Math100C VIII

C23,34,35,26

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Topic: Quantitative analysis of differential equations

Slope Fields

Recall the differential equations

$$\frac{dy}{dt} = y(1-y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

1. What is $\frac{dy}{dx}$ when $t = 3$?

Slope Fields

Recall the differential equations

$$\frac{dy}{dt} = y(1 - y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

2. What is $\frac{dy}{dx}$ for the following y values?

$(a)\ y = \frac{1}{8}$

$(b)\ y = \frac{1}{2}$

$(c)\ y = \frac{7}{8}$

$(d)\ y = 2$

Slope Fields

Recall the differential equations

$$\frac{dy}{dt} = y(1-y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

3. For what y-values is $\frac{dy}{dx} = 0$

Slope Fields

Recall the differential equations

$$\frac{dy}{dt} = y(1 - y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

4. **Definition**: A *slope field* records values for $\frac{dy}{dt}$ on a y vs t axis.

Slope Fields

Recall the differential equations

$$\frac{dy}{dt} = y(1-y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

5. Draw the axes for a slope field for the differentia equation, and draw slopes for $y = 0, y = 1$ and $y = 2$.

https://aeb019.hosted.uark.edu/dfield.html

Slope Fields

Recall the differential equations

$$\frac{dy}{dt} = y(1 - y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

6. Sketch some solutions to the differential equation using your slope field.

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Slope Fields

Recall the differential equations

$$\frac{dy}{dt} = y(1-y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

7. Suppose you are given the initial condition $y(0) = \frac{1}{4}$. Sketch the solutions satisfying the condition.

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Phase Portraits and stability

Recall the differential equations

$$\frac{dy}{dt} = y(1 - y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

1. **Definition**: A *steady state* is a state in which a system is not changing. For example, a solution $y(t)$ to a differential equation is a steady state solution if y remains constant for all t.

Phase Portraits and stability

Recall the differential equations

$$\frac{dy}{dt} = y(1 - y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

2. What are the steady states for the given differential equation?

Phase Portraits and stability

Recall the differential equations

$$\frac{dy}{dt} = y(1-y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

3. We find steady state by setting $\frac{dy}{dx} = 0$.

Phase Portraits and stability

Recall the differential equations

$$\frac{dy}{dt} = y(1-y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

4. Partition a horizontal axis using the steady state. Determine the sign $\frac{dy}{dt}$ between the steady states.

Phase Portraits and stability

Recall the differential equations

$$\frac{dy}{dt} = y(1-y)$$

From Week 6 small classes. (In that small class, you sketched a solution $y = \frac{1}{1+e^{-x}}$.)

5. **Definition**: A *phase line* records values for $\frac{dy}{dt}$ on a horizontal y axis. A steady state solution is *stable* if all nearby solutions tend to it. (Otherwise, we say the steady state is *unstable* or *semi-stable*)

Phase Portraits and stability

Now consider the augmented differential equation

$$\frac{dy}{dt} = y(1-y) - H$$

6. Draw the slope field and phase line for three different values of H: $H = \frac{1}{8}$, $H = \frac{1}{4}$ and $H = \frac{1}{2}$.

If the differential equation describes a population y (with respect to time t) and H is a term that represents a “constant harvest”, what is the overall effect of H?

Additional problems:

1. Exercises 13.2, 13.3, 13.5, 13.6, 13.7, 13.12 in Differential calculus for the life sciences
2. Draw slope fields and phase lines for the differential equations: (a) $y' = -3y$ (b) $y' = y - y^3$ (c) $y' = y - y^2$ (d) $y' = y^2 + y^3$

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