

# Note of Fast Runner

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## 1 About systems and methods

### 1.1 Requirements - system

- List of assumptions
- Capture the required parameters (i.e. how to normalize the systems)
  - Resonance
  - Nonlinear elastic components
    - \* a set of linear components for multiple modes?
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### 1.2 Requirements - method

- Applicable to complex system (e.g. for the designed mechanism)
- Nondimensionlization (so that it can be used for robots with different scales)
- Stability analysis
- Robustness

### 1.3 Remarks

- Impact does not cause velocity change on runner with massless leg!
- In SCS, to simulate massless leg, it is better to use only one body, and manipulate the relation between the contact point and the body in controller instead.

### 1.4 ToDo

- Rearrange/updating references for fastRunner
- Check if the foot is sliding
- Check optimization tools ihmcc have
  - parameter optimization tool using Gradient Decent or GA
- Ask Cris about the parameter range/selection

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\*Last update: June 22, 2018

## 1.5 Questions

### Direction

- Should I exclude the gyroscopic-based stabilization?
- Eigen values of linearized system, Poincare map analysis, anything else I should study for the stability analysis?
- The linkage between the control in simulation and mechanism design
  - Parameters
  - How to design a mechanism can emulate PD control?

### General Utilities

- Any solver for nonlinear program IHMC used?
- Any trajectory optimization package IHMC used?
- Methods to get stable Reciprocating Spoked Runner?

### Past simulations

- Why the abstract runner (in spoked runner project) can be stabilized in x direction?

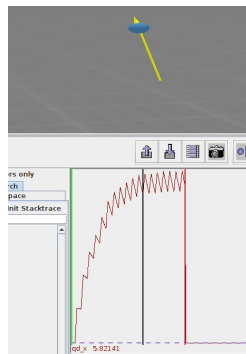
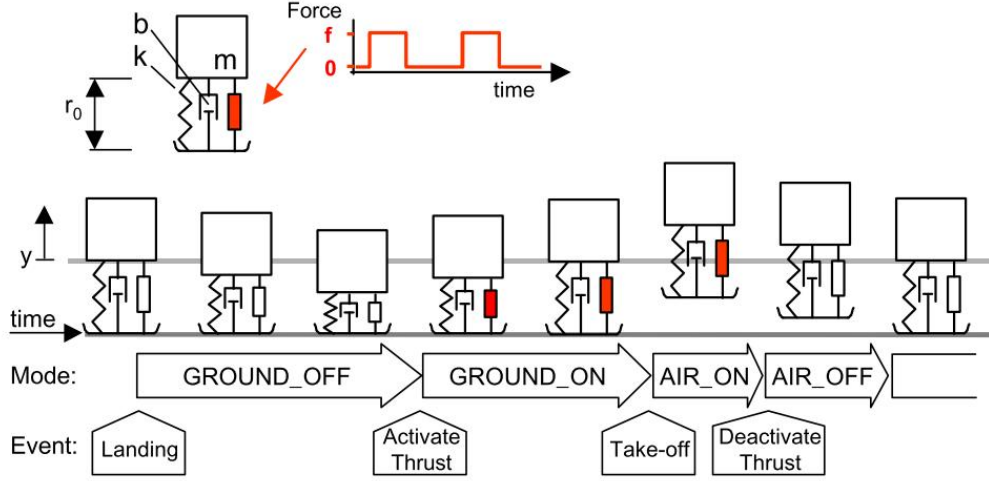


Figure 1: The Abstract Runner

- The simulation setup is really robust for a large set of initial conditions/throttle angles
- It turns out its because the added wind resistance dissipate a lot of energies.
- Methods to get stable Reciprocating Spoked Runner?
- What is the line private static final long serialVersionUID for?

## 2 Pitch Stability of an Vertically Open-loop Hopper

### 2.1 Jorge Cham's Dissertation - openloop control of 1DOF vertical hopper



**Figure 3-1.** The vertical hopping model used for analysis. The hopper's leg consists of a spring, a damper and a force element which is active according to a binary motor pattern. The figure shows a sample trajectory of the hopper, the different modes that it goes through, and the events that trigger the transitions between the modes.

Figure 2: The schematic of a 1 DOF hopper [11]

#### 2.1.1 Equation of motion

Using the model as shown in Fig. 2, during the stand phase (i.e.  $y \leq 0$ ), the equation of motion can be expressed as:

$$m\ddot{y} = -b\dot{y} - ky - mg + f$$

where  $m$  is the mass,  $b$  is the damping,  $k$  is the stiffness,  $f$  is the control input. Normalized by weight, the equation becomes

$$\ddot{y} = -b/m\dot{y} - k/my - g + f/m$$

Expressed in state space form:

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ -g + f/m \end{bmatrix} \quad (1)$$

or equivalently

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\xi\omega \end{bmatrix} X + \begin{bmatrix} 0 \\ -g + f_n(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_p & -k_d \end{bmatrix} X + \begin{bmatrix} 0 \\ -g + f_n(t) \end{bmatrix} \quad (2)$$

where  $X \triangleq [y, \dot{y}]^T$ . When the hopper is in the air (i.e.  $y > 0$ , flight phase),

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ -g \end{bmatrix} \quad (3)$$

Define the force of an open-loop motor pattern

$$f_n(t) = \begin{cases} f/m, & \text{if } t_{off} < t < t_{off} + t_{on}. \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

## 2.2 System Dynamics

Use the state space of  $z$  motion form 2 with a simplified open-loop force input:

$$\begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -kp_z & -kd_z \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ -g + f_n(t) \end{bmatrix} \quad (5)$$

where

$$f_n(t) = \begin{cases} f_n \triangleq f/m, & \text{if } t_{flight} < t < t_{flight} + t_{contact}. \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

To further simplify the problem, assuming  $f_n(t)$  is much more dominant than  $-kp_z z - kd_z \dot{z} - g$  so that:

$$\begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} \approx \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ f_n(t) \end{bmatrix} \quad (7)$$

Assumptions:

- $f_n(t)$ <sup>1</sup> can induce stable vertical hopping motion.
- $t_0$  starts when the foot leaves the ground.
- $t_{flight} + t_{contact} = T$ ,  $t_{contact} = \alpha$ , and  $T > \alpha$

Then the pitch dynamics with feedback control can be expressed as:

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ -f_n(t)m/I\Delta x \end{bmatrix} \quad (8)$$

### 2.2.1 Poincare Section

Denote the state at the  $n^{th}$  step Poincare section  $\theta_n, \dot{\theta}_n$  (defined at the start of the flight phase). Then we can calculate the state at Poincare section at the  $n+1^{th}$  step:

$$\dot{\theta}_{n+1} = \dot{\theta}_n - \frac{f}{I} \Delta x t_{contact} \quad (9)$$

$$\theta_{n_{touchDown}} = \theta_n + \dot{\theta}_n t_{flight}$$

$$\dot{\theta}_{n_{touchDown}} = \dot{\theta}_n$$

$$\begin{aligned} \theta_{n+1} &= \theta_n + \dot{\theta}_n t_{flight} + \dot{\theta}_n t_{contact} - \frac{1}{2} \frac{f}{I} \Delta x t_{contact}^2 \\ &= \theta_n + T \dot{\theta}_n - \frac{1}{2} \frac{f}{I} \alpha^2 \Delta x \end{aligned} \quad (10)$$

### 2.2.2 Poincare Map of Pitch Dynamics with Proportional Control

By designing a proportional control such that  $\Delta x = k\phi_n$  and defining  $K = \frac{1}{2} \frac{f}{I} k$ , Eq. 9 and Eq.10 can be expressed as follows:

$$\begin{aligned} \theta_{n+1} &= \theta_n - \alpha^2 K \theta_n + T \dot{\theta}_n \\ \dot{\theta}_{n+1} &= \dot{\theta}_n - 2\alpha K \theta_n \end{aligned}$$

Arranged them in the state space equation, we can get a discrete map  $M$  (i.e. Poincare Map, with set of difference equations):

$$\begin{bmatrix} \theta_{n+1} \\ \dot{\theta}_{n+1} \end{bmatrix} = \begin{bmatrix} 1 - \alpha^2 K & T \\ -2\alpha K & 1 \end{bmatrix} \begin{bmatrix} \theta_n \\ \dot{\theta}_n \end{bmatrix} = M \begin{bmatrix} \theta_n \\ \dot{\theta}_n \end{bmatrix} \quad (11)$$

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<sup>1</sup>Conceptually, the  $f_n(t)$  can be treated as a force applied from a nonlinear component which connects the massless leg to the body (so there is no velocity change happen at foot strike)

### Eigen value analysis

To analyze the stability of the equation in 11, we need to check whether the eigen values of Poincare map  $M$  are within the unit cycle. Similar to the Routh-Herwitz method for the continuous map, we can use Jury Stability Test (Ogata, 1985)<sup>2</sup>, which states that a discrete system of two dimensions with the characteristic equations  $P(z)$  of the form:

$$P(z) = a_0 z^2 + a_1 z + a_2$$

where  $a_0 > 0$ , is stable if the following conditions are all satisfied:

$$\begin{aligned} |a_2| &< a_0 \\ a_0 + a_1 + a_2 &> 0 \\ a_0 - a_1 + a_2 &> 0 \\ |(a_0 + a_2)(a_2 - a_0)| &> |a_1(a_0 - a_1)| \end{aligned}$$

For a Jacobian of the form

$$J = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}$$

The characteristics equation can be expressed as follows:

$$P(z) = z^2 - (J_1 + J_4)z + (J_1 J_4 - J_2 J_3)$$

Substituting into the stable conditions stated above,

$$|(J_1 J_4 - J_2 J_3)| < 1 \tag{12}$$

$$1 - (J_1 + J_4) + (J_1 J_4 - J_2 J_3) > 0 \tag{13}$$

$$1 + (J_1 + J_4) + (J_1 J_4 - J_2 J_3) > 0 \tag{14}$$

$$|(1 + (J_1 J_4 - J_2 J_3))((J_1 J_4 - J_2 J_3) - 1)| > |(J_1 + J_4)(1 + (J_1 + J_4))| \tag{15}$$

### Check condition Eq.12:

First assuming  $1 - \alpha^2 K + 2T\alpha K > 0$

$$\begin{aligned} 1 - \alpha^2 K + 2T\alpha K &< 1 \\ \rightarrow -\alpha^2 K + 2T\alpha K &< 0 \\ \rightarrow \alpha K(-\alpha + 2T) &< 0 \end{aligned}$$

Since  $\alpha > 0$ ,  $K > 0$ , and  $T > \alpha$ , the assumption cannot satisfy the condition.

Next, assuming  $1 - \alpha^2 K + 2T\alpha K < 0$  :

$$\begin{aligned} 1 - \alpha^2 K + 2T\alpha K &> -1 \\ \rightarrow -1 + \alpha^2 K - 2T\alpha K &< 1 \\ \rightarrow \alpha K(\alpha - 2T) &< 2 \end{aligned}$$

Since  $T > \alpha$ , the condition can always be satisfied, as long as the following condition is satisfied:

$$(J_1 J_4 - J_2 J_3) = (1 - \alpha^2 K + 2T\alpha K) < 0$$

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<sup>2</sup>contents quoted from [11]

Combine conditions above we can get a new inequality as follows:

$$-1 < (J_1 J_4 - J_2 J_3) = (1 - \alpha^2 K + 2T\alpha K) < 0 \quad (16)$$

**Check condition Eq.13:**

$$\begin{aligned} 1 - (1 - \alpha^2 K + 1) + (1 - \alpha^2 K + 2T\alpha K) &> 0 \\ &\rightarrow 2T\alpha K > 0 \end{aligned}$$

From the last inequality we can get the condition is always hold.

**Check condition Eq.14:**

$$\begin{aligned} 1 + (1 - \alpha^2 K + 1) + (1 - \alpha^2 K + 2T\alpha K) &> 0 \\ &\rightarrow 4 - 2\alpha^2 K + 2T\alpha K > 0 \\ &\rightarrow 4 + \alpha K(-2\alpha + 2T) > 0 \end{aligned}$$

From the last inequality we can get the condition is always hold.

**Check condition Eq.15:**

Based on Eq. 16, the left hand side of Eq. 15 can be rearranged as :

$$|(det(M) + 1)(det(M) - 1)| = |det(M)^2 - 1| = 1 - det(M)^2$$

From Eq. 13 and 14 we can got  $(J_1 + J_4) > 0$ , therefore the right hand side of Eq. 15 can be rearranged as:

$$|(J_1 + J_4)(J_1 + J_4 + 1)| = (J_1 + J_4)(J_1 + J_4 + 1)$$

Therefore the Eq. 15 can be expressed as follows:

$$1 - det(M)^2 > tr(M)(tr(M) + 1)$$

where  $det(M) = \prod_i \lambda_i = (J_1 J_4 - J_2 J_3)$  is the determinant of matrix  $M$  and  $tr(M) = \sum_i \lambda_i = (J_1 + J_4)$  is the trace of the matrix  $M$ .

### To sum up

For the (Poincare) stability, the following conditions need to be satisfied:

$$-1 < det(M) < 0 \quad (17)$$

$$0 < tr(M)(tr(M) + 1) < 1 - det(M)^2 \quad (18)$$

where

$$\begin{aligned} det(M) &= 1 - \alpha^2 K + 2T\alpha K \\ tr(M) &= 2 - \alpha^2 K \\ K &= \frac{1}{2} \frac{f_n}{I} k \end{aligned}$$

### Result

After check the sign of the  $det(M)$ , it was found that  $det(M)$  always  $> 0$ :

$$1 - \alpha^2 K + 2T\alpha K = 1 + \alpha K(-\alpha + 2T) > 0$$

Therefore, it is concluded that proportional control with this system setup cannot stablize the pitch dynamics.

### 2.2.3 Poincare Map of Pitch Dynamics with PD Control

By designing a PD control such that  $\Delta x = k_p \theta_n + k_d \dot{\theta}_n$  and defining  $K = \frac{1}{2} \frac{f_n}{I} k_p$ ,  $C = \frac{1}{2} \frac{f_n}{I} k_d$ , Eq. 9 and Eq.10 can be expressed as follows:

$$\begin{aligned}\theta_{n+1} &= \theta_n - \alpha^2 K \theta_n + T \dot{\theta}_n - \alpha^2 C \dot{\theta}_n \\ \dot{\theta}_{n+1} &= \dot{\theta}_n - 2\alpha K \theta_n - 2\alpha C \dot{\theta}_n\end{aligned}$$

Arranged them in the state space equation, we can get a discrete map  $M_{pd}$ :

$$\begin{bmatrix} \theta_{n+1} \\ \dot{\theta}_{n+1} \end{bmatrix} = \begin{bmatrix} 1 - \alpha^2 K & T - \alpha^2 C \\ -2\alpha K & 1 - 2\alpha C \end{bmatrix} \begin{bmatrix} \theta_n \\ \dot{\theta}_n \end{bmatrix} = M_{pd} \begin{bmatrix} \theta_n \\ \dot{\theta}_n \end{bmatrix} \quad (19)$$

### 2.2.4 Analytical Solution for Eq.7

Start from  $t_0$  (the beginning of the flight phase), assuming  $Z = [0, \dot{z}_0]^T$ , then we can get:

$$z(t_{flight}) = \dot{z}_0 t_{flight} - 1/2 g t_{flight}^2 = 0 \quad (20)$$

$$\dot{z}(t_{flight}) = \dot{z}_0 - g t_{flight} = -\dot{z}_0 \quad (21)$$

where a constraint for the  $\dot{z}_0$  can be derived:

$$\dot{z}_0 = 1/2 g t_{flight} \quad (22)$$

$$(23)$$

Then we can derive the solution at the end of the touch down:

$$z(1) = -\dot{z}_0 t_{contact} + (f/m - g) t_{contact}^2 = 0 \quad (24)$$

$$\dot{z}(1) = -\dot{z}_0 + (f/m - g) t_{contact} = \dot{z}_0 \quad (25)$$

where another constraint for the  $\dot{z}_0$  can be derived:

$$\dot{z}_0 = 1/2 (f/m - g) t_{contact} \quad (26)$$

**Period  $T$ , contact force  $f$  and  $t_{contact}$  are dependent** From Eqs. 26 and 22 we can get

$$\begin{aligned} 1/2 g t_{flight} &= 1/2 (f/m - g) t_{contact} \\ \rightarrow t_{flight} &= (f/mg - 1) t_{contact} \\ \rightarrow t_{flight} + t_{contact} &= T = (f/mg) t_{contact} \end{aligned}$$



### 3 Simulations

#### 3.1 1 DOF Vertical Hopper with Open-loop Control[11]

##### System Setup

- Body mass  $m = 1$  kg with massless leg,  $l = 1$  m.
- Spring parameters:  $\omega_n = 30$  rad/s,  $\xi = 0.15$  (or equivalently,  $kp = 900, kd = 9$ )
- Static initial condition, COM height = 1.3 m (foot to ground = 0.3 m)
- Open-loop external force:

$$f_n(t) = \begin{cases} f_n \in \mathbb{C}, & \text{if } t \in t_{on}. \\ 0, & \text{otherwise.} \end{cases}$$

- $t_{on}$ : The duration of actuator activation, starts when the spring reaches the maximum compression, ends when the contact point leave the ground.

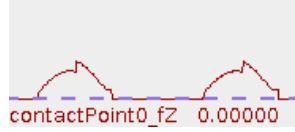


Figure 3: Ground reaction force when  $f_n = 10$  N

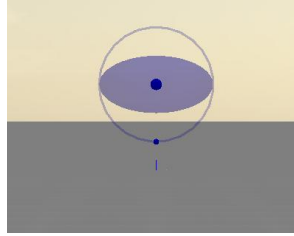


Figure 4: The vertical hopper, the blue dot at the bottom is the contact point of the massless leg.

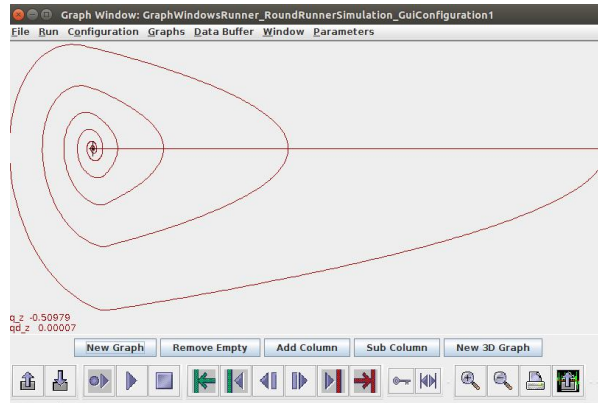


Figure 5: Phase portrait (stable spiral) of  $f = 1$  N, period 0 sec

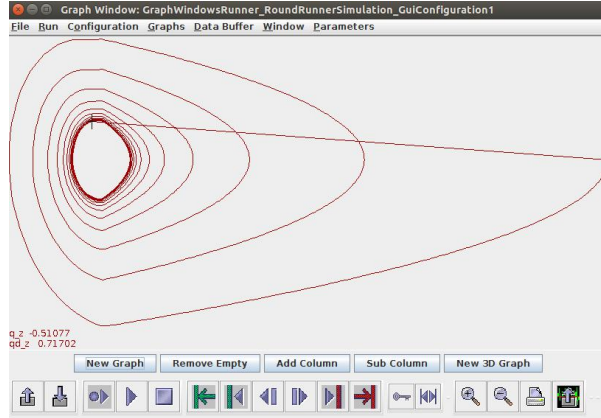


Figure 6: Phase portrait (stable limit cycle) of  $f = 10$  N, period 0.27sec, (closer to the damped natural period  $\cong 0.3295$  sec)

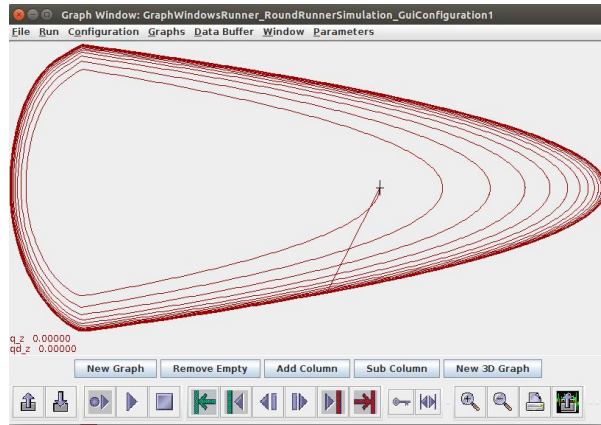


Figure 7: Phase portrait (stable limit cycle) of  $f = 50$  N, period 0.859 sec

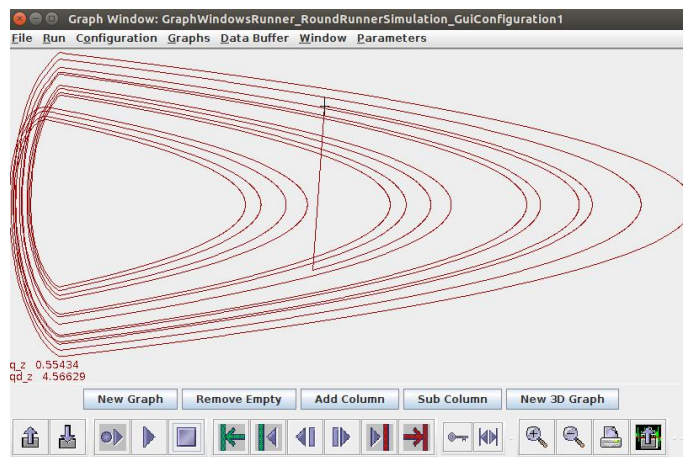


Figure 8: Phase portrait of  $f = 100$  N, no stable limit cycle evolved (might be bifurcation).

## Plan

- Go through and reuse the Poincare analysis in spokedReader package.

- Could be a good case for me to learn how to use parameterOptimizer (or other constrained nonlinear program solver) to get IC/parameters for a stable/optimal gait.

### 3.2 Abstract Runner with Open-loop Normal Force and Closed-loop Pitch Angle Control

#### System Setup

- Body mass  $m = 10$ ,  $I_{yy} = 10$  with massless leg,  $l = 1$ .
- Reuse the vertical hopper above, change the initial condition to  $\theta = 0.2$
- No force applied in the x direction,  $\dot{x}_0$  can be 0 (hopper) or a constant (runner).
- Similar to the abstract runner (Fig. 9), enforces the on/off timing of ground reaction force  $f_n(t)$ :

$$f_n(t) = \begin{cases} (f_n + u)|f_n \in \mathbb{C}, & \text{if } t \in t_{on}. \\ 0, & \text{otherwise.} \end{cases}$$

where  $f_n = \alpha * mg$ ,  $\alpha \in \mathbb{C}$ ,  $u$  is the force from PD control,  $kp_z = 80, kd_z = 6$ .  $kp_{pitch} = 80, kd_{pitch} = 6$

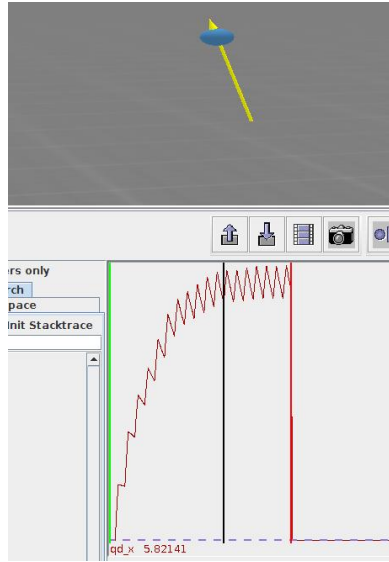


Figure 9: The Abstract Runner

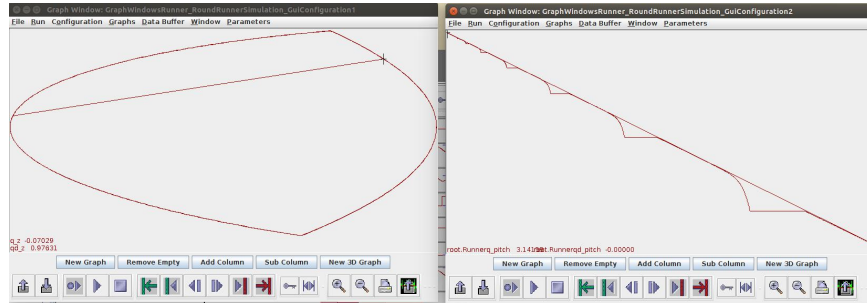


Figure 10: The phase portrait of the abstract runner: phase portrait (left) of body  $z$  movement  $[q_z, qd_z]^T$  and the pitch motion (right, the movement is converging to the origin in the upper-left corner) .

## Plan

- Link it to the Math from Jerry's note (analysis of a linear Poincare map) to get the boundaries of stable parameters.

### 3.3 Spoked Runner with Massless Legs

#### System Setup

- $m = 15$ ,  $I_{yy} = 10$ ,  $l = 4$ ,  $r_{penetration} = 0.3$  (the distance the virtual wheel penetrate into the ground)
- Adjustable spoke leg number
- Fixed rotation rate w.r.t inertial frame
- Setup of contact force: PD control
  - w.r.t to world frame
  - w.r.t to inertial frame (virtual pivot point)
- Assuming no friction (Could be an bad idea?)

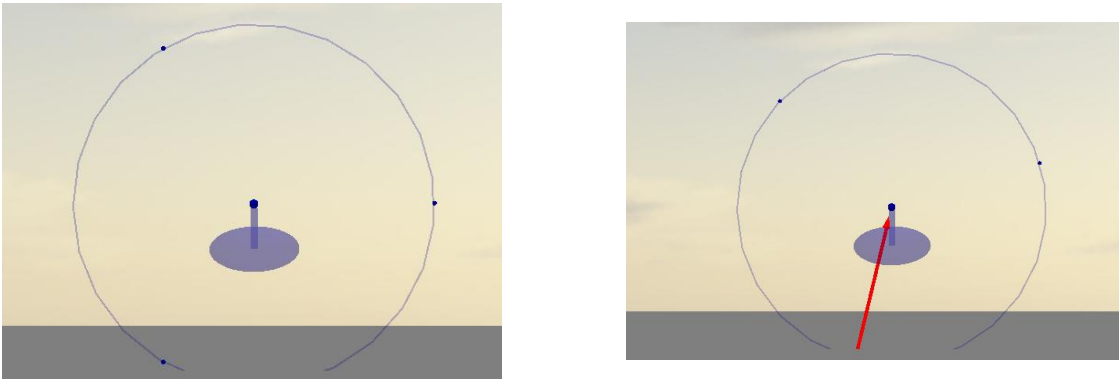


Figure 11: The Spoked Runner with three legs

## Plan

- Smoothly change the leg length, or the rotational speed of the virtual wheel, and observe the system response.
- Learn how to use GUI for parameter adjustment with SCS.

## 4 Code implementation

### 4.1 Modeling and Parameters

Main idea: a virtual wheel (as the massless leg) with radius  $r_{wheel}$  penetrate the ground for a distance  $r_{pen}$  where a external force point  $pe$  is attached on it. A body (with mass  $m$  and inertia  $I_{yy}$ ) is attached to the center of wheel. Using PD control to interpret contact force when  $p_e$  is under the ground.

#### 06/07 First prototype (Not used now)

- Joint numbers: 2
- Joint types: Floating planer joint for virtual wheel and pin joint for the body link.
- Contact point type: External force point
- Virtual wheel rotation: set proper initial condition for virtual wheel (also need a large inertia to make it nearly constant).

Contact force: Assuming the ground height is 0,

$$F_z = kp(0 - pe_z) + kd(0 - ve_z) \quad (27)$$

$$\phi = atan2(pe_x, r_{wheel} - pe_z) \quad (28)$$

$$F_x = F_z tan(\phi) \quad (29)$$

where  $ve$  is the velocity vector of the contact point  $pe$ ,  $kp$  and  $kd$  are the PD control parameters.  $F_x$  is calculated so that the vector of ground reaction force  $[F_x, F_y, F_z]^T$  will point towards the virtual pivot (the center of the virtual wheel).

Assessments:

- Need to set a non-zero inertia of massless virtual wheel (for numerical stability), otherwise the simulation will diverge.
- The inertia of virtual wheel need to be a large one for constant rotational speed.
- Suggestions: remove the massless link, attach the external force point to the body and change its position in the controller every time step.

#### 06/08 Round Runner

- Joint numbers: 1
- Joint types: Floating planer joint for the body link.
- Contact point type: External force point
- Virtual wheel rotation: Assigning the external force point location with respect to the joint in an open loop manner.
- Contact force: Assuming the ground height is 0,

$$F_z = kp(0 - pe_z) + kd(0 - ve_z) \quad (30)$$

$$\phi = atan2(pe_x, r_{wheel} - pe_z) \quad (31)$$

$$F_x = F_z tan(\phi) \quad (32)$$

where  $ve$  is the velocity vector of the contact point  $pe$ ,  $kp$  and  $kd$  are the PD control parameters.  $F_x$  is calculated so that the vector of ground reaction force  $[F_x, F_y, F_z]^T$  will point towards the virtual pivot (the center of the virtual wheel).

Assessments:

- The ground reaction force looks better, while the energy is not balanced (after a while it will move towards the negative  $x$  direction)
- The inertia of virtual wheel need to be a large one for constant rotational speed.
- Suggestions: Use the ground contact point (instead of external force point) to see how it goes.

#### 06/11 Round Runner(with Ground Contact Point)

- Joint numbers: 1
- Joint types: Floating planer joint for the body link.
- Contact point type: Ground contact point, linear contact model<sup>1</sup>
- Virtual wheel rotation: Assigning the external force point location with respect to the joint in an open loop manner.
- **Contact point number** Parameterized, currently set to 3-6 points.
- Contact force: using built-in functionalities, only assigning the  $kp$ ,  $kd$  (PD parameters in the  $z$  direction),  $kp_x$ , and  $kd_x$  (PD parameters in the  $x/y$  directions).

Assessments:

- Was able to generate a stable walking. Contact point has sliding.
- Due to setting up stiffness and damping for  $x$  and  $z$  separately, the force is not always point towards the virtual pivot.

#### 06/12 Round Runner(with External Contact Point Point)

- Implement the same one as 06/11, but replace the ground contact point to the external one (because it is more complex for ground contact point to adjust stiffness/damping as parameters.)
- implement the linear ground contact model basically.

#### 06/13 Round Runner

- Parameterize contact point numbers
- Adding enum for switching between different setup: contact point type and the corresponding ground reaction force calculation: (w.r.t to the world frame or inertia frame.)

#### 06/16 Round Runner (vertical hopper)

- Adding vertical hopper with open-loop force control
- Playing with open-loop force magnitudes for different stability conditions

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<sup>1</sup>Disable the hardening stiffness in  $z$  direction by setting `groundStiffeningLength` to `Double.NEGATIVE_INFINITY`

## 5 Info might be useful

### 5.1 Going through references

1. Compare different terrestrial locomotions: Some parameters of the walk are not speed- dependent. The swing duration is a constant time parameter [1].
2. Trunk plays an important role during walking (birds) [2].
3. The use of these drives (Resonance drives, with adaptive control) allows increasing machine's quickness several times and decreasing energy expenses simultaneously 10-50 times [3].
4. Light weight leg (ostrich vs. moa) can run faster[5]. Also a famous allometric equation:

$$Y = M^{3/4} \quad (33)$$

where  $M$  is the body mass,  $Y$  is the metabolic rate.

5. Human's walking may not be really self-optimized: the preferred speed maybe different from the energetically optimal speed[8].
6. It is concluded that the most important adjustment to the bodys spring system to accommodate higher stride frequencies is that leg spring becomes stiffer [19].
7. magic equations for imd force (ostrich) [26]
8. gait frequency was reported to be highly correlated with the resonant frequency of the mass-spring model [30]
9. WABIAN, why you are here? [31]

### 5.2 Categories

1. Nonlinear oscillators/components [3, 6, 9, 10, 12, 28, 39];
2. zoology, biomechanics of animals: [1, 2, 4, 5, 16]
3. Bio-inspired robots: [7, 32]
4. Reference I should read: [11, 15, 27, 28]
5. Article not found (or not free)[4].
6. Robots in 3D: [13]
7. Stability analysis (Monocycle, linearized system) [14] (Limit cycle) [11, 27] dimensionless [41]
8. Biology/Anatomical structure [17, 20]
9. Light weight fast robot [18, 25]
10. take a look again [21]
11. mechanism design of robot [22]
12. quadruped reference [23] MIT Cheetah[37]
13. human energy cost, resonance usage [24, 8, 38, 40]
14. walking parameterization [29, 21, 42]
15. human-animal differences [15]
16. open-loop robot [33], passive robot [35, 34, 36]

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