

**THE UNIVERSITY OF BRITISH COLUMBIA**

**Math 312 Section 951**

**Calculators are allowed**

**No cell phones or information sheets**

**Test begins at 10:00 am and ends at 10:50am**

TEST #4

August 2, 2023

NAME

STUDENT NUMBER

1. Create valid ISBN-10 codes in each of the following cases by replacing the symbol ? by the correct digit. Show your calculations.

(a) 0-19-8?3804-9

(b) 91-554-2125-?

2. Let  $\phi(x)$  be Euler's  $\phi$  function.

(a) Find  $\phi(891)$  and  $\phi(4125)$

(b) Find all integers such that  $\phi(x) = 6$ . Show your work.

3. Show that 25 is a strong pseudoprime to the base 7.

4. Show that if  $p$  and  $q$  are distinct primes, then

$$p^{(q-1)} + q^{(p-1)} = 1 \pmod{pq}.$$

5. (a) Using Fermat's little theorem, find the last digit in the base 7 expansion of  $3^{(100)}$ .
- (b) What is the remainder when  $40!$  is divided by 1763?