In [104]:

```
import numpy as np
import math
import scipy as scp
import scipy.stats as ss
import matplotlib.pyplot as plt
```

In [105]:

```
class Option_param():
    """
    Option class wants the option parameters:
    S0 = current stock price
    K = Strike price
    T = time to maturity

    exercise = European or American
    """
    def __init__(self, S0=100, K=100, T=1, payoff="call", exercise="Europ
ean"):
        self.S0 = S0
        self.K = K
        self.T = T
        self.Delta = None
        self.Gamma = None
        self.Theta = None
        self.Vega = None
        self.Rho = None

        if (exercise=="European" or exercise=="American"):
            self.exercise = exercise
        else:
            raise ValueError("invalid type. Set 'European' or 'American'"
)

        if (payoff=="call" or payoff=="put"):
            self.payoff = payoff
        else:
            raise ValueError("invalid type. Set 'call' or 'put'")
```

In [106]:

```
class Diffusion_process():
    """
    Class for the diffusion process:
    r = continuously compounded risk-free interest rate (% p.a.)
    q = continuously compounded dividend yield (% p.a.)
    sig = volatility (% p.a.)
    """
    def __init__(self, r=0.1, q=0.0, sig=0.2):
        self.r = r
        if (q<0):
            raise ValueError("Dividend must be positive")
        else:
            self.q = q

        if (sig<=0):
            raise ValueError("sig must be positive")
        else:
            self.sig = sig

    def exp_RV(self, S0, T, N):
        W = ss.norm.rvs( (self.r-0.5*self.sig**2)*T , np.sqrt(T)*self.sig
, N)
        S_T = S0 * np.exp(W)
        return S_T
```

In [107]:

```
class Option_pricer():

    def __init__(self, Option_info, Process_info ):
        """
        Process_info:  of type Diffusion_process. It contains (r,mu, sig)
i.e.  interest rate, drift coefficient, diffusion coefficient

        Option_info:  of type Option_param. It contains (S0,K,T) i.e. cur
rent price, strike, maturity in years
        """
        self.r = Process_info.r           # interest rate
        self.q = Process_info.q
        self.sig = Process_info.sig       # diffusion coefficient
        self.S0 = Option_info.S0          # current price
        self.K = Option_info.K            # strike
        self.T = Option_info.T            # maturity in years
        self.exercise = Option_info.exercise
        self.payoff = Option_info.payoff


    def payoff_f(self, S):
        if self.payoff == "call":
            Payoff = np.maximum( S - self.K, 0 )
        elif self.payoff == "put":
            Payoff = np.maximum( self.K - S, 0 )
        return Payoff


    def Binomial_Tree(self, N=10000):
        dT = float(self.T) / N                             # Delta t
        u = np.exp(self.sig * np.sqrt(dT))                 # up factor
        d = 1.0 / u                                    # down factor

        V = np.zeros(N+1)                              # initialize the pr
ice vector
        S_T = np.array( [(self.S0 * u**j * d**(N - j)) for j in range(N +
1)] )  # price S_T at time T

        a = np.exp((self.r - self.q ) * dT)     # risk free compound retur
n
        p = (a - d)/ (u - d)  # risk neutral up probability
        q = 1.0 - p           # risk neutral down probability

        if self.payoff =="call":
            V[:] = np.maximum(S_T-self.K, 0.0)
        elif self.payoff =="put":
            V[:] = np.maximum(self.K-S_T, 0.0)
        else: raise ValueError("invalid type. Set 'call' or 'put'")


        if self.exercise == "American":
            for i in range(N-1, -1, -1):
                V[:-1] = np.exp(-self.r*dT) * (p * V[1:] + q * V[:-1])
# the price vector is overwritten at each step
```

```
                S_T = S_T * u                    # it is a tricky way to
 obtain the price at the previous time step
                if self.payoff=="call":
                    V = np.maximum( V, S_T-self.K )
                elif self.payoff=="put":
                    V = np.maximum( V, self.K-S_T )

            return ("American", V[0])

        elif self.exercise == "European":
            for i in range(N-1, -1, -1):
                V[:-1] = np.exp(-self.r*dT) * (p * V[1:] + q * V[:-1])
# the price vector is overwritten at each step

            return ("European",  V[0])

        else: raise ValueError("invalid type. Set 'American' or 'Europea
n'")


    def Monte_Carlo_Simulation(self, N=10000, paths=10000, order=2):
        """
        Monte Carlo Simulation for pricing American options

        N = number of time steps
        paths = number of generated paths
        order = order of the polynomial for the regression
        """
        dt = self.T/(N-1)            # time interval
        df = np.exp(-self.r * dt)  # discount factor per time time interv
al

        X0 = np.zeros((paths,1))
        increments = ss.norm.rvs(loc=(self.r - self.q - self.sig**2/2)*dt
, scale=np.sqrt(dt)*self.sig, size=(paths,N-1))
        X = np.concatenate((X0,increments), axis=1).cumsum(1)
        S = self.S0 * np.exp(X)

        if self.exercise == "American":
            if self.payoff =="put":
                H = np.maximum(self.K - S, 0)    # intrinsic values for pu
t option
                V = np.zeros_like(H)             # value matrix
                V[:,-1] = H[:,-1]

                # Valuation by Monte Carlo Simulation
                for t in range(N-2, 0, -1):
                    good_paths = H[:,t] > 0
                    rg = np.polyfit( S[good_paths, t], V[good_paths, t+1]
* df, order)    # polynomial regression
                    C = np.polyval( rg, S[good_paths,t] )
# evaluation of regression

                    exercise = np.zeros( len(good_paths), dtype=bool)
                    exercise[good_paths] = H[good_paths,t] > C
```

```
                V[exercise,t] = H[exercise,t]
                V[exercise,t+1:] = 0
                discount_path = (V[:,t] == 0)
                V[discount_path,t] = V[discount_path,t+1] * df

            V0 = np.mean(V[:,1]) * df  #
            return ("American" , V0)

        elif self.payoff =="call":
            H = np.maximum(S - self.K, 0)    # intrinsic values for pu
t option
            V = np.zeros_like(H)             # value matrix
            V[:,-1] = H[:,-1]

            # Valuation by Monte Carlo Simulation
            for t in range(N-2, 0, -1):
                good_paths = H[:,t] > 0
                if t==N-2: type(good_paths)
                rg = np.polyfit( S[good_paths, t], V[good_paths, t+1]
* df, order)    # polynomial regression
                C = np.polyval( rg, S[good_paths,t] )
# evaluation of regression

                exercise = np.zeros( len(good_paths), dtype=bool)
                exercise[good_paths] = H[good_paths,t] > C

                V[exercise,t] = H[exercise,t]
                V[exercise,t+1:] = 0
                discount_path = (V[:,t] == 0)
                V[discount_path,t] = V[discount_path,t+1] * df

            V0 = np.mean(V[:,1]) * df  #
            return("American" , V0)

        else: raise ValueError("invalid type. Set 'call' or 'put'")

    elif self.exercise == "European":
        if self.payoff =="put":
            H = np.maximum(self.K - S, 0)    # intrinsic values for pu
t option
            V = np.zeros_like(H)             # value matrix
            V[:,-1] = H[:,-1]

            # Valuation by Monte Carlo Simulation
            for t in range(N-2, 0, -1):
                V[:,t] = V[:,t+1] * df

            V0 = np.mean(V[:,1]) * df  #
            return("European" , V0)

        elif self.payoff =="call":
            H = np.maximum(S - self.K, 0)    # intrinsic values for pu
t option
            V = np.zeros_like(H)             # value matrix
            V[:,-1] = H[:,-1]
```

```
                # Valuation by Monte Carlo Simulation
                for t in range(N-2, 0, -1):
                    V[:,t] = V[:,t+1] * df

                V0 = np.mean(V[:,1]) * df  #
                return("European" , V0)

            else: raise ValueError("invalid type. Set 'call' or 'put'")

        else: raise ValueError("invalid type. Set 'American' or 'Europea
n'")



    def Black_Scholes(self):
        """ Black Scholes closed formula:
            payoff: call or put.
            S0: float.    initial stock/index level.
            K: float strike price.
            T: float maturity (in year fractions).
            r: float constant risk-free short rate.
            sigma: volatility factor in diffusion term. """

        d1 = (np.log(self.S0/self.K) + (self.r - self.q + self.sig**2 / 2
) * self.T) / (self.sig * np.sqrt(self.T))
        d2 = (np.log(self.S0/self.K) + (self.r - self.q - self.sig**2 / 2
) * self.T) / (self.sig * np.sqrt(self.T))

        if self.payoff=='call':
            return ("European", self.S0 * np.exp(-self.q * self.T) * ss.n
orm.cdf( d1 ) - self.K * np.exp(-self.r * self.T) * ss.norm.cdf( d2 ))
        elif self.payoff=='put':
            return ("European", self.K * np.exp(-self.r * self.T) * ss.no
rm.cdf( -d2 ) - self.S0 * np.exp(-self.q * self.T) * ss.norm.cdf( -d1 ))
        else:
            raise ValueError("invalid type. Set 'call' or 'put'")



    def plot(self):

        if self.exercise == "European":
            fig, axs = plt.subplots(2, figsize=(10,10))
            fig.tight_layout(pad = 7)

            K = np.arange(self.K*0.1,self.K*1.9, self.K*0.1)
            if self.payoff == "call":
                payoff = np.array(np.maximum(self.S0-K, 0))
            if self.payoff == "put":
                payoff = np.array(np.maximum(K-self.S0, 0))
            axs[0].plot(K, payoff, label = "exercise price at S0")


            price = np.array([])
            for k in np.arange(self.K*0.1,self.K*1.9, self.K*0.1):
                opt_param = Option_param(S0=self.S0, K=k, T=self.T, exerc
```

```
ise=self.exercise, payoff=self.payoff)
                diff_param = Diffusion_process(r=self.r, sig=self.sig)
                option = Option_pricer(opt_param, diff_param)
                price = np.append(price, option.Black_Scholes()[1])

            axs[0].plot(K, price, label = "Black_Scholes Pricing")
            axs[0].set_title("Spot price = " + str(self.S0))
            axs[0].set(xlabel = "Strike price (K)", ylabel = "Option Valu
e")
            axs[0].legend()



            S0 = np.arange(self.S0*0.1,self.S0*1.9, self.S0*0.1)
            if self.payoff == "call":
                payoff = np.array(np.maximum(S0-self.K, 0))
            if self.payoff == "put":
                payoff = np.array(np.maximum(self.K-S0, 0))
            axs[1].plot(S0, payoff, label = "exercise price at S0")


            price = np.array([])
            for S in np.arange(self.S0*0.1,self.S0*1.9, self.S0*0.1):
                opt_param = Option_param(S0=S, K=self.K, T=self.T, exerci
se=self.exercise, payoff=self.payoff)
                diff_param = Diffusion_process(r=self.r, sig=self.sig)
                option = Option_pricer(opt_param, diff_param)
                price = np.append(price, option.Black_Scholes()[1])

            axs[1].plot(S0, price, label = "Black_Scholes Pricing")
            axs[1].set_title("Strike price = " + str(self.S0))
            axs[1].set(xlabel = "Spot price (S0)", ylabel = "Option Valu
e")
            axs[1].legend()

        elif self.exercise == "American":
            fig, axs = plt.subplots(2, figsize=(10,10))
            fig.tight_layout(pad = 7)

            K = np.arange(self.K*0.1,self.K*1.9, self.K*0.1)
            if self.payoff == "call":
                payoff = np.array(np.maximum(self.S0-K, 0))
            if self.payoff == "put":
                payoff = np.array(np.maximum(K-self.S0, 0))
            axs[0].plot(K, payoff, label = "exercise price at S0")


            price = np.array([])
            for k in np.arange(self.K*0.1,self.K*1.9, self.K*0.1):
                opt_param = Option_param(S0=self.S0, K=k, T=self.T, exerc
ise=self.exercise, payoff=self.payoff)
                diff_param = Diffusion_process(r=self.r, sig=self.sig)
                option = Option_pricer(opt_param, diff_param)
                price = np.append(price, option.Binomial_Tree()[1])

            axs[0].plot(K, price, label = "Binomial_Tree Pricing")
```

```
            axs[0].set_title("Spot price = " + str(self.S0))
            axs[0].set(xlabel = "Strike price (K)", ylabel = "Option Value")
            axs[0].legend()



            S0 = np.arange(self.S0*0.1,self.S0*1.9, self.S0*0.1)
            if self.payoff == "call":
                payoff = np.array(np.maximum(S0-self.K, 0))
            if self.payoff == "put":
                payoff = np.array(np.maximum(self.K-S0, 0))
            axs[1].plot(S0, payoff, label = "exercise price at S0")


            price = np.array([])
            for S in np.arange(self.S0*0.1,self.S0*1.9, self.S0*0.1):
                opt_param = Option_param(S0=S, K=self.K, T=self.T, exercise=self.exercise, payoff=self.payoff)
                diff_param = Diffusion_process(r=self.r, sig=self.sig)
                option = Option_pricer(opt_param, diff_param)
                price = np.append(price, option.Binomial_Tree()[1])

            axs[1].plot(S0, price, label = "Binomial_Tree Pricing")
            axs[1].set_title("Strike price = " + str(self.S0))
            axs[1].set(xlabel = "Spot price (S0)", ylabel = "Option Value")
            axs[1].legend()


        else: raise ValueError("invalid type. Set 'American' or 'European'")



    def greeks(self):
        d1 = (np.log(self.S0/self.K) + (self.r - self.q + self.sig**2 / 2) * self.T) / (self.sig * np.sqrt(self.T))
        d2 = (np.log(self.S0/self.K) + (self.r - self.q - self.sig**2 / 2) * self.T) / (self.sig * np.sqrt(self.T))

        if self.payoff == "call":
            self.Delta = np.exp(-self.q * self.T) * ss.norm.cdf(d1)
            self.Gamma = (np.exp(-self.q * self.T - (d1**2)/2))/(self.S0 * self.sig * np.sqrt(self.T) * np.sqrt(2 * math.pi))
            self.Theta = (1/(self.T*365)) * (-((self.S0 * self.sig * np.exp(-self.q * self.T - (d1**2)/2))/(2 * np.sqrt(2 * self.T * math.pi))) - self.r * self.K * np.exp(-self.r * self.T) * ss.norm.cdf(d2) + self.q * self.S0 * np.exp(-self.q * self.T) * ss.norm.cdf(d1))
            self.Vega  = (1/100) * self.S0 * np.exp(-self.q * self.T - (d1**2)/2) * np.sqrt(self.T) * (1/np.sqrt(2 * math.pi))
            self.Rho   = (1/100) * self.K * self.T * np.exp(-self.r * self.T) * ss.norm.cdf(d2)

        elif self.payoff == "put":
            self.Delta = np.exp(-self.q * self.T) * (ss.norm.cdf(d1)-1)
```

```
            self.Gamma = (np.exp(-self.q * self.T - (d1**2)/2))/(self.S0 * self.sig * np.sqrt(self.T) * np.sqrt(2 * math.pi))
            self.Theta = (1/(self.T*365)) * (-((self.S0 * self.sig * np.exp(-self.q * self.T - (d1**2)/2))/(2 * np.sqrt(2 * self.T * math.pi))) + self.r * self.K * np.exp(-self.r * self.T) * ss.norm.cdf(-d2) - self.q * self.S0 * np.exp(-self.q * self.T) * ss.norm.cdf(-d1))
            self.Vega  = (1/100) * self.S0 * np.exp(-self.q * self.T - (d1**2)/2) * np.sqrt(self.T) * (1/np.sqrt(2 * math.pi))
            self.Rho   = (-1/100) * self.K * self.T * np.exp(-self.r * self.T) * ss.norm.cdf(-d2)

        else: raise ValueError("invalid type. Set 'call' or 'put'")

        print("Delta: ", self.Delta)
        print("Gamma: ", self.Gamma)
        print("Theta: ", self.Theta)
        print("Vega: " , self.Vega )
        print("Rho: "  , self.Rho  )
        print("\n")
```

In [108]:

```
def implied_vol(payoff, option_price, S0, K, T, r, q):
    # apply bisection method to get the implied volatility by solving the
BSM function
    precision = 0.00001
    upper_vol = 500.0
    lower_vol = 0.0001
    iteration = 0

    opt_param = Option_param(S0=S0, K=K, T=T, exercise="European", payoff
=payoff)
    diff_param = Diffusion_process(r=r, q=q, sig=250.0)
    option = Option_pricer(opt_param, diff_param)

    while 1:
        iteration +=1
        mid_vol = (upper_vol + lower_vol)/2.0
        option.sig = mid_vol
        price = option.Black_Scholes()[1]
        if payoff == "call":
            option.sig = lower_vol
            lower_price = option.Black_Scholes()[1]
            if (lower_price - option_price) * (price - option_price) > 0:
                lower_vol = mid_vol
            else:
                upper_vol = mid_vol
            if abs(price - option_price) < precision: break
            if iteration > 100: raise ValueError("Computational error occ
ured. ")

        elif payoff == "put":
            option.sig = upper_vol
            upper_price = option.Black_Scholes()[1]
            if (upper_price - option_price) * (price - option_price) > 0:
                upper_vol = mid_vol
            else:
                lower_vol = mid_vol
            if abs(price - option_price) < precision: break
            if iteration > 100: raise ValueError("Computational error occ
ured. ")

    return mid_vol
```

In [109]:

```
def greeks_hedge (*option):
    """
    if Delta is being hedged:      number of options needed = Number of h
edged Greeks
    if Delta is not being hedged:  number of options needed = Number of h
edged Greeks + 1
    """
    num_option = 0
    for opt in option:
        num_option += 1
        print("Option",num_option,":")
        opt.greeks()

    number = ['first', 'second', 'thirth', 'fourth', 'fifth']

    num_greeks = int(input("How many greeks are going to hedge: "))
    greeks = [np.array([],dtype = str)]
    for n in range(num_greeks):
        greeks = np.append(greeks, input("The " + number[n] + " hedging g
reek: "))


    stock_Delta = np.array([1])
    array_Delta = np.append(stock_Delta, [option[index].Delta for index i
n range(num_option-1)])
    stock_Gamma = np.array([0])
    array_Gamma = np.append(stock_Gamma, [option[index].Gamma for index i
n range(num_option-1)])
    stock_Theta = np.array([0])
    array_Theta = np.append(stock_Theta, [option[index].Theta for index i
n range(num_option-1)])
    stock_Vega  = np.array([0])
    array_Vega  = np.append(stock_Vega , [option[index].Vega  for index i
n range(num_option-1)])
    stock_Rho = np.array([0])
    array_Rho   = np.append(stock_Rho  , [option[index].Rho   for index i
n range(num_option-1)])

    matrix_Left  = np.array([])
    matrix_Right = np.array([])

    for g in greeks:
        if g == "Delta":
            matrix_Left  = np.append(matrix_Left, array_Delta)
            matrix_Right = np.append(matrix_Right, -option[-1].Delta)
        if g == "Gamma":
            matrix_Left  = np.append(matrix_Left, array_Gamma)
            matrix_Right = np.append(matrix_Right, -option[-1].Gamma)
        if g == "Theta":
            matrix_Left  = np.append(matrix_Left, array_Theta)
            matrix_Right = np.append(matrix_Right, -option[-1].Theta)
        if g == "Vega":
            matrix_Left  = np.append(matrix_Left, array_Vega)
            matrix_Right = np.append(matrix_Right, -option[-1].Vega)
```

```
        if g == "Rho":
            matrix_Left  = np.append(matrix_Left, array_Rho)
            matrix_Right = np.append(matrix_Right, -option[-1].Rho)


    matrix_Left.shape = (num_greeks,num_option)
    x = np.linalg.lstsq(matrix_Left, matrix_Right)
    result = np.append(x[0],1)
    for i in range(num_option+1):
        if i==0:
            print("\nPortfolio: ")
            print("Stock      : ", result[i])
        else:
            print("Option ",i,": ", result[i])



    matrix_greeks = np.array([np.append(array_Delta, option[-1].Delta), n
p.append(array_Gamma, option[-1].Gamma), np.append(array_Theta, option[-1
].Theta), np.append(array_Vega, option[-1].Vega), np.append(array_Rho, op
tion[-1].Rho)])
    portfolio_greeks = matrix_greeks.dot(result)
    print("\nThe Portfolio's Greeks: ")
    print("Delta: ", portfolio_greeks[0])
    print("Gamma: ", portfolio_greeks[1])
    print("Theta: ", portfolio_greeks[2])
    print("Vega : ", portfolio_greeks[3])
    print("Rho  : ", portfolio_greeks[4])
```

In [110]:

```
implied_vol(payoff="call", option_price=5.17174691, S0=100, K=110, T=1, r
=0.13, q=0.09)
```

Out[110]:

```
0.1999999687580392
```

In [111]:

```
implied_vol(payoff="put", option_price=0.10336389, S0=100, K=70, T=1, r=
0.13, q=0.02)
```

Out[111]:

```
0.21991443538814784
```

In [112]:

```
implied_vol(payoff="call", option_price=6.0128267, S0=100, K=110, T=1, r=0.13, q=0.07)
```

Out[112]:

```
0.20028961002081633
```

In [113]:

```
# Creates the option_1
opt_param = Option_param(S0=100, K=110, T=1, exercise="European", payoff="call")
diff_param = Diffusion_process(r=0.13, q=0.09, sig=0.2)
option_1 = Option_pricer(opt_param, diff_param)
```

In [114]:

```
# Creates the option_2
opt_param = Option_param(S0=100, K=70, T=1, exercise="European", payoff="put")
diff_param = Diffusion_process(r=0.13, q=0.02, sig=0.22)
option_2 = Option_pricer(opt_param, diff_param)
```

In [115]:

```
# Creates the option_3
opt_param = Option_param(S0=100, K=110, T=1, exercise="European", payoff="call")
diff_param = Diffusion_process(r=0.13, q=0.07, sig=0.2)
option_3 = Option_pricer(opt_param, diff_param)
```

In [116]:

```
option_1.Binomial_Tree()
```

Out[116]:

```
('European', 5.17173982431996)
```

In [117]:

```
option_1.Monte_Carlo_Simulation()
```

Out[117]:

```
('European', 5.296825291527466)
```

In [118]:

```
option_1.Black_Scholes()
```

Out[118]:

```
('European', 5.171746910724259)
```

In [119]:

```
option_2.Binomial_Tree()
```

Out[119]:

```
('European', 0.10361881912995327)
```

In [120]:

```
option_2.Monte_Carlo_Simulation()
```

Out[120]:

```
('European', 0.11649074315647964)
```

In [121]:

```
option_2.Black_Scholes()
```

Out[121]:

```
('European', 0.10363892841819022)
```

In [122]:

```
option_3.Binomial_Tree()
```

Out[122]:

```
('European', 6.002053141851009)
```

In [123]:

```
option_3.Monte_Carlo_Simulation()
```

Out[123]:

```
('European', 5.992604805106601)
```

In [124]:

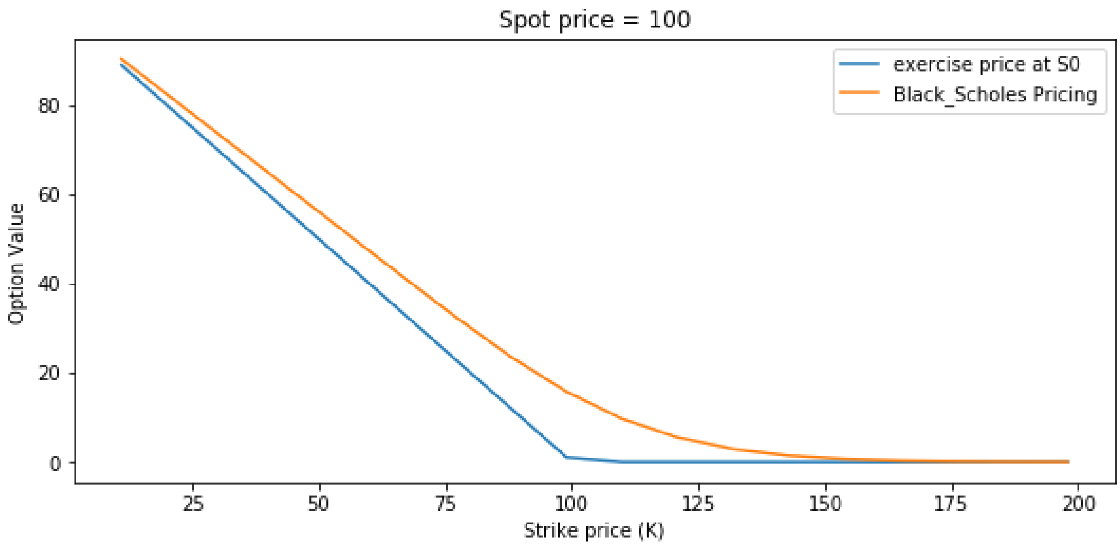
```
option_3.Black_Scholes()
```

Out[124]:

```
('European', 6.002075719878697)
```

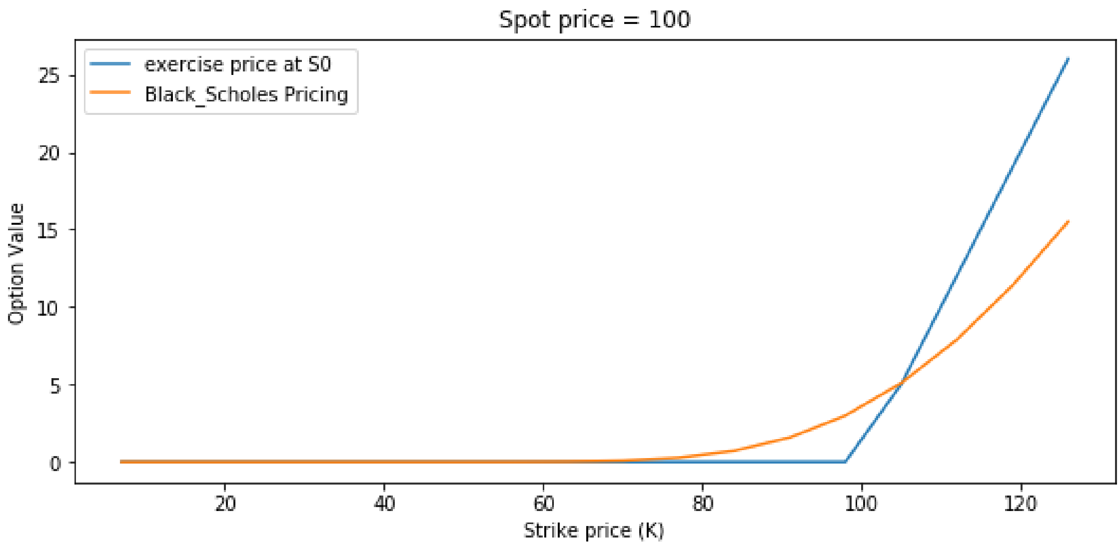
In [125]:

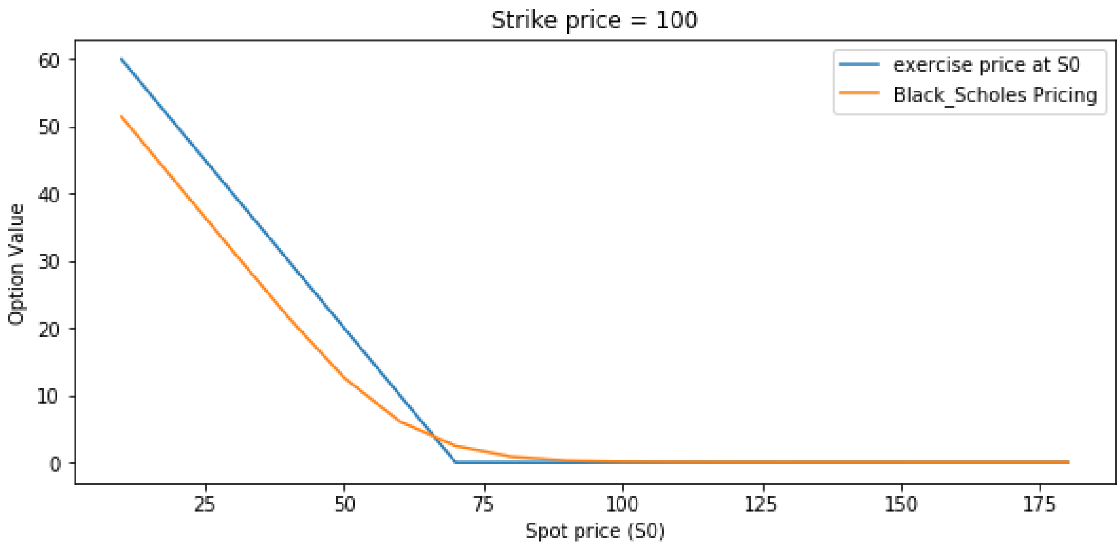
```
option_1.plot()
```



In [126]:

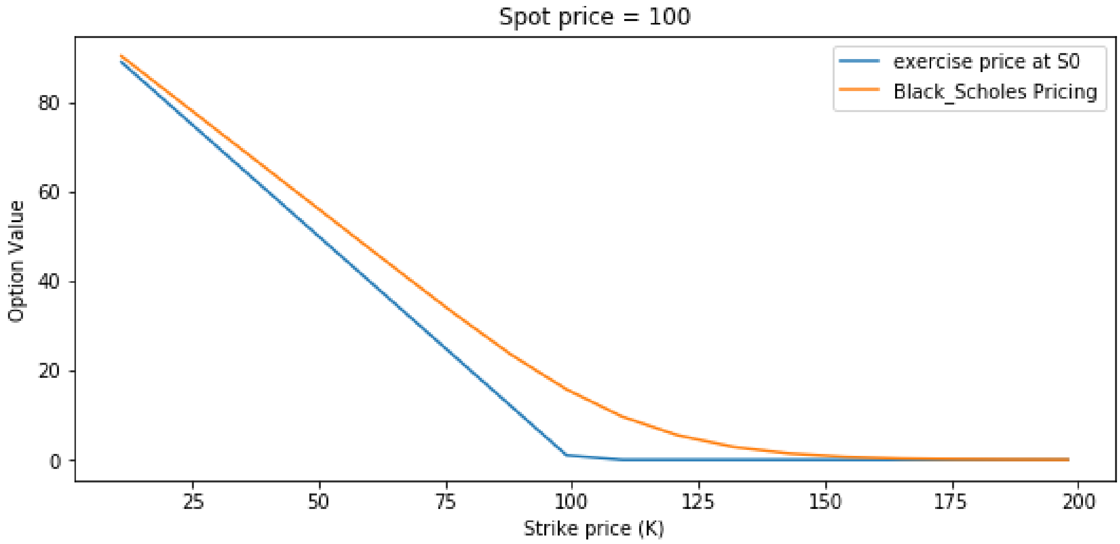
```
option_2.plot()
```





In [127]:

```
option_3.plot()
```



In [141]:

```
greeks_hedge(option_1,option_2,option_3)
```

```
Option 1 :
Delta:  0.39292696684251743
Gamma:  0.01794837106324773
Theta:  -0.01229878077157661
Vega:  0.35896742126495457
Rho:  0.34120949773527487


Option 2 :
Delta:  -0.012578198789316201
Gamma:  0.0014748209310516587
Theta:  -0.0005618449806511207
Vega:  0.03244606048313649
Rho:  -0.013614588073498057


Option 3 :
Delta:  0.43774995733591904
Gamma:  0.01854415142268068
Theta:  -0.015219342972513475
Vega:  0.37088302845361354
Rho:  0.37772920013713207


How many greeks are going to hedge: 3
The first hedging greek: Delta
The second hedging greek: Theta
The thirth hedging greek: Rho

Portfolio:
Stock     :  0.001956661742301191
Option  1 :  -1.1678427311703725
Option  2 :  -1.5240880948644884
Option  3 :  1.0

The Portfolio's Greeks:
Delta:  0.0
Gamma:  -0.004664480282954627
Theta:  -4.336808689942018e-17
Vega :  -0.09778511970523812
Rho  :  5.551115123125783e-17

/Users/kenneth/anaconda3/lib/python3.7/site-packages/ipykerne
l_launcher.py:53: FutureWarning: `rcond` parameter will chang
e to the default of machine precision times ``max(M, N)`` whe
re M and N are the input matrix dimensions.
To use the future default and silence this warning we advise
to pass `rcond=None`, to keep using the old, explicitly pass
`rcond=-1`.
```

In [ ]:

```

```