Theorem **Corollary 1**

If $f'(x) = 0$ for each x in an open interval (a,b), then f is constant on (a,b).

Proof

We will show that given any two distinct points x_1 and x_2 in (a,b), $f(x_1) = f(x_2)$. Since $x_1 \neq x_2$, one is less than the other; we can assume $x_1 < x_2$. The fact that f is differentiable on all of (a,b) means that f is continuous on $[x_1, x_2]$ and differentiable on (x_1, x_2), so by the MVT, there is a point $c \in (x_1, x_2)$ such that

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

But since f' is 0 throughout (a,b), this means that

$$f(x_2) - f(x_1) = f'(c) \cdot (x_2 - x_1) = 0 \cdot (x_2 - x_1) = 0,$$

so $f(x_1) = f(x_2)$.