Creating Distributions from Data

Frequency Distributions for Categorical Data

Relative Frequency and Percent Frequency Distributions

Frequency Distributions for Quantitative Data

Histograms

Cumulative Distributions

Creating Distributions from Data

- **Frequency distribution:**

  A summary of data that shows the number (frequency) of observations in each of several nonoverlapping classes, typically referred to as **bins.**

Number of Pets	Frequency
1-2	7
3-4	3
5-6	3
7-8	2

Creating Distributions from Data

- **Relative frequency distribution:**
  - A tabular summary of data showing the relative frequency for each bin.
- **Percent frequency distribution:**
  - Summarizes the percent frequency of the data for each bin.
  - Used to provide estimates of the relative likelihoods of different values of a random variable.

Number of Pets	Frequency	Relative Frequency	
1	150	37.5%	← 150/400 = 37.5%
2	90	22.5%	← 90/400 = 22.5%
3	110	27.5%	← 110/400 = 27.5%
4	30	7.5%	← 30/400 = 7.5%
5	20	5.0%	← 20/400 = 5.0%

Creating Distributions from Data

Table 2.5: Relative Frequency and Percent Frequency Distributions of Soft Drink Purchases

Soft Drink	Relative Frequency	Percent Frequency (%)
Coca-Cola	0.38	38
Diet Coke	0.16	16
Dr. Pepper	0.10	10
Pepsi	0.26	26
Sprite	0.10	10
Total	1.00	100

Creating Distributions from Data

- Three steps necessary to define the classes for a frequency distribution with quantitative data:
  1. Determine the number of nonoverlapping bins.
  2. Determine the width of each bin.
  3. Determine the bin limits.

APPROXIMATE BIN WIDTH

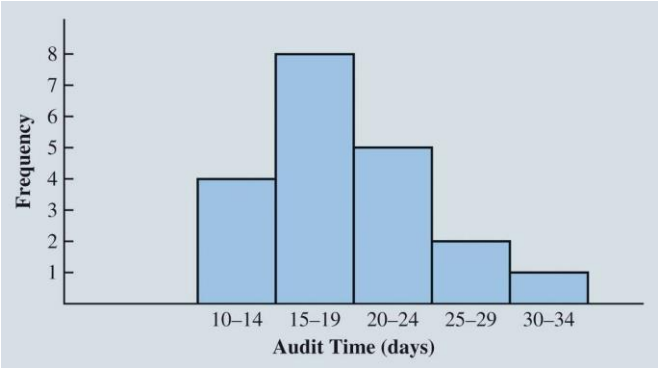
$$\frac{\text{Largest data value} - \text{smallest data value}}{\text{Number of bins}} \qquad (2.1)$$

Creating Distributions from Data

- **Histogram:** A common graphical presentation of quantitative data.
- Constructed by placing the variable of interest on the horizontal axis
- Frequency measure (absolute frequency, relative frequency, or percent frequency) on the vertical axis.

Creating Distributions from Data

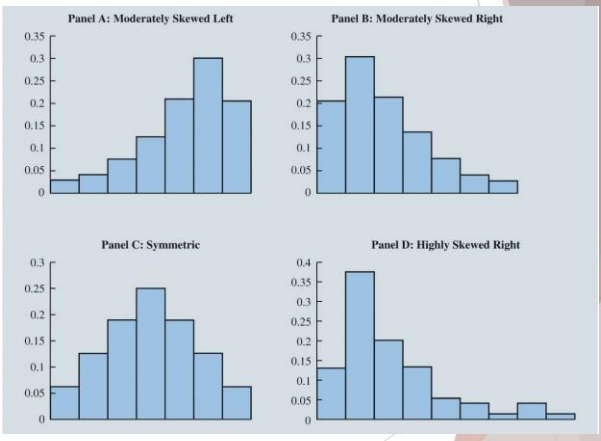
Figure 2.12: Histogram for the Audit Time Data



Creating Distributions from Data

Histograms (cont.):

- Histograms provide information about the shape, or form, of a distribution.
- **Skewness**: Lack of symmetry.
- Skewness is an important characteristic of the shape of a distribution.



Creating Distributions from Data

- **Cumulative frequency distribution**: A variation of the frequency distribution that provides another tabular summary of quantitative data.
  - Uses the number of classes, class widths, and class limits developed for the frequency distribution.
  - Shows the number of data items with values less than or equal to the upper class limit of each class.

Creating Distributions from Data

Table 2.3: Data from a Sample of 50 Soft Drink Purchases

Coca-Cola	Sprite	Pepsi
Diet Coke	Coca-Cola	Coca-Cola
Pepsi	Diet Coke	Coca-Cola
Diet Coke	Coca-Cola	Coca-Cola
Coca-Cola	Diet Coke	Pepsi
Coca-Cola	Coca-Cola	Dr. Pepper
Dr. Pepper	Sprite	Coca-Cola
Diet Coke	Pepsi	Diet Coke
Pepsi	Coca-Cola	Pepsi
Pepsi	Coca-Cola	Pepsi
Coca-Cola	Coca-Cola	Pepsi
Dr. Pepper	Pepsi	Pepsi
Sprite	Coca-Cola	Coca-Cola
Coca-Cola	Sprite	Dr. Pepper
Diet Coke	Dr. Pepper	Pepsi
Coca-Cola	Pepsi	Sprite
Coca-Cola	Diet Coke	

Creating Distributions from Data

Table 2.4: Frequency Distribution of Soft Drink Purchases

Soft Drink	Frequency
Coca-Cola	19
Diet Coke	8
Dr. Pepper	5
Pepsi	13
Sprite	5
Total	50

- The frequency distribution summarizes information about the popularity of the five soft drinks:
  - Coca-Cola is the leader.
  - Pepsi is second.
  - Diet Coke is third.
  - Sprite and Dr. Pepper are tied for fourth.

Creating Distributions from Data

Figure 2.10: Creating a Frequency Distribution for Soft Drinks Data in Excel

	A	B	C	D	E
1	Sample Data			Bins	
2	Coca-Cola	Coca-Cola		Coca-Cola	19
3	Diet Coke	Sprite		Diet Coke	8
4	Pepsi	Pepsi		Dr. Pepper	5
5	Diet Coke	Coca-Cola		Pepsi	13
6	Coca-Cola	Pepsi		Sprite	5
7	Coca-Cola	Sprite			
8	Dr. Pepper	Dr. Pepper			
9	Diet Coke	Pepsi			
10	Pepsi	Diet Coke			
11	Pepsi	Pepsi			
12	Coca-Cola	Coca-Cola			
13	Dr. Pepper	Coca-Cola			
14	Sprite	Diet Coke			
15	Coca-Cola	Pepsi			
16	Diet Coke	Pepsi			
17	Coca-Cola	Pepsi			
18	Coca-Cola	Coca-Cola			
19	Diet Coke	Dr. Pepper			
20	Coca-Cola	Sprite			
21	Coca-Cola	Coca-Cola			
22	Coca-Cola	Coca-Cola			
23	Sprite	Pepsi			
24	Coca-Cola	Dr. Pepper			
25	Coca-Cola	Pepsi			
26	Diet Coke	Pepsi			

Creating Distributions from Data

Table 2.6: Year-End Audit Times (Days)

12	14	19	18
15	15	18	17
20	27	22	23
22	21	33	28
14	18	16	13

Creating Distributions from Data

Table 2.7: Frequency, Relative Frequency, and Percent Frequency Distributions for the Audit Time Data

Audit Times (days)	Frequency	Relative Frequency	Percent Frequency
10–14	4	0.20	20
15–19	8	0.40	40
20–24	5	0.25	25
25–29	2	0.10	10
30–34	1	0.05	5

Creating Distributions from Data

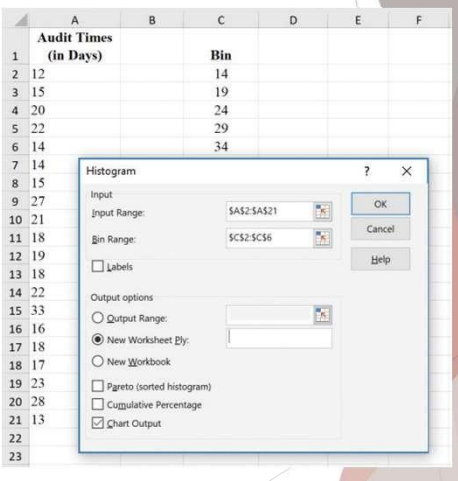
Figure 2.11: Using Excel to Generate a Frequency Distribution for Audit Times Data

	A	B	C	D
1	Audit Times (in Days)		Bin	Frequency
2	12		14	=FREQUENCY(A2:A21,C2:C6)
3	15		19	=FREQUENCY(A2:A21,C2:C6)
4	20		24	=FREQUENCY(A2:A21,C2:C6)
5	22		29	=FREQUENCY(A2:A21,C2:C6)
6	14		34	=FREQUENCY(A2:A21,C2:C6)
7	14			
8	15			
9	27			
10	21			
11	18			
12	19			
13	18			
14	22			
15	33			
16	16			
17	18			
18	17			
19	23			
20	28			
21	13			

	A	B	C	D
1	Audit Times (in Days)		Bin	Frequency
2	12		14	4
3	15		19	8
4	20		24	5
5	22		29	2
6	14		34	1
7	14			
8	15			
9	27			
10	21			
11	18			
12	19			
13	18			
14	22			
15	33			
16	16			
17	18			
18	17			
19	23			
20	28			
21	13			

Creating Distributions from Data

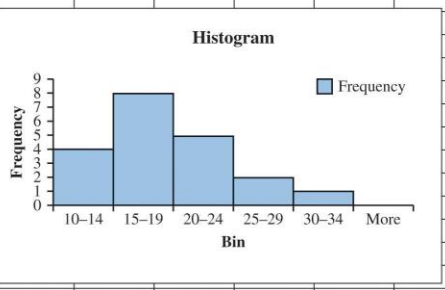
Figure 2.13: Creating a Histogram for the Audit Time Data Using Data Analysis Toolpak in Excel



Creating Distributions from Data

Figure 2.14: Completed Histogram for the Audit Time Data Using Data Analysis ToolPak in Excel

	A	B	C	D	E	F	G	H	I	J
1	*Bin*	*Frequency*								
2	10–14	4								
3	15–19	8								
4	20–24	5								
5	25–29	2								
6	30–34	1								
7	More	0								
8										
9										
10										
11										
12										
13										
14										



Creating Distributions from Data

Table 2.8: Cumulative Frequency, Cumulative Relative Frequency, and Cumulative Percent Frequency Distributions for the Audit Time Data

Audit Time (days)	Cumulative Frequency	Cumulative Relative Frequency	Cumulative Percent Frequency
Less than or equal to 14	4	0.20	20
Less than or equal to 19	12	0.60	60
Less than or equal to 24	17	0.85	85
Less than or equal to 29	19	0.95	95
Less than or equal to 34	20	1.00	100

Measures of Location

Mean (Arithmetic Mean)

Median

Mode

Geometric Mean

Measures of Location

Mean/Arithmetic Mean

Average value for a variable.

The mean is denoted by $\bar{x}$.

n = sample size.

x_1 = variable of x for the first observation.

x_2 = variable of x for the second observation.

x_i = variable of x for the i*th* observation.

SAMPLE MEAN

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad (2.2)$$

Measures of Location

Computation of Sample Mean

- Illustration: Computation of the mean home selling price for the sample of 12 home sales.

$$\bar{x} = \frac{\sum x_i}{n} = \frac{x_1 + x_2 + \cdots + x_{12}}{12}$$

$$= \frac{138{,}000 + 254{,}000 + \cdots 456{,}250}{12}$$

$$= \frac{2{,}639{,}250}{12} = 219{,}937.50$$

Measures of Location

- Median:

  Value in the middle when the data are arranged in ascending order.

  - Middle value, for an odd number of observations.
  - Average of two middle values, for an even number of observations.

Measures of Location

Computation of Sample Median

- Illustration: When the number of observations are odd:
- Consider the class size data for a sample of five college classes:

46 54 42 46 32

- Arrange the class size data in ascending order:

32 42 46 46 54

- Middlemost value in the data set = 46.
- Median is 46.

Measures of Location

- Illustration: When the number of observations are even:
  - Consider the data on home sales in Cincinnati, Ohio, Suburb (Table 2.9).
  - Arrange the data in ascending order:

108,000 138,000 138,000 142,000 186,000 199,500 208,000 254,000 254,000 257,500 298,000 456,250

  - Median = average of two middle values:

$$\text{Median} = \frac{199{,}500 + 208{,}000}{2} = 203{,}750$$

Measures of Location

- **Mode:**

Value that occurs most frequently in a data set.

- Consider the class size data:

32 42 46 46 54

- Observe: 46 is the only value that occurs more than once.
  - Mode is 46.
- Multimodal data: Data contain at least two modes.
- Bimodal data: Data contain exactly two modes.

Measures of Location

- **Geometric mean:**

A measure of location that is calculated by finding the *n*th root of the product of *n* values

- Used in analyzing growth rates in financial data.

SAMPLE GEOMETRIC MEAN

$$\bar{x}_g = \sqrt[n]{(x_1)(x_2)\cdots(x_n)} = [(x_1)(x_2)\cdots(x_n)]^{1/n} \qquad \textbf{(2.3)}$$

Measures of Location

- We will determine the mean rate of growth for the fund over the 10-year period.

Percentage Annual Returns and Growth Factors for the Mutual Fund Data:

Year	Return (%)	Growth Factor
1	−22.1	0.779
2	28.7	1.287
3	10.9	1.109
4	4.9	1.049
5	15.8	1.158
6	5.5	1.055
7	−37.0	0.630
8	26.5	1.265
9	15.1	1.151
10	2.1	1.021

Measures of Location

Computation of Geometric Mean:

- Product of the growth factors:

$$\$100(0.779)(1.287)(1.109)(1.049)(1.158)(1.055)(0.630)(1.265)(1.151)(1.021)$$
$$= \$100(1.335) = \$133.45$$

- Geometric mean of the growth factors:

$$\bar{x}_g = \sqrt[10]{1.335} = 1.029.$$

- Conclude that annual returns grew at an average annual rate of $(1.029 - 1)100\%$ or 2.9%.

Measures of Location

Figure 2.16: Calculating the Mean, Median, and Modes for the Home Sales Data using Excel

	A	B	C	D	E
1	Home Sale	Selling Price ($)			
2	1	138,000		Mean:	=AVERAGE(B2:B13)
3	2	254,000		Median:	=MEDIAN(B2:B13)
4	3	186,000		Mode 1:	=MODE.MULT(B2:B13)
5	4	257,500		Mode 2:	=MODE.MULT(B2:B13)
6	5	108,000			
7	6	254,000			
8	7	138,000			
9	8	298,000			
10	9	199,500			
11	10	208,000			
12	11	142,000			
13	12	456,250			

	A	B	C	D	E
1	Home Sale	Selling Price ($)			
2	1	138,000		Mean:	$ 219,937.50
3	2	254,000		Median:	$ 203,750.00
4	3	186,000		Mode 1:	$ 138,000.00
5	4	257,500		Mode 2:	$ 254,000.00
6	5	108,000			
7	6	254,000			
8	7	138,000			
9	8	298,000			
10	9	199,500			
11	10	208,000			
12	11	142,000			
13	12	456,250			

Measures of Location

Figure 2.17: Calculating the Geometric Mean for the Mutual Fund Data Using Excel

	A	B	C	D
1	Year	Return (%)	Growth Factor	
2	1	–22.1	0.779	
3	2	28.7	1.287	
4	3	10.9	1.109	
5	4	4.9	1.049	
6	5	15.8	1.158	
7	6	5.5	1.055	
8	7	–37.0	0.630	
9	8	26.5	1.265	
10	9	15.1	1.151	
11	10	2.1	1.021	
12				
13		Geometric Mean:	1.029	
14				