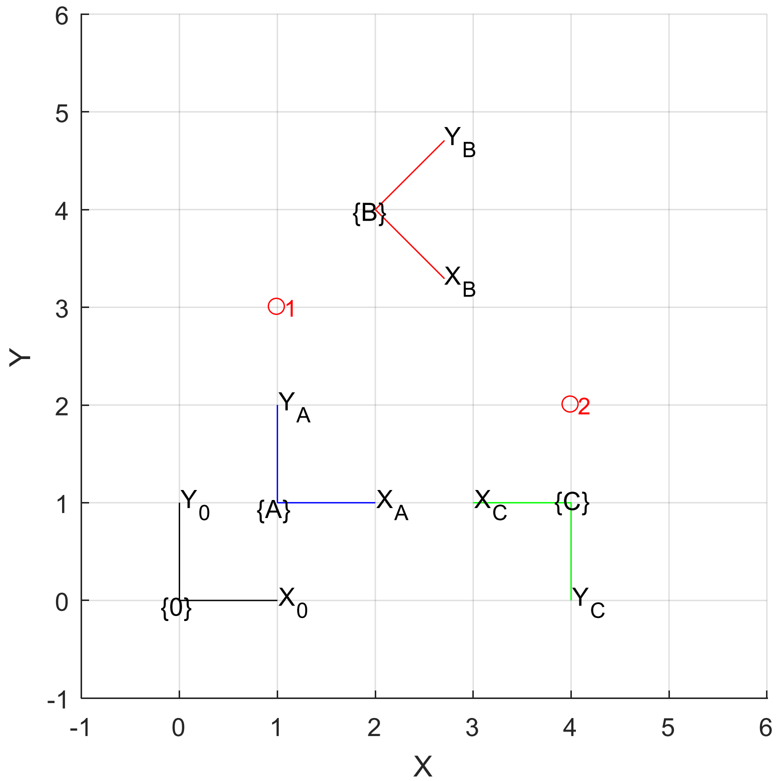
Exercises – Representing pose in 2D

OsloMet ELVE3610 Introduction to Robotics

1) Consider the following diagram



a) Using the diagram, determine the positions of the points 1 and 2, denoted by red circles in the following reference frames
   a. Determine vectors ${}^{O}p_1$, ${}^{A}p_1$, ${}^{B}p_1$, ${}^{C}p_1$
   b. Determine vectors ${}^{O}p_2$, ${}^{A}p_2$, ${}^{B}p_2$, ${}^{C}p_2$

b) Determine the homogeneous transformation matrices ${}^{O}T_A$, ${}^{O}T_B$, ${}^{O}T_C$

c) Determine the homogeneous transformation matrices ${}^{B}T_C$, ${}^{C}T_A$, ${}^{A}T_B$

d) Using the obtained transformation matrices ${}^{O}T_A$, ${}^{O}T_B$, ${}^{O}T_C$, and ${}^{O}p_1$, ${}^{O}p_2$ from the diagram, determine the position vectors
   a. ${}^{A}p_1$, ${}^{B}p_1$, ${}^{C}p_1$
   b. ${}^{A}p_2$, ${}^{B}p_2$, ${}^{C}p_2$

   and verify that the results are the same as in (a). HINT: For instance, to determine ${}^{A}p_1$ use the transformation formula ${}^{A}\tilde{p}_1 = {}^{A}T_O\, {}^{O}\tilde{p}_1 = \left({}^{O}T_A\right)^{-1}\, {}^{O}\tilde{p}_1$, where $\tilde{p}$ denotes the homogeneous representation of vector p.

2) Determine if the following are valid rotation matrices. HINT: A rotation matrix satisfies $\det(R) = 1$ and $R^T R = I$

   a. $R = \begin{bmatrix} 0.995 & -0.0998 \\ 0.0998 & 0.995 \end{bmatrix}$
   b. $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
   c. $R = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
   d. $R = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$
   e. $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
   f. $R = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$
   g. $R = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$
   h. $R = \begin{bmatrix} -0.5 & -0.866 \\ 0.866 & -0.5 \end{bmatrix}$
   i. $R = \begin{bmatrix} -0.5 & -0.866 \\ 0.866 & 0.5 \end{bmatrix}$

3) Consider a rotation matrix

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

   a) Show that $R(0) = I_2$ where I_2 is the identity matrix of dimension 2.
   b) Show that $\det(R(\theta)) = +1$ for any angle θ
   c) Show that $R(\theta)^T R(\theta) = I_2$
   d) Show that $R(-\theta) = R(\theta)^T$
   e) Show that $R(a)R(b) = R(a + b)$
   f) Show that the columns of $R(\theta)$ are orthonormal (orthogonal and unit norm)
   g) Show that the rows of $R(\theta)$ are orthonormal

4) Consider the transforms

$$T_1 = \begin{bmatrix} R_1 & t_1 \\ 0_{1\times 2} & 1 \end{bmatrix}, T_2 = \begin{bmatrix} R_2 & t_2 \\ 0_{1\times 2} & 1 \end{bmatrix}$$

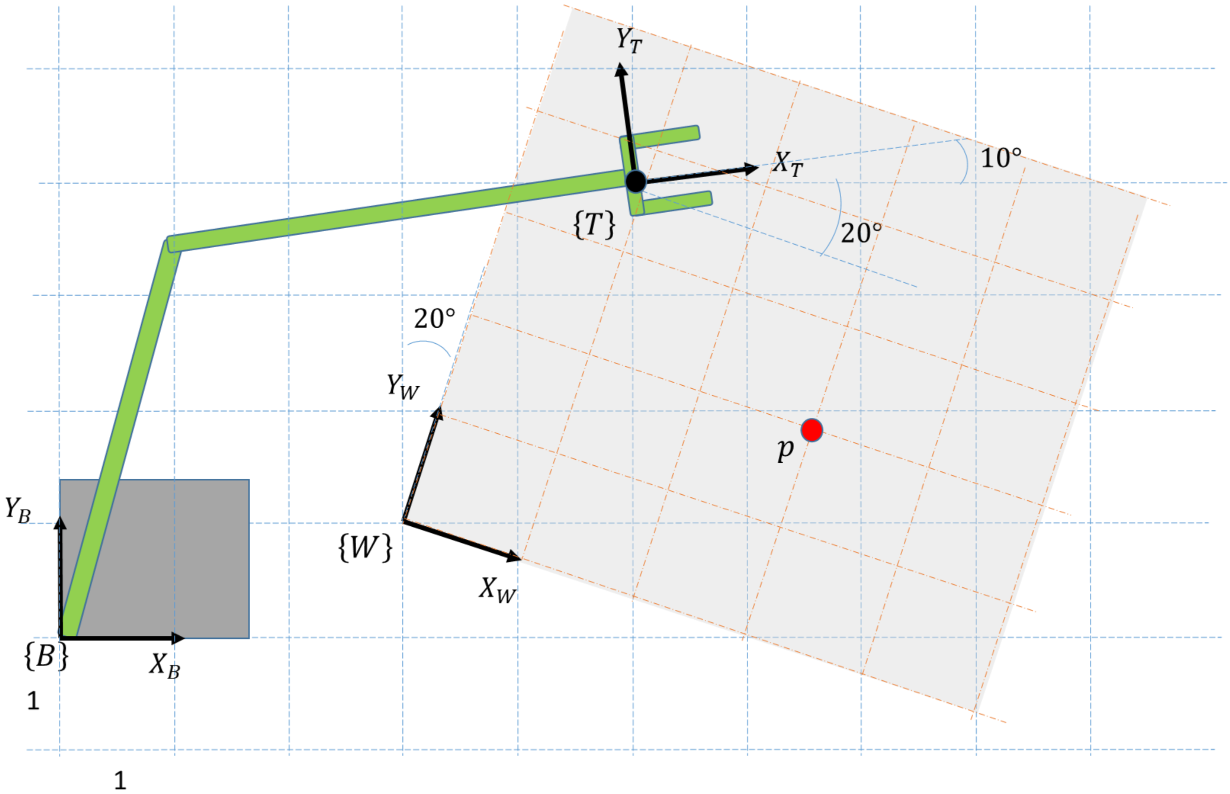
Where

$$t_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, t_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}, 0_{1\times 2} = [0 \quad 0]$$

Show that $T_1 T_2 = \begin{bmatrix} R_1 R_2 & t_1 + R_1 t_2 \\ 0_{1\times 2} & 1 \end{bmatrix}$

5) Show that $T^{-1} = \begin{bmatrix} R^T & -R^T t \\ 0_{1\times 2} & 1 \end{bmatrix}$ is the inverse transform of $T = \begin{bmatrix} R & t \\ 0_{1\times 2} & 1 \end{bmatrix}$. HINT: Show that $TT^{-1} = I_3$.

6) Consider the following diagram:



a) Determine the associated homogeneous transformation matrices ${}^{B}T_{W}$, ${}^{B}T_{T}$

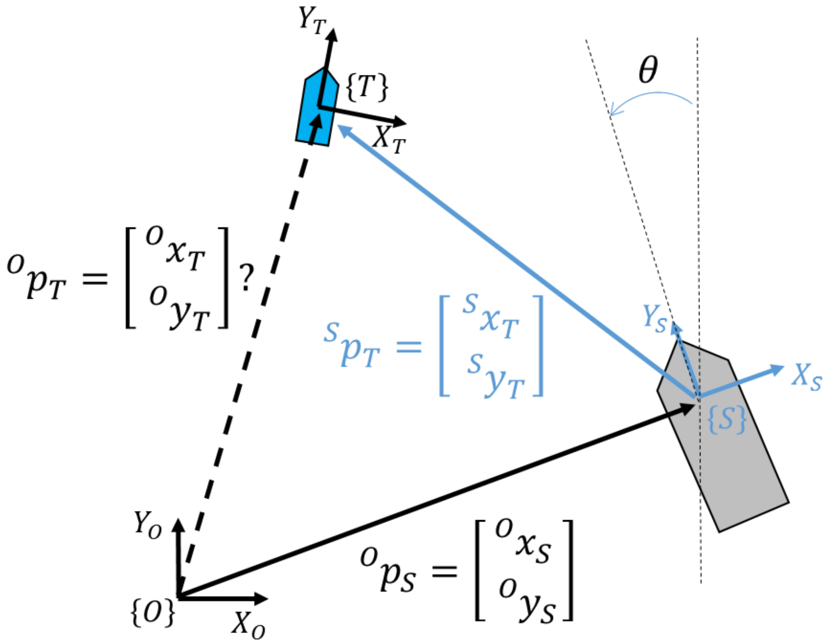
b) Using ${}^{B}T_{W}$, ${}^{B}T_{T}$ determine ${}^{W}T_{T}$

c) Determine the coordinates of the point p in frame $\{W\}$ denoted by ${}^{W}p$

d) Using ${}^{B}T_{W}$, ${}^{B}p$

RADAR target localization

Consider the problem of determining the position of a target from relative position measurements obtained from a ship mounted radar system. The objective is to determine the position of a target $^{O}p_T$ with respect to a reference frame $\{O\}$ when only the relative position measured with respect to a RADAR antenna on a ship is known. The position of the ship with respect to the origin reference frame is denoted by $^{O}p_S$.



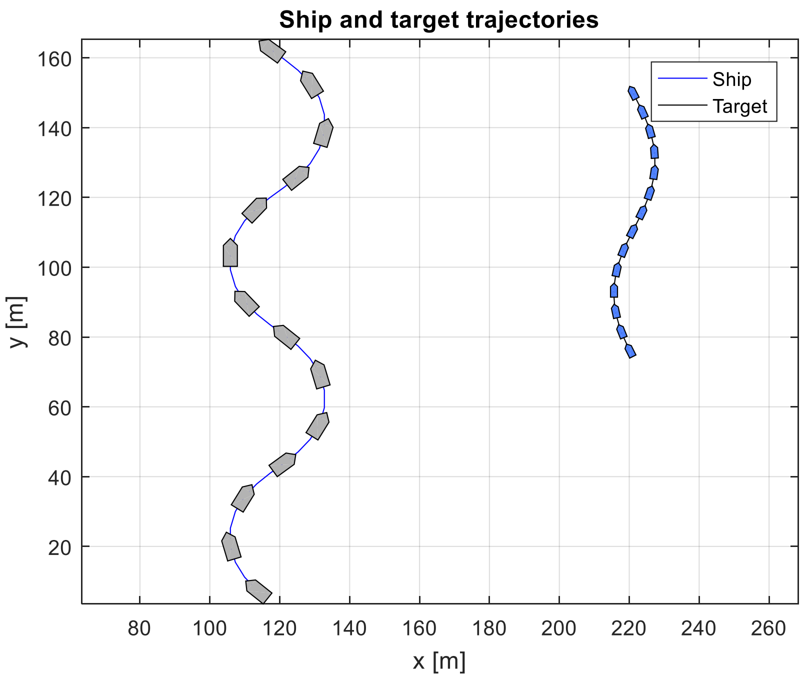
You have a file with measurement data “radar_data.csv” in canvas which contains the following 5 columns:

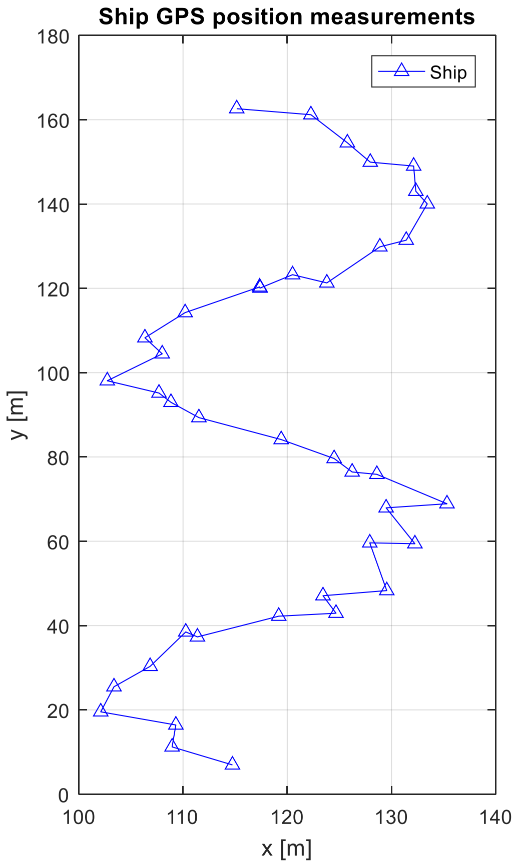
1. $^{O}x_S$: X position of ship with respect to $\{O\}$ in meters.
2. $^{O}x_S$: Y position of the ship with respect to $\{O\}$ in meters
3. θ : Rotation angle from $\{O\}$ to $\{S\}$ in degrees.
4. $^{S}x_T$: X position of target with respect to $\{S\}$ in meters
5. $^{S}x_T$: Y position of the target with respect to $\{S\}$ in meters

The file contains 40 data samples. (NOTE: Angle θ is related to the ship heading ψ by $\theta = -\psi$. We have chosen for simplicity X as East and Y as North which makes a counterclockwise rotation θ as positive. The standard in marine navigation is to take X as North, and Y as East, which makes a clockwise rotation positive)

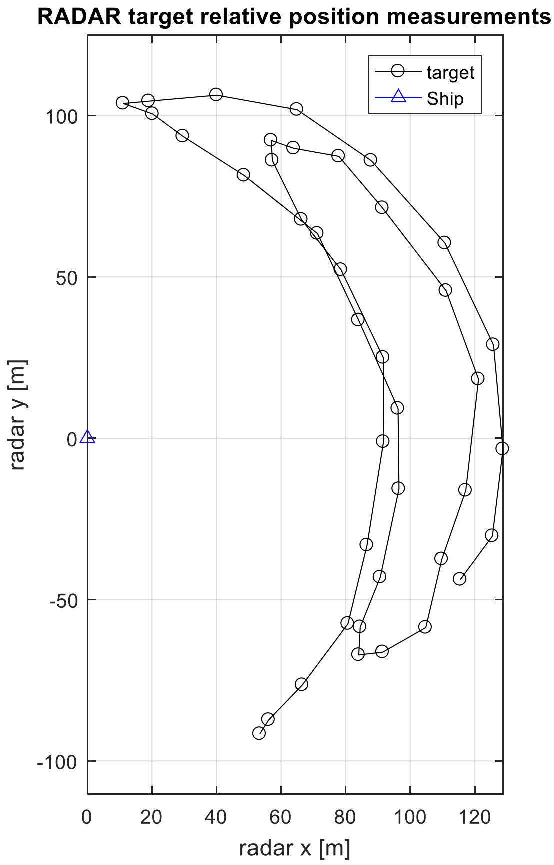
1) Write a python script that reads the measurement data file and plots the following information:
   a. Ship position
   b. Ship heading
   c. Relative target position measurements

2) Write a python script that uses the available measurements to estimate the trajectory of the target with respect to the origin reference frame $\{O\}$. Plot the estimated target trajectory in world reference frame. HINT: Use the relationship ${}^{O}\tilde{p}_T = {}^{O}T_S\, {}^{S}\tilde{p}_T$ or ${}^{O}p_T = {}^{O}R_S\, {}^{S}p_T + {}^{O}p_S$ where ${}^{O}R_S = \begin{bmatrix} cos\,\theta & -\sin\theta \\ sin\,\theta & cos\,\theta \end{bmatrix}$.

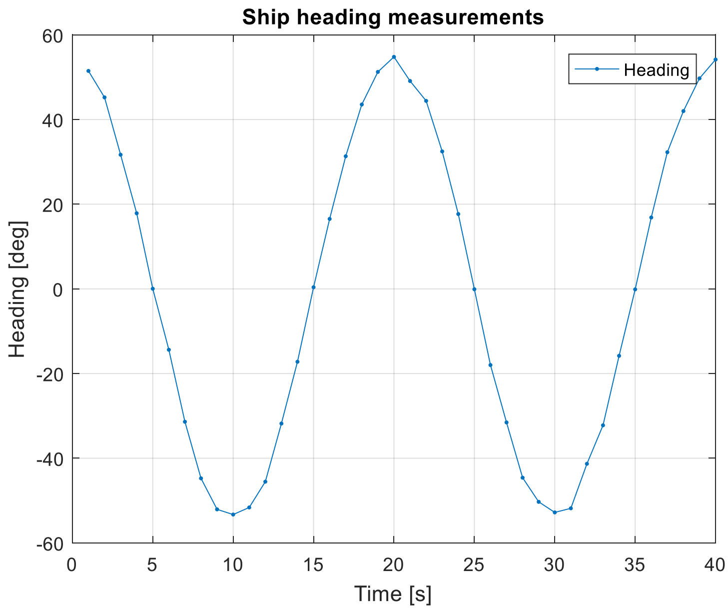




Ship GPS position measurements
Ship
y [m]
x [m]
180
160
140
120
100
80
60
40
20
0
100
110
120
130
140



RADAR target relative position measurements
target
Ship
radar y [m]
radar x [m]
100
50
0
-50
-100
0
20
40
60
80
100
120



Ship heading measurements
Heading
Heading [deg]
Time [s]
60
40
20
0
-20
-40
-60
0
5
10
15
20
25
30
35
40