

Chapter 2. Signals and Signal Space

Communication Theory - 2026

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EE / KNU

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DEFINITION: SIGNAL AND SYSTEM

Signal

A signal is a set of information or data.

The signals are functions of the independent variable **time t**.

- Examples: Audio signals, video signals, sensor data, etc.

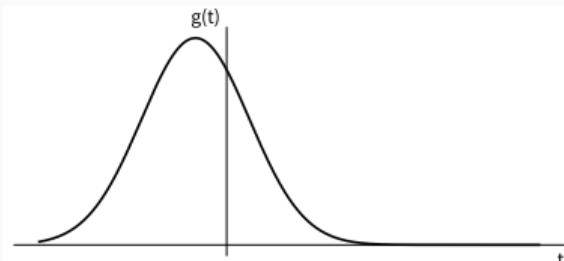
System

Signals may be processed further by systems, which may modify them or extract additional information from them.

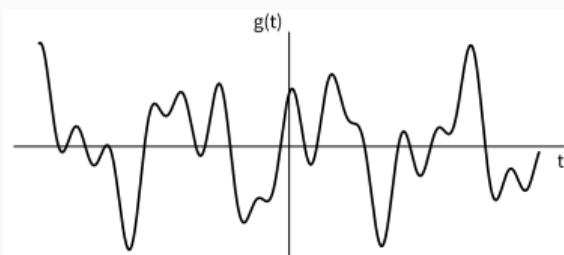
Thus, a system is an entity that processes signals (**inputs**) to yield another set of signals (**outputs**).

- For example, an antiaircraft radar system processes the received signals (inputs) to determine the position and velocity of an aircraft (outputs).
- More examples: Amplifiers, filters, modulators, demodulators, etc.

ENERGY VS POWER SIGNALS



(a) Signal with finite energy



(b) Signal with finite power

Fig. 2.1: Examples of signals

Energy Signal

A signal is said to be an energy signal if its energy is finite and its average power approaches zero.

$$E = \int_{-\infty}^{\infty} |g(t)|^2 dt < \infty, \quad P \rightarrow 0$$

Power Signal

A signal is said to be a power signal if its average power is finite and its energy approaches infinite.

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt < \infty, \quad E \rightarrow \infty$$

UNITS OF SIGNAL POWER

- The standard units of signal energy and power are the "joule" and the "watt".
- However, in practice, it is often customary to use logarithmic scales to describe signal power.
- A signal with average power of P watts has power of either P_{dBW} or P_{dBm} .

$$P_{dBW} = [10 \cdot \log_{10} P] \text{ dBW}$$

$$P_{dBm} = [30 + 10 \cdot \log_{10} P] \text{ dBm}$$

- For example,

$$P_{dBm} = -30 \text{ dBm} = 10^{-6} \text{ W.}$$

EXAMPLE 2.1

Q. Determine the suitable measures of the signals in Fig. 2.2.

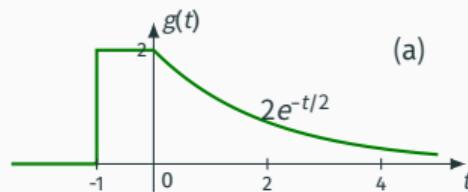


Fig. 2.2 (a)

Energy signal. Power approaches 0 as $|t| \rightarrow \infty$.

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-1}^0 (2)^2 dt + \int_0^{\infty} 4e^{-t} dt = 4 + 4 = 8$$

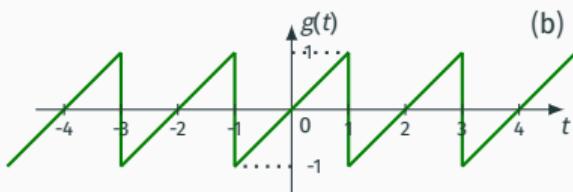


Fig. 2.2 (b)

Power signal. Averaging $|g|^2(t)$ over an infinitely large interval is equivalent to averaging it over one period (2 seconds).

$$\begin{aligned} P_g &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt = \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt \\ &= \frac{1}{2} \int_{-1}^1 t^2 dt = \frac{1}{3} \end{aligned}$$

Fig. 2.2: Signal for Example

CLASSIFICATION OF SIGNALS

1. Continuous time and discrete time signals
2. Analog and digital signals
3. Periodic and aperiodic signals
4. Energy and power signals
5. Deterministic and probabilistic signals

FIGURE 2.3 CONTINUOUS VS DISCRETE TIME SIGNALS

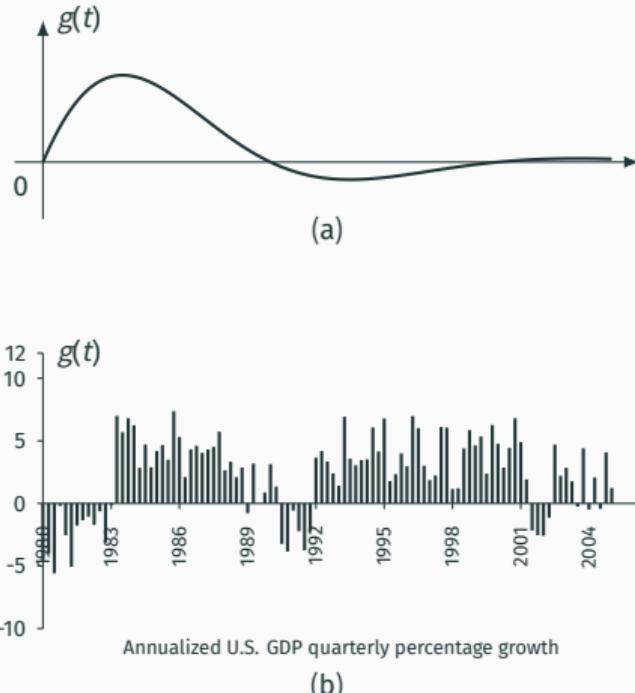


Fig. 2.3 (a)

Continuous time signals are specified for every value of time t . Many examples including:

- Audio recordings in analog media like LP, magnetic cassette, or reel-to-reel tapes.
- Signals received through AM/FM radio channel.

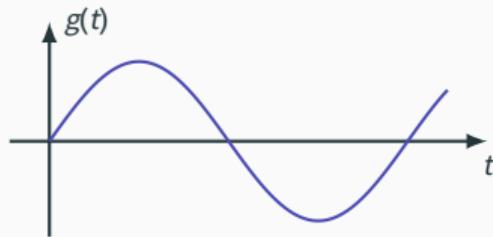
Fig. 2.3 (b)

Discrete time signals are specified only at discrete points of $t = nT$. Many examples including:

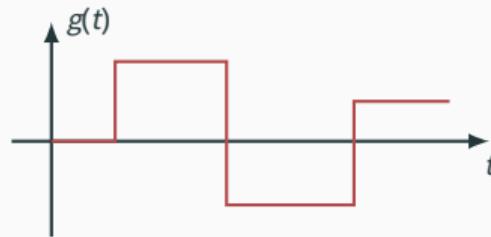
- The quarterly gross domestic product (GDP), stock market daily averages, and monthly sales of a corporation.
- Audio signals formatted by MP3, HE-AAC, FLAC, or ALAC.

Fig. 2.3: Continuous vs Discrete Time Signals

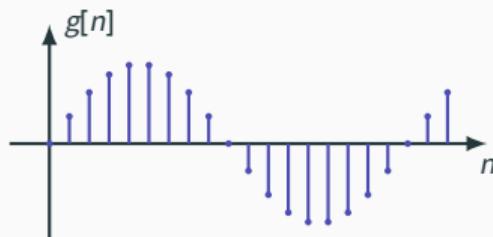
FIGURE 2.4 CLASSIFICATION OF SIGNALS



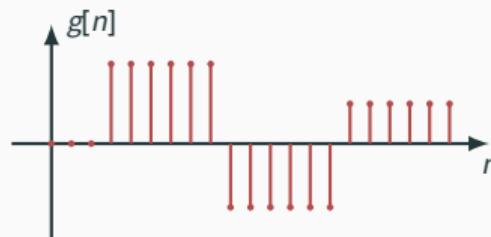
(a) Analog, Continuous-time



(b) Digital, Continuous-time



(c) Analog, Discrete-time



(d) Digital, Discrete-time

Fig. 2.4: Classification of signals based on amplitude and time domains

PERIODIC AND APERIODIC SIGNALS

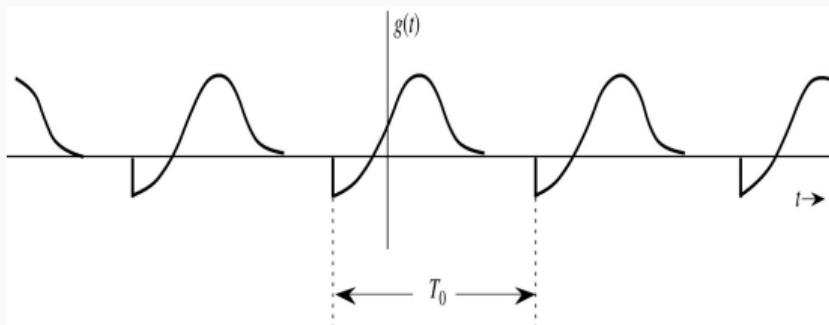


Fig. 2.5: Periodic signal of period T_0

Fig. 2.5

A signal $g(t)$ is **periodic** if there exists a positive constant T_0 . The smallest value of T_0 in Eq. (2.0) is the **period** of $g(t)$.

$$g(t) = g(t + T_0) \quad \text{for all } t \quad (2.5)$$

ENERGY VS POWER SIGNALS

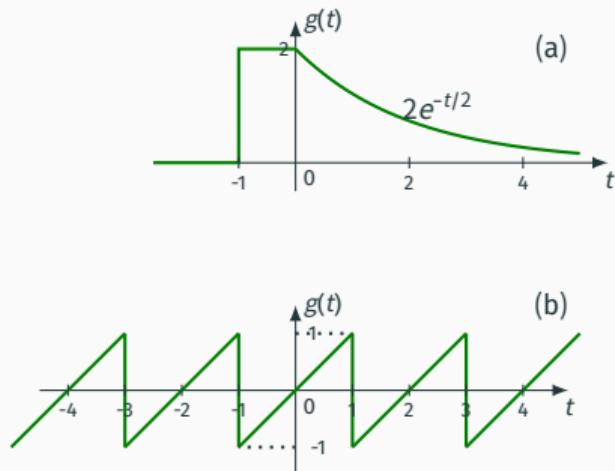


Fig. 2.2: Energy vs Power Signals

(Recap)

Fig. 2.2(a) Energy Signal

A signal is said to be an energy signal if its energy is finite and its average power approaches zero.

$$\int_{-\infty}^{\infty} |g(t)|^2 dt < \infty \quad (2.6)$$

Fig. 2.2(b) Power Signal

A signal is said to be a power signal if its average power is finite and its energy approaches infinite.

$$0 < \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt < \infty \quad (2.7)$$

DETERMINISTIC AND RANDOM SIGNALS

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$$g(t) = \quad (2.8)$$

TIME SHIFTING A SIGNAL

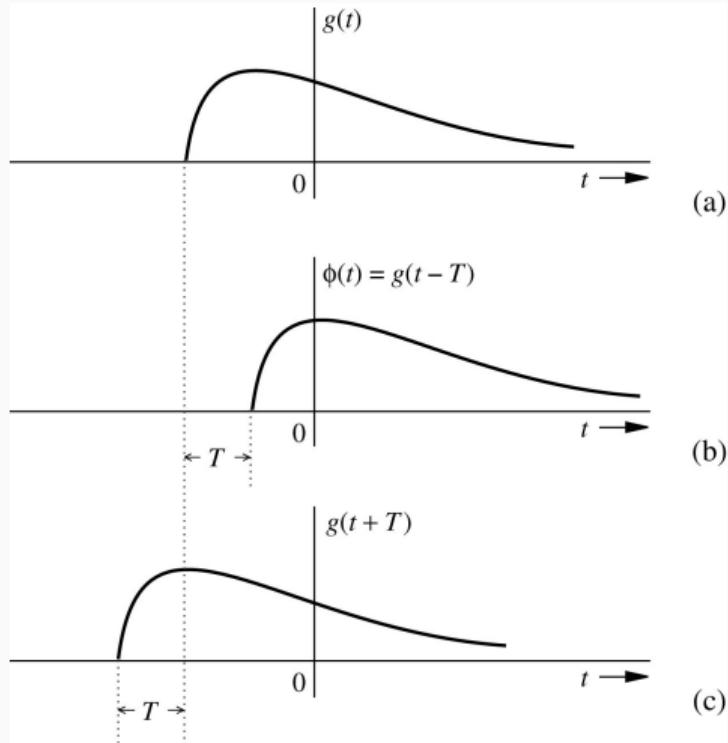


Fig. 2.6: Time shifting a signal

Fig. 2.6 Time shifting a signal

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$$g(t) = \dots \quad (2.9)$$

TIME SCALING A SIGNAL

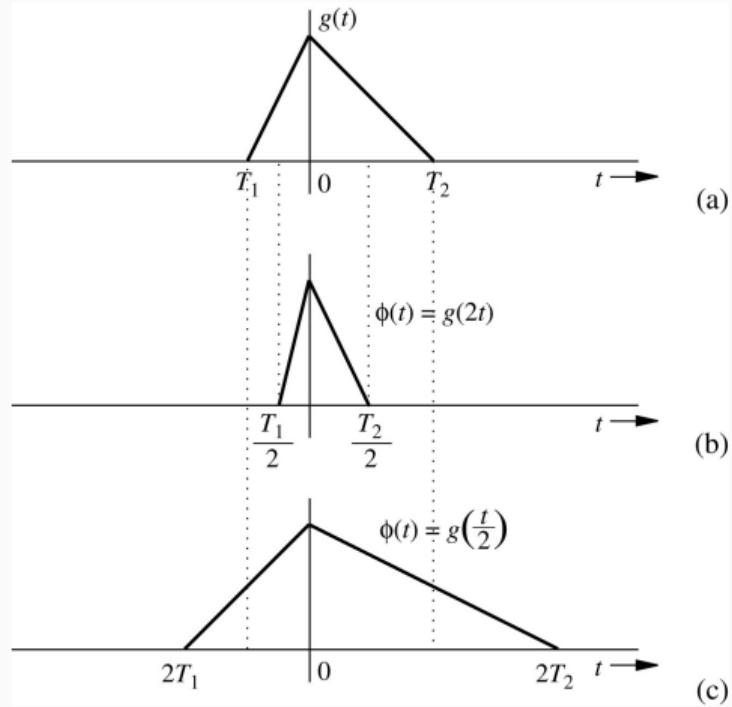


Fig. 2.7 Time scaling a signal

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$$g(t) = \dots \quad (2.10)$$

Fig. 2.7: Time scaling a signal

EXAMPLES OF TIME COMPRESSION AND TIME EXPANSION OF SIGNALS

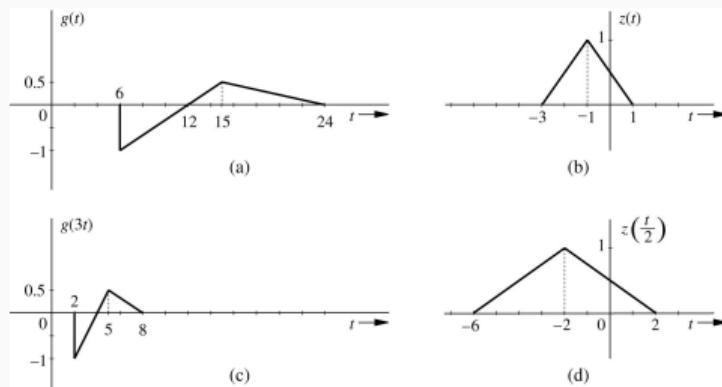
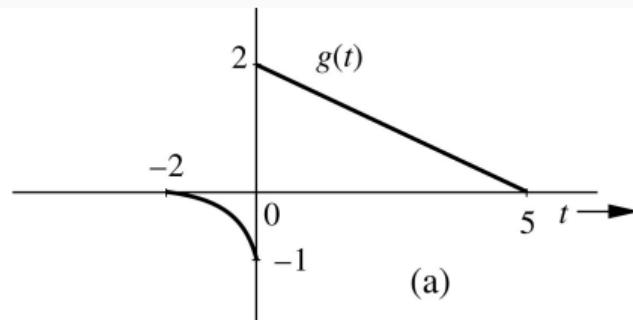


Fig. 2.8: Examples of time compression and time expansion of signals

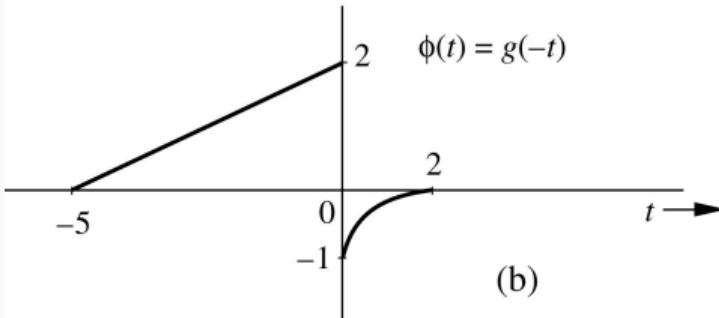
Fig. 2.8

$$g(t) = \dots \quad (2.11)$$

TIME INVERSION (REFLECTION) OF A SIGNAL



(a)



(b)

Fig. 2.9

$$g(t) = \dots \quad (2.12)$$

Fig. 2.9: Time inversion (reflection) of a signal

EXAMPLE OF TIME INVERSION

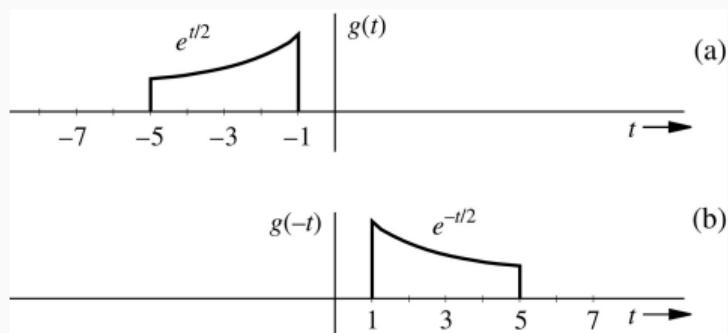


Fig. 2.10: Example of time inversion

Fig. 2.10

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$$g(t) = \dots \quad (2.13)$$

(A) UNIT IMPULSE AND (B) ITS APPROXIMATION



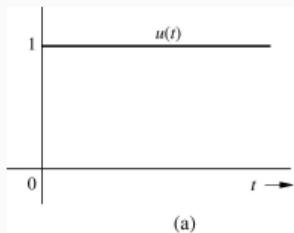
Fig. 2.11: (a) Unit impulse and (b) its approximation

Fig. 2.11

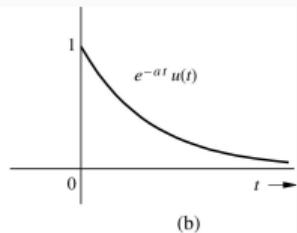
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$$g(t) = \quad (2.14)$$

(A) UNIT STEP FUNCTION $u(t)$. (B) CAUSAL EXPONENTIAL $e^{-at}u(t)$



(a)



(b)

Fig. 2.12: (a) Unit step function $u(t)$. (b) Causal exponential $e^{-at}u(t)$

Fig. 2.12

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$$g(t) = \quad (2.15)$$

SIGNALS VERSUS VECTORS

COMPONENT (PROJECTION) OF A VECTOR ALONG ANOTHER VECTOR

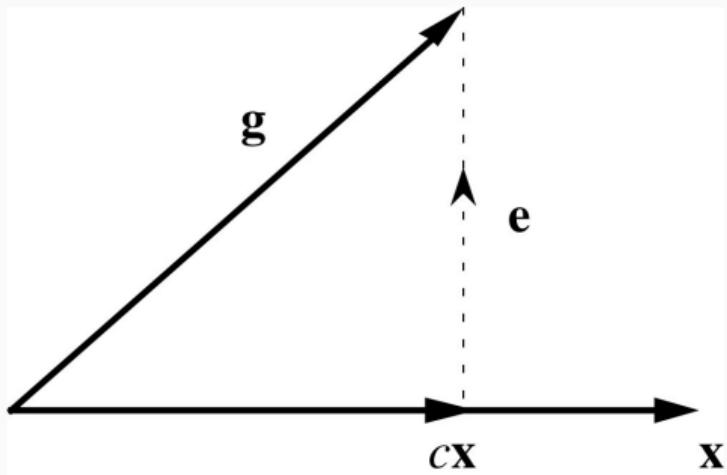


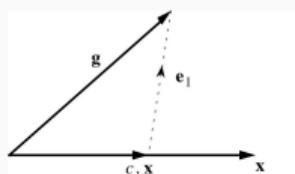
Fig. 2.13: Component (projection) of a vector along another vector

Fig. 2.13

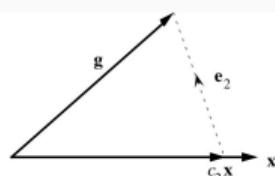
⋮
⋮

$$g(t) = \quad (2.16)$$

APPROXIMATIONS OF A VECTOR IN TERMS OF ANOTHER VECTOR



(a)



(b)

Fig. 2.14: Approximations of a vector in terms of another vector

Fig. 2.14

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$$g(t) = \quad (2.17)$$

DECOMPOSITION OF A SIGNAL AND SIGNAL COMPONENTS

APPROXIMATION OF SQUARE SIGNAL IN TERMS OF A SINGLE SINUSOID

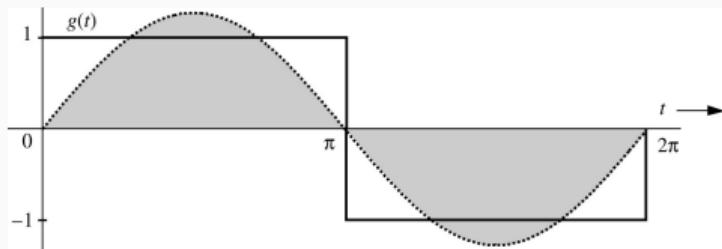


Fig. 2.15: Approximation of square signal in terms of a single sinusoid

Fig. 2.15

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$$g(t) = \dots \quad (2.18)$$

COMPLEX SIGNAL SPACE AND ORTHOGONALITY

ENERGY OF THE SUM OF ORTHOGONAL SIGNALS

SIGNALS FOR EXAMPLE 2.6

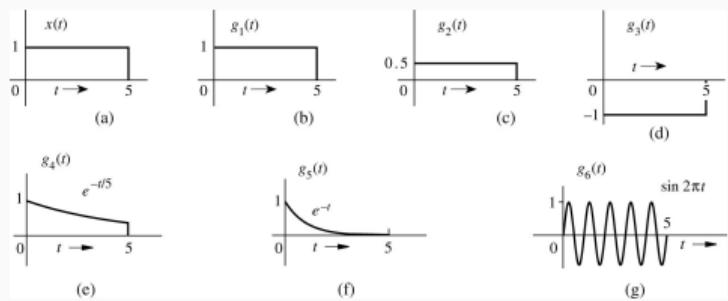


Fig. 2.16: Signals for Example 2.6

Fig. 2.16

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$$g(t) = \quad (2.19)$$

CORRELATION OF SIGNALS

CORRELATION FUNCTIONS

PHYSICAL EXPLANATION OF THE AUTO-CORRELATION FUNCTION

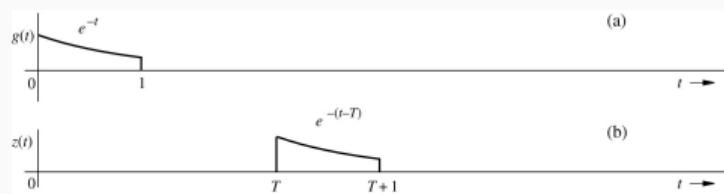


Fig. 2.17: Physical explanation of the auto-correlation function

Fig. 2.17

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$$g(t) = \quad (2.20)$$

REPRESENTATION OF A VECTOR IN THREE-DIMENSIONAL SPACE

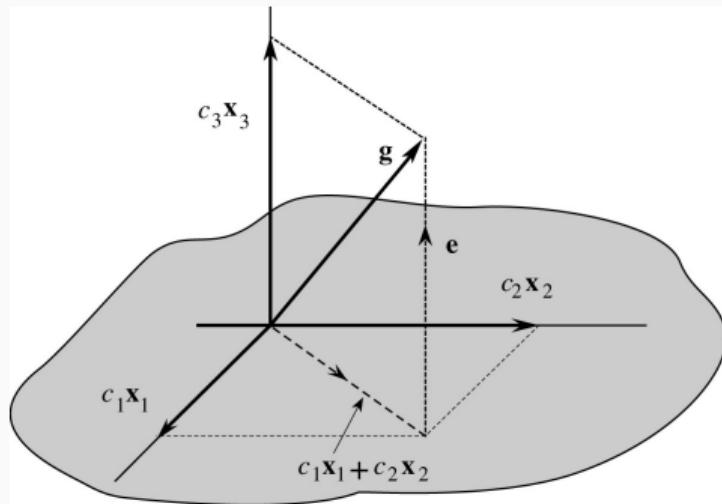


Fig. 2.18: Representation of a vector in three-dimensional space

Fig. 2.18

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$$g(t) = \quad (2.21)$$

ORTHOGONAL VECTOR SPACE

ORTHOGONAL SIGNAL SPACE

PARSEVAL'S THEOREM

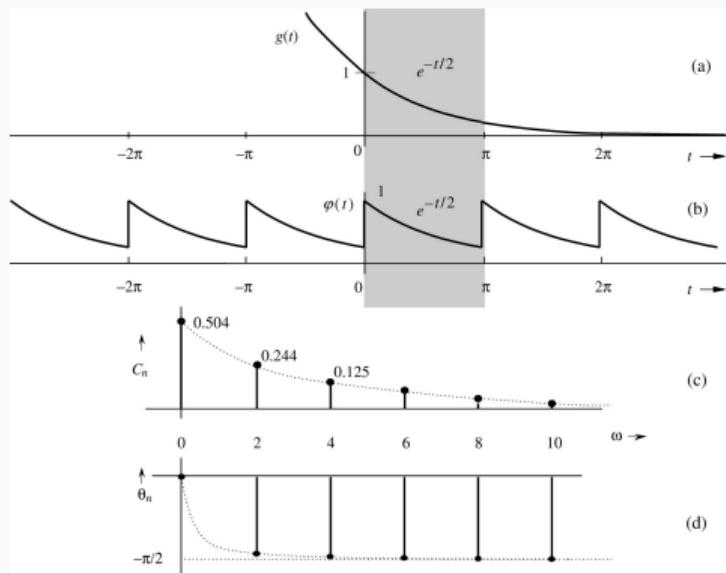


Fig. 2.19:

Fig. 2.19

$$g(t) = \quad (2.22)$$

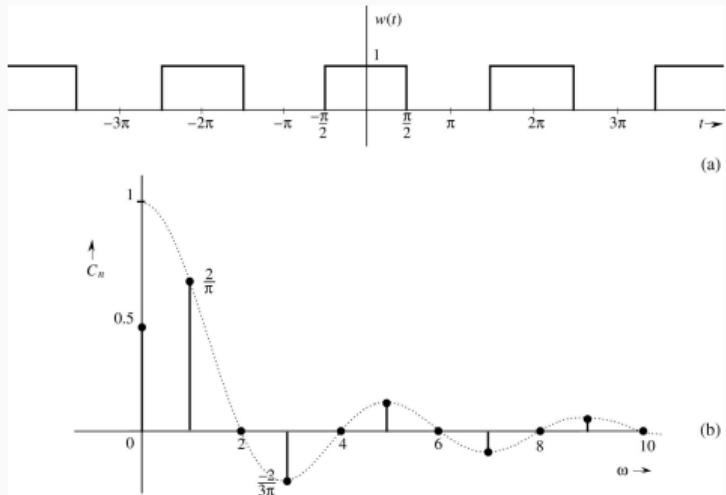


Fig. 2.20:

Fig. 2.20

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$$g(t) = \quad (2.23)$$

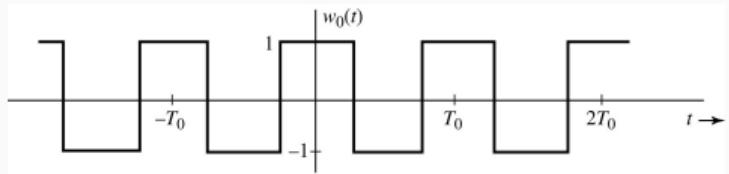


Fig. 2.21: .

Fig. 2.21

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$$g(t) = \quad (2.24)$$

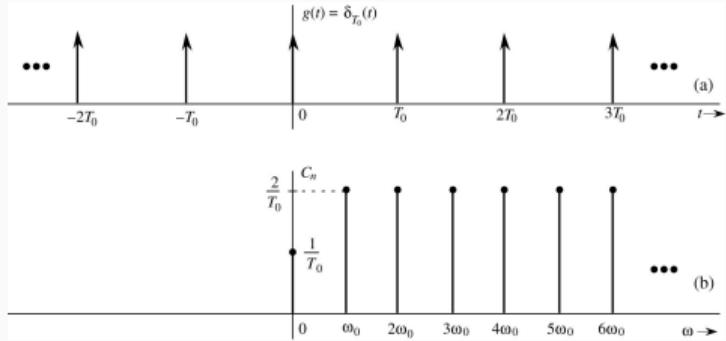


Fig. 2.22:

Fig. 2.22

$$g(t) = \dots \quad (2.25)$$

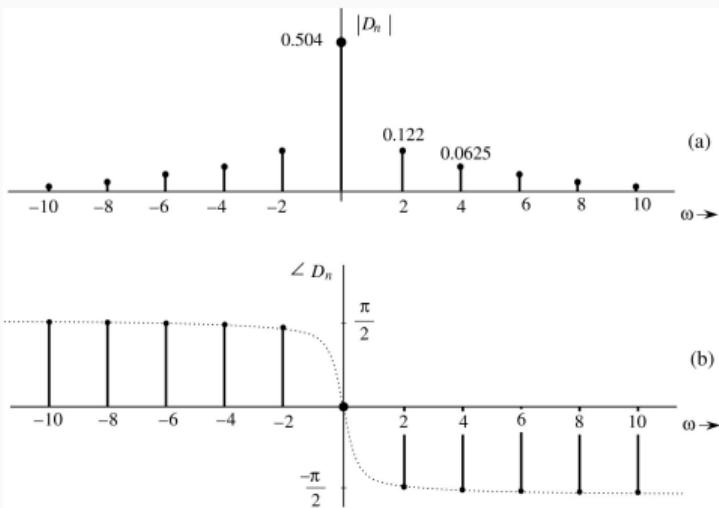


Fig. 2.23

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$$g(t) = \quad (2.26)$$

Fig. 2.23: .

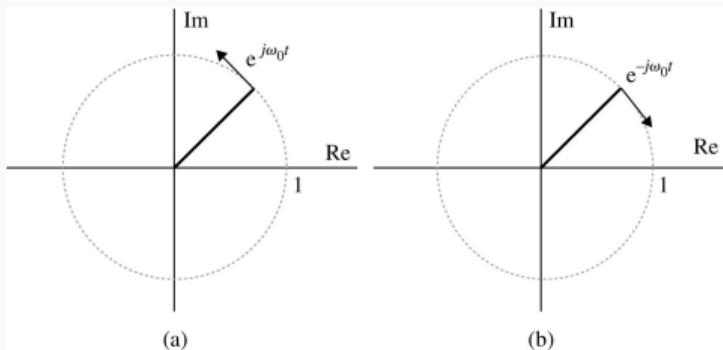


Fig. 2.24: .

Fig. 2.24

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$$g(t) = \quad (2.27)$$

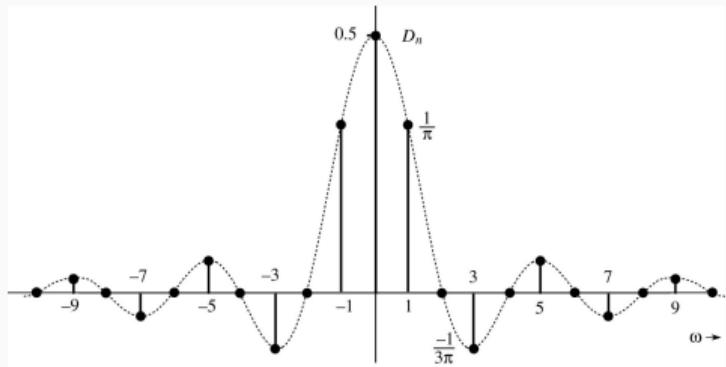


Fig. 2.25: .

Fig. 2.25

$$g(t) = \dots \quad (2.28)$$

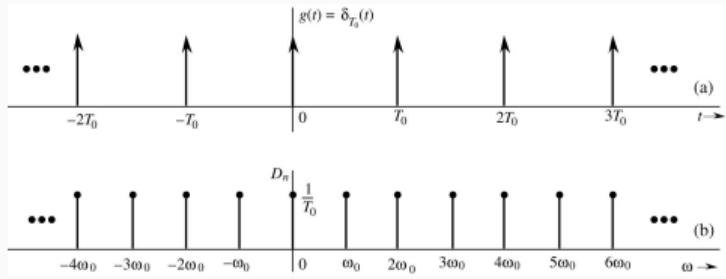


Fig. 2.26:

Fig. 2.26

$$g(t) = \dots \quad (2.29)$$

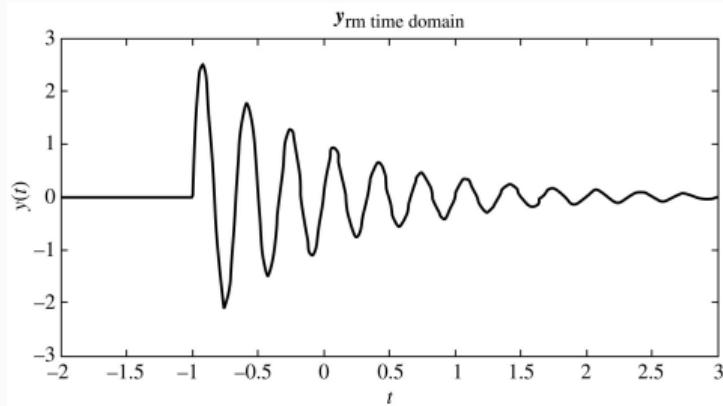


Fig. 2.27: .

Fig. 2.27

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$$g(t) = \quad (2.30)$$

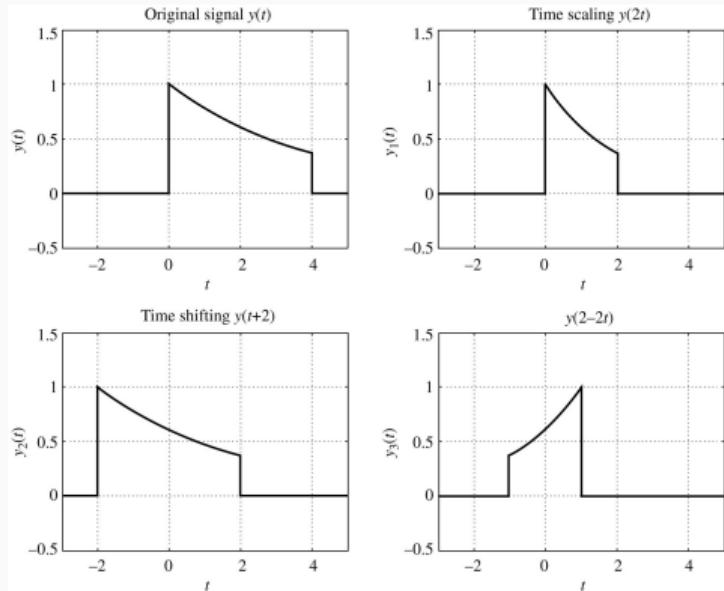


Fig. 2.28

Fig. 2.28

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$$g(t) = \quad (2.31)$$

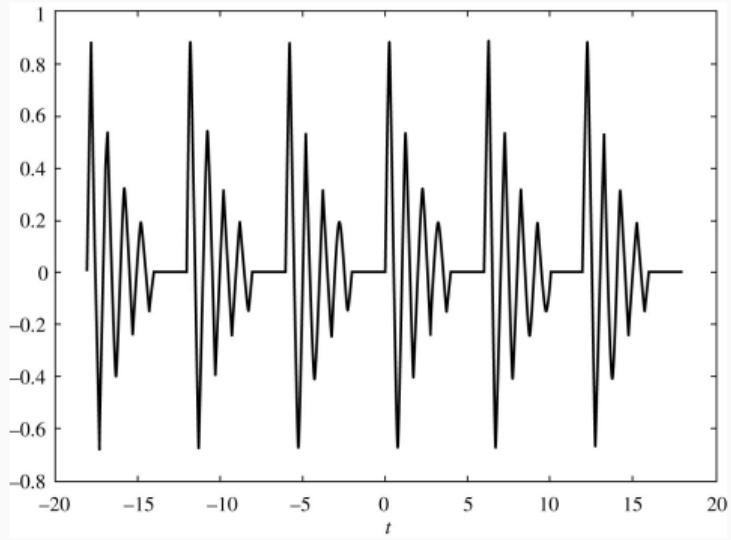


Fig. 2.29: .

Fig. 2.29

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$$g(t) = \quad (2.32)$$

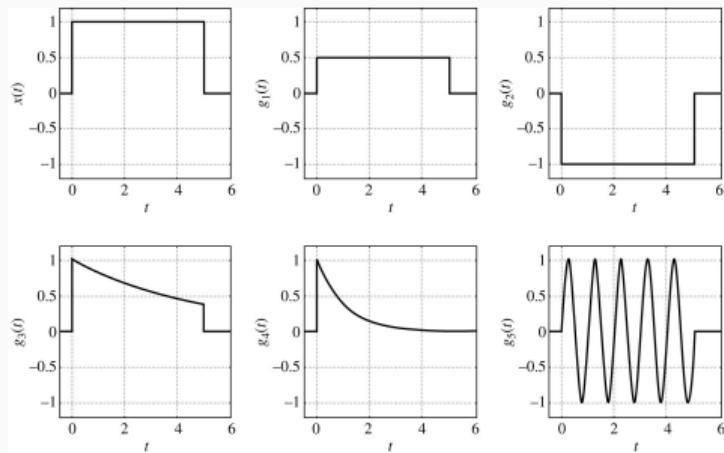


Fig. 2.30: .

Fig. 2.30

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$$g(t) = \quad (2.33)$$

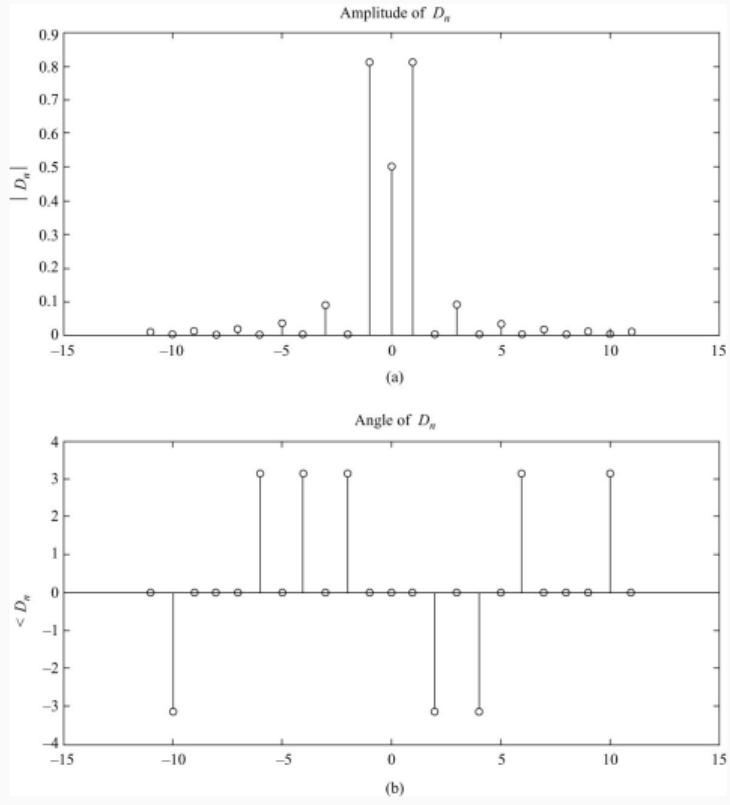


Fig. 2.31

$$g(t) = \quad (2.34)$$

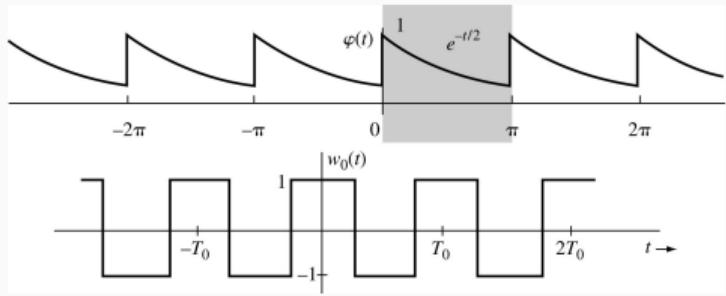
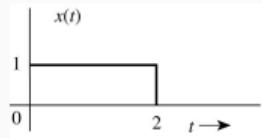


Fig. 2.32:

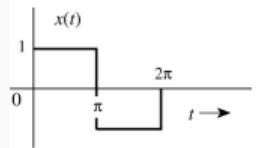
Fig. 2.32

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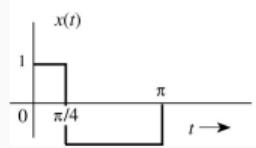
$$g(t) = \quad (2.35)$$



(a)



(b)



(c)

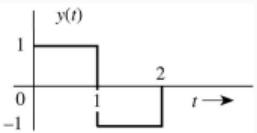
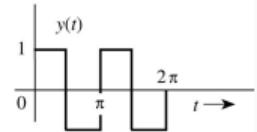
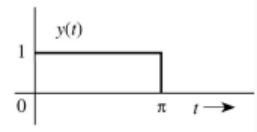


Fig. 2.33



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$$g(t) = \quad (2.36)$$

Fig. 2.33: .

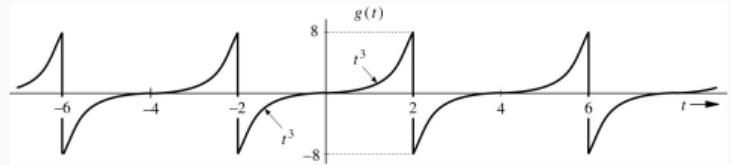


Fig. 2.34: .

Fig. 2.34

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$$g(t) = \quad (2.37)$$

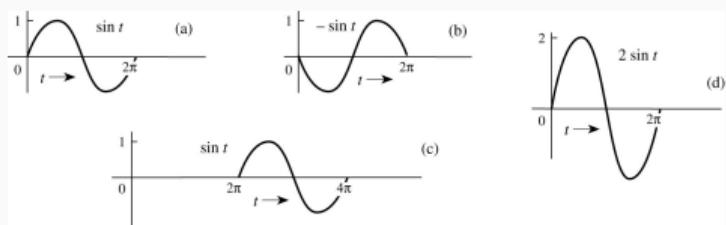


Fig. 2.35: .

Fig. 2.35

$$g(t) = \quad (2.38)$$

SUMMARY
