**Mathematics of Deep Learning Project Proposal (Madeleine Kearns)**

*Exploring the Limits of Lazy Training in Neural Networks*

Neural networks are becoming increasingly popular in machine learning due to their ability to handle complex tasks, such as image and speech recognition, natural language processing, and predictive modelling. One of the challenges in neural network training is finding an optimal architecture, which requires finding the optimal number of neurons, layers, and activation functions. However, the search for the optimal architecture can be time-consuming and computationally expensive, especially for large datasets. This is largely due to a lack of theoretical understanding of the inner functioning of neural networks, which leads to a trial-and-error approach to tailoring a network to an application. In this project, I aim to investigate the limits of lazy training. This is a novel technique in neural network training, which aims to reduce the computational cost of training deep neural networks. The idea is to start with a large number of neurons in the network and then gradually reduce the number of neurons until the weights of the network start to move significantly. When a network is initialised with many neurons, the weights of the network tend to stay close to their initial (often random) initialisation, still achieving good accuracy and not demonstrating large amounts of overfitting. By finding the limit of how far this can be exploited, it may be possible to achieve a more efficient and faster training process, which can be useful for large datasets. However, as the theoretical underpinning of lazy training is not well-understood, there is a lack of empirical evidence to support its effectiveness. Work will also go into researching other causes for the lack of weight movement, to investigate whether it can be explained by another cause (e.g., due to the implicit choice of scaling causing the model to behave as its linearisation around the initialisation and therefore be equivalent to learning with a positive definite kernel, as discussed in the Chizat/Bach 2018 paper)

The main objective of this project is to investigate the limits of lazy training in neural networks using Python. Specifically, I aim to investigate the following questions:

* How far can the number of neurons in a neural network be reduced before the weights start to move significantly?
* What is the impact of lazy training on the performance of the neural network? Can it be responsible for their success in high dimensional tasks?
* How does the performance of lazy training compare with the ArcCosine kernel problem?

As the focus of the project is on the underpinning theory of neural networks, rather than on fitting a network to a specific dataset, I will use a small dataset of 100 points. This is likely to be synthetically generated. The initial network will be severely overparameterised with more than 1000 neurons and be randomly initialised. The network will be fitted and evaluated, before gradually the number of neurons is reduced. Plotting the movement of the weights (potentially as their Euclidean distance moved between different numbers of neurons) will allow identification of the minimum number of neurons for which the network weights stay approximately equal to their random initialisation. Network performance can also be evaluated through accuracy, precision, and recall.

I will use Python and PyTorch throughout the project. I will experiment with different learning rates and stopping criteria to find an optimal approach.