Sampling Distributions

Sampling Distribution of $\overline{x}$

Sampling Distribution of $\overline{p}$

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Sampling Distributions

- Chose a random sample of 30 managers and calculated
  - Sample Mean = $\bar{x}$
  - Sample Proportion = $\bar{p}$

J	K	L
Sample Mean	Sample SD	Proportion of Competing Training (Sample)
72372.71	3839.873894	0.666666667

- $\bar{x}$ is the point estimator of the population mean, μ
  - Point Estimate: $\bar{x}$ = $72,372.71
- $\bar{p}$ is the point estimator of the population proportion, p
  - Point Estimate: $\bar{p}$ = 0.67

What would happen if we chose a different sample of 30 managers?*

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Sampling Distributions

If we picked 30 random managers 500 times, the results might look something like this:

Sample Number	Sample Mean ($\bar{x}$)	Sample Proportion ($\bar{p}$)
1	71,814	0.63
2	72,670	0.70
3	71,780	0.67
4	71,588	0.53
.	.	.
.	.	.
.	.	.
500	71,752	0.50

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Sampling Distributions

Make a frequency and relative frequency distribution of the $\bar{x}$ results:

Mean Annual Salary ($)	Frequency	Relative Frequency
69,500.00-69,999.99	2	0.004
70,000.00-70,499.99	16	0.032
70,500.00-70,999.99	52	0.104
71,000.00-71,499.99	101	0.202
71,500.00-71,999.99	133	0.266
72,000.00-72,499.99	110	0.220
72,500.00-72,999.99	54	0.108
73,000.00-73,499.99	26	0.052
73,500.00-73,999.99	6	0.012
Totals:	**500**	**1.000**

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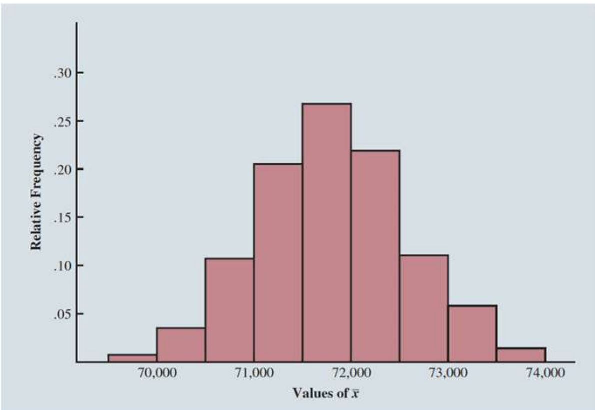
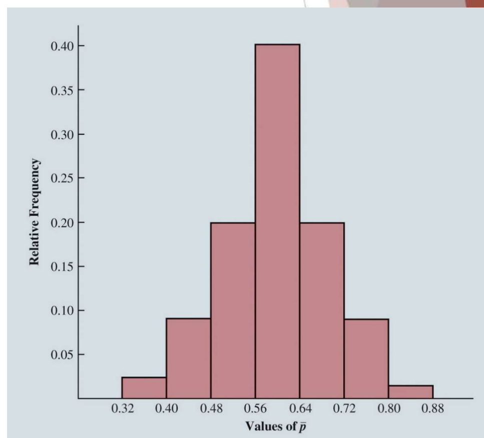
Sampling Distributions

Use the frequency distribution to make a histogram:

$\bar{x}$ $\bar{p}$





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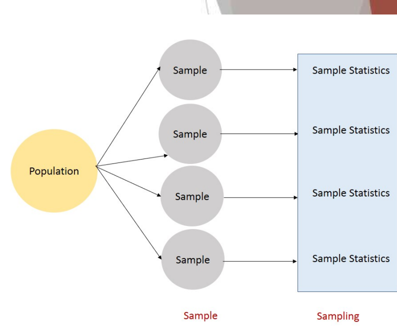
Sampling Distributions



- $\bar{x}$ and $\bar{p}$ are random variables so they have:
  - A Mean, or Expected Value
  - A Standard Deviation
  - Probability Distribution or **Sampling Distribution**
    - Each Probability Distribution has a **characteristic shape** or **form**
    - Knowing the **Sampling Distribution** of $\bar{x}$, and its properties helps us gauge how close the sample mean, $\bar{x}$ is to the true Population Mean, $\bar{\mu}$

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Sampling Distributions

- The Expected Value of the sample mean $\bar{x}$
  - Is the mean average of all possible values of $\bar{x}$ that can be generated by the various simple random samples
- Turns out:
  - The average of all the $\bar{x}$'s we could get by sampling 30 managers over and over actually equals the Population Mean, $\bar{\mu}$

EXPECTED VALUE OF $\bar{x}$

$$E(\bar{x}) = \mu \tag{6.1}$$

where

$E(\bar{x})$ = the expected value of $\bar{x}$
μ = the population mean

What is it called when the expected value of a point estimator equals the population parameter?*

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Sampling Distributions

Sample Number	Sample Mean ($\bar{x}$)	Sample Proportion ($\bar{p}$)
1	71,814	0.63
2	72,670	0.70
3	71,780	0.67
4	71,588	0.53
.	.	.
.	.	.
.	.	.
500	71,752	0.50

$$\frac{71{,}814 + 72{,}670 + 71{,}780 + 71{,}588 + \ ... + 71{,}752}{500} = \$71{,}800 = \ \mu = Population\ Mean$$

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Sampling Distributions

The formula for the standard deviation of x̄ depends on whether the population is finite or infinite.

Using the following notation:
$\sigma_{\bar{x}}$ = the standard deviation of $\bar{x}$, or the standard error of the mean.
σ = the standard deviation of the population.
n = the sample size.
N = the population size.

STANDARD DEVIATION OF $\bar{x}$

Finite Population

$$\sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Infinite Population

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \qquad \textbf{(6.2)}$$

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Sampling Distributions

- **Finite population correction factor:**

$$\sqrt{\frac{N-n}{N-1}}$$

Finite Population

$$s_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\left(\frac{s}{\sqrt{n}}\right)$$

- In many practical sampling situations,
  - The finite population correction factor is close to 1
  - So, the difference between the finite and infinite standard deviations is negligible.

In general, you can use *Infinite Population* $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ when, $\frac{n}{N} < 0.05$

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Sampling Distributions

ESTIMATED STANDARD DEVIATION OF $\bar{x}$

Finite Population	*Infinite Population*	
$s_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\left(\frac{s}{\sqrt{n}}\right)$	$s_{\bar{x}} = \left(\frac{s}{\sqrt{n}}\right)$	**(6.3)**

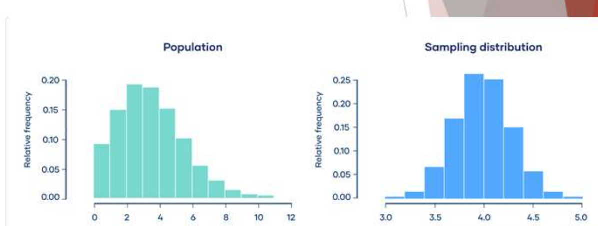
Estimated standard error: $s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{3,348}{\sqrt{30}} = 611.3.$

True standard error: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4,000}{\sqrt{30}} = 730.3.$

The difference between $s_{\bar{x}}$ and $\sigma_{\bar{x}}$ is due to sampling error.

Sampling Distributions

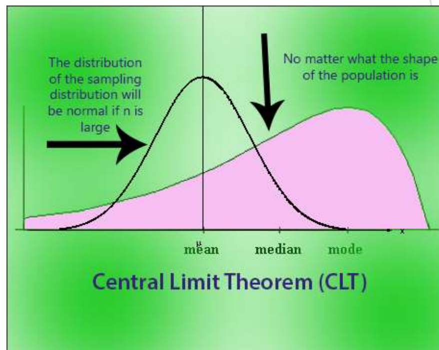
- When the population has a normal distribution,
  - The sampling distribution of $\bar{x}$ is normally distributed for any sample size
- When the population does not have a normal distribution
  - The central limit theorem is helpful in identifying the shape of the sampling distribution of $\bar{x}$



Sampling Distributions

- **Central Limit Theorem**
  - Distribution of the sample mean $\bar{x}$ can be approximated by a normal distribution as the sample size becomes large.

$$n \to \infty$$



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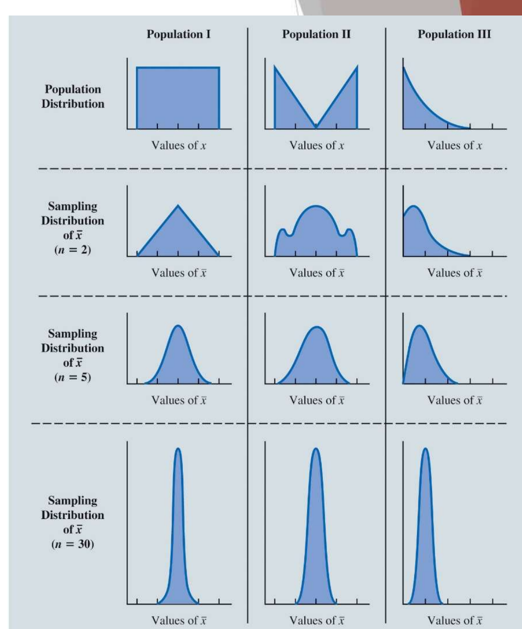
Sampling Distributions

Central Limit Theorem for Three Populations

- Top panel shows that none of the populations are normally distributed.
- Bottom three panels show the shape of the sampling distribution for samples n = 2, n = 5, and n = 30.
- For sample size of 30 or more, it looks closer to normal
- We can assume then, for a sample of 30 or more, the sampling distribution can be approximated by normal distribution.



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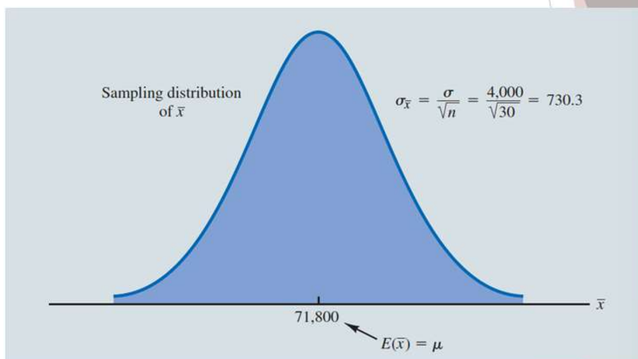
Sampling Distributions

Sampling Distribution of $\bar{x}$ for the Mean Annual Salary of a Simple Random Sample of 30 EAI Employees



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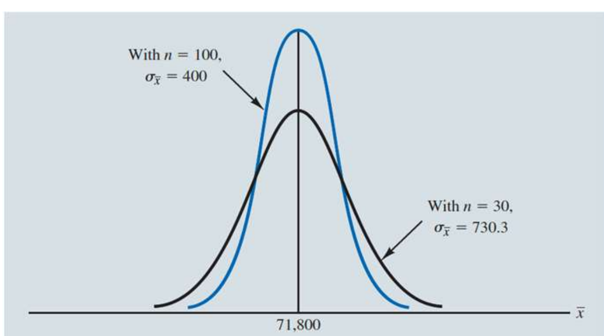
Sampling Distributions

A Comparison of the Sampling Distributions of $\bar{x}$ for Simple Random Samples of: $n = 30$ and $n = 100$ EAI Employees.



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Sampling Distributions

Sampling Distribution of $\bar{p}$:

The sample proportion $\bar{p}$ is the point estimator of the population proportion p.

The formula for computing the sample proportion is:

$$\bar{p} = \frac{x}{n}$$

where

x = the number of elements in the sample that possess the characteristic of interest.

n = sample size.

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Sampling Distributions

Sampling distribution of $\bar{p}$: The sampling distribution of $\bar{p}$ is the probability distribution of all possible values of the sample proportion $\bar{p}$.

EXPECTED VALUE OF $\bar{p}$

$$E(\bar{p}) = p \qquad \textbf{(6.4)}$$

where

$E(\bar{p})$ = the expected value of $\bar{p}$
p = the population proportion

STANDARD DEVIATION OF $\bar{p}$

Finite Population | *Infinite Population*

$$\sigma_{\bar{p}} = \sqrt{\frac{N-n}{N-1}}\sqrt{\frac{p(1-p)}{n}} \qquad \sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} \qquad \textbf{(6.5)}$$

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Sampling Distributions

ESTIMATED STANDARD DEVIATION OF $\bar{p}$

Finite Population

$$s_{\bar{p}} = \sqrt{\frac{N-n}{N-1}}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \tag{6.6}$$

Infinite Population

$$s_{\bar{p}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

The sampling distribution of $\overline{p}$ can be approximated by a normal distribution whenever $np \geq 5$ and $n(1-p) \geq 5$.

For our example: $\bar{p} = 0.63$, n = 30

$n\bar{p} = 30(0.63) = 18.9 > 5$ *and* $n(1-p) = 30(0.3\bar{7}) = 11.1 > 5$

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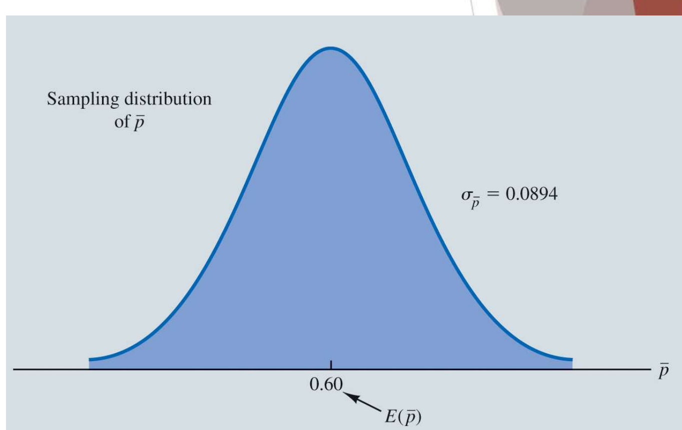
Sampling Distributions

Sampling Distribution of $\bar{p}$ for the Proportion of EAI Employees Who Participated in the Management Training Program



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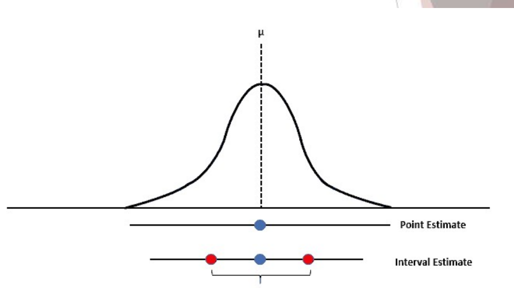
Interval Estimation

Interval Estimation of the Population Mean

Interval Estimation of the Population Proportion

Interval Estimation

- Point estimators are not perfect!
  - They do not provide the exact value of the population parameter
- An **interval estimate**
  - computed by adding and subtracting a value, called the **margin of error**, to the point estimate.
- The general form of an interval estimate is:



Point estimate $\pm$ Margin of error

Interval Estimation

Interval Estimation of the Population Mean:

- An interval estimate provides information about how close the point estimate is to the value of the population parameter.
- General form of an interval estimate of a population mean is:

$$\bar{x} \pm \text{ Margin of error}$$

- General form of an interval estimate of a population proportion is:

$$\bar{p} \pm \text{ Margin of error}$$

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Interval Estimation

Interval Estimation of the Population Mean (cont.):

For any normally distributed random variable:

- 90% of the values lie within 1.645 standard deviations of the mean.
- 95% of the values lie within 1.960 standard deviations of the mean.
- 99% of the values lie within 2.576 standard deviations of the mean.

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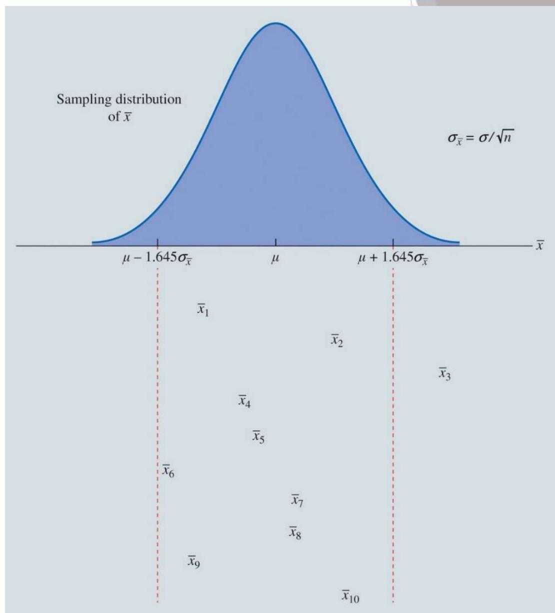
Interval Estimation

Sampling Distribution of the Sample Mean

How many of the $\bar{x}'s$ are in between the lines? What does this mean?

- Remember, we do not generally know the population standard deviation, σ
  - We have to use the sample data to estimate:
    - σ and μ
  - This introduces more uncertainty about the distribution values of $\bar{x}$.



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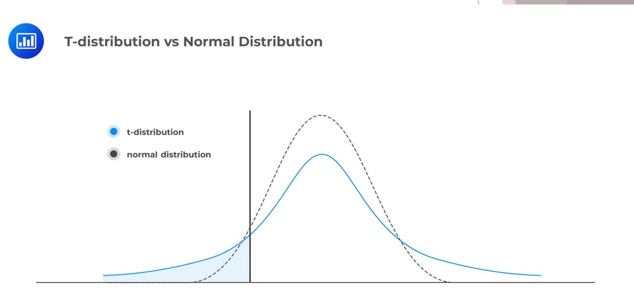
Interval Estimation

- To address this additional source of uncertainty
  - Use a probability distribution known as the *t* **distribution:**
    - A family of similar probability distributions.
    - The shape of each depends on a the **degrees of freedom.**
    - Similar in shape to the **standard normal distribution**, but wider.



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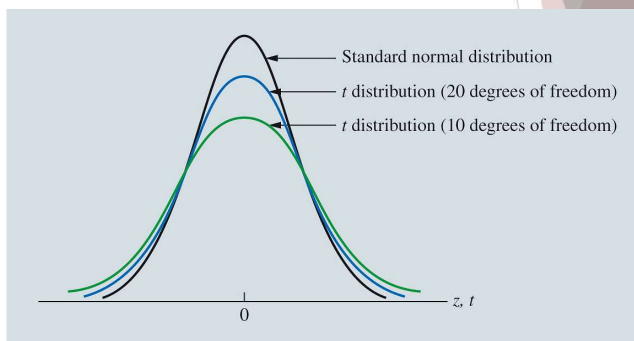
Interval Estimation

Comparison of the Standard Normal Distribution with t Distributions with 10 and 20 Degrees of Freedom

As the degrees of freedom increase, the t distribution narrows, its peak becomes higher, and it becomes more similar to the standard normal distribution.



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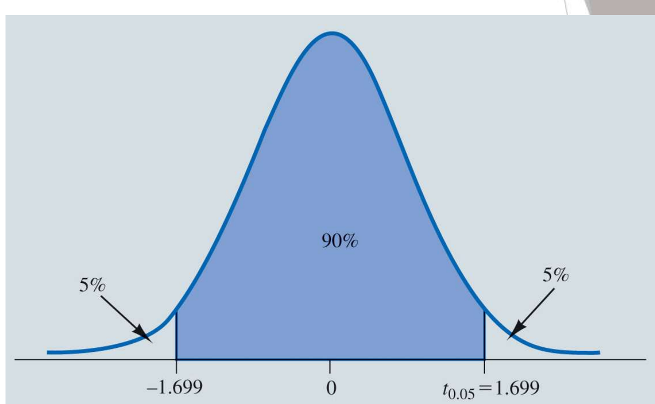
Interval Estimation

t Distribution with 29 Degrees of Freedom



Use Excel's T.INV.2T function to find the value from a t distribution such that 95% of the distribution is included in the interval $\pm t$ for 29 degrees of freedom.*

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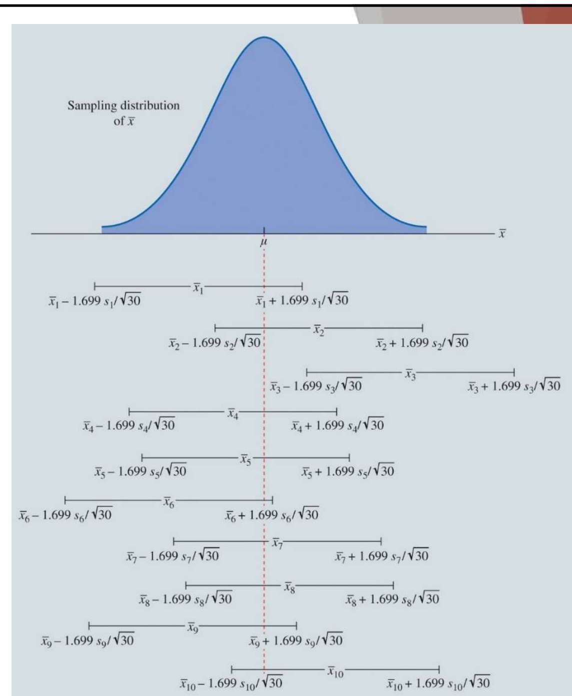
Interval Estimation

Intervals Formed Around Sample Means
from 10 Independent Random Samples

- Approximately 90% of all the intervals constructed will contain the population mean
- We are approximately 90% confident that the interval will include the population mean:
  - The value of 0.90 is referred to as the **confidence coefficient.**
  - The interval is called the 90% **confidence interval**.



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Interval Estimation

- The **level of significance**
  - is the probability that the interval estimation procedure will generate an interval that does not contain the population mean:

$$\alpha = \text{ level of significance } = 1 - \text{ confidence coefficient}$$

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