Math100C VI

C23,34,35,26

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Topic: Curve sketching and logistic function

Characteristics from $f(x)$

Let $f(x) = \frac{1}{1+e^{-kx}}$ where $k > 0$ is a constant

One characteristic we can determine from $f(x)$ itself (i.e., not its derivatives) is its domain.

What are other characteristics we can determine from $f(x)$ itself?

Characteristics from $f(x)$

Let $f(x) = \frac{1}{1+e^{-kx}}$ where $k > 0$ is a constant

Find the intercepts, horizontal asymptotes and vertical asymptotes.

Characteristics from $f'(x)$

Let $f(x) = \frac{1}{1+e^{-kx}}$ where $k > 0$ is a constant

Find $f'(x)$.

Characteristics from $f'(x)$

Let $f(x) = \frac{1}{1+e^{-kx}}$ where $k > 0$ is a constant

Where is $f(x)$ increasing? Where is it deceasing?

Characteristics from $f''(x)$

Let $f(x) = \frac{1}{1+e^{-kx}}$ where $k > 0$ is a constant

Find $f''(x)$.

$$f'(x) = \frac{ke^{-kx}}{(1+e^{-kx})^2}$$

$$f''(x) = \frac{-k^2e^{-kx}\left(1+e^{-kx}\right) + 2k^2e^{-2kx}}{(1+e^{-kx})^3}$$

$$= \frac{k^2e^{-kx}\left(e^{-kx}-1\right)}{(1+e^{-kx})^3}$$

Characteristics from $f''(x)$

Let $f(x) = \frac{1}{1+e^{-kx}}$ where $k > 0$ is a constant

Where is $f(x)$ concave up? Where is it concave down?

Set

$$f"(x) = 0$$

We have

$$\frac{k^2 e^{-kx}\left(e^{-kx} - 1\right)}{(1 + e^{-kx})^3} = 0$$

$$e^{-kx} - 1 = 0$$

$$k = 0$$

Draw out $f(x)$

Let $f(x) = \frac{1}{1+e^{-kx}}$ where $k > 0$ is a constant

Take all the information determined previously and draw a large sketch of $f(x)$ on a sheet of flipchart paper. Keep your work hidden from other groups!

https://www.desmos.com/calculator/mubiicq45q

Problems

1. Can you think of a function, maybe in your intended specialization, that might have a similar graph?

Let $f(x) = \frac{1}{1+e^{-kx}}$ where $k > 0$ is a constant.

2. What does the parameter k represent? In particular, which of the posted graphs do you think has the biggest k?

Additional problems

1. The 1 in the numerator is also a parameter, just one that is set to equal 1. What would happen to the graph if we changed the 1 to a 2.

2. $f(x)$ satisfies the differential equation $f'(x) = kf(x)\big(1 - f(x)\big)$. Can you confirm this?

3. Draw a set of axes with $f(x)$ as your horizontal axis and $f'(x)$ as your vertical axis. Then sketch the graph of $f'(x) = kf(x)\big(1 - f(x)\big)$. (Hint: it is a parabola.) What characteristics determined in this small class can you determine using this graph?

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