Math312 Test #3 Solutions

July 27, 2023

Typo correction: In solutions to test #2, question 4a, the solution is $x = (-11) \times 11 + 47t, y = (-11) \times (-7) - 30t$. Similarly the s in question 4b should be t.

1a

$$3x = 6 \mod 9$$

Using division rule for modulo

$$x = 2 \mod 3$$

So the solutions is

$$x = 2, 5, 8 \mod 9$$

1a

$$17x = 14 \mod 21$$

$$5 \times 17x = 5 \times 14 \mod 21$$

$$x = 7 \mod 21$$

2a Since

$$5 = 0 \mod 5$$

$$11 = 1 \mod 5$$

$$2 = 2 \mod 5$$

$$3 = 3 \mod 5$$

$$19 = 4 \mod 5$$

Therefore $\{2, 3, 5, 11, 19\}$ is a complete system of residues $\mod 5$.

2b

$$\{f_0 = 0, f_1 = 1, f_3 = 2, f_4 = 3, f_5 = 5, f_6 = 8, f_9 = 34, f_{11} = 89, 6, 7\}$$

forms a complete system of residues $\mod 10$

3 The solution is

$$x = 7 \times 13 \times 17m_1 + 8 \times 11 \times 17m_2 + 9 \times 11 \times 13m_3 \mod 11 \times 13 \times 17$$

where m_1, m_2, m_3 are solutions of the following equations

$$13 \times 17m_1 = 1 \mod 11$$

$$11 \times 17m_2 = 1 \mod 13$$

$$11 \times 13m_3 = 1 \mod 17$$

After solving these equations, we have

$$m_1 = 1 \mod 11$$

$$m_2 = 8 \mod 13$$

$$m_3 = 5 \mod 17$$

Therefore, the solution to the system of linear congruences is

$$x = 7 \times 13 \times 17 \times 1 + 8 \times 11 \times 17 \times 8 + 9 \times 11 \times 13 \times 5 \mod 11 \times 13 \times 17$$

which is

$$x = 502 \mod 2431$$

4 The lowest common multiple of $6, 10, 15$ is $2 \times 3 \times 5 = 30$. The orginal system of linear congruences is equivalent to,

$$x = 1 \mod 2$$

$$x = 2 \mod 3$$

$$x = 3 \mod 5$$

The solution to the above system of linear congruences is

$$x = 1 \times 3 \times 5m_1 + 2 \times 2 \times 5m_2 + 3 \times 2 \times 3m_3 \mod 2 \times 3 \times 5$$

where m_1, m_2, m_3 are solutions of the following equations

$$3 \times 5m_1 = 1 \mod 2$$

$$2 \times 5m_2 = 1 \mod 3$$

$$2 \times 3m_3 = 1 \mod 5$$

After solving these equations, we have

$$m_1 = 1 \mod 2$$

$$m_2 = 1 \mod 3$$

$$m_3 = 1 \mod 5$$

Therefore, the solution to the system of linear congruences is

$$x = 23 \mod 30$$

5a It is easy to spot that $x = -2, y = 1$ is a solution to $5x + 11y = 1$. So the general solution to $5x + 11y = n$ is

$$x = -2n + 11t, y = n - 5t$$

For x, y to be nonnegative integers, we require that

$$5t \leq n \leq \frac{11t}{2}$$

When $t = 7$, $n = 35, 36, 37, 38$ while when $t = 8$, $n = 40, 41, 42, 43, 44$. Therefore, the largest $n = 39$.

5b It is easy to spot that $x = -1, y = 4$ is a solution of

$$5x + 11y = 39$$

Therefore, the general solution is

$$x = -1 + 11t, y = 4 - 5t, t \in \mathbb{Z}$$