THE UNIVERSITY OF BRITISH COLUMBIA

Math 312 Section 951

Calculators are allowed
No cell phones or information sheets
Test begins at 10:00 am and ends at 10:50am

TEST #5   August 9, 2023

NAME

STUDENT NUMBER

1. Let $M_n = 2^n - 1$ be the n-th Mersenne number. Prove that if M_n is prime, then so is n.

2. If n is composite, prove that $\phi(n)$ is less than or equal to $n - n^{1/2}$.

3. (a) Decrypt the ciphertext message LFDPH LVDZL FRQTX HUHG, which has been encrypted using the Caesar cipher. Show work on the first 5 letters but for full credit be sure to decrypt the entire 19 letter message.

(b) Encrypt the message BITCOIN using the affine transformation $C = (2P + 4) \pmod{26}$.

4. What is the ciphertext that is produced when RSA encryption with the key $(e, n) = (7, 2627)$ is used to encrypt the message DRAMAS?

5. (a) Find all positive integers whose sum of divisors is 24. Show your work.

   (b) Find the smallest positive integer with number of divisors equal to 14. Show your work.