Math101C: Integral Calculus

Volumes

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Small Class IV for C15,18,22,24

Outline

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Volume of a cone

Examples

1 Construct a cone using the paper, scissors, and tape provided.

Volume of a cone

Examples

2. What kind of info can we gather from the cone using our ruler?
3. Find the area of the base of the cone.
4. Find the area of the cross section halfway down the cone.
5. Orient the cone so it's point down. Consider an axis through the center of the cone with $x = 0$ at the point. Suppose we had the cross sectional area at a number of points along the cone, $x_1, x_2, x_3, ..., x_n$ with Δx being the distance between each point. How do we find the true volume?
   https://www.geogebra.org/m/g8QE7eHc
6. For any height x, find the cross sectional area of your cone. Then compute the total volume of your cone!
7. Let's compare our cone to see whether our calculated volume

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Takeaway

Integrals can be used for more than just area – we can also compute volumes.

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Volume of revolution

Examples

1. Find the volume of the surface obtained from revolving the function $y = \sqrt{x}$ around the x axis from $x = 0$ to $x = 4$. Use the following steps
   i. Draw a picture and label a small cross section of volume.
   ii. Find the area $A(x)$ of this disk.
   iii. Set up an integral that describes the required volume.
   iv. Integrate!

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Volume of revolution

Takeaways

We can find the volume of a 3D object providing we can find the cross sectional area as a function of x: $\int_a^b A(x)dx$. For a surface of revolution, the cross sectional area is a disk as we get $\int_a^b \pi r^2(x)dx$.

Outline

Volume of a pyramid

Examples

- Find the volume of a square based pyramid with height h and base length b.
  https://www.whitman.edu/mathematics/calculus_online/section09.0

Volume of a pyramid

Takeaways

Not every cross section will be a circle, but if we can find the cross sectional area as an explicit function of, say, height, we can integrate to find the volume.

Addtional Problems

- CLP-2 Section 1.6: Q1-Q2, Q4a, Q7, Q8, Q14, Q16, Q17, Q20a, Q22

For Additional Problems I

E. Yeager, J. Feldman, A. Rechnitzer
CLP-2 Integral Calculus Exercise
https://personal.math.ubc.ca/~CLP/CLP2/clp_2_ic_problems.pd