Math101C: Integral Calculus

Derivatives and Integrals in Applications

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Small Class II for C15,18,22,24

Outline

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1. Problems and Takeaways
   - Displacement as area
   - Moving between acceleration, velocity and displacement
   - Functions defined via integrals
2. Additional Problems

Problems

Examples

1. You're out for a walk/run and start at a slow pace of $1m/s$. (Average human walking speed is around $1.3m/s$, running is more like $5m/s$.) At some point in your first minute you see something that causes you to change your speed. You check your speed each minute for 4 minutes (and each time it's different, for different reasons). Draw a set of axes and graph the speed against time.
2. How far (roughly) did you go over the course of your journey?

Takeaway

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We can “undo” a derivative by finding the area under the curve.

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1. Suppose a ball (or brick, or anything you like) moves subject to gravity. Maybe we drop it, maybe we throw it. In any case, the acceleration will be constant: $a(t) = -g$. Find the velocity $v(t)$.
2. Now find the position $x(t)$.
3. Suppose further that we threw the ball upwards with an initial velocity of $v(0) = v_0$ and that the ball has an initial position of $x(0) = x_0$. What should the constants c_1 and c_2 be?

Takeaways

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- We can "undo" a derivative by taking an indefinite integral. In particular, with a little integration, we can understand the motion of a ball subject to gravity with any initial velocity and position.
- Initial conditions will determine constants of integration

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Problems

Examples

1. The Error function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

   is an important function in probability and statistics. Does this function have any critical points?

2. Is $\operatorname{erf}(x)$ increasing or decreasing? What happens to the slope of our function as $x \to \infty$and $x \to -\infty$? Where is the slope of the error function the largest? Make a rough sketch of $\operatorname{erf}(x)$.

Addtional Problems

- Repeat the analysis in section 1.2 with linearly increasing acceleration $a(t) = c_1 t$. Think constantly increasing your pressure on the accelerator while driving
- Consider the Fresnel functions

$$S(x) = \int_0^x \sin(t^2)dt, \; C(x) = \int_0^x \cos(t^2)dt$$

  which come from optics (but also find applications in highways, railways, and rollercoasters). Find the critical points of these functions.
- Section 1.2 can be done using definite integrals instead of indefinite integrals. Try setting the bounds correctly to get what we expect (you'll need a technique called "substitution" which we haven't seen yet). Hint: you're looking to achieve

$$\int_{v_0}^{v} dw = \int_0^t$$

For Additional Problems I

E. Yeager, J. Feldman, A. Rechnitzer
CLP-2 Integral Calculus Exercise
https://personal.math.ubc.ca/~CLP/CLP2/clp_2_ic_problems.pd