Math100C VII

C23,34,35,26

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Topic: Optimization

Problems and takeaways

Think of your favorite candy and consider the utility $u = \sqrt{x}$ where x is the number of candy.

Problems and takeaways

Definition: *Utility* is the measure of welfare (happiness, satisfaction) of a person/entity. In this case u can be how happy you'd be if you had x pieces of candy.

Problems and takeaways

How happy are you if you have 9 pieces of candy? What about 16? What about 25? Why does it take so many more pieces to get from happiness 4 to 5 than it took from happiness 3 to 4.

Problems and takeaways

What's another candy you like? We'll use y to denote the number you have of this second candy. Say your utility considering both is $u = \sqrt{xy}$.

Problems and takeaways

The first candy costs $\$3$ each and the second $\$2$ each. You have a total $\$24$. Ultimately, we'd like to know how much of each candy to buy in order to maximize happiness subject to our budgetary constraint. Let's do this in several steps.

Problems and takeaways

Write down an equation that represents the budget constraint. It should involve your budget as well as the costs of each candy.

Problems and takeaways

What’s the largest value of x we can have? What’s the smallest? What are the bound for y? Feel free to draw the graph.

Problems and takeaways

We'd like to maximize $u = \sqrt{xy}$ but in its current form we're going to have trouble. What do we need to do first? Can we turn this into a more well-defined optimization problem?

Problems and takeaways

Now this looks like a problem we've seen in large. Maximize $u(x)$ on this domain. First differentiate.

Problems and takeaways

Find critical and singular points of $u(x)$.

Problems and takeaways

Are there any other points we need to check?

Problems and takeaways

What is the x and y pair that produces the greatest happiness? What is our maximum happiness.

Problems and takeaways

This problem can be stated more generally by assigning parameter values to the budge and costs of each item. That is find x and y that maximize $u = \sqrt{xy}$ subject to the budge constraint $I = px + qy$ where I is the fixed budget and p and q are the unit costs.

Problems and takeaways

Definition: The x and y values that maximize the utility subject to a budge constraint are called *Marshallian demand*.

They determine the demand for both items as a function of costs and budget.

Problems and takeaways

An alternative but equally interesting problem is one of Hicksian demand. Suppose that you will not settle for happiness less than, say, $u = 10$. What values of x and y give the minimum budget that will assure this happiness?

(a) Once again, take $u = \sqrt{xy}$ and $I = 3x + 2y$, but this time set $u = 10$ and minimize I.

(b) Write I as a function of only one variable, say $I(x)$.

(c) What is the domain of $I(x)$? Is it closed? (Hint: It's not!)

(d) Find critical and singular points. Are we sure that critical point is a minimum?

(e) Compute $\lim_{x \to 0^+} I(x)$ and $\lim_{x \to \infty} I(x)$. Argue why the critical point you found in (d) should, in fact, be a minimum.

(f) Find both x and y that minimize I. In general, this pair is called the Hicksian demand. This determines how much demand there is for each item as a function of the costs and minimum required utility.

Problems and takeaways

Repeat the above problem with parameters. That is, take $M = \sqrt{xy}$ and $I = px + qy$.

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