Math101C: Integral Calculus

Alternating series test, absolute and conditional convergence

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Small Class VIII for C15,18,22,24

Outline

1. Problems and takeaways
   - Alternating series test
   - Absolute and conditional convergence
2. Additional Problems

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Alternating series test

Examples

1 We're going to construct a series. Your job is to try to make the series diverge while still following the instructions. Write down a positive number. Choose another number, smaller than the first and subtract it. Choose a third number, smaller than the second and add it to your total. Continue choosing smaller and smaller numbers alternating between adding and subtracting. Does your sequence appear to converge?

For an example desmos

Alternating series test

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Examples

2 We need a few things to make this alternating series converge. What are they? What do we always need to happen for a series to converge?

Alternating series test

3 Takeaway (Alternating series test)

Let $\{A_n\}_{n=1}^{\infty}$ be a sequence satisfying

i $A_n \geq 0$ for all $n \geq 1$

ii $A_{n+1} \leq A_n$ for all $n \geq 1$ (that is, A_n is monotonically decreasing)

iii $\lim_{n\to\infty} A_n = 0$

Then the series $\sum_{n=1}^{\infty}(-1)^{n+1}A_n$ converges.

Alternating series test

Examples

4 Think back to the large class this week. Does $\sum_{n=1}^{\infty} \frac{1}{n}$ converge or diverge? Multiple choice:

A) Yes
B) No
C) I don't remember

Does $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ converge or diverge?

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Outline

Absolute convergence

Examples

1. Consider $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$. Does $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converge or diverge?
   - A) Yes
   - B) No
   - C) I don't remember
2. Does the inclusion of this minus sign help or hinder the convergence of this series?
   - A) Help
   - B) Hinder
   - C) Neither
   - D) I'm not sure

Absolute convergence

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Absolute convergence

3 Takeaway

The minus sign will not disrupt this convergence. If $\sum_{n=1}^{\infty} |a_n|$ converges, then so too will $\sum_{n=1}^{\infty} a_n$.

Definition (4 Absolute convergence)

If $\sum_{n=1}^{\infty} |a_n|$ converges we say that $\sum_{n=1}^{\infty} a_n$ is *absolutely convergent*.

5 Absolute convergence implies convergence. Does this work the other way around? Can you find an example of a convergent sequence that is not absolutely convergent?

Conditional covergence

Definition (6 Conditional covergence)

If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ does not, we say that $\sum_{n=1}^{\infty} a_n$ is *condtionally convergent*.

Conditional convergence

Examples

7 Determine whether $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converges or diverges.

8 Last class we saw first hand the convergence of the geometric series $\sum_{n=1}^{\infty} \frac{1}{4^n}$ (the one with the triangles). This geometric series has $r = \frac{1}{4}$. We only spent explicit time dealing with $0 < r < 1$ but our formula works perfectly well for $-1 < r < 0$. Show that geometric series with $-1 < r < 0$ are absolutely convergent.

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Addtional Problems

1. Ordering of alternating series: have a look at Example 3.4.6 in the optional section 3.4.2 in the CLP-2 text. The alternating harmonic series
$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}$$
converge conditionally. We usually expect to be able to change the order of terms in addition, but that is not the case here. Trying to do so produces some absurdities
2. CLP-2 Problem book section 3.3: Q10
3. CLP-2 Problem book section 3.4: Q1, Q2, Q5, Q9, Q10, Q11, Q12

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For Additional Problems I

E. Yeager, J. Feldman, A. Rechnitzer
CLP-2 Integral Calculus Exercise
https://personal.math.ubc.ca/~CLP/CLP2/clp_2_ic_problems.pd