Chapter 2. Signals and Signal Space

Communication Theory - 2026

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EE / KNU

Table of Contents

1. Size of A Signal
2. Classification of Signals
3. Unit Impulse Signal
4. Signals versus Vectors
5. Correlation of Signals
6. Orthogonal Signal Set
7. The Exponential Fourier Series
8. Frequency Allocation Details

Definition: Signal and System

Signal

A signal is a set of information or data.
The signals are functions of the independent variable **time t**.

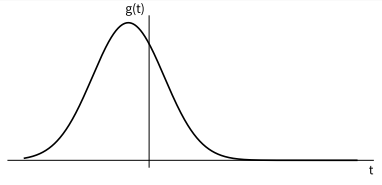
- Examples: Audio signals, video signals, sensor data, etc.

System

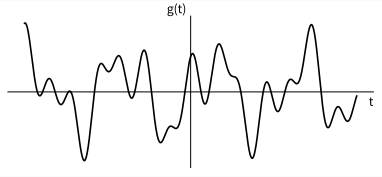
Signals may be processed further by systems, which may modify them or extract additional infromation from them.
Thus, a system is an entity that processes signals (**inputs**) to yield another set of signals (**outputs**).

- For example, an antiaircraft radar system processes the received signals (inputs) to determine the position and velocity of an aircraft (outputs).
- More examples: Amplifiers, filters, modulators, demodulators, etc.

Energy vs Power Signals



(a) Signal with finite energy



(b) Signal with finite power

Fig. 2.1: *Examples of signals*

Energy Signal

A signal is said to be an energy signal if its energy is finite and its average power approaches zero.

$$E = \int_{-\infty}^{\infty} |g(t)|^2 dt < \infty, \quad P \to 0$$

Power Signal

A signal is said to be a power signal if its average power is finite and its energy approaches infinite.

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt < \infty, \quad E \to \infty$$

Units of Signal Power

- The standard units of signal energy and power are the ”joule” and the ”watt”.
- However, in practice, it is often customary to use logarithmic scales to describe signal power.
- A signal with average power of P watts has power of either P_{dBw} or P_{dBm}.

$$P_{dBw} = [10 \cdot \log_{10} P] \text{ dBw}$$

$$P_{dBm} = [30 + 10 \cdot \log_{10} P] \text{ dBm}$$

- For example,
  $P_{dBm} = -30$ dBm $= 10^{-6}$W.

Q. Determine the suitable measures of the signals in Fig. 2.2.

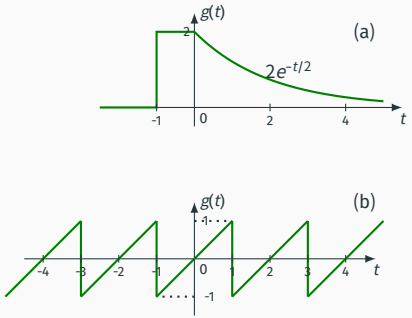


Fig. 2.2: *Signal for Example*

Fig. 2.2 (a)

Energy signal. Power approaches 0 as $|t| \to \infty$.

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2\,dt = \int_{-1}^{0} (2)^2\,dt + \int_{0}^{\infty} 4e^{-t}\,dt = 4 + 4 = 8$$

Fig. 2.2 (b)

Power signal. Averaging $|g|^2(t)$ over an infinitely large interval is equivalent to averaging it over one period (2 seconds).

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2\,dt = \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2\,dt$$

$$= \frac{1}{2} \int_{-1}^{1} t^2\,dt = \frac{1}{3}$$

Classification of Signals

1. Continuous time and discrete time signals
2. Analog and digital signals
3. Periodic and aperiodic signals
4. Energy and power signals
5. Deterministic and probabilistic signals

Figure 2.3 Continuous vs Discrete Time Signals

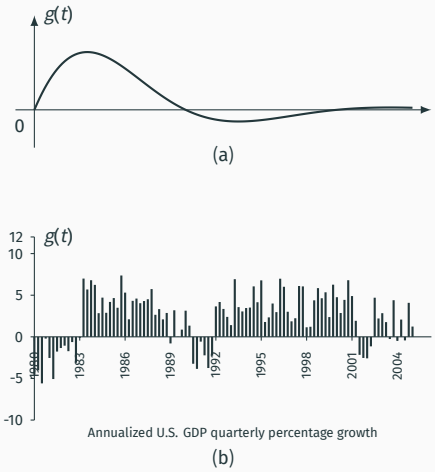


Fig. 2.3: *Continuous vs Discrete Time Signals*

Fig. 2.3 (a)

Continous time signals are specified for every value of time *t*. Many examples including:

- Audio recordings in analog media like LP, magnetic cassette, or reel-to-reel tapes.
- Signals received through AM/FM radio channel.

Fig. 2.3 (b)

Discrete time signals are specified only at discrete points of $t = nT$. Many examples including:

- The quarterly gross domestic product (GDP), stock market daily averages, and monthly sales of a corporation.
- Audio signals formatted by MP3, HE-AAC, FLAC, or ALAC.

Figure 2.4 Classification of Signals

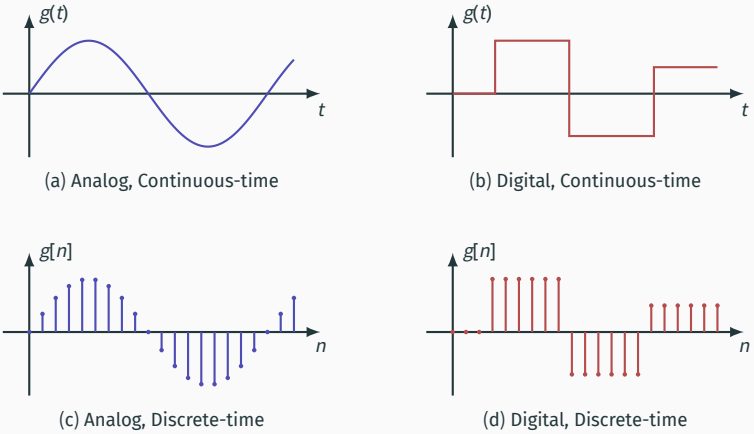


Fig. 2.4: *Classification of signals based on amplitude and time domains*

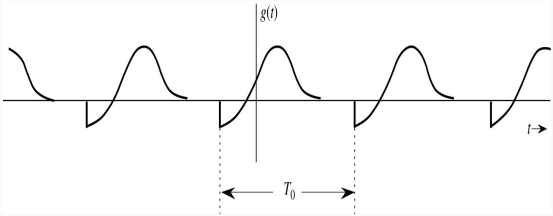


Fig. 2.5: *Periodic signal of period T_0*

Fig. 2.5

A signal $g(t)$ is **periodic** if there exists a positive constant T_0. The smallest value of T_0 in Eq. (2.0) is the **period** of $g(t)$.

$$g(t) = g(t + T_0) \quad \text{for all t} \qquad (2.5)$$

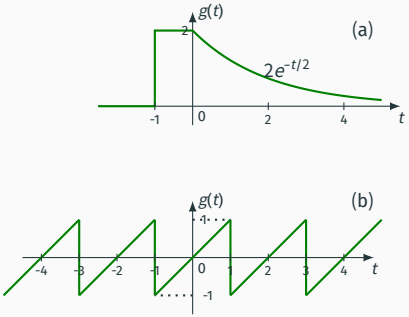


Fig. 2.2: *Energy vs Power Signals (Recap)*

Fig. 2.2(a) Energy Signal

A signal is said to be an energy signal if its energy is finite and its average power approaches zero.

$$\int_{-\infty}^{\infty} |g(t)|^2 dt < \infty \tag{2.6}$$

Fig. 2.2(b) Power Signal

A signal is said to be a power signal if its average power is finite and its energy approaches infinite.

$$0 < \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt < \infty \tag{2.7}$$

$$g(t) = \qquad (2.8)$$

Time shifting a signal

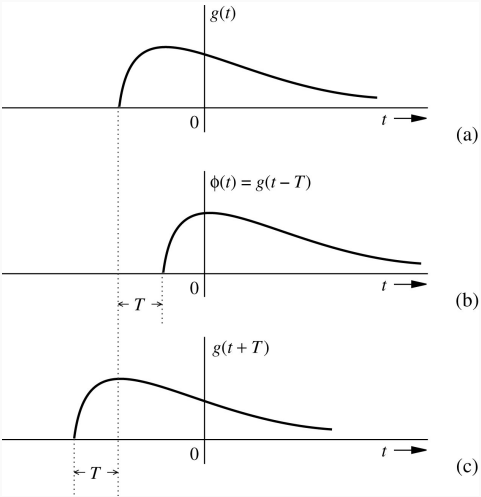


Fig. 2.6: *Time shifting a signal*

Fig. 2.6 Time shifting a signal

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$g(t) =$ (2.9)

Time scaling a signal

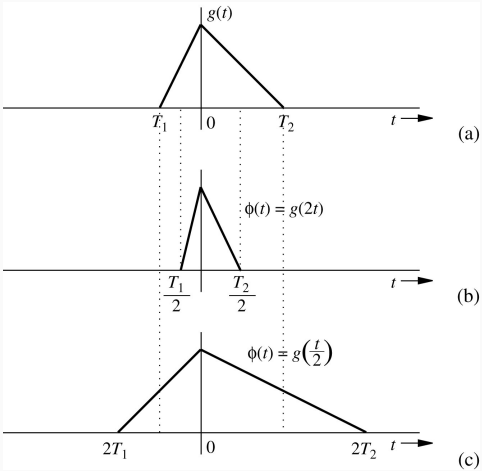

Fig. 2.7: *Time scaling a signal*

Fig. 2.7 Time scaling a signal

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$g(t) =$ (2.10)

Examples of time compression and time expansion of signals

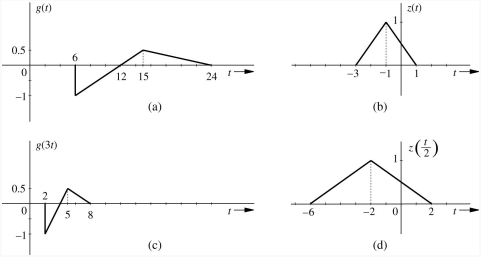

Fig. 2.8: *Examples of time compression and time expansion of signals*

Fig. 2.8

$$g(t) = \qquad (2.11)$$

Time inversion (reflection) of a signal

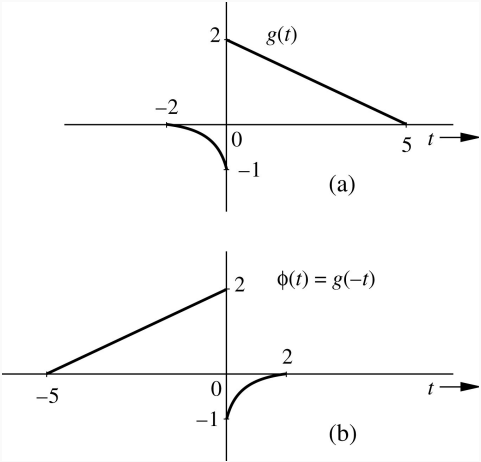

Fig. 2.9: *Time inversion (reflection) of a signal*

Fig. 2.9

$g(t) =$ (2.12)

Example of time inversion

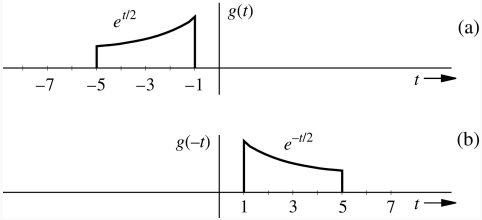

Fig. 2.10: *Example of time inversion*

Fig. 2.10

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$g(t) =$ (2.13)

(a) Unit impulse and (b) its approximation

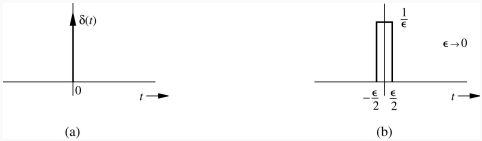

Fig. 2.11: *(a) Unit impulse and (b) its approximation*

Fig. 2.11

$g(t) =$ (2.14)

(a) Unit step function $u(t)$. (b) Causal exponential $e^{-at}u(t)$

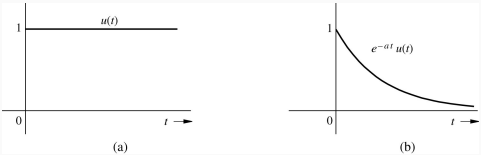

Fig. 2.12: *(a) Unit step function $u(t)$. (b) Causal exponential $e^{-at}u(t)$*

Fig. 2.12

$g(t) =$ (2.15)

Signals versus Vectors

Component (projection) of a vector along another vector

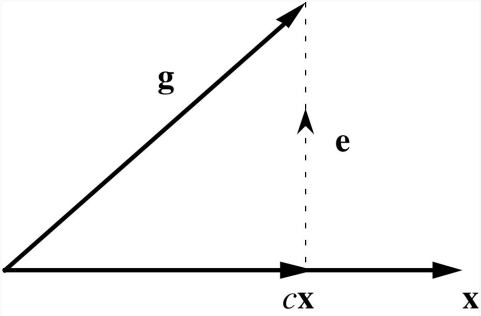

Fig. 2.13: *Component (projection) of a vector along another vector*

Fig. 2.13

$g(t) =$ (2.16)

Approximations of a vector in terms of another vector

Fig. 2.14: *Approximations of a vector in terms of another vector*

Fig. 2.14

$$g(t) = \qquad (2.17)$$

Decomposition of a Signal and Signal Components

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Approximation of square signal in terms of a single sinusoid

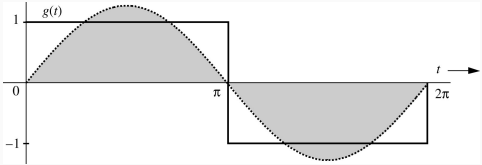

Fig. 2.15: *Approximation of square signal in terms of a single sinusoid*

Fig. 2.15

$$g(t) = \qquad (2.18)$$

Complex Signal Space and Orthogonality

Energy of the Sum of Orthogonal Signals

Signals for Example 2.6

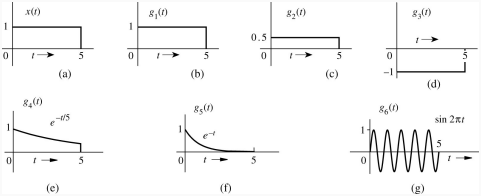

Fig. 2.16: *Signals for Example 2.6*

Fig. 2.16

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$g(t) =$ (2.19)

Correlation of Signals

Correlation Functions

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Physical explanation of the auto-correlation function

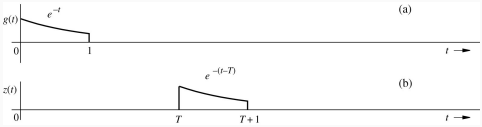

Fig. 2.17: *Physical explanation of the auto-correlation function*

Fig. 2.17

$g(t) =$ (2.20)

Representation of a vector in three-dimensional space

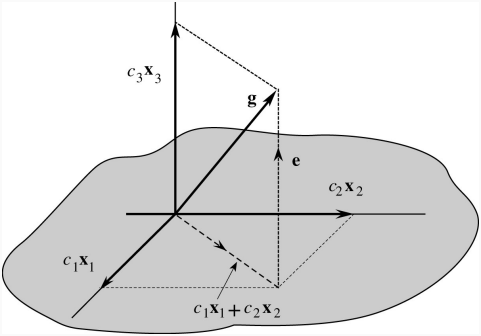

Fig. 2.18: *Representation of a vector in three-dimensional space*

Fig. 2.18

$g(t) =$ (2.21)

Orthogonal Vector Space

Orthogonal Signal Space

Parseval's Theorem

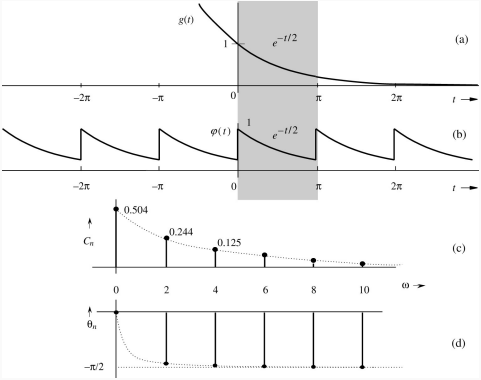

Fig. 2.19: .

Fig. 2.19

$g(t)$ = (2.22)

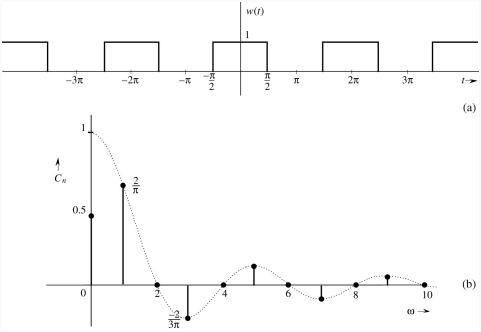

Fig. 2.20: .

Fig. 2.20

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$g(t)$ = (2.23)

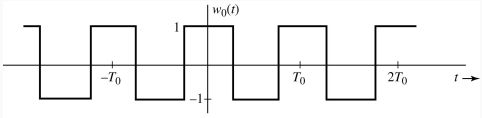

Fig. 2.21: .

Fig. 2.21

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$g(t) =$ (2.24)

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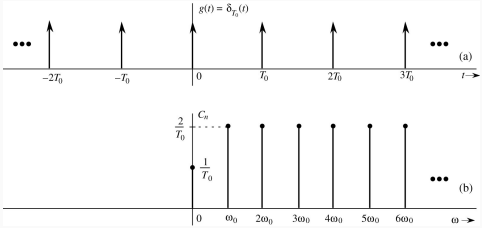

Fig. 2.22: .

Fig. 2.22

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$g(t) =$ (2.25)

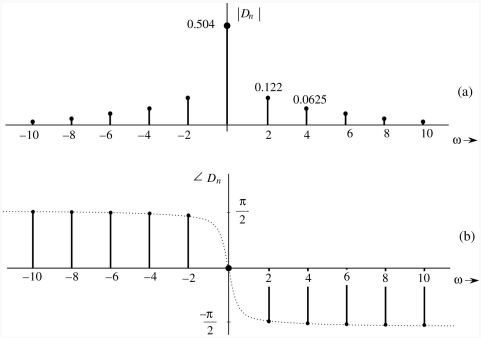

Fig. 2.23: .

Fig. 2.23

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$g(t) =$ (2.26)

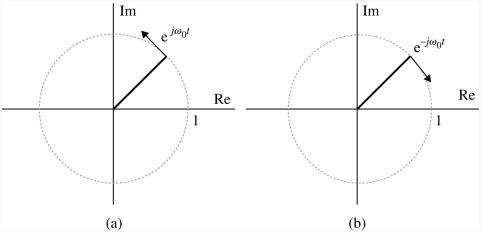

Fig. 2.24: .

Fig. 2.24

$g(t) =$ (2.27)

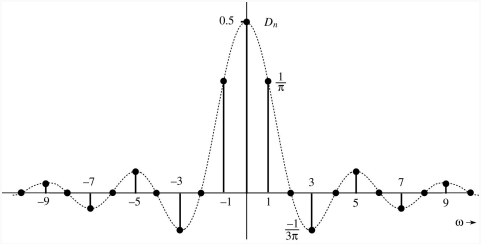

Fig. 2.25: .

Fig. 2.25

$g(t) =$ (2.28)

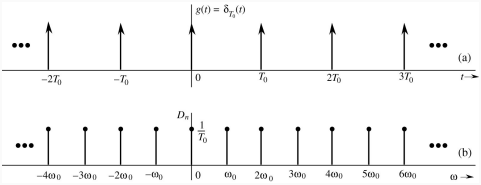

Fig. 2.26: .

Fig. 2.26

$g(t) =$ (2.29)

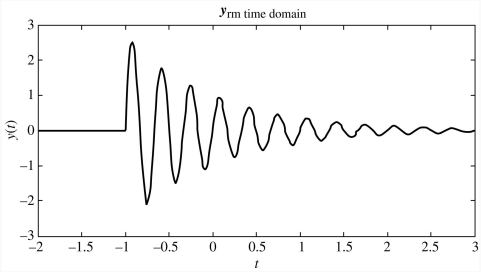

Fig. 2.27: .

Fig. 2.27

$g(t) =$ (2.30)

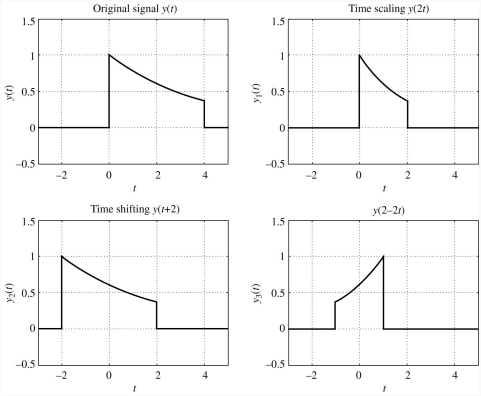

Fig. 2.28

Fig. 2.28

$g(t) =$ (2.31)

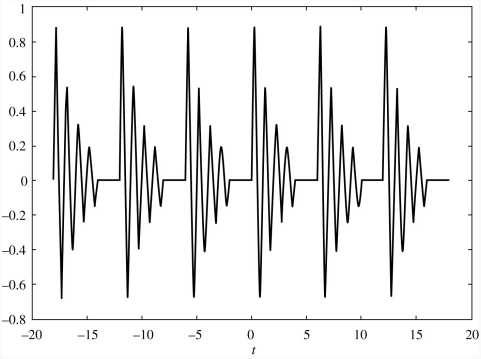

Fig. 2.29: .

Fig. 2.29

$g(t) =$ (2.32)

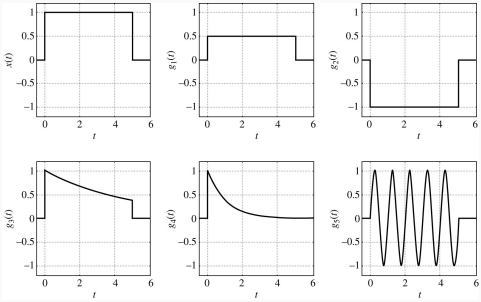

Fig. 2.30: .

Fig. 2.30

$g(t)$ = (2.33)

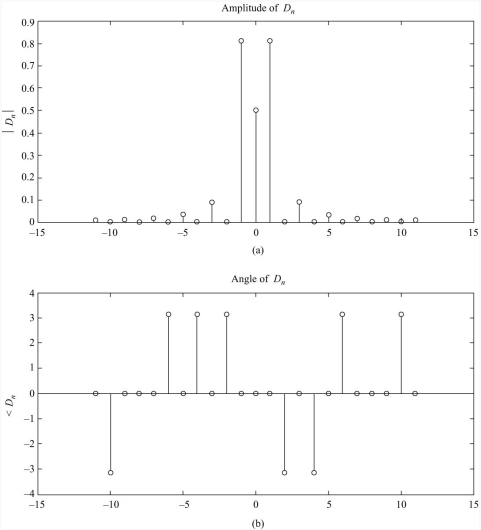

Fig. 2.31: .

Fig. 2.31

$g(t) =$ (2.34)

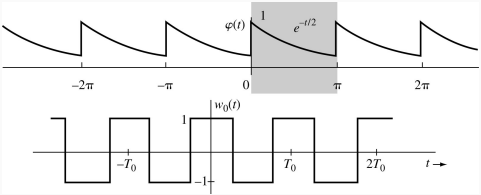

Fig. 2.32: .

Fig. 2.32

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$g(t) =$ (2.35)

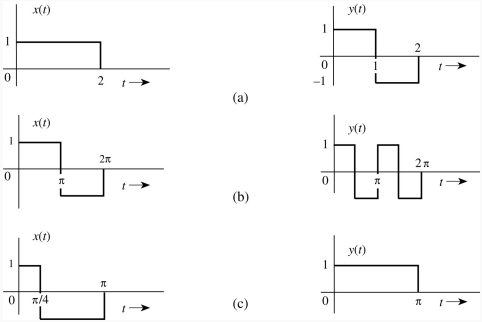

Fig. 2.33: .

Fig. 2.33

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$g(t)$ = (2.36)

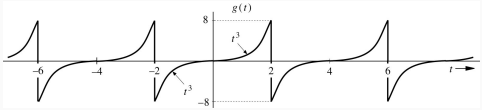

Fig. 2.34: .

Fig. 2.34

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$g(t) =$ (2.37)

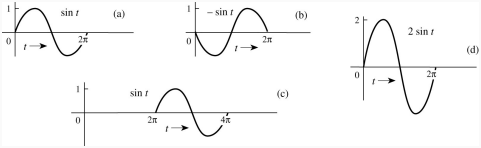


Fig. 2.35: .

Fig. 2.35

$g(t) =$ (2.38)

1. Low Frequency Bands (VLF ~ MF)

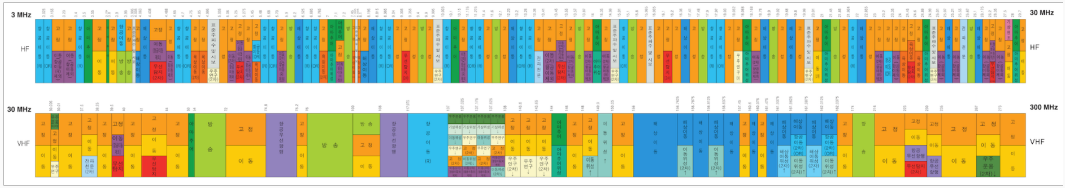
Key Services

- **VLF/LF**: 해상/잠수함 통신, 항공 무선 표지 (Beacon)
- **MF**: AM Radio (535~1605 kHz), 해상 이동 업무

Characteristics

- **지표파(Ground Wave)**: 지표면을 따라 멀리 전파됨
- 회절성이 매우 강함 (산/건물 통과 유리)
- 안테나 크기가 매우 커야 함 (λ가 길기 때문)

2. Mid-Range Frequency (HF ~ VHF)



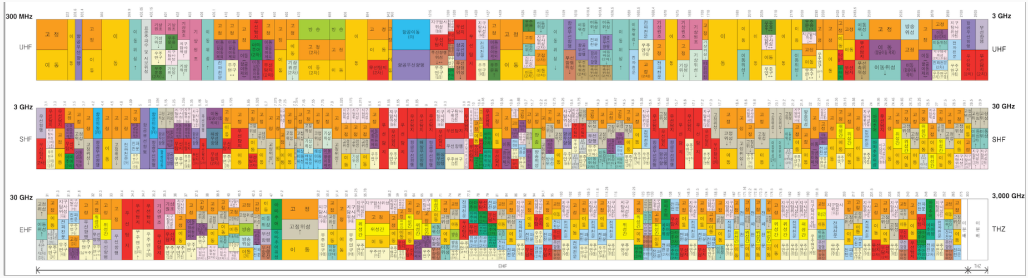
Key Services

- **HF**: 단파 방송, 아마추어 무선(HAM), 비상 통신
- **VHF**: FM Radio (88~108 MHz), 지상파 DMB, 업무용 무전기

Characteristics

- **Sky Wave (HF)**: 전리층 반사를 이용한 원거리(국제) 통신
- **Line-of-Sight (VHF)**: 직진성 시작, 장애물 영향 받음
- 음질이 좋고 잡음에 강해짐

3. High Frequency Bands (UHF ~ EHF)



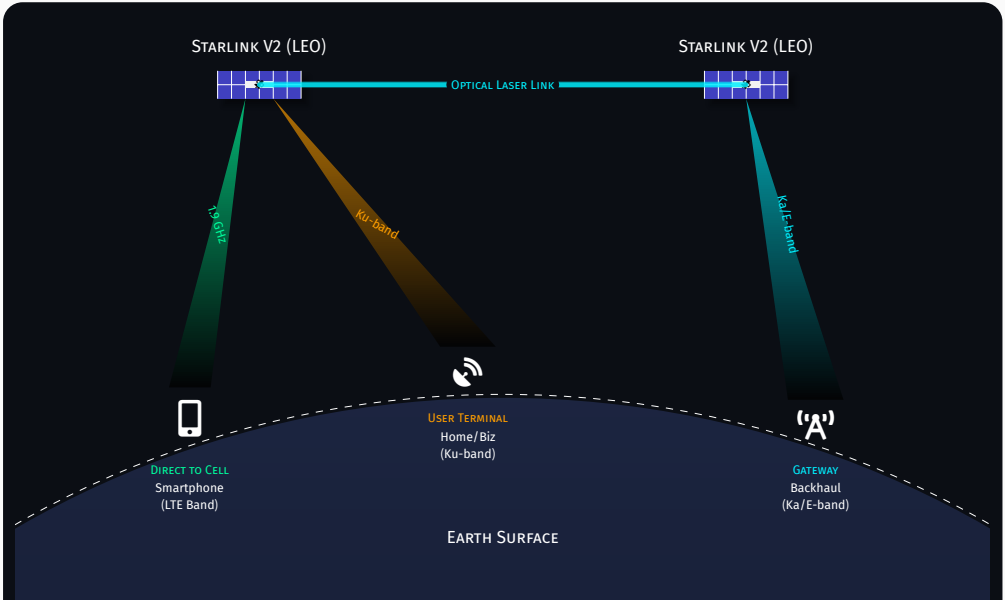
Key Services (Most Active)

- **UHF**: Mobile (LTE/5G), Wi-Fi(2.4G), GPS, DTV
- **SHF/EHF**: Satellite, Radar, 5G mmWave, Wi-Fi 6E/7

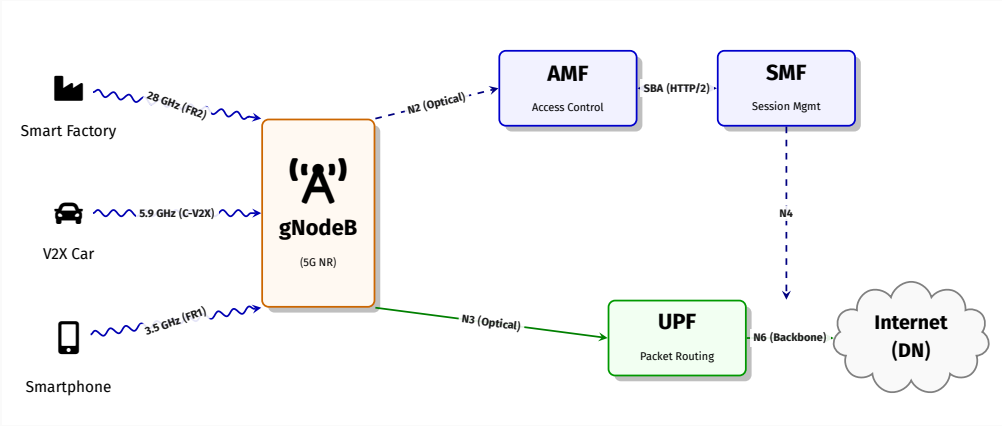
Characteristics

- 빛에 가까운 강한 직진성
- **Wide Bandwidth**: 초고속 대용량 데이터 전송 가능
- 장애물(비, 벽)에 의한 감쇄 심함

Starlink Network Architecture



5G Terrestrial Network Architecture (PLMN)



Underwater Communication: SONAR System Architecture

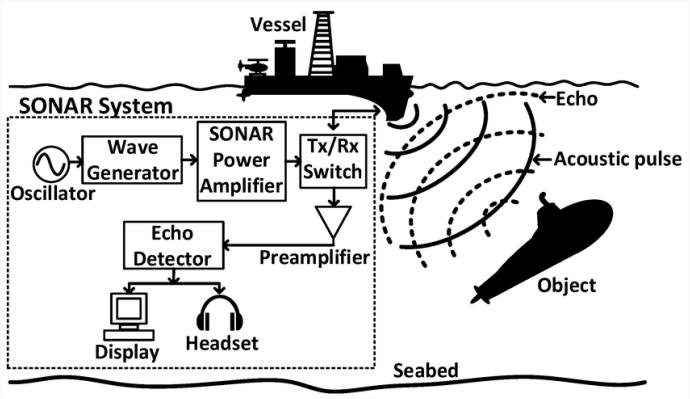


Fig. 2.36: *General Architecture of SONAR System*

Source: MDPI J. Mar. Sci. Eng. 2023, 11(7), 1279

Summary