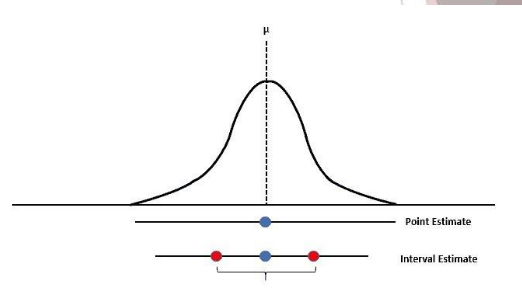
Interval Estimation Continued

- Point estimators are not perfect!
- An **interval estimate** is used to hopefully capture the true value

Point estimate ± Margin of error

$\bar{x}$ ± Margin of error

$\bar{p}$ ± Margin of error



Interval Estimation

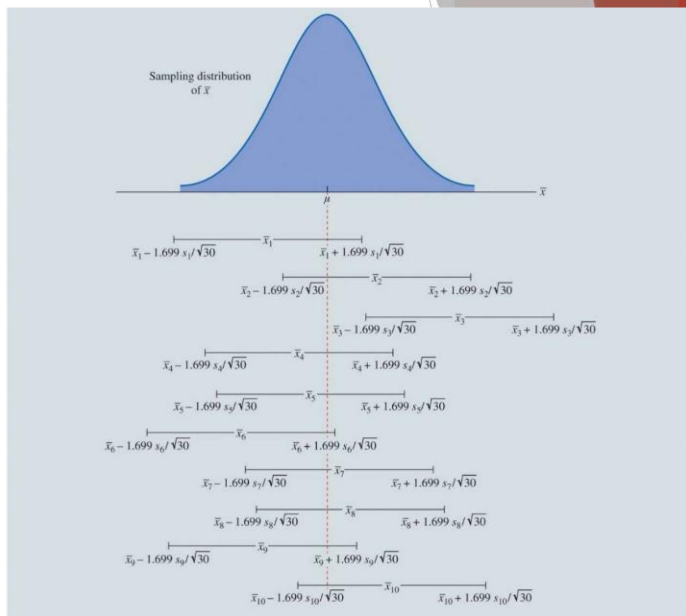
Interval Estimation of the Population Mean (cont.):

For any normally distributed random variable:

- 90% of the values lie within 1.645 standard deviations of the mean.
- 95% of the values lie within 1.960 standard deviations of the mean.
- 99% of the values lie within 2.576 standard deviations of the mean.

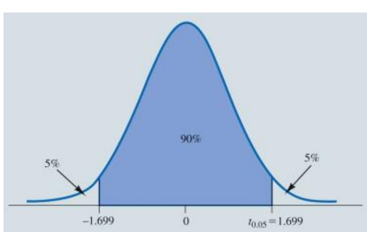
EAI Managers

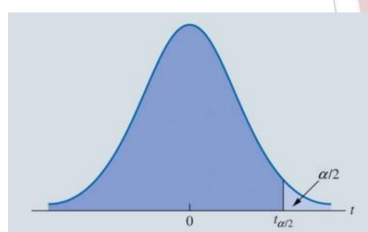
- Recall:
  - N = 30 managers
  - Sample mean salary (x) = $71,814
  - Sample SD (s) = $3,340
- $x \pm 1.699(3340/\sqrt{30})$
  - = $70,778 to $72,850
- The True Population Mean = $71,800



Interval Estimation

t Distribution with $\frac{\alpha}{2}$ Area or Probability in the Upper Tail





INTERVAL ESTIMATE OF A POPULATION MEAN

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}, \qquad \textbf{(6.7)}$$

where s is the sample standard deviation, α is the level of significance, and $t_{\alpha/2}$ is the t value providing an area of $\alpha/2$ in the upper tail of the t distribution with $n - 1$ degrees of freedom.

Another Example

Table 6.5: Credit Card Balances for a Sample of 70 Households

9,430	14,661	7,159	9,071	9,691	11,032
7,535	12,195	8,137	3,603	11,448	6,525
4,078	10,544	9,467	16,804	8,279	5,239
5,604	13,659	12,595	13,479	5,649	6,195
5,179	7,061	7,917	14,044	11,298	12,584
4,416	6,245	11,346	6,817	4,353	15,415
10,676	13,021	12,806	6,845	3,467	15,917
1,627	9,719	4,972	10,493	6,191	12,591
10,112	2,200	11,356	615	12,851	9,743
6,567	10,746	7,117	13,627	5,337	10,324
13,627	12,744	9,465	12,557	8,372	
18,719	5,742	19,263	6,232	7,445	

Interval Estimation

Figure 6.13: 95% Confidence Interval for Credit Card Balances

	A	B	C	D
1	NewBalance		*NewBalance*	
2	9430			
3	7535		Mean	9312
4	4078		Standard Error	478.9281
5	5604		Median	9466
6	5179		Mode	13627
7	4416		Standard Deviation	4007
8	10676		Sample Variance	16056048
9	1627		Kurtosis	−0.2960
10	10112		Skewness	0.1879
11	6567		Range	18648
12	13627		Minimum	615
13	18719		Maximum	19263
14	14661		Sum	651840
15	12195		Count	70
16	10544		Confidence Level(95.0%)	955
17	13659			
18	7061		**Point Estimate**	=D3
19	6245		**Lower Limit**	=D18−D16
20	13021		**Upper Limit**	=D3+D16
70	9743			
71	10324			
72				

	A	B	C	D	E	F
1	NewBalance		*NewBalance*			
2	9430					
3	7535		Mean	9312	Point Estimate	
4	4078		Standard Error	478.9281		
5	5604		Median	9466		
6	5179		Mode	13627		
7	4416		Standard Deviation	4007		
8	10676		Sample Variance	16056048		
9	1627		Kurtosis	−0.2960		
10	10112		Skewness	0.1879		
11	6567		Range	18648		
12	13627		Minimum	615		
13	18719		Maximum	19263		
14	14661		Sum	651840		
15	12195		Count	70		
16	10544		Confidence Level(95.0%)	955	Margin of Error	
17	13659					
18	7061		**Point Estimate**	9312		
19	6245		**Lower Limit**	8357		
20	13021		**Upper Limit**	10267		
70	9743					
71	10324					
72						

Interval Estimation

Interval Estimation of the Population Proportion:

$\overline{p}$ ± Margin of error

The sampling distribution of $\overline{p}$ plays a key role in computing the margin of error in the interval estimate.

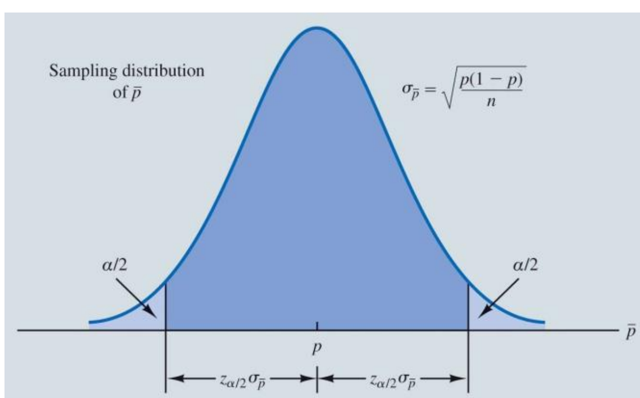
INTERVAL ESTIMATE OF A POPULATION PROPORTION

$$\overline{p} \pm z_{\alpha/2}\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}, \qquad \textbf{(6.10)}$$

where α is the level of significance and $z_{\alpha/2}$ is the z value providing an area of $\alpha/2$ in the upper tail of the standard normal distribution.

Interval Estimation

Figure 6.14: Normal Approximation of the Sampling Distribution of $\overline{p}$



Interval Estimation

Figure 6.15: 95% Confidence Interval for Survey of 900 Women Golfers

Are you satisfied with your tee times?

	A	B	C	D
1	Response		Interval Estimate of a Population Proportion	
2	Yes			
3	No		Sample Size	=COUNTA(A2:A901)
4	Yes		Response of Interest	Yes
5	Yes		Count for Response	=COUNTIF(A2:A901,D4)
6	No		Sample Proportion	=D5/D3
7	No			
8	No		Confidence Coefficient	0.95
9	Yes		Level of Significance (alpha)	=1−D8
10	Yes		*z* Value	=NORM.S.INV(1−D9/2)
11	Yes			
12	No		Standard Error	=SQRT(D6*(1−D6)/D3)
13	No		Margin of Error	=D10*D12
14	Yes			
15	No		Point Estimate	=D6
16	No		Lower Limit	=D15−D13
17	Yes		Upper Limit	=D15+D13
18	No			
900	Yes			
901	Yes			
902				

	A	B	C	D	E	F	G
1	Response		Interval Estimate of a Population Proportion				
2	Yes						
3	No		Sample Size	900	Enter Yes as the Response of Interest		
4	Yes		Response of Interest	Yes			
5	Yes		Count for Response	396			
6	No		Sample Proportion	0.44			
7	No						
8	No		Confidence Coefficient	0.95			
9	Yes		Level of Significance	0.05			
10	Yes		*z* Value	1.96			
11	Yes						
12	No		Standard Error	0.0165			
13	No		Margin of Error	0.0324			
14	Yes						
15	No		Point Estimate	0.44			
16	No		Lower Limit	0.4076			
17	Yes		Upper Limit	0.4724			
18	No						
900	Yes						
901	Yes						
902							

Hypothesis Tests

Developing Null and Alternative Hypothesis

Type I and Type II Errors

Hypothesis Test of the Population Mean

Hypothesis Test of the Population Proportion

Hypothesis Tests

- Statistically deciding if a statement about a parameter should be accepted or rejected
  - The average mpg of a vehicle is <= 24
  - The average Gatorade in a bottle is at least (>=) 67.6 ounces
- **Null hypothesis**
  - The tentative conjecture
- **Alternative hypothesis**
  - The opposite of what is stated in the null hypothesis
- Using data from a sample, we can test the validity of the two competing statements about a population.

Hypothesis Tests

Developing Null and Alternative Hypotheses:

- Context is KEY!
  - The context determines how the hypotheses should be stated
- **Ask:**
  - What is the purpose of collecting the sample?
  - What conclusions are we hoping to make?

Hypothesis Tests

- **The Alternative as a Research Hypothesis**
  - Current car gets 24 mpg
  - New fuel system
    - Better than 24 mpg
  - Several cars are built with new fuel system and tested

$$H_0: \mu \leq 24$$
$$H_a: \mu > 24$$

- Make the alternative the conclusion the research hopes to support

Hypothesis Tests

- **The Null Hypothesis as a Conjecture to be Challenged**
  - Bottle label states: 67.6 fl ounces
  - Assume correct if average fill is at least 67.6 fl ounces
  - Gather sample and test

$$H_0: \mu \geq 67.6$$
$$H_a: \mu < 67.6$$

Hypothesis Tests

- **The Null Hypothesis as a Conjecture to be Challenged**
  - **(From Company Perspective)**
  - Bottle label states: 67.6 fl ounces
  - Don't want to underfill or overfill bottles
  - Gather sample and test

$$H_0: \mu = 67.6$$
$$H_a: \mu \neq 67.6$$

Hypothesis Tests

- Depending upon the situation, hypothesis tests about a population parameter may take one of three forms:

$$H_0: \mu \geq \mu_0 \qquad H_0: \mu \leq \mu_0 \qquad H_0: \mu = \mu_0$$
$$H_a: \mu < \mu_0 \qquad H_a: \mu > \mu_0 \qquad H_a: \mu \neq \mu_0$$

  - First two forms are called one-tailed tests.
  - Third form is called a two-tailed test.

Hypothesis Tests

Type I and Type II Errors:

	H_0 True	H_a True
Do Not Reject H_0	Correct Conclusion	Type II Error
Reject H_0	Type I Error	Correct Conclusion

- Example: $H_0: \mu \le 24$ $H_a: \mu > 24$
  - Type 1: Researchers say the MPG on the new system is better than 24, when its really not. Man is innocent, but he gets sentenced to life in prison
  - Type 2: Researchers say the MPG on new system is no better than the old, when it really is. Man is guilty, but he gets set free.

Hypothesis Tests

- **Level of Significance:**
  - Probability of making a Type 1 Error
    - The **level of significance (Alpha) or if Confidence level - 95%**
    - **Alpha = 5%**
  - Usually, hypothesis tests control for Type I errors
    - Potentially Worse conclusion
  - Type II errors can be controlled for
    - **Usually just say "Fail to Reject Ho"**

Hypothesis Tests

Hypothesis Test of the Population Mean:

- **One tailed tests** about a population mean take one of the following forms:

Lower-Tail Test	Upper-Tail Test
$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$
$H_a: \mu < \mu_0$	$H_a: \mu > \mu_0$

1. Develop the null and alternative hypothesis for the test.
2. Specify the level of significance for the test.
3. Collect the sample data and compute the value of what is called a test statistic.

Example

- Hilltop Coffee
  - States each can of coffee contains 3 lbs of coffee
- Federal Trade Commission (FTC) wants to check
  - Alpha = 0.01 (1%)

$$H_0: \mu \geq 3$$
$$H_a: \mu < 3$$

- Test Statistic for Hypothesis Test About a Population Mean
  - Does x deviate from the hypothesized μ enough to justify rejecting the Null Hypothesis?

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

$$t = \frac{\overline{x} - \mu_0}{s_{\overline{x}}} = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{\overline{x} - 3}{0.028}$$

Example

- We find out our sample mean is 2.92

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{2.92 - 3}{0.17/\sqrt{36}} = -2.824$$

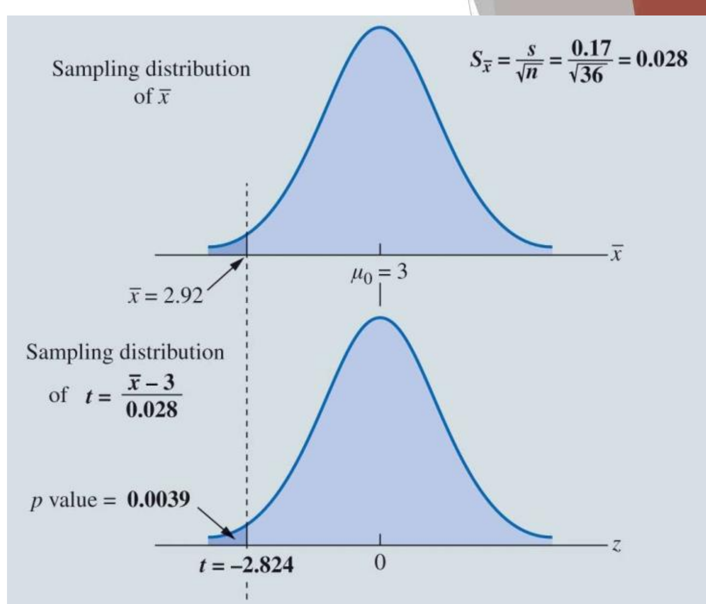
- Test Statistic = -2.824
- Is this small enough to lead us to reject the Null?
  - Is there support that the cans of coffee do not have 3lbs of coffee?

How small must the test statistic *t* be before we choose to reject the null hypothesis?

- ***P* Value:**
  - Probability, assuming the Null is true, of obtaining a random sample of size n that results in a test statistic at least as extreme as the one observed in the current sample

Hypothesis Tests

Figure 6.19: p Value for the Hilltop Coffee Study When $x = 2.92$ and $s = 0.17$



Alpha = 0.01

P value = 0.0039

- The probability of obtaining a value of 2.92 lbs or less when the null hypothesis is true is 0.0039 (.39%)
- Since this is less than 0.01, we reject the Null.
- There is statistical evidence that the cans of coffee do not have 3lbs in them

Hypothesis Tests

Figure 6.18: Hypothesis Test about a Population Mean

CoffeeTest.xlsx

$$= \text{T.DIST(test statistic, degrees of freedom, cumulative)}.$$

	A	B	C	D
1	Weight		Hypothesis Test about a Population Mean	
2	3.15			
3	2.76			
4	3.18		Sample Size	=COUNT(A2:A37)
5	2.77		Sample Mean	=AVERAGE(A2:A37)
6	2.86		Sample Standard Deviation	=STDEV.S(A2:A37)
7	2.66			
8	2.86		Hypothesized Value	3
9	2.54			
10	3.02		Standard Error	=D6/SQRT(D4)
11	3.13		Test Statistic *t*	=(D5-D8)/D10
12	2.94		Degrees of Freedom	=D4-1
13	2.74			
14	2.84		*p* value (Lower Tail)	=T.DIST(D11,D12,TRUE)
15	2.6		*p* value (Upper Tail)	=1-D14
16	2.94		*p* value (Two Tail)	=2*MIN(D14,D15)
17	2.93			
18	3.18			
19	2.95			
20	2.86			
21	2.91			
22	2.96			
23	3.14			
24	2.65			
25	2.77			
26	2.96			
27	3.1			
28	2.82			
29	3.05			
30	2.94			
31	2.82			
32	3.21			
33	3.11			
34	2.9			
35	3.05			
36	2.93			
37	2.89			

	A	B	C	D
1	Weight		Hypothesis Test about a Population Mean	
2	3.15			
3	2.76			
4	3.18		Sample Size	36
5	2.77		Sample Mean	2.92
6	2.86		Sample Standard Deviation	0.170
7	2.66			
8	2.86		Hypothesized Value	3
9	2.54			
10	3.02		Standard Error	0.028
11	3.13		Test Statistic *t*	−2.824
12	2.94		Degrees of Freedom	35
13	2.74			
14	2.84		*p* value (Lower Tail)	0.0039
15	2.60		*p* value (Upper Tail)	0.9961
16	2.94		*p* value (Two Tail)	0.0078

Hypothesis Tests

- The level of significance indicates the strength of evidence that is needed in the sample data before rejection of the null hypothesis.

REJECTION RULE

$$\text{Reject } H_0 \text{ if } p\,\text{value} \leq \alpha$$

- Different decision makers may express different opinions concerning the cost of making a Type I error and may choose a different level of significance.
- Providing the *p* value as part of the hypothesis testing results allows decision makers to compare the reported *p* value to his or her own level of significance.
  - Typically less than 0.1 (10%) is widely accepted.

Hypothesis Tests

- Upper-tail test:
  - Using the *t* distribution
  - Compute the probability that *t* is greater than or equal to the value of the test statistic (area in the upper tail).

Upper-Tail Test

$$H_0: \mu \le \mu_0$$
$$H_a: \mu > \mu_0$$

In hypothesis testing, the general form for a **two-tailed test** about population mean is:

$$H_0: \mu = \mu_0$$
$$H_a: \mu \neq \mu_0$$

Example

- Holiday Toys
  - Expected demand for new toy
    - 40 units per retail outlet
  - Survey 25 retailers - anticipated order quantity

$$H_0: \mu = 40$$
$$H_a: \mu \neq 40$$

  - If Null rejected – reevaluate production plan

- From the sample
  - x = 37.4 and SD = 11.79 units
- Test Statistic

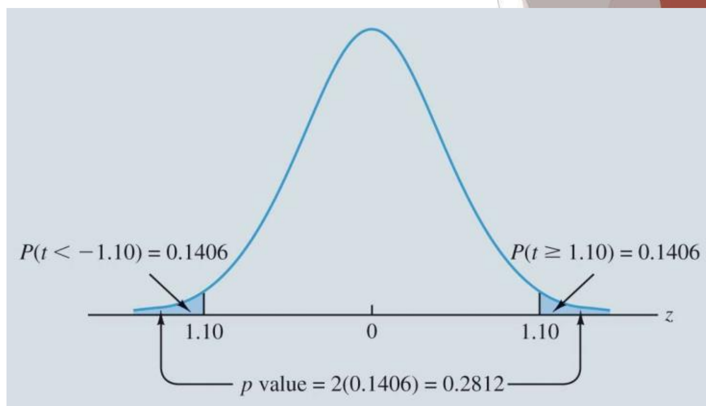
$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{37.4 - 40}{11.79/\sqrt{25}} = -1.10$$

- Two-Tailed Test
  - Must find the probability of obtaining a value for test statistic that is at least as likely as -1.10

$$P(t \le -1.10) + P(t \ge 1.10).$$

Hypothesis Tests

Figure 6.20: *p* Value for the Holiday Toys Two-Tailed Hypothesis Test

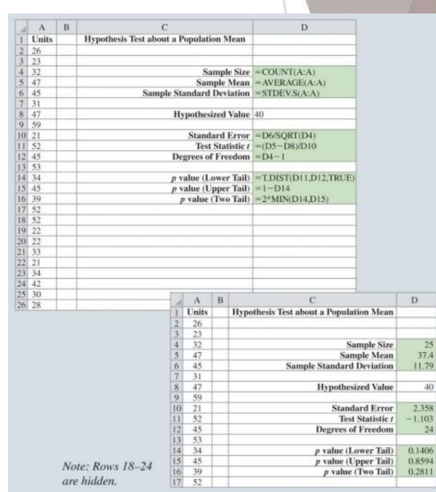


- Computation of *p* Values for Two-Tailed Tests:
  1. Compute the value of the test statistic.
  2. Compute the p value for one of the tail areas.
  3. Double the probability (or tail area) from step 2 to obtain the final *p* value.
- Conclusion:
  - 0.2812 > 0.05
  - Fail to Reject the Null –
  - Holiday Toys can make 40 toys for each retail location

Hypothesis Tests

Figure 6.21: Two-Tailed Hypothesis Test for Holiday Toys

OrdersTest.xlsx



	A	B	C	D
1	Units		Hypothesis Test about a Population Mean	
2	26			
3	23			
4	32		Sample Size	=COUNT(A:A)
5	47		Sample Mean	=AVERAGE(A:A)
6	45		Sample Standard Deviation	=STDEV.S(A:A)
7	31			
8	47		Hypothesized Value	40
9	59			
10	21		Standard Error	=D6/SQRT(D4)
11	52		Test Statistic *t*	=(D5−D8)/D10
12	45		Degrees of Freedom	=D4−1
13	53			
14	34		*p* value (Lower Tail)	=T.DIST(D11,D12,TRUE)
15	45		*p* value (Upper Tail)	=1−D14
16	39		*p* value (Two Tail)	=2*MIN(D14,D15)
17	52			
18	52			
19	22			
20	22			
21	33			
22	21			
23	34			
24	42			
25	30			
26	28			

	A	B	C	D
1	Units		Hypothesis Test about a Population Mean	
2	26			
3	23			
4	32		Sample Size	25
5	47		Sample Mean	37.4
6	45		Sample Standard Deviation	11.79
7	31			
8	47		Hypothesized Value	40
9	59			
10	21		Standard Error	2.358
11	52		Test Statistic *t*	−1.103
12	45		Degrees of Freedom	24
13	53			
14	34		*p* value (Lower Tail)	0.1406
15	45		*p* value (Upper Tail)	0.8594
16	39		*p* value (Two Tail)	0.2811
17	52			

Note: Rows 18–24 are hidden.

Hypothesis Tests

Table 6.7: Summary of Hypothesis Tests About a Population Mean

	Lower-Tail Test	Upper-Tail Test	Two-Tailed Test
Hypotheses	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test Statistic	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
***p* Value**	=T.DIST(t,n − 1, TRUE)	=1 − T.DIST(t,n − 1, TRUE)	=2*MIN(T.DIST(t,n − 1, TRUE),1 − T.DIST(t,n − 1, TRUE))

Hypothesis Tests

Steps of Hypothesis Testing:

Step 1. Develop the null and alternative hypotheses.

Step 2. Specify the level of significance.

Step 3. Collect the sample data and compute the value of the test statistic.

Step 4. Use the value of the test statistic to compute the *p* value.

Step 5. Reject

H_0 if the p □□.

Step 6. Interpret the statistical conclusion in the context of the application.

Hypothesis Tests

A CONFIDENCE INTERVAL APPROACH TO TESTING A HYPOTHESIS OF THE FORM

$$H_0\colon \mu = \mu_0$$
$$H_a\colon \mu \neq \mu_0$$

1. Select a simple random sample from the population and use the value of the sample mean $\bar{x}$ to develop the confidence interval for the population mean μ.

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$$

2. If the confidence interval contains the hypothesized value μ_0, do not reject H_0. Otherwise, reject[3] H_0.

Hypothesis Tests

Hypothesis Test of the Population Proportion:

- The three forms for a hypothesis test about a population proportion are:

$H_0\colon p \geq p_0$	$H_0\colon p \leq p_0$	$H_0\colon p = p_0$
$H_a\colon p < p_0$	$H_a\colon p > p_0$	$H_a\colon p \neq p_0$

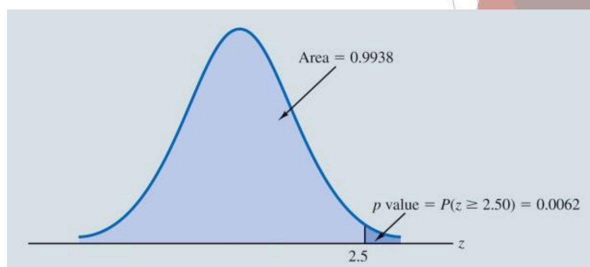
- The first form is called a lower-tail test.
- The second form is called an upper-tail test.
- The third form is called a two-tailed test.

Hypothesis Tests

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION PROPORTION

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{6.13}$$

Figure 6.22: Calculation of the *p* Value for the Pine Creek Hypothesis Test



Hypothesis Tests

Figure 6.23: Hypothesis Test for Pine Creek Golf Course

Womengolf.xlsx

	A	B	C	D
1	**Golfer**		**Hypothesis Test about a Population Proportion**	
2	Female			
3	Male		**Sample Size**	=COUNTA(A2:A401)
4	Female		**Response of Interest**	Female
5	Male		**Count for Response**	=COUNTIF(A2:A401,D4)
6	Male		**Sample Proportion**	=D5/D3
7	Female			
8	Male		**Hypothesized Value**	0.2
9	Male			
10	Female		**Standard Error**	=SQRT(D8*(1−D8)/D3)
11	Male		**Test Statistic *z***	=(D6−D8)/D10
12	Male			
13	Male		***p* value (Lower Tail)**	=NORM.S.DIST(D11,TRUE)
14	Male		***p* value (Upper Tail)**	=1−D13
15	Male		***p* value (Two Tail)**	=2*MIN(D13,D14)
16	Female			
400	Male			
401	Male			
402				

	A	B	C	D	E	F
1	**Golfer**		**Hypothesis Test about a Population Proportion**			
2	Female					
3	Male		**Sample Size**	400		
4	Female		**Response of Interest**	Female		
5	Male		**Count for Response**	100		
6	Male		**Sample Proportion**	0.25		
7	Female					
8	Male		**Hypothesized Value**	0.20		
9	Male					
10	Female		**Standard Error**	0.02		
11	Male		**Test Statistic *z***	2.5000		
12	Male					
13	Male		***p* value (Lower Tail)**	0.9938		
14	Male		***p* value (Upper Tail)**	0.0062		
15	Male		***p* value (Two Tail)**	0.0124		
16	Female					
400	Male					
401	Male					
402						

Hypothesis Tests

Table 6.8: Summary of Hypothesis Tests About a Population Proportion

	Lower-Tail Test	Upper-Tail Test	Two-Tailed Test
Hypotheses	$H_0: p \geq p_0$ $H_a: p < p_0$	$H_0: p \leq p_0$ $H_a: p > p_0$	$H_0: p = p_0$ $H_a: p \neq p_0$
Test Statistic	$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
***p* Value**	=NORM.S.DIST (z, TRUE)	=1 − NORM.S.DIST (z, TRUE)	2*MIN(NORM.S.DIST(z, TRUE), 1 − NORM.S.DIST(z, TRUE))