Math101C: Integral Calculus

Numerical Integration

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Small Class V for C15,18,22,24

Outline

1. Problems and takeaways
   - Implementing numerical methods using an online calculator
   - Implementing numerical methods with spreadsheets
2. Additional Problems

Outline

Numerical Methods by Geogebra

Examples

1. In GeoGebra, find under "tools" then "more" the "Freehand Shape" function. *https://www.geogebra.org/calculator*
2. Draw a freehand function defined on the interval $[-5, 5]$. Start with a simple function, whose area you can approximate easily by inspection. Make a guess for what you think the area should be.

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Numerical Methods by Geogebra

Examples

3 Use the "Algebra" tab to enter an equation to approximate the integral of your function from -5 to 5 using the Trapezoidal method with $n = 5$. Recall

$$\int_a^b f(x)dx \approx \frac{\Delta x}{2}(f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n))$$

Numerical Methods by Geogebra

Examples

4 Now approximate $\int_{-5}^{5} f(x)dx$ using $n = 20$. You'll want to make use of the "Sum" function. Use the function as follows. Next

$$\sum_{i=a}^{n} f(i) = Sum(f(i), i, a, n)$$

where the first entry is the summand, the second is the counter, the third is the start, and the fourth is the finish. Try this first on a sum that you already know such as $\sum_{i=1}^{10} i^2$.

https://www.geogebra.org/calculator/qgtgftwz *https://www.geogebra.org/calculator/sasatqv6*

Numerical Methods by Geogebra

Takeaway

The trapezoidal method can be implemented on functions that are not defined explicitly so long as we can evaluate points.

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Numerical Methods by Spreadsheets

Examples

1. Our goals is to approximate $\int_{-2}^{2} \sin x^2 dx \approx 1.60955$ using all three of our numerical methods via spreadsheet. Note that there's no technique that will allow us to integrate this exactly.
2. Take $n = 20$. Compute Δx.

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Numerical Methods by Spreadsheets

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3 Recall Trapezoidal method above as well as our other two methods. Right-Riemann sum

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(a+i\Delta x)\Delta x = \Delta x(f(x_1)+f(x_2)+...+f(x_n))$$

and Simpson's rule

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4)$$

$$+... + 4f(x_{n-1}) + f(x_n))$$

Numerical Methods by Spreadsheets

Examples

4 Using a spreadsheet program (Excel, LibreOffice Calc, Google Sheets) prepare the following columns:

- Column A: this is your counter i. It starts at 0 and goes up to $n = 20$.
- Column B: this is your x_i. Recall $x_i = a + i\Delta x$. You can use column A in place of i.
- Column C: $f(x_i)$. You can use column B together with the built-in function $SIN()$.

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Numerical Methods by Spreadsheets

Examples

- Columns D, E, F: these are the coefficients for the three methods right-Riemann sum, trapezoidal rule, and Simpson's rule respectively. You can enter these by hand but try finding a faster way. For Simpson's method, you might use the $MOD()$ function on column A to tell you whether i is even or odd. You can add 1 and then multiply by 2 to get the coefficients you're looking for.
- Columns G, H, I: these are the coefficients for your respective method multiplied by the functions value.
- Once you have all the columns set up, you can use the $SUM()$ function on columns G, H, I, and apply the correct multiplication to find your desired value!

https://docs.google.com/spreadsheets/d/1d73d02jB4sb_qtIoKtvMWKNXk8GAF_6NfsV8XBNIUd4/edit?usp

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Numerical Methods by Spreadsheets

Examples

5 Which method performs better in this case? Is that what you expected?

https://www.wolframalpha.com/input?i=int+from+-2+to+2+of+sin%7Bx%5E2%7D+dx+using+trapezoid+method+with+n%3D20

Numerical Methods by Spreadsheets

Takeaways

We can use spreadsheets to implement our schemes and approximate integrals numerically.

Addtional Problems

- Imagine you wanted to explain to a friend (or a computer) how to implement one of our numerical methods. Write out a list of instructions, a numerical recipe, if you will, to give to your friend. If you know some computing, you can use sudo-code (or any language you prefer).
- Using your spreadsheet, approximate $\int_{-5}^{5} \sin x^2 dx$ using Simpson's method. Sketch the graph of $\sin x^2$. Should your numerical methods be more or less accurate farther away from $x = 0$. What about the behaviour of this function will be difficult for our numerical methods to capture?

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For Additional Problems I

E. Yeager, J. Feldman, A. Rechnitzer
CLP-2 Integral Calculus Exercise
https://personal.math.ubc.ca/~CLP/CLP2/clp_2_ic_problems.pd