

# Math100C X

C23,34,35,26

Shikun Nie, PhD student, Department of Mathematics, UBC

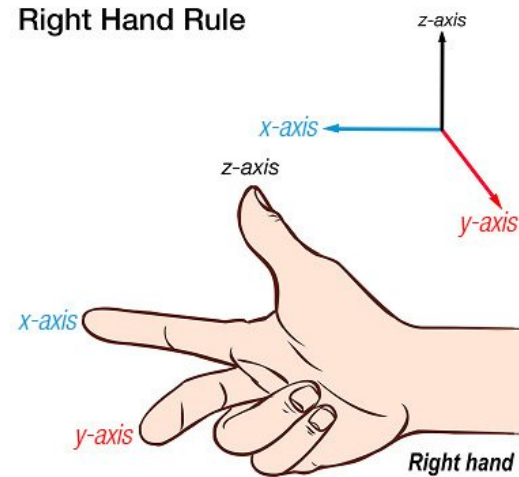


# Topic: Sketching functions of two variables



# Plotting in 3D

1. Fold a loose piece of paper to represent the  $x$ - $y$ - $z$  axes. Draw lines in the creases to represent the  $x$  axis,  $y$  axis and  $z$  axis.



## Plotting in 3D

2. Plot the points  $(1,0,0)$ ,  $(0,1,0)$  and  $(0,0,1)$ , marking them with dots. Once you are done, plot the additional points  $(2,2,0)$ ,  $(2,0,2)$  and  $(0,2,2)$



## Plotting in 3D

3. Using a pen or other small object represent the point  $(3,2,1)$  in space, for example by hold the base of the pen at the origin and the writing end at the point.



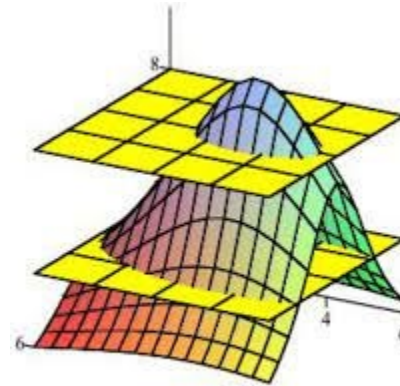
## Sketching level curves and traces

1. Consider  $z = x^2 + y^2$ . We want to get a picture of this function but need to sneak up on it.
2. To make things easier, let's get rid of one variable. Set  $z = 4$  and plot the resulting equations in 2D. Do the same with  $z = 1$ ,  $z = 0$  and  $z = -1$ .



# Sketching level curves and traces

3. **Definition:** A *level curve* is 2D horizontal slice of a larger 3D surface. We find these by setting  $z$  equal to a constant.
4. We can get a good sense of what a function or surface in 3D looks like by plotting level curves.



The planes  $z=4$  and  $z=8$  "slicing" the surface

## Sketching level curves and traces

5. So far, we know our surface doesn't have points for  $z < 0$ , is a single point at  $z = 0$  and is a tower of increasing larger circles for  $z > 0$ . How does the circle change? In our surface (a) a bowl (b) a cone (c) a wormhole?





## Sketching level curves and traces

6. To get a sense of which it is, let's plot 2D curves while fixing values for  $x$  and  $y$  this time.

Sketch when  $y = 0$ ,  $x = 0$ ,  $x = 1$  and  $y = 2$ .



## Sketching level curves and traces

7. **Definition:** A *trace* is just like a level curve only we can set any variable equal to a constant. Traces are also very useful in understanding the shape of a surface.



## Sketching level curves and traces

8. Now what shape is our surface  $z = x^2 + y^2$

<https://www.geogebra.org/3d?lang=en>



# Resources

Textbook sections:

Section 1.3 of Optimal. Integral, Likely: <http://personal.math.ubc.ca/~elyse/OIL/>

Additional problems:

Optimal, Integral, Likely Practice Book Section 1.3: Q1, Q2, Q3, Q7





THE UNIVERSITY OF BRITISH COLUMBIA