# Chapter 4. Exponents and logarithms

## 1. Exponential and Logarithm funtions

**Definition 1:** A function of the form $f(x) = ab^x$ where $a \neq 0$, $b > 0$ and $b \neq 1$ is an **exponential function** with base $b$.

**Definition 2:** A **logarithms function** with base $b$ is the inverse of an exponential function of the form $y = b^x$ where $x$ and $b$ are positive numbers and $b \neq 1$, $y = \log_b x$ if and only if $b^y = x$.

## 2. Propierties of Logarithms

**Basic Properties of logarithms**: If $x$ and $b$ are positives numbers and $b \neq 0$.

1. $\log_b 1 = 0$ since $b^0 = 1$
2. $\log_b b = 1$ since $b^1 = b$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x$

## 3. Properties of exponential and logarithms

| Property | of Logarithms (*) | of Exponents |
|---|---|---|
| Equality | $\log_b m = \log_b n \iff m = n$ | $b^m = b^n \iff m = n$ |
| Product | $\log_b mn = \log_b m + \log_b n$ | $b^m \cdot b^n = b^{m+n}$ |
| Quotient | $\log_b \frac{m}{n} = \log_b m - \log_b n$ | $\frac{b^m}{b^n} = b^{m-n}$ |
| Power | $\log_b m^p = p \log_b m$ | $(b^m)^p = b^{mp}$ |

(*) where $m > 0$ and $n > 0$

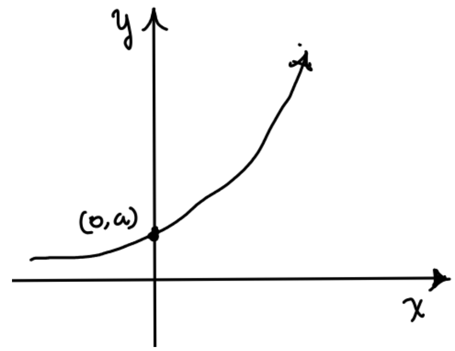
**Natural logarithm:** Is defined by $\log_e(x) = \ln(x)$. Where $e = 2.71828182846\ldots$

**References in the book:** From pages 127 to 148.

# 4. Analysis of exponential functions

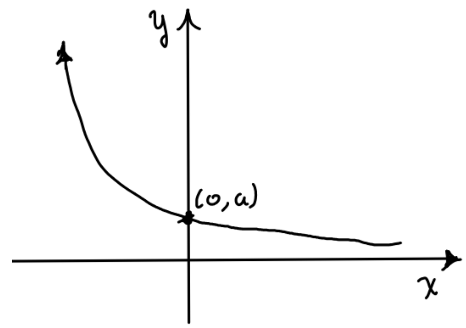
$f(x) = ab^x$ with $a > 0$ and $b > 1$

Exponential
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Asymptote: The line $x$
Increasing



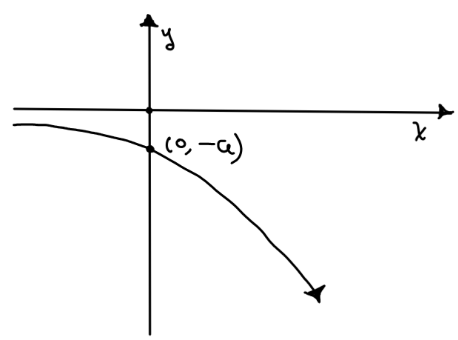

$f(x) = ab^x$ with $a > 1$ and $0 < b < 1$

Exponential
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Asymptote: The line $x$
Decreasing



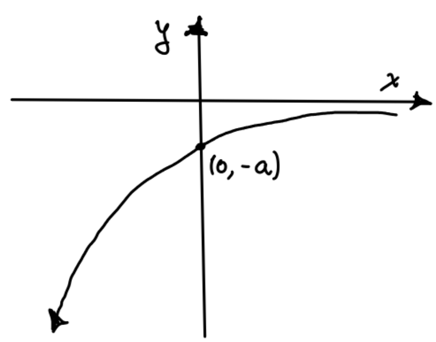

$f(x) = -ab^x$ with $b > 1$

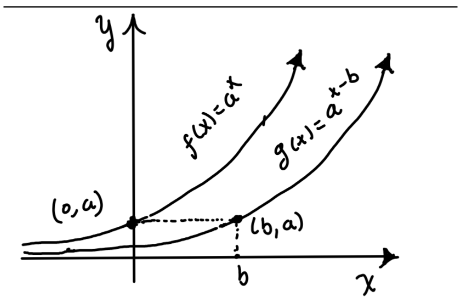
Exponential
Domain: $(-\infty, \infty)$
Range: $(-\infty, 0)$
Asymptote: The line $x$
Decreasing




$f(x) = -ab^x$ with $0 < b < 1$

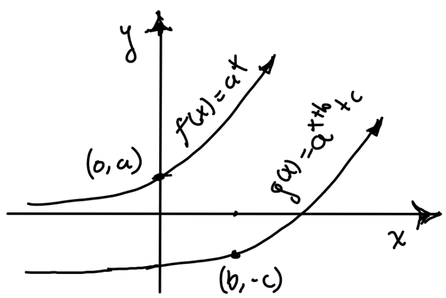
Exponential
Domain: $(-\infty, \infty)$
Range: $(-\infty, 0)$
Asymptote: The line $x$
Increasing






$f(x) = ad^{x-b}$ with $a > 0$

Exponential
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Asymptote: The line $x$
Increasing



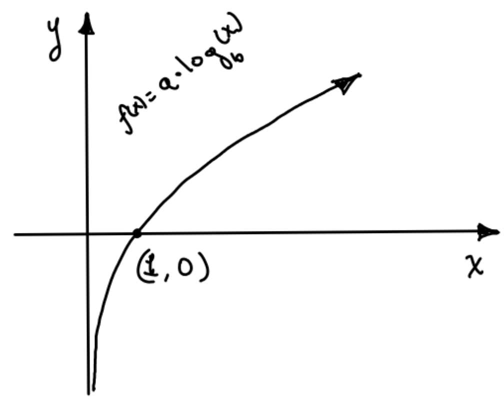

$g(x) = ad^{x+b} + c$ with $a > 0$

Exponential
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Asymptote: The line $c$
Increasing

## 5. Analysis of logarithms functions

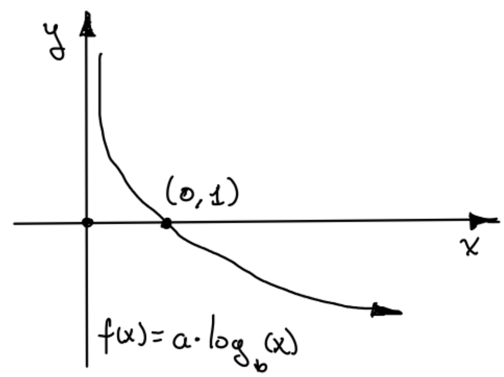
$f(x) = a\log_b(x)$ with $b > 0$

Logarithmic
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Asymptote: The line $x$
Increasing



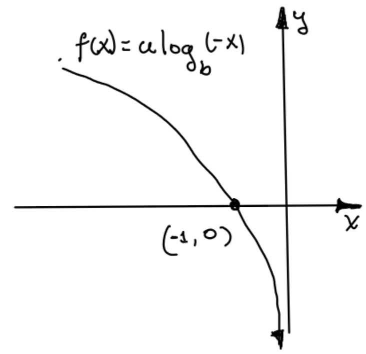

$f(x) = a\log_b(x)$ with $0 < b < 1$

Logarithmic
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Asymptote: The line $x$
Decreasing



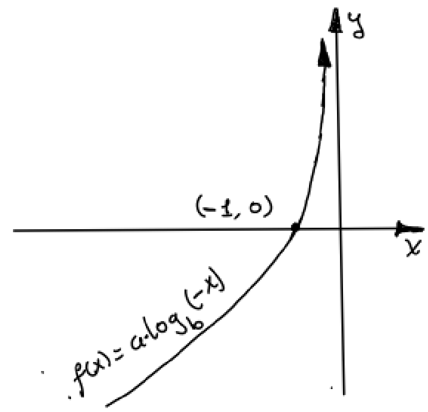
$f(x) = a\log_b(-x)$ with $b > 1$

Logarithmic
Domain: $(-\infty, 0)$
Range: $(-\infty, +\infty)$
Asymptote: The line $x$
Decreasing



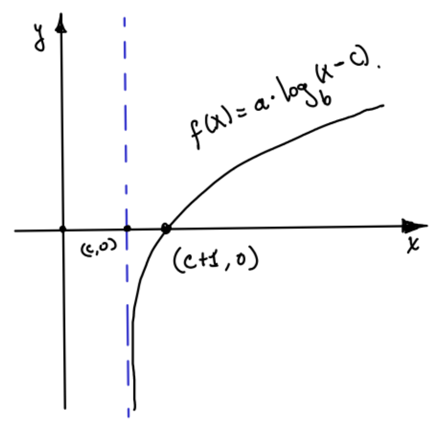

$f(x) = a\log_b(-x)$ with $0 < b < 1$

Logarithmic
Domain: $(-\infty, 0)$
Range: $(-\infty, +\infty)$
Asymptote: The line $x$
Increasing



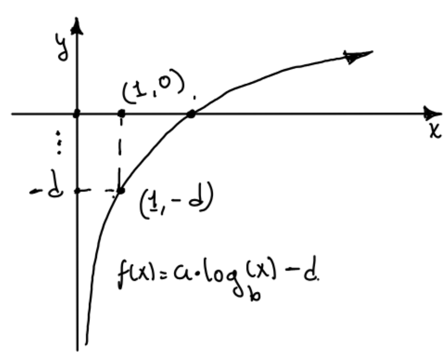

$f(x) = a\log_b(x - c)$ with $b > 1$

Logarithmic
Domain: $(c, \infty)$
Range: $(-\infty, \infty)$
Asymptote: The line $c$
Increasing




$f(x) = a\log_b(x) - d$ with $b > 1$

Logarithmic
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Asymptote: The line $x$
Increasing



Comparison of exponential ($a^x$) and logarithmic ($a\log_b(x)$) functions

Asymptote: The line $g(x) = x$