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THE UNIVERSITY OF BRITISH COLUMBIA

Math 312 Section 101

Calculators are allowed

No cell phones or information sheets

Test begins at 10:00 am and ends at 10:50am

TEST #1

July 12, 2023

NAME

STUDENT NUMBER

1.(a) Is it true that $3^n > n^3$ for $n > 3$? If so, use induction to prove it. If not give a counterexample.

1.(b). Find integers a and b such that $0 < a < 9$ and $|ax - b| < 1/8$, when $x = e$. Show by calculation that you have made correct choices for a and b , when $x = e$.

2. (a) Show that if p_k is the k th prime, then p_n is less than or equal to $(p_1 p_2 \dots p_{(n-1)}) + 1$ for all integers $n > 2$.

(b) Prove that $f_{n+3} - f_n = 2f_{n+1}$ whenever n is a positive integer.

3(a) Show that if a is an integer, then 3 divides $a^3 - a$.

3(b) Show by induction that the sum of the cubes of three consecutive integers is divisible by 3.

4(a) Subtract $(12345)_7$ from $(54321)_7$ Work in base 7.

4(b) Calculate EBE times BC in base 16. Show your work.

5. The Fibonacci sequence begins with $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$ and every term afterwards is the sum of the two predecessors.

(a) By calculation find the value of $n > 0$ such that f_n has 0 in its units position.

(b) By calculation find the value of $n > 0$ such that f_n has 6 in its units position.