Note of Fast Runner

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1 System dynamics

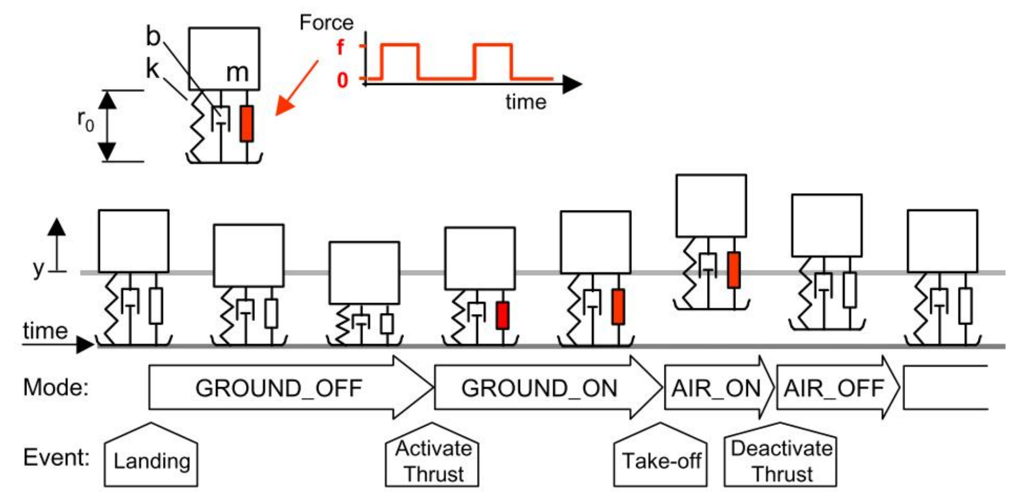

Figure 3-1. The vertical hopping model used for analysis. The hopper's leg consists of a spring, a damper and a force element which is active according to a binary motor pattern. The figure shows a sample trajectory of the hopper, the different modes that it goes through, and the events that trigger the transitions between the modes.

Figure 1: The schematic of a 1 DOF hopper [11]

1.1 Sequence

{AIR_OFF, GROUND_OFF, GROUND_ON, AIR_ON}

1.2 Equation of motion

Using the model as shown in Fig. 1, during the stand phase (i.e. $y \leq 0$), the equation of motion can be expressed as:

$$m\ddot{y} = -b\dot{y} + -ky - mg + f$$

where m is the mass, b is the damping, k is the stiffness, f is the control input. Normalized by weight, the equation becomes

$$\ddot{y} = -b/m\dot{y} + -k/my - g + f/m$$

Expressed in state space form:

$$\begin{bmatrix}\dot{y}\\ \ddot{y}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -k/m & -b/m\end{bmatrix}\begin{bmatrix}y\\ \dot{y}\end{bmatrix} + \begin{bmatrix}0\\ -g+f/m\end{bmatrix} \tag{1}$$

or equivalently

$$\dot{X} = \begin{bmatrix}0 & 1\\ -\omega^2 & -2\xi\omega\end{bmatrix} X + \begin{bmatrix}0\\ -g+f_n(t)\end{bmatrix} = AX + B \tag{2}$$

where $X \triangleq [y, \dot{y}]^T$. When the hopper is in the air (i.e. $y > 0$, flight phase),

$$\dot{X} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} X + \begin{bmatrix}0\\ -g\end{bmatrix} \tag{3}$$

Define the force of an open-loop motor pattern

$$f_n(t) = \begin{cases} f/m, & \text{if } t_{off} < t < t_{off} + t_{on}. \\ 0, & \text{otherwise.}\end{cases} \tag{4}$$

Solutions

For (3):

$$X(t) = \begin{bmatrix}1 & t\\ 0 & 0\end{bmatrix} X_0 + \begin{bmatrix}t^2/2\\ t\end{bmatrix}(-g) \tag{5}$$

For (2) when actuator is on:

$$X(t) = e^{At}(X_0 - X_{eq_{on}}) + X_{eq_{on}} \tag{6}$$

For (2) when actuator is off:

$$X(t) = e^{At}(X_0 - X_{eq_{off}}) + X_{eq_{off}} \tag{7}$$

where $X_{eq_{on}}$ and $X_{eq_{off}}$ are the equilibrium states:

$$X_{eq_{on}} = [\frac{f_n - g}{\omega^2}, 0]^T \tag{8}$$

$$X_{eq_{off}} = [\frac{-g}{\omega^2}, 0]^T \tag{9}$$

1.3 Stability Analysis

1.3.1 Eigen values

For (3), eigen values are ± 1, in inherently unstable. (Why this does not matter? Because the contact)
For (2), eigen values are $-\xi\omega \pm \omega\sqrt{(\xi^2 - 1)} = -\omega(\xi \pm \sqrt{\xi^2 - 1}) = -\omega(\xi \pm i\sqrt{1 - \xi^2})$ As long as $\omega$and $\xi$are larger than zero, the system is stable.

1.3.2 Poincare Method

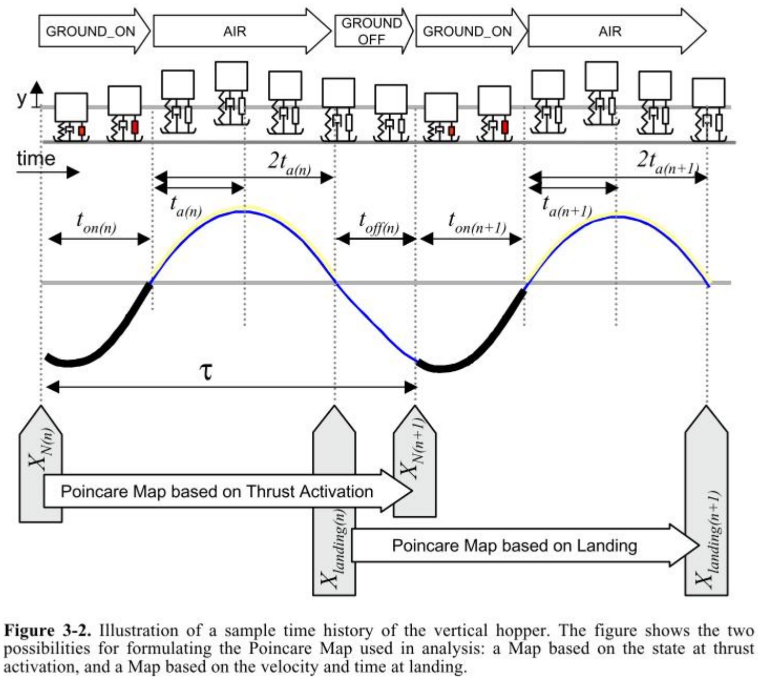

Figure 2: The modes of the hopper [11]

Assumptions:

- the period is T
- two modes need to be checked
- $X(0) = X_{N_n}$ where n indicates the n^{th} trajectory

Using Equations 6, we can derive

$$X(t_{on_n}) = e^{At_{on_n}}(X_{N_n} - X_{eq_{on}}) + X_{eq_{on}}$$

Use the fact that

$$X(t_{on_n} + 2t_{a_n}) = -X(t_{on_n})$$

Then we can calculate the $X_{N_{n+1}}$ as follows:

$$\begin{aligned}
X_{N_{n+1}} &= e^{A(T-2t_{a_n}-t_{on_n})}(-X(t_{on_n}) - X_{eq_{off}}) + X_{eq_{off}} \\
X_{N_{n+1}} &= e^{A(T-2t_{a_n}-t_{on_n})}(-e^{At_{on_n}}(X_{N_n} - X_{eq_{on}}) - X_{eq_{on}}) - X_{eq_{off}}) + X_{eq_{off}} \\
&= X_{eq_{off}} - e^{A(T-2t_{a_n})}(X_{N_n} - X_{e_{on}}) - e^{A(T-2t_{a_n}-t_{on_n})}(X_{eq_{on}} + X_{eq_{off}})
\end{aligned} \tag{10}$$

About the second switch surface $X_{landing_n}$,

$$X_{landing_n} = -X(t_{on_n}) = -e^{At_{on_n}}(X_{N_n} - X_{eq_{on}}) - X_{eq_{on}} \tag{11}$$

2 Code implementation

2.1 Modeling and Parameters

Main idea: a virtual wheel (as the massless leg) with radius r_{wheel} penetrate the ground for a distance r_{pen} where a external force point pe is attached on it. A body (with mass m and inertia Iyy) is attached to the center of wheel. Using PD control to interpret contact force when p_e is under the ground.

06/07 First prototype (Not used now)

- Joint numbers: 2
- Joint types: Floating planer joint for virtual wheel and pin joint for the body link.
- Contact point type: External force point
- Virtual wheel rotation: set proper initial condition for virtual wheel (also need a large inertia to make it nearly constant).

Contact force: Assuming the ground height is 0,

$$F_z = kp(0 - pe_z) + kd(0 - ve_z) \tag{12}$$

$$\phi = atan2(pe_x, r_{wheel} - pe_z) \tag{13}$$

$$F_x = F_z tan(\phi) \tag{14}$$

where ve is the velocity vector of the contact point pe, kp and kd are the PD control parameters. F_x is calculated so that the vector of ground reaction force $[F_x, F_y, F_z]^T$ will point towards the virtual pivot (the center of the virtual wheel).

Assessments:

- Need to set a non-zero inertia of massless virtual wheel (for numerical stability), otherwise the simulation will diverge.
- The inertia of virtual wheel need to be a large one for constant rotational speed.
- Suggestions: remove the massless link, attach the external force point to the body and change its position in the controller every time step.

06/08 Round Runner

- Joint numbers: 1
- Joint types: Floating planer joint for the body link.
- Contact point type: External force point
- Virtual wheel rotation: Assigning the external force point location with respect to the joint in an open loop manner.
- Contact force: Assuming the ground height is 0,

$$F_z = kp(0 - pe_z) + kd(0 - ve_z) \tag{15}$$

$$\phi = atan2(pe_x, r_{wheel} - pe_z) \tag{16}$$

$$F_x = F_z tan(\phi) \tag{17}$$

where ve is the velocity vector of the contact point pe, kp and kd are the PD control parameters. F_x is calculated so that the vector of ground reaction force $[F_x, F_y, F_z]^T$ will point towards the virtual pivot (the center of the virtual wheel).

Assessments:

- The ground reaction force looks better, while the energy is not balanced (after a while it will move towards the negative x direction)
- The inertia of virtual wheel need to be a large one for constant rotational speed.
- Suggestions: Use the ground contact point (instead of external force point) to see how it goes.

06/11 Round Runner(with Ground Contact Point)

- Joint numbers: 1
- Joint types: Floating planer joint for the body link.
- Contact point type: Ground contact point, linear contact model[1]
- Virtual wheel rotation: Assigning the external force point location with respect to the joint in an open loop manner.
- **Contact point number** Parameterized, currently set to 3-6 points.
- Contact force: using built-in functionalities, only assigning the kp, kd (PD parameters in the z direction), kp_x, and kd_x (PD parameters in the x/y directions).

Assessments:

- Was able to generate a stable walking. Contact point has sliding.
- Due to setting up stiffness and damping for x and z separately, the force is not always point towards the virtual pivot.

3 Info mightbe useful

3.1 Going through references

1. Compare different terrestiral locomotions: Some parameters of the walk are not speed- dependent. The swing duration is a constant time parameter [1].
2. Trunk plays an important role during walking (birds) [2].
3. The use of these drives (Resonance drives, with adaptive control) allows increasing machine's quickness several times and decreasing energy expenses simultaneously 10-50 times [3].
4. Light weight leg (ostrich vs. moa) can run faster[5]. Also a famous allometric equation:

   $$Y = M^{3/4} \tag{18}$$

   where M is the body mass, Y is the metabolic rate.
5. Human's walking may not be really self-optimized: the preferred speed maybe different from the energetically optimal speed[8].
6. It is concluded that the most important adjustment to the bodys spring system to accommodate higher stride frequencies is that leg spring becomes stiffer [19].
7. magic equations for imd force (ostrich) [26]
8. gait frequency was reported to be highly correlated with the resonant frequency of the mass-spring model [30]
9. WABIAN, why you are here? [31]

[1]Disable the hardening stiffness in z direction by setting groundStiffeningLength to Double.NEGATIVE_INFINITY

3.2 Categories

1. Nonlinear oscillators/components [3, 6, 9, 10, 12, 28, 39];
2. zoology, biomechanics of animals: [1, 2, 4, 5, 16]
3. Bio-inspired robots: [7, 32]
4. Reference I should read: [11, 15, 27, 28]
5. Article not found (or not free)[4].
6. Robots in 3D: [13]
7. Stability analysis (Monocycle, linearized system) [14] (Limit cycle) [11, 27] dimensionless [41]
8. Biology/Anatomical structure [17, 20]
9. Light weight fast robot [18, 25]
10. take a look again [21]
11. mechanism design of robot [22]
12. quadruped reference [23] MIT Cheetah[37]
13. human energy cost, resonance usage [24, 8, 38, 40]
14. walking parameterization [29, 21, 42]
15. human-animal differences [15]
16. open-loop robot [33], passive robot [35, 34, 36]

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