

# Business Analytics

Chapter 4 Probability



## Why is Probability Important?

# It Can Save Lives, too!

## Chile 2010

- ▶ San Jose copper and gold mine caved in, trapping 33 men 2000 ft underground
  - ▶ Contacted NASA to help
  - ▶ NASA Team: Engineer, 2 Doctors, and a psychologist

## No historical data

- ▶ Developed subjective probability estimates:
  - ▶ Success vs. Failure of rescue methods

## Results?

- ▶ Based on probabilities the team designed:
  - ▶ 13 ft, 924 steel rescue capsule that brought up miners 1 at a time
  - ▶ All miners were rescued - some after 68 days underground



# Introduction

## ▶ UNCERTAINTY

- ▶ We don't know everything
- ▶ We wish we did



## ▶ Probability:

- ▶ is the numerical measure of the likelihood that an event will occur.
- ▶ Often communicated through a probability distribution
- ▶ Helpful in providing additional information about an event.
- ▶ Can be used to help a decision maker evaluate possible actions and determine best course of action.

## Events and Probabilities

## Events and Probabilities

- ▶ A random experiment:
  - ▶ is a process that generates well-defined outcomes.
- ▶ The sample space for a random experiment:
  - ▶ All possible outcomes
- ▶ Examples:
  - ▶ A coin toss - Sample Space = Heads, Tails
  - ▶ Rolling a die - Sample Space = 1, 2, 3, 4, 5, 6
- ▶ An event is defined as a collection of outcomes.



## Events and Probabilities

Other Examples: Random Experiments and Experimental Outcomes

Random Experiment	Experimental Outcomes
Toss a coin	Head, tail
Roll a die	1, 2, 3, 4, 5, 6
Conduct a sales call	Purchase, no purchase
Hold a particular share of stock for one year	Price of stock goes up, price of stock goes down, no change in stock price
Reduce price of product	Demand goes up, demand goes down, no change in demand

## Events and Probabilities

► California Power & Light Company (CP&L).

- CP&L is starting a project designed to increase the generating capacity of one of its plants in southern California.
- Analysis of similar construction projects indicates that the possible completion times for the project are 8, 9, 10, 11, and 12 months.



► Events:

- Event that project is completed in 10 months or less
  - $C = \{8, 9, 10\}$
- Event that project is completed in less than 10 months
  - $C = \{8, 9\}$
- Event that project is completed in more than 10 months
- Event that project is completed between 9 and 11 months

## Events and Probabilities

- ▶ If there have been 40 already completed projects, then based on previous data,
- ▶ What are the probabilities of each outcome?

Completion Time (months)	No. of Past Projects Having This Completion Time	Probability of Outcome
8	6	$6/40 = 0.15$
9	10	$10/40 = 0.25$
10	12	$12/40 = 0.30$
11	6	$6/40 = 0.15$
12	6	$6/40 = 0.15$
Total	40	1.00

## Events and Probabilities

How to calculate probability of an event?

- ▶ The probability of an event:
  - ▶ is equal to the sum of probabilities of outcomes for the event.
- ▶ CP&L example: Let  $C$  be the event that the project is completed in 10 months or less,
- ▶ The probability of event  $C$ ,

$$P(C) = P(8) + P(9) + P(10) = 0.15 + 0.25 + 0.30 = 0.70$$

- ▶ We can tell CP&L management that there is a 0.70 probability (70% chance) that the project will be completed in 10 months or less.

## Some Basic Relationships of Probability

Complement of an Event

Addition Law

## Some Basic Relationships of Probability

### Complement of an Event:

- ▶ Given an event  $A$ , the **complement of  $A$** :
  - ▶ is defined to be the event consisting of all outcomes that are *not* in  $A$ .

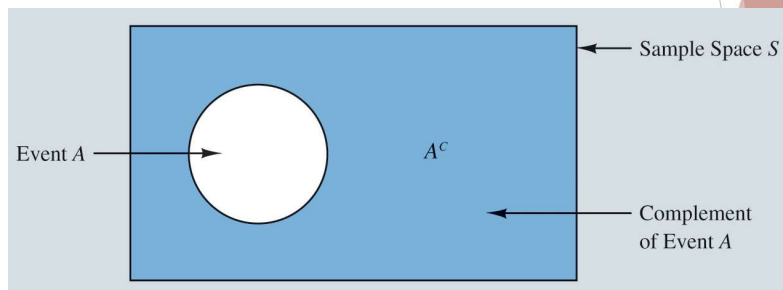
#### Example:

Sample space: Roll a Die

$S = \{1, 2, 3, 4, 5, 6\}$

Event:  $A = \{1, 2, 3, 4\}$

$A$  complement =  $A^c = \{5, 6\}$



## Some Basic Relationships of Probability

In any probability application, either event  $A$  or its complement  $A^c$  must occur!

► Example:

- Roll a Dice
  - $S = \{1, 2, 3, 4, 5, 6\}$
  - $A = \text{Roll an Even} = \{2, 4, 6\}$
  - $A^c = \text{Roll an Odd} = \{1, 3, 5\}$
- If you don't roll an even #, you must roll an odd #!

### COMPUTING PROBABILITY USING THE COMPLEMENT

$$P(A) = 1 - P(A^c) \quad (4.1)$$

The probability of an event  $A$  can be computed easily if the probability of its complement is known.

## Some Basic Relationships of Probability

### Union of 2 Events

- Given two events  $A$  and  $B$ , the **union of  $A$  and  $B$** 
  - is defined as the event containing all outcomes belonging to  $A$  or  $B$  or both.
- The union of  $A$  and  $B$  is denoted by  $A \cup B$ .

Example:

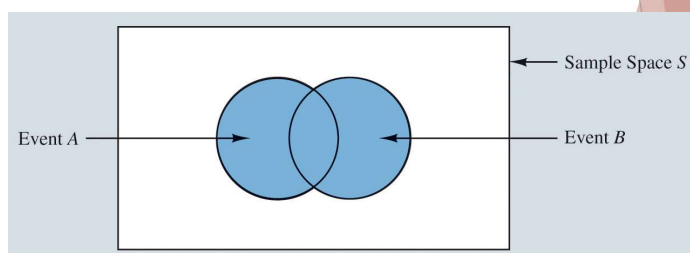
Sample space: Roll a Die

$S = \{1, 2, 3, 4, 5, 6\}$

Event:  $A = \{1, 2, 3, 4\}$

Event  $B = \{5\}$

$A \cup B = \{1, 2, 3, 4, 5\}$



## Some Basic Relationships of Probability

### Intersection of 2 Events

- ▶ Given two events  $A$  and  $B$ , the **intersection of  $A$  and  $B$** 
  - ▶ is the event containing the outcomes that belong to both  $A$  and  $B$ . The union of  $A$  and  $B$  is denoted by
- ▶ The intersection of  $A$  and  $B$  is denoted by  $A \cap B$ .

#### Example:

Sample space: Roll a Die

$S = \{1, 2, 3, 4, 5, 6\}$

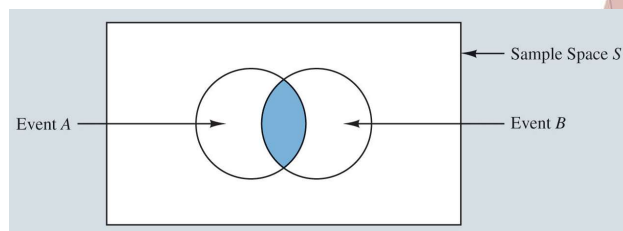
Event:  $A = \{1, 2, 3, 4\}$

Event  $B = \{2, 4, 6\}$

Event  $C = \{5\}$

$A \cap B = \{2, 4\}$

$A \cap C = \{\emptyset\}$



## Some Basic Relationships of Probability

What is the probability that at least one of two events will occur?

What is the probability that Event A OR Event B will happen?

### ADDITION LAW

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (4.2)$$

- ▶ A special case arises for **mutually exclusive events**:
  - ▶ If the occurrence of one event precludes the occurrence of the other.
  - ▶ If the events have no outcomes in common.



## Some Basic Relationships of Probability

### ADDITION LAW

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (4.2)$$

► Example:

- Sample space: Roll a Die
- $S = \{1, 2, 3, 4, 5, 6\}$
- Event:  $A = \{1, 2, 3, 4\}$
- Event  $B = \{2, 4, 6\}$
- Event  $C = \{5\}$

$$P(A) = 4/6 = .667$$

$$P(B) = 3/6 = .5$$

$$P(A \cap B) = 2/6 = .333$$

$$P(A \cup B) = .667 + .5 - .333 = .834 = 5/6$$

$= P(1, 2, 3, 4, 6)$

- What is the  $P(A \cup B)$ ?

## Some Basic Relationships of Probability

- A special case arises for mutually exclusive events:

- If the events have no outcomes in common.

► Example:

- Sample space: Roll a Die
- $S = \{1, 2, 3, 4, 5, 6\}$
- Event:  $A = \{1, 2, 3, 4\}$
- Event  $B = \{5, 6\}$

$$P(A) = 4/6 = .667$$

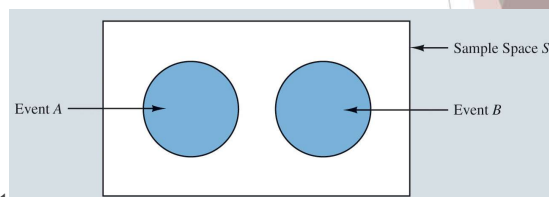
$$P(B) = 2/6 = .333$$

$$P(A \cap B) = 0/6 = 0$$

$$P(A \cup B) = .667 + .333 - 0 = 1$$

- What is the  $P(A \cup B)$ ?

### Mutually Exclusive Events



### ADDITION LAW FOR MUTUALLY EXCLUSIVE EVENTS

$$P(A \cup B) = P(A) + P(B)$$

# Conditional Probability

Independent Events  
Multiplication Law  
Bayes' Theorem

## Conditional Probability

- ▶ **Conditional probability:**
  - ▶ When the probability of one event is dependent on whether some related event has already occurred.
- ▶ **Illustration: Lancaster Savings and Loan:**

Does the probability of a customer defaulting on a mortgage differ by marital status?



## Conditional Probability

- ▶ Does the probability of a customer defaulting on a mortgage differ by marital status?
- ▶  $S$  = event that a customer is single
- ▶  $M$  = event that a customer is married
- ▶  $D$  = event that a customer defaulted on their mortgage
- ▶  $D^c$  = event that a customer did not default on their mortgage

**Table 4.3** Subset of Data from 300 Home Mortgages of Customers at Lancaster Savings and Loan

Customer No.	Age	Marital Status	Annual Income	Mortgage Amount	Payments per Year	Total Amount Paid	Default on Mortgage?
1	37	Single	\$ 172,125.70	\$ 473,402.96	24	\$ 581,885.13	Yes
2	31	Single	\$ 108,571.04	\$ 300,468.60	12	\$ 489,320.38	No
3	37	Married	\$ 124,136.41	\$ 330,664.24	24	\$ 493,541.93	Yes
4	24	Married	\$ 79,614.04	\$ 230,222.94	24	\$ 449,682.09	Yes
5	27	Single	\$ 68,087.33	\$ 282,203.53	12	\$ 520,581.82	No
6	30	Married	\$ 59,959.80	\$ 251,242.70	24	\$ 356,711.58	Yes
7	41	Single	\$ 99,394.05	\$ 282,737.29	12	\$ 524,053.46	No
8	29	Single	\$ 38,527.35	\$ 238,125.19	12	\$ 468,595.99	No
9	31	Married	\$ 112,078.62	\$ 297,133.24	24	\$ 399,617.40	Yes
10	36	Single	\$ 224,899.71	\$ 622,578.74	12	\$ 1,233,002.14	No
11	31	Married	\$ 27,945.36	\$ 215,440.31	24	\$ 285,900.10	Yes
12	40	Single	\$ 48,929.74	\$ 252,885.10	12	\$ 336,574.63	No
13	39	Married	\$ 82,810.92	\$ 183,045.16	12	\$ 262,537.23	No
14	31	Single	\$ 68,216.88	\$ 165,309.34	12	\$ 253,633.17	No

## Conditional Probability

Figure 4.5: PivotTable for Marital Status and Whether Customer Defaults on Mortgage

	A	B	C	D	E
1					
2					
3	Count of Customer Number	Column Labels			
4	Row Labels	NO	YES	Grand Total	
5	MARRIED	64	79	143	
6	SINGLE	116	41	157	
7	Grand Total	180	120	300	
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					
21					
22					
23					
24					
25					
26					
27					
28					
29					

**PivotTable Fields**

Choose fields to add to report:

Search

☒ Customer Number  
☐ Age  
☐ Marital Status  
☐ Annual Income  
☐ Mortgage Amount  
☐ Payments Per Year  
☐ Total Amount Paid  
☒ Default on Mortgage?

MORE TABLES...

Drag fields between areas below:

**Filters:**  
 (Empty)

**Columns:**  
 Default on Mortgage?

**Rows:**  
 Marital Status

**Values:**  
 Count of Customer Number

## Conditional Probability

Crosstabulation of Marital Status and if Customer Defaults on Mortgage

Marital Status	No Default	Default	Total
Married	64	79	143
Single	116	41	157
Total	180	120	300

The probability that a customer defaults on his or her mortgage is  
 $120/300 = 0.4$ .

The probability that a customer does not default on his or her mortgage is  
 $1 - 0.4 = 0.6$  or  $180/300$ .

## Conditional Probability

- ▶ **Joint probabilities:**
  - ▶ When values give the probability of the intersection of two events
- ▶ **Marginal probabilities:**
  - ▶ Sum of the joint probabilities in the corresponding row or column of the joint probability table.
- ▶ Conditional probabilities can be computed as the ratio of joint probability to a marginal probability.

### CONDITIONAL PROBABILITY

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (4.3)$$

or

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad (4.4)$$

## Conditional Probability

Table 4.5: Joint Probability Table for Customer Mortgage Prepayments

Joint Probabilities		No Default ( $D^c$ )	Default ( $D$ )	Total
Married ( $M$ )		0.2133	0.2633	0.4766
Single ( $S$ )		0.3867	0.1367	0.5234
Total		0.6000	0.4000	1.0000

Marginal Probabilities

## Conditional Probability

Figure 4.6: Using Excel PivotTable to Calculate Conditional Probabilities

	NO	YES	Grand Total
MARRIED	44.76%	55.24%	100.00%
SINGLE	73.89%	26.11%	100.00%
Grand Total	60.00%	40.00%	100.00%

**PivotTable Fields**

Choose fields to add to report:

- ☒ Customer Number
- ☐ Age
- ☒ Marital Status
- ☐ Annual Income
- ☐ Mortgage Amount
- ☐ Payments Per Year
- ☐ Total Amount Paid
- ☒ Default on Mortgage?

MORE TABLES...

Drag fields between areas below:

**FILTERS**

**COLUMNS**  
Default on Mortgage?

**ROWS**  
Marital Status

**VALUES**  
Count of Customer Number