Faculty of Applied Science

School of Mechatronics Systems Engineering

MSE 211 - Computational Methods for Engineers

Instructor : Dr. Ahad Armin

Lab 4 - Mathematical Modeling and Simulation

Prepared By :

Lab Group 3 - Monday

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Abstract

The kinematic behaviour of a pendulum can be determined numerically using the fourth-order Runge-Kutta method (RK4) to avoid having to find the analytical solution, as the pendulum's behaviour is predicted with a second-order differential equation. This model is compared to experimental data, which is used to determine a relationship between the initial angle and the maximum linear velocity of the pendulum.

Introduction

The objective of this lab is to model and simulate the oscillation of a pendulum using numerical methods. This model is compared to an analytical solution and experimental data. In this lab, an IP02 Linear Servo Base Unit by Quanser was used to hold the pendulum and report angle and angular velocity data.

Results and Discussion

Part 1

This part involves testing different initial conditions to measure the maximum linear velocity for each test case. Linear regression is then used to find the relationship between the initial angle θ and the maximum linear velocity f(θ). Doing the regression by hand requires the following formulae:

$$a_1 = \frac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2} \qquad a_0 = \bar{y} - a_1 \bar{x}$$

Using the Curve Fitting Toolbox in Matlab, the max velocity of each initial angle as a function of the initial angle was plotted. The expression found by linear regression was determined to be

$$f(\theta) = 6.013\theta + 54.06$$

This expression matches what was found by hand, and can be used to calculate the maximum angular velocity of the pendulum for any given initial angle θ. The product of this expression is then multiplied by the length of the pendulum to get the maximum linear velocity of the pendulum. The maximum calculated velocity was determined to be 148.369 m/s, and the maximum measured velocity was 153.1 m/s. The error between these two values is 3.095%.

Part 2

Equation 1 in the lab manual can be converted into a system of first-order differential equations with an appropriate substitution:

$$\frac{d\theta}{dt} = \omega, \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

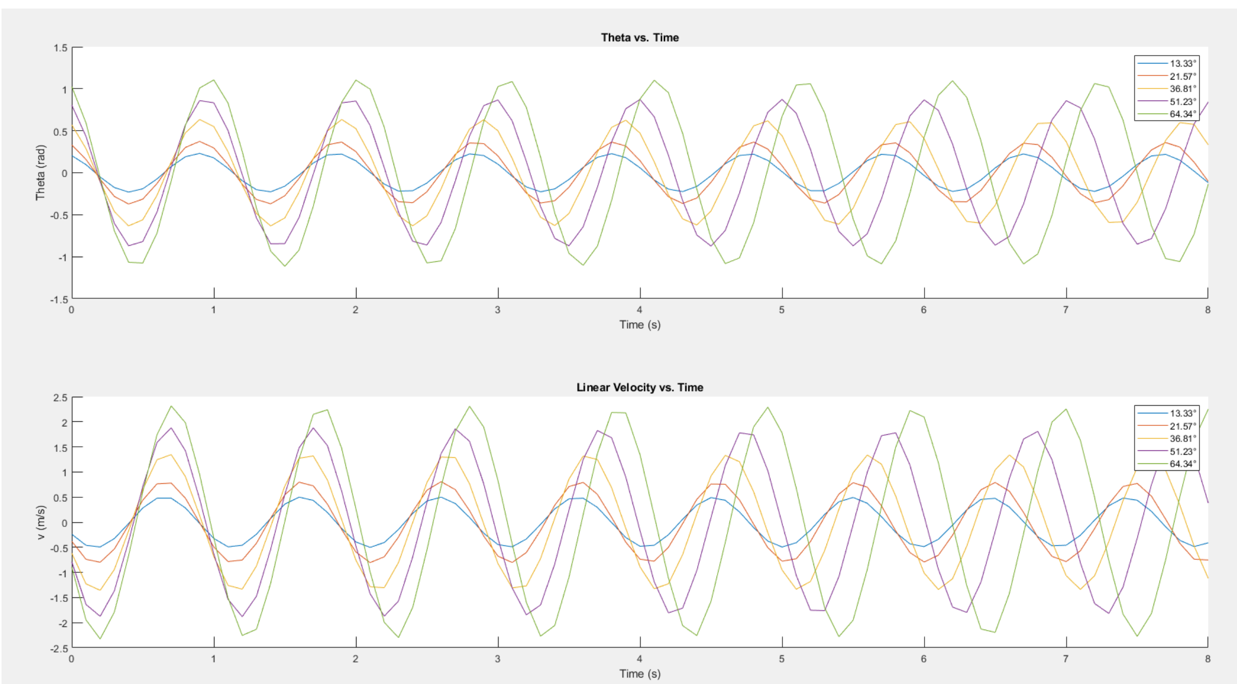
With these variables defined, the system of equations in vector form appears as follows:

$$\left[\frac{d\theta}{dt}, \frac{d\omega}{dt}\right] = \left[\omega, \quad -\frac{mgr}{(I_G + mr^2)} sin(\theta)\right]$$

Applying the values from Table 2 of the lab manual, this system becomes

$$\left[\frac{d\theta}{dt}, \frac{d\omega}{dt}\right] = \left[\omega, \quad -\frac{0.222}{5.21\times10^{-3}} sin(\theta)\right]$$

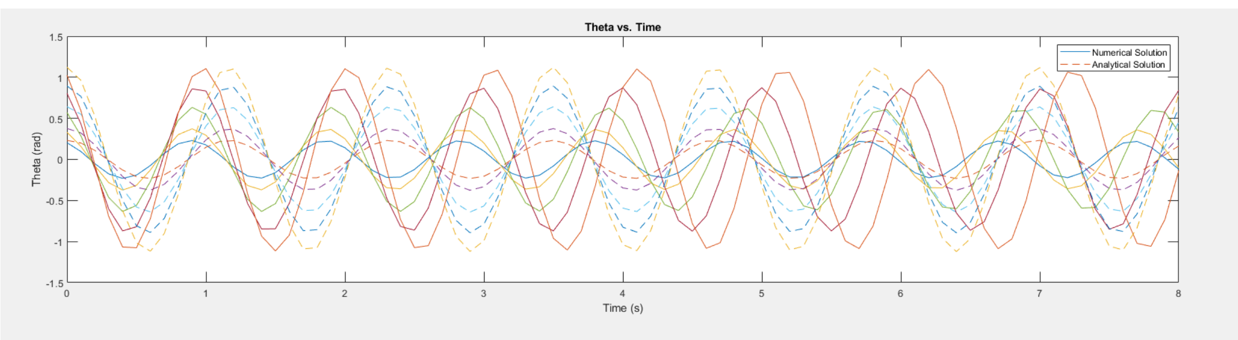
This system can be solved using the Fourth-Order Runge-Kutta method to numerically estimate the angular velocity, ω, of the pendulum, which can be used to estimate the maximum linear velocity of the pendulum. This produces the following graphs:



Trial	$\theta_{max}\ (degrees)$	$v_{max}\ (m/s)$
1	**13.19**	**9.62**
2	**21.37**	**15.55**
3	**36.33**	**25.95**
4	**50.04**	**36.26**
5	**63.32**	**44.66**

Discussion

It is interesting to note that the results from part 2 disagree with the results from part 1. Comparing the part 2 results to an analytical solution for a simple pendulum yields the insight that perhaps there is an issue with the data collection system used in part 1, or a unit conversion that was missed in the initial setup. Figure () shows a graph comparing the analytical solution to the numerical solution.



Additionally, the supplied ODE is for a simple pendulum, and it neglects the impact of friction forces, thus losing the attenuation characteristic of the part 1 data.

Conclusion

The kinematic behaviour of a simple pendulum can be determined analytically, but this strategy does not leverage computational resources effectively. Therefore, to model the pendulum, it is useful to decompose the given 2nd-order differential equation into two first order differential equations and apply a numerical method such as RK4 to numerically estimate the behaviour of the pendulum.

Appendix

Appendix 1: Full MATLAB code for Part 2 can be found here.