Test Cases for the Influence Diagram Form Implementation of Multivariate Gaussian Distributions

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1 Definition of the Forms

1.1 Covariance Form

1.2 Influence Diagram Form

2 Algorithms for Converting from One Form to Another

The algorithms for converting form one form to another are in the sections indicated by the table below.

		Covariance	Influence Diagram
From Form	Covariance	N/A	2.2.1
	Influence Diagram	2.1.1	N/A

2.2 Converting to Covariance Form

2.2.1 Converting Influence Diagram Form to Covariance Form

2.2.1.1 Test Case

Input

Domain = {X1, X2, X3, X4}

$\mu = [0\ 0\ 0\ 0]^T$

$$B = \begin{bmatrix} 0 & 0.5 & -1.75 & -0.125 \\ 0 & 0 & 5 & 0.5 \\ 0 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} 16 \\ 1 \\ 36 \\ 49 \end{bmatrix}$$

Output

Domain = {X1, X2, X3, X4}

$\mu_X = [0\ 0\ 0\ 0]^T$

$$\Sigma_X = \begin{bmatrix} 16 & 8 & 12 & -4 \\ 8 & 5 & 11 & -4 \\ 12 & 11 & 70 & -31 \\ -4 & -4 & -31 & 63 \end{bmatrix}$$

2.2.1.2 Test Case

Input

Domain = {X1, X2, X3, X4, X5, X6}

$\mu = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$B = \begin{bmatrix} 0 & 0.5 & -1.75 & -0.125 & 1 & 0.5 \\ 0 & 0 & 5 & 0.5 & -1 & 0.5 \\ 0 & 0 & 0 & -0.5 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} 16 \\ 1 \\ 36 \\ 49 \\ 4 \\ 25 \end{bmatrix}$$

Output

Domain = {X1, X2, X3, X4, X5, X6}

$\mu_X = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$\Sigma_X = \begin{bmatrix} 16 & 8 & 12 & -4 & 16 & 12 \\ 8 & 5 & 11 & -4 & 10 & 4 \\ 12 & 11 & 70 & -31 & 40 & -19 \\ -4 & -4 & -31 & 63 & 32 & 59 \\ 16 & 10 & 40 & 32 & 82 & 50 \\ 12 & 4 & -19 & 59 & 50 & 97 \end{bmatrix}$$

2.2.1.3 Test Case

Input

Domain = {X1, X2, X3, X4, X5, X6}

$\mu = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$B = \begin{bmatrix} 0 & 0.5 & 0 & 1.75 & -0.7987805 & -0.9363173 \\ 0 & 0 & 2 & -3.3548387 & 2.59581882 & 0.8922853 \\ 0 & 0 & 0 & 0.67741935 & 0.65853659 & 0.8558952 \\ 0 & 0 & 0 & 0 & -0.543554 & 0.0713246 \\ 0 & 0 & 0 & 0 & 0 & -0.8922853 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} 16 \\ 1 \\ 62 \\ 55.548 \\ 14.362 \\ 3.5662 \end{bmatrix}$$

Output

Domain = {X1, X2, X3, X4, X5, X6}

$\mu_X = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$\Sigma_X = \begin{bmatrix} 16 & 8 & 16 & 12 & 12 & -4 \\ 8 & 5 & 10 & 4 & 11 & -4 \\ 16 & 10 & 82 & 50 & 40 & 32 \\ 12 & 4 & 50 & 97 & -19 & 59 \\ 12 & 11 & 40 & -19 & 70 & -31 \\ -4 & -4 & 32 & 59 & -31 & 63 \end{bmatrix}$$

2.2.1.4 Test Case

Input

Domain = {X1, X2, X3, X4, X5, X6}

$\mu = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.4428571 & 1.01826616 & 0.18324152 \\ 0 & 0 & 0 & 0 & 1.00898811 & 1.0266744 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} \infty \\ \infty \\ 70 \\ 49.271 \\ 0 \\ 0 \end{bmatrix}$$

Output

Domain = {X1, X2, X3, X4, X5, X6}

$\mu_X = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$\Sigma_X = \begin{bmatrix} \infty & 0 & 0 & 0 & 0 & 0 \\ 0 & \infty & 0 & 0 & 0 & 0 \\ 0 & 0 & 70 & -31 & 40 & -19 \\ 0 & 0 & -31 & 63 & 32 & 59 \\ 0 & 0 & 40 & 32 & 73.0182661641055 & 40.1832415192808 \\ 0 & 0 & -19 & 59 & 40.1832415192808 & 57.0922006378658 \end{bmatrix}$$

2.2.1.5 Test Case

Input

Domain = {X1, X2, X3, X4, X5, X6}

$\mu = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.55031766 & 1.1930593 & -0.212938 \\ 0 & 0 & 0 & 0 & -1.1725067 & 1.18328841 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} \infty \\ \infty \\ 73.018 \\ 34.979 \\ 0 \\ 0 \end{bmatrix}$$

Output

Domain = {X1, X2, X3, X4, X5, X6}

$\mu_X = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$\Sigma_X = \begin{bmatrix} \infty & 0 & 0 & 0 & 0 & 0 \\ 0 & \infty & 0 & 0 & 0 & 0 \\ 0 & 0 & 73.0182661641055 & 40.1832415192808 & 40 & 32 \\ 0 & 0 & 40.1832415192808 & 57.0922006378658 & -19 & 59 \\ 0 & 0 & 40 & -19 & 70 & -31 \\ 0 & 0 & 32 & 59 & -31 & 63 \end{bmatrix}$$

2.3 Converting to Influence Diagram Form

2.3.1 Converting Covariance Form to Influence Diagram Form

2.3.1.1 Test Case

Input

Domain = {Y}

$\mu_Y = 0$

$\Sigma_Y = \infty$

Output

Domain = {Y}

$\mu = [0]$

$B = [0]$

$v = [\infty]$

2.3.1.2 Test Case

Input

Domain = {X1, X2, X3, X4}

$\mu_X = [0\ 0\ 0\ 0]^T$

$$\Sigma_X = \begin{bmatrix} 16 & 8 & 12 & -4 \\ 8 & 5 & 11 & -4 \\ 12 & 11 & 70 & -31 \\ -4 & -4 & -31 & 63 \end{bmatrix}$$

Output

Domain = {X1, X2, X3, X4}

$\mu = [0\ 0\ 0\ 0]^T$

$$B = \begin{bmatrix} 0 & 0.5 & -1.75 & -0.125 \\ 0 & 0 & 5 & 0.5 \\ 0 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} 16 \\ 1 \\ 36 \\ 49 \end{bmatrix}$$

2.3.1.3 Test Case

Input

Domain = {X1, X2, X3, X4, X5, X6}

$\mu_X = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$\Sigma_X = \begin{bmatrix} 16 & 8 & 12 & -4 & 16 & 12 \\ 8 & 5 & 11 & -4 & 10 & 4 \\ 12 & 11 & 70 & -31 & 40 & -19 \\ -4 & -4 & -31 & 63 & 32 & 59 \\ 16 & 10 & 40 & 32 & 82 & 50 \\ 12 & 4 & -19 & 59 & 50 & 97 \end{bmatrix}$$

Output

Domain = {X1, X2, X3, X4, X5, X6}

$\mu = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$B = \begin{bmatrix} 0 & 0.5 & -1.75 & -0.125 & 1 & 0.5 \\ 0 & 0 & 5 & 0.5 & -1 & 0.5 \\ 0 & 0 & 0 & -0.5 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} 16 \\ 1 \\ 36 \\ 49 \\ 4 \\ 25 \end{bmatrix}$$

2.3.1.4 Test Case

Input

Domain = {X1, X2, X3, X4, X5, X6}

$\mu_X = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$\Sigma_X = \begin{bmatrix} 16 & 8 & 16 & 12 & 12 & -4 \\ 8 & 5 & 10 & 4 & 11 & -4 \\ 16 & 10 & 82 & 50 & 40 & 32 \\ 12 & 4 & 50 & 97 & -19 & 59 \\ 12 & 11 & 40 & -19 & 70 & -31 \\ -4 & -4 & 32 & 59 & -31 & 63 \end{bmatrix}$$

Output

Domain = {X1, X2, X3, X4, X5, X6}

$\mu = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$B = \begin{bmatrix} 0 & 0.5 & 0 & 1.75 & -0.7987805 & -0.9363173 \\ 0 & 0 & 2 & -3.3548387 & 2.59581882 & 0.8922853 \\ 0 & 0 & 0 & 0.67741935 & 0.65853659 & 0.8558952 \\ 0 & 0 & 0 & 0 & -0.543554 & 0.0713246 \\ 0 & 0 & 0 & 0 & 0 & -0.8922853 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} 16 \\ 1 \\ 62 \\ 55.548 \\ 14.362 \\ 3.5662 \end{bmatrix}$$

2.3.1.5 Test Case

Input

Domain = {X1, X2, X3, X4, X5, X6}

p = 1

$\mu_X = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$\Sigma_X = \begin{bmatrix} \infty & 0 & 0 & 0 & 0 & 0 \\ 0 & \infty & 0 & 0 & 0 & 0 \\ 0 & 0 & 70 & -31 & 40 & -19 \\ 0 & 0 & -31 & 63 & 32 & 59 \\ 0 & 0 & 40 & 32 & 73.0182661641055 & 40.1832415192808 \\ 0 & 0 & -19 & 59 & 40.1832415192808 & 57.0922006378658 \end{bmatrix}$$

Output

Domain = {X1, X2, X3, X4, X5, X6}

$\mu = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.4428571 & 1.01826616 & 0.18324152 \\ 0 & 0 & 0 & 0 & 1.00898811 & 1.0266744 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} \infty \\ \infty \\ 70 \\ 49.271 \\ 0 \\ 0 \end{bmatrix}$$

2.3.1.6 Test Case

Input

Domain = {X1, X2, X3, X4, X5, X6}

$\mu_X = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$\Sigma_X = \begin{bmatrix} \infty & 0 & 0 & 0 & 0 & 0 \\ 0 & \infty & 0 & 0 & 0 & 0 \\ 0 & 0 & 73.0182661641055 & 40.1832415192808 & 40 & 32 \\ 0 & 0 & 40.1832415192808 & 57.0922006378658 & -19 & 59 \\ 0 & 0 & 40 & -19 & 70 & -31 \\ 0 & 0 & 32 & 59 & -31 & 63 \end{bmatrix}$$

Output

Domain = {X1, X2, X3, X4, X5, X6}

$\mu = [0\ 0\ 0\ 0\ 0\ 0]^T$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.55031766 & 1.1930593 & -0.212938 \\ 0 & 0 & 0 & 0 & -1.1725067 & 1.18328841 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} \infty \\ \infty \\ 73.018 \\ 34.979 \\ 0 \\ 0 \end{bmatrix}$$

3 Methods Applicable to Influence Diagram Form

3.1 Enter Evidence in Influence Diagram

3.1.1 Test Case

Input

Domain = $\{X_1, X_2\}$

$n_0 = 1, n_1 = 1, n_2 = 0$

$\mu = [0\ 0]^T$

$B = [0\ 1$

$\quad 0\ 0]$

$v = [\infty\ 1]^T$

Evidence: $X_2 = 20$

Output

Domain = $\{X_1, X_2\}$

$\mu = [20\ 0]^T$

$B = [0\ 0$

$\quad 0\ 0]$

$v = [1\ 0]^T$

3.1.2 Test Case

Input

Domain = $\{X_1, X_2\}$

$n_0 = 1, n_1 = 1, n_2 = 0$

$\mu = [0\ 0]^T$

$B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$v = [\infty\ 1]^T$

Evidence: $X_2 = 80$

Output

Domain = $\{X_1, X_2\}$

$\mu = [80\ 0]^T$

$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$v = [1\ 0]^T$

3.1.3 Test Case

Input

Domain = $\{X_1, X_2\}$

$n_0 = 1, n_1 = 2, n_2 = 0$

$$\boldsymbol{\mu} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\ \boldsymbol{B} = \begin{bmatrix} 0 & 1 & 1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\ \boldsymbol{v} = \begin{bmatrix} \infty \\ 1 \\ 1 \end{bmatrix}$$

Evidence $X_2 = [20\ z]^T$

Output

Domain = $\{X_1, X_2\}$

$$\boldsymbol{\mu} = \begin{bmatrix} 20 + 2/5(z-10) \\ 0 \\ 0 \end{bmatrix}\ \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\ \mathbf{v} = \begin{bmatrix} 4/5 \\ 0 \\ 0 \end{bmatrix}$$

3.1.4 Test Case

Input

Domain = $\{X_1, X_2\}$

$n_0 = 1, n_1 = 2, n_2 = 0$

$$\boldsymbol{\mu} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\ \boldsymbol{B} = \begin{bmatrix} 0 & 1 & 1/4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\ \boldsymbol{v} = \begin{bmatrix} \infty \\ 1 \\ 1 \end{bmatrix}$$

Evidence $X_2 = [80\ z]^T$

Output

Domain = { {X_1, X_2}

$$\boldsymbol{\mu} = \begin{bmatrix} 80 + \left(\frac{4}{17}\right)(z-20) \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} \frac{16}{17} \\ 0 \\ 0 \end{bmatrix}$$

3.2 Node Removal for Vector Nodes in Gaussian Influence Diagrams

3.2.1 Test Case

Input

pred = {X1}, **tgt** = {X2}, **succ** = {X3}

$n_0 = 1$, $n_1 = 1$, $n_2 = 1$

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{\mathbf{pred}} \\ \boldsymbol{\mu}_{\mathbf{tgt}} \\ \boldsymbol{\mu}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} 16 \\ 1 \\ 36 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & -1.75 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

Output

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{\mathbf{pred}} \\ \boldsymbol{\mu}_{\mathbf{tgt}} \\ \boldsymbol{\mu}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \\ 61 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.75 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

3.2.2 Test Case

Input

pred = {X1}, **tgt** = {X2, X3}, **succ** = {X4}

$n_0 = 1$, $n_1 = 2$, $n_2 = 1$

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{\mathbf{pred}} \\ \boldsymbol{\mu}_{\mathbf{tgt}} \\ \boldsymbol{\mu}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} 16 \\ 1 \\ 36 \\ 49 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & -1.75 & -0.125 \\ 0 & 0 & 5 & 0.5 \\ 0 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Output

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{\mathbf{pred}} \\ \boldsymbol{\mu}_{\mathbf{tgt}} \\ \boldsymbol{\mu}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \\ 0 \\ 62 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -0.25 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3.2.3 Test Case

Input

pred = {X1,X2}, **tgt** = {X3, X4}, **succ** = {X5,X6}
$n_0 = 2$, $n_1 = 2$, $n_2 = 2$

$$\boldsymbol{\mu}=\begin{bmatrix}\boldsymbol{\mu}_{\mathbf{pred}}\\ \boldsymbol{\mu}_{\mathbf{tgt}}\\ \boldsymbol{\mu}_{\mathbf{succ}}\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}$$

$$\mathbf{v}=\begin{bmatrix}\mathbf{v}_{\mathbf{pred}}\\ \mathbf{v}_{\mathbf{tgt}}\\ \mathbf{v}_{\mathbf{succ}}\end{bmatrix}=\begin{bmatrix}16\\1\\36\\49\\4\\25\end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix}\mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}}\\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}}\\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}}\end{bmatrix}=\begin{bmatrix}0 & 0.5 & -1.75 & -0.125 & 1 & 0.5\\ 0 & 0 & 5 & 0.5 & -1 & 0.5\\ 0 & 0 & 0 & -0.5 & 1 & -0.5\\ 0 & 0 & 0 & 0 & 1 & 0.5\\ 0 & 0 & 0 & 0 & 0 & 0.5\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

Output

$$\boldsymbol{\mu}=\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}$$

$$\boldsymbol{v}=\begin{bmatrix}16\\1\\0\\0\\62\\55.548\end{bmatrix}$$

$$\boldsymbol{B}=\begin{bmatrix}0 & 0.5 & 0 & 0 & 0 & 1.75\\ 0 & 0 & 0 & 0 & 2 & -3.35483871\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0.677419355\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

3.2.4 Test Case

Input

pred = {X1,X2}, **tgt** = {X3, X4}, **succ** = {X5,X6}

$n_0 = 2$, $n_1 = 2$, $n_2 = 2$

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{\mathbf{pred}} \\ \boldsymbol{\mu}_{\mathbf{tgt}} \\ \boldsymbol{\mu}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} \infty \\ \infty \\ 70 \\ 49.271 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.4428571 & 1.01826616 & 0.18324152 \\ 0 & 0 & 0 & 0 & 1.00898811 & 1.0266744 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Output

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{\mathbf{pred}} \\ \boldsymbol{\mu}_{\mathbf{tgt}} \\ \boldsymbol{\mu}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \end{bmatrix} = \begin{bmatrix} \infty \\ \infty \\ 0 \\ 0 \\ 73.018 \\ 34.979 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.55031766 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3.3 Arc Reversal Between Vector Nodes in Gaussian Influence Diagrams

3.3.1 Test Case

Input

pred = {X1}, **tgt** = {X2}, **succ** = {X3}, **gc** = {}
$n_0 = 1$, $n_1 = 1$, $n_2 = 1$ $n_3 = 0$

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{\mathbf{pred}} \\ \boldsymbol{\mu}_{\mathbf{tgt}} \\ \boldsymbol{\mu}_{\mathbf{succ}} \\ \boldsymbol{\mu}_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \\ \mathbf{v}_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} 16 \\ 1 \\ 36 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} & \mathbf{B}_{\mathbf{pred,gc}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} & \mathbf{B}_{\mathbf{tgt,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} & \mathbf{B}_{\mathbf{succ,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{gc,gc}} \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & -1.75 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

Output

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{\mathbf{pred}} \\ \boldsymbol{\mu}_{\mathbf{tgt}} \\ \boldsymbol{\mu}_{\mathbf{succ}} \\ \boldsymbol{\mu}_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \\ \mathbf{v}_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} 16 \\ 0.5902 \\ 61 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} & \mathbf{B}_{\mathbf{pred,gc}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} & \mathbf{B}_{\mathbf{tgt,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} & \mathbf{B}_{\mathbf{succ,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{gc,gc}} \end{bmatrix} = \begin{bmatrix} 0 & 0.43852459 & 0.75 \\ 0 & 0 & 0 \\ 0 & 0.08196721 & 0 \end{bmatrix}$$

3.3.2 Test Case

Input

pred = {X1}, **tgt** = {X2, X3}, **succ** = {X4}, **gc** = {}
$n_0 = 1$, $n_1 = 2$, $n_2 = 1$, $n_3 = 0$

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{\mathbf{pred}} \\ \boldsymbol{\mu}_{\mathbf{tgt}} \\ \boldsymbol{\mu}_{\mathbf{succ}} \\ \boldsymbol{\mu}_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \\ \mathbf{v}_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} 16 \\ 1 \\ 36 \\ 49 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} & \mathbf{B}_{\mathbf{pred,gc}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} & \mathbf{B}_{\mathbf{tgt,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} & \mathbf{B}_{\mathbf{succ,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{gc,gc}} \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & -1.75 & -0.125 \\ 0 & 0 & 5 & 0.5 \\ 0 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Output

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu_0} \\ \boldsymbol{\mu_1} \\ \boldsymbol{\mu_2} \\ \boldsymbol{\mu_3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{v} = \begin{bmatrix} 16 \\ 0.9355 \\ 30.414 \\ 62 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0.491933548 & -1.5172414 & -0.25 \\ 0 & 0 & 4.37931034 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -0.0322581 & -0.3103448 & 0 \end{bmatrix}$$

3.3.3 Test Case

Input

pred = {X1, X2}, **tgt** = {X3}, **succ** = {X4}, **gc** = {}

$n_0 = 2$, $n_1 = 1$, $n_2 = 1$, $n_3 = 0$

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_{\mathbf{pred}} \\ \mu_{\mathbf{tgt}} \\ \mu_{\mathbf{succ}} \\ \mu_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \\ \mathbf{v}_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} 16 \\ 1 \\ 36 \\ 49 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} & \mathbf{B}_{\mathbf{pred,gc}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} & \mathbf{B}_{\mathbf{tgt,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} & \mathbf{B}_{\mathbf{succ,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{gc,gc}} \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & -1.75 & -0.125 \\ 0 & 0 & 5 & 0.5 \\ 0 & 0 & 0 & -0.5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Output

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_{\mathbf{pred}} \\ \mu_{\mathbf{tgt}} \\ \mu_{\mathbf{succ}} \\ \mu_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \\ \mathbf{v}_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} 16 \\ 1 \\ 30.414 \\ 58 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} & \mathbf{B}_{\mathbf{pred,gc}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} & \mathbf{B}_{\mathbf{tgt,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} & \mathbf{B}_{\mathbf{succ,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{gc,gc}} \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & -1.5172414 & 0.75 \\ 0 & 0 & 4.37931034 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.3103448 & 0 \end{bmatrix}$$

3.3.4 Test Case

Input

pred = {X1,X2}, **tgt** = {X3, X4}, **succ** = {X5,X6}, **gc** = {}

$n_0 = 2$, $n_1 = 2$, $n_2 = 2$, $n_3 = 0$

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{\text{pred}} \\ \boldsymbol{\mu}_{\text{tgt}} \\ \boldsymbol{\mu}_{\text{succ}} \\ \boldsymbol{\mu}_{\text{gc}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\text{pred}} \\ \mathbf{v}_{\text{tgt}} \\ \mathbf{v}_{\text{succ}} \\ \mathbf{v}_{\text{gc}} \end{bmatrix} = \begin{bmatrix} 16 \\ 1 \\ 36 \\ 49 \\ 4 \\ 25 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\text{pred,pred}} & \mathbf{B}_{\text{pred,tgt}} & \mathbf{B}_{\text{pred,succ}} & \mathbf{B}_{\text{pred,gc}} \\ \mathbf{0} & \mathbf{B}_{\text{tgt,tgt}} & \mathbf{B}_{\text{tgt,succ}} & \mathbf{B}_{\text{tgt,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\text{succ,succ}} & \mathbf{B}_{\text{succ,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\text{gc,gc}} \end{bmatrix} = \begin{bmatrix} 0 & 0.5 & -1.75 & -0.125 & 1 & 0.5 \\ 0 & 0 & 5 & 0.5 & -1 & 0.5 \\ 0 & 0 & 0 & -0.5 & 1 & -0.5 \\ 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Output

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_0 \\ \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \\ \boldsymbol{\mu}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{v} = \begin{bmatrix} 16 \\ 1 \\ 14.362 \\ 3.5662 \\ 62 \\ 55.548 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0.5 & -0.7987805 & -0.9363173 & 0 & 1.75 \\ 0 & 0 & 2.59581882 & 0.8922853 & 2 & -3.3548387 \\ 0 & 0 & 0 & -0.8922853 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.65853659 & 0.8558952 & 0 & 0.677419355 \\ 0 & 0 & -0.543554 & 0.0713246 & 0 & 0 \end{bmatrix}$$

3.3.5 Test Case

Input

$\mathbf{pred} = \{X1,X2\}$, $\mathbf{tgt} = \{X3, X4\}$, $\mathbf{succ} = \{X5,X6\}$, $\mathbf{gc} = \{\}$

$n_0 = 2$, $n_1 = 2$, $n_2 = 2$, $n_3 = 0$

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_{\mathbf{pred}} \\ \boldsymbol{\mu}_{\mathbf{tgt}} \\ \boldsymbol{\mu}_{\mathbf{succ}} \\ \boldsymbol{\mu}_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_{\mathbf{pred}} \\ \mathbf{v}_{\mathbf{tgt}} \\ \mathbf{v}_{\mathbf{succ}} \\ \mathbf{v}_{\mathbf{gc}} \end{bmatrix} = \begin{bmatrix} \infty \\ \infty \\ 70 \\ 49.271 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{\mathbf{pred,pred}} & \mathbf{B}_{\mathbf{pred,tgt}} & \mathbf{B}_{\mathbf{pred,succ}} & \mathbf{B}_{\mathbf{pred,gc}} \\ \mathbf{0} & \mathbf{B}_{\mathbf{tgt,tgt}} & \mathbf{B}_{\mathbf{tgt,succ}} & \mathbf{B}_{\mathbf{tgt,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{succ,succ}} & \mathbf{B}_{\mathbf{succ,gc}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathbf{gc,gc}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.4428571 & 1.01826616 & 0.18324152 \\ 0 & 0 & 0 & 0 & 1.00898811 & 1.0266744 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Output

$$\boldsymbol{\mu} = \begin{bmatrix} \boldsymbol{\mu}_0 \\ \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \\ \boldsymbol{\mu}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{v} = \begin{bmatrix} \infty \\ \infty \\ 0 \\ 0 \\ 73.018 \\ 34.979 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.009195 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.1930593 & 0.991092 & 0 & 0.5503149 \\ 0 & 0 & -1.1725067 & 0 & 0 & 0 \end{bmatrix}$$

3.4 Implementation of Kalman Filtering Using the Influence Diagram Form

3.4.1 Mathematical Model

$Dynamic\ Process$
$x(k+1) = \Phi(k)x(k) + \Gamma(k)w(k)\ for\ k = 0, \dots, N.$

$Measurement\ Process$
$z(k) = \mathrm{H}(k)x(k) + v(k)\ for\ k = 0, \dots, N.$

$Probabilistic\ Structure$
$E[x(0)] = \mu_0$
$Cov[x(0)] = \mathrm{P}_0$
$E[w(k)] = 0\ for\ k = 0, \dots, N.$
$Cov[w(j), w(k)] = \delta_{jk}\mathrm{Q}_k\ for\ k = 0, \dots, N.\ \mathrm{Q}_k\ are\ diagonal\ for\ k = 0, \dots, N.$
$Cov[x(0), w(0)] = 0.$
$E[v(k)] = 0\ for\ k = 0, \dots, N.$
$Cov[v(j), v(k)] = \delta_{jk}\mathrm{R}_k\ for\ k = 0, \dots, N.\ \mathrm{R}_k\ are\ diagonal\ for\ k = 0, \dots, N.$
$Cov[w(j), v(k)] = 0\ for\ j = 1, \dots, N\ and\ for\ k = 0, \dots, N.$
$Cov[x(0), v(k)] = 0\ for\ k = 0, \dots, N.$

$Dimensions\ of\ Vectors$
$x(k)\epsilon R^n, w(k)\epsilon R^r, z(k)\epsilon R^p, v(k)\epsilon R^p$

3.4.2 Measurement Update for Measurement Z(K)

3.4.2.1 Test Case – Kalman Filter for Dummies

(http://bilgin.esme.org/BitsAndBytes/KalmanFilterforDummies)

Input

Time step (k)	**z**(k) with p=1	**μ**(k)	**B**(k) or Σ(k)	**v**(k)	$\mathbf{R}_k$	**H(k)** with p=1 and n=1
0	0.39	0	1	0	0.1	1
1	0.5	0.355	0	0.091	0.1	1

Output

Time step (k)	$\begin{bmatrix}\boldsymbol{\mu}_1\\ \mathbf{0}\end{bmatrix}$	$\begin{bmatrix}\boldsymbol{v}_1\\ \mathbf{0}\end{bmatrix}$	$\begin{bmatrix}\boldsymbol{B}_{11} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}$
0	$\begin{bmatrix}0.355\\ 0\end{bmatrix}$	$\begin{bmatrix}0.091\\ 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$
1	$\begin{bmatrix}0.424\\ 0\end{bmatrix}$	$\begin{bmatrix}0.048\\ 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

3.4.2.2 Test Case

Example 5.5 (Discrete-Time Implementation with Numerical Values), p. 195-196 from Kalman Filtering: Theory and Practice Using MATLAB, 4th Edition

Input

Time step (k)	**z**(k) with p=2	**μ**(k)	Σ(k)	**v**(k)	**R**$_k$	**H(k)** with p=2 and n=2
0	$\begin{bmatrix}3\\4\end{bmatrix}$	$\begin{bmatrix}1\\2\end{bmatrix}$	$\begin{bmatrix}4 & 1\\1 & 9\end{bmatrix}$	$\begin{bmatrix}0\\0\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 4\end{bmatrix}$	$\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}$

Output (after converting influence diagram representation output $\begin{bmatrix}\boldsymbol{v}_1\\\mathbf{0}\end{bmatrix}, \begin{bmatrix}\boldsymbol{B}_{11} & \mathbf{0}\\\mathbf{0} & \mathbf{0}\end{bmatrix}$ to covariance representation $\begin{bmatrix}\boldsymbol{\Sigma}_{11} & \mathbf{0}\\\mathbf{0} & \mathbf{0}\end{bmatrix}$)

Time step (k)	$\begin{bmatrix}\boldsymbol{\mu}_1\\\mathbf{0}\end{bmatrix}$	$\begin{bmatrix}\boldsymbol{\Sigma}_{11} & \mathbf{0}\\\mathbf{0} & \mathbf{0}\end{bmatrix}$
0	$\begin{bmatrix}\frac{467}{361}\\\frac{2819}{1444}\\0\\0\end{bmatrix} = \begin{bmatrix}1.29362881\\1.51592798\\0\\0\end{bmatrix}$	$\begin{bmatrix}\frac{144}{361} & \frac{1}{361} & 0 & 0\\\frac{1}{361} & \frac{351}{1444} & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix} = \begin{bmatrix}0.398891967 & 0.002770083 & 0 & 0\\0.002770083 & 0.243074792 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}$

3.4.2.3 Test Case

Example 5.7 (Resonator Tracking with sample interval equal to 1 s), p. 174-176, 225, 596. from Kalman Filtering: Theory and Practice Using MATLAB, 4th Edition

Input

Time step (k)	**z**(k) with p=1	**μ**(k)	Σ(k)	**v**(k)	**R**$_k$	**H(k)** with p=1 and n=2
0	[0.0101]	$\begin{bmatrix}0.0101\\0.1188\end{bmatrix}$	$\begin{bmatrix}0.01071225 & 0.017495523\\0.017495523 & 2.04175521\end{bmatrix}$	$\begin{bmatrix}0\\0\end{bmatrix}$	[0.01]	[1 0]

Output (after converting influence diagram representation output $\begin{bmatrix}\boldsymbol{v}_1\\\mathbf{0}\end{bmatrix}, \begin{bmatrix}\boldsymbol{B}_{11} & \mathbf{0}\\\mathbf{0} & \mathbf{0}\end{bmatrix}$ to covariance representation $\begin{bmatrix}\boldsymbol{\Sigma}_{11} & \mathbf{0}\\\mathbf{0} & \mathbf{0}\end{bmatrix}$)

Time step (k)	$\begin{bmatrix} \boldsymbol{\mu}_1 \\ \mathbf{0} \end{bmatrix}$	$\begin{bmatrix} \boldsymbol{\Sigma}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$
0	$\begin{bmatrix} 0.0101 \\ 0.1188 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.00516961 & 0.008445032 & 0 \\ 0.008445032 & 2.02692169 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3.4.3 Time Update From X(K) to X(K+1)

3.4.3.1 Test Case

Example 5.7 (Resonator Tracking with sample interval equal to 1 s), p. 174-176, 225, 596. from Kalman Filtering: Theory and Practice Using MATLAB, 4th Edition

Input from exam57.m

Σ(k) with n=2
$\begin{bmatrix} 0.00516961 & 0.008445032 \\ 0.008445032 & 2.02692169 \end{bmatrix}$

Let (**B, v**) be the output of Cov_to_ID (**Σ**(k)**, B**, **v**, **P**).

(1) Then set $\mathbf{B}(\mathrm{k}) = \boldsymbol{B}$ and set $\boldsymbol{v}(\mathrm{k}) = \boldsymbol{v}$.

Influence Diagram Input

μ(k) with n=2	$\boldsymbol{B}$(k)	$\boldsymbol{v}$(k)	**Φ**(k) with n=2	**Γ**(k) with n=2 and r=2	**Q** with r=2
$\begin{bmatrix} 0.0101 \\ 0.1188 \end{bmatrix}$	See (1)	See (1)	$\begin{bmatrix} 1.0191 & 0.0099 \\ -0.2474 & 0.9994 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.002 & 0.002 \\ 0.002 & \mathbf{0}.438 \end{bmatrix}$.

Call Table 7 algorithm using these inputs.

Output of Table 7 (after converting influence diagram representation $\begin{bmatrix} \mathbf{0} \\ \boldsymbol{v}''_2 \end{bmatrix}, \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{B}''_{22} \end{bmatrix}$ to covariance representation $\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{22} \end{bmatrix}$)

Time step (k)	$\begin{bmatrix} \mathbf{0} \\ \boldsymbol{\mu}''_2 \end{bmatrix}$	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{22} \end{bmatrix}$
0	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.01146903 \\ 0.11622998 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.007738039 & 0.02933158 \\ 0 & 0 & 0 & 0 & 0.02933158 & 2.458630434 \end{bmatrix}$

3.4.4 Kalman Filter Update at Time Step K

3.4.4.1 Test Case

Example 5.7 (Resonator Tracking with sample interval equal to 1 s), p. 174-176, 225, 596. from Kalman Filtering: Theory and Practice Using MATLAB, 4th Edition

Input from exam57.m

Influence Diagram Input

Time step (k)	**z**(k) with p=1	**μ**(k) with n=2	Σ(k) with n=2	**v**(k) with n=2	**R**$_k$ with p=1	**H(k)** with p=1 and n=2	**Φ**(k) with n=2	**Γ**(k) with n=2 and r=2	**Q** with r=2
0	[0.010	$\begin{bmatrix} 0.010 \\ 0.118 \end{bmatrix}$	$\begin{bmatrix} 0.01071225 & 0.01 \\ 0.017495523 & 2.0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	[0.01	[1 (	$\begin{bmatrix} 1.0191 & 0 \\ -0.2474 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & (\\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.002 & 0. \\ 0.002 & \mathbf{0.} \end{bmatrix}$

Output (after converting influence diagram representation output $\begin{bmatrix} \mathbf{0} \\ \boldsymbol{v'}_{2} \end{bmatrix}, \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{B'}_{22} \end{bmatrix}$ to covariance representation $\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma'}_{22} \end{bmatrix}$)

Time step (k)	$\begin{bmatrix} \mathbf{0} \\ \boldsymbol{\mu'}_{2} \end{bmatrix}$	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma'}_{22} \end{bmatrix}$
0	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.01146903 \\ 0.11622998 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.007738039 & 0.02933158 \\ 0 & 0 & 0 & 0 & 0.02933158 & 2.458630434 \end{bmatrix}$

4 Interfacing MATLAB Sensor Fusion Tracking Toolbox to Influence Diagram

4.1 Mathematical Models

Table 1. Mapping from Bryson and Ho to MATLAB trackingKF object and correct function

Bryson and Ho (1975, 360)	MATLAB trackingKF equations (trackingKF, 2020)	Bryson and Ho (1975, 360)	MATLAB trackingKF properties (trackingKF, 2020)	MATLAB correct and predict variables (correct 2020, predict 2020)
Dynamic Process				
$x(k+1) = \Phi(k)x(k) + \Gamma(k)w(k)\ for\ k = 0, \ldots, N.$	$x_{k+1} = F_k x_k + G_k u_k + v_k$	$x(k)$	x_k	
		$\Phi(k)$	StateTransitionModel = F_k	
		null	ControlModel = G_k	
		$\Gamma(k)w(k)$	v_k	
Measurement Process				
$z(k) = \mathrm{H}(k)x(k) + v(k)\ for\ k = 0, \ldots, N.$	$z_k = H_k x_k + w_k$	$z(k)$	z_k	zmeas
		$\mathrm{H}(k)$	MeasurementModel = H_k	
		$x(k)$	x_k	
		$v(k)$	w_k	
Probabilistic Structure				
$E[x(0)] = \mu_0$				
$Cov[x(0)] = \mathrm{P}_0$				
$E[w(k)] = 0\ for\ k = 0, \ldots, N.$	*Implicit*			

Bryson and Ho (1975, 360)	MATLAB trackingKF equations (trackingKF, 2020)	Bryson and Ho (1975, 360)	MATLAB trackingKF properties (trackingKF, 2020)	MATLAB correct and predict variables (correct 2020, predict 2020)
$Cov[w(j), w(k)] = \delta_{jk} \mathrm{Q}_k \; for \; k = 0, \dots, N.$	*Implicit*	Q	ProcessNoise = $Q_{trackingKF} = \Gamma(k) Q_k \Gamma^t(k)$	$\boldsymbol{Q} = Q_{trackingKF}$
$Cov[x(0), w(0)] = 0.$	*Implicit*			
$E[v(k)] = 0 \; for \; k = 0, \dots, N.$	*Implicit*			
$Cov[v(j), v(k)] = \delta_{jk} \mathrm{R}_k \; for \; k = 0, \dots, N. \; \mathrm{R}_k \; are \; diagonal \; for \; k = 0, \dots, N.$	*Implicit*	R	MeasurementNoise = R	zcov
$Cov[w(j), v(k)] = 0 \; for \; j = 1, \dots, N \; and \; for \; k = 0, \dots, N.$	*Implicit*			
$Cov[x(0), v(k)] = 0 \; for \; k = 0, \dots, N.$	*Implicit*			

Table 2. Dimensions of Vectors for Bryson and Ho and for MATLAB trackingKF object

Bryson and Ho (1975, 360)	MATLAB trackingKF equations (trackingKF, 2020)
$x(k) \epsilon R^n, w(k) \epsilon R^r, z(k) \epsilon R^p, v(k) \epsilon R^p$	$x_k \epsilon R^M, u_k \epsilon R^L, v_k \epsilon R^M, z_k \epsilon R^N, w_k \epsilon R^N$

4.2 *Kalman Filter Update at Time Step K*

4.2.1 Test Case

Example 5.7 (Resonator Tracking with sample interval equal to 1 s), p. 174-176, 225, 596. from Kalman Filtering: Theory and Practice Using MATLAB, 4th Edition

trackingKF and correct inputs

zmeas with N=1	**μ** with M=2	***P*** with M=2	**v** with M =2	**zcov** with N=1	H with N=1 and M=2	F with M=2	$\boldsymbol{Q}_{trackingKF}$ with M=2
$[0.0101$	$\begin{bmatrix} 0.0101 \\ 0.1188 \end{bmatrix}$	$\begin{bmatrix} 0.01071225 & 0.0174 \\ 0.017495523 & 2.0417 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$[0.01]$	$[1 \quad 0$	$\begin{bmatrix} 1.0191 & 0.0 \\ -0.2474 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.002 & 0.0 \\ 0.002 & \mathbf{0}.4 \end{bmatrix}$

Output (after converting influence diagram representation output $\begin{bmatrix} \mathbf{0} \\ \boldsymbol{v'}_{\mathbf{2}} \end{bmatrix}, \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{B'}_{\mathbf{22}} \end{bmatrix}$ to covariance representation $\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma'}_{\mathbf{22}} \end{bmatrix}$)

Time step (k)	$\begin{bmatrix} \mathbf{0} \\ \boldsymbol{\mu'}_{\mathbf{2}} \end{bmatrix}$	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma'}_{\mathbf{22}} \end{bmatrix}$
0	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.01146903 \\ 0.11622998 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.007738039 & 0.02933158 \\ 0 & 0 & 0 & 0 & 0.02933158 & 2.458630434 \end{bmatrix}$

$$[\mathrm{xcorr}, \mathrm{Pcorr}] = [\boldsymbol{\mu'}_{\mathbf{2}}, \boldsymbol{\Sigma'}_{\mathbf{22}}] = \left[\begin{bmatrix} 0.01146903 \\ 0.11622998 \end{bmatrix}, \begin{bmatrix} 0.007738039 & 0.02933158 \\ 0.02933158 & 2.458630434 \end{bmatrix}\right]$$