Math 100C

Section C24,34,35,36

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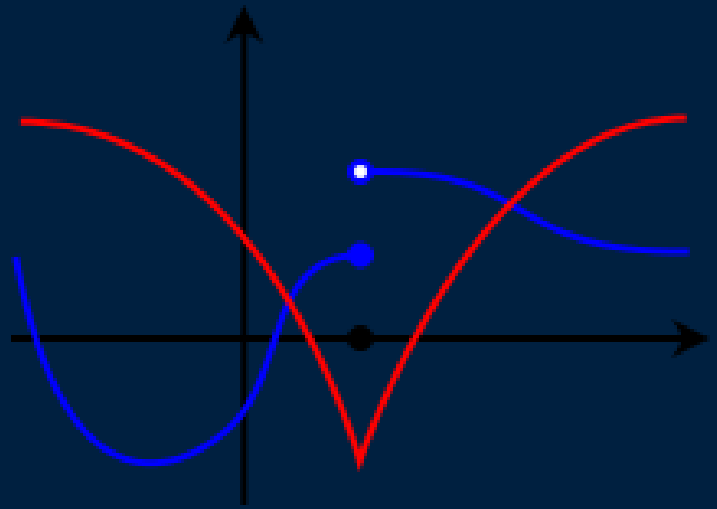
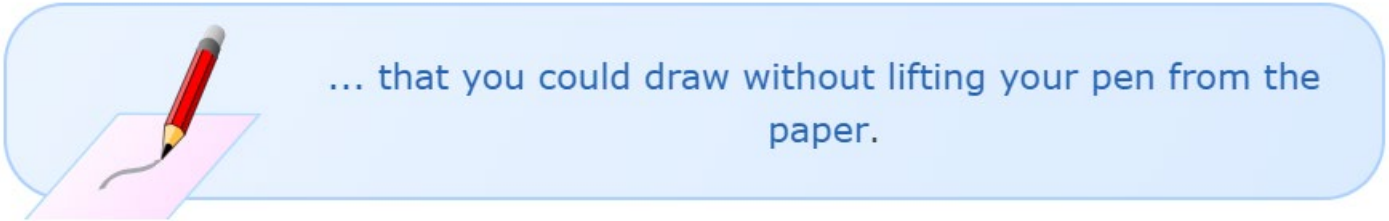
Topic: Continuity



Intuitive Definition of a Continuous Function

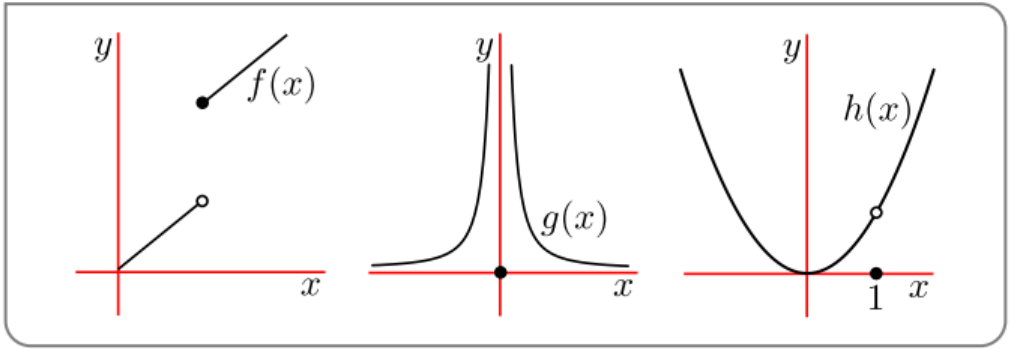
- **A function is continuous if it is a single unbroken curve**
- **A function is continuous if we can draw the graph from start to finish without ever once picking up your pencil**



Example of Discontinuous Functions

- **Jump discontinuity**
- **Infinite discontinuity**
- **Removable discontinuity, i.e. hole**

Consider the functions drawn below



These are

$$f(x) = \begin{cases} x & x < 1 \\ x+2 & x \geqslant 1 \end{cases} \qquad g(x) = \begin{cases} 1/x^2 & x \neq 0 \\ 0 & x = 0 \end{cases} \qquad h(x) = \begin{cases} \frac{x^3-x^2}{x-1} & x \neq 1 \\ 0 & x = 1 \end{cases}$$

Reflections

- What is $\lim_{x \to 2} x^3$? Why can we just “plug it in”?

- What is the *formal* mathematical definition of continuous function? **(CLP Textbook)**

https://personal.math.ubc.ca/~CLP/CLP1/clp_1_dc_text.pdf

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- Draw a function whose domain is all real numbers that is *not* continuous.

- What is a real-life example of discontinuous function?

Problems

- Sketch

$$f(x) = \frac{x^3 - 2x^2}{x - 2}$$

- Find a such that $g(x)$ is continuous

$$g(x) = \begin{cases} \dfrac{x^3 - 2x^2}{x - 2}, \text{if } x \neq 2 \\ a, \text{if } x = 2 \end{cases}$$

- Find k such that $f(x)$ is continuous

$$f(x) = \begin{cases} 8 - kx, \text{if } x < k \\ x^2, \text{if } x \geq k \end{cases}$$

Group Activity

- Design a function with exactly two discontinuities. It can contain familiar functions, and it can be defined piecewise. Your task will be to find the discontinuities in another team's function.

- Trade functions with another team and find their discontinuities!

Additional Problems

- CLP1-Problem book section 1.6: **Q1,Q7-11,Q13-18, Q20**

https://personal.math.ubc.ca/~CLP/CLP1/clp_1_dc_problems.pdf

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