

Understanding Adversarial Training (AT): An Introductory yet Rigorous Guide

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Abstract

Adversarial Training (AT) is a cornerstone defense mechanism in modern machine learning that seeks to improve model robustness against worst-case, intentionally crafted perturbations—so-called adversarial examples. This tutorial-style article aims to provide beginners with an accessible yet academically grounded introduction to AT: its motivation, formal foundations, algorithmic implementations, and practical considerations.

1 Introduction

Deep neural networks achieve remarkable performance on many tasks, but are vulnerable to small, adversarially designed perturbations that cause misclassification. *Adversarial Training* (AT) directly incorporates such perturbations into the training process, teaching models to withstand these worst-case inputs. Unlike data augmentation with random noise, AT focuses on *worst-case* perturbations within a specified norm ball.

2 Adversarial Examples Recap

Given a classifier $f_\theta : \mathbb{R}^d \rightarrow \{1, \dots, K\}$ and a clean example (x, y) , an *adversarial example* is

$$x_{\text{adv}} = x + \delta \quad \text{s.t.} \quad \|\delta\|_p \leq \epsilon \quad \text{and} \quad f_\theta(x + \delta) \neq y,$$

where $\epsilon > 0$ is the perturbation budget. Common choices of p include $p = \infty$ (pixel-wise bound) and $p = 2$ (Euclidean bound).

3 Core Idea of Adversarial Training

AT solves a *minimax* optimization:

$$\min_{\theta} \mathbb{E}_{(x,y) \sim \mathcal{D}} \left[\max_{\|\delta\|_p \leq \epsilon} \ell(f_\theta(x + \delta), y) \right], \quad (1)$$

where $\ell(\hat{y}, y)$ is a loss function (e.g., cross-entropy). Intuitively:

- The inner maximization finds the worst perturbation δ (adversarial example).
- The outer minimization updates model parameters θ to reduce loss on these worst-cases.

4 Algorithmic Implementation

A practical instantiation uses Projected Gradient Descent (PGD) to approximate the inner maximization. Pseudocode:

Algorithm 1: PGD-based Adversarial Training

1. **Initialize:** $\theta \leftarrow$ random, perturbation steps T , step-size α .

2. **Repeat until convergence:**

(a) Sample minibatch $\{(x_i, y_i)\}_{i=1}^m$.

(b) **Generate adversarial examples:** for each i ,

$$x_i^{(0)} \leftarrow x_i + \xi, \quad \xi \sim \text{Uniform}(-\epsilon, \epsilon),$$

$$x_i^{(t+1)} \leftarrow \Pi_{\|\delta\|_p \leq \epsilon} \left(x_i^{(t)} + \alpha \text{sign}(\nabla_x \ell(f_\theta(x_i^{(t)}), y_i)) \right),$$

for $t = 0, \dots, T-1$.

(c) **Update model:**

$$\theta \leftarrow \theta - \eta \nabla_\theta \frac{1}{m} \sum_{i=1}^m \ell(f_\theta(x_i^{(T)}), y_i).$$

5 Practical Considerations

5.1 Computational Cost

Adversarial Training roughly multiplies training time by $(T+1)$ due to inner-maximization iterations. Typical values: $T = 7-10$.

5.2 Hyperparameters

- ϵ : Perturbation budget (e.g. $8/255$ for images in $[0, 1]$).
- T : Number of PGD steps; tradeoff between robustness and runtime.
- α : PGD step-size, often set to ϵ/T .

5.3 Loss Functions

While cross-entropy is standard, recent variants (e.g. TRADES [?]) add a margin term to better balance robustness and accuracy.

6 Benefits and Limitations

Benefits

- Provides strong empirical defenses against white-box attacks.
- Theoretical connections to certifiable robustness under certain norms.

Limitations

- High computational overhead.
- May overfit to specific attack patterns (e.g. ℓ_∞ PGD) and be vulnerable to unseen attacks.

7 Extensions and Further Reading

- **TRADES** (Zhang et al., 2019): Introduces a trade-off between accuracy and robustness via a regularization term.
- **Fast AT** (Wong et al., 2020): Uses single-step adversaries with appropriate random initialization for efficiency.
- **Certified Defenses** (Cohen et al., 2019): Offers probabilistic robustness guarantees via randomized smoothing.

8 Conclusion

Adversarial Training remains the most widely adopted method for defending deep models against worst-case perturbations. By integrating inner maximization into the learning loop, AT teaches models to recognize and correctly classify adversarial examples, trading additional computation for enhanced reliability.

References

- Madry, A., Makelov, A., Schmidt, L., Tsipras, D., & Vladu, A. (2018). *Towards deep learning models resistant to adversarial attacks*.
- Zhang, H., Yu, Y., Jiao, J., Xing, E., El Ghaoui, L., & Jordan, M. (2019). *Theoretically principled trade-off between robustness and accuracy*.
- Wong, E., & Kolter, J. Z. (2020). *Fast is better than free: Revisiting adversarial training*.