Assignment 1

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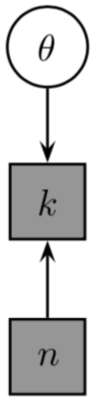
Github: https://github.com/KennethEnevoldsen/Advanced-Cognitive-Modelling/tree/master/assignment_1

Explain the beta-binomial model for inferring rates. Present the theta, alpha, and beta parameters, and their interpretation. You may refer to the chicken example, or one of your own. Use figures to support your answer where appropriate.

Compare the beta binomial and learning model covered in class.Explain the concepts of model and parameter recovery, and present the results of model and parameter recovery studies for the beta binomial and learning models. Use figures where relevant. What can you say about these models from these studies?

The beta binomial model is one of the simplest models of interest in bayesian modelling. In this assignment we used it to infer the skill of a chicken sexer, a man with the occupation of sexing chickens. We will check each chicken sexer over a number of trials, n.

The first model which we will present is the fixed model where we assume the persons probability



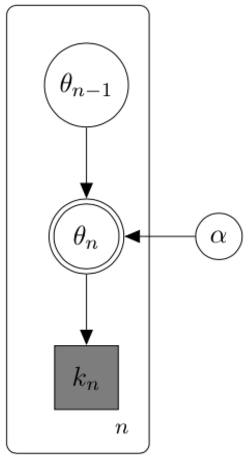
$$\theta \sim \mathrm{Beta}(1, 1)$$
$$k \sim \mathrm{Binomial}(\theta, n)$$

Figure 1: Graphical model for inferring the skill, θ of the chicken sexer, where k is the number of successes. Source: Lee and Wagenmaker (2013).

of correctly classifying the sex of a chicken, θ is fixed and can be a value from 0-1. This results model specified in figure 1. The second model assume the chicken sexer learns per trial n from a given starting point θ_1 with the given learning model:

$$\theta_n = \theta_{n-1}^{\frac{1}{1+\alpha}}$$

Where α is the learning rate, i.e. how much . Which results in the graphical model seen in figure 2.



$$\alpha \sim \text{normal}(0,1)$$
$$\theta_n = \theta_{n-1}^{\frac{1}{1+\alpha}}$$
$$\theta_1 \sim \text{normal}(0,1)$$
$$k_n \sim \text{binomial}(\theta,1)$$

Figure 2: A Graphical model for inferring the skill, of the chicken sexer at time n, where k is the number of successes.

Question: How do I denote that it is a truncated normal distribution in a graphical model?

For each of these model we generated data for a total of 100 trials and afterwards fit both of the models. Afterwards the model which best fit the data, operationalised as the lowest BIC, was determined. This process was repeated 100 times as to make it possible to derive the following confusion matrix.

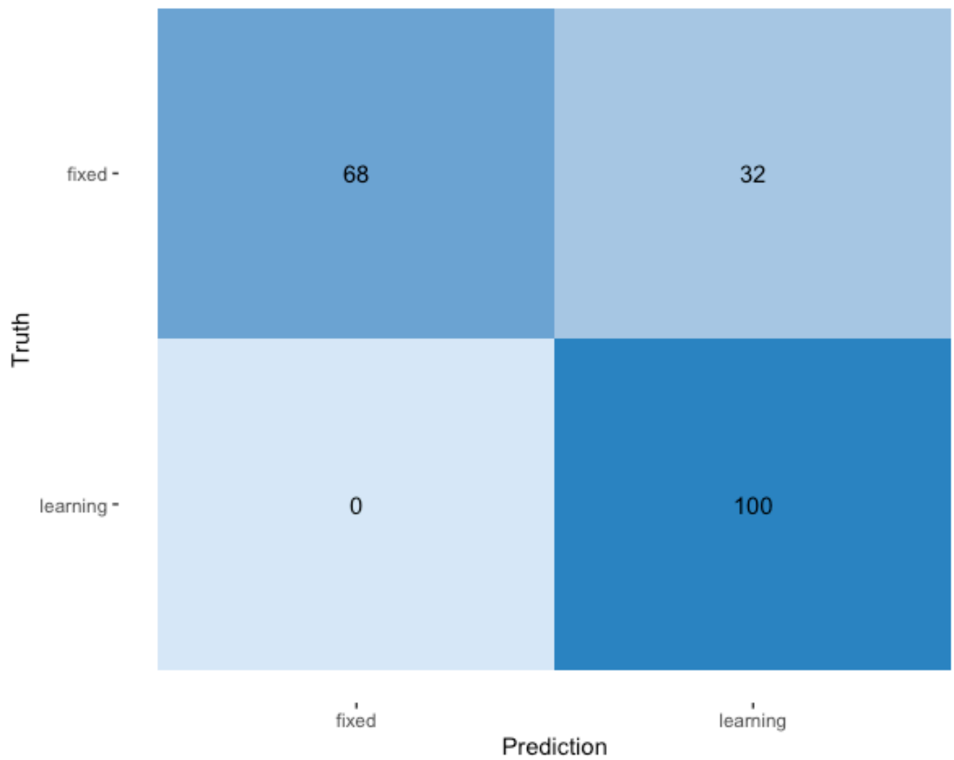


Figure 3: A confusion matrix showing how many times the model was it owns best fit. Darker shades of blue indicate higher numbers.

From figure 3 we see that each model in general is its own best fit, however seemingly the learning model is also a decent fit to the for the fixed model, which seems reasonable as the learning model with $\alpha = 0$ is equivalent to fixed model. Furthermore for each of the simulations a maximum a posteriori (MAP) estimation was conducted to estimate the difference between the learning for each of the hidden states. As can be seen in figure 4 the fixed model is properly able to recover it starting parameter. On the contrary the learning model was shown unable to recover its starting parameters except for very low values of α (see figure 5). Consequently, it is possible to distinguish between these two model, albeit the learning model can (with $\alpha = 0$) fully describe

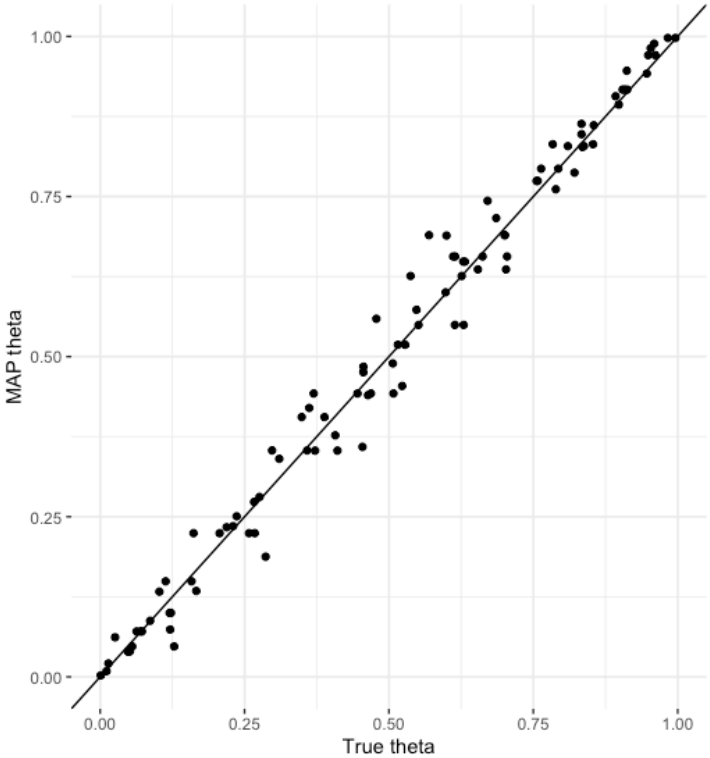


Figure 4: A figure plotting the difference between the true θ and the estimated θ. The estimated states is estimated using MAP.

the fixed model. Consequently it is unlikely, given a certain degree of noise, that these model will be distinguishable in practice.

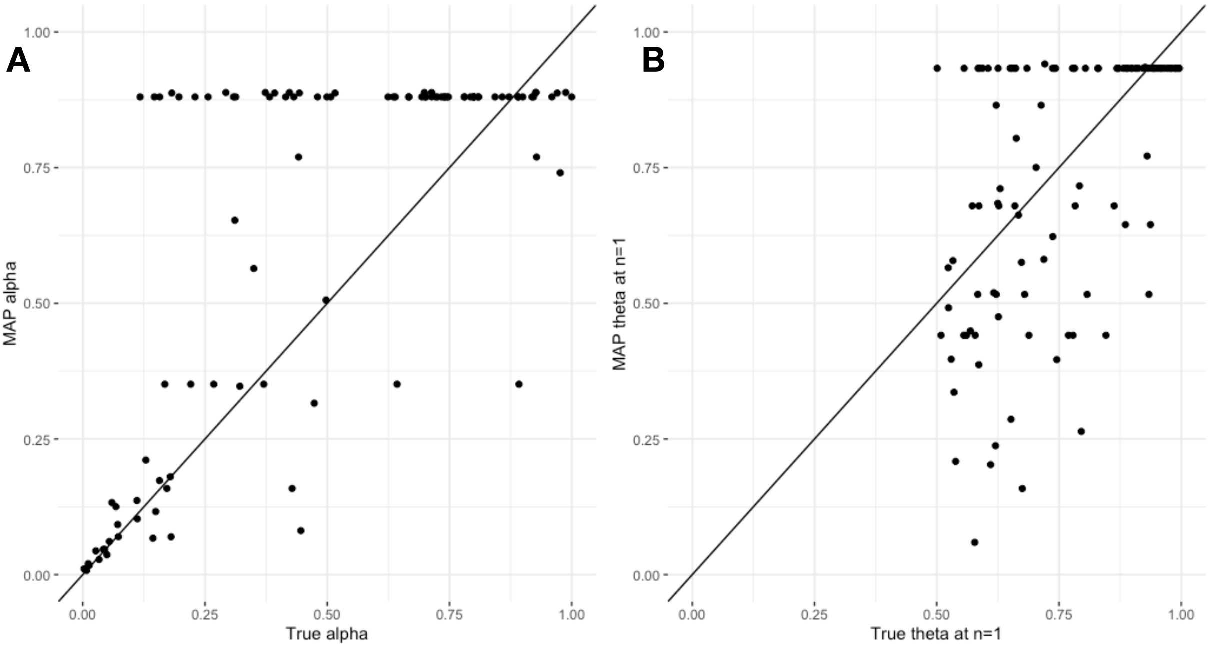


Figure 5: A) a figure plotting the difference between the true α and the estimated α. B) a figure showing the difference between the true θ_1 and the estimated θ_1. The estimated states is estimated using MAP. Note that during the simulation θ_1 is sampled from a uniform distribution from 0.5 to 1.