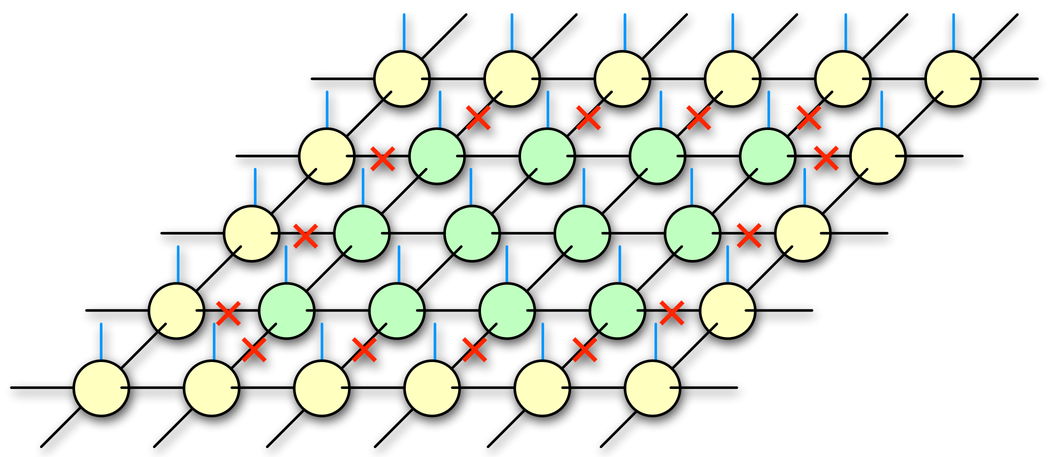


MPS

MERA

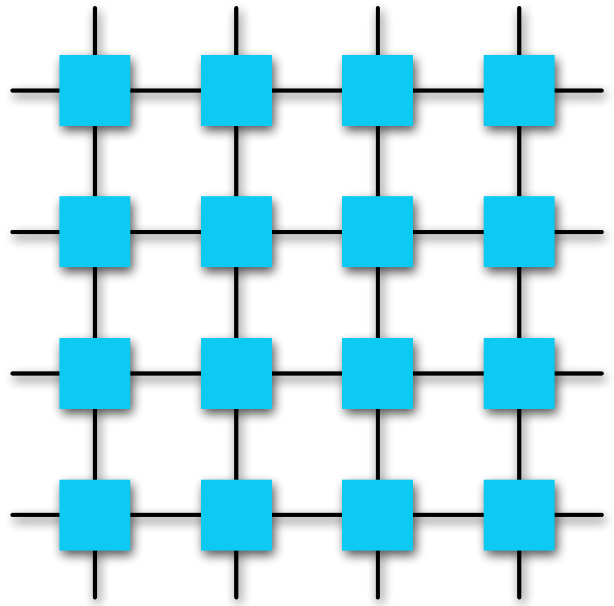


2D quantum state



PEPS(TPS)

2D classical partition function



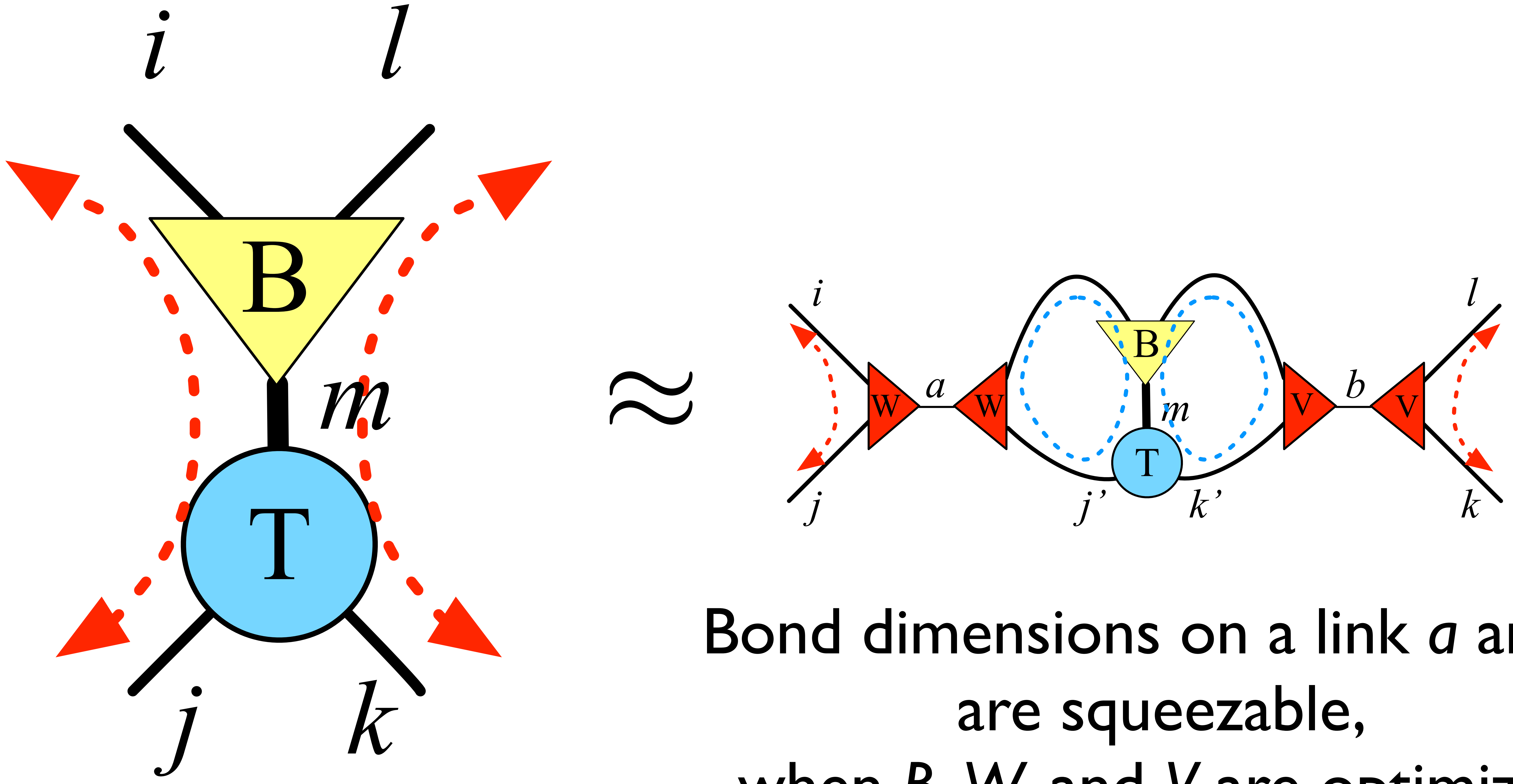
Entanglements flow in a link

Tensor network algorithm

TEBD, CTM,TRG, HOTRG, …

Entanglement branching operator

Split of a composite entanglement flow in a link



Bond dimensions on a link *a* and *b* are squeezable, when *B*, *W*, and *V* are optimized

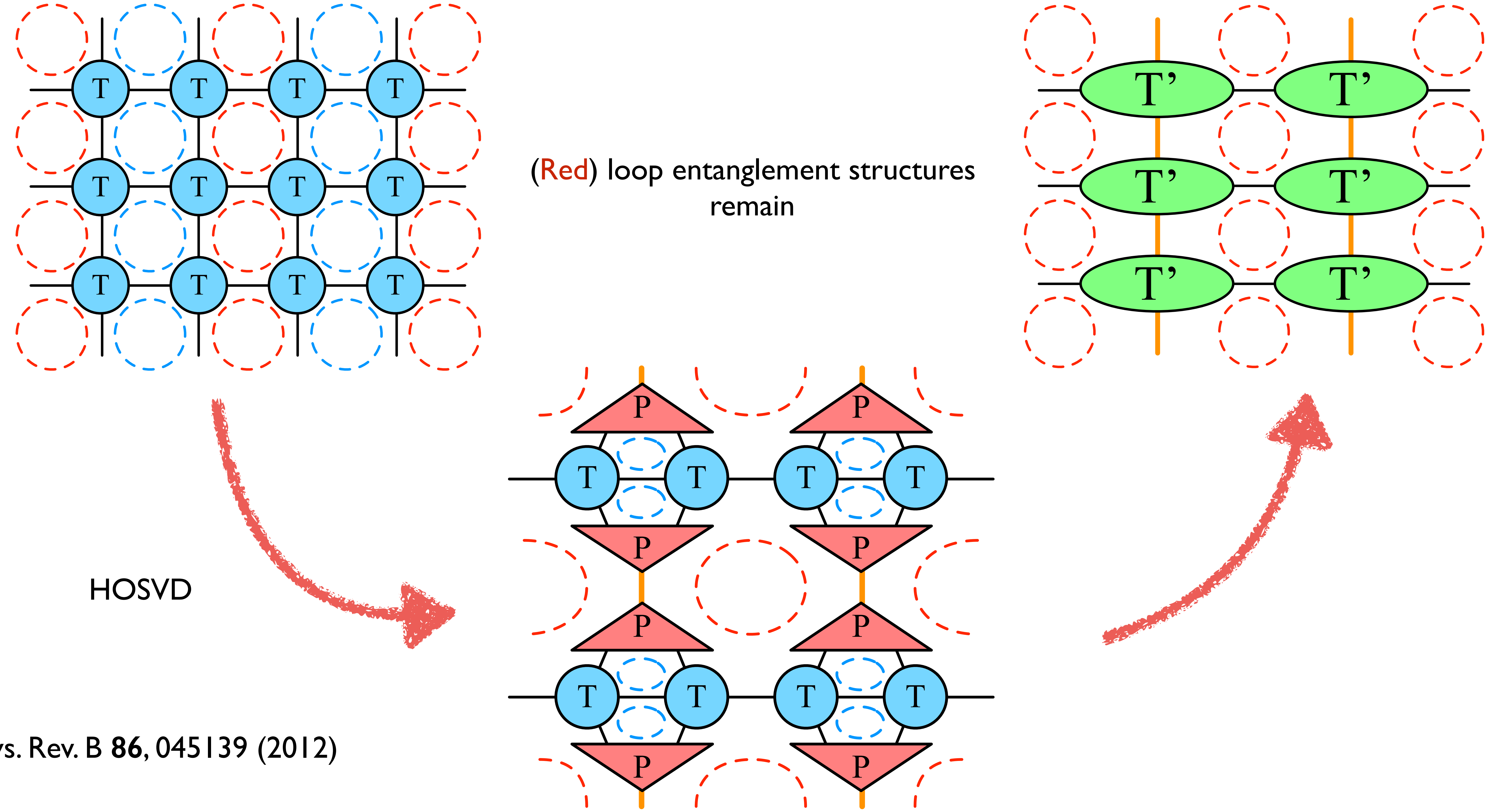
Improvement of HOTRG by entanglement branching

- **Necessary condition of a proper real-space RG**

Gu and Wen, Phys. Rev. B **80**, 155131 (2009)
Evenly and Vidal, Phys. Rev. Lett. **115**, 180405 (2015)

  - erase entanglements under a renormalized scale ⟶ TNR based on TRG (not HOTRG)

- **HOTRG algorithm**



Xie et al., Phys. Rev. B **86**, 045139 (2012)

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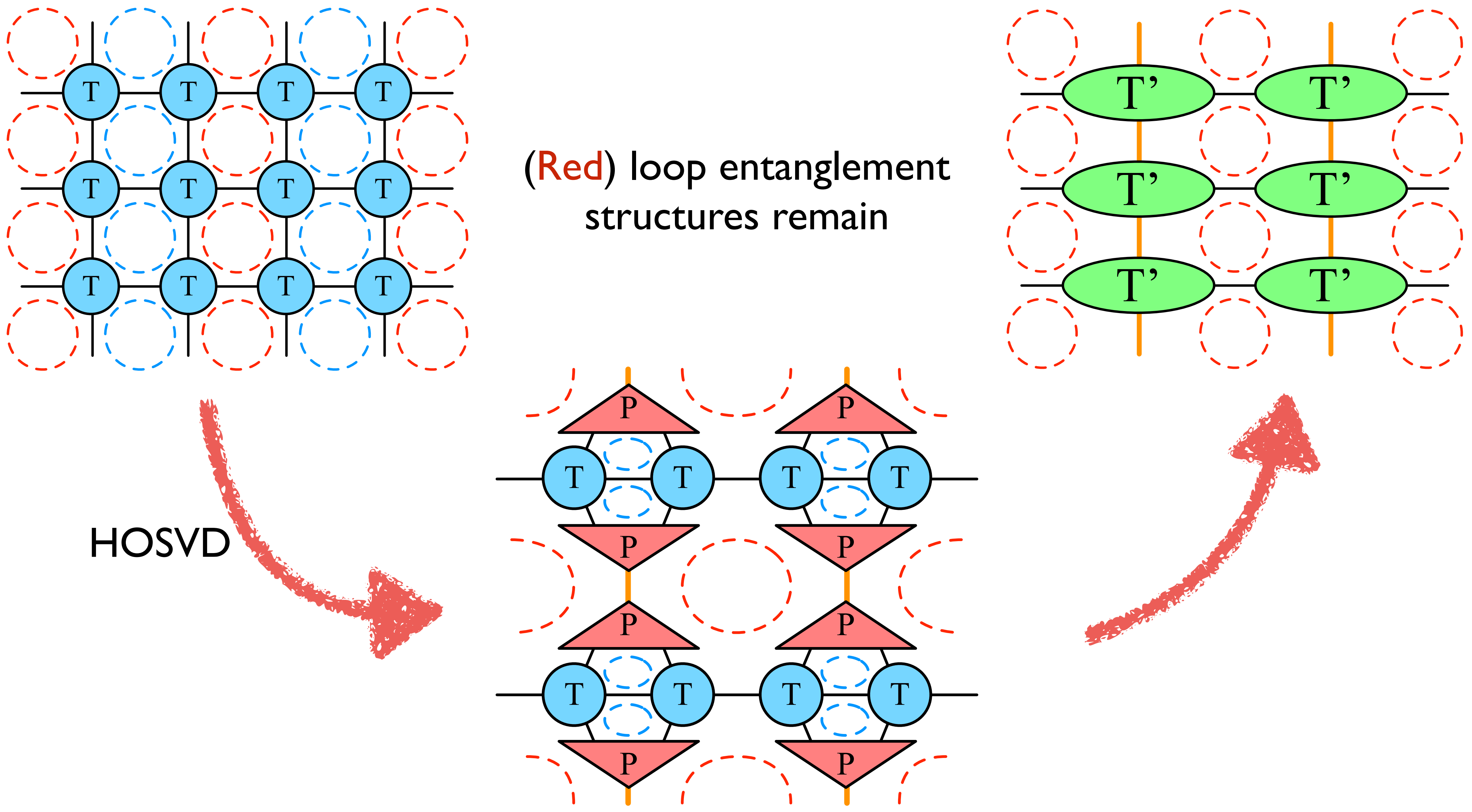

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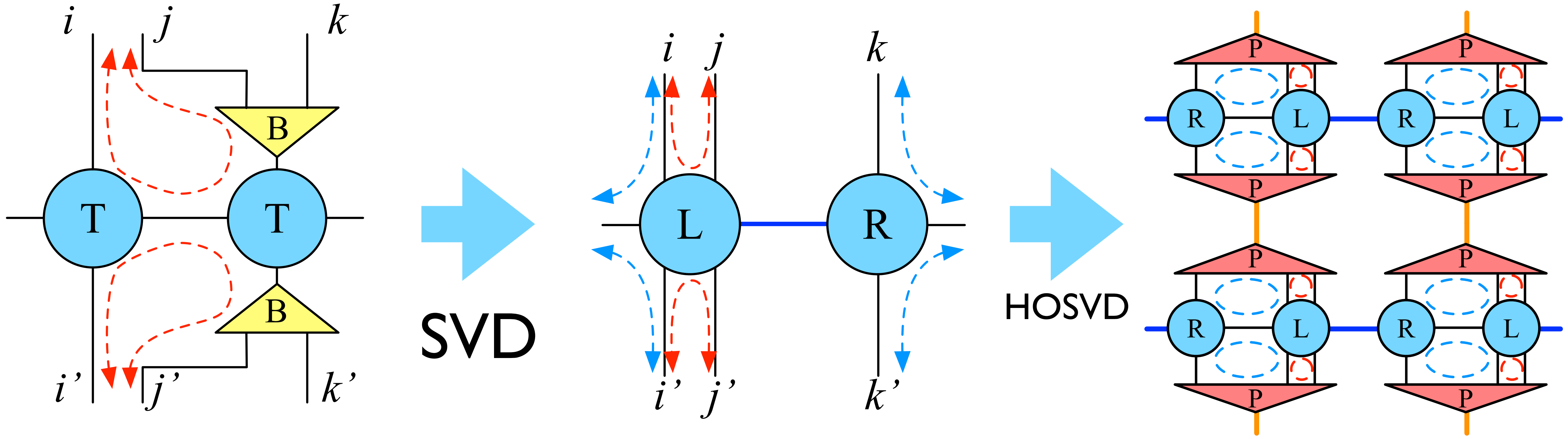
- **Pick up a red entanglement flow**



There is no entanglement between *L* and *R*.

Gather loop entanglement structures in the combination of *R* and *L*.

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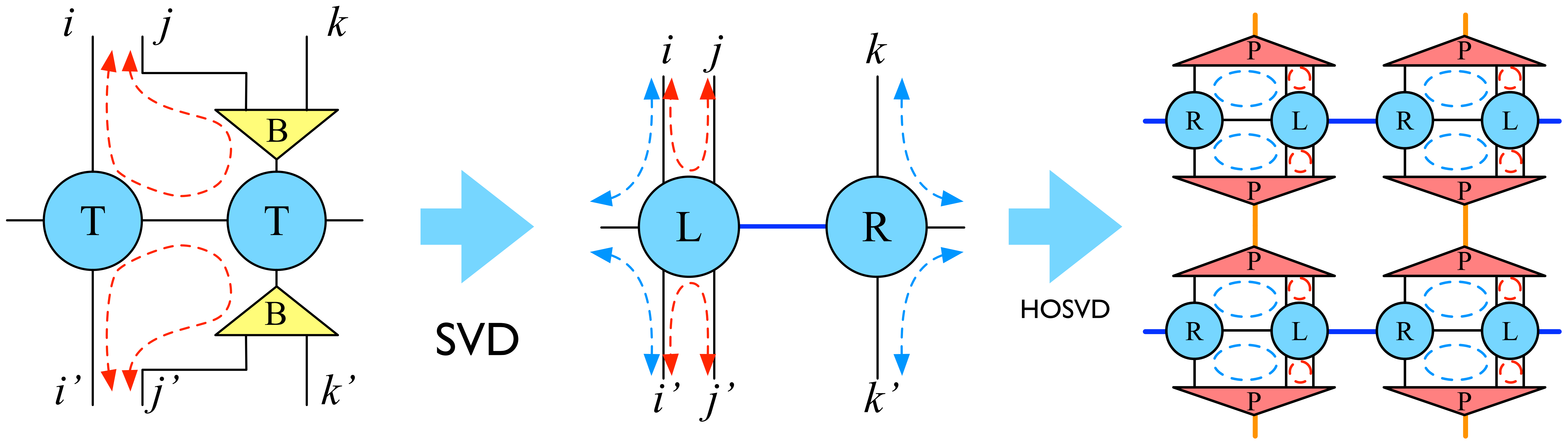

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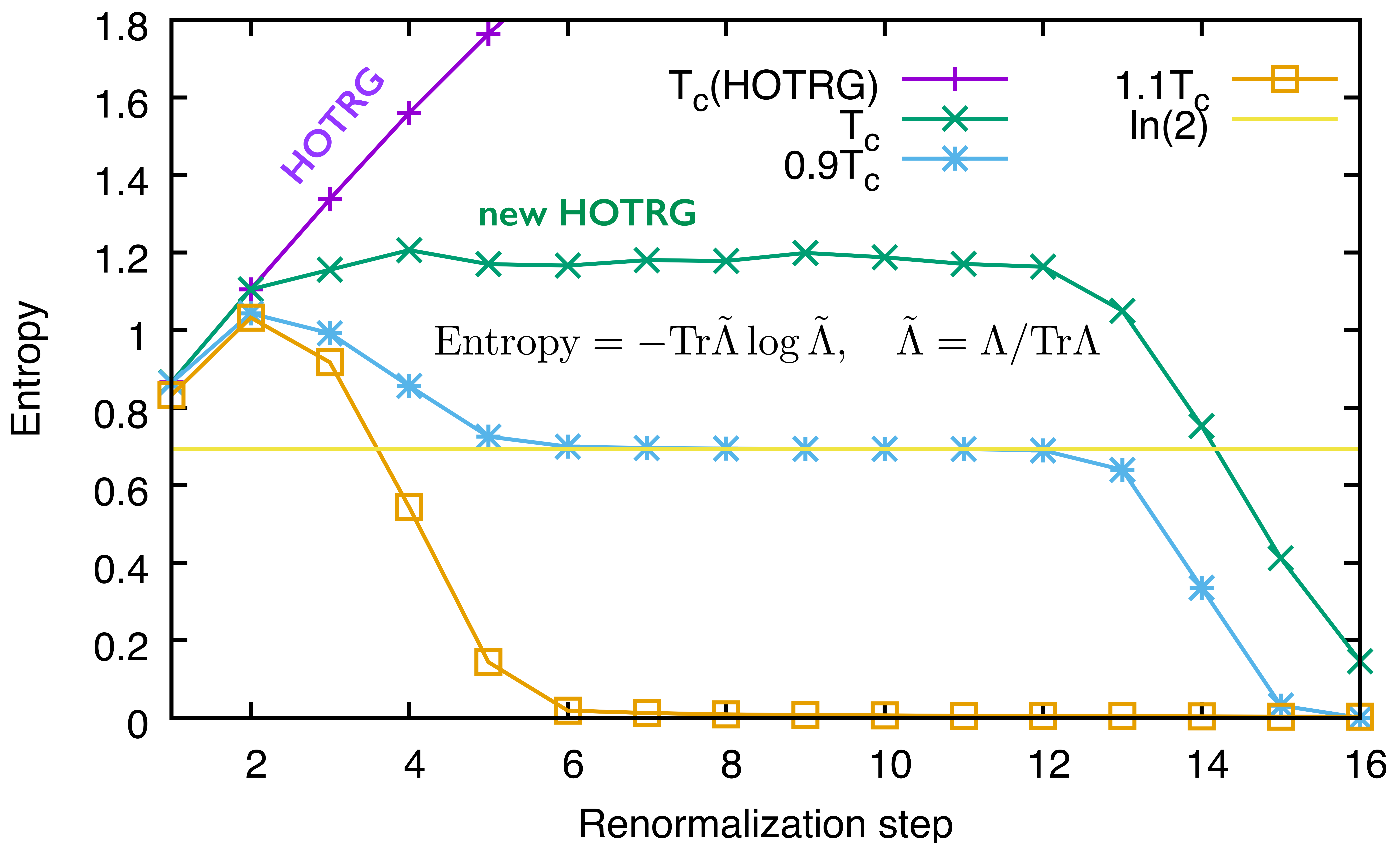
Example: HOTRG of 2D Ising model



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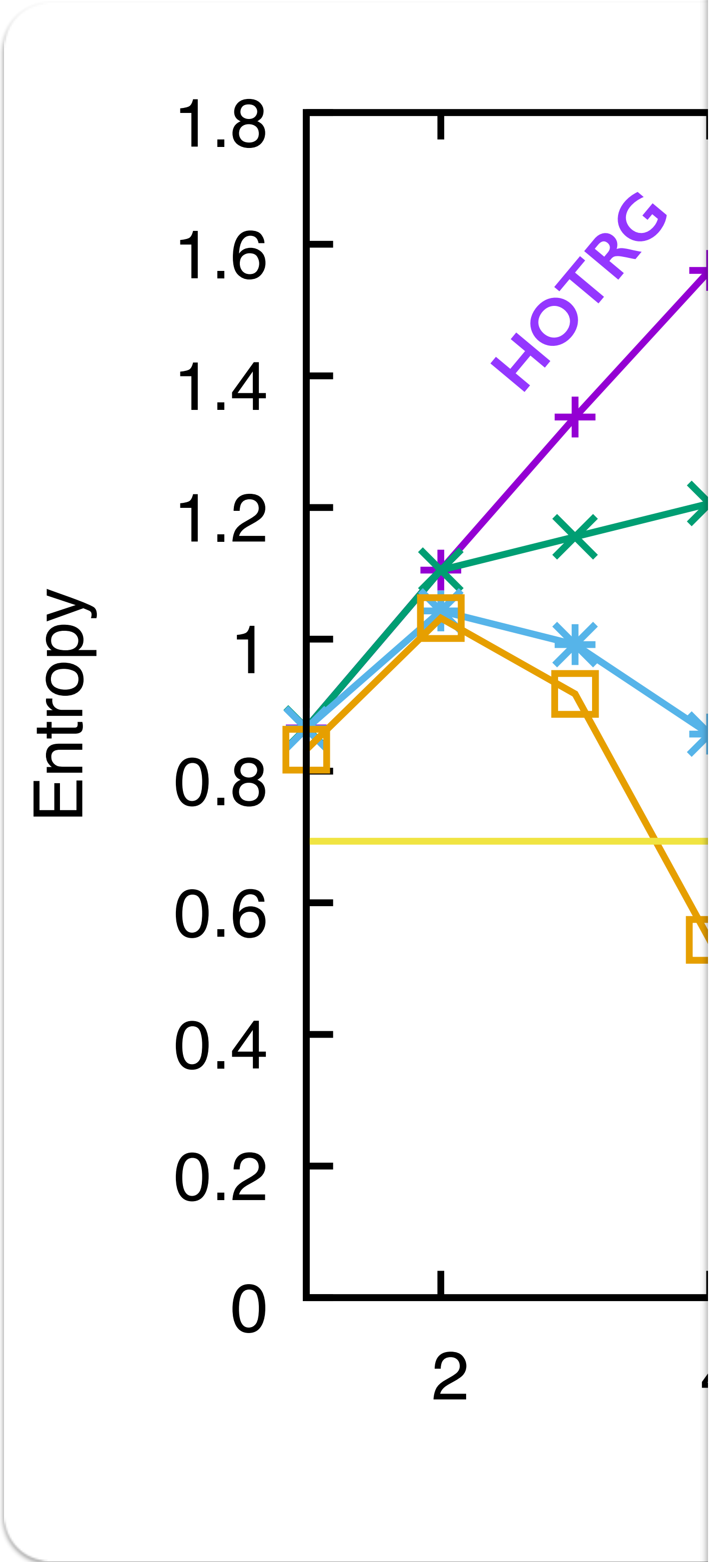
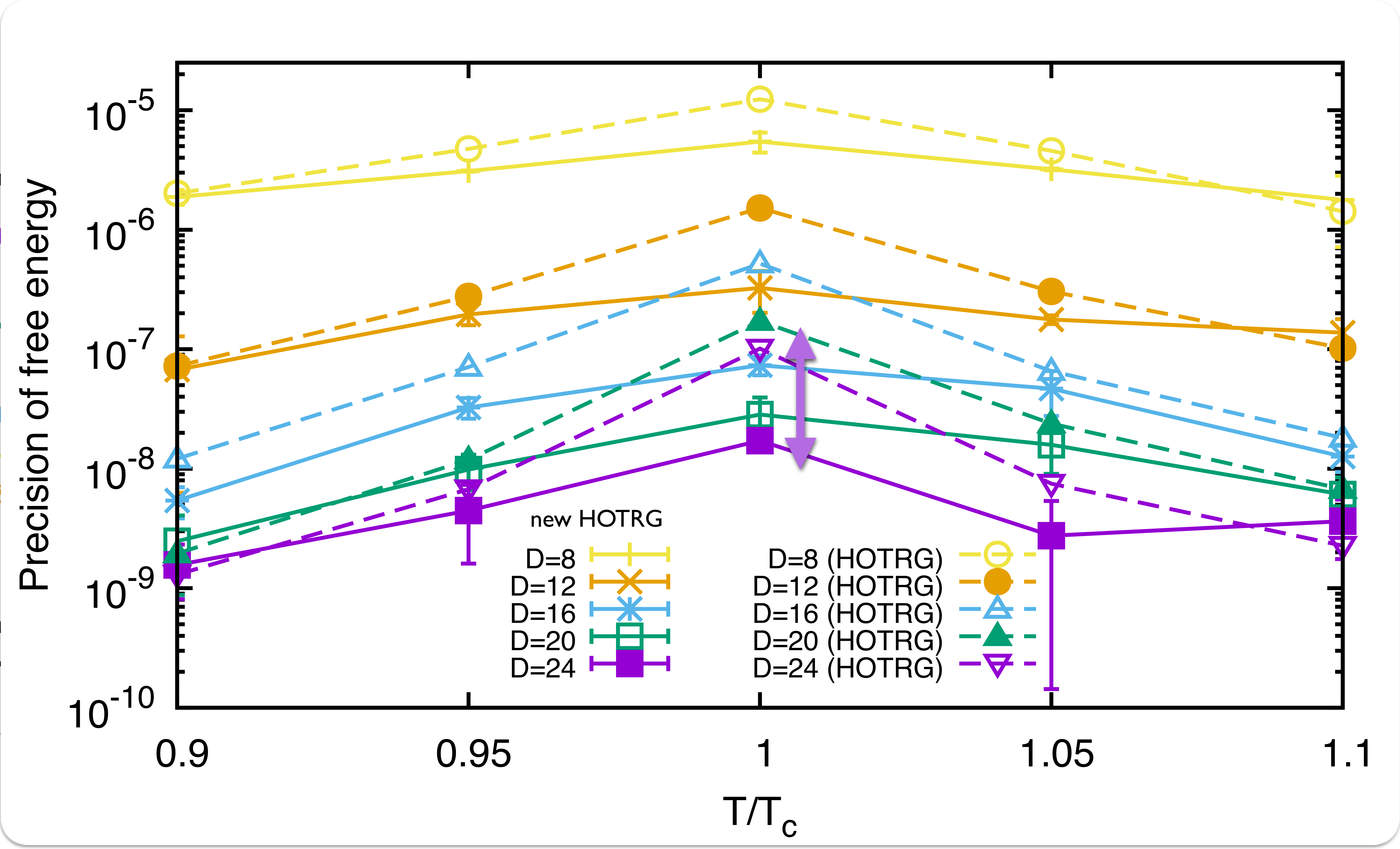
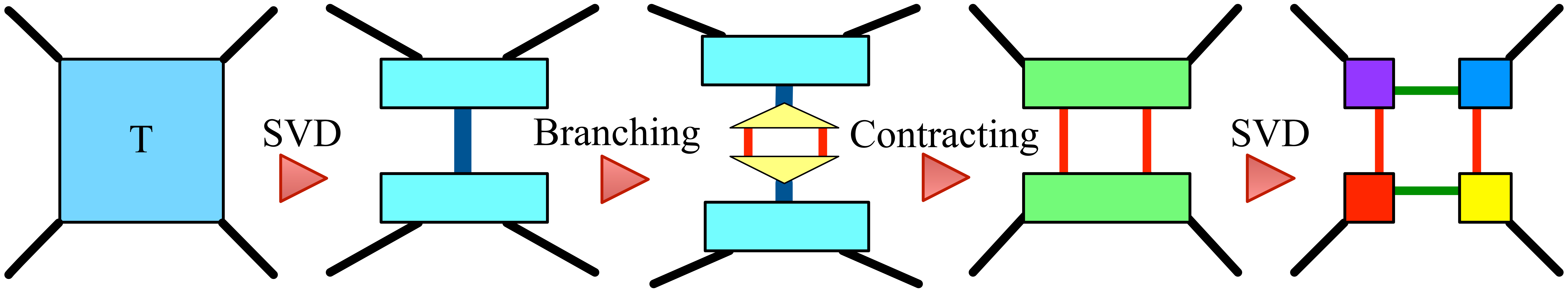


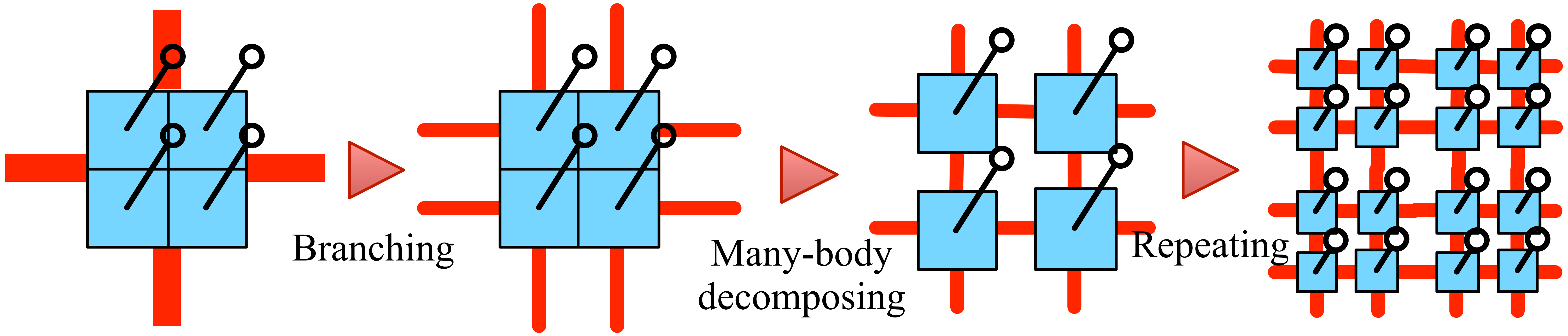


Many-body decomposition and derivation of PEPS

- **Tensor decomposition**
  - Matrix-based decomposition yields only a two-body tensor network
  - Many-body decomposition by entanglement branching



- **Derivation of PEPS based on many-body decomposition**

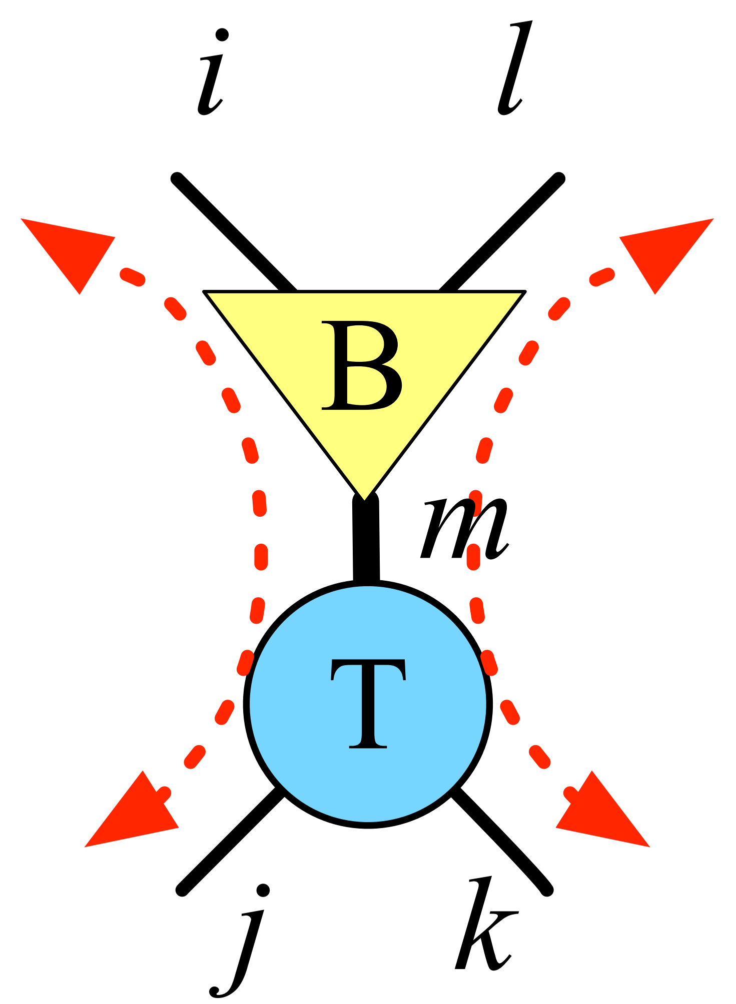


If the area law of entanglement entropy holds, bond dimensions of a derived PEPS are finite

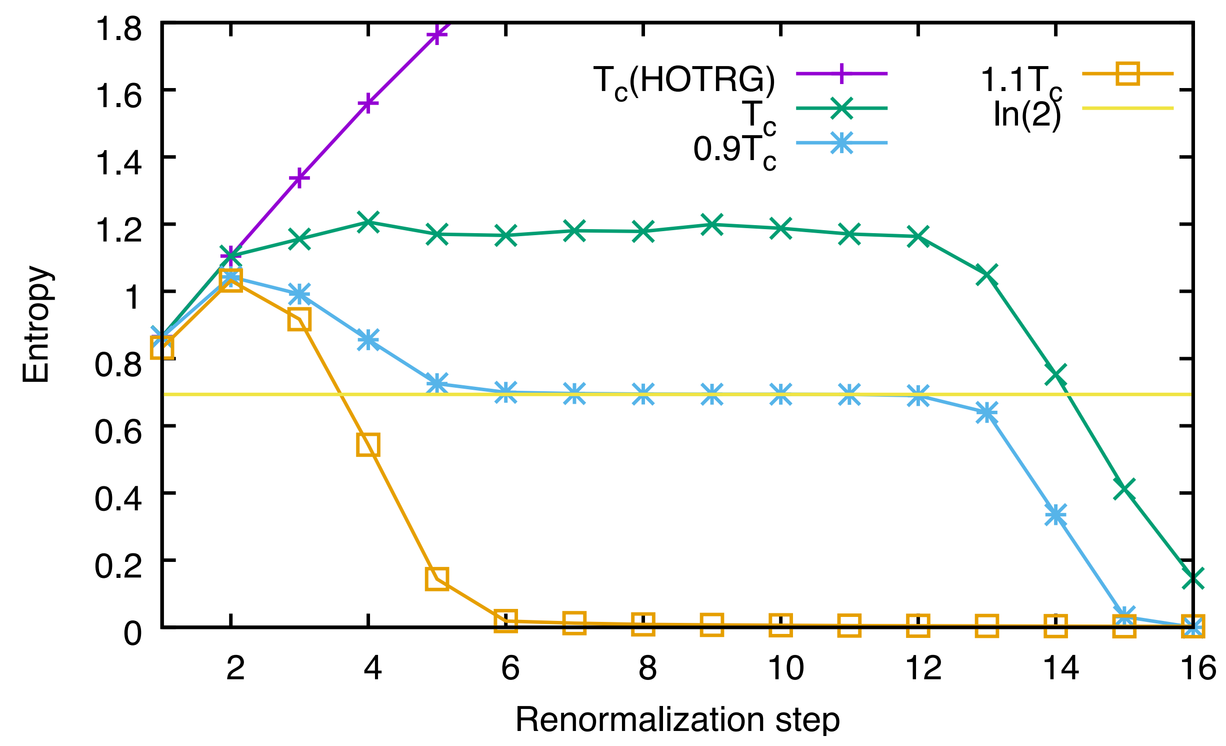
The metric in PEPS is related to entanglement strength

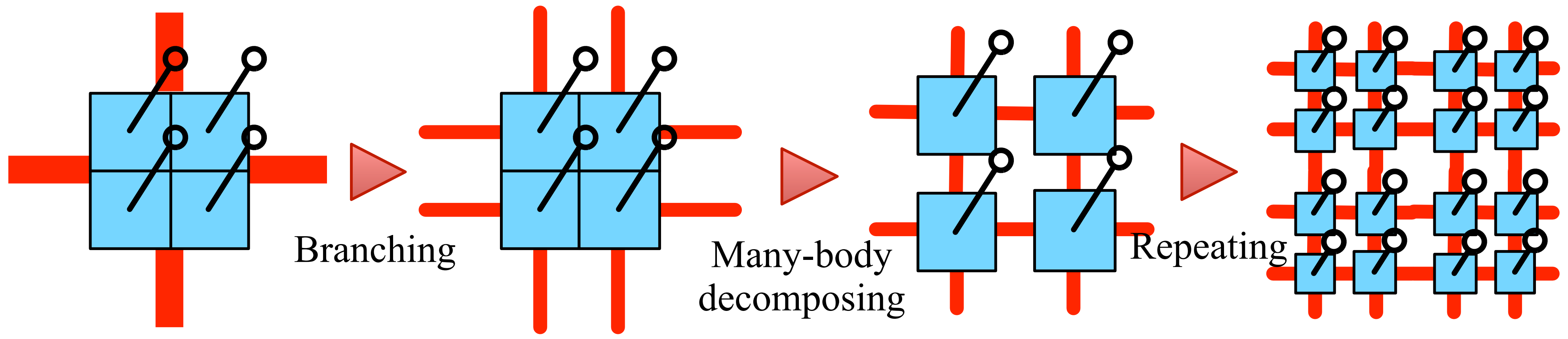
Summary

- **Entanglement branching operator**
  - split of a composite entanglement flow in a link
  - optimization problem by squeezing operators for EB operator
    - iteration method can be applied



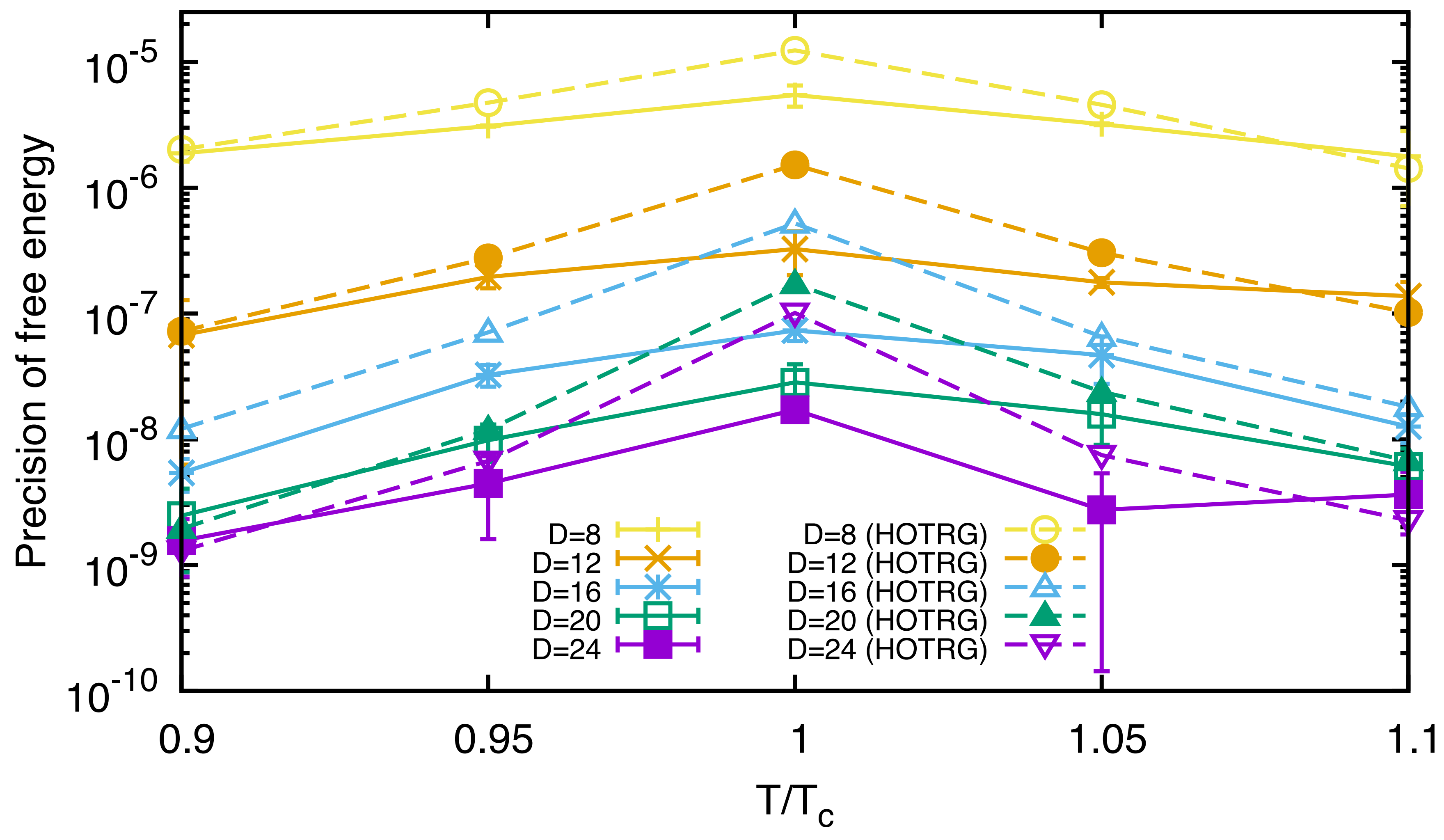
- **Applications of entanglement branching operators**
  - improvement of HOTRG
    - proper RG
    - new tensor network state
  - many-body decomposition
    - derivation of PEPS



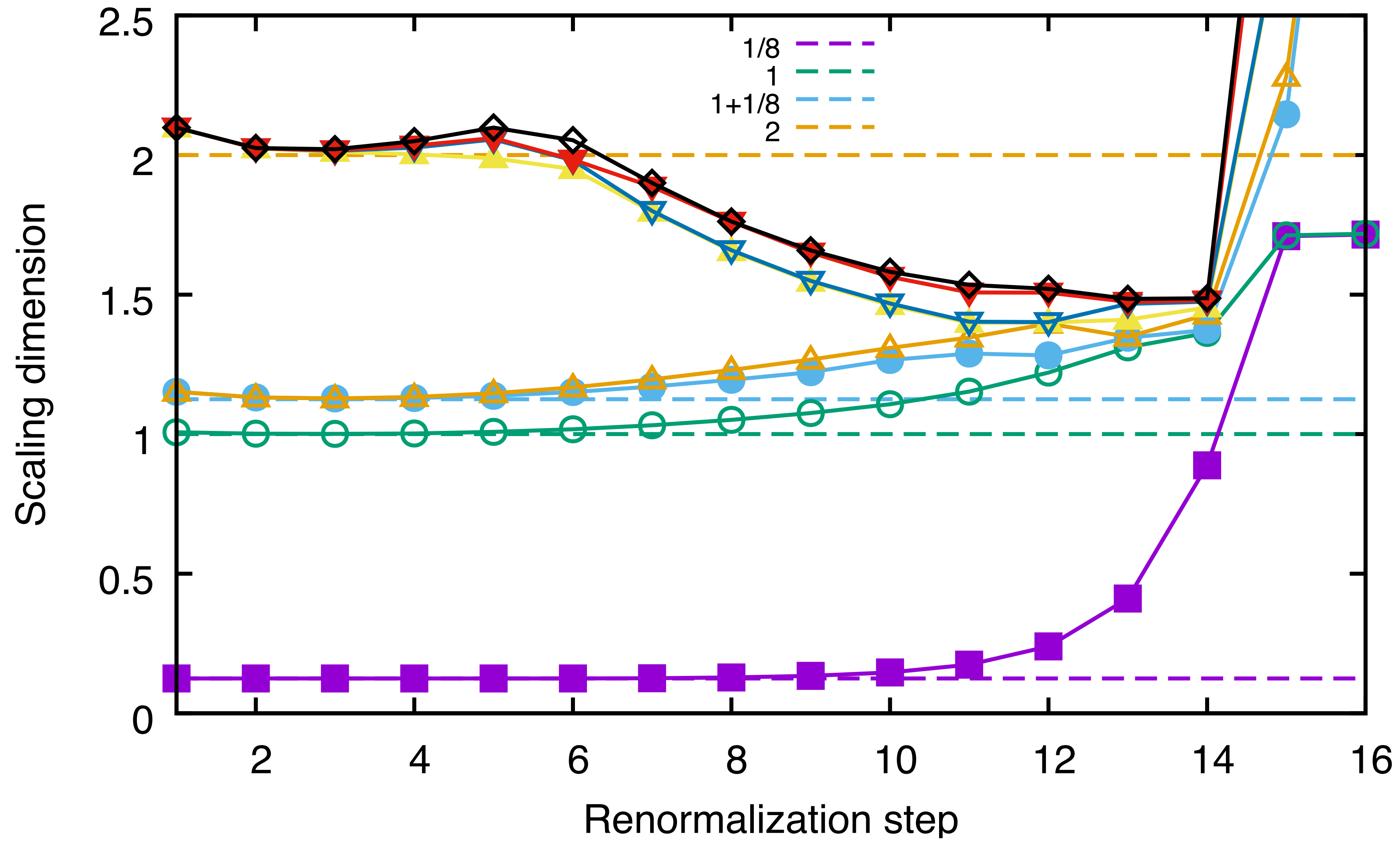


Reference: Kenji Harada, "Entanglement branching operator", Phys. Rev. B **97**, 045124 (2018)

Example: HOTRG of 2D Ising model



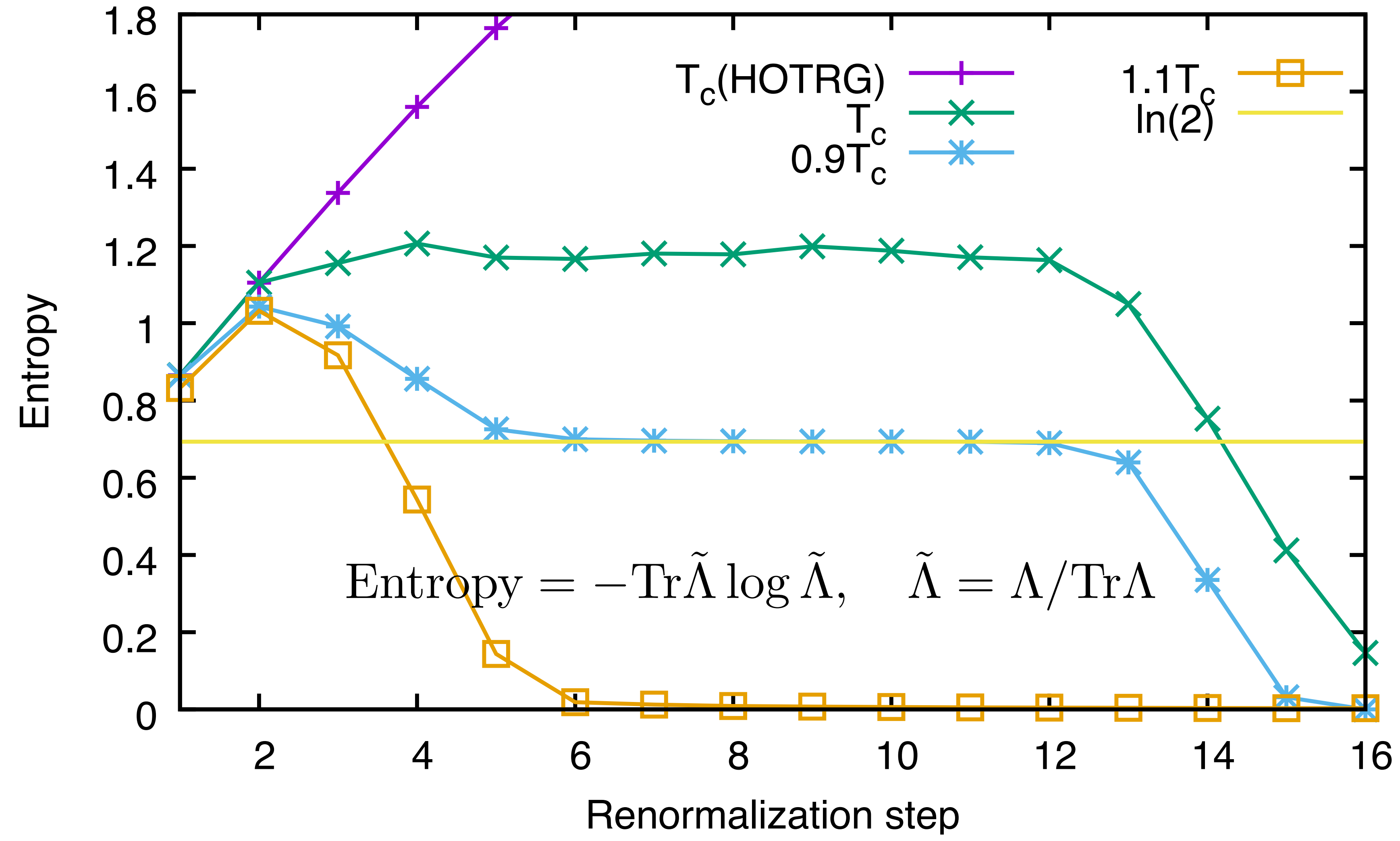
(a) HOTRG



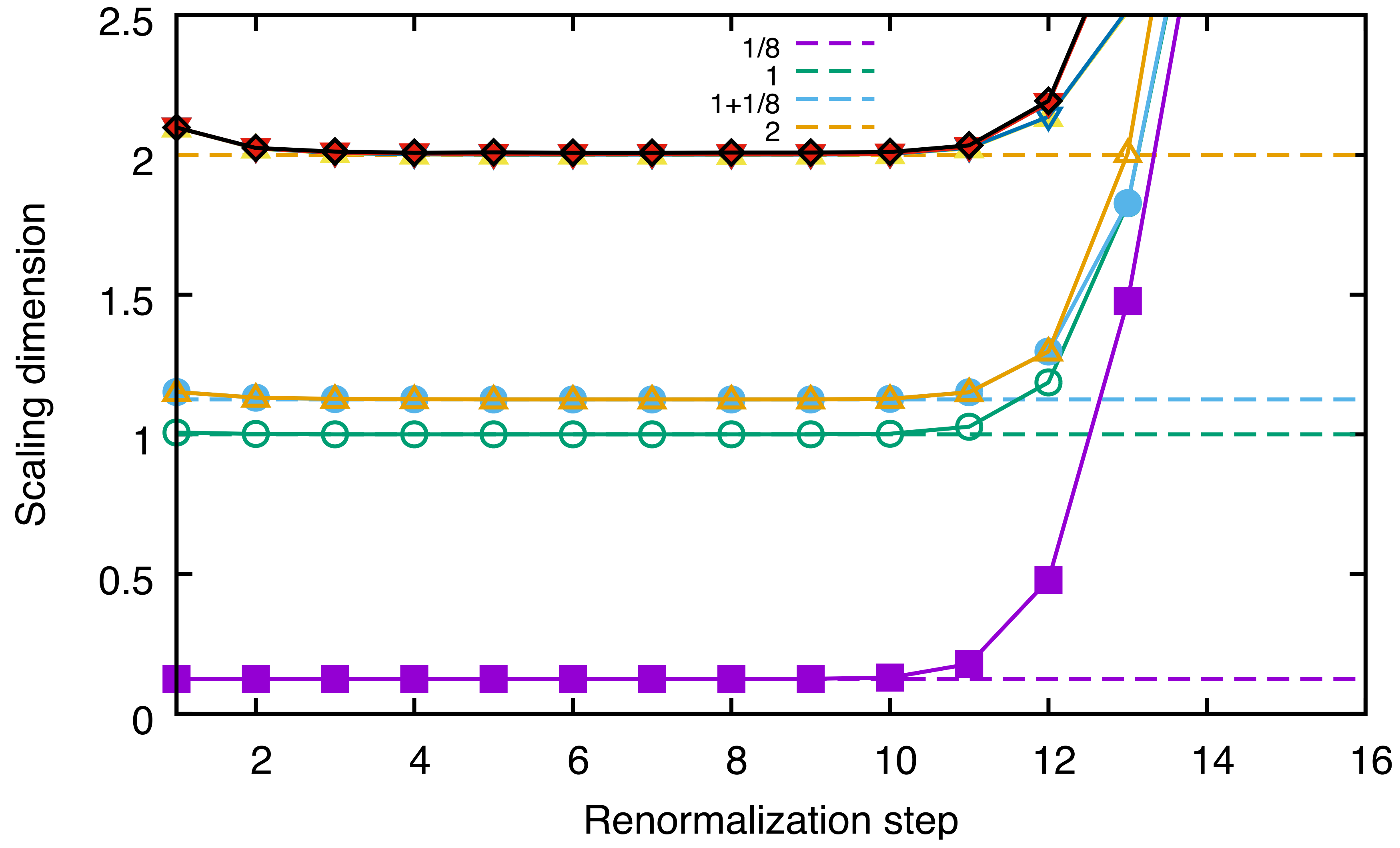
Scaling dimensions by $D = 24$

spins	exact	HOTRG with EB op. $2^{16} \sim 2^{20}$	TNR 2^{18}
c	0.5	0.49996(2)	0.50001
σ	0.125	0.12515(3)	0.1250004
ϵ	1	1.0002(1)	1.00009
	1.125	1.1250(1)	1.12492
	1.125	1.1252(1)	1.12510
	2	2.0009(2)	1.99922
	2	2.0013(2)	1.99986
	2	2.0029(4)	2.00006
	2	2.008(1)	2.00006

$$\Delta_i = -\frac{1}{2\pi}\log(\lambda_i/\lambda_0)$$



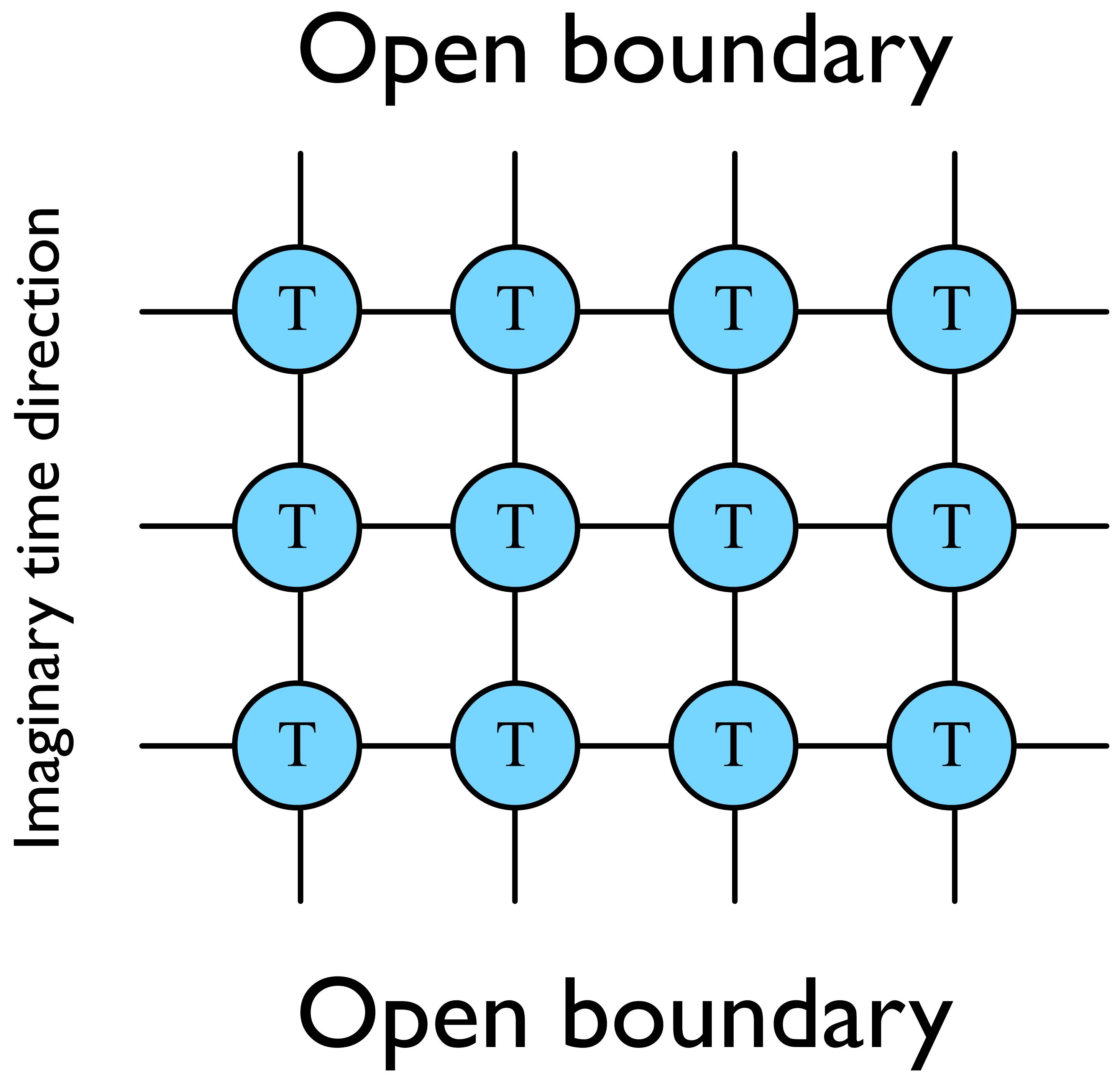
(b) HOTRG with EB op.

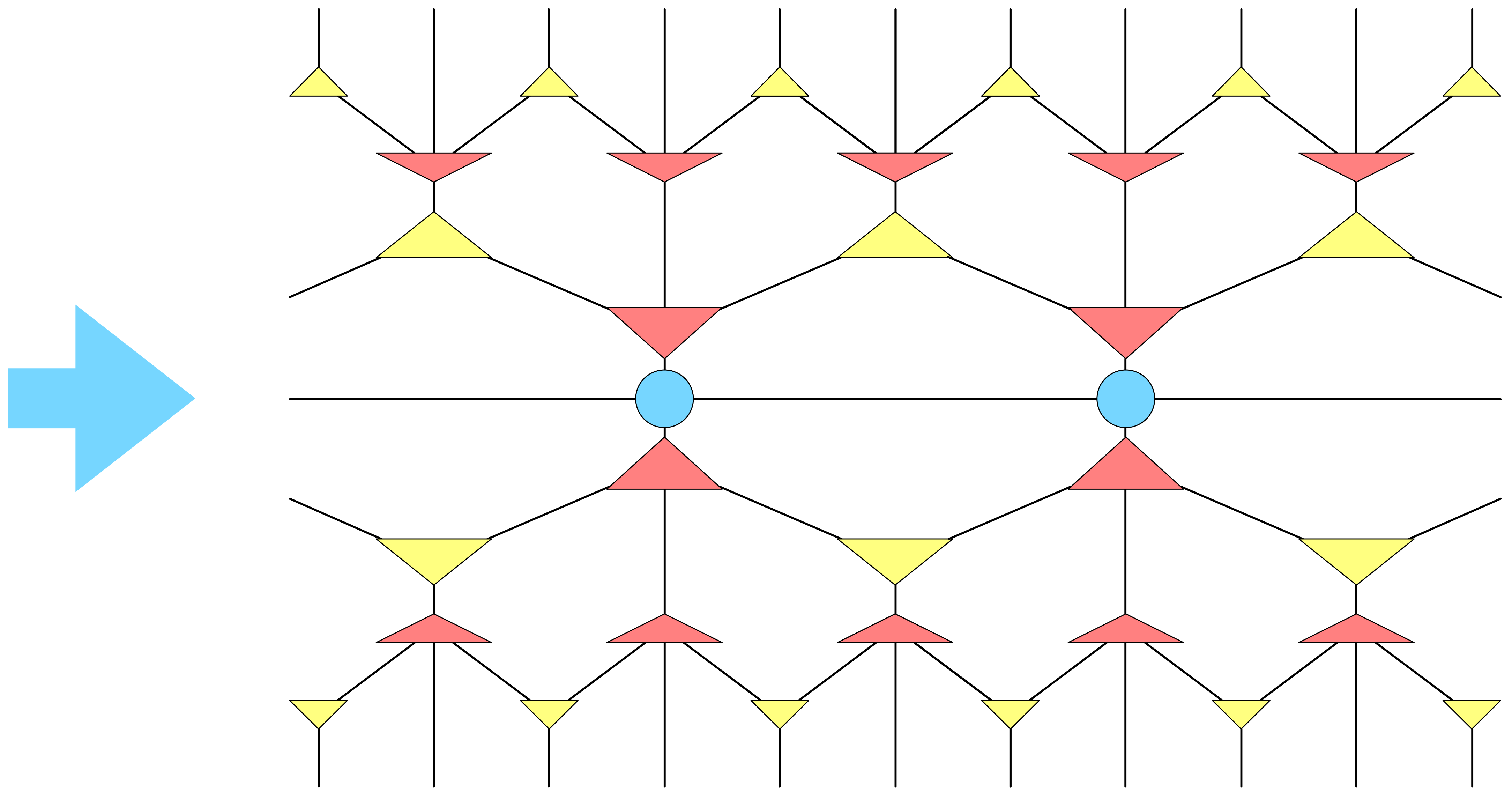


Reference: K.H., Phys. Rev. B **97**, 045124 (2018),
Evenly and Vidal, Phys. Rev. Lett. **115**, 180405 (2015)

New tensor network state as like MERA

- **Repeating a new HOTRG procedure to a tensor network representation of a density operator**





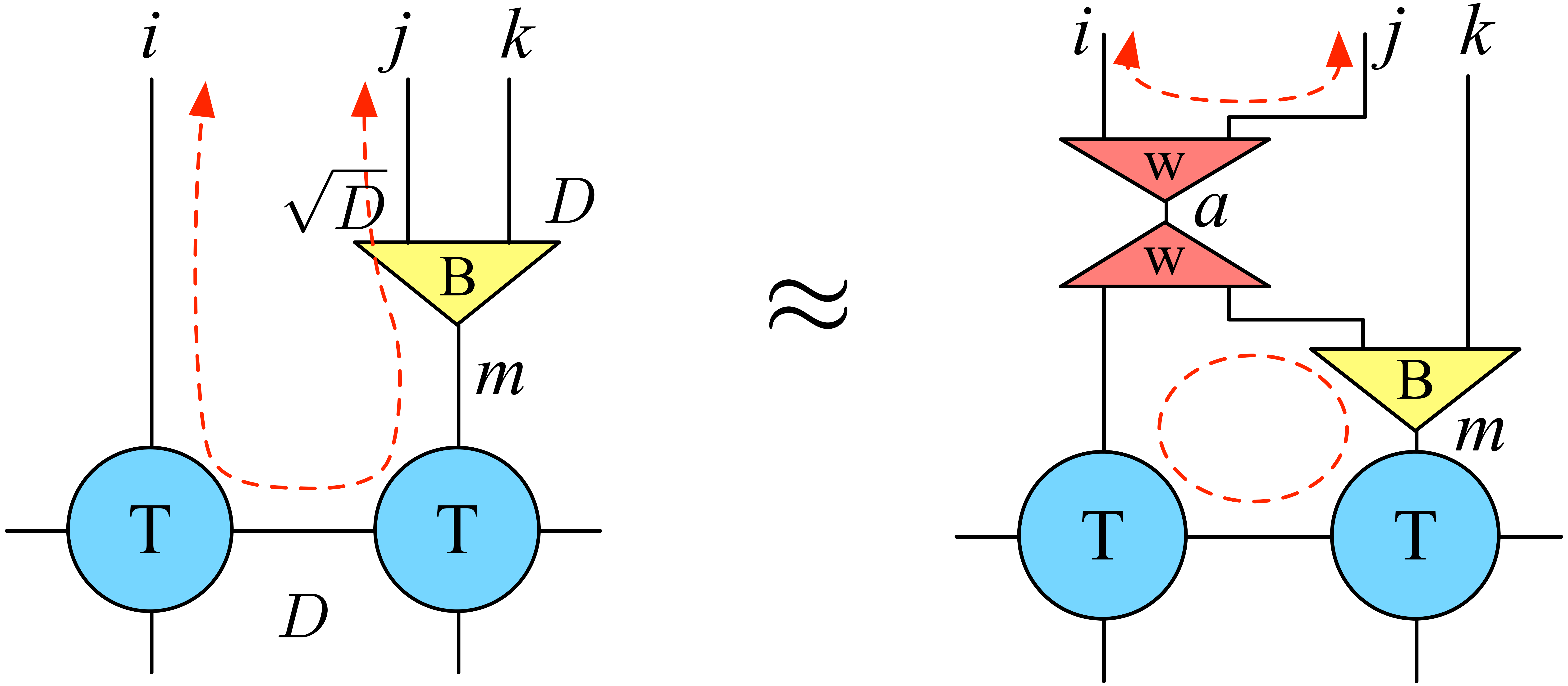
New tensor network

Log correction of E.E. : ok!

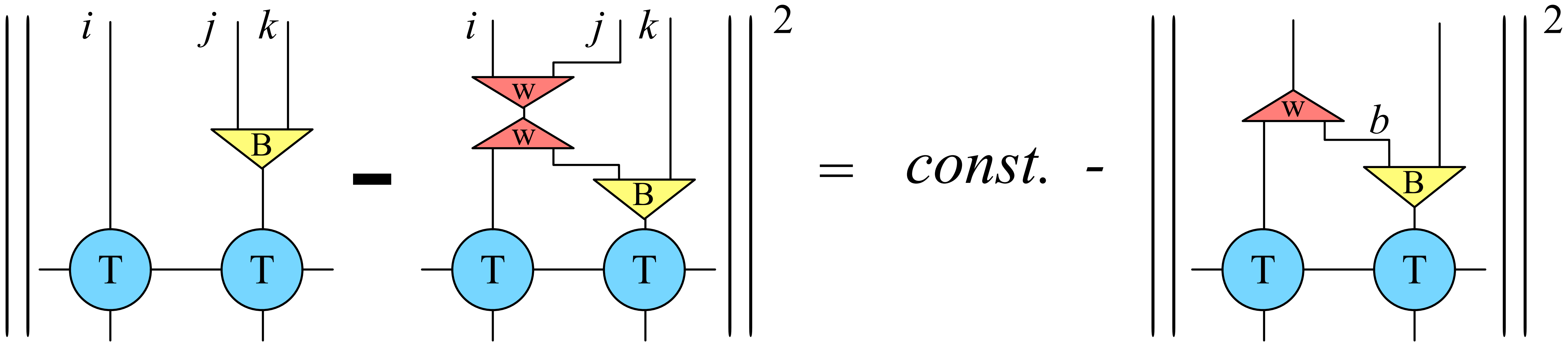
Splitting the shortest entanglement flow

Entanglement branching

Squeezing operators



Optimization problem for B and w



Iteration method to solve an optimization problem

Algorithm

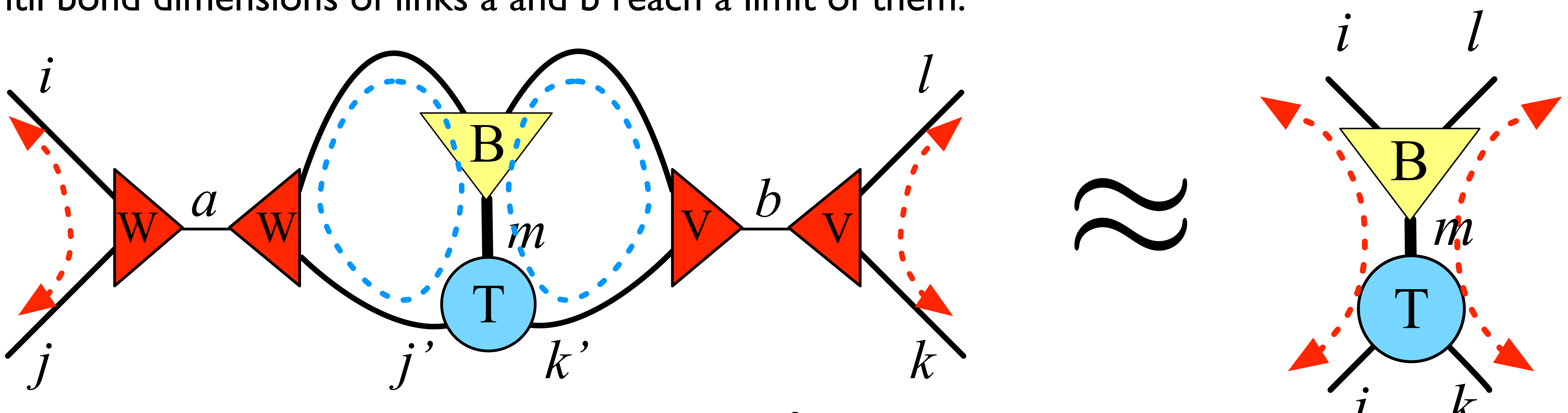
(1) Initialize B randomly.

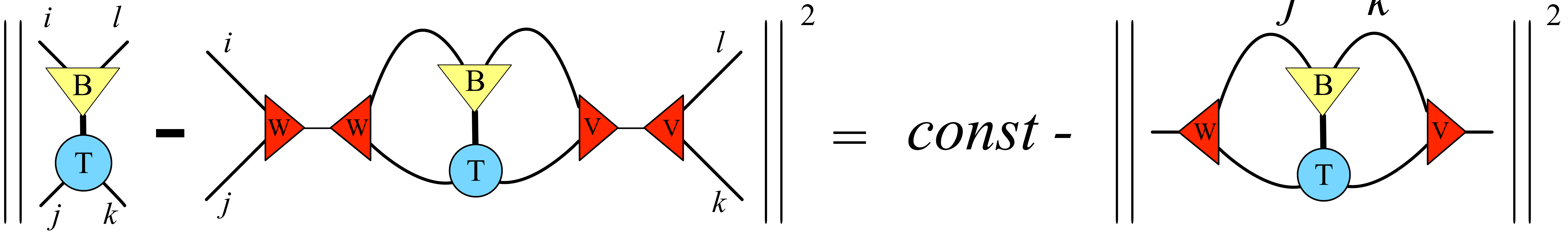
(2) Set the values of bond dimension of links a and b 1, and initialize w and v randomly.

(3) Iteratively update B, w, and v to minimize the squared distance.

(4) Increase bond dimensions of links a and b, and extend bond dimensions of w and v. New elements of w and v are initialized as zero, but other elements are unchanged.

(5) Go back to (3), until bond dimensions of links a and b reach a limit of them.

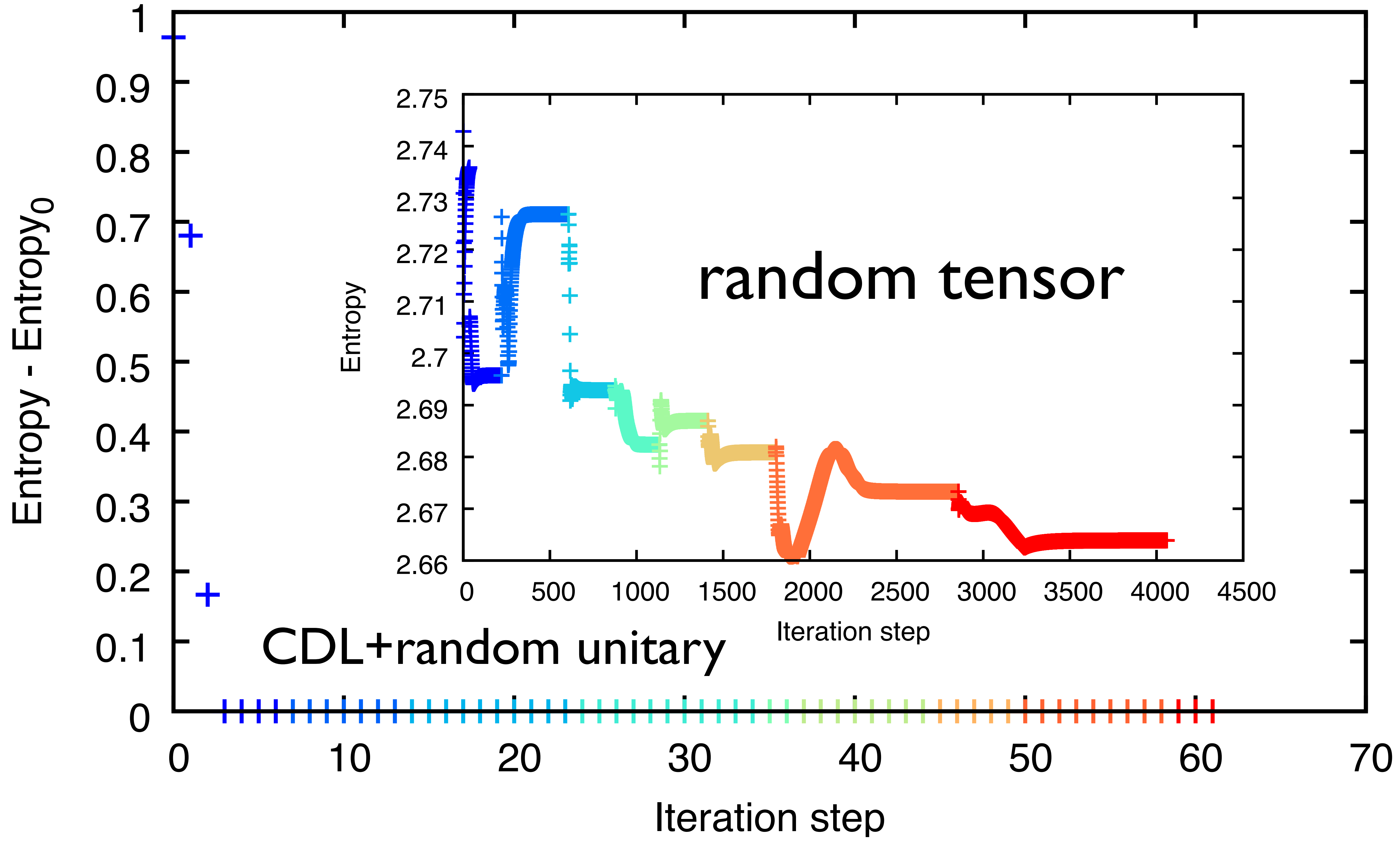


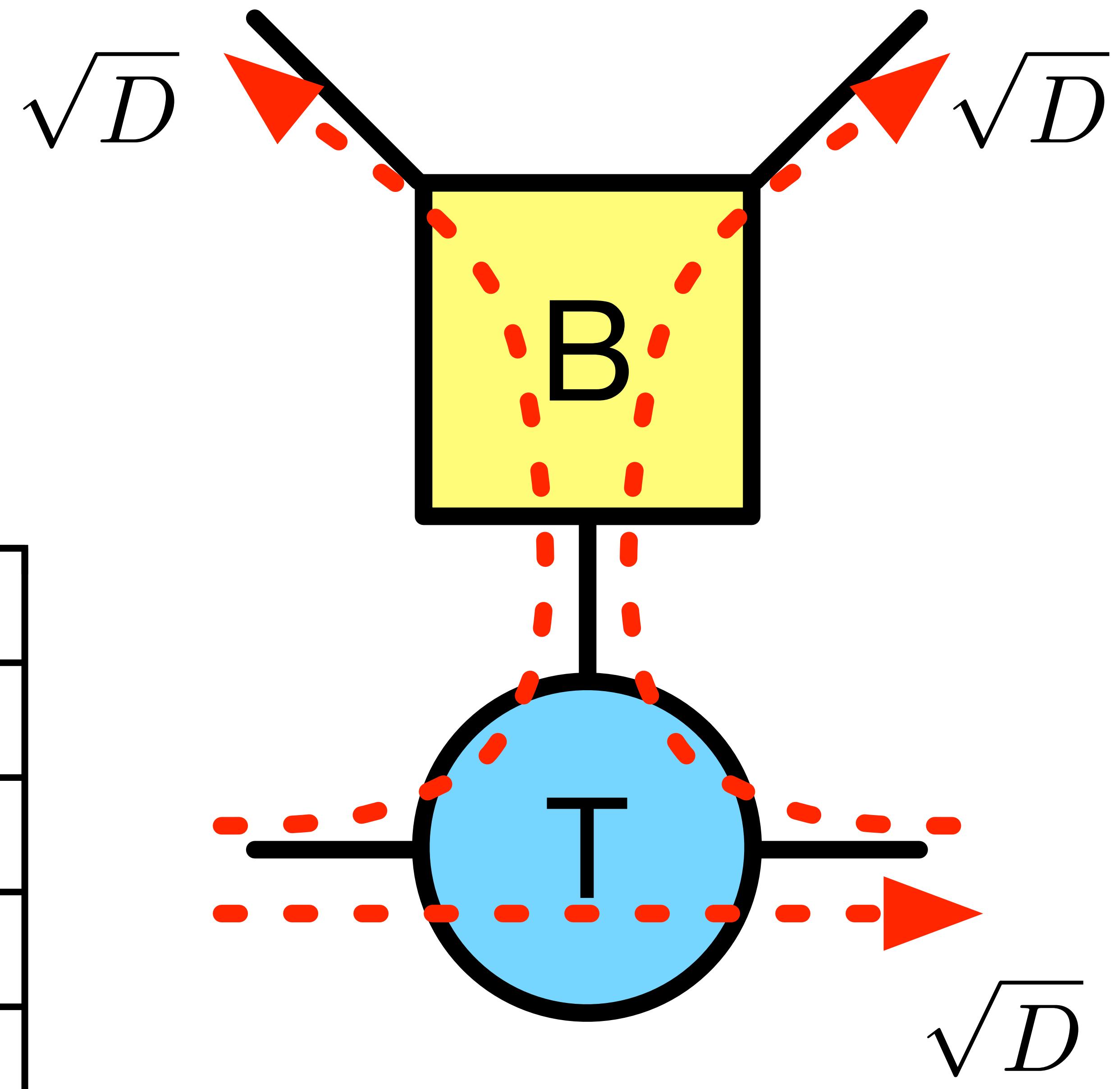


Optimization process of branching operator

- **Corner Double Line (CDL) tensor + random unitary**

$$T_{i_1 i_2, j_1 j_2, k_1 k_2} = U_{i_1 i_2, i'_1 i'_2} \delta_{i'_1, j_2} \delta_{i'_2, k_2} \delta_{j_1, k_1}$$



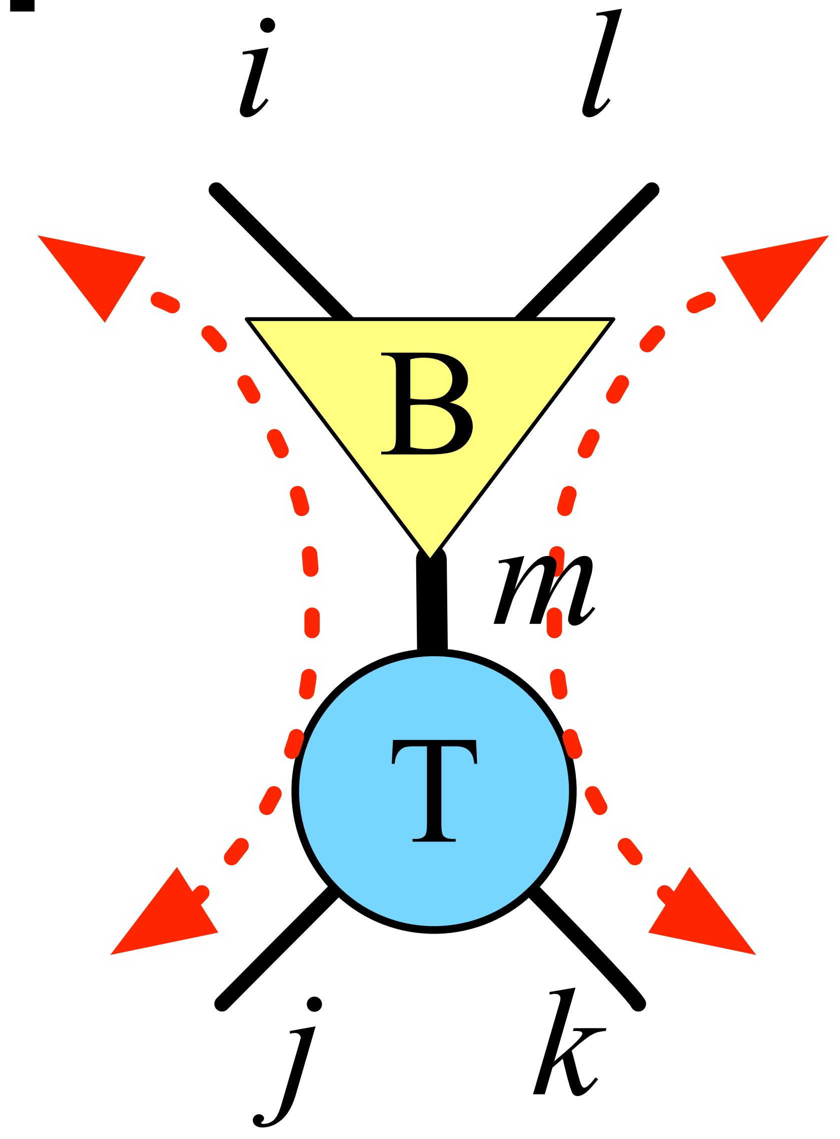


$$\text{Entropy} = -\mathrm{Tr}\tilde{\Lambda}\log\tilde{\Lambda},$$

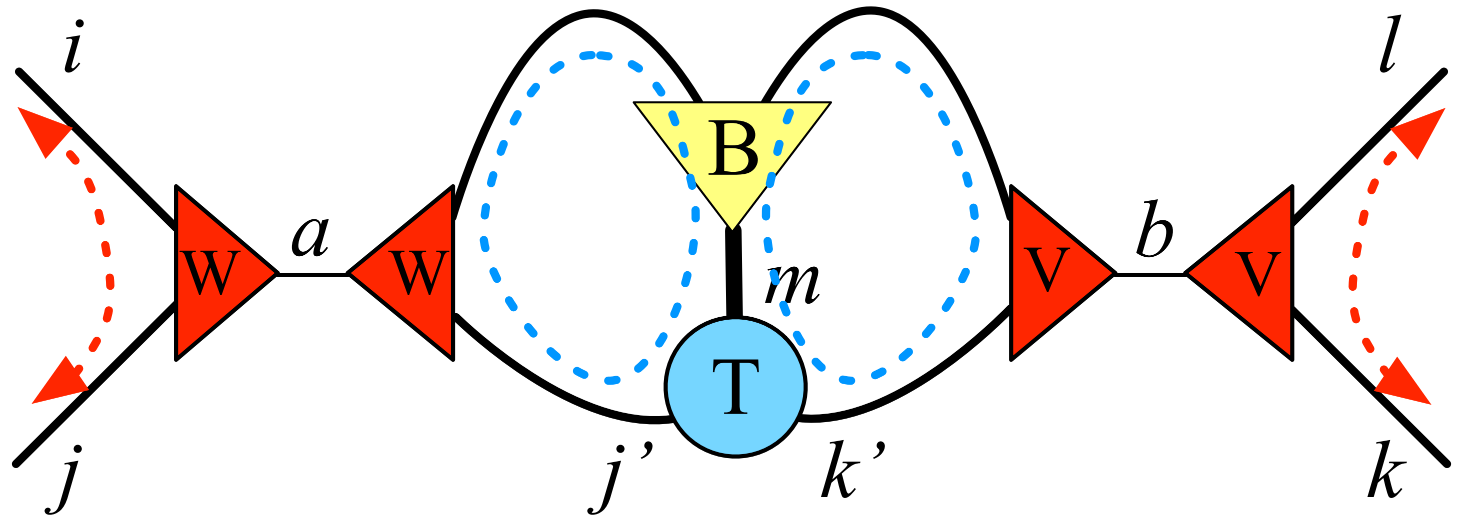
$$\tilde{\Lambda} = \Lambda/\mathrm{Tr}\Lambda$$

Entanglement branching operator

- **Split of a composite entanglement flow in a link**



$\approx$



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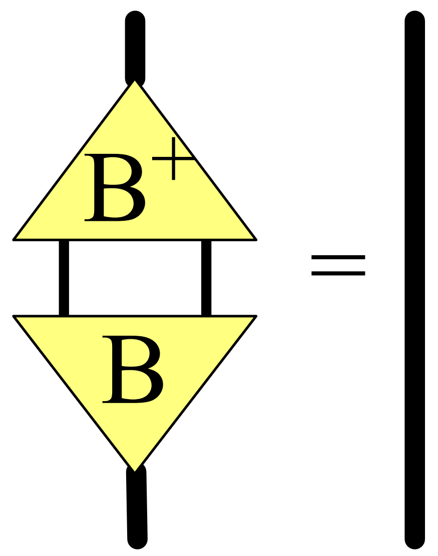
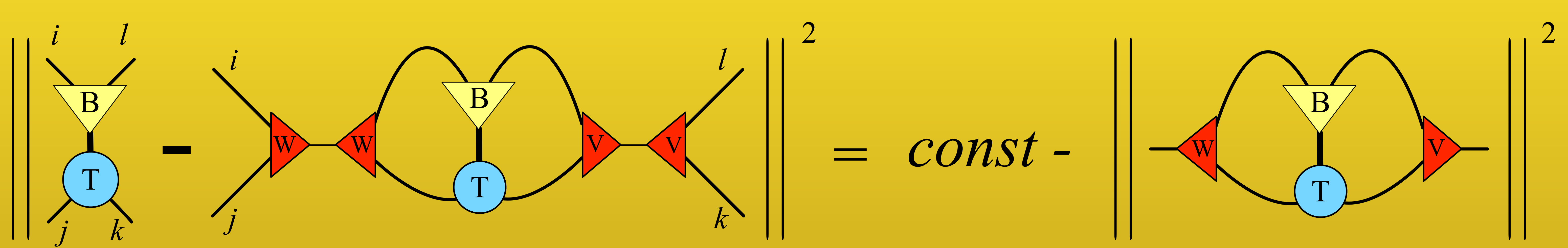


The pair of branching operators can be freely inserted on a link

Minimization of a distance between two tensor networks



solvable by applying an iteration method