RMBI 3110 Topic 7: Value at Risk and Short Fall Risk

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Before our coding, we need to import some packages to handle the data. The packages used are:

1. Pandas
2. Numpy
3. matplotlib

In [1]:

```
# Data analysis and manipulation tool
import pandas as pd

# Numerical computing of array and matrix
import numpy as np

# Provides a MATLAB-like way of plotting
import matplotlib.pyplot as plt
```

1. Data Preprocessing

In [2]:

```
# Import the data set 'apple.csv'
# Transform the first column as the row (index_col=0)
apple=pd.read_csv("data/apple.csv",index_col=0)
```

In [3]:

```
apple
```

Out[3]:

	Open	High	Low	Close	Adj Close	Volume

Date	Open	High	Low	Close	Adj Close	Volume
Date						
2012-11-07	20.494286	20.519285	19.848215	19.928572	17.373989	793648800
2012-11-08	20.022499	20.079643	19.117500	19.205357	16.743479	1056146000
2012-11-09	19.300714	19.817142	19.061428	19.537857	17.033361	929913600
2012-11-12	19.791071	19.803572	19.237499	19.386786	16.901649	515802000
2012-11-13	19.246786	19.660000	19.155714	19.389286	16.903831	532949200
...	...	...	...	...	...	...
2020-10-12	120.059998	125.180000	119.279999	124.400002	124.400002	240226800
2020-10-13	125.269997	125.389999	119.650002	121.099998	121.099998	262330500
2020-10-14	121.000000	123.029999	119.620003	121.190002	121.190002	151062300
2020-10-15	118.720001	121.199997	118.150002	120.709999	120.709999	112559200
2020-10-16	121.279999	121.550003	118.809998	119.019997	119.019997	115073800

2000 rows × 6 columns

In [4]:

```
# Summary of basic statistical details
apple.describe()
```

Out[4]:

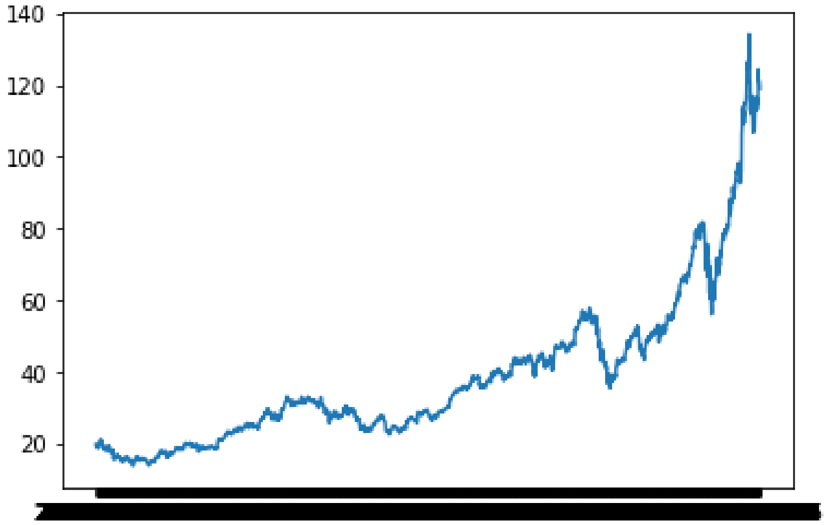
	Open	High	Low	Close	Adj Close	Vo
count	2000.000000	2000.000000	2000.000000	2000.000000	2000.000000	2.000000
mean	38.581717	38.982451	38.189815	38.603343	37.060424	2.014358
std	21.554297	21.873378	21.215109	21.569187	22.144028	1.478722
min	13.856071	14.271428	13.753572	13.947500	12.230474	3.247000
25%	24.255625	24.469999	24.016250	24.254375	22.456524	1.068141
50%	31.753750	31.962500	31.472501	31.708750	29.107528	1.538082
75%	46.906877	47.232501	46.486874	46.923126	45.705216	2.444863
max	137.589996	137.979996	130.529999	134.179993	134.179993	1.460852

In [5]:

```
# Time-series plot of Apple's closing price
```

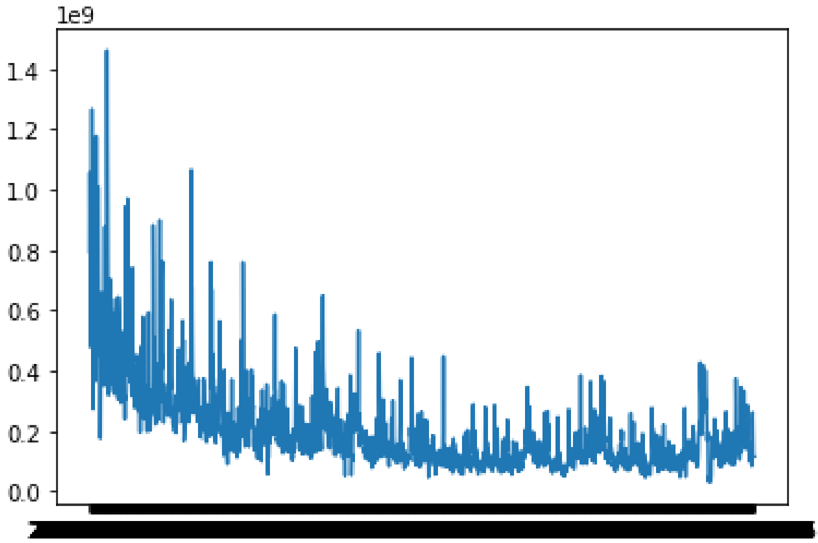
```
plt.plot(apple['Close'])
```

Out[5]: [<matplotlib.lines.Line2D at 0x116113c90>]



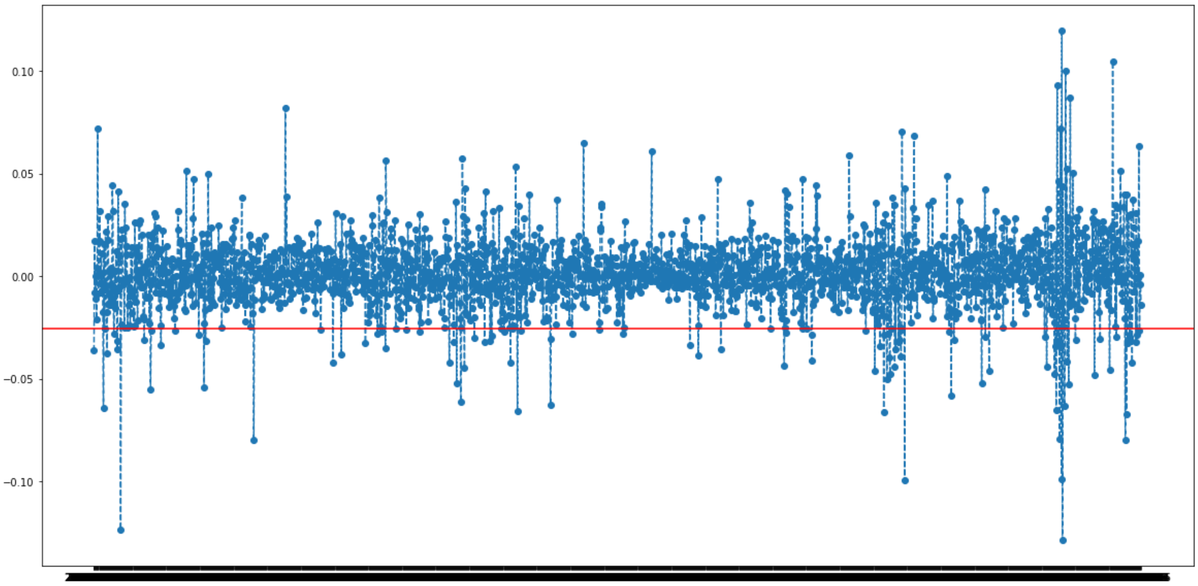
In [6]:
```
# Time-series plot of Apple's Volume
plt.plot(apple['Volume'])
```

Out[6]: [<matplotlib.lines.Line2D at 0x1154380d0>]



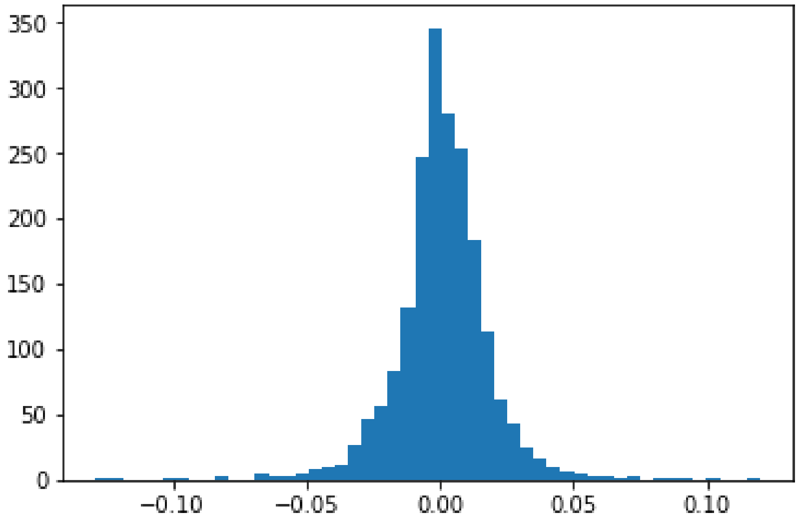
In [7]:
```
# Create a figure with size=(20, 10)
plt.figure(figsize=(20,10))

# Compute the simple daily return of Apple
apple['Return']=apple['Close'].pct_change()

# Plot the daily return of Apple
plt.plot(apple['Return'], linestyle='--', marker='o')

# Indicate the 5% Quantile of daily return of Apple
plt.axhline(apple['Return'].quantile(0.05), color="red")
```

Out[7]: <matplotlib.lines.Line2D at 0x11b3e4d50>



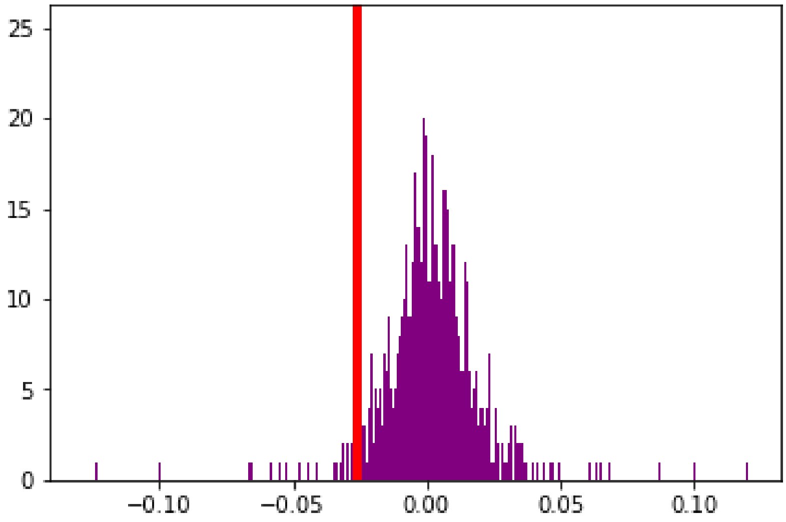
In [8]:
```
# Histogram of daily return of Apple
# no. of bins: 50
plt.hist(apple["Return"],bins=50)
plt.show()
```

```
/Users/ccmakad/.local/lib/python3.7/site-packages/numpy/lib/histogram
s.py:829: RuntimeWarning: invalid value encountered in greater_equal
  keep = (tmp_a >= first_edge)
/Users/ccmakad/.local/lib/python3.7/site-packages/numpy/lib/histogram
s.py:830: RuntimeWarning: invalid value encountered in less_equal
  keep &= (tmp_a <= last_edge)
```



In [11]:
```
# Histogram of daily return of Apple
# no. of bins: 1000
plt.hist(apple['Return'].dropna(), bins=1000, color='purple')

# Indicate the 5% Quantile of daily return of Apple
plt.axvline(x=apple['Return'].dropna().quantile(0.05), linewidth=4, co
```

Out[11]: <matplotlib.lines.Line2D at 0x12716d650>



In [12]:
```
# 5% Quantile of daily return of Apple
apple['Return'].quantile(0.05)
```

Out[12]: -0.02575171592698291

$1-\alpha=0.95$ and $VaR_{0.05}=0.0258$

In [13]:
```
# Import norm to use normal distribution
from scipy.stats import norm

# For standard normal r.v. P(Z<-2.33)=0.01
# Return the -2.33
norm.ppf(0.01)
```

Out[13]: -2.3263478740408408

1. Nonparametric Estimation of Risk Measure

In [14]:
```
# Value at Risk(VaR)
VaR=-apple['Return'].quantile(0.05)
VaR
```

Out[14]: 0.02575171592698291

In [15]:
```
# Shape of DataFrame 'apple'
apple.shape
```

Out[15]: (2000, 7)

In [16]:
```
# Shape of DataFrame 'apple' after filtering
apple[apple["Return"]<-VaR].shape
```

Out[16]:
```
(100, 7)
```

In [17]:
```
# Daily return of apple less than -VaR
apple[apple["Return"]<-VaR]["Return"]
```

Out[17]:
```
Date
2012-11-08   -0.036290
2012-12-05   -0.064357
2012-12-14   -0.037569
2013-01-04   -0.027855
2013-01-14   -0.035653
                ...
2020-09-18   -0.031720
2020-09-23   -0.041946
2020-10-02   -0.032280
2020-10-06   -0.028669
2020-10-13   -0.026527
Name: Return, Length: 100, dtype: float64
```

In [18]:
```
# CVaR: mean of daily return less than -VaR
CVaR=-apple[apple["Return"]<-VaR]["Return"].mean()
CVaR
```

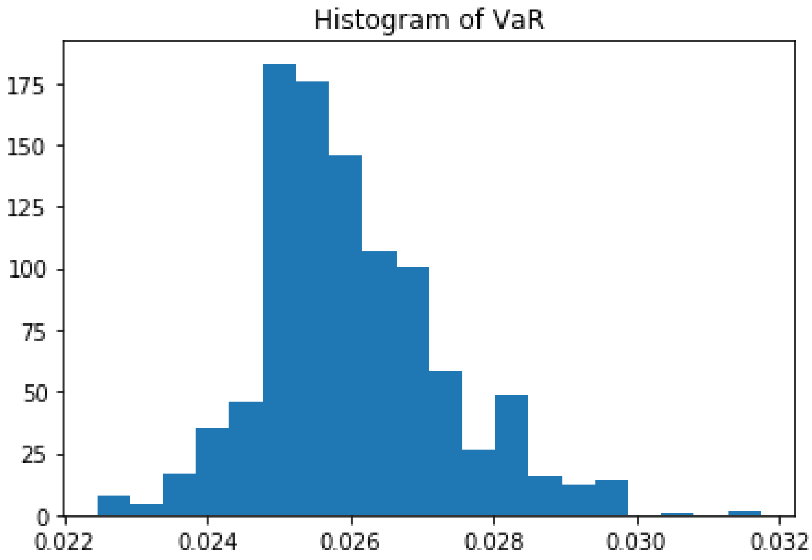
Out[18]:
```
0.042060646441772785
```

1.1 Standard Error of Estimation

In [19]:
```
# Shape of DataFrame 'apple'
apple.shape
```

Out[19]:
```
(2000, 7)
```

In [20]:
```
# Empty list for saving VaR
VaRList=[]

# For loop to resample a single dataset to create many simulated sampl
for n in range(1000):
    returnsample=apple["Return"].sample(n=1000, replace=True)
    # Save the negative 5% quantile to 'VaRList'
    VaRList.append(-returnsample.quantile(0.05))

# Histogram of VaR
# no. of bins: 20
plt.hist(VaRList, bins=20)

# Title
plt.title("Histogram of VaR")
plt.show()
```



In [22]:

```
# Standard error of VaR
SE_VaR=pd.Series(VaRList).std()
SE_VaR
```

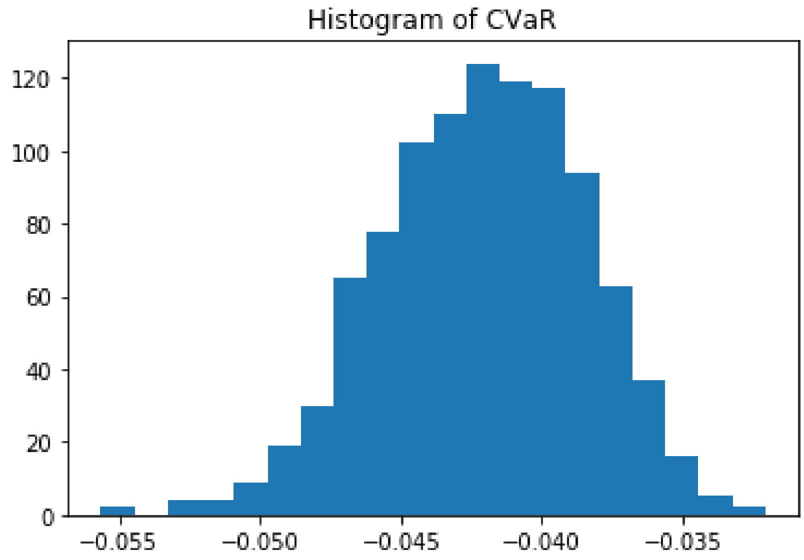
Out[22]: 0.0013057099808697504

In [ ]:

```
####Seperate
```

In [21]:

```
# Empty list for saving CVaR
CVaRList=[]

# For loop to resample a single dataset to create many simulated sampl
for n in range(1000):
    returnsample=apple["Return"].sample(n=1000, replace=True)
    var=-returnsample.quantile(0.05)
    # Save the CVaR to 'CVaRList'
    CVaRList.append(returnsample[returnsample<-var].mean())

# Histogram of CVaR
# no. of bins: 20
plt.hist(CVaRList,bins=20)

# Title
plt.title("Histogram of CVaR")
plt.show()
```



In [23]:

```
# Standard error of CVaR
SE_CVaR=pd.Series(CVaRList).std()
SE_CVaR
```

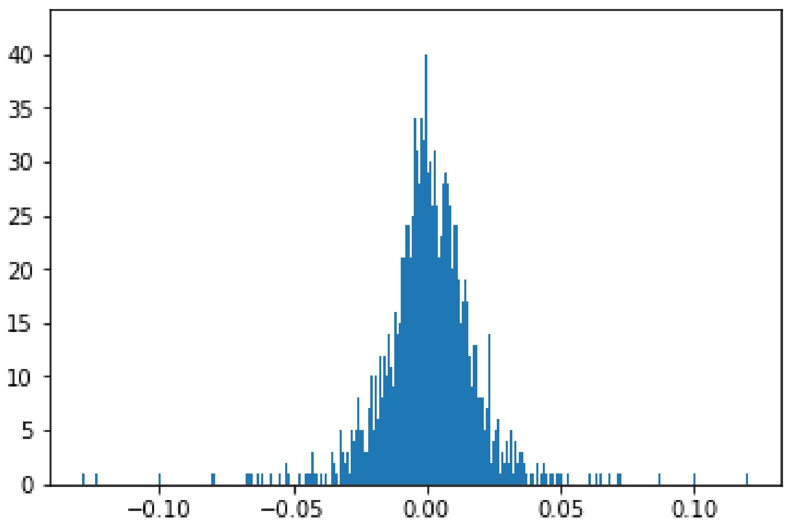
Out[23]: 0.0035954149241852027

2. Parametric Estimation of Risk Measure

In [24]:

```
# Histogram of daily return of Apple
# no. of bins: 500
plt.hist(apple["Return"],bins=500)
plt.show()
```

```
/Users/ccmakad/.local/lib/python3.7/site-packages/numpy/lib/histogram
s.py:829: RuntimeWarning: invalid value encountered in greater_equal
  keep = (tmp_a >= first_edge)
/Users/ccmakad/.local/lib/python3.7/site-packages/numpy/lib/histogram
s.py:830: RuntimeWarning: invalid value encountered in less_equal
  keep &= (tmp_a <= last_edge)
```



In [25]:

```
# Mean of daily return of Apple
mean=apple["Return"].mean()
```

```
mean
```

Out[25]: 0.0010584931758779558

In [26]:
```
# SD of daily return of Apple
std=apple["Return"].std()
std
```

Out[26]: 0.01809210334154013

In [27]:
```
# Import norm to use normal distribution
from scipy.stats import norm

# For standard normal r.v. P(X < mean-1.645*std)=0.05
norm.ppf(0.05, mean, std)
```

Out[27]: -0.02870036862463519

In [28]:
```
# VaR
VaR=-norm.ppf(0.05, mean, std)
VaR
```

Out[28]: 0.02870036862463519

2.1 Parametric Estimation with Monte-Carlo Simulation

z is standard normal, what is mean of $\sqrt{z^2+1}$?

In [29]:
```
# Generate 1000000 values from Standard Normal Distribution r.v.
z=np.random.randn(1000000)

# Generate 1000000 values of (z^2 + 1)^0.5
X=(z**2+1)**0.5

# Mean of 1000000 values of (z^2 + 1)^0.5
X.mean()
```

Out[29]: 1.3542990113314888

In [31]:
```
# Mean of daily return of Apple
meanDelta=apple['Return'].mean()

# SD of daily return of Apple
sigmaDelta=apple['Return'].std()
```

In [32]:
```
# Generate 1000000 values from Standard Normal Distribution r.v.
epsilon=np.random.randn(10000)
```

```
# Daily return of 'A Stock Model' in the lecture slide
dailyReturn=meanDelta+sigmaDelta*epsilon
```

Daily VaR at 95% level is

In [33]:

```
# VaR of Daily return of 'A Stock Model' in the lecture slide
VaR=-pd.Series(dailyReturn).quantile(0.05)
VaR
```

Out[33]: 0.028452873932614 86

To calculate CVaR, we need to get

In [34]:

```
# Daily return of 'A Stock Model' less than -VaR
lossReturn=dailyReturn[dailyReturn<-VaR]

lossReturn.shape, dailyReturn.shape
```

Out[34]: ((500,), (10000,))

In [35]:

```
# Daily return of 'A Stock Model' less than -VaR
lossReturn=dailyReturn[dailyReturn<-VaR]

# CVaR: Negative mean of Daily return of 'A Stock Model' less than -Va
CVaR=-lossReturn.mean()
CVaR
```

Out[35]: 0.03601740740744003

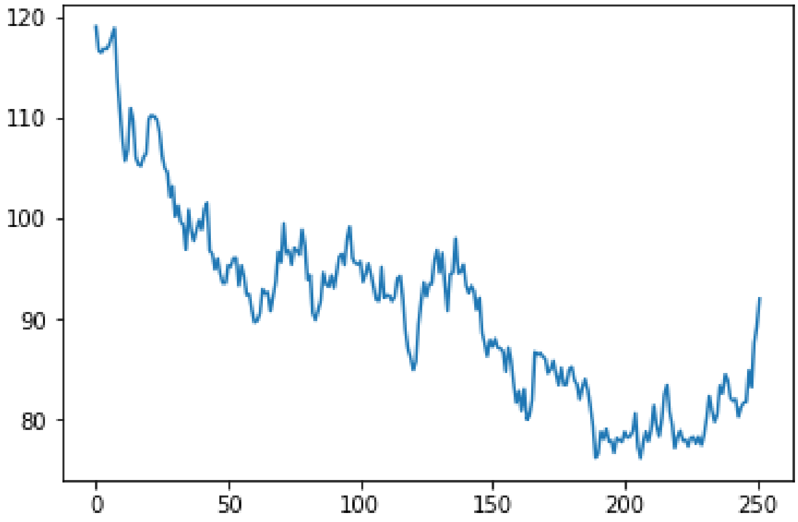
In [36]:

```
# The last price of Apple's closing price
S0=apple['Close'][-1]

# Create an array with 252 zeros
price_simmulation=np.zeros(252)

# Assign the first item as S0
price_simmulation[0]=S0

# For loop to do simmulation
for day in range(1,252):
    price_simmulation[day]=price_simmulation[day-1]+price_simmulation[
```

In [37]:

```
# Plot: price simmulation
plt.plot(price_simmulation)
```

Out[37]: [<matplotlib.lines.Line2D at 0x1a29b5b110>]



Hence we should generate a lot of paths.

In [40]:

```
# PriceGeneration function of above code
def PriceGeneration(S0, T, mu, sigma):
    # Create an array with T zeros
    price_simmulation=np.zeros(T)

    # Assign the first item as S0
    price_simmulation[0]=S0

    # For loop to do simmulation
    for day in range(1,T):
        price_simmulation[day]=price_simmulation[day-1]+price_simmulat
    return price_simmulation
```

In [41]:

```
# Create an empty DataFrame
Stockpaths=pd.DataFrame()

# For loop to generate a lot of paths
for path in range(10000):
    Stockpaths[path]=PriceGeneration(apple['Close'][-1], 252, meanDelt

# First five rows
Stockpaths.head()
```

Out[41]:

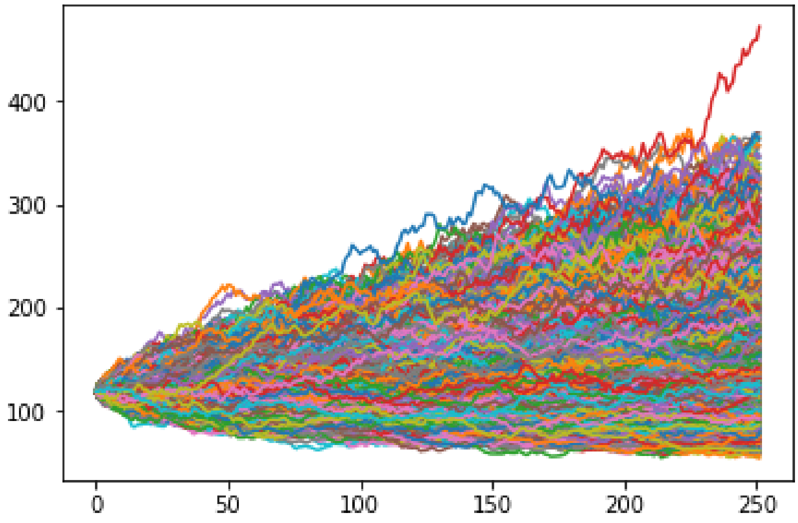
	0	1	2	3	4	5	
0	119.019997	119.019997	119.019997	119.019997	119.019997	119.019997	119.0199
1	119.449417	115.579153	117.094279	116.063598	119.104630	119.210989	121.9135
2	120.177828	114.399736	116.551326	113.792149	116.642568	120.535652	120.2320
3	122.700561	116.732300	118.466779	112.106294	122.009760	123.116707	120.818€
4	123.681928	115.599699	118.227926	114.232442	122.991073	123.402301	119.969€

5 rows × 10000 columns

In [42]:

```
# Plot of 10000 paths
```

```
plt.plot(Stockpaths)
plt.show()
```



In [43]:

```
# The last column of DataFrame 'Stockpaths'
Price_T=Stockpaths.iloc[-1,:]

# Compute the return
Return_T=(Price_T-S0)/S0

# VaR: negative 5% quantile
VaR=-Return_T.quantile(0.05)
VaR
```

Out[43]: 0.22083229491456488

In [44]:

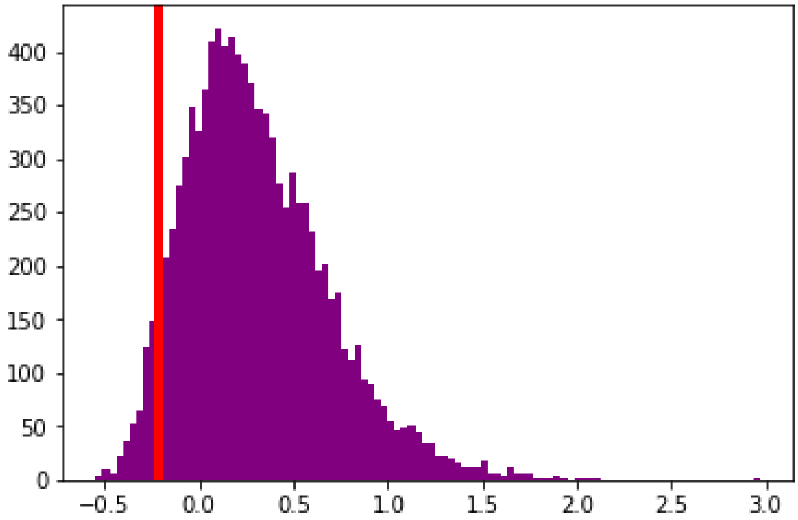
```
# Return less than -VaR
loss_T=Return_T[Return_T<-VaR]

# CVaR: Negative mean of Return less than -VaR
CVaR=-loss_T.mean()
CVaR
```

Out[44]: 0.29911316900097246

In [45]:

```
# Histogram of return
plt.hist(Return_T, color='purple', bins=100)

# Indicate the 5% quantile
plt.axvline(x=Return_T.quantile(0.05), linewidth=4, color='r')
```

Out[45]: <matplotlib.lines.Line2D at 0x1a390a20d0>



2.2 Independent Return ?

In [46]:
```
# Shift down the daily return one row
apple["ReturnLag1"]=apple['Return'].shift(1)

# First five rows of "Return","ReturnLag1"
apple[["Return","ReturnLag1"]].head()
```

Out[46]:

Date	Return	ReturnLag1
2012-11-07	NaN	NaN
2012-11-08	-0.036290	NaN
2012-11-09	0.017313	-0.036290
2012-11-12	-0.007732	0.017313
2012-11-13	0.000129	-0.007732

In [47]:
```
# correlation table of "Return","ReturnLag1"
apple[["Return","ReturnLag1"]].corr()
```

Out[47]:

	Return	ReturnLag1
Return	1.000000	-0.071349
ReturnLag1	-0.071349	1.000000

In [48]:
```
# Autocorrelation of the daily return of apple
apple["Return"].autocorr()
```

Out[48]:
```
-0.07134860427315212
```

2.3. Normal Return ?

In [55]:

```
# Sample 3 data of daily return of apple
apple["Return"].sample(n=3, replace=True)
```

Out[55]:

```
Date
2013-07-30    0.012350
2020-05-05    0.015009
2017-12-19   -0.010656
Name: Return, dtype: float64
```

In [56]:

```
# Sample 1 data of daily return of apple
apple["Return"].sample(n=1, replace=True).values[0]
```

Out[56]:

```
0.025314423907359496
```

In [57]:

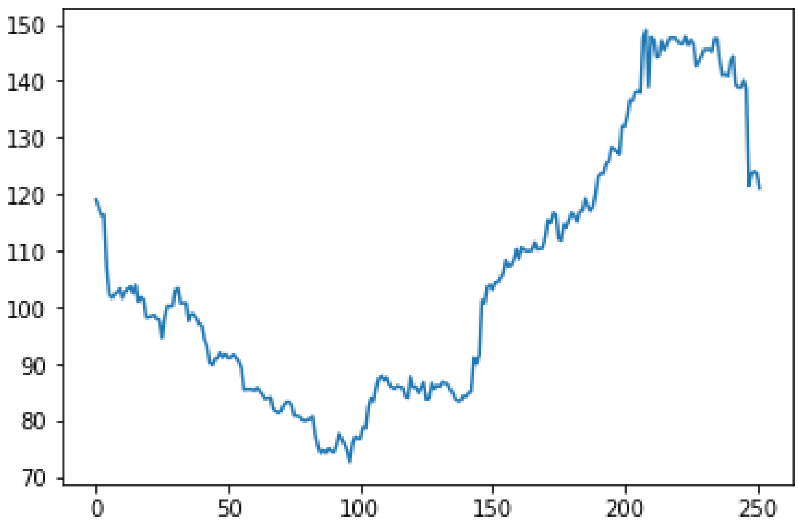
```
# The last price of Apple's closing price
S0=apple['Close'][-1]

# Create an array with 252 zeros
price_simmulation=np.zeros(252)

# Assign the first item as S0
price_simmulation[0]=S0

# For loop to do simmulation
for day in range(1,252):
    # Sample 1 data of daily return of apple
    r=apple["Return"].sample(n=1, replace=True).values[0]
    price_simmulation[day]=price_simmulation[day-1]+price_simmulation[
```

In [58]:

```
# Plot of price simmulation
plt.plot(price_simmulation)
```

Out[58]:

```
[<matplotlib.lines.Line2D at 0x1a390d9090>]
```



In [59]:

```
# PriceGenerationWithBS function of above code
def PriceGenerationWithBS(S0,T,mu,sigma):
    # Create an array with T zeros
    price_simmulation=np.zeros(T)
```

```
    # Assign the first item as S0
    price_simmulation[0]=S0

    # For loop to do simmulation
    for day in range(1,T):
        # Sample 1 data of daily return of apple
        r=apple["Return"].sample(n=1, replace=True).values[0]
        price_simmulation[day]=price_simmulation[day-1]+price_simmulat
    return price_simmulation
```

In [60]:

```
# Empty DataFrame for storing a lot of paths
Stockpaths=pd.DataFrame()

# For loop the generate the paths
for path in range(10000):
    Stockpaths[path]=PriceGenerationWithBS(apple['Close'][-1],252,mear

# First five rows
Stockpaths.head()
```

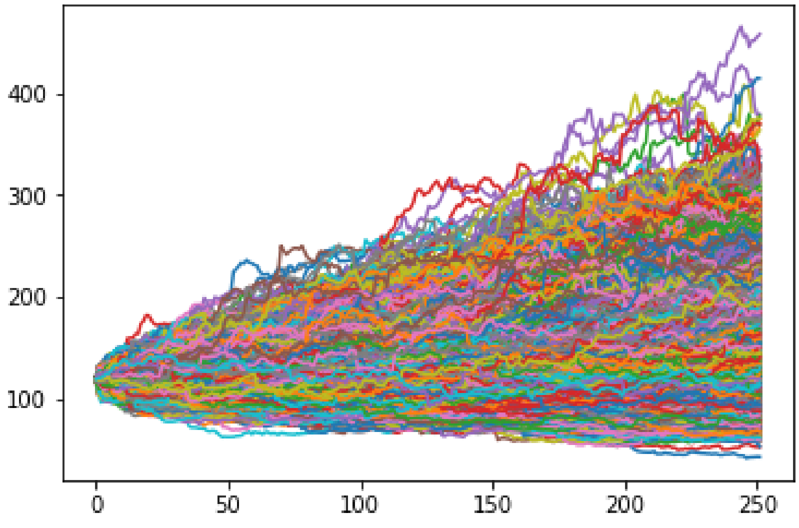
Out[60]:

	0	1	2	3	4	5	
0	119.019997	119.019997	119.019997	119.019997	119.019997	119.019997	119.019
1	118.533478	120.271035	118.040605	117.616858	124.643798	120.379074	121.411
2	121.051226	120.891653	115.796049	116.603922	124.848825	121.675535	122.303
3	119.587588	121.570340	118.267691	116.165290	123.059390	121.517179	122.118
4	119.959408	122.042369	115.403490	117.470452	127.967804	120.505647	123.348

5 rows × 10000 columns

In [61]:

```
# Plot of 10000 paths
plt.plot(Stockpaths)
plt.show()
```



In [62]:

```
# The last column of DataFrame 'Stockpaths'
```

```
Price_T=Stockpaths.iloc[-1,:]

# Compute the return
Return_T=(Price_T-S0)/S0

# VaR: negative 5% quantile
VaR=-Return_T.quantile(0.05)
VaR
```

Out[62]: 0.22109871024911504

In [63]:
```
# Return less than -VaR
loss_T=Return_T[Return_T<-VaR]

# CVaR: Negative mean of Return less than -VaR
CVaR=-loss_T.mean()
CVaR
```

Out[63]: 0.3047707470611795

3. VaR and CVaR for Portfolio Investment

In [64]:
```
# Import data set google.csv
# Change the dafault row to Columns 'Date'
google=pd.read_csv("data/google.csv",index_col=0)

# Empty DataFrame to store closing price of Apple and Google
Portfolio=pd.DataFrame()
Portfolio['APPL']=apple['Close']
Portfolio['GOOG']=google['Close']
Portfolio['Wealth']=Portfolio['APPL']+Portfolio['GOOG']
```

In [66]:
```
# First five rows
Portfolio.head()
```

Out[66]:

Date	APPL	GOOG	Wealth
2012-11-07	19.928572	332.314392	352.242964
2012-11-08	19.205357	324.927094	344.132450
2012-11-09	19.537857	330.277039	349.814896
2012-11-12	19.386786	331.706696	351.093481
2012-11-13	19.389286	328.294464	347.683750

In [67]:
```
# Compute the daily return of portfolio
Portfolio['Return']=Portfolio['Wealth'].pct_change()
```

```
# First five rows
Portfolio.head()
```

Out[67]:

Date	APPL	GOOG	Wealth	Return
2012-11-07	19.928572	332.314392	352.242964	NaN
2012-11-08	19.205357	324.927094	344.132450	-0.023025
2012-11-09	19.537857	330.277039	349.814896	0.016512
2012-11-12	19.386786	331.706696	351.093481	0.003655
2012-11-13	19.389286	328.294464	347.683750	-0.009712

In [68]:

```
# VaR of daily return of portfolio
VaR=-Portfolio['Return'].quantile(0.05)
VaR
```

Out[68]: 0.02266836729305749

In [69]:

```
# CVaR of daily return of portfolio
CVaR=-Portfolio[Portfolio['Return']<-VaR]["Return"].mean()
CVaR
```

Out[69]: 0.03580158893448394 4

In [70]:

```
# Compute the daily return of Apple
Portfolio['Areturn']=Portfolio['APPL'].pct_change()

# Compute the daily return of Google
Portfolio['Greturn']=Portfolio['GOOG'].pct_change()

# Mean of the daily return of Apple
mu_apple=Portfolio['Areturn'].mean()

# SD of the daily return of Apple
sigma_apple=Portfolio['Areturn'].std(ddof=1)

# Mean of the daily return of Google
mu_google=Portfolio['Greturn'].mean()

# SD of the daily return of Google
sigma_google=Portfolio['Greturn'].std(ddof=1)

# Correlation between 'Areturn' and 'Greturn'
corr=Portfolio[['Areturn','Greturn']].corr().iloc[0,1]
```

In [72]:

```
# PriceGeneration function of above code
def PortfolioGeneration(S0,T,mu,sigma,rho):
    # price1: an array with T zeros
    price1=np.zeros(T)
```

```
    # price2: an array with T zeros
    price2=np.zeros(T)

    # portfolio: an array with T zeros
    portfolio=np.zeros(T)

    # Assign the first item of price1 as S0[0]
    price1[0]=S0[0]

    # Assign the first item of price2 as S0[1]
    price2[0]=S0[1]

    for day in range(1,T):
        # ep1: Generate one value from Standard normal r.v.
        ep1=np.random.randn(1)
        # ep2: Generate one value from Standard normal r.v.
        ep2=np.random.randn(1)
        price1[day]=price1[day-1]+price1[day-1]*(mu[0]+sigma[0]*ep1)
        price2[day]=price2[day-1]+price2[day-1]*(mu[1]+sigma[1]*(rho*e
    return price1+price2
```

In [73]:

```
# The last price of Apple and Google
S0=[Portfolio['APPL'][-1], Portfolio['GOOG'][-1]]
```

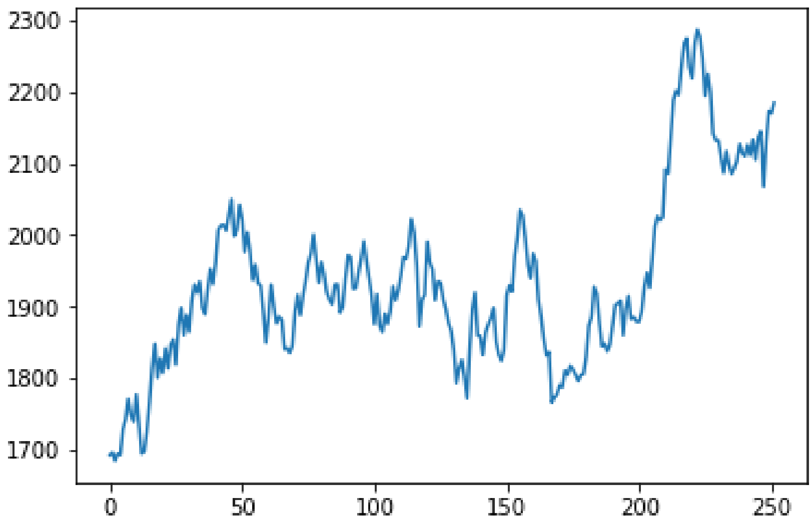
In [74]:

```
# Mean of the daily return of Apple and Google
mu=[mu_apple,mu_google]

# SD of the daily return of Apple and Google
sigma=[sigma_apple,sigma_google]

# Apply the PriceGeneration function
a_portfolio=PortfolioGeneration(S0,252,mu,sigma,corr)
```

In [75]:

```
# Plot of portfolio
plt.plot(a_portfolio)
```

Out[75]: [<matplotlib.lines.Line2D at 0x1a2fd1ea90>]



Next we generate 10000 paths of portfolio

In [76]:

```
# Empty DataFrame for storing the paths
Portfliopaths=pd.DataFrame()

# For loop to generate 10000 paths
for path in range(10000):
    Portfliopaths[path]=PortfolioGeneration(S0,252,mu,sigma,corr)

# First five rows
Portfliopaths.head()
```

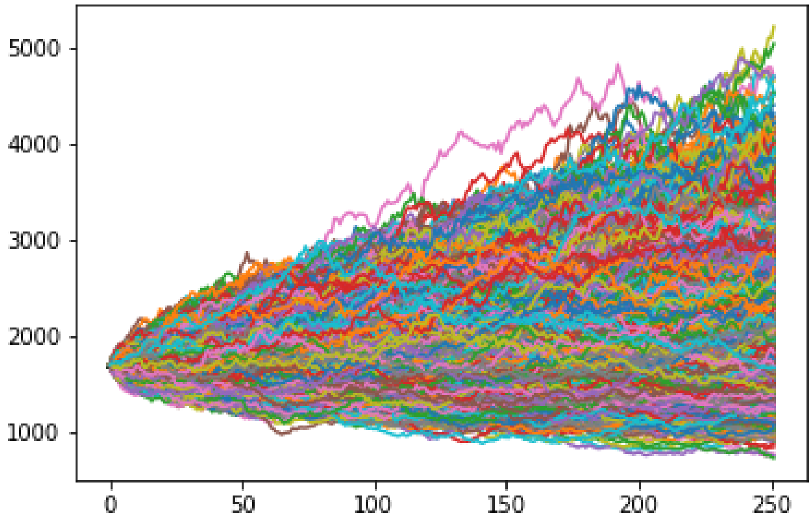
Out[76]:

	0	1	2	3	4	5
0	1692.030006	1692.030006	1692.030006	1692.030006	1692.030006	1692.030006
1	1633.355376	1674.710143	1694.443611	1676.915528	1693.354254	1716.393652
2	1625.104813	1693.199981	1687.355178	1690.848224	1722.731510	1699.533734
3	1621.578096	1709.533577	1693.982399	1704.463544	1749.729769	1708.850814
4	1620.268403	1666.032335	1677.050117	1720.383449	1764.592944	1694.889036

5 rows × 10000 columns

In [77]:

```
# Plot of 10000 paths
plt.plot(Portfliopaths)
plt.show()
```



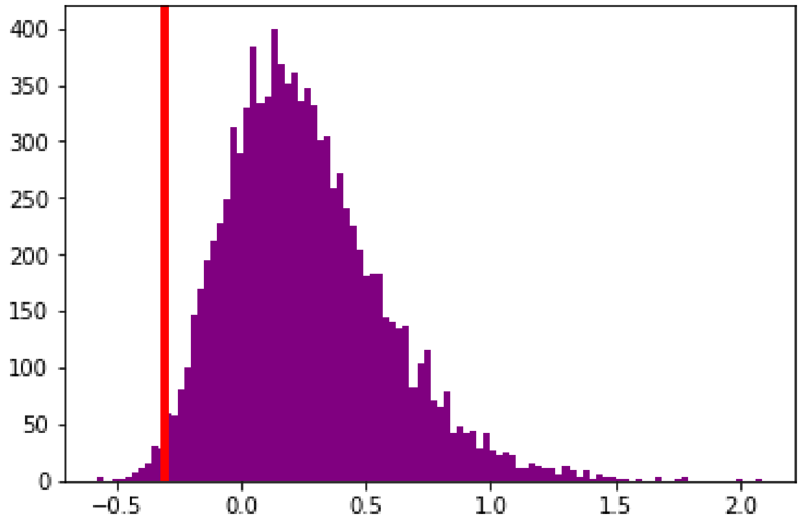
In [78]:

```
# The last path of Portflio paths
Portfolio_T=Portfliopaths.iloc[-1,:]

# Compute the return of the last path
# S0[0]: The last price of Apple
# S0[1]: The last price of Google
Return_T=(Portfolio_T-(S0[0]+S0[1]))/(S0[0]+S0[1])

# VaR: negative 5% quantile
VaR=-Return_T.quantile(0.05)
VaR
```

Out[78]: 0.18290531329160703

In [79]:
```
# Histogram of the return of the last path
# no. of bins: 100
plt.hist(Return_T,color='purple',bins=100)

# Indicate the 1% quantile
plt.axvline(x=Return_T.quantile(0.01), linewidth=4, color='r')
```

Out[79]: <matplotlib.lines.Line2D at 0x1a3077c110>



In [ ]:
```
# CVaR of the return of the last path
CVaR=-Return_T[Return_T<-VaR].mean()
CVaR
```