Math100C IV

C23,34,35,26

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# Trigonometric functions and their derivatives

Definition of trigo functions

- What do you recall about how the functions $\sin\theta$, $\cos\theta$ and $\tan\theta$ are defined?
(i)   Side length ratio
(ii)   Unit circle
(iii) Graph

Derivatives of trigo functions

- Can you determine the derivatives of the functions from the graph of the functions?

Hint: the derivatives of $f(x)$ at a particular point x_0 is the slope of the tangent line at x_0
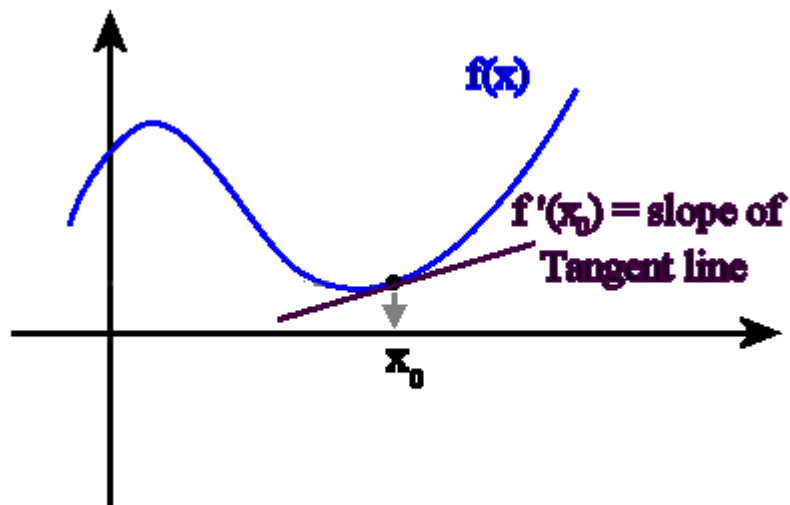
Derivatives of trigo functions

$$\frac{d}{d\theta} \sin \theta = \cos \theta$$

$$\frac{d}{d\theta} \cos \theta = -\sin \theta$$

$$\frac{d}{d\theta} \tan \theta = \frac{1}{\cos^2 \theta}$$

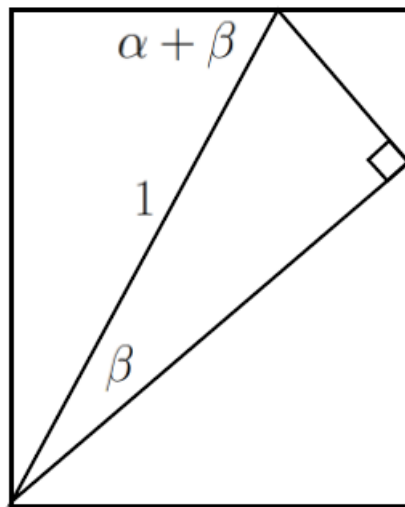However, can you prove it to convince yourself these rules are true?

https://www.desmos.com/calculator/kpfk2qupbg

Derivatives of trigo functions

- Write the definition of $\frac{d}{d\theta}\sin\theta$ and $\frac{d}{d\theta}\cos\theta$ using the limit definition of derivatives.

- What tool(s) do you think you need to resolve the limit?

Derivatives of trigo functions

- Determine the lengths of the missing sides

Derivatives of trigo functions

- Compound angle formula

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

- Challenge yourself to prove the following

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Derivatives of trigo functions

- Use the addition formulas to write $\frac{d}{d\theta}\sin\theta$ and $\frac{d}{d\theta}\cos\theta$ in terms of $\sin\theta$ and $\cos\theta$. (You may still have unresolved limits in your answers.)

- Without using a formal proof, how would you convince someone of the following limits?

(a)

$$\lim_{h\to 0}\frac{\sin h}{h} = 1$$

(b)

$$\lim_{h\to 0}\frac{\cos h - 1}{h} = 0$$

- Apply these limits to the work already done on $\frac{d}{d\theta}\sin\theta$ and $\frac{d}{d\theta}\cos\theta$

Additional Problems

- Find $\frac{d}{d\theta} \tan \theta$. (Hint: you do not need the limit definition of derivatives.)
- Three secondary trigonometric functions are

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Find their derivatives.

- CLP-1 Problem Book Section 2.8: Q3-Q15