Math100C XI

C23,34,35,26

Shikun Nie, PhD student, Department of Mathematics, UBC

Topic: Lagrange multiplier

Optimization

1. Recall from week 7 that we considered the utility (say, your happiness) of having a combination of x (a quantity of one candy) and y (a quantity of a second candy). We optimized $u = \sqrt{xy}$ subject to the budget constraint $24 = 3x + 2y$ (we have 24 dollars to spend, and the candies cost 3 and 2 dollars each, respectively). We found that the max utility is achieved for $x = 4$ and $y = 6$.

Optimization by Lagrange multiplier

2. We can solve this problem a second way using the method of Lagrange multipliers. What does the method say? Suppose we have an objective function we wish to optimized $u(x, y)$ and a constraint function $g(x, y) = c$ where c is a constant. The local extrema of $u(x, y)$ subject to a constraint $g(x, y) = c$ occurs when all of:

$$u_x(a, b) = \lambda g_x(a, b)$$
$$u_y(a, b) = \lambda g_y(a, b)$$
$$g(a, b) = c$$

The constant λ is the Lagrange multiplier. Let's solve our problem again but using this method.

Optimization by Lagrange multiplier

3. This seems complicated, why would we do this? What could go wrong when we try to eliminate one variable instead?

Reflection

1. There are a few mathematical ideas in this course that cut across topics and appear at multiple points along the way. Can anyone think of some of these ideas?

Reflection

2. In this course, you’ve worked to develop a lot of skills beyond mathematical ones. Name a few.

Reflection

3. What’s one thing you’ll remember about this course? Having completed the course, how would you describe differential calculus to someone who hasn’t taken it yet.

Additional problems

1. Find the local max and min of the function $f(x, y) = xy + 14$ on the curve $x^2 + y^2 = 18$. Solve this problem two ways: (1) by eliminating a variable and (2) by using Lagrange multipliers.
2. Find the local max and min of the function $f(x, y) = xy$ subject to the constraint $x^2 - 2xy + 5y^2 = 1$.
3. Optimal, Integral, Likely Practice Book Section 2.5: Q1, Q3, Q4, Q5, Q6, Q7.

THE UNIVERSITY OF BRITISH COLUMBIA