

# Math101C: Integral Calculus

## Properties of Definite Integrals

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University of British Columbia

Small Class I for C15,18,22,24



# Outline

## 1 Course Information

- About Me and Our TA
- Contacts and Small Class Participations

## 2 Problems and Takeaways

- Facts of Definite Integrals
- Arithmetics of Definite Integrals



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# About Me

- 2010-2014, **Bachelor of Science (Honors)**, *National University of Singapore*, majoring in chemistry, minor in mathematics and environmental chemistry
- 2019-2021, **Master of Science**, *National University of Singapore*, Mathematics
- 2021-now, **Doctor of Philosophy** (in progress), *University of British Columbia*, in the field of mathematical biology, my interests are in single molecule localization microscopy.
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# About Our TA

- Section 24: Syed Nazif Ishrak
- Section 22: Sogand Golshahian
- Section 18: Ruilong Liu
- Section 15: Ivan (Yuan) Gao





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# Office Hours

- Monday 2:00pm to 3:00pm @ IBLC 461
- Tuesday 2:00pm to 3:00pm @ ANGU 339



# Participation

Assessment	Percent
WeBWork Assignment $\times 10$	10%
WeBWork Quizzes $\times 10$	10%
Written Assignment $\times 5$	20%
<b>Participation</b>	10% ( <b>attendance</b> , practice exams)
Final	50%

Table: Composition of Provisional Course Grade

- Attendance is compulsory unless with forgivable reason
  - Being late for more than 15 minutes is considered as absent
  - Unwillingness to work on group activities in class may result in absence
- Extra bonus or penalty may be applied based on active class contributions.



# Forming Groups

- Form a team of 4
- Introduce yourselves. Share one thing you remember enjoying about your last mathematics course
- Come up with a team name!





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# Property 1

## Fact

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

*Question: Write an English sentence explaining why this property is true.*



# Property 2

## Definition

The definite integral of  $f(x)$  with lower bound  $a$  and upper bound  $b$  is the limit of right Riemann sum, i.e.

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(a + i\Delta x) \Delta x$$

where  $\Delta x = \frac{b-a}{n}$ .



## Property 2 (Cont'd)

### Examples

(a) Write down the Riemann sum (right-hand side) that corresponds to the integral

$$\int_2^5 x^2 dx$$

(b) Write down the Riemann sum corresponding to the integral

$$\int_5^2 x^2 dx$$

(c) How do they compare? Any guesses?



## Property 2 (Cont'd)

Fact

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$



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## Property 3

### Problem

Sketch  $x^2$  on the same axis as  $2x^2$  for  $x$  values between 0 and 3. How does the area under the curve of  $x^2$  compare to the area under the curve of  $2x^2$ .



## Property 3 (Cont'd)

### Takeaway

$$\int_a^b cf(x)dx = c \int_a^b f(x)dx$$

where  $c$  is a constant.

This property is inherited from the summation property.

$$\sum_{i=1}^n cf(x_i) = c \sum_{i=1}^n f(x_i)$$





# Property 4

## Problem

Consider

$$\int_0^3 (x^2 + 2) dx.$$

We'd like to say that

$$\int_0^3 (x^2 + 2) dx = \int_0^3 x^2 dx + \int_0^3 2 dx$$

Draw a picture to illustrate this truth.



## Property 4 (Cont'd)

### Takeaway

$$\int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

where  $c$  is a constant.

This property is inherited from the summation property.

$$\sum_{i=1}^n f(x_i) + g(x_i) = \sum_{i=1}^n f(x_i) + \sum_{i=1}^n g(x_i)$$



# Summary

1

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

2

$$\int_a^b f(x)dx = - \int_b^a f(x)dx$$

3

$$\int_a^b cf(x)dx = c \int_a^b f(x)dx$$

4

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$



# For Additional Problems I

CLP-2 Section 1.2: Q1, Q4-Q10

E. Yeager, J. Feldman, A. Rechnitzer *CLP-2 Integral Calculus Exercise*

[https://personal.math.ubc.ca/~CLP/CLP2/clp\\_2\\_ic\\_problems.pdf](https://personal.math.ubc.ca/~CLP/CLP2/clp_2_ic_problems.pdf)

