Math101C: Integral Calculus

Integration by partial fraction decomposition

S. Nie[1]

[1]Department of Mathematics
University of British Columbia

Small Class III for C15,18,22,24

Outline

Outline

the idea

Examples

- Which would you rather integrate?
  1. $$\int \left(\frac{1}{x-1} - \frac{1}{2x+1} \right) dx$$
  2. $$\int \left(\frac{x+2}{2x^2 - x - 1} \right) dx$$
  3. Neither
  4. Both

the idea

Examples

- Alright, let's solve the integral in 1

1. $$\int \left(\frac{1}{x-1} - \frac{1}{2x+1} \right) dx$$

the idea

Examples

- What happens if we find a common denominator and add the fractions in integral 1

the idea

Takeaway

Integral 2 would be doable but we need a way to "undo" finding a common denominator. This method is called the method of *partial fractions*.

Outline

big example

Examples

- Consider $\int \left(\frac{7x+13}{(2x+5)(x-2)} \right) dx$. Start by supposing our function can be written in the following way

$$\frac{7x+13}{(2x+5)(x-2)} = \frac{A}{2x+5} + \frac{B}{x-2}$$

- Our goal is to find constants A and B
- How do we solve for A and B

big example

Examples

- We can also now compute the integral

$$\int \left(\frac{7x+13}{(2x+5)(x-2)} \right) dx$$

Takeaways

Takeaways

- When doing partial fractions, selecting convenient x values can simplify your algebra.
- Write the big fraction as two simpler fractions and solve for the numerator.

Outline

in context

Examples

- Compute $\int \sec x dx$. Let's transform this in a few steps
  - Step 1: Transform the integral into $\int \frac{\cos x}{\cos^2 x} dx$.
  - Step 2: Assuming step 1, apply the substitution $u = \sin x$ to transform the integral into $-\int \frac{1}{u^2-1} du$.

in context

Examples

- Now we can compute the integral through method of partial fraction.

$$\int \sec x dx$$

Takeaways

Takeaways

- Sometimes we combine partial fractions with other techniques.

Addtional Problems

- CLP-2 Section 1.10: Q7, Q8, Q11, Q12, Q18, Q19, Q24, Q26

For Additional Problems I

E. Yeager, J. Feldman, A. Rechnitzer
CLP-2 Integral Calculus Exercise
https://personal.math.ubc.ca/~CLP/CLP2/clp_2_ic_problems.pd