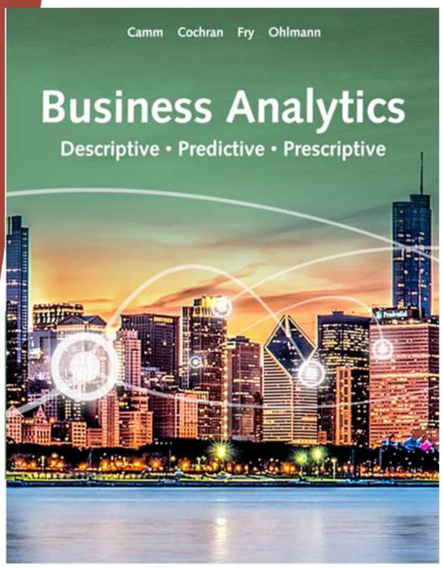

Business Analytics

Chapter 7 Linear Regression

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Introduction

- Managerial decisions based on:
  - Relationship between two or more variables:
- Example:
  - **Paying for ads** vs. **getting sales**
- Intuition can be useful but…
- If you have data
  - **USE regression analysis** to show how the variables are related.

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Simple Linear Regression Model

Regression Model

Estimated Regression Equation

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Simple Linear Regression Model

Best Fit Linear Regression

$$y = \beta_0 + \beta_1 x$$

Best Fit Regression Model:

- The equation that describes how y is related to x with an intercept.
  - Slope = β_1
  - Intercept = β_0
  - Y = dependent variable
  - X = independent variable

Slope Intercept Form

$$y = mx + b$$

Equation of a line:

- The equation that describes how y is related to x with an intercept
  - Slope = m
  - Intercept = b
  - Y = dependent variable
  - X = independent variable

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Simple Linear Regression Model

- The equation that describes the "True Relationship" between y and x and an error term.

SIMPLE LINEAR REGRESSION MODEL

$$y = \beta_0 + \beta_1 x + \varepsilon$$

Simple Linear Regression Model:

- Parameters: The characteristics of the population, β_0 and β_1.
  - Slope = β_1
  - Intercept = β_0
  - ε = error term
    - Variability in y that cannot be explained by the relationship between x and y
    - Assume ε is normally distributed with mean 0 and constant variance

Example:

How is travel time (y) of a delivery truck related to number miles traveled (x)

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Simple Linear Regression Model

Estimated Regression Equation:

- The "true" parameter values are usually not known
  - must be estimated using sample data.
- Sample Statistics - b_0 and b_1 are calculated as estimates of β_0 and β_1
  - We plug in b_0 and b_1 and drop the error term
    - Expected value of ε is = 0

ESTIMATED SIMPLE LINEAR REGRESSION EQUATION

$$\hat{y} = b_0 + b_1 x \quad \textbf{(7.2)}$$

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Simple Linear Regression Model

In the estimated simple linear regression equation:

$$\hat{y} = b_0 + b_1 x$$

$\hat{y}$ = Estimate for the mean value of y corresponding to a given value of x.

b_0 = Estimated y-intercept.

b_1 = Estimated slope.

- The graph of the estimated simple linear regression equation is called the estimated regression line.
- "In general, $\hat{y}$ is the point estimator of $E(y|\,x)$, the mean value of y for a given value of x

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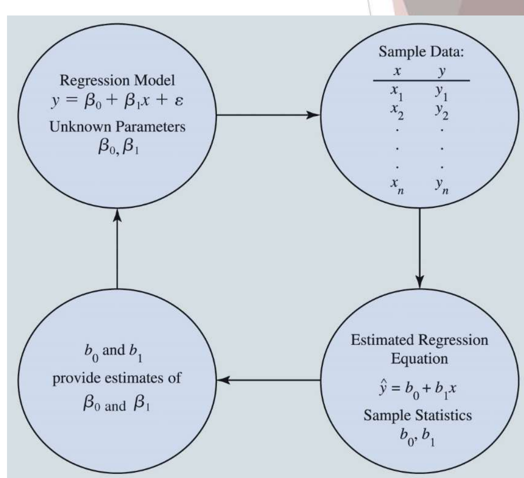
Simple Linear Regression Model

Example

- Butler Trucking
- How is travel time (y) of a delivery truck related to number miles traveled (x)
- We need data….

The Estimation Process in Simple Linear Regression



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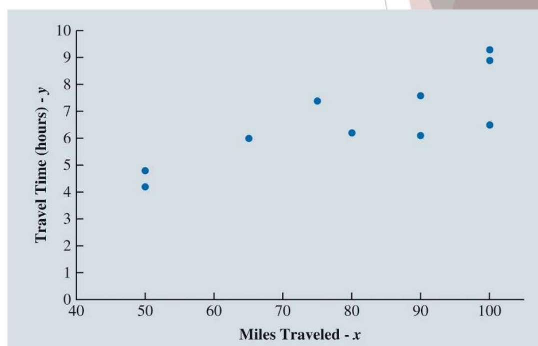
Least Squares Method

Miles Traveled and Travel Time for 10 Butler Trucking Company Driving Assignments

Driving Assignment *i*	*x* = Miles Traveled	*y* = Travel Time (hours)
1	100	9.3
2	50	4.8
3	50	8.9
4	100	6.5
5	50	4.2
6	80	6.2
7	75	7.4
8	65	6.0
9	90	7.6
10	90	6.1



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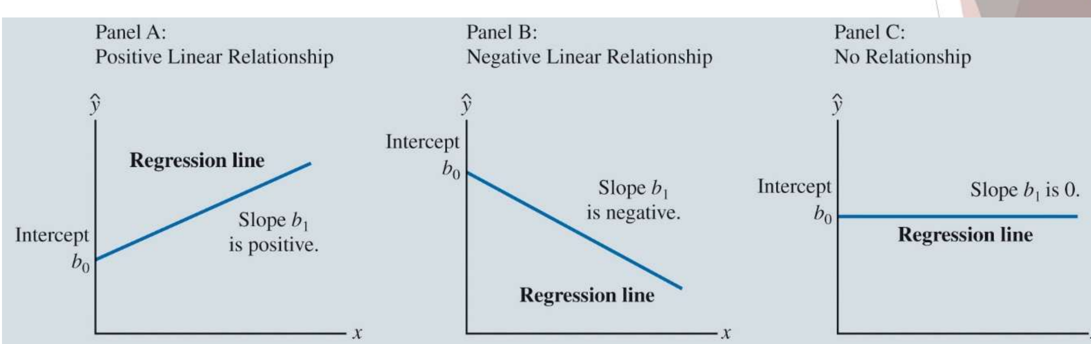
Simple Linear Regression Model

Figure 7.2: Possible Regression Lines in Simple Linear Regression



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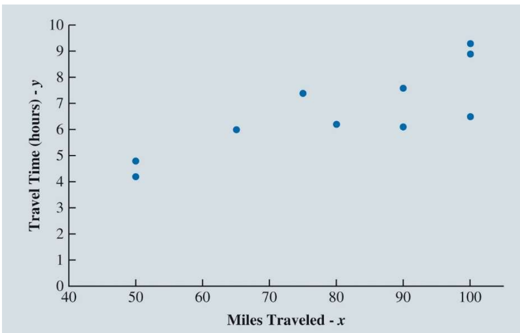
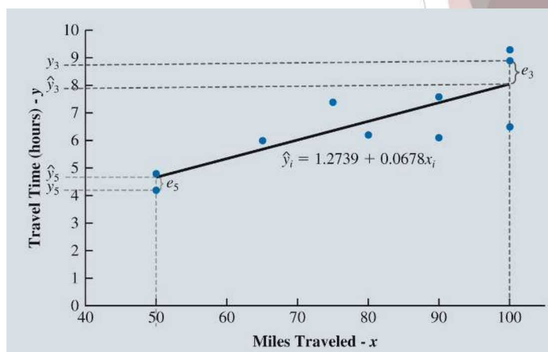
Least Squares Method

Scatter Chart of Miles Traveled and Travel Time for Butler Trucking Company Driving Assignments with Regression Line Superimposed





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Least Squares Method

Least Squares Estimates of the Regression Parameters:

- For the Butler Trucking Company data
  - Estimated slope of b_1 = 0.0678
  - Estimated y-intercept of b_0= 1.2739

The estimated simple linear regression model:

$$\hat{y} = 1.2739 + 0.0678x_1$$

What do these numbers mean?

- b_1 = 0.0678 – As the trip increases 1 more mile,
  - the average travel time increases 0.0678 hours (4 Minutes)
- b_0= 1.2739 – When the trip is 0 miles,
  - the estimated travel time is 1.2 hours (76 minutes)

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Examples

What is the expected travel time (y) of a delivery truck that travels 75 miles on deliveries?

= 1.2739 + 0.0678 (75) = 6.35 hours

What is the expected travel time (y) of a delivery truck that travels 100 miles on deliveries?

= 1.2739 + 0.0678 (100) = 8.05 hours

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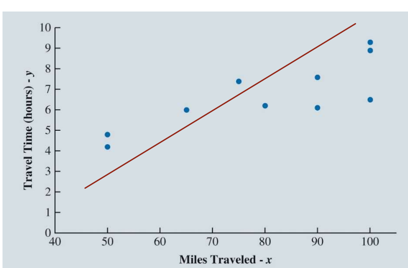
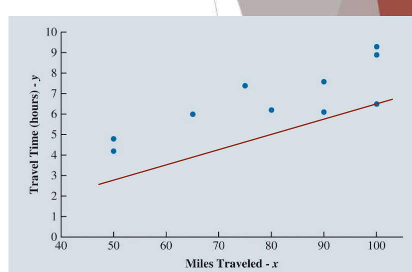
How do we know what line is best?

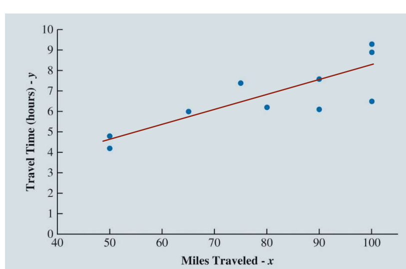
A.



B.



C.



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Least Squares Method

Least Squares Estimates of the Regression Parameters

Using Excel's Chart Tools to Compute the Estimated Regression Equation

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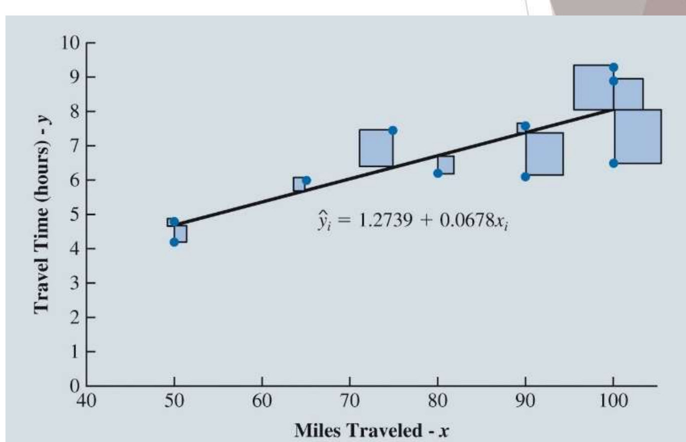
Least Squares Method

Least Squares Method

1. Measure the difference between each y value (y_i data point) and the estimated y value on the regression line ($\hat{y}_i$)
   Denoted as $e_i = y_i - \hat{y}_i$
   (Called Residual)
2. Square the differences
3. Add up the squared differences
4. The minimum sum of square differences is the best-fit line



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Least Squares Method

LEAST SQUARES EQUATION

$$\min \sum_{i=1}^{n} e_i^2 = \min \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \min \sum_{i=1}^{n} (y_1 - b_0 - b_1 x_1)^2 \quad \textbf{(7.4)}$$

where

y_i = observed value of the dependent variable for the i^{th} observation
$\hat{y}_i$ = predicted value of the dependent variable for the i^{th} observation
n = total number of observations

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Least Squares Method

Table 7.2: Predicted Travel Time and Residuals for 10 Butler Trucking Company Driving Assignments

Driving Assignment i	x = Miles Traveled	y = Travel Time (hours)	$\hat{y}_i = b_0 + b_1 x_i$	$e_i = y_i - \hat{y}_i$	e_i^2
1	100	9.3	8.0565	1.2435	1.5463
2	50	4.8	4.6652	0.1348	0.0182
3	100	8.9	8.0565	0.8435	0.7115
4	100	6.5	8.0565	−1.5565	2.4227
5	50	4.2	4.6652	−0.4652	0.2164
6	80	6.2	6.7000	−0.5000	0.2500
7	75	7.4	6.3609	1.0391	1.0797
8	65	6.0	5.6826	0.3174	0.1007
9	90	7.6	7.3783	0.2217	0.0492
10	90	6.1	7.3783	−1.2783	1.6341
	Totals	67.0	67.0000	0.0000	8.0288

What do you notice about the actual y values (data points) and the estimated y values ($\hat{y}_i$)

What do you notice about the sum of residuals or error terms?

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Least Squares Method – Differential Calculus

Slope Equation

$$b_1 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

y-Intercept Equation

$$b_0 = \overline{y} - b_1\overline{x}$$

$x_i =$ value of the independent variable for the *i*th observation.
$y_i =$ value of the dependent variable for the *i*th observation.
$\overline{x} =$ mean value for the independent variable.
$\overline{y} =$ mean value for the dependent variable.
$n =$ total number of observations.

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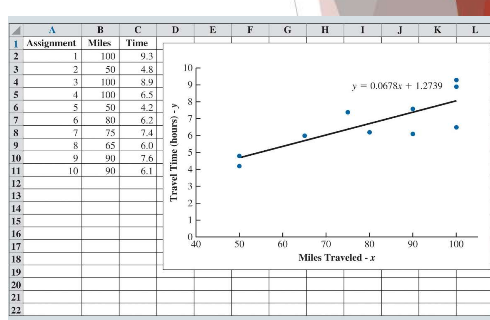
Least Squares Method

Using Excel's Chart Tools to Compute the Estimated Regression Equation:

- After constructing a scatter chart with Excel's chart tools:
    1. Right-click on any data point and select **Add Trendline.**
    2. When the **Format Trendline** task pane appears:
        - Select **Linear** in the **Trendline Options** area.
        - Select **Display Equation on chart** in the **Trendline Options** area.



	A	B	C
1	Assignment	Miles	Time
2	1	100	9.3
3	2	50	4.8
4	3	100	8.9
5	4	100	6.5
6	5	50	4.2
7	6	80	6.2
8	7	75	7.4
9	8	65	6.0
10	9	90	7.6
11	10	90	6.1

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