

# Chapter 2. Signals and Signal Space

*Communication Theory - 2026*

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EE / KNU

1. SIZE OF A SIGNAL

2. CLASSIFICATION OF SIGNALS

3. UNIT IMPULSE SIGNAL

4. SIGNALS VERSUS VECTORS

5. CORRELATION OF SIGNALS

6. ORTHOGONAL SIGNAL SET

7. THE EXPONENTIAL FOURIER SERIES

# DEFINITION: SIGNAL AND SYSTEM

## Signal

A signal is a set of information or data.

The signals are functions of the independent variable **time  $t$** .

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- Examples: Audio signals, video signals, sensor data, etc.

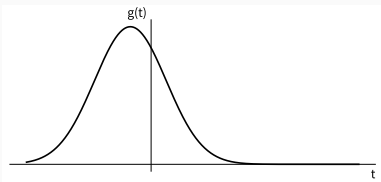
## System

Signals may be processed further by systems, which may modify them or extract additional information from them.

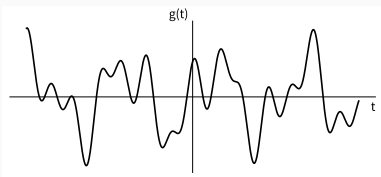
Thus, a system is an entity that processes signals (**inputs**) to yield another set of signals (**outputs**).

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- For example, an anti-aircraft radar system processes the received signals (inputs) to determine the position and velocity of an aircraft (outputs).
  - More examples: Amplifiers, filters, modulators, demodulators, etc.

# ENERGY VS POWER SIGNALS



(a) Signal with finite energy



(b) Signal with finite power

**Fig. 2.1:** Examples of signals

## Energy Signal

A signal is said to be an energy signal if its energy is finite and its average power approaches zero.

$$E = \int_{-\infty}^{\infty} |g(t)|^2 dt < \infty, \quad P \rightarrow 0$$

## Power Signal

A signal is said to be a power signal if its average power is finite and its energy approaches infinite.

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt < \infty, \quad E \rightarrow \infty$$

## UNITS OF SIGNAL POWER

- The standard units of signal energy and power are the "joule" and the "watt".
- However, in practice, it is often customary to use logarithmic scales to describe signal power.
- A signal with average power of  $P$  watts has power of either  $P_{dBW}$  or  $P_{dBm}$ .

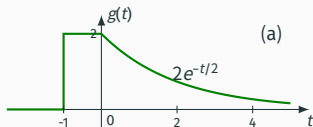
$$P_{dBW} = [10 \cdot \log_{10} P] \text{ dBW}$$

$$P_{dBm} = [30 + 10 \cdot \log_{10} P] \text{ dBm}$$

- For example,  
 $P_{dBm} = -30 \text{ dBm} = 10^{-6} \text{ W}.$

## EXAMPLE 2.1

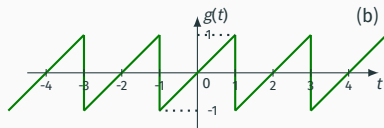
**Q. Determine the suitable measures of the signals in Fig. 2.2.**



**Fig. 2.2 (a)**

Energy signal. Power approaches 0 as  $|t| \rightarrow \infty$ .

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-1}^0 (2)^2 dt + \int_0^{\infty} 4e^{-t} dt = 4 + 4 = 8$$



**Fig. 2.2 (b)**

Power signal. Averaging  $|g|^2(t)$  over an infinitely large interval is equivalent to averaging it over one period (2 seconds).

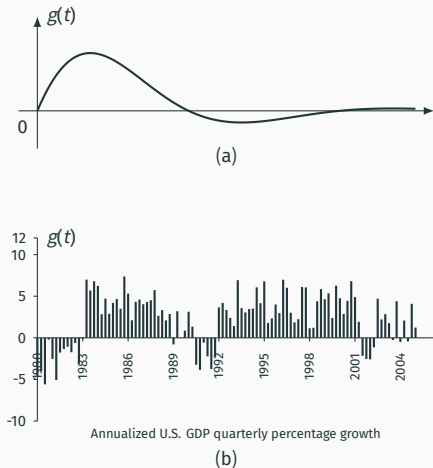
$$\begin{aligned} P_g &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt = \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt \\ &= \frac{1}{2} \int_{-1}^1 t^2 dt = \frac{1}{3} \end{aligned}$$

**Fig. 2.2:** Signal for Example

# CLASSIFICATION OF SIGNALS

1. Continuous time and discrete time signals
2. Analog and digital signals
3. Periodic and aperiodic signals
4. Energy and power signals
5. Deterministic and probabilistic signals

## FIGURE 2.3 CONTINUOUS VS DISCRETE TIME SIGNALS



**Fig. 2.3 (a)**

**Continuous time signals** are specified for every value of time  $t$ . Many examples including:

- Audio recordings in analog media like LP, magnetic cassette, or reel-to-reel tapes.
- Signals received through AM/FM radio channel.

**Fig. 2.3 (b)**

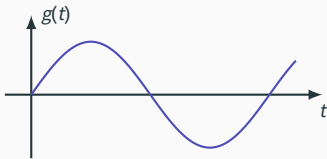
**Discrete time signals** are specified only at discrete points of  $t = nT$ . Many examples including:

- The quarterly gross domestic product (GDP), stock market daily averages, and monthly sales of a corporation.
- Audio signals formatted by MP3, HE-AAC, FLAC, or ALAC.

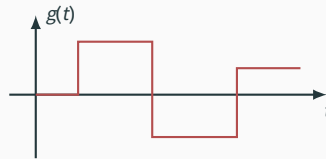
**Fig. 2.3:** *Continuous vs Discrete Time Signals*



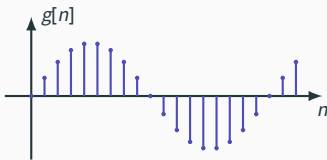
## FIGURE 2.4 CLASSIFICATION OF SIGNALS



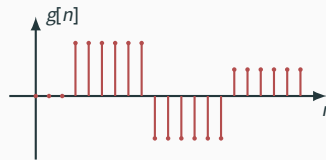
(a) Analog, Continuous-time



(b) Digital, Continuous-time



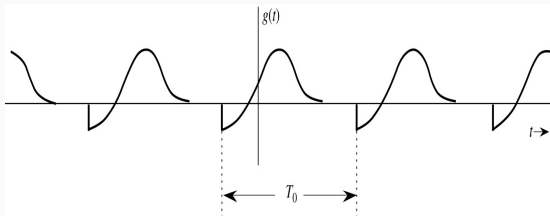
(c) Analog, Discrete-time



(d) Digital, Discrete-time

**Fig. 2.4:** Classification of signals based on amplitude and time domains

# PERIODIC AND APERIODIC SIGNALS



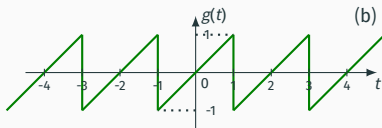
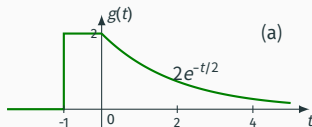
**Fig. 2.5:** Periodic signal of period  $T_0$

**Fig. 2.5**

A signal  $g(t)$  is **periodic** if there exists a positive constant  $T_0$ . The smallest value of  $T_0$  in Eq. (2.0) is the **period** of  $g(t)$ .

$$g(t) = g(t + T_0) \quad \text{for all } t \quad (2.5)$$

# ENERGY VS POWER SIGNALS



**Fig. 2.2:** *Energy vs Power Signals*  
(Recap)

## Fig. 2.2(a) Energy Signal

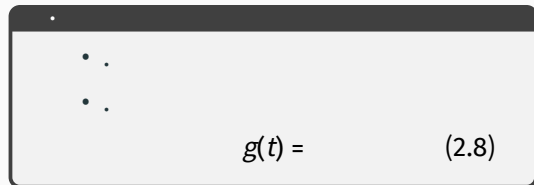
A signal is said to be an energy signal if its energy is finite and its average power approaches zero.

$$\int_{-\infty}^{\infty} |g(t)|^2 dt < \infty \quad (2.6)$$

## Fig. 2.2(b) Power Signal

A signal is said to be a power signal if its average power is finite and its energy approaches infinite.

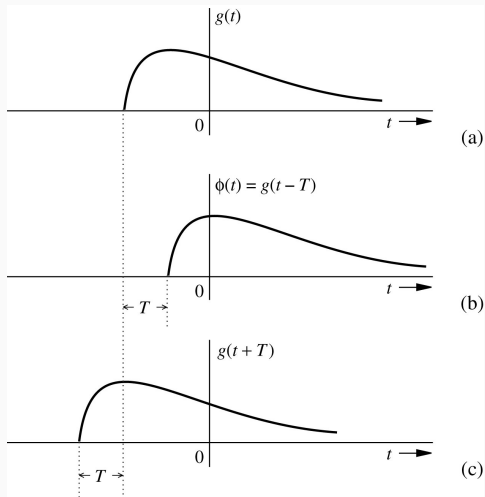
$$0 < \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt < \infty \quad (2.7)$$



A light gray rectangular box with a dark gray border and rounded corners. Inside the box, on the left side, there are two vertically aligned dots, each followed by a period (• .). To the right of these dots, the equation  $g(t) =$  is displayed, followed by the label (2.8) on the far right.

$$g(t) = \quad (2.8)$$

# TIME SHIFTING A SIGNAL



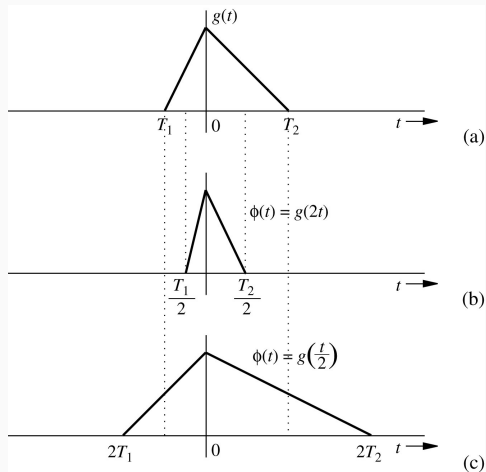
**Fig. 2.6:** Time shifting a signal

**Fig. 2.6** Time shifting a signal

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$$g(t) = \quad (2.9)$$

# TIME SCALING A SIGNAL



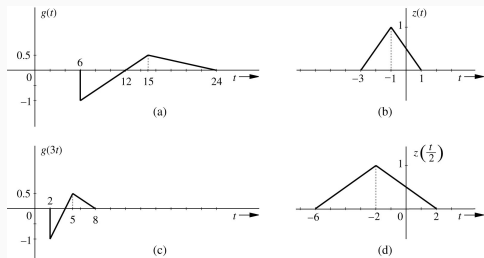
**Fig. 2.7:** Time scaling a signal

**Fig. 2.7** Time scaling a signal

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$$g(t) = \quad (2.10)$$

# EXAMPLES OF TIME COMPRESSION AND TIME EXPANSION OF SIGNALS



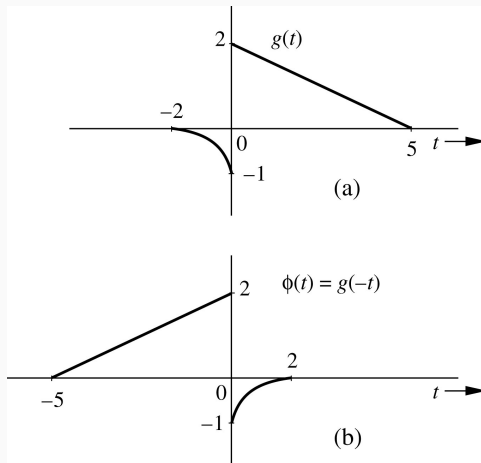
**Fig. 2.8:** Examples of time compression and time expansion of signals

**Fig. 2.8**

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$$g(t) = \quad (2.11)$$

# TIME INVERSION (REFLECTION) OF A SIGNAL



**Fig. 2.9:** Time inversion (reflection) of a signal

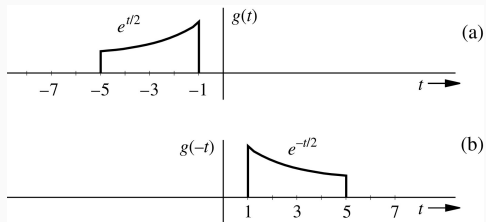
**Fig. 2.9**

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$$g(t) = \quad (2.12)$$



## EXAMPLE OF TIME INVERSION



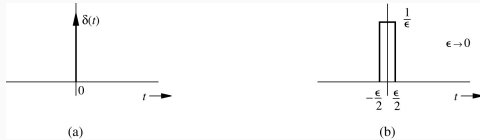
**Fig. 2.10:** Example of time inversion

**Fig. 2.10**

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$$g(t) = \quad (2.13)$$

## (A) UNIT IMPULSE AND (B) ITS APPROXIMATION



**Fig. 2.11:** (a) Unit impulse and (b) its approximation

**Fig. 2.11**

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$$g(t) = \quad (2.14)$$

(A) UNIT STEP FUNCTION  $u(t)$ . (B) CAUSAL EXPONENTIAL  $e^{-at}u(t)$



**Fig. 2.12:** (a) Unit step function  $u(t)$ . (b) Causal exponential  $e^{-at}u(t)$

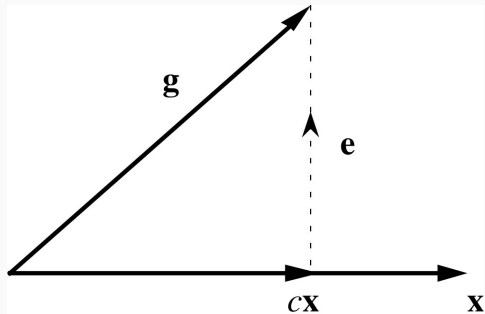
**Fig. 2.12**

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$$g(t) = \quad (2.15)$$

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## COMPONENT (PROJECTION) OF A VECTOR ALONG ANOTHER VECTOR



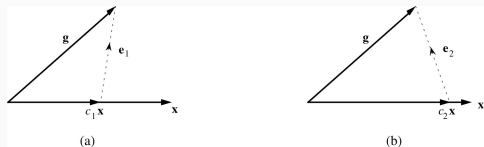
**Fig. 2.13:** Component (projection) of a vector along another vector

**Fig. 2.13**

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$$g(t) = \quad (2.16)$$

# APPROXIMATIONS OF A VECTOR IN TERMS OF ANOTHER VECTOR



**Fig. 2.14:** Approximations of a vector in terms of another vector

**Fig. 2.14**

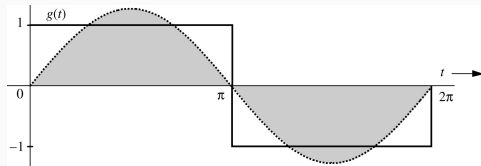
• .  
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$$g(t) = \quad (2.17)$$

# DECOMPOSITION OF A SIGNAL AND SIGNAL COMPONENTS

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# APPROXIMATION OF SQUARE SIGNAL IN TERMS OF A SINGLE SINUSOID



**Fig. 2.15:** Approximation of square signal in terms of a single sinusoid

**Fig. 2.15**

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$$g(t) = \quad (2.18)$$

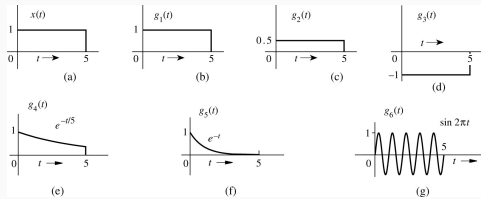


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## ENERGY OF THE SUM OF ORTHOGONAL SIGNALS

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# SIGNALS FOR EXAMPLE 2.6



**Fig. 2.16:** Signals for Example 2.6

**Fig. 2.16**

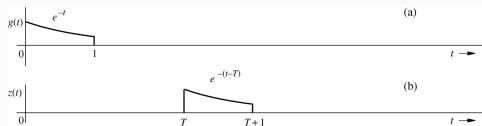
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$$g(t) = \quad (2.19)$$

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# PHYSICAL EXPLANATION OF THE AUTO-CORRELATION FUNCTION



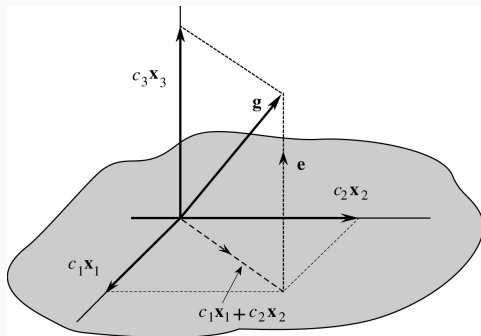
**Fig. 2.17:** Physical explanation of the auto-correlation function

**Fig. 2.17**

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$$g(t) = \quad (2.20)$$

# REPRESENTATION OF A VECTOR IN THREE-DIMENSIONAL SPACE



**Fig. 2.18:** Representation of a vector in three-dimensional space

**Fig. 2.18**

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$$g(t) = \quad (2.21)$$

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# PARSEVAL'S THEOREM

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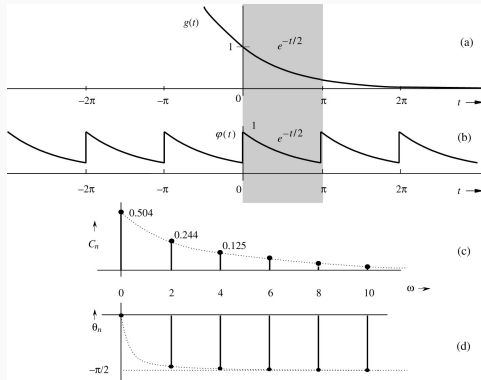


Fig. 2.19: .

Fig. 2.19

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$$g(t) = \quad (2.22)$$

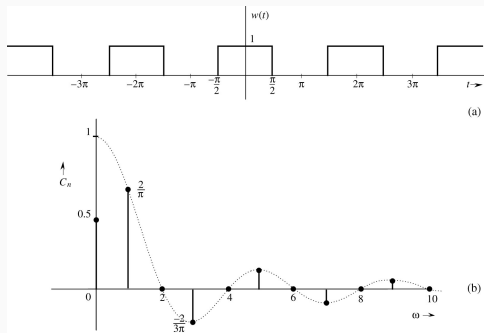
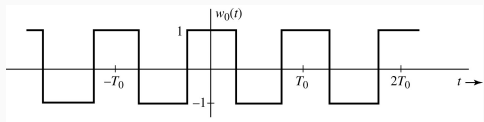


Fig. 2.20: .

Fig. 2.20

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$$g(t) = \quad (2.23)$$



**Fig. 2.21:** .

**Fig. 2.21**

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$$g(t) = \quad (2.24)$$

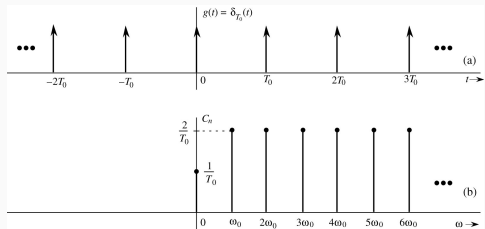


Fig. 2.22: .

Fig. 2.22



$$g(t) = \quad (2.25)$$

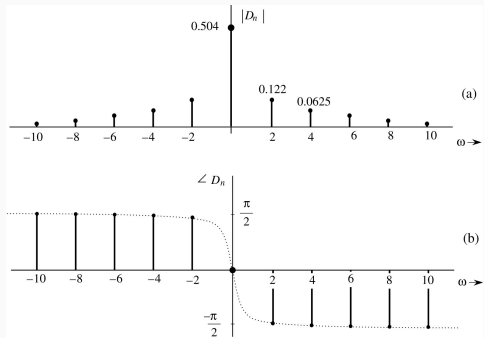


Fig. 2.23: .

Fig. 2.23



$$g(t) = \quad (2.26)$$

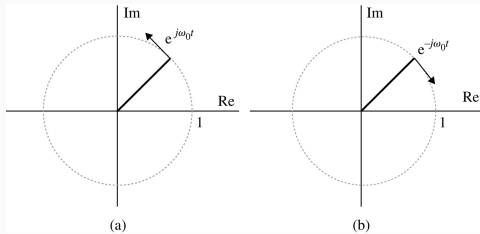


Fig. 2.24: .

Fig. 2.24

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$$g(t) = \quad (2.27)$$



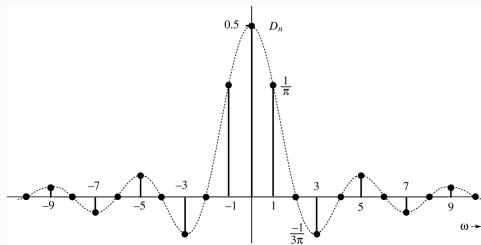


Fig. 2.25: .

Fig. 2.25

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$$g(t) = \quad (2.28)$$

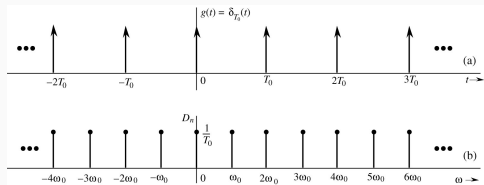
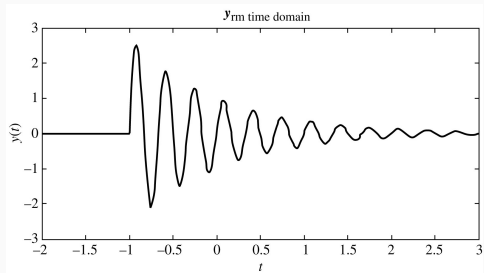


Fig. 2.26: .

Fig. 2.26



$$g(t) = \quad (2.29)$$

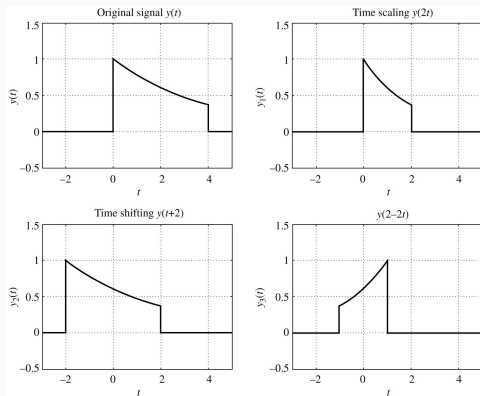


**Fig. 2.27:** .

**Fig. 2.27**

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$$g(t) = \quad (2.30)$$



**Fig. 2.28**

**Fig. 2.28**

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$$g(t) = \quad (2.31)$$

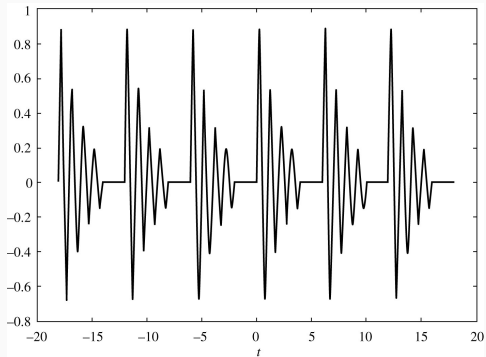


Fig. 2.29: .

Fig. 2.29

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$$g(t) = \quad (2.32)$$

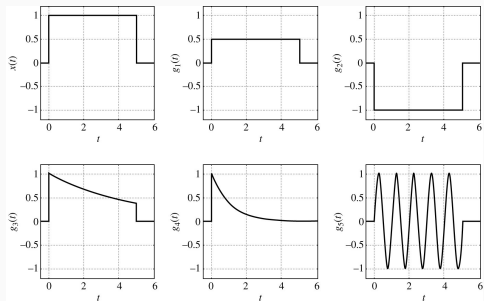


Fig. 2.30: .

Fig. 2.30

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$$g(t) = \quad (2.33)$$

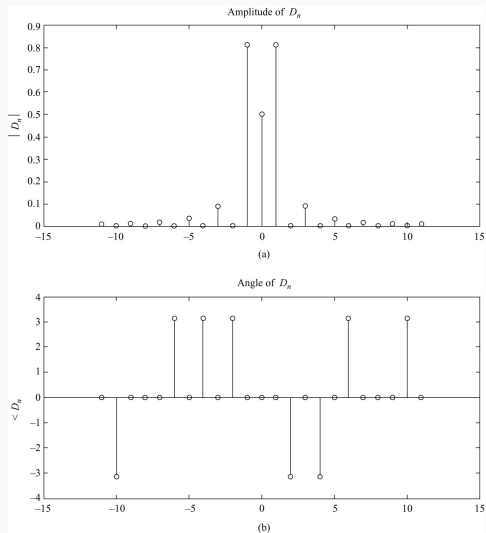


Fig. 2.31: .

Fig. 2.31



$$g(t) = \quad (2.34)$$

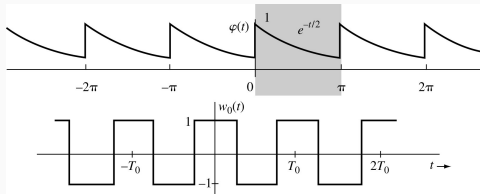


Fig. 2.32: .

Fig. 2.32

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$$g(t) = \quad (2.35)$$



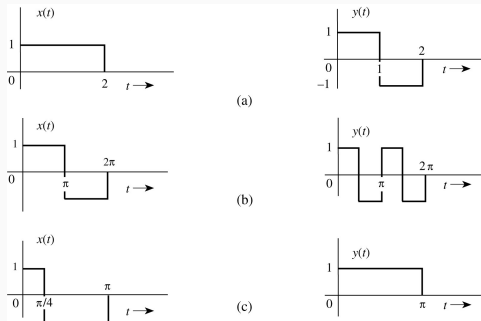


Fig. 2.33: .

Fig. 2.33

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$$g(t) = \quad (2.36)$$

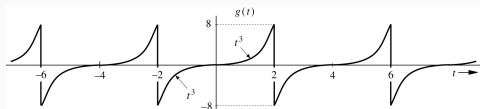
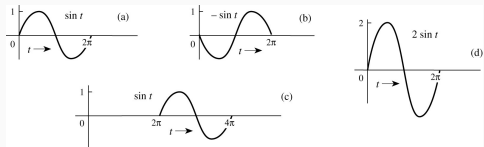


Fig. 2.34: .

Fig. 2.34

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$$g(t) = \quad (2.37)$$



**Fig. 2.35:** .

**Fig. 2.35**

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$$g(t) = \quad (2.38)$$

