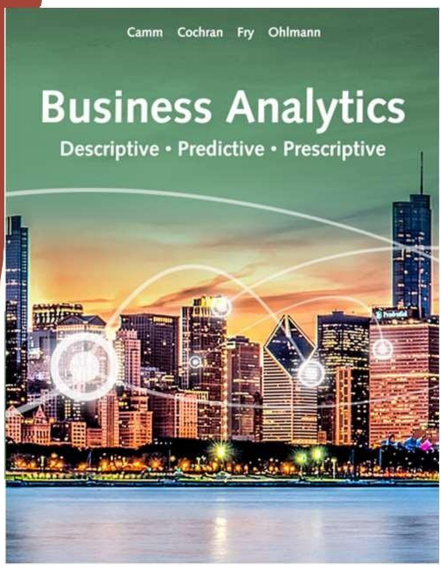



# Business Analytics

Chapter 4 Probability

HAMLINE UNIVERSITY SCHOOL OF BUSINESS

## Why is Probability Important?

# It Can Save Lives, too!

**Chile 2010**

- San Jose copper and gold mine caved in, trapping 33 men 2000 ft underground
  - Contacted NASA to help
  - NASA Team: Engineer, 2 Doctors, and a psychologist

**No historical data**

- Developed subjective probability estimates:
  - Success vs. Failure of rescue methods

**Results?**

- Based on probabilities the team designed:
  - 13 ft, 924 steel rescue capsule that brought up miners 1 at a time
  - All miners were rescued – some after 68 days underground

# Introduction

- UNCERTAINTY
  - We don't know everything
  - We wish we did
- **Probability:**
  - is the numerical measure of the likelihood that an event will occur.
  - Often communicated through a probability distribution
  - Helpful in providing additional information about an event.
  - Can be used to help a decision maker evaluate possible actions and determine best course of action.

# Events and Probabilities

## Events and Probabilities

- A **random experiment:**
  - is a process that generates well-defined outcomes.
- **The sample space** for a random experiment:
  - All possible outcomes
- Examples:
  - A coin toss – Sample Space = Heads, Tails
  - Rolling a die – Sample Space = 1, 2, 3, 4, 5, 6
- An **event** is defined as a collection of outcomes.

# Events and Probabilities

Other Examples: Random Experiments and Experimental Outcomes

| Random Experiment | Experimental Outcomes |
|---|---|
| Toss a coin | Head, tail |
| Roll a die | 1, 2, 3, 4, 5, 6 |
| Conduct a sales call | Purchase, no purchase |
| Hold a particular share of stock for one year | Price of stock goes up, price of stock goes down, no change in stock price |
| Reduce price of product | Demand goes up, demand goes down, no change in demand |

# Events and Probabilities

- California Power & Light Company (CP&L).
  - CP&L is starting a project designed to increase the generating capacity of one of its plants in southern California.
  - Analysis of similar construction projects indicates that the possible completion times for the project are 8, 9, 10, 11, and 12 months.

- Events:
  - Event that project is completed in 10 months or less
    - C = {8, 9, 10}
  - Event that project is completed in less than 10 months
    - C = {8, 9}
  - Event that project is completed in more than 10 months
  - Event that project is completed between 9 and 11 months

## Events and Probabilities

- If there have been 40 already completed projects, then based on previous data,
- **What are the probabilities of each outcome?**

| Completion Time (months) | No. of Past Projects Having This Completion Time | Probability of Outcome |
|---|---|---|
| 8 | 6 | 6/40 = 0.15 |
| 9 | 10 | 10/40 = 0.25 |
| 10 | 12 | 12/40 = 0.30 |
| 11 | 6 | 6/40 = 0.15 |
| 12 | 6 | 6/40 = 0.15 |
| Total | 40 | 1.00 |

## Events and Probabilities

**How to calculate probability of an event?**

- The **probability of an event:**
  - is equal to the sum of probabilities of outcomes for the event.
- CP&L example: Let $C$ be the event that the project is completed in 10 months or less,
- The probability of event $C$,

$$P(C) = P(8) + P(9) + P(10) = 0.15 + 0.25 + 0.30 = 0.70$$

- We can tell CP&L management that there is a 0.70 probability (70% chance) that the project will be completed in 10 months or less.

# Some Basic Relationships of Probability

Complement of an Event

Addition Law

## Some Basic Relationships of Probability

**Complement of an Event:**

- Given an event $A$, the **complement of A:**
  - is defined to be the event consisting of all outcomes that are *not* in $A$.

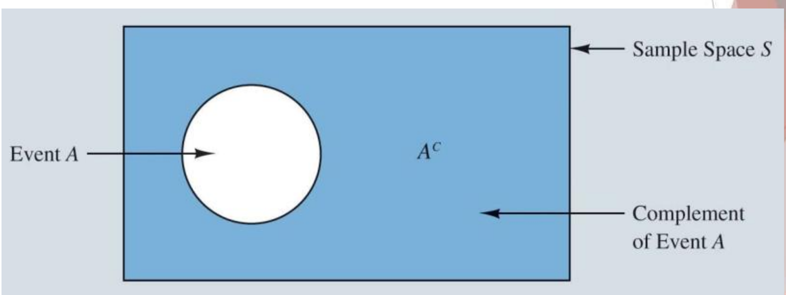
**Example:**

*Sample space: Roll a Die*

*S = {1, 2, 3, 4, 5, 6}*

*Event: A = {1, 2, 3, 4}*

A complement = $A^C = \{5, 6\}$



# Some Basic Relationships of Probability

In any probability application, either event A or its complement $A^c$ must occur!

- Example:
  - Roll a Dice
  - S = {1, 2, 3, 4, 5, 6}
  - A = Roll an Even = {2, 4, 6}
  - A^c = Roll an Odd = {1, 3, 5}

If you don't roll and even #, you must roll an odd #!

**COMPUTING PROBABILITY USING THE COMPLEMENT**

$$P(A) = 1 - P(A^C) \quad \textbf{(4.1)}$$

The probability of an event *A* can be computed easily if the probability of its complement is known.

# Some Basic Relationships of Probability

**Union of 2 Events**

- Given two events *A* and *B*, the **union of** ***A*** **and** ***B***
  - is defined as the event containing all outcomes belonging to *A* or *B* or both.
- The union of *A* and *B* is denoted by $A \cup B$.

**Example:**

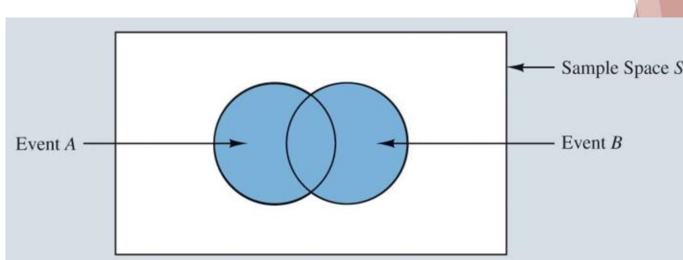
*Sample space: Roll a Die*

*S = {1, 2, 3, 4, 5, 6}*

*Event: A = {1, 2, 3, 4}*

*Event B = {5}*

A U B = {1, 2, 3, 4, 5}



## Some Basic Relationships of Probability

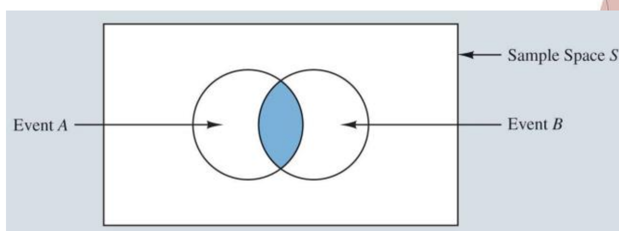
**Intersection of 2 Events**

- Given two events *A* and *B*, the **intersection of *A* and *B***
  - is the event containing the outcomes that belong to both *A* and *B*. The union of *A* and *B* is denoted by
- The intersection of *A* and *B* is denoted by $A \cap B$.

**Example:**

*Sample space: Roll a Die*
*S = {1, 2, 3, 4, 5, 6}*
*Event: A = {1, 2, 3, 4}*
*Event B = {2, 4, 6}*
*Event C = {5}*

A ∩ B = {2,4}
A ∩ C = {ø}




## Some Basic Relationships of Probability

**What is the probability that at least one of two events will occur?**
**What is the probability that Event A OR Event B will happen?**

**ADDITION LAW**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \textbf{(4.2)}$$

- A special case arises for **mutually exclusive events:**
  - If the occurrence of one event precludes the occurrence of the other.
  - If the events have no outcomes in common.

# Some Basic Relationships of Probability

**ADDITION LAW**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad \textbf{(4.2)}$$

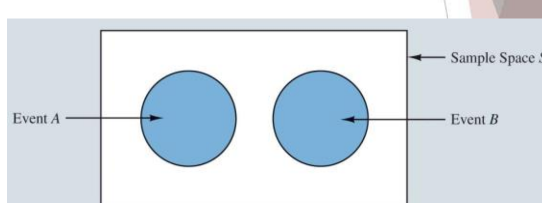
- Example:
  - Sample space: Roll a Die
  - S = {1, 2, 3, 4, 5, 6}
  - Event: A = {1, 2, 3, 4}
  - Event B = {2, 4, 6}
  - Event C = {5}
- What is the P(A U B)?

P(A) = 4/6 = .667
P(B) = 3/6 = .5
P(A n B) = 2/6 = .333
P(A U B) = .667 + .5 - .333 = .834 = 5/6
=P (1,2,3,4,6)

# Some Basic Relationships of Probability

- A special case arises for **mutually exclusive events:**
  - If the events have no outcomes in common.

**Mutually Exclusive Events**




- Example:
  - Sample space: Roll a Die
  - S = {1, 2, 3, 4, 5, 6}
  - Event: A = {1, 2, 3, 4}
  - Event B = {5, 6}
- What is the P(A U C)?

P(A) = 4/6 = .667
P(B) = 2/6 = .333
P(A n B) = 0/6 = 0
P(A U C) = .667 + .333 - 0 = 1

**ADDITION LAW FOR MUTUALLY EXCLUSIVE EVENTS**

$$P(A \cup B) = P(A) + P(B)$$

# Conditional Probability

Independent Events
Multiplication Law
Bayes' Theorem

## Conditional Probability

- **Conditional probability:**
  - When the probability of one event is dependent on whether some related event has already occurred.
- Illustration: Lancaster Savings and Loan:

**Does the probability of a customer defaulting on a mortgage differ by marital status?**

# Conditional Probability

- **Does the probability of a customer defaulting on a mortgage differ by marital status?**
- S = event that a customer is single
- M = event that a customer is married
- D = event that a customer defaulted on their mortgage
- D^C = event that a customer did not default on their mortgage

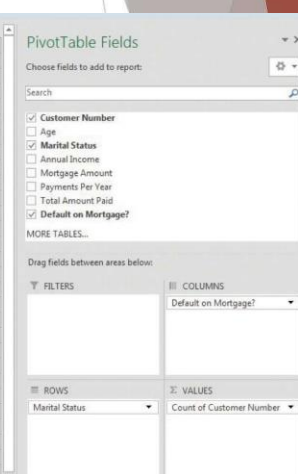
**Table 4.3** Subset of Data from 300 Home Mortgages of Customers at Lancaster Savings and Loan

| Customer No. | Age | Marital Status | Annual Income | Mortgage Amount | Payments per Year | Total Amount Paid | Default on Mortgage? |
|---|---|---|---|---|---|---|---|
| 1 | 37 | Single | $ 172,125.70 | $ 473,402.96 | 24 | $ 581,885.13 | Yes |
| 2 | 31 | Single | $ 108,571.04 | $ 300,468.60 | 12 | $ 489,320.38 | No |
| 3 | 37 | Married | $ 124,136.41 | $ 330,664.24 | 24 | $ 493,541.93 | Yes |
| 4 | 24 | Married | $ 79,614.04 | $ 230,222.94 | 24 | $ 449,682.09 | Yes |
| 5 | 27 | Single | $ 68,087.33 | $ 282,203.53 | 12 | $ 520,581.82 | No |
| 6 | 30 | Married | $ 59,959.80 | $ 251,242.70 | 24 | $ 356,711.58 | Yes |
| 7 | 41 | Single | $ 99,394.05 | $ 282,737.29 | 12 | $ 524,053.46 | No |
| 8 | 29 | Single | $ 38,527.35 | $ 238,125.19 | 12 | $ 468,595.99 | No |
| 9 | 31 | Married | $ 112,078.62 | $ 297,133.24 | 24 | $ 399,617.40 | Yes |
| 10 | 36 | Single | $ 224,899.71 | $ 622,578.74 | 12 | $ 1,233,002.14 | No |
| 11 | 31 | Married | $ 27,945.36 | $ 215,440.31 | 24 | $ 285,900.10 | Yes |
| 12 | 40 | Single | $ 48,929.74 | $ 252,885.10 | 12 | $ 336,574.63 | No |
| 13 | 39 | Married | $ 82,810.92 | $ 183,045.16 | 12 | $ 262,537.23 | No |
| 14 | 31 | Single | $ 68,216.88 | $ 165,309.34 | 12 | $ 253,633.17 | No |

# Conditional Probability

Figure 4.5: PivotTable for Marital Status and Whether Customer Defaults on Mortgage

| Count of Customer Number | Column Labels | | |
|---|---|---|---|
| Row Labels | NO | YES | Grand Total |
| MARRIED | 64 | 79 | 143 |
| SINGLE | 116 | 41 | 157 |
| Grand Total | 180 | 120 | 300 |



# Conditional Probability

Crosstabulation of Marital Status and if Customer Defaults on Mortgage

| Marital Status | No Default | Default | Total |
|---|---|---|---|
| Married | 64 | 79 | 143 |
| Single | 116 | 41 | 157 |
| Total | 180 | 120 | 300 |

The probability that a customer defaults on his or her mortgage is

$120/300 = 0.4.$

The probability that a customer does not default on his or her mortgage is

$1 - 0.4 = 0.6$ or $180/300$.

# Conditional Probability

- **Joint probabilities:**
  - When values give the probability of the intersection of two events
- **Marginal probabilities:**
  - Sum of the joint probabilities in the corresponding row or column of the joint probability table.
- Conditional probabilities can be computed as the ratio of joint probability to a marginal probability.

**CONDITIONAL PROBABILITY**

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \quad \textbf{(4.3)}$$

or

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \quad \textbf{(4.4)}$$

## Conditional Probability

Table 4.5: Joint Probability Table for Customer Mortgage Prepayments

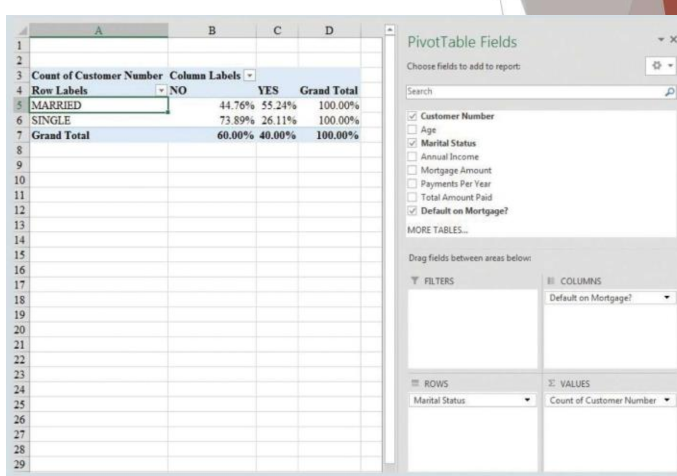
Joint Probabilities

| | No Default ($D^C$) | Default ($D$) | Total |
|---|---|---|---|
| Married ($M$) | 0.2133 | 0.2633 | *0.4766* |
| Single ($S$) | 0.3867 | 0.1367 | *0.5234* |
| *Total* | *0.6000* | *0.4000* | *1.0000* |

Marginal Probabilities

## Conditional Probability

Figure 4.6: Using Excel PivotTable to Calculate Conditional Probabilities



| Count of Customer Number | Column Labels | | |
|---|---|---|---|
| Row Labels | NO | YES | Grand Total |
| MARRIED | 44.76% | 55.24% | 100.00% |
| SINGLE | 73.89% | 26.11% | 100.00% |
| Grand Total | 60.00% | 40.00% | 100.00% |