In this section, we explain how to perform statistical analysis for extrasolar gaseous objects to understand their formation and evolution process and show how to deal with the selection effect of radial velocity measurements by which samples constructed for this study were detected. We also explain how we constructed the samples used in this study, determining the boundary between gas dwarfs such as Neptune-like planets and gaseous giants.

2.1 Statistical analysis

In this study, we first examined whether the distributions of semi-major axes and planet masses for gaseous objects arise from the selection effect of radial velocity measurements or from the dependency of the planetary formation and evolution process on the disk metallicity, constructing samples named as “common-biased samples” that minimize an impact of the selection effect on the distributions, as to be discussed in Section 2.2. Given that the measurement errors follow a normal distribution, we sampled the host star metallicities and companion masses and divided the common-biased samples into two by a host-star metallicity. Using the “anderson\_ksamp” module in Python, we compared the divided sub-samples in terms of planet mass and semi-major axis with two-sample Anderson-Darling test. Calculating the p-values derived from the two-sample Anderson-Darling test as a function of the host star metallicity, we searched for a boundary metallicity that divides the common-biased samples such that the distributions of companion masses and semi-major axes for the two sub-samples are most different. We iterated this procedure 1000 times and finally evaluated how much different the two common-biased sub-samples are in terms of the semi-major axis and planet mass at the boundary metallicity. This result is shown in Section 3.1.

Next, we explored how many populations exist in extrasolar gaseous objects discovered so far to investigate what the upper mass limit of the core-accreted planets is; we re-examined whether only two populations exist in the extrasolar gaseous objects, as shown in several previous studies (Ribas & Miralda-Escude 2007; Santos et al. 2017; Schlaufman 2018). Using the “GaussianMixture” package in Python, we applied two-dimensional Gaussian mixture model to the diagram of host star metallicities versus companion masses and for the common-biased samples. The number of the Gaussian mixture models used for this cluster analysis ranges from 1 to 8. We determined the number of the components of the best Gaussian mixture model based on the Bayesian Information Criterion as well as to which each common-biased sample belong. Sampling the host star metallicities and companion masses, we repeated this procedure 1000 times. This result is introduced in Section 3.2.

2.2. Common-biased samples

In order to reveal the distributions of companion mass and orbital properties for companions orbiting various host-star metallicities, we gathered extrasolar gaseous objects discovered by radial velocity observations that precisely determine lower limit of companion mass, semi-major axis and eccentricity. The gathered objects are referred to as “original samples” in this paper. Considering that there may exist the relation between the planetary-formation processes and the host-star metallicity, as discussed in Section 1, it is preferable that the accuracies and terms of the radial velocity measurements, which detected the original samples, are independent of the host-star metallicity. This is because the original samples detected via radial velocity measurements are influenced by two selection effects: (i) limited sensitivity to long-period planets owing to short observation terms and (ii) limited sensitivity to low-mass planets owing to a lack of measurement precision in radial velocity measurement. The semi-major axis, ajmax , and lower mass limit, Mp sin ijmin, of the detectable companion can be determined by the accuracy and term of the radial velocity measurements as below (Torres et al. 2008 ),

Equation (1)

where, M\_ , P , and e are the stellar mass, the orbital period and eccentricity of the companion, respectively. The region in which a companion can be detected is derived for each radial velocity measurement based on Equation (1). Figure 1 (a) compares the detectable probabilities of a companion with radial velocity measurements in a diagram of period versus lower limit of companion mass for the metal-rich and -poor regions. Note that the boundary metallicity was fixed to 0 dex. As shown in Figure 1, the accuracies of the radial velocity observations for the metal-poor original samples are clearly worse than those for the metal-rich ones. In contrast, the detectable semi-major axis for the original samples orbiting the metal-rich stars are slightly wider compared to those of the metal-poor samples. Thus, the selection effect of the radial velocity measurements depends on the host-star metallicity and affect the distributions of masses and semi-major axes for the two original sub-samples orbiting the metal-rich and -poor samples.

Focusing on a fact that the distributions of masses and semi-major axes for the original samples discovered in the metal-rich (-poor) region are biased with the selection effect of the radial velocity measurements for the metal-rich (-poor) stars, we can minimize the impact of the different selection effects on the original samples through filtering the metal-rich (-poor) original samples with the selection effect in the metal-poor (-rich); the selection biases in the metal-rich and -poor stars were equalized (see Figure 1 (b)). The filtering process judges whether each original sample simply satisfy the following criteria:

Equation (3)

Equation (4)

Now, we refer the filtered samples to as “common-biased samples.”

2.3. Preparation of samples

The original samples considered in this study are limited to companion objects detected by the radial velocity observations, allowing the orbital parameters to be characterized and lower limit of the companion mass to be determined. Essentially, the original samples are selected from those labeled “Radial Velocity” in the “detection method” column of the Extrasolar Planet Encyclopedia catalog as of the end of June 2018 (Schneider et al. 2011). The radial velocities of the host stars orbited by the original samples, and the orbital periods and eccentricities of the original samples are also collected from the same catalog. The SWEET-Cat catalog was referred to for the metallicity and mass of the host star (Santos et al. 2013 ; Sousa et al. 2018 ); this catalog presents the uniformly derived stellar parameters of the planet host stars. For some of the original samples that are not listed in the SWEET-Cat catalog, the metallicities and masses measured by the Geneva-Copenhagen catalog (Casagrande et al. 2011) the Padova database (Girardi et al. 2000), and the BaSTI stellar model (Hidalgo et al. 2018 ) were applied and were calibrated by using regression lines determined from the correlations between the values in the SWEET-Cat catalog and those in the three catalogs to minimize measurement biases for host-star metallicities and masses. Using the stellar mass and the lower limit of companion mass was newly calculated based on Equation (2) because the host-star masses were revised.

The measurement accuracy and observation term for the radial velocity measurement of each original sample as the indicators of the selection effect were extracted from the exoplanets.org catalog. According to the Kepler’s third law shown in Equation (1), the observation term and stellar mass provide the upper limit on the semi-major axis of a detectable companion with the radial velocity measurement of each original sample. Using the derived maximum semi-major axis, host-star mass and measurement accuracy, the lower limit on the mass of the detectable companion was derived based on Equation (1).

2.4. Boundary between gas giants and Neptune-like planets

The gaseous objects were extracted from all the samples in the Extrasolar Planet Encyclopedia catalog in order to remove the impact of low-mass samples, such as Neptune-mass planets (gas dwarfs) and super-Earths, on this analysis. First, we determined the boundary mass between the gaseous and gas-dwarf objects from a perspective of both theory and observation. According to a previous study (Ida & Lin 2004a ), gas-dwarf objects, which primarily consist of heavy-core objects such as Neptune and Uranus, have the potential to grow to the extent allowed by the core building materials inside their semi-major axes. This growth occurs via giant impacts in the inner region of the disk after the disk gas dissipates. However, this core growth is limited by the scattering effect of the heavy core increasing with greater distances from the central star. Therefore, the mass of a gas-dwarf object reaches a maximum at the semi-major axis, where the scattering effect begins to limit the core growth. Given that the ratio of collision-to-ejection probabilities for the heavy core is 0.1 and the core density is 1 g/cm3, the upper mass limit of the gas-dwarf object is approximately 0.1 MJ for dust surface densities of 3 times the Minimum Solar Nebulae Model value (MMSN).

From a standpoint of the observation, a boundary between gas giants and gas dwarfs at four times the Earth's radius has been observationally revealed by the Kepler data (Buchhave et al. 2012 ). From the empirical planetary mass-radius relation (e.g., Bashi et al. 2017):

Equation (3)

we found that the boundary of planetary mass is about 30 times the Earth's mass, corresponding to 0.1 MJ. Thus, 0.1 MJ was applied in this study as the boundary mass between gas giants and gas dwarfs. The numbers of samples and their planetary systems considered in this study are 623 and 520, respectively.