Math100C III

C23,34,35,26

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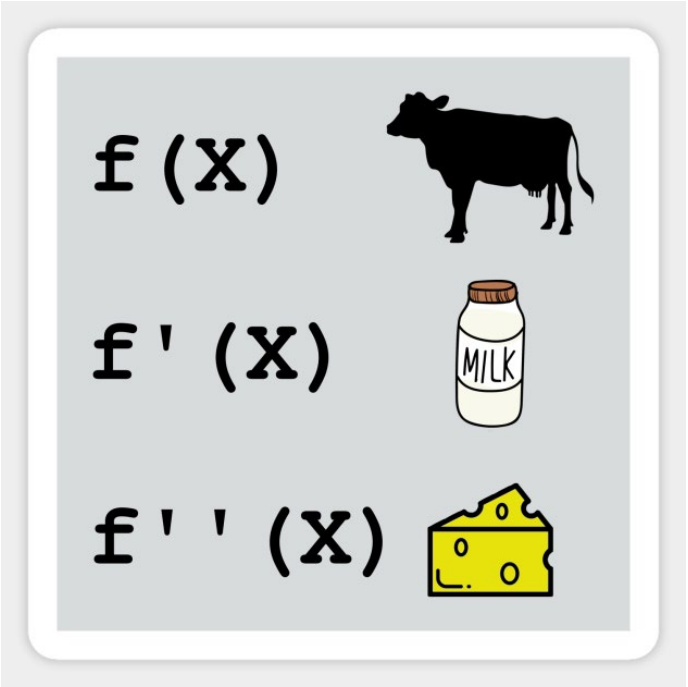
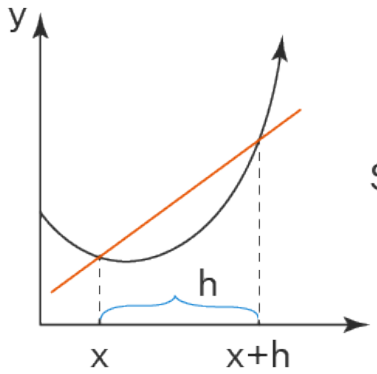
Derivatives

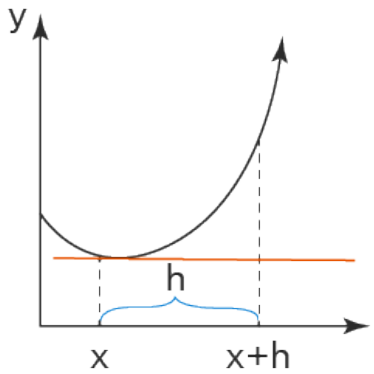


Definition of Derivatives



$$\text{Slope of Secant} = \frac{f(x+h) - f(x)}{h}$$

("Difference quotient")



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(if limit exists)

Proof the derivatives of b^x

- what is the graph of b^x?

(i) $0 < b < 1$

(ii) $b > 1$

- What is the limit definition of $f'(0)$ if $f(x) = b^x$? Simplify your answer by using the hint:

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

- Proof the following by the definition of derivatives

If

$$f(x) = b^x$$

Then

$$f'(x) = b^x \ln b$$

Proof the derivatives of b^x

- What is the slope of $f(x) = e^x$ at $x = 0$? At $x = 1$? At $x = -2$?

Problems and takeaways

- ***Definition***: A differential equation is an equation involving unknown function and its derivatives. Unlike algebraic functions where the solutions are numbers, the solutions to differential equations are functions.

- What is one function that satisfies $y'(t) = y(t)$?

- Suppose $y(t)$ represents the size of a population (e.g. of bacteria, deer, foxes, etc.) at time t, and satisfies the differential equation $y'(t) = y(t)$. What happens toe the population over time?

- Can you propose another solution to the differential equation $y'(t) = y(t)$?

Problems and takeaways

- Imagine you have deer populations, population A and population B. Both populations satisfy the differential equation $y'(t) = y(t)$, where $y(t)$ is the population at time t. However, population A starts off with 200 deer while population B starts off with 300 deer. How many deer are in population B at the moment population A has 500 deer?

- Sketch the graphs of population A and B on a single set of axes, indicating when the population A is at 200 and 500, and when the population is at 300 and 750.

Additional Problems

- Confirm, using the limit definition of derivatives, that $y(t) = Ce^t$, where C is a constant, is a solution to the differential equation $y'(t) = y(t)$

- Propose a differential equation that describes exponential decay, where the derivative of the solution $f(x)$ to the differential equation is both negative and proportional to $f(x)$.

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