ISOCRATIC CONDITIONS (i.e, $\phi = \text{const}$)

$$\left\{1 + F_{a0} \cdot \exp\left[G\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right]\exp(-S_s\phi)\right\}\frac{\partial \vec{C}}{\partial t} + u\frac{\partial \vec{C}}{\partial z} - \frac{Lu}{2N}\cdot\frac{\partial^2 \vec{C}}{\partial z^2} = 0$$

$$F(t, \vec{\eta}, \vec{\eta}_t, \vec{\eta}_z, \vec{\eta}_{zz}, \vec{\beta}) = 0 \qquad \vec{\eta} = C \quad \vec{\eta}_t = \frac{\partial C}{\partial t} \quad \vec{\eta}_z = \frac{\partial C}{\partial z}$$

$$\vec{\eta}_{zz} = \frac{\partial^2 C}{\partial z^2} \qquad \vec{\beta} = [F_{a0}, G, S_s]$$

$$\frac{dF}{d\beta} = \frac{\partial \vec{F}}{\partial t}\cdot\overbrace{\cancel{\frac{\partial t}{\partial \vec{\beta}}}}^{\vec{0}} + \overbrace{\cancel{\frac{\partial \vec{F}}{\partial \vec{\eta}}}}^{\vec{0}}\cdot\frac{\partial \vec{\eta}}{\partial \vec{\beta}} + \frac{\partial \vec{F}}{\partial \vec{\eta}_t}\cdot\frac{\partial \vec{\eta}_t}{\partial \vec{\beta}} + \frac{\partial \vec{F}}{\partial \vec{\eta}_z}\cdot\frac{\partial \vec{\eta}_z}{\partial \vec{\beta}} + \frac{\partial \vec{F}}{\partial \vec{\eta}_{zz}}\cdot\frac{\partial \vec{\eta}_{zz}}{\partial \vec{\beta}} + \frac{\partial \vec{F}}{\partial \vec{\beta}}\cdot\overbrace{\cancel{\frac{\partial \vec{\beta}}{\partial \vec{\beta}}}}^{I}$$

$$\frac{\partial \vec{F}}{\partial \vec{\eta}_t} = 1 + F_{a0}\cdot\exp\left[G\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right]\exp(-S_s\phi)$$

$$\frac{\partial \vec{F}}{\partial \vec{\eta}_z} = u$$

$$\frac{\partial \vec{F}}{\partial \vec{\eta}_{zz}} = -\frac{Lu}{2N}$$

$$\frac{\partial \vec{F}}{\partial \vec{\beta}} = \Bigg[\left\{\exp\left[G\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right]\exp(-S_s\phi)\right\}\frac{\partial \vec{C}}{\partial t}$$

$$\left\{F_{a0}\cdot\exp\left[G\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right]\exp(-S_s\phi)\cdot\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right\}\frac{\partial \vec{C}}{\partial t}$$

$$\left\{-F_{a0}\cdot\exp\left[G\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right]\exp(-S_s\phi)\cdot\phi\right\}\frac{\partial \vec{C}}{\partial t}\Bigg]^T$$

$$\frac{d\vec{F}}{d\beta} = \vec{0} + \vec{0} + \left\{1 + F_{a0}\cdot\exp\left[G\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right]\exp(-S_s\phi)\right\}\cdot\left[\frac{\partial}{\partial F_{a0}}\left(\frac{\partial \vec{C}}{\partial t}\right) \ \frac{\partial}{\partial G}\left(\frac{\partial \vec{C}}{\partial t}\right) \ \frac{\partial}{\partial S_S}\cdot\frac{\partial \vec{C}}{\partial t}\right]$$

$$+ u\left[\frac{\partial}{\partial z}\left(\frac{\partial \vec{C}}{\partial F_a}\right) \ \frac{\partial}{\partial z}\left(\frac{\partial \vec{C}}{\partial G}\right) \ \frac{\partial}{\partial z}\left(\frac{\partial \vec{C}}{\partial S_s}\right)\right]$$

$$- \frac{Lu}{2N}\cdot\left[\frac{\partial}{\partial z^2}\left(\frac{\partial \vec{C}}{\partial F_a}\right) \ \frac{\partial}{\partial z^2}\left(\frac{\partial \vec{C}}{\partial G}\right) \ \frac{\partial}{\partial z^2}\left(\frac{\partial \vec{C}}{\partial S_s}\right)\right] + \frac{\partial \vec{F}}{\partial \vec{\beta}}$$

SENSITIVITY EQUATIONS:

① $$\left\{1+F_{a0}\cdot \exp\left[G\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right]\cdot \exp(-S_s\phi)\right\}\frac{\partial}{\partial t}\cdot\left(\frac{\partial \vec{C}}{\partial F_{a0}}\right)+u\,\frac{\partial}{\partial z}\cdot\left(\frac{\partial \vec{C}}{\partial F_{a0}}\right)$$
$$-\frac{L_u}{2N}\cdot\frac{\partial}{\partial z^2}\cdot\left(\frac{\partial^2 \vec{C}}{\partial F_{a0}}\right)+\left\{\exp\left[G\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right]\exp(-S_s\phi)\right\}\frac{\partial \vec{C}}{\partial t}=0$$

② $$\left\{1+F_{a0}\cdot \exp\left[G\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right]\exp(-S_s\phi)\right\}\cdot\frac{\partial}{\partial t}\left(\frac{\partial \vec{C}}{\partial G_T}\right)+u\,\frac{\partial}{\partial z}\left(\frac{\partial \vec{C}}{\partial G_T}\right)$$
$$-\frac{L_u}{2N}\cdot\frac{\partial}{\partial z^2}\left(\frac{\partial^2 \vec{C}}{\partial G_T}\right)+\left\{F_{a0}\cdot \exp\left[G\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right]\exp(-S_s\phi)\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right\}\frac{\partial \vec{C}}{\partial t}=0$$

③ $$\left\{1+F_{a0}\exp\left[G\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right]\exp(-S_s\phi)\right\}\cdot\frac{\partial}{\partial t}\left(\frac{\partial \vec{C}}{\partial S_s}\right)+u\,\frac{\partial}{\partial z}\left(\frac{\partial \vec{C}}{\partial S_s}\right)$$
$$-\frac{L_u}{2N}\cdot\frac{\partial}{\partial z^2}\left(\frac{\partial^2 \vec{C}}{\partial S_s}\right)+\left\{-F_{a0}\cdot\exp\left[G\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)\right]\exp(-S_s\phi)\,\phi\right\}\frac{\partial \vec{C}}{\partial t}=0$$

INLET CONDITION (DANKWERTS)

$$uC-\mathcal{D}\frac{\partial C}{\partial z}-uC_0=0$$

$$\frac{\partial \vec{F}}{\partial \beta}=\frac{\partial \vec{F}}{\partial t}\cdot\overset{\vec{0}}{\cancel{\frac{\partial t}{\partial \beta}}}+\frac{\partial \vec{F}}{\partial \vec{\eta}}\cdot\frac{\partial \vec{\eta}}{\partial \beta}+\overset{\vec{0}}{\cancel{\frac{\partial \vec{F}}{\partial \vec{\eta}_t}}}\cdot\frac{\partial \vec{\eta}_t}{\partial \beta}+\frac{\partial \vec{F}}{\partial \vec{\eta}_z}\cdot\frac{\partial \vec{\eta}_z}{\partial \beta}+\overset{\vec{0}}{\cancel{\frac{\partial \vec{F}}{\partial \vec{\eta}_{zz}}}}\cdot\frac{\partial \vec{\eta}_{zz}}{\partial \beta}+\frac{\partial \vec{F}}{\partial \beta}\cdot\overset{\vec{I}}{\cancel{\frac{\partial \vec{\beta}}{\partial \beta}}}$$

$$\frac{dF}{d\beta}=\boxed{u\cdot\frac{\partial c}{\partial \beta}+(-\mathcal{D})\cdot\frac{\partial}{\partial z}\left(\frac{\partial c}{\partial \beta}\right)=0}$$

INLET CONDITION (DIRICHLET)

$$C=C_0 \Rightarrow C-C_0=0$$

$$\frac{dF}{d\beta}=\frac{\partial \vec{F}}{\partial \vec{\eta}}\cdot\frac{\partial \vec{\eta}}{\partial \beta}=1\cdot\frac{\partial c}{\partial \beta}=\boxed{\frac{\partial c}{\partial \beta}=0}$$

OUTLET CONDITION

$$\frac{\partial c}{\partial z}=0$$

$$\boxed{\frac{\partial}{\partial z}\cdot\left(\frac{\partial c}{\partial \beta}\right)=0}$$