Exercises – Representing pose in 3D

ELVE3610 Introduction to Robotics

1) Consider the following diagrams

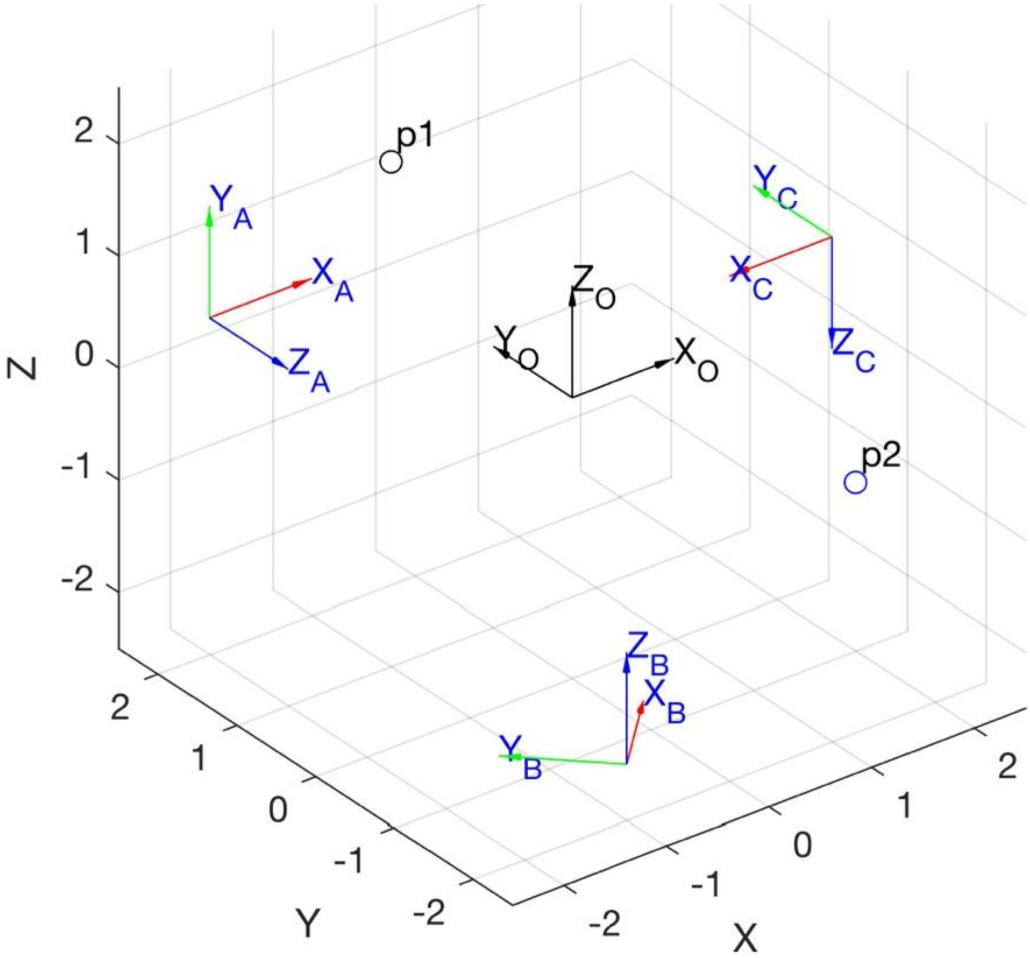

Figure 1: 3D view

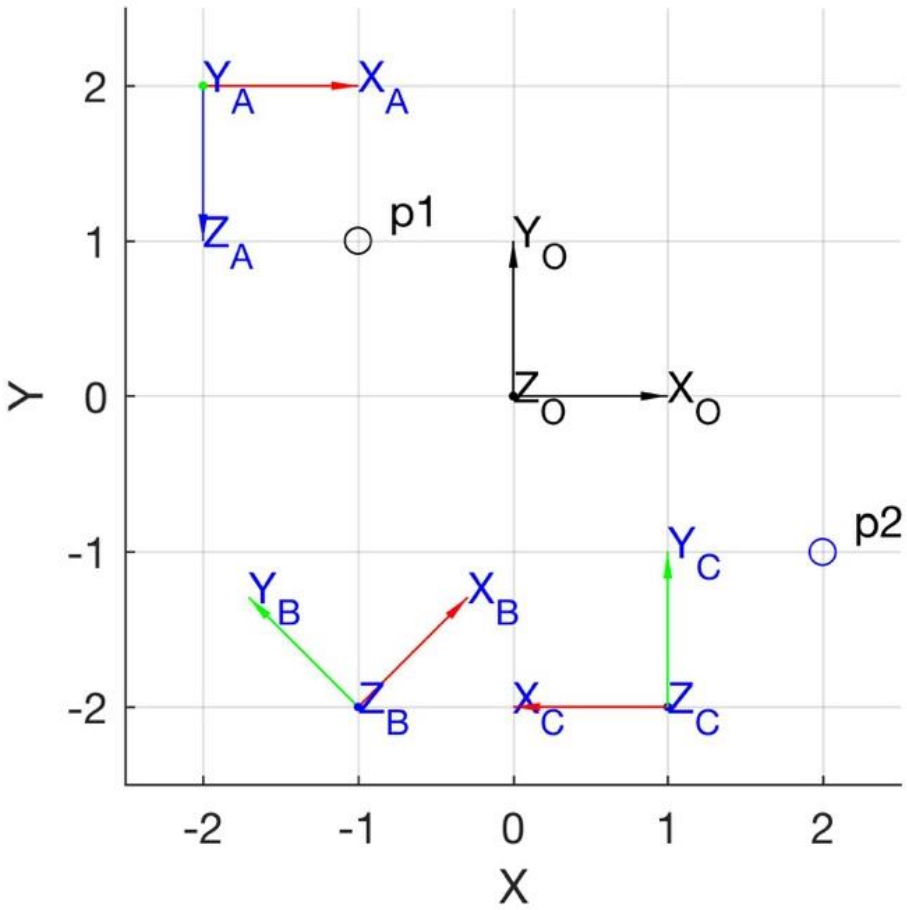

Figure 2: Top view

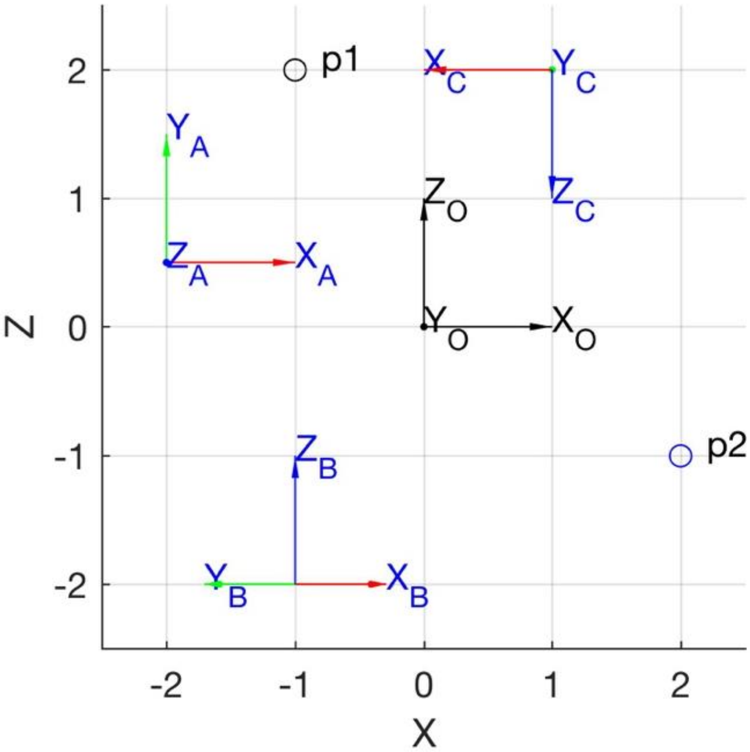

Figure 3:Projection on Y axis

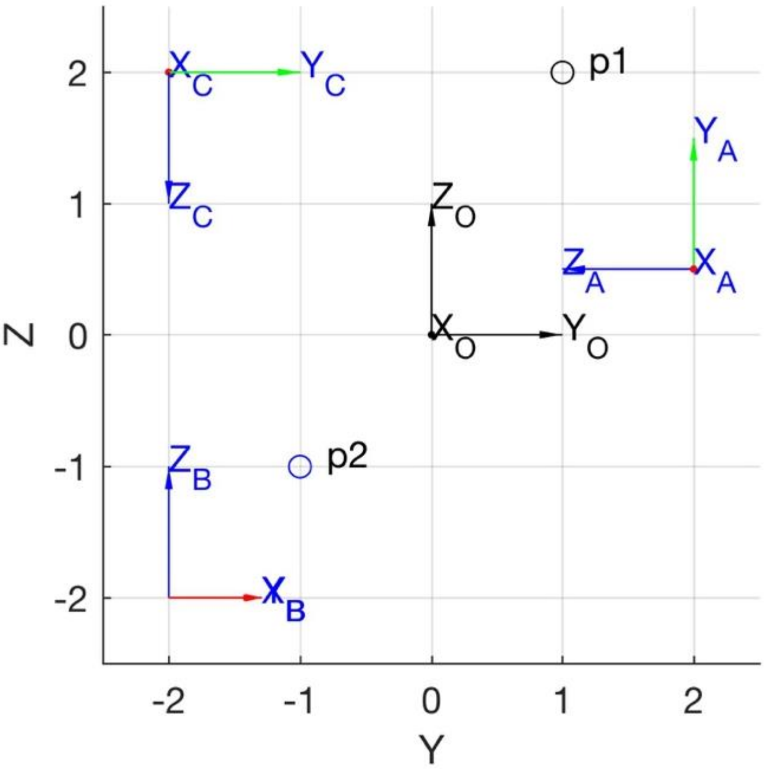

Figure 4: Projection on X axis

a) Using the diagrams, determine the positions of the points 1 and 2, denoted by circles in the following reference frames
   a. Determine vectors ${}^{O}p_1,\ {}^{A}p_1,\ {}^{B}p_1,\ {}^{C}p_1$
   b. Determine vectors ${}^{O}p_2,\ {}^{A}p_2,\ {}^{B}p_2,\ {}^{C}p_2$
b) Determine the homogeneous transformation matrices ${}^{O}T_A,\ {}^{O}T_B,\ {}^{O}T_C$
c) Determine the homogeneous transformation matrices ${}^{B}T_C,\ {}^{C}T_A,\ {}^{A}T_B$
d) Using the obtained transformation matrices ${}^{O}T_A,\ {}^{O}T_B,\ {}^{O}T_C$, and ${}^{O}p_1,\ {}^{O}p_2$ from the diagram, determine the position vectors
   a. ${}^{A}p_1,\ {}^{B}p_1,\ {}^{C}p_1$
   b. ${}^{A}p_2,\ {}^{B}p_2,\ {}^{C}p_2$

   and verify that the results are the same as in (a). HINT: For instance, to determine ${}^{A}p_1$ use the transformation formula ${}^{A}\tilde{p}_1 = {}^{A}T_O\,{}^{O}\tilde{p}_1 = \left({}^{O}T_A\right)^{-1}\,{}^{O}\tilde{p}_1$, where $\tilde{p}$ denotes the homogeneous representation of vector p.

2) Determine if the following are valid rotation matrices. HINT: A rotation matrix satisfies $\det(R) = 1$ and $R^T R = I$. In numpy you can use:

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np.linalg.det(R)
R.dot(R.T)
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   a. $R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
   b. $R = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$
   c. $R = \begin{bmatrix} -0.5 & 0 & 0.866 \\ 0 & 1 & 0 \\ 0.866 & 0 & -0.5 \end{bmatrix}$

3) Consider a rotation matrix

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a) Show that $R(0) = I_2$ where I_2 is the identity matrix of dimension 2.
b) Show that $\det\big(R(\theta)\big) = +1$ for any angle θ
c) Show that $R(\theta)^T R(\theta) = I_2$
d) Show that the columns of $R(\theta)$ are orthonormal (orthogonal and unit norm)
e) Show that the rows of $R(\theta)$ are orthonormal

4) Consider the transforms

$$T_1 = \begin{bmatrix} R_1 & t_1 \\ 0_{1\times 3} & 1 \end{bmatrix}, T_2 = \begin{bmatrix} R_2 & t_2 \\ 0_{1\times 3} & 1 \end{bmatrix}$$

Where

$$t_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, t_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}, 0_{1\times 3} = [0 \quad 0 \quad 0]$$

Show that $T_1 T_2 = \begin{bmatrix} R_1 R_2 & t_1 + R_1 t_2 \\ 0_{1\times 2} & 1 \end{bmatrix}$

5) Show that $T^{-1} = \begin{bmatrix} R^T & -R^T t \\ 0_{1\times 3} & 1 \end{bmatrix}$ is the inverse transform of $T = \begin{bmatrix} R & t \\ 0_{1\times 3} & 1 \end{bmatrix}$. HINT: Show that $TT^{-1} = I_3$.

6) Given the roll, pitch, yaw angles $(\theta_r, \theta_p, \theta_y)$ the corresponding rotation matrix is given by $R = R_x(\theta_r) R_y(\theta_p)\, R_z(\theta_y)$ where

$$R_x(\theta_r) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_r & -\sin\theta_r \\ 0 & \sin\theta_r & \cos\theta_r \end{bmatrix}$$

$$R_y(\theta_p) = \begin{bmatrix} \cos\theta_p & 0 & \sin\theta_p \\ 0 & 1 & 0 \\ -\sin\theta_p & 0 & \cos\theta_p \end{bmatrix}$$

$$R_z(\theta_y) = \begin{bmatrix} \cos\theta_y & -\sin\theta_y & 0 \\ \sin\theta_y & \cos\theta_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Write a function in python that returns this rotation R matrix with roll, pitch, yaw angles as an input.

7) Convert the following roll, pitch, and yaw angles $(\theta_r, \theta_p, \theta_y) = (\frac{\pi}{4}, \frac{\pi}{6}, -\frac{\pi}{2})$
   a. Determine equivalent rotation matrix and plot it
   b. Determine the equivalent unit quaternion
   c. Determine the equivalent vector-angle representation

8) Given a rotation of $\theta = \pi/5$ along axis $= (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)$
   a. Determine the equivalent rotation matrix and plot it
   b. Determine equivalent unit quaternion