Channel Estimation and Joint Beamforming Design for Multi-IRS MIMO Systems

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Introduction

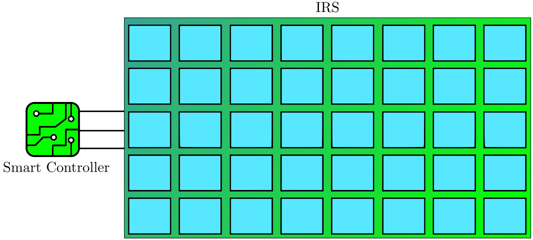
- Intelligent Reflective Surfaces (IRSs) as a promising candidate to Beyond Fifth Generation (B5G) and Sixth Generation (6G) technologies.
- Scaling law with the number of reflective elements.
- Technical difficulties: Absence of Radio Frequency (RF) chains.

Introduction

- The main contributions of this work are
  1. A multi-IRS aided MIMO system scenario and show that it reaches higher SE compared to single-IRS aided MIMO system.
  2. Two pilot-assisted protocols to obtain the channel estimation for the proposed scenario.
  3. System design recommendations for the proposed protocols.
- The scientific contributions present in this study have been partially published with the following bibliographic information
  1. **K. B. dos A. Benicio**, B. Sokal, A. L. F. de Almeida, "Channel Estimation and Joint Beamforming Design for Multi-IRS MIMO systems", in *Brazilian Symposium on Telecommunications and Signal Processing*, Sep. 2021.

Introduction
What is an IRS?

- A IRS is a 2D-panel composed of N passive reflective elements.

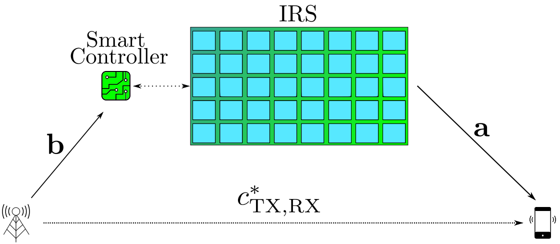


- Phase and amplitude response [1]

$$s_n = \beta_n e^{j\theta_n}, \tag{1}$$
$$\beta_n = \{0, 1\}, \forall n \in \{1, N\}. \tag{2}$$

- **Centralized**: A single IRS contains all the available reflective elements of the system.
- **Decentralized**: Multiples IRS contains only a fraction of the total number of available reflective elements.

Single-IRS aided SISO Systems



- Received signal model [1]

$$y = \left(\mathbf{a}^{\mathrm{H}}\mathrm{diag}\left(\mathbf{s}\right)\mathbf{b} + c^*_{\mathrm{TX,RX}}\right)x + v \in \mathbb{C}. \tag{3}$$

1. $\mathbf{a} \in \mathbb{C}^{N\times 1} \rightarrow$ Channel between IRS and RX,
2. $\mathbf{b} \in \mathbb{C}^{N\times 1} \rightarrow$ Channel between TX and IRS,
3. $c^*_{TX,RX} \rightarrow$ Direct link between TX and RX,
4. $x \rightarrow$ Transmitted pilot signal,
5. $v \rightarrow$ AWGN component.

Single-IRS aided SISO Systems

- IRS phase-shift matrix

$$\mathrm{diag}(\mathbf{s}) = \begin{bmatrix} \beta_1 e^{j\theta_1} & 0 & \cdots & 0 \\ 0 & \beta_2 e^{j\theta_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta_N e^{j\theta_N} \end{bmatrix} \in \mathbb{C}^{N\times N}. \quad (4)$$

- Optimum IRS phase-shift

$$\theta_n^{(\mathrm{opt})} = \zeta - (\phi_n + \psi_n), \forall n \in \{1, \cdots, N\}. \quad (5)$$

where ζ, ϕ ,and ψ are respectively the phases of $c^*_{\mathrm{TX,RX}}$, **a**, and **b**.

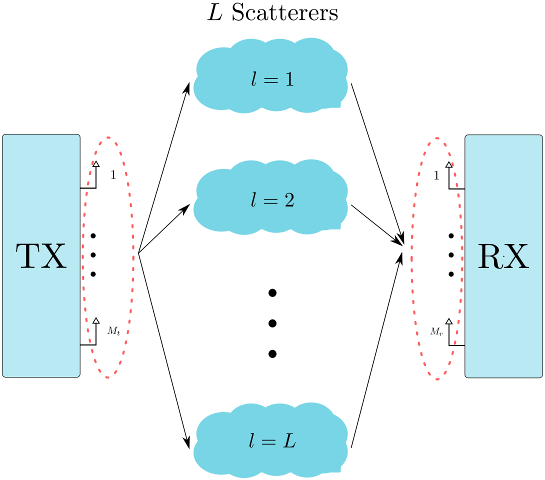
- Scaling SNR at the receiver with Equation 3

$$\gamma = \frac{\left|\left| \sum_{n=1}^{N} e^{j\theta_n^{\mathrm{opt}}} a_n^* b_n \right|\right|^2}{\sigma^2}, \quad (6)$$

$$\gamma = N^2 ||\mathbf{a}||_2 ||\mathbf{b}||_2. \quad (7)$$

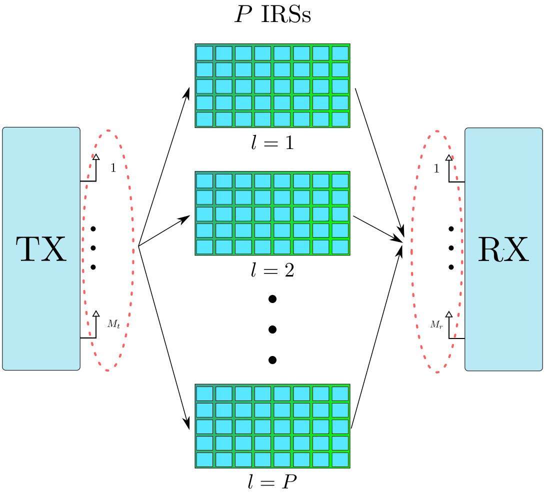
System Model

MIMO Systems with L Scatterers



$$\mathbf{y}_k = \mathbf{A}^{(\mathrm{rx})}\mathrm{diag}(\boldsymbol{\gamma})\mathbf{B}^{(\mathrm{tx})}\mathbf{x}_k \in \mathbb{C}^{M_r \times 1}. \tag{8}$$

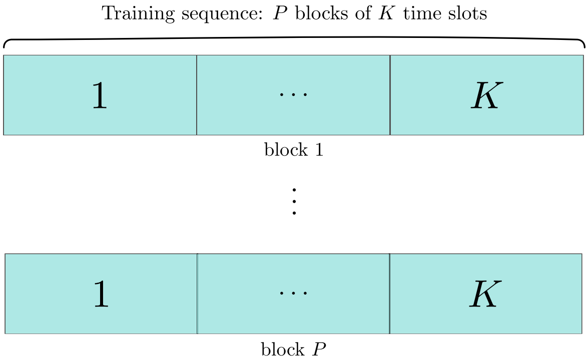
System Model
Multi-IRS aided MIMO Systems



$$\mathbf{y}_k = \mathbf{A}^{(\mathrm{rx})}\mathrm{diag}(\boldsymbol{\gamma})\mathrm{diag}(\boldsymbol{\mu})\mathbf{B}^{(\mathrm{tx})}\mathbf{x}_k \in \mathbb{C}^{M_r \times 1}. \quad (9)$$

System Model

Multi-IRS aided MIMO Systems: Training Protocol



System Model
Multi-IRS aided MIMO Systems

- Received pilot signal from the first IRS at the kth time slot

$$\mathbf{y}_k = \mathbf{A}^{(\mathrm{rx})}\mathrm{diag}(\boldsymbol{\gamma})\mathrm{diag}(\boldsymbol{\mu})\mathbf{B}^{(\mathrm{tx})}\mathbf{x}_k \in \mathbb{C}^{M_r \times 1}, \tag{10}$$

$$\mu_p = \mathbf{a}_p^{(\mathrm{irs})\mathrm{T}}\mathrm{diag}_k\left(\mathbf{S}^{(p)}\right)\mathbf{b}_p^{(\mathrm{irs})}, \forall p \in \{1, \cdots, P\}, \tag{11}$$

$$\mathbf{y}_k = \sum_{p=1}^{P} \gamma_p \mathbf{a}_p^{(\mathrm{rx})}\mathbf{a}_p^{(\mathrm{irs})\mathrm{T}}\mathrm{diag}_k\left(\mathbf{S}^{(p)}\right)\mathbf{b}_p^{(\mathrm{irs})}\mathbf{b}_p^{(\mathrm{tx})\mathrm{T}}\mathbf{x}_k + \mathbf{v}_k \in \mathbb{C}^{M_r \times 1}. \tag{12}$$

1. $\mathbf{a}_p^{(rx)}$ and $\mathbf{b}_p^{(tx)}$ $\rightarrow$ Steering vectors RX and TX, respectively.
2. $\mathbf{a}_p^{(irs)}$ and $\mathbf{b}_p^{(irs)}$ $\rightarrow$ Steering vectors of the IRS.
3. $\mathrm{diag}_k(\mathbf{S}^{(p)})$ $\rightarrow$ IRS phase-shift matrix.
4. γ_p and μ_p $\rightarrow$ Path loss component and IRS gain.
5. $\mathbf{x}_k$ $\rightarrow$ Transmitted pilot signal.
6. $\mathbf{v}_k$ $\rightarrow$ AWGN noise vector.

- IRS phase-shift matrix

$$\operatorname{diag}_k\left(\mathbf{S}^{(p)}\right) = \begin{bmatrix} \beta_{1,k}^{(p)} e^{j\theta_{1,k}^{(p)}} & 0 & \cdots & 0 \\ 0 & \beta_{2,k}^{(p)} e^{j\theta_{2,k}^{(p)}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta_{N,k}^{(p)} e^{j\theta_{N,k}^{(p)}} \end{bmatrix} \in \mathbb{C}^{N\times N}. \quad (13)$$

System Model

Multi-IRS aided MIMO Systems: Perfect Absorbers *vs.* Regular Scatterers

- Perfect Absorbers (**PA**): Metamaterials capable of perfectly absorbing impinging electromagnetic waves [2]. The IRS will not contribute to the channel interference and the reflective coefficient is $\beta_n = 0, \forall n \in \{1, N\}$.

- Regular Scatterers (**RS**): The practical IRS is not capable of perfectly absorber impinging electromagnetic waves. The IRS will contribute to the channel interference and the reflective coefficient is $\beta_n = 1, \forall n \in \{1, N\}$.

System Model
Multi-IRS aided MIMO Systems: Perfect Absorbers

- Received pilot signal at the kth time slot for the first block

$$\mathbf{y}_k^{(1)} = \gamma_1 \mathbf{a}_1^{(\mathrm{rx})} \mathbf{a}_1^{(\mathrm{irs})\mathrm{T}} \mathrm{diag}_k \left(\mathbf{S}^{(1)} \right) \mathbf{b}_1^{(\mathrm{irs})} \mathbf{b}_1^{(\mathrm{tx})\mathrm{T}} \mathbf{x}_k$$

$$+ \underbrace{\cancelto{0}{\sum_{p=2}^{P} \gamma_p \mathbf{a}_p^{(\mathrm{rx})} \mathbf{a}_p^{(\mathrm{irs})\mathrm{T}} \mathrm{diag}_k \left(\mathbf{S}^{(p)} \right) \mathbf{b}_p^{(\mathrm{irs})} \mathbf{b}_p^{(\mathrm{tx})\mathrm{T}} \mathbf{x}_k}}_{\text{interference}} \in \mathbb{C}^{M_r \times 1}. \tag{14}$$

- Rewriting Equation 14

$$\mathbf{y}_k^{(1)} = \left(\mathbf{s}_k^{(1)\mathrm{T}} \otimes \mathbf{x}_k^{\mathrm{T}} \otimes \mathbf{I}_{M_r} \right) \mathrm{vec} \left(\gamma_1 (\mathbf{b}_1^{(\mathrm{tx})} \otimes \mathbf{a}_1^{(\mathrm{rx})}) (\mathbf{b}_1^{(\mathrm{irs})\mathrm{T}} \diamond \mathbf{a}_1^{(\mathrm{irs})\mathrm{T}}) \right) \in \mathbb{C}^{M_r \times 1}. \tag{15}$$

- Stacking the K pilot signals

$$\bar{\mathbf{y}}_1 = \underbrace{\left[\left(\mathbf{S}^{(1)} \diamond \mathbf{X} \right) \otimes \mathbf{I}_{M_r} \right]}_{\mathbf{C}_{PA}} \underbrace{\mathrm{vec} \left((\mathbf{b}_1^{(\mathrm{tx})} \otimes \mathbf{a}_1^{(\mathrm{rx})}) (\mathbf{b}_1^{(\mathrm{irs})\mathrm{T}} \diamond \mathbf{a}_1^{(\mathrm{irs})\mathrm{T}}) \right)}_{\mathbf{z}^{(1)}} \in \mathbb{C}^{M_r K \times 1}. \tag{16}$$

System Model

Multi-IRS aided MIMO Systems: Perfect Absorbers

- Adding the noise vector

$$\bar{\mathbf{y}}_1 = \mathbf{C}_{\text{PA}}\mathbf{z}^{(1)} + \mathbf{v}^{(1)} \in \mathbb{C}^{M_r K \times 1}. \tag{17}$$

- Least square estimate problem from Equation 17

$$\hat{\mathbf{z}}^{(1)} \approx \mathbf{C}_{\text{PA}}^{\text{H}}\bar{\mathbf{y}}_1 \in \mathbb{C}^{M_r K \times 1},\ K \geq M_t N. \tag{18}$$

- Channel and path loss estimation

$$\hat{\mathbf{Z}}^{(1)} \approx \gamma_1(\mathbf{b}_1^{(\text{tx})} \otimes \mathbf{a}_1^{(\text{rx})})(\mathbf{b}_1^{(\text{irs})\text{T}} \diamond \mathbf{a}_1^{(\text{irs})\text{T}}) \in \mathbb{C}^{M_r M_t \times N}, \tag{19}$$

$$\hat{\mathbf{Z}}^{(1)} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\text{H}} \approx \gamma_1 \mathbf{f}_1 \mathbf{r}_1^{\text{T}} \in \mathbb{C}^{M_r M_t \times N}, \tag{20}$$

$$\hat{\mathbf{f}_1} \approx \mathbf{u}_1^{(1)} \in \mathbb{C}^{M_r M_t \times 1}, \tag{21}$$

$$\hat{\mathbf{r}_1} \approx \mathbf{v}_1^{(1)} \in \mathbb{C}^{1 \times N}. \tag{22}$$

- Passive beamforming

$$\mathbf{s}_{\text{opt}}^{(1)} = e^{-j\angle\hat{\mathbf{r}}_1} \in \mathbb{C}^{N \times 1} \tag{23}$$

System Model

Multi-IRS aided MIMO Systems: Regular Scatterers

- Received pilot signal at the kth time slot for the first block

$$\mathbf{y}_k^{(1)} = \gamma_1 \mathbf{a}_1^{(\mathrm{rx})} \mathbf{a}_1^{(\mathrm{irs})\mathrm{T}} \mathrm{diag}_k \left(\mathbf{S}^{(1)} \right) \mathbf{b}_1^{(\mathrm{irs})} \mathbf{b}_1^{(\mathrm{tx})\mathrm{T}} \mathbf{x}$$

$$+ \underbrace{\sum_{p=2}^{P} \gamma_p \mathbf{a}_p^{(\mathrm{rx})} \mathbf{a}_p^{(\mathrm{irs})\mathrm{T}} \mathrm{diag}_k \left(\mathbf{S}^{(p)} \right) \mathbf{b}_p^{(\mathrm{irs})} \mathbf{b}_p^{(\mathrm{tx})\mathrm{T}}}_{\text{interference}} \mathbf{x} \in \mathbb{C}^{M_r \times 1}. \tag{24}$$

- Rewriting Equation 24

$$\mathbf{y}_k^{(1)} = \left(\mathbf{x}^{\mathrm{T}} \otimes \mathbf{I}_{M_r} \right) \left(\gamma_1 (\mathbf{b}_1^{(\mathrm{irs})} \mathbf{b}_1^{(\mathrm{tx})\mathrm{T}})^{\mathrm{T}} \diamond (\mathbf{a}_1^{(\mathrm{rx})} \mathbf{a}_1^{(\mathrm{irs})\mathrm{T}}) \mathbf{s}_k^{(1)} + \mathbf{h}_1 \in \mathbb{C}^{M_r \times 1}.$$

- Eliminating the interference by subtracting two consecutive vectors

$$\mathbf{y}_k^{(1)} - \mathbf{y}_{k-1}^{(1)} = \left(\mathbf{x}^{\mathrm{T}} \otimes \mathbf{I}_{M_r} \right) \left(\gamma_1 (\mathbf{b}_1^{(\mathrm{irs})} \mathbf{b}_1^{(\mathrm{tx})\mathrm{T}})^{\mathrm{T}} \diamond (\mathbf{a}_1^{(\mathrm{rx})} \mathbf{a}_1^{(\mathrm{irs})\mathrm{T}}) \right) \underbrace{\left(\mathbf{s}_{k+1}^{(1)} - \mathbf{s}_k^{(1)} \right)}_{\mathbf{w}_k^{(1)}}$$

$$+ \cancelto{0}{\left(\mathbf{x}^{\mathrm{T}} \otimes \mathbf{I}_{M_r} \right) (\mathbf{h}_1 - \mathbf{h}_1)}, \in \mathbb{C}^{M_r \times 1}. \tag{25}$$

System Model

Multi-IRS aided MIMO Systems: Regular Scatterers

- Stacking the $K-1$ pilot signals

$$\bar{\mathbf{y}}_1 = \underbrace{\left[\mathbf{W}^{(1)} \otimes \left(\mathbf{X}^{\mathrm{T}} \otimes \mathbf{I}_{M_r}\right)\right]}_{\mathbf{C}_{RS}} \underbrace{\mathrm{vec}\left(\left(\mathbf{b}_1^{(\mathrm{tx})} \otimes \mathbf{a}_1^{(\mathrm{rx})}\right)\left(\mathbf{b}_1^{(\mathrm{irs})\mathrm{T}} \diamond \mathbf{a}_1^{(\mathrm{irs})\mathrm{T}}\right)\right)}_{\mathbf{z}^{(1)}} \in \mathbb{C}^{M_r(K-1)\times 1}.$$

- Combined phase-shift matrix

$$\mathbf{W}^{(1)} = \begin{bmatrix} | & | & & | \\ \mathbf{s}_2^{(1)} - \mathbf{s}_1^{(1)} & \mathbf{s}_3^{(1)} - \mathbf{s}_2^{(1)} & \cdots & \mathbf{s}_K^{(1)} - \mathbf{s}_{K-1}^{(1)} \\ | & | & & | \end{bmatrix} \in \mathbb{C}^{N\times(K-1)}. \quad (26)$$

- Adding the noise vector

$$\bar{\mathbf{y}}_1 = \mathbf{C}_{\mathrm{RS}}\mathbf{z}^{(1)} + \mathbf{v}^{(1)} \in \mathbb{C}^{M_r(K-1)\times 1}. \quad (27)$$

- Least square estimate problem from Equation 27

$$\hat{\mathbf{z}}^{(1)} \approx \mathbf{C}_{\mathrm{RS}}^{+}\bar{\mathbf{y}}_1 \in \mathbb{C}^{M_r(K-1)\times 1},\ K \geq M_t N + 1. \quad (28)$$

Algorithm 1 Channel Estimation and Beamforming

for p = $1{:}P$ **do**

Estimate $\mathbf{z}^{(p)}$ as

$$\hat{\mathbf{z}}^{(p)} = \mathbf{C}^{+}\overline{\mathbf{y}}_p \in \mathbb{C}^{M_r M_t N \times 1},$$

Define $\hat{\mathbf{Z}}_p = \mathrm{unvec}_{M_r M_t \times N}\left(\hat{\mathbf{z}}^{(p)}\right)$ and compute the SVD as $\hat{\mathbf{Z}}_p = \mathbf{U}^{(p)}\mathbf{\Sigma}^{(p)}\mathbf{V}^{(p)\mathrm{H}}$ to obtain the estimates

$$\hat{\mathbf{f}}_p = \mathbf{u}_1^{(p)} \in \mathbb{C}^{M_r M_t \times 1}, \hat{\mathbf{r}}_p \quad = \mathbf{v}_1^{(p)*} \in \mathbb{C}^{N \times 1},$$

Set the optimum phase-shifts for the pth IRS as

$$\mathbf{s}_{\mathrm{opt}}^{(p)} = e^{-j\angle \hat{\mathbf{r}}_p} \in \mathbb{C}^{N \times 1},$$

Define $\hat{\mathbf{F}}_p = \mathrm{unvec}_{M_r \times M_t}\left(\hat{\mathbf{f}}_p\right)$ and compute its SVD as $\hat{\mathbf{F}}_p = \mathbf{U}^{\mathrm{f}(p)}\mathbf{\Sigma}^{\mathrm{f}(p)}\mathbf{V}^{\mathrm{f}(p)\mathrm{H}}$. Calculate

$$\hat{\mathbf{a}}_p^{(\mathrm{rx})} = \mathbf{u}_1^{\mathrm{f}(p)} \in \mathbb{C}^{M_r \times 1}, \hat{\mathbf{b}}_p^{(\mathrm{tx})} = \mathbf{v}_1^{\mathrm{f}(p)*} \in \mathbb{C}^{M_t \times 1}, \hat{\gamma}_p = \mathbb{E}\{\mathbf{z}^{\hat{(p)}} \oslash \mathrm{vec}(\hat{\mathbf{f}}_p \hat{\mathbf{r}}_p^{\mathrm{T}})\}.$$

end

return $\hat{\mathbf{A}}^{(\mathrm{rx})} = \left[\hat{\mathbf{a}}_1^{(\mathrm{rx})}, \ldots, \hat{\mathbf{a}}_P^{(\mathrm{rx})}\right], \hat{\mathbf{B}}^{(\mathrm{tx})} = \left[\hat{\mathbf{b}}_1^{(\mathrm{tx})}, \ldots, \hat{\mathbf{b}}_P^{(\mathrm{tx})}\right],$
$\hat{\boldsymbol{\gamma}} = [\hat{\gamma}_1, \cdots, \hat{\gamma}_P]$ and $\mathbf{s}_{\mathrm{opt}}$.

Define $\hat{\mathbf{H}} = \hat{\mathbf{A}}^{(\mathrm{rx})}\mathrm{diag}(\hat{\boldsymbol{\gamma}})\hat{\mathbf{B}}^{(\mathrm{tx})\mathrm{T}} \in \mathbb{C}^{M_r \times M_t}$ and compute its SVD as $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{H}}$. Calculate active beamforming as

$$\mathbf{W} = \mathbf{U} \in \mathbb{C}^{M_r \times P}, \mathbf{Q} = \mathbf{V} \in \mathbb{C}^{M_t \times P}.$$

Results

Simulation Scenario

- $\mathbb{E}\{\mathbf{x}^{(\mathrm{d})}\mathbf{x}^{(\mathrm{d})H}\} = 1$
- $\gamma_p \sim \mathcal{CN}(0, 1)$
- AoA, AoD $\sim \mathcal{U}(-\pi, \pi)$
- IRS's Angles $\sim \mathcal{U}(-\pi/2, \pi/2)$
- SNR $= 1/\sigma^2_{\text{noise}}$, $\text{SNR}_{training} = 25$ dB

The spectral efficiency is calculated as

$$\text{SE (bps/Hz)} = \log_2 \left[\det \left(\mathbf{I}_P + \frac{\mathbf{H}_{\text{eq}}\mathbf{R}_{xx}\mathbf{H}^{\text{H}}_{\text{eq}}}{\sigma^2_{\text{noise}}} \right) \right], \tag{29}$$

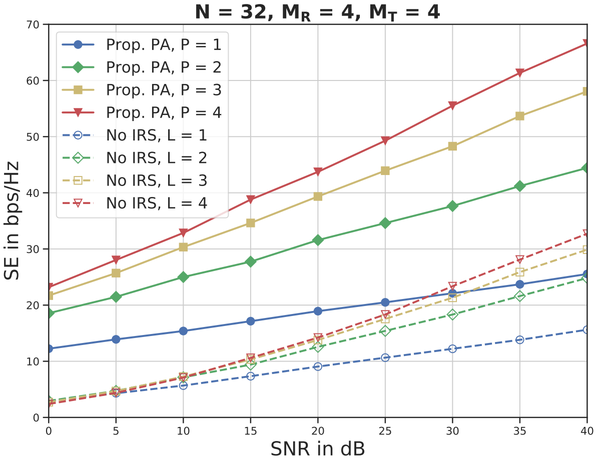
where

$$\mathbf{H}_{\text{eq}} = \mathbf{W}^{\text{H}}\mathbf{A}^{\text{rx}}\text{diag}(\lambda)\text{diag}(\mu)\mathbf{B}^{(\text{tx})\text{T}}\mathbf{Q}, \tag{30}$$

$$\mathbf{R}_{xx} = \mathbb{E}\{\mathbf{x}^{(\mathrm{d})}\mathbf{x}^{(\mathrm{d})H}\}, \tag{31}$$

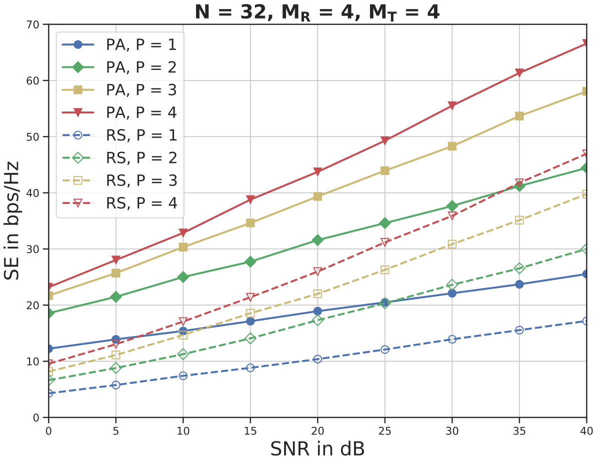
$$\text{trace}\left(\mathbf{Q}\mathbf{R}_{xx}\mathbf{Q}^{\text{H}}\right) = 1. \tag{32}$$

Results
Multi-IRS aided MIMO System: Number of IRSs

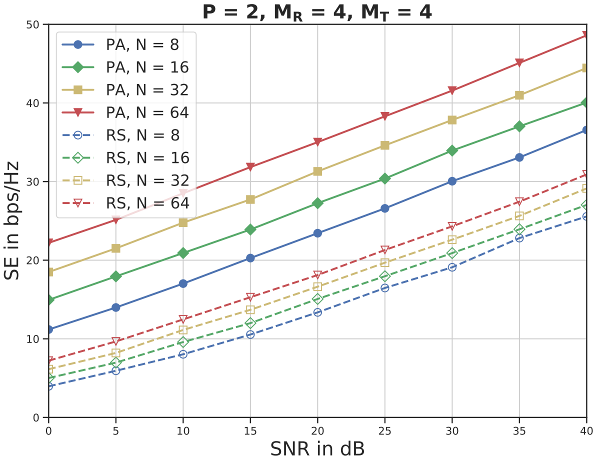


Results

Multi-IRS aided MIMO System: Number of IRSs

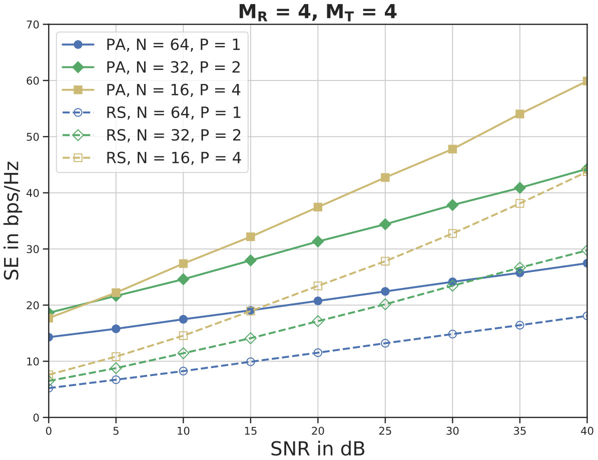


Results
Multi-IRS aided MIMO System: Number of Reflective Elements



Results

Multi-IRS aided MIMO System: Constant Rate ($N \cdot P$)



Conclusions

- Investigation of a way to optimize the proposed transmission protocol to reduce the training overhead.
- Validation of the proposed scenario using current technologies, e.g., relaying protocols, as benchmark for our simulations.
- Investigation of performance degradation in a multi-IRS deployment scenario if we consider practical constraints to the IRSs.
- Semi-unitary design for $\mathbf{C}_{RS}$ of the RS scenario.

References

[1] Q. Wu, S. Zhang, B. Zheng, C. You, and R. Zhang, "Intelligent reflecting surface aided wireless communications: A tutorial," *IEEE Transactions on Communications*, 2021.

[2] T. Badloe, J. Mun, and J. Rho, "Metasurfaces-based absorption and reflection control: perfect absorbers and reflectors," *Journal of Nanomaterials*, vol. 2017, 2017.

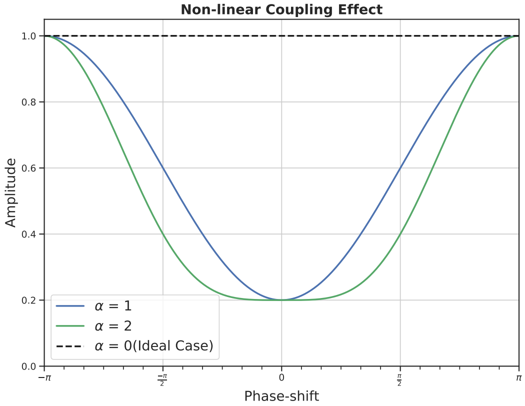
[3] S. Abeywickrama, R. Zhang, Q. Wu, and C. Yuen, "Intelligent reflecting surface: Practical phase shift model and beamforming optimization," *IEEE Transactions on Communications*, vol. 68, no. 9, pp. 5849–5863, 2020.

Thank you for your presence!

Appendix
Coupling Effect

- Coupling between phase-shift and amplitude [3]

$$\beta_n(\theta_n) = (1 - \beta_{\min})\Big(\frac{\sin(\theta_n - \phi) + 1}{2}\Big)^{\alpha} + \beta_{\min}. \tag{33}$$



Appendix
Quantization Error

- Phase-shift quantization

$$K_\theta = 2^{\text{Bits}}, \tag{34}$$

$$\Delta\theta = 2\pi / K_\theta. \tag{35}$$