B16 Software Engineering

Structured Programming

Lecture 1: Software engineering

Dr Andrea Vedaldi
4 lectures, Hilary Term

For lecture notes, tutorial sheets, and updates see
http://www.robots.ox.ac.uk/~vedaldi/teach.html

B16: Software Engineering

Four Parts

Structured Programming

- Algorithms, data structures, complexity, correctness
- Structured programming languages: C

Object Oriented Programming

- Object-oriented programming
- Object-oriented languages: C++

Design patterns

- Tested program structures to solve typical problems

Operating Systems

- Hardware abstraction and virtualisation
- Programming the hardware

B16 Part 1: Structured Programming

Software engineering principles

- Design, modularity, abstraction, encapsulation, etc.

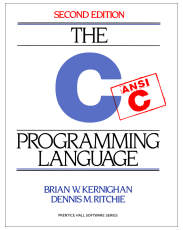
Structured programming languages

- Interpreted (MATLAB) vs compiled (C) languages
- **Control flow**
  - Sequencing, alternation, iteration
  - Functions and libraries
- **Data**
  - Data types: primitive, aggregate, and compound
  - Local and global variables, parameters
  - The heap and the stack

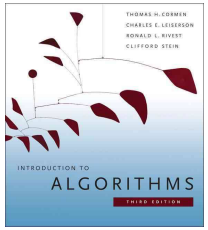
Algorithms

- Proving algorithm **correctness** by mathematical induction
- Time and space **complexity**
- **Recursion**

Texts



The C programming language, 2nd edition
Kernaghan & Ritchie



Introduction to algorithms
Cormen, Leiserson, Rivest, Stein

Lecture 1 outline

The challenge of building software

- Why we care about software
- The size and complexity of code

Software engineering

- The aims and scope of software engineering
- Abstraction and modularity
- Design, validation & verification

Structured programming

- Structuring programs by using abstractions in a programming language
- Types of languages: imperative vs declarative
- Fundamental abstractions

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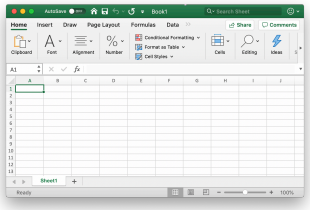
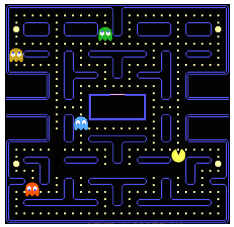
Software is ubiquitous

Business / productivity



Communication

Entertainment



Software in engineering: Design & Control

From nuclear reactors ...

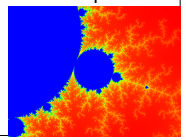
... to cars.

The “size” of software

SLOC: number of source lines of code

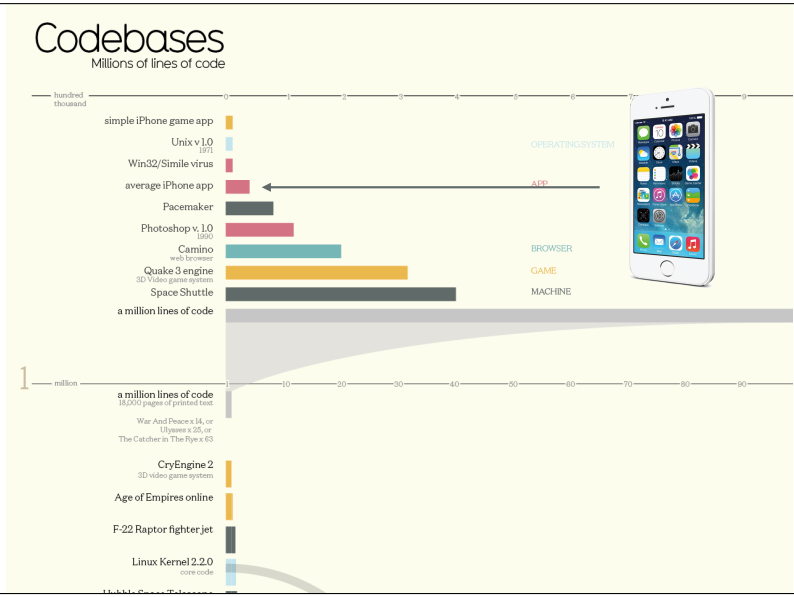
```
/*
c program:
--------------------------------
 1. draws Mandelbrot set for Fc(z)=z*z +c
 using Mandelbrot algorithm ( boolean escape time )
--------------------------------
2. technique of creating ppm file is  based on the code of Claudio Rocchini
http://en.wikipedia.org/wiki/Image:Color_complex_plot.jpg
create 24 bit color graphic file ,  portable pixmap file = PPM
see http://en.wikipedia.org/wiki/Portable_pixmap
to see the file use external application ( graphic viewer)
 */
#include <stdio.h>
#include <math.h>
int main()
{
        /* screen ( integer) coordinate */
        int iX,iY;
        const int iXmax = 800;
        const int iYmax = 800;
        /* world ( double) coordinate = parameter plane*/
        double Cx,Cy;
        const double CxMin=-2.5;
        const double CxMax=1.5;
        const double CyMin=-2.0;
        const double CyMax=2.0;
        /* */
        double PixelWidth=(CxMax-CxMin)/iXmax;
        double PixelHeight=(CyMax-CyMin)/iYmax;
        /* color component ( R or G or B) is coded from 0 to 255 */
        /* it is 24 bit color RGB file */
        const int MaxColorComponentValue=255;
        FILE * fp;
        char *filename="new1.ppm";
        char *comment="# ";/* comment should start with # */
        static unsigned char color[3];
        /* Z=Zx+Zy*i  ;   Z0 = 0 */
        double Zx, Zy;
        double Zx2, Zy2; /* Zx2=Zx*Zx;  Zy2=Zy*Zy  */
        /*  */
        int Iteration;
        const int IterationMax=200;
        /* bail-out value , radius of circle ;  */
        const double EscapeRadius=2;
        double ER2=EscapeRadius*EscapeRadius;
        /*create new file,give it a name and open it in binary mode  */
        fp= fopen(filename,"wb"); /* b -  binary mode */
        /*write ASCII header to the file*/
        fprintf(fp,"P6\n %s\n %d\n %d\n %d\n",comment,iXmax,iYmax,MaxColorComponentValue);
        /* compute and write image data bytes to the file*/
        for(iY=0;iY<iYmax;iY++)
        {
                Cy=CyMin + iY*PixelHeight;
                if (fabs(Cy)< PixelHeight/2) Cy=0.0; /* Main antenna */
                for(iX=0;iX<iXmax;iX++)
                {
                        Cx=CxMin + iX*PixelWidth;
                        /* initial value of orbit = critical point Z= 0 */
                        Zx=0.0;
                        Zy=0.0;
                        Zx2=Zx*Zx;
                        Zy2=Zy*Zy;
                        /* */
                        for (Iteration=0;Iteration<IterationMax && ((Zx2+Zy2)<ER2);Iteration++)
                        {
                            Zy=2*Zx*Zy + Cy;
                            Zx=Zx2-Zy2 +Cx;
                            Zx2=Zx*Zx;
                            Zy2=Zy*Zy;
                        };
                        /* compute  pixel color (24 bit = 3 bytes) */
                        if (Iteration==IterationMax)
                        {  /*  interior of Mandelbrot set = black */
                           color[0]=0;
                           color[1]=0;
                           color[2]=0;
                        }
                      else
                        {  /* exterior of Mandelbrot set = white */
                            color[0]=255; /* Red*/
                            color[1]=255;  /* Green */
                            color[2]=255;/* Blue */
                        };
                        /*write color to the file*/
                        fwrite(color,1,3,fp);
                }
        }
        fclose(fp);
        return 0;
}
```

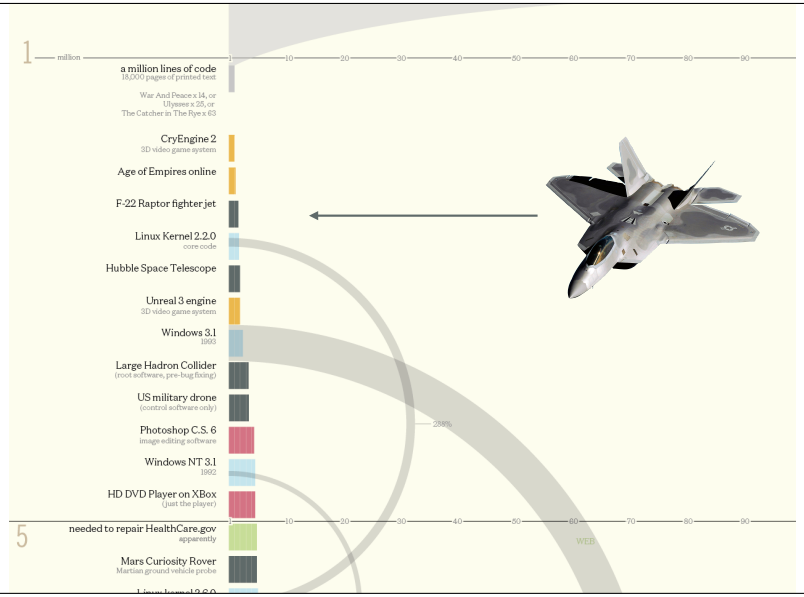
output:

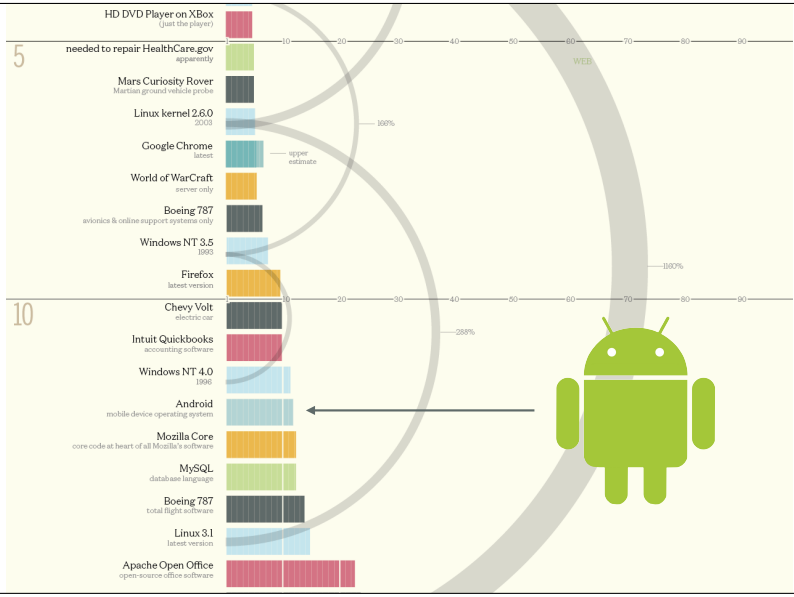


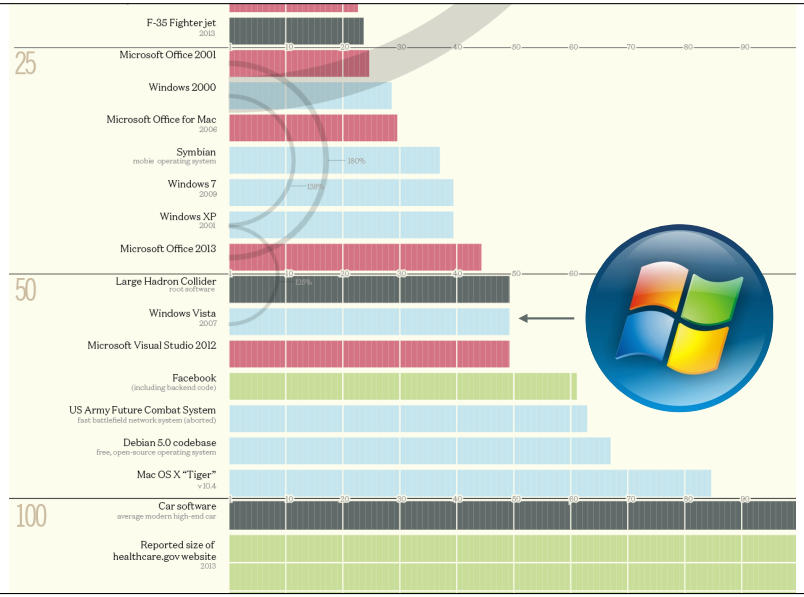
About 90 SLOCs of C code

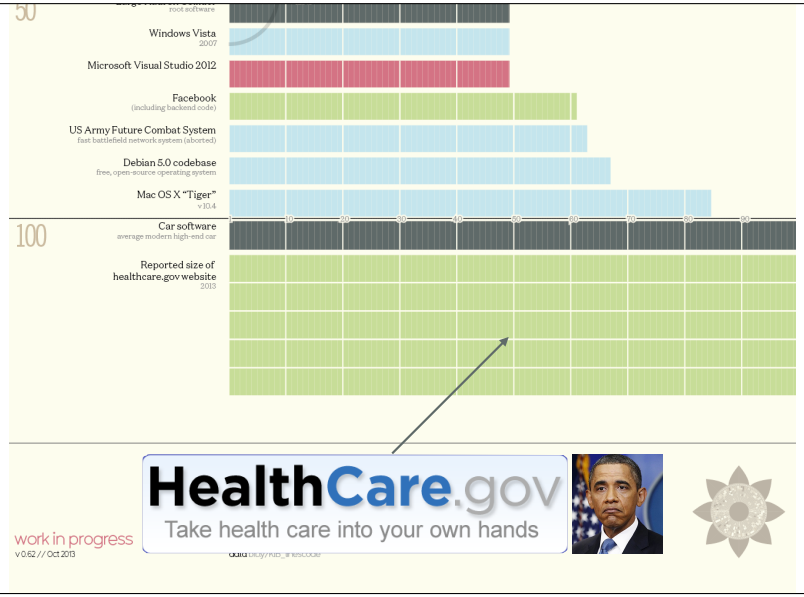
Source: https://www.rosettacode.org/wiki/Mandelbrot_set#C











What 130K lines of code look like

Apollo-11, 1969

Margaret Hamilton,
director of software engineering

Code available here: https://github.com/chrislgarry/Apollo-11/

Lecture 1 outline

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- The size and complexity of code

Software engineering

- The aims and scope of software engineering
- Abstraction and modularity
- Design, validation & verification

Structured programming

- Structuring programs by using abstractions in a programming language
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Software engineering

Aims

Software engineering seeks principles and methodologies to make programs that are:

- **Usable**
  Meet their requirements, including being acceptable by the users
- **Dependable**
  Reliable, secure, safe
- **Maintainable**
  Can be updated with minimal effort
- **Efficient**
  Run as fast as possible with limited resources

Scope

Software engineering is concerned with all aspects of software production. It includes:

- **Theory**
  Computability, algorithms, correctness, complexity, formal languages
- **Tools and best practices**
  Specific programming languages, programming environments, developer tools to build, debug, and analyse programs
- **Management**
  Processes of software creation & maintenance

Abstraction and modularity

The complexity of software is reduced via:

- **Modularity**
  Decomposing the system into smaller components
- **Abstraction**
  Each component's behaviour has a simple description, independent of the other components and of the internal implementation

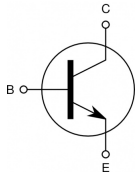
Benefits:

- **Understandability**
  Individual components are simple and easy to understand
- **Reuse**
  The same component can be used in many applications (e.g. transistors)
- **Isolating changes**
  The implementation of a component (e.g. transistor materials) can be changed as long as the behaviour (e.g. electrical properties) does not

Abstraction and modularity

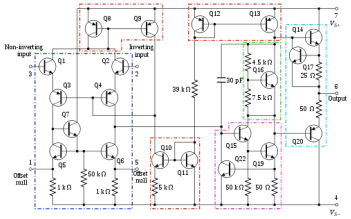
Examples

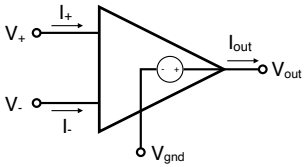
transistor



$I_C = \beta I_B$,
$I_E = I_C + I_B$,
$V_{EB} = 60$ mV

operational amplifier



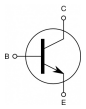


$V_+ = V_-, I_+ = 0$ A, $I_- = 0$ A

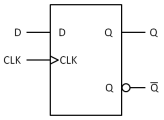
Abstraction and modularity

A hierarchy of increasingly-powerful abstractions

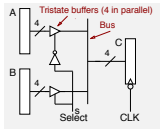
transistors



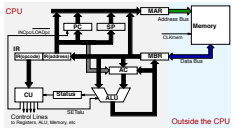
flip-flops



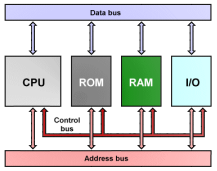
registers



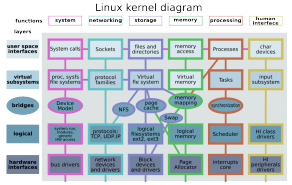
CPU



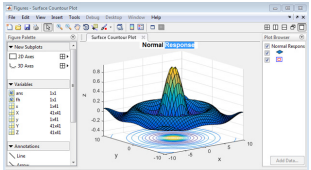
computer



virtual machine (OS)



application / interpreter

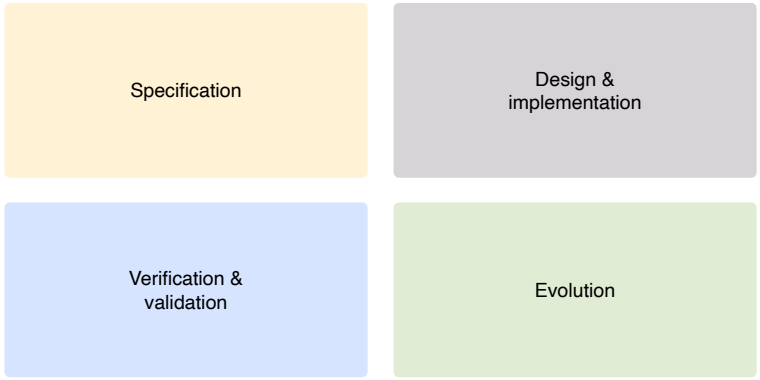


[Some images from Micro-controller, Murray]

Producing software

*A **software process** is a set of related activities that leads to the production of a software product [Sommerville].*

Key activities of a software process:



Design & implementation

Design

Formal

- collect the requirements
- break down the problem in sub-problems
- map them to software sub-modules

Informal (aka "extreme programming")

- quickly implement & iterate over a prototype
- incorporate user feedback at each iteration

Implementation

Top-down

- implement first the most abstract module
- use placeholders (mock-ups) for sub-modules

Bottom-up

- start from elementary module
- then build more complex ones using them

Verification & validation

Verification

Squash the bugs:

- Black-box: test what the code does
- White-box: inspect the code (possibly automatically)

Validation

Test the code in the real world and get feedback

More often than not, the *requirements* are wrong (i.e. you don't know exactly how the software should behave to be useful to the user).

Testing strategies

- Top-down: test that the system behaves correctly overall. If not, dig into your code to find the problem.
- Bottom-up: create **unit tests** to test individual sub-modules, starting from the smallest ones. This is extremely useful!

Test coverage

- Exhaustive testing is impossible
- Pick representative examples of “normal” inputs as well as “corner cases”
- Example: Test a function to compute the tangent
  - normal input: `tan(1.1)`
  - corner cases: `tan(-pi/2)`, `tan(0)`, `tan(pi/2)`

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Imperative languages

The most common programming languages are **imperative**.

An **imperative program** is a **list of instructions** to be executed in a specified order.

Different imperative languages are characterised by different **abstractions**.

Machine Code (Intel x86)

```
0:   55
1:   48 89 e5
4:   c7 45 fc 01 00 00 00
b:   c7 45 f8 02 00 00 00
12:  8b 45 fc
15:  03 45 f8
18:  89 45 f4
1b:  8b 45 f4
1e:  5d
1f:  c3
```

Machine Language

```
pushq   %rbp
movq    %rsp, %rbp
movl    $1, -4(%rbp)
movl    $2, -8(%rbp)
movl    -4(%rbp), %eax
addl    -8(%rbp), %eax
movl    %eax, 
movl    -12(%r
popq    %rbp
retq
```

Machine language's main abstraction are mnemonics (readable names for instructions, registers, etc.)

Imperative languages

Abstractions can have a massive impact on the ease of use, understandability, maintainability, power, and efficiency of programming languages.

Look at these three versions of the **same program**:

Machine Code (Intel x86)

```
0:   55
1:   48 89 e5
4:   c7 45 fc 01 00 00 00
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pushq   %rbp
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movl    $2, -8(%rbp)
movl    -4(%rbp), %eax
addl    -8(%rbp), %eax
movl    %eax, -12(%rbp)
movl    -12(%rbp), %eax
popq    %rbp
retq
```

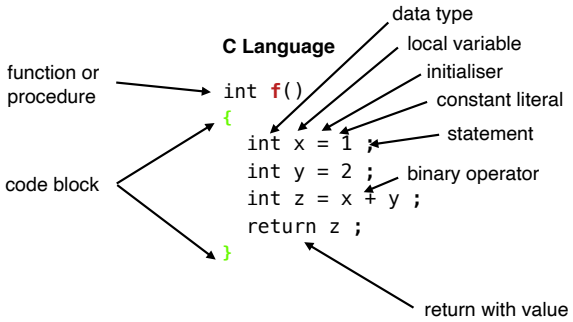
C Language

```
int f()
{
  int x = 1 ;
  int y = 2 ;
  int z = x + y ;
  return z ;
}
```

Imperative languages

Abstractions have a massive impact on the ease of use, understandability, maintainability, power, and efficiency of programming languages.

Here's a preview of some of the abstractions we are going to learn in this course:



Declarative (functional) languages

A **declarative program** specifies the desired behaviour of the program, but not how this is achieved in term of elementary steps.

Regular expression (declarative)

```
[a-Z]*
```

This means that the program should match any string consisting only of letters from 'a' to 'Z'.

It says what the program should do.

C language (imperative)

```
bool f(char const * str)
{
  bool match = true ;
  while (*str) {
     match &= ('a' <= *str
               && *str <= 'Z') ;
     str ++ ;
  }
  return match ;
}
```

This is a C implementation of the same program.

It specifies how to solve the problem in terms of elementary steps.

Structure in imperative languages

An overview of fundamental abstractions

Data

Data types: elementary, aggregate and compound.

Variables.

Control flow

Blocks, conditionals, loops, switches.

Statements.

Procedural languages

Functions, data scoping, encapsulation, recursion.

Object-oriented programming (Part II of the course)

Attach behaviour to data.

Hello world!

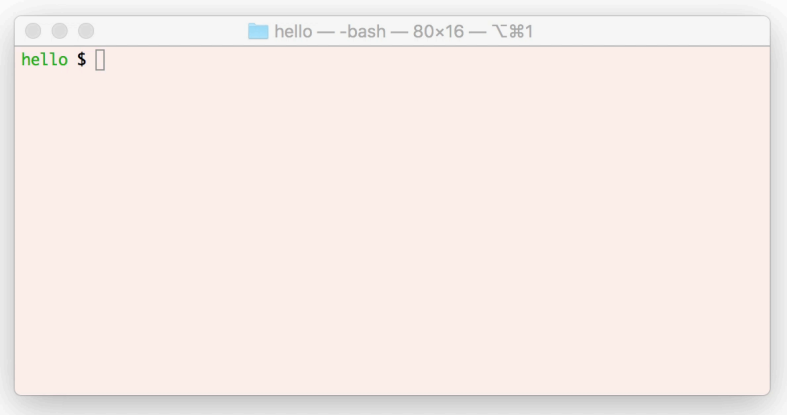
Getting started with C programming

```c
#include <stdio.h>

int main(int argc, char** argv)
{
  printf("Hello, world!\n") ;
  return 0 ;
}
```

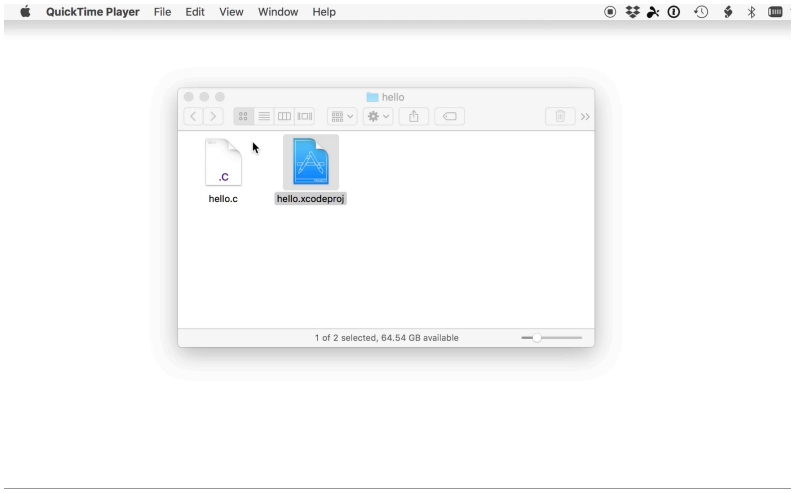
Hello world!

Getting started with C programming



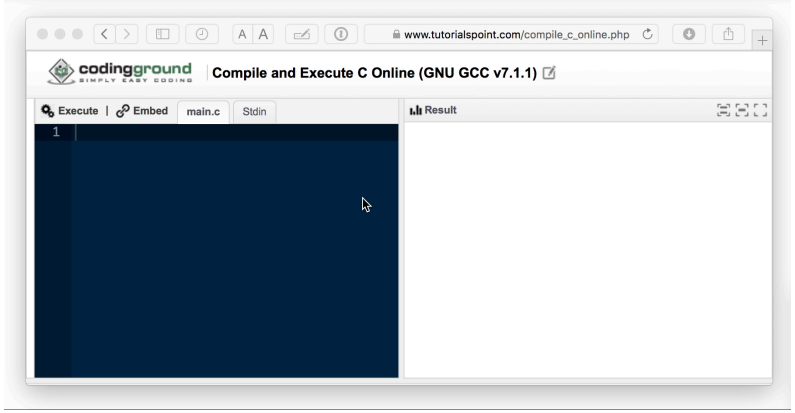
Hello world!

Getting started with C programming



Hello world!

Getting started with C programming



B16 Software Engineering

Structured Programming

Lecture 2: Control flow, variables, procedures, and modules

Dr Andrea Vedaldi
4 lectures, Hilary Term

For lecture notes, tutorial sheets, and updates see
http://www.robots.ox.ac.uk/~vedaldi/teach.html

Lecture 2 outline

Control flow

- Imperative languages
- Goto (considered harmful)
- Blocks, conditionals, loops

State

- Variable
- Data types
- Static vs dynamic typing

Compiled vs interpreted language

- MATLAB functions, subfunctions, toolboxes
- C/C++ declaration, definition, objective, and executable files

Practical notes

- Clean vs obfuscated code
- Avoid cut & paste

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Control flow

An **imperative program** is a list of statements (instructions) to execute

- Statements are executed sequentially
- The **program counter** (PC) is a register pointing to the current instructions
- It is incremented to move to the next instruction

```
10 sleep eight hours
11 wake up
12 have breakfast
```

PC 12

Branching statements

- Allow for non-sequential execution, conditionally on the state of the program
- Branching is performed by resetting the program counter

```
13 if today is Saturday then goto 10
14 leave home
```

Goto with line numbers (avoid)

Control flow using **goto**

- BASIC language example:

```
10 i ← 0
20 i ← i + 1
30 print i, " squared is ", i * i
40 if i >= 10 then goto 60
50 goto 20
60 print "that's all folks!"
```

To insert a line of code, one uses an intermediate line number (make sure to leave some spaces in the numbers for later!)

```
10 i ← 0
20 i ← i + 1
30 print i, " squared is ", i * i
35 print i, " cubed is ", i * i * i
40 if i >= 10 then goto 60
50 goto 20
60 print "that's all folks!"
```

Machines like this really existed!

1982-1994

Goto with labels (better, but still avoid)

Labels are an abstraction of line numbers and simplify the use of goto

- A label is just a name given to a statement in the sequence

```
      i ← 0
more: i ← i + 1
      print i, " squared is ", i * i
      if i >= 10 then goto end
      goto more
end:  print "that's all folks!"
```

we call this "spaghetti code"

[Dijkstra, E. W. (1968). Letters to the editor: **go to statement considered harmful**. Communications of the ACM, 11(3), 147-148.]

Structured control flow

Goto is (almost) never used. Any program can be expressed in terms of three simple control structures [Böhm-Jacopini 66]:

Blocks: sequences of executable statements

- { do_this ; do_that ; do_something_else ; }

Conditionals: execute a block if a condition is true

- if (condition) { }

Loops: keep executing a block until a condition remains true

- while (condition) { }

Spaghetti monster

```
      i ← 0
more: i ← i + 1
      print i, " squared is ", i * i
      if i >= 10 then goto end
      goto more
end:  print "that's all folks!"
```

Structured program

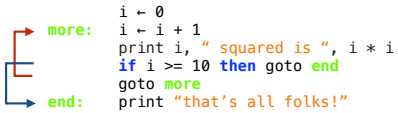
```
{
    i ← 0
    while (i < 10)
    {
        i ← i + 1
        print i, " squared is ", i*i
    }
    print "that's all folks!"
}
```

Structured control flow

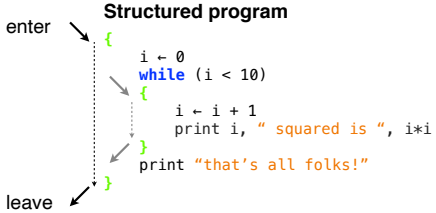
The code is much easier to understand because each block has

- only **one entry point** at the beginning
- only **one exit point** at the end

Spaghetti monster



Structured program



Structured control flow: procedures

A way to create a software module or component is to wrap a sequence of statements in a **procedure**.

```
procedure print_n_squared_numbers(n) {
    i ← 0
    while (i < n)
    {
        i ← i + 1
        print i, " squared is ", i*i ;
    }
    print "that's all folks!"
}
```

```
procedure do_something () {
    print "first four squares"
    print_n_squared_numbers(4) ;
}

procedure do_something_more () {
    print "first ten squares"
    print_n_squared_numbers(10) ;
}
```

A procedure implements a reusable functionality (behaviour) **hiding** the internal implementation details.

Examples

- `y = tan(x) // compute the tangent of a number`
- `printf("a string") // display a string on the screen`
- `window = createWindow() // create a new window on the display`
- `destroyWindow(window) // destroy it`

Structured control flow: procedures

A way to create a software module or component is to wrap a sequence of statements in a procedure:

```
procedure print_n_squared_numbers(n) {
    i ← 0
    while (i < n)
    {
        i ← i + 1
        print i, " squared is ", i*i ;
    }
    print "that's all folks!"
}
```

Calling the procedure from two other procedures:

```
procedure do_something () {
    print "first four squares"
    print_n_squared_numbers(4) ;
}

procedure do_something_more () {
    print "first ten squares"
    print_n_squared_numbers(10) ;
}
```

A **procedure** implements a reusable functionality (behaviour) hiding the internal implementation details.

Examples of procedures:

- `y = tan(x) // compute the tangent of a number`
- `printf("a string") // display a string on the screen`
- `window = createWindow() // create a new window on the display`
- `destroyWindow(window) // destroy it`

MATLAB vs C

C version

```
#include <stdio.h>

void print_n_squared_numbers(int n)
{
    int i = 0 ;
    while (i < n) {
        i = i + 1 ;
        printf("%d squared is %d\n",i,i*i);
    }
    printf("that s all folks!\n");
}

/* the program entry point
   is called main */
int main(int argc, char **argv)
{
    print_n_squared_numbers(10) ;
    return 0 ;
}
```

MATLAB version

```
function print_n_squared_numbers(n)
    i = 0 ;
    while i < n
        i = i + 1 ;
        fprintf('%d squared is %d\n',i,i*i);
    end
    fprintf('that's all folks!\n');
end

% Example usage
print_n_squared_numbers(10) ;
```

Both MATLAB and C are **imperative** and **procedural**.

MATLAB is **interpreted**, C/C++ is **compiled**.

MATLAB is **dynamically typed**, C/C++ is **statically typed**.

Lecture 2 outline

Control flow

- Imperative languages
- Goto (considered harmful)
- Blocks, conditionals, loops

State

- Variable
- Data types
- Static vs dynamic typing

Compiled vs interpreted language

- MATLAB functions, subfunctions, toolboxes
- C/C++ declaration, definition, objective, and executable files

Practical notes

- Clean vs obfuscated code
- Avoid cut & paste

Statements and the state

Program state = program counter + content of memory

Executing a statement **changes the state**

- Updates the program counter
- Almost always modifies the content of the memory as well

Example

```
50 * x * x + y ;  // result is not remembered, no effect
z = 50 * x * x + y ; // write the result to the variable z
```

If a statement does not alter the content of the memory, it has essentially no effect.

Exceptions:

- wasting time
- in MATLAB, displaying a value on the screen
- other side effects

Memory and variables

A computer's memory is just a large array of words (strings of bits):


```
001F034C: 66 72 65 64
```

The **meaning** depends on how it is interpreted:

32 bit integer	1718773092
4 characters	"fred"
floating point	*1.68302e+022*

Write a 32-bit integer to memory in C

```
*((int *) 0x001F034C) = 1718773092 ;
```

Structured version: using a **variable**

```
milesToGo = 1718773092 ;
```

To a first approximation, a **variable** is just a **name** given to a **memory address** (and a data type)

Data types

Data type

A (data) **type** specifies

- a set of **possible values**
  e.g. integers in the range –2,147,483,648 to 2,147,483,647, character strings
- **operations** involving variables of that type
  e.g. create, assign, sum, multiply, divide, print, convert to float, ...

Data type representation

A data type representation specifies how values are actually stored in memory
e.g. integer is usually represented as a string of 32 bits, or four consecutive bytes, in binary notation

This is a another example of abstraction
You never have to think how MATLAB represents numbers to use them!

Examples

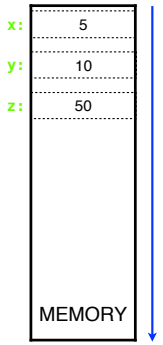
Most programming languages support several primitive data types:

- MATLAB: numeric arrays (characters, integer, single and double precision), logical arrays, cell arrays, ...
- C: various integer types, character types, floating point types, arrays, strings, ...

Dynamic data typing

Consider the following MATLAB fragment

```
% x, y, and z are stored as 64-bit float
x = 5 ;
y = 10 ;
z = x * y ;
```



Each variable stores both:

- the **address** of the data in memory and
- the **type** of the data

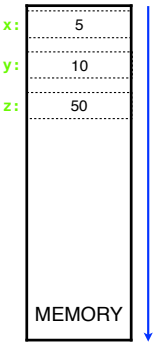
Use the MATLAB command **whos** to get a list of variables and their types (classes):

```
  Name      Size            Bytes  Class     Attributes

  x         1x1                 8  double
  y         1x1                 8  double
  z         1x1                 8  double
```

Dynamic data typing

Consider the following MATLAB fragment

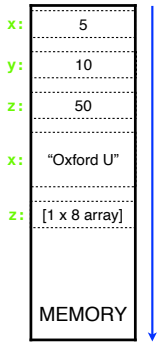
```
% x, y, and z are stored as 64-bit float
x = 5 ;
y = 10 ;
z = x * y ;
```



Dynamic data typing

Consider the following MATLAB fragment

```
% x, y, and z are stored as 64-bit float
x = 5 ;
y = 10 ;
z = x * y ;

% now reassign x and z
x = 'Oxford U' ;
z = x * y ;
```



Now variable **x** refers to

- a new memory block and
- a different data type

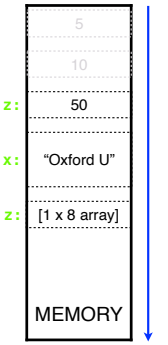
In MATLAB, the data type associated to a variable can be determined only at run-time, i.e. when the program is executed

This is called **dynamic typing**

Dynamic data typing

Consider the following MATLAB fragment

```
% x, y, and z are stored as 64-bit float
x = 5 ;
y = 10 ;
z = x * y ;

% now reassign x and z
x = 'Oxford U' ;
z = x * y ;
```



Each variable stores both:

- the **address** of the data in memory and
- the **type** of the data

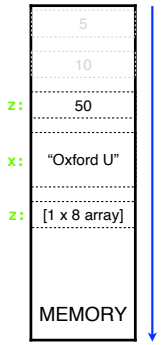
Use the MATLAB command **who** to get a list of variables and their types (classes):

```
  Name      Size            Bytes  Class     Attributes

  x         1x8                16  char
  y         1x1                 8  double
  z         1x8                64  double
```

Dynamic data typing

Consider the following MATLAB fragment

```
% x, y, and z are stored as 64-bit float
x = 5 ;
y = 10 ;
z = x * y ;

% now reassign x and z
x = 'Oxford U' ;
z = x * y ;
```



What is **z**?

```
  Name      Size            Bytes  Class     Attributes
  …
  z         1x8                64  double
```

Two operations are involved in calculating **z**:

- **promotion**: the string **x** is reinterpreted as an array of 1×8 64-bit floats
- **vector-matrix mult.**: the scalar **y** is multiplied by this array

Overhead in **dynamic** data typing

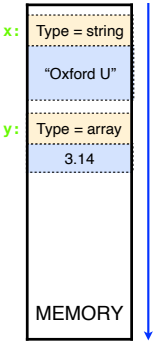
In dynamic data typing each a variable is contains to both the **actual data** record as well as **metadata** describing its type.

While usually this is not a problem, in some cases the overhead may be significant.

Example

MATLAB uses about 80 bytes to store the data type descriptor:

- Storing **one array of 1 million numbers** uses 80 + 8 * 1e6 bytes (~ 7.6 MB, efficiency ~ 100%)
- Storing **1 million arrays of 1 number each** uses (80 + 8) * 1e6 bytes (~ 83 MB, efficiency ~ 9%)



Static data typing and variable declaration

In C variables must be declared before they can be used

A **declaration** assigns statically a data type to a variable

Examples

```
int anInteger ; /* usually 32 bits length */
unsigned int anUnsignedInteger ;
char aCharacter ;
double aFloat ;
int32_t a32BitInteger ; /* C99 and C++ */
int16_t a16BitInteger ;
```

Statically-typed variables

- have a well defined type before the program is run
- incorporate constraints on how a variable can be used

Static typing allows for

- smaller run-time overhead in handling variables
- better error checking before the program is run

Lecture 2 outline

Control flow

- Imperative languages
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- Blocks, conditionals, loops

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- Data types
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Compiled vs interpreted language

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Practical notes

- Clean vs obfuscated code
- Avoid cut & paste

Compiled vs **interpreted** languages

MATLAB is an **interpreted language**

- a MATLAB program is executed by an interpreter
- the interpreter converts the text file in instructions on the fly
- significant overhead at run-time

C and C++ are **compiled languages**

- a compiler reads and transforms the text file into an executable before it can be executed
- no overhead at run-time
- the compiler can spot some programming error before the program runs (e.g. type errors)

Example. Compiling the following fragment generates an error because the multiplication of an integer and a pointer (see later) is not defined:

```
int * aPointerToInt = 0 ;
int anInt = 10 ;
int anotherInt = anInt * aPointerToInt ;
```

```
error-pointer-by-integer.c:7: error: invalid operands to binary * (have 'int *' and 'int')
```

MATLAB: program organisation

MATLAB procedures are called **functions**

A MATLAB function is stored in a homonymous file with a `.m` extension

file: `print_ten_squared_numbers.m`

```
function print_ten_squared_numbers(n)
  i = 0 ;
  while i < n
     i = i + 1 ;
     fprintf('%d squared is %d\n',i,i*i);
  end
  fprintf('that's all folks!\n');
end
```

file: `my_script.m`

```
% demonstrates the use of a
function
print_ten_squared_numbers()
```

A `.m` file can also contain a *script*.

A script does not define a function. It is more similar to cutting & pasting code in to the MATLAB prompt.

MATLAB: program organisation

MATLAB procedures are called **functions**

A MATLAB function is stored in a homonymous file with a `.m` extension

file: `print_ten_squared_numbers.m`

```
function print_ten_squared_numbers(n)
  i = 0 ;
  while i < n
     i = i + 1 ;
     fprintf('%d squared is %d\n',i,i*i);
  end
  thats_all() ;
end

function thats_all()
  fprintf('that's all folks!\n');
end
```

file: `my_script.m`

```
% demonstrates the use of a
function
print_ten_squared_numbers()
```

An .m file defines a function that can be accessed by functions and scripts in other files.

A .m file can contain also any number of **local functions**.

Local functions are only visible from the file where they are defined.[1]

[1]Advanced techniques allow to pass references to local functions, so that they *can* be called from other files.

MATLAB: grouping related functions

Put related functions into a given directory:

Directory: `drawing/`
  `drawAnArc.m`
  `drawAnArrow.m`
  `drawACircle.m`
Directory: `math/`
  `tan.m`
  `atan.m`
  `sqrt.m`
Directory: `pde/`
  `euler.m`

Use MATLAB `addpath()` function to include directories in MATLAB search path and make these functions visible.

MATLAB Toolboxes are just collections of functions organised in directories.

C/C++: program organisation

C/C++ explicitly support the notion of modules.

A module has two parts:

- the **declaration** (`.h`), defining the interface of the functions
  i.e. the function names and the types of the input and output arguments
- the **definition** (`.c`), containing the actual implementation of the functions

file: `usefulstuff.h`

```
void print_n_squared_numbers(int n) ;
int get_an_awesome_number() ;
```

file: `usefulstuff.c`

```
#include "usefulstuff.h"
#include <stdio.h>

void print_n_squared_numbers(int n) {
  int i = 0 ;
  while (i < n) {
    i = i + 1 ;
    printf("%d squared is %d\n",i,i*i);
  }
  printf("that s all folks!\n");
}

int get_an_awesome_number() {
   return 42 ;
}
```

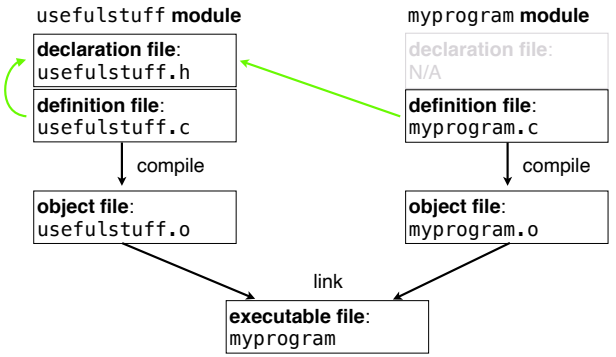
file: `myprogram.c`

```
#include "usefulstuff.h"
int main(int argc, char** argv) {
  print_n_squared_numbers(10) ;
}
```

C/C++: compiling a program

Run the **compiler** `cc`
Each `.c` file is **compiled** into an object file `.o`
This is the binary translation of a module

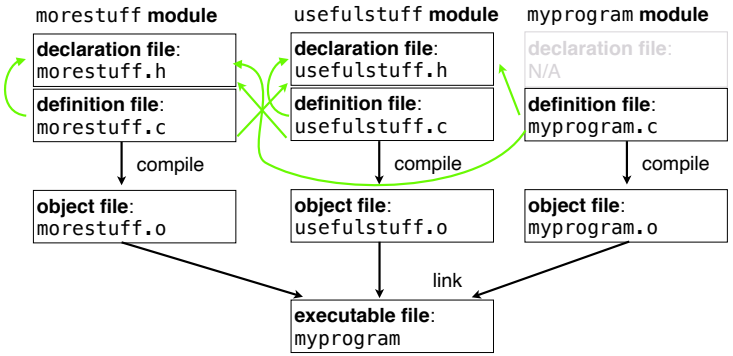
Run the **linker**, usually also implemented in `cc`
The `.o` files are merged to produce an executable file



C/C++: compiling a program

Run the **compiler** `cc`
Each `.c` file is **compiled** into an object file `.o`
This is the binary translation of a module

Run the **linker**, usually also implemented in `cc`
The `.o` files are merged to produce an executable file



More on declaring, defining, and calling functions

Declaration of the function prototype

```
void print_n_squared_numbers(int n) ;
```

Definition of the function implementation

```
void print_n_squared_numbers(int n)
{
   /* do something */
}
```

formal parameter

Invocation of the function

```
print_n_squared_numbers(10) ;
```

actual parameter

Declaring a function

- defines its **prototype** = name of the functions + type of input/output parameters
- thus specifies the **interface**
  the compiler needs to know only the prototype to call the function

Defining a function specifies its **implementation**. In the definition, the parameters are said to be *formal* as their value is not determined until the function is called.

Calling a function starts executing the function body. Now the parameters are said to be *actual* because they are assigned a value.

Return value(s)

In C functions have a single output value, assigned by the `return` statement.

Definition of the function

```c
int get_awesome_number()
{
   return 42 ;
}
```

Invocation of the function

```c
int x ;
x = get_awesome_number() ;
/* x is now 42 */
```

In MATLAB functions have an arbitrary number of output values.

- They get their value from homonymous variables.

Definition of the function

```
function [a,b,c] = get_many_numbers()
   a = 42 ;
   b = 3.14 ;
   c = +inf ;
   return ;
end
```

Invocation of the function

```
% x gets 42, y 3.14, and z +inf
[x,y,z] = get_many_numbers();

% get eigenvectors and eigenvalues
[V, D] = eig(A) ;
```

Lecture 2 outline

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- Imperative languages
- Goto (considered harmful)
- Blocks, conditionals, loops

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Practical notes

- Clean vs obfuscated code
- Avoid cut & paste

Some practical notes

The **look is important**

- Use meaningful *variable names*
- Use *comments* to supplement the meaning
- *Indent code* for each block/loop

Avoid cutting & pasting code

- Use functions to encapsulate logic that can be reused
- Duplicating code is a recipe to disaster because when you need to change the code, you need to change all the copies!

Top-down vs **bottom-up**

- Design top-down
- Code bottom-up or top-down, or a combination

Obfuscated code (don't)

Here is a valid C program (http://en.wikipedia.org/wiki/Obfuscation_(software))

```
char M[3],A,Z,E=40,J[40],T[40];main(C){for(*J=A=scanf("%d",&C);
--          E;          J[          E]          =T
[E   ]=  E)  printf("._");  for(;(A-=Z=!Z)  ||  (printf("\n|"
)    ,   A   =           39          ,C          --
)    ;   Z   ||     printf   (M   ))M[Z]=Z[A-(E   =A[J-Z])&&!C
&    A   ==            T[                        A]
|6<<27<rand()|||!C&!Z?J[T[E]=T[A]]=E,J[T[A]=A-Z]=A,"_.":"  |"];}
```

Can you figure out what this program do?

B16 Software Engineering

Structured Programming

Lecture 3: Scope, dynamic memory, pointers, references, recursion, stack, compounds

Dr Andrea Vedaldi
4 lectures, Hilary Term

For lecture notes, tutorial sheets, and updates see
http://www.robots.ox.ac.uk/~vedaldi/teach.html

Lecture 3 outline

Scope

- Local and global variables
- Modularisation and side effects

Dynamic memory and pointers

- Memory organisation, dynamically allocating memory in the heap
- Pointers, dereferencing, referencing, references
- Passing by values or reference, side-effects

Recursion

- Procedures that call themselves
- Recursion and local variables
- The stack and stack frames

Passing functions as parameters

Compound data types: structures

Lecture 3 outline

Scope

- Local and global variables
- Modularisation and side effects

Dynamic memory and pointers

- Memory organisation, dynamically allocating memory in the heap
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- Passing by values or reference, side-effects

Recursion

- Procedures that call themselves
- Recursion and local variables
- The stack and stack frames

Passing functions as parameters

Compound data types: structures

The scope of a variable

The **scope of a variable** is the context in which the variable can be used.

The scope of a **local variable** is the function where the variable is defined. Usually, local variables are created when the function is entered, and destroyed when it is left.

Global variables can be accessed by all functions. They are created when the program starts, and destroyed when it ends.

MATLAB example

```
function x = myFunction(n)
   m = 10 ;
   x = m * n ;
end

% test script
myFunction(5) % 50
m = 20 ;
myFunction(5) % still 50!
```

`m` here denotes a variable local to `myFunction`

`m` here denotes a variable local to the test script

The two variables are distinct and accessible only from the respective context.

MATLAB global variables

MATLAB strongly discourages the use of global variables.

When they are really needed, they must be declared by the **global** operator.

```
function x = myFunction(n)
   global m ;
   x = m * n ;
end

% test script
global m ;
m = 10 ;
myFunction(5) % 50
m = 20 ;
myFunction(5) % 100
```

global lets the variable **m** refer to a data record stored in the global memory area.

Now the two variables are the same.

You can always use MATLAB whos command to check your variables:

```
Name        Size              Bytes  Class     Attributes
m           1x1                   8  double    global
```

C/C++ global variables

Declaring a global variable

A variable is implicitly **global** if *declared outside of any function.*

Question: which part of the program is responsible for initialising **m** ?

file: `myfunction.c`

```
#include "myfunction.h"
#include <stdio.h>

int m ; /* global */

int myFunction(int n) {
   return m * n ;
}
```

Scope

A global variable defined in a module is visible only to the functions of that module.

To make the variable visible from other modules it must be declared in the **.h** file, exactly like functions.

Furthermore, the **export** keyword must be used.

file: `myfunction.h`

```
/* global decleration */
export int m ;

int myFunction(int n) ;
```

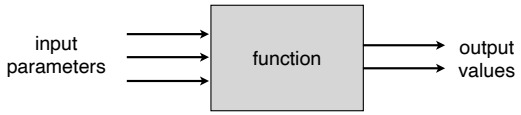
Procedure as functions

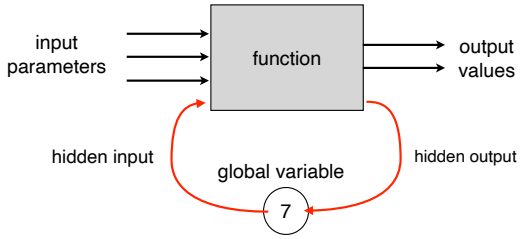
Procedures are often intended as **mathematical functions**:

- Then **only effect** of calling a procedure is to compute and return an output value.
- The output value depends only on the value of the input parameters.
- This is sometimes called having the **function semantics**

Side-effects break the function-like semantics

- e.g. a global variable is an implicit input/ output parameter





Side-effects

A procedure is useful only if its behaviour is easy to predict and understand.

This is particularly desirable in software librares:

- e.g. C/C++ **`math.h`** (**`tan`**, **`cos`**, ...), MATLAB toolboxes

In practice, many procedures have **useful side-effects**:

- reading a file, displaying a message, generating an error, ...
- allocating and returning a new memory block
- reading / writing a global variable
- operating on data in the caller scope by means of references (see later)
- ...

A clean interface design (and documentation) is essential to control these side-effects.

Lecture 3 outline

Scope

- Local and global variables
- Modularisation and side effects

Dynamic memory and pointers

- Memory organisation, dynamically allocating memory in the heap
- Pointers, dereferencing, referencing, references
- Passing by values or reference, side-effects

Recursion

- Procedures that call themselves
- Recursion and local variables
- The stack and stack frames

Passing functions as parameters

Compound data types: structures

Memory organisation

A structured program organises the memory into four areas:

The **code area** stores the program instructions.

- The OS prevents the program from changing it.

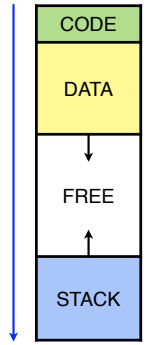
The **data (or heap) area** contains dynamically allocated records.

- Implicit in MATLAB, using malloc() in C.
- It grows towards the bottom as more memory is allocated.

The **stack area** is used to handle recursive procedure calls and local variables.

The **free area is** memory not yet assigned to a particular purpose.

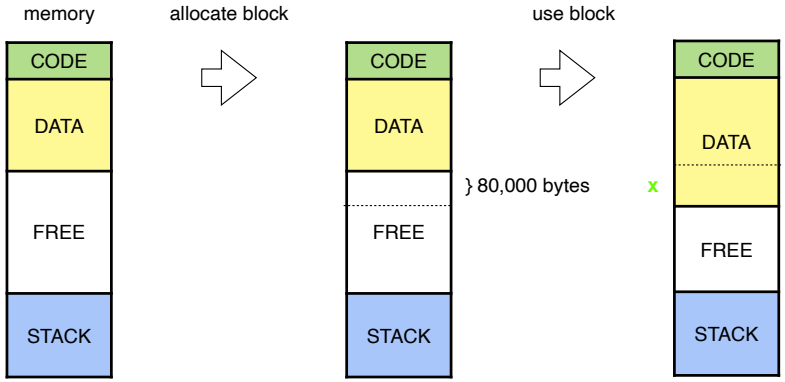
address space model



Dynamic memory allocation (in MATLAB)

In MATLAB dynamic memory allocation is implicit.

```
% allocate 80,000 bytes to store an array of 10,000 double
x = zeros(100,100) ;
```



Dynamic memory allocation (in C)

In C/C++ dynamic memory allocation is **explicit**.

In C a new memory block is obtained by calling the **malloc()** function.
Allocated memory must be disposed by calling **free**; otherwise the memory is **leaked**.

The output of **malloc** is the address of the allocated memory block.

An *address* is stored into a variable of type **pointer to T**.

```
/* declare a pointer x to a double */
double * x ;

/* allocate a double (eight bytes) and store the address in x */
x = malloc(8) ;

/* better: use sizeof to get the required size */
x = malloc(sizeof(double)) ;

/* write to the memory pointed by x */
*x = 3.14 ;

/* free the memory once done */
free(x) ;
```

Pointers and dereferencing

A **pointer to T** is a variable containing the address to a record of type T. Its type is denoted T *

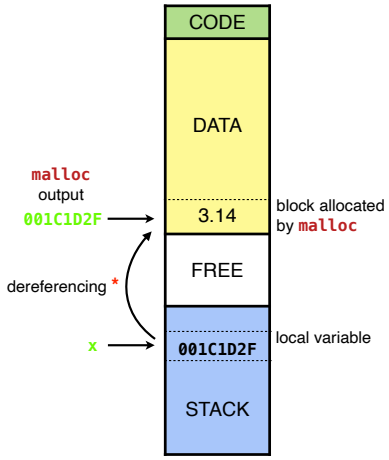
```
/* Declare and assign a pointer to double */
double *x ;
x = malloc(sizeof(double)) ;

/* Dereference x to access the pointed memory */
*x = 3.14 ;

/* This changes the pointer, not the
pointed data. */
x = 42 ;

/* This crashes the program
because x does not contain
the address of a valid
memory block anymore */
free(x)
```

The operator * is called **dereferencing**. It allows accessing the value pointed by the pointer.



Null pointers

By convention, the memory address **0** (0x0000000) is reserved.

A **null pointer** is a pointer with value **0** (denoted by NULL).

Null pointers are commonly used to represents particular states. For example:

- **malloc()** returns NULL if the requested memory block cannot be allocated because the memory is exhausted (an error condition).
- in a linked list a NULL pointer may be used to denote the end of the list (see Lecture 4).

Note that writing to a null pointer (or as a matter of fact to any address not corresponding to a properly allocated memory block) crashes the program (or worse!). For example:

```
int *myPointer = NULL ;
*myPointer = 42 ; /* crash */
```

Referencing

Pointers can be **copied**:

```
unsigned int * x = malloc(sizeof(unsigned int)) ;
unsigned int * y = x ;
```

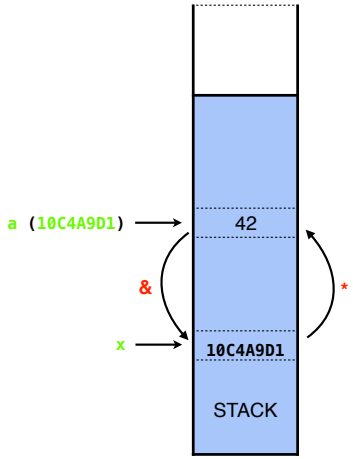
Pointers can also **point to a local variable.**

```
unsigned int a = 42 ;
unsigned int * x = &a ;
```

The operator & is called **referencing**. It returns the address of a variable.

Now the same data record can be accessed by using the variable or the pointer:

```
/* these three instructions have the same effect */
a = 56 ;
*x = 56 ;
*(&a) = 56 ;
```



Pointers as input arguments

By using pointers, a procedure **can access data that belongs to the caller**.

Example. A procedure that swaps the value of two variables:

```
void swap(int a, int b) {
    int temp = a ;
    a = b ;
    b = temp ;
}

/* example usage */
int x = 10 ;
int y = 20 ;
swap(x,y) ;
/* x = 10, y = 20 */
```

The function has no effect because calling it *a and b are copies of x and y. x and y remain unaffected.*

```
void swap(int *a, int *b) {
    int temp = *a ;
    *a = *b ;
    *b = temp ;
}

/* example usage */
int x = 10 ;
int y = 20 ;
swap(&x,&y) ;
/* x = 20, y = 10 */
```

By passing pointers, the function can access the variables x and y in the caller and can swap them.

Pass by value *vs* reference

C++ (but not C) can pass parameters by **reference** instead of **value**

Think of references as *implicit pointers*

```
void swap(int *a, int *b) {
    int temp = *a ;
    *a = *b ;
    *b = temp ;
}

/* example usage */
int x = 10 ;
int y = 20 ;
swap(&x,&y) ;
/* x = 20, y = 10 */
```

Using **pointers** (C or C++).

```
void swap(int &a, int &b) {
    int temp = a ;
    a = b ;
    b = temp ;
}

/* example usage */
int x = 10 ;
int y = 20 ;
swap(x,y) ;
/* x = 20, y = 10 */
```

Using **references** (C++ only)

Pointers and references: why?

Uses. Pointers/references are powerful:

- they allow a procedure to access data of the caller (e.g. `swap()`)
- they allow to pass data to a procedure avoiding copying (faster)
  E.g. think of passing a 1000-dimensional vector
- they allow to construct interlinked data structures
  E.g. lists, trees, containers in general

Caveats. Pointers/references allow **side effects**:

- they make a procedure behaviour harder to understand
- they make programming errors harder to find
  An error inside a procedure may affect the caller in unpredictable ways

In MATLAB

- there are (almost) no *references* nor *pointers*
- it is only possible to assign or copy the *value* of variables
  Under the hood, however, all data are passed by reference. The pass-by-value semantic is ensured by sharing copies as much as possible.

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Passing functions as parameters

Compound data types: structures

Recursion

Recursion is one of the most powerful ideas in computer programming

- Algorithmic techniques such as divide & conquer map directly to recursion
- Many data structures are recursive (e.g. trees)
- Procedures can also be called recursively

Example: computing the factorial of *n*

Mathematical definition

$$\text{fact}(n) = \begin{cases} 1, & n = 1, \\ n \ \text{fact}(n-1), & n > 1. \end{cases}$$

Corresponding C function

```
int fact(int n)
{
   int m ;
   if (n == 1) return 1 ;
   m = n * fact(n - 1) ;
   return m ;
}
```

Recursion and local variables


```
int fact(int n)
{
   int m ;
   if (n == 1) return 1 ;
   m = n * fact(n - 1) ;
   return m ;
}
```

Each time a function is re-entered a new copy of the local variables **m** and **n** is created.

The local variables constitute the "private" state of the function. Each execution has its own state.

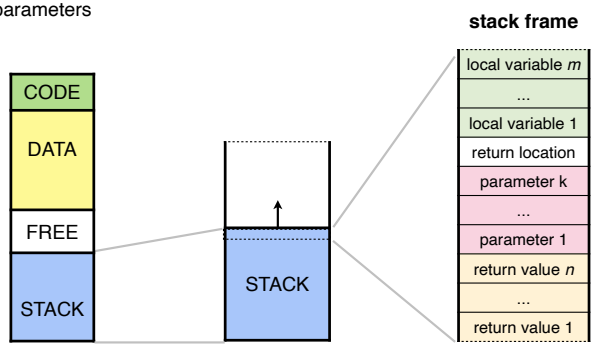
In this manner, recursive calls do not interfere with each other.
And memory for local variable is allocated only when needed.

Recursion and the stack

The **stack** is a memory area used to handle (recursive) calls to procedures.

A **stack frame** is *pushed* on top of the stack when a procedure is *entered*, and *popped* when it is *left*. It contains:

- a return location (PC)
  to enable resuming the caller upon completion of the procedure
- the input and output parameters
- the local variables

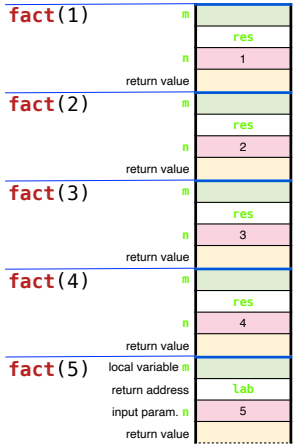


Example of recursive calls


```
    int fact(int n)
    {
        int m ;
        if (n == 1) return 1 ;
res:    m = n * fact(n - 1) ;
        return m ;
    }

    /* example usage */
    x = fact(5) ;

lab:printf("fact(5) is %d", x) ;
```

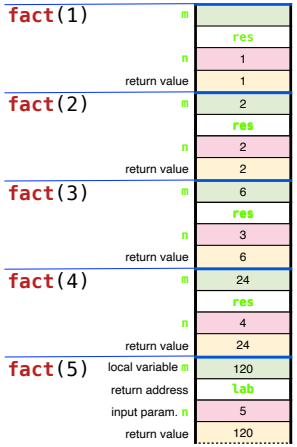


Example of recursive calls


```
    int fact(int n)
    {
        int m ;
        if (n == 1) return 1 ;
res:    m = n * fact(n - 1) ;
        return m ;
    }

    /* example usage */
    x = fact(5) ;

lab:printf("fact(5) is %d", x) ;
```



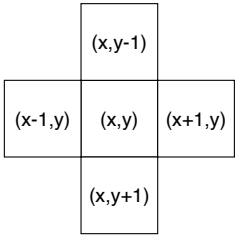
Recursion: a more advanced example

Multiple recursion. A procedure can call itself multiple times.

This examples paints the image region of colour `old_colour` containing the pixel (x, y) with `new_colour`.

```
const int SIZE = 256 ;
int im[SIZE][SIZE] ;

void fill (int x, int y, int old_colour, int new_colour)
{
    if (x >= 0 && x < SIZE && y >=0 && y < SIZE) {
      if (im[y][x] == old_colour) {
        im[y][x] = new_colour;
        fill (x-1,y,old_colour,new_colour);
        fill (x+1,y,old_colour,new_colour);
        fill (x,y-1,old_colour,new_colour);
        fill (x,y+1,old_colour,new_colour);
      }
    }
    return;
}
```



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Passing functions as parameters

Compound data types: structures

Passing functions as parameters

A function can be passed as a parameter of another function to change its behaviour.

Example. Consider implementing an algorithm for the numerical solution of a first order ODE

$$\dot{y}(t) = -y(t), \quad t \geq 0$$

The *Euler method* chooses a step size h and an initial condition y_0 and then uses the iteration:

$$y(0) = y_0$$
$$y(hn) = -hy(h(n-1)) + y(h(n-1)), \quad n = 1,2,\ldots,N-1$$

MATLAB implementation:

```
function y = solve(y0, h, N)
      y = zeros(1, N) ;
      y(1) = y0 ;
      for n = 2:N
         ydot = - y(n-1) ;
         y(n) = y(n-1) + h * ydot ;
      end
end
```

Passing functions as parameters (in MATLAB)

More in general, there is one ODE problem for each function F:

$$\dot{y}(t) = F(y(t)), \quad t \geq 0$$

The Euler solver needs to be modified:

$$y(0) = y_0$$
$$y(hn) = hF(y(h(n-1))) + y(h(n-1)), \quad n = 1,2,\ldots,N-1$$

To avoid writing a new program for each F pass the latter as a parameter:

```
function y = solve(F, y0, h, N)
    y = zeros(1, N) ;
    y(1) = y0 ;
    for n = 2:N
        ydot = F(y(n-1)) ;
        y(n) = y(n-1) + h * ydot ;
    end
end
```

```
% Example usage
function myF(y)
  ydot = -y ;
end

solve(@myF, Y0, h, N) ;
```

The `@` operator returns a **handle to a function**. A handle is similar to a pointer.

Passing functions as parameters (in C)

In C one uses a pointer to a function:

```c
double myF(double y) {
    return -y ;
}

/* this is not equivalent to MATLAB version as it integrates only one
step */
double solve(double func(double), double y, double h)
{
    double ydot = func(y) ;
    return y + h * ydot ;
}

int main()
{
    int n ;
    double y[200], h = 0.05;
    y[0] = 1.0;
    for (n = 1 ; n < 200 ; n++) {
        y[i] = solve(myF, y[n-1], h);
        printf("Y[%d] = %f\n", n, y[n]);
    }
}
```

This declares a parameter `func`.

The type of `func` is "pointer to a function that takes a double as input and returns a double as output".

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Compound data types: structures

Custom and structured data types

All languages support natively a number of **primitive types**

- C/C++: `char`, `int`, `float`, `double`, ...
- MATLAB: arrays of `char`, `int16`,`int32`, `single`, `double`, ...

Most languages support defining novel data types. Often these are **compound types** combining primitive types.

- C: array and structures (`struct`)
- C++: array, structures (`struct`), and classes (`class`)
- MATLAB: cell arrays `{}`, structures, and classes (`class`)

Structures can be used to group related information together into a single data record.

Classes add a behavior to structures in term of a custom set of operations that can be applied to the data (see the next lecture series).

MATLAB structures

A **structure** is a compound data type which comprises related data into a single entity.

In MATLAB, a structure is defined by assigning a variable using the `.` operator.

Example: create and assigns a new variable **`person`**:

```
person.name = 'Isaac' ;
person.surname = 'Asimov' ;
person.age = 66 ;
person.occupation = 'writer' ;
```

The variable **`person`** is a structure with the following fields: `name`, `surname`, `age` and `occupation`.

Structures can contain other structures, recursively:

```
person.address.city = 'New York' ;
person.address.zipCode = '12345' ;
```

C/C++ structures: `struct`

In C, each new type of structure must be declared before a corresponding variable can be defined and assigned:

Declaration

```
struct Complex_ {
  double re ;
  double im ;
} ;
```

Definition and assignment

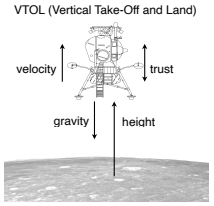
```
struct Complex_ c ;
c.re = 1.0 ;
c.im = 0.0 ;
```

typedef can be used as a shorthand:

```
struct Complex_ {
  double re ;
  double im ;
} ;
typedef struct Complex_
Complex ;
```

```
Complex c ;
c.re = 1.0 ;
c.im = 0.0 ;
```

Example: VTOL state



VTOL state:

- height, velocity, mass (numbers)
- landed (bool)

Use a single structure to store all numbers

- data encapsulation
- abstraction

Example

```
typedef struct
{
    double position ;
    double velocity ;
    double mass ;
    bool landed ;
} VTOLState ;
```

Using structures in C/C++

Creating a structure

```
% As a local variable
VTOLState state ;
state.position = 10 ;
state.velocity = 5 ;
state.mass = 1000 ;
state.landed = false ;
```

```
% Dynamically
VTOLState * statePtr ;
statePtr = malloc(sizeof(VLTOState));
statePtr->position = 10 ;
statePtr->velocity = 5 ;
statePtr->mass = 1000 ;
statePtr->landed = false ;
```

Note: x**->** combines dereferencing and structure access. It is the same as (*x)**.**

Passing a structure to a function

```
% By value
double getThrust(VTOLState
state) ;

double t = getThrust(state);
```

```
% By a pointer
double getThrust(VTOLState
*statePtr) ;

double t = getThrust(statePtr) ;

double t = getThrust(&state) ;
```

B16 Software Engineering

Structured Programming

Lecture 4: Programs = Algorithms + Data structures

Dr Andrea Vedaldi
4 lectures, Hilary Term

For lecture notes, tutorial sheets, and updates see
http://www.robots.ox.ac.uk/~vedaldi/teach.html

Lecture 4 outline

Arrays

- In MATLAB and C
- Pointer arithmetic

Sorting

- The sorting problem
- Insertion sort
- Algorithmic complexity

Divide & conquer

- Solving problems recursively
- Merge sort
- Bisection root finding

Linked list

- Search, insertion, deletion

Trees

- Binary search trees

Graphs

- Minimum spanning tree

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Arrays

An **array** is a data structure containing a numbered (indexed) collection of items of a single data type.

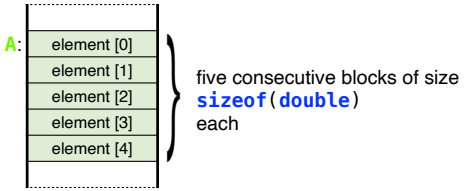
- In MATLAB arrays are primitive types.
- In C, arrays are compound types. Furthermore, C arrays are much more limited than MATLAB's.

```
/* Define, initlaise, and access an array of three integers in C */
int a[3] = {10, 20, 30} ;
int sum = a[0] + a[1] + a[2];

/* Arrays of custom data types are supported too */
VTOLState states[100];
for (t = 1 ; t < 100 ; t++) {
   states[t].position = states[t-1].position + states[t-1].velocity + 0.5*g;
   states[t].velocity = states[t-1].velocity + g
      - getThrust(states[t-1], burnRate) / states[t-1].mass;
   states[t].mass = states[t-1].mass - burnRate * escapeVelocity ;
}
```

Array representation in C

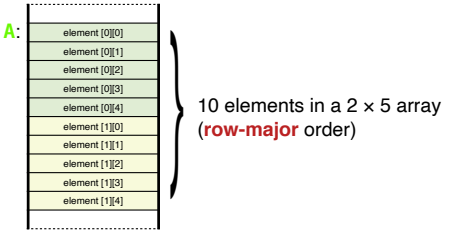
In C an array is represented as a sequence of records at consecutive memory addresses.

```
/* array of five doubles */
double A [5] ;

/* get a pointer to
the third element */
double * pt = &A[2] ;
```



Two (and more) dimensional arrays are simply arrays of arrays.

```
/* A 2x5 array */
double A[2][5] ;
```



Static vs dynamic arrays in C

This C statement defines an array a of five integers

```
int A[5]  ;
```

The size is **static** because it is specified before the program is compiled. What if the size needs to be adjusted at run-time?

The solution is to **allocate dynamically** the required memory:

```
int arraySize = 5 ;
int *A = malloc(sizeof(int) * arraySize) ;
```

Note that A is declared as a **pointer** to an `int`, not as an array. However, the array access operator `[]` can still be used. E.g. `A[1] = 2`

Pointer math: `A[n]` is the same as `*(A + n)`

- E.g. `A[0]` is the same as dereferencing the pointer `*A`
- Under the hood, the address stored by a is incremented by `n * sizeof(int)` to account for the size of the pointed elements

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Sorting

Problem: sort an array of numbers in non-decreasing order.

There are many algorithms to do this: bubble sort, merge sort, quick sort, ...

We will consider three aspects:

- Describing the algorithm formally.
- Proving its correctness.
- Evaluating its efficiency.

We start from the **insertion sort algorithm**

- Input: an array of numbers.
- Output: the numbers sorted in non-decreasing order.
- Algorithm: initially the sorted output array is empty. At each step, remove an element from the input array and insert it into the output array at the right place.

See http://www.sorting-algorithms.com/ for illustrations

Insertion

The **insertion** procedure extends a sorted array by inserting a new element into it:

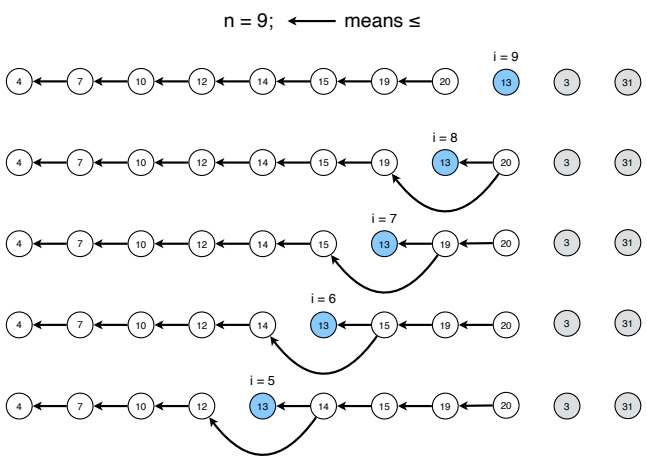
```
% Input: array A of size ≥ n such that A[1] <= ... <= A[n-1]
% Output: permuted array such that A[1] <= ... <= A[n-1] <= A[n]
function A = insert(A, n)
    i = n
    % the invariant is true here
    while i > 1 and A[i-1] > A[i]
        swap(A[i-1], A[i])
        i = i - 1
        % the invariant is true here
    end
end
```

I.e., start with the first n-1 elements sorted, end with n

A **loop invariant** is a property that is valid before each loop execution starts. It is usually proved by induction. For `insert()` the loop invariant is:

- `A[1] ≤ A[2] ≤ ... ≤ A[i-1] ≤ A[i+1] ≤ ... ≤ A[n]` and
- `A[i] ≤ A[i+1]`

Insertion: example



Insertion sort


```
% Input: an array A with n elements
% Output: array A such that A[1] <= A[2] <= ... <= A[n]
function A = insertionSort(A, n)
   i = 1
   % the invariant is true here (A)
   while i < n
      i = i + 1
      A = insert(A, i)
      % the invariant is true here (B)
   end
end
```

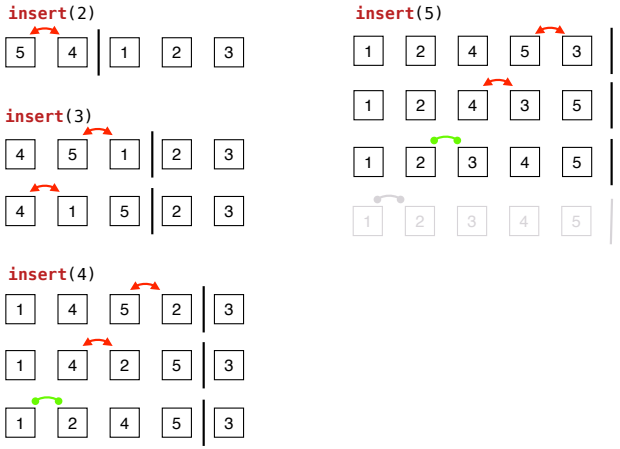
Loop invariant: the first `i` elements are sorted: `A[1]` ≤ `A[2]` ≤ `...` ≤ `A[i]`

Proof by induction

- *base case* (A) i = 1: A[1] is sorted
- *inductive step* (B) i ≥ 1: at iteration `i` the **`insert()`** procedure sorts `A[1], ..., A[i]` provided that `A[1], ..., A[i-1]` are sorted. The latter is given by the invariant at iteration `i - 1`.

Insertion sort: example

Task: sort 5 4 1 2 3



Algorithmic complexity

The **time complexity** of an algorithm is the *maximum number of elementary operations* $f(n)$ *required to process an input of size* n. Its **space complexity** is the *maximum amount of memory* required.

It often suffices to determine the **order of the complexity** $g(n)$: linear n, squared n^2, polynomial n^k, logarithmic $\log(n)$, exponential $\exp(n)$, ... We say that the order of $f(n)$ is $g(n)$, and we write $f(n) = O(g(n))$, if:

$$\exists a, n_0 : \forall n \geq n_0 : f(n) \leq a g(n)$$

Example: insertion sort

- The size of the input is the number n of elements to sort.
- The space complexity is $O(n)$ as the algorithm stores only the elements and a constant number of local variables.
- The time complexity of **`insert()`** is $O(m)$ as the while loop is executed at most m times. The time complexity of **`insertionSort()`** is $O(n^2)$ because

$$\sum_{m=1}^{n} m = \frac{(n+1)n}{2} = O(n^2)$$

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Divide & conquer

- Solving problems recursively
- Merge sort
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Divide and conquer

Divide and conquer is a *recursive strategy* applicable to the solution of a wide variety of problems.

The idea is to split each problem instance into two or more smaller parts, solve those, and recombine the results.

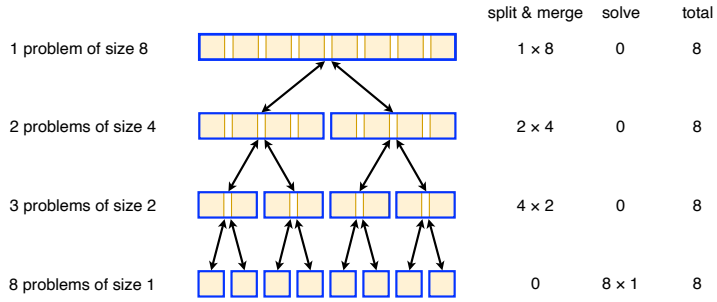
```
% Divide and conquer pseudocode
solution = solve(problem)
  If problem is easy, compute solution
  Else
    Subdivide problem into subproblem1, subproblem2, ...
    sol1 = solve(subproblem1), sol2 = solve(subproblem2), ...
    Get solution by combining sol1, sol2, ...
```

Note the **recursive call**. Divide and conquer is naturally implemented as a recursive procedure.

Some of the best known and most famous (and useful) algorithms are of this form, notably quicksort and the Fast Fourier Transform (FFT).

Complexity of divide and conquer

Assume that the cost of splitting and merging a subproblem of size m is $O(m)$ (linear) and that the cost of solving a subproblem of size $m = 1$ is $O(1)$.



Given a problem of size n, at each level $O(n)$ work is done in order to split&merge or solve subproblems. Since there are $\log_2(n)$ levels the total cost is

$$O(n \log_2 n)$$

Merge sort

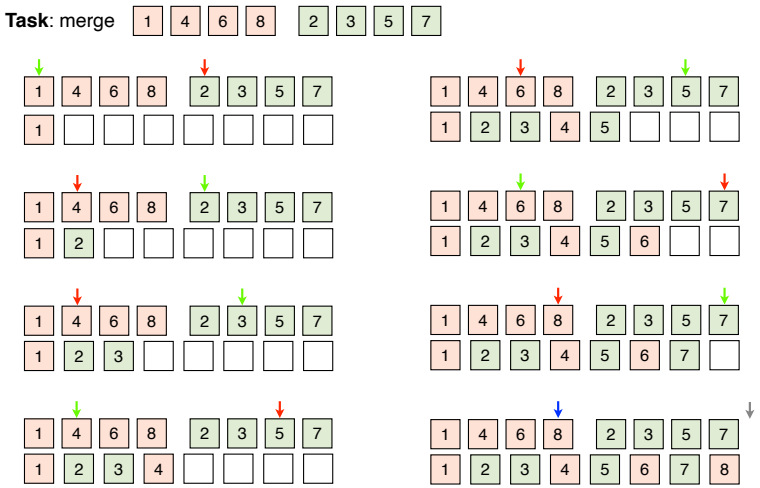
The **merge sort** algorithm sorts an array A by divide and conquer:

- **Split:** divide A into two halves A1 and A2.
- **Merge**: iteratively remove from the beginning of the sorted A1 and A2 the smallest element and append it to A.
- **Base case**: if A has one element only it is sorted.

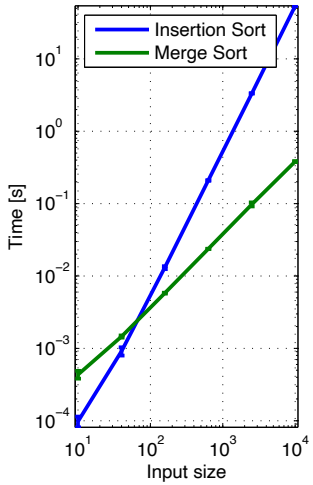
```
function A = mergeSort(A)
  n = length(A)
  % nothing to do if one element
  if n == 1 then return A
  % split into half
  k = floor(n / 2)
  A1 = A(1:k)
  A2 = A(k+1:end)
  % solve subproblems
  A1 = mergeSort(A1)
  A2 = mergeSort(A2)
  % merge solutions
  return merge(A1, A2)
end
```

```
function A = merge(A1,A2)
  i1 = 1, i2 = 1
  m1 = length(A1), m2 = length(A1)
  while i1 <= m1 and i2 <= m2
    if A1[i1] <= A2[i2]
      A[i1+i2-1] = A1[i1], i1 = i1 + 1
    else
      A[i1+i2-1] = A2[i2], i2 = i2 + 1
    end
  end
  while i1 <= m1
    A[i1+i2-1] = A1[i1], i1 = i1 + 1
  end
  while i2 <= m2
    A[i1+i2-1] = A2[i2], i2 = i2 + 1
  end
end
```

Merge sort: merging example



Insertion *vs* Merge Sort



The two sorting algorithms have **different complexities**:

- insertion: $O(n^2)$
- merge: $O(n \log(n))$

Plotting time vs size in loglog coordinates should give a line of slope:

- 2 for insertion sort
- ~ 1 for merge sort

This is verified experimentally in the figure.

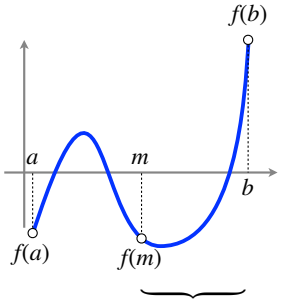
Root finding

Problem: find a root of a non-linear scalar function $f(x)$)
i.e. a value of x such that $f(x) = 0$.

Assumption: $f(x)$ is a continuous function defined in the interval $[a, b]$;
furthermore, $f(a)f(b) < 0$.

The **bisection algorithm** is a divide and conquer strategy to solve this problem.

```
function bisect(f, a, b)
  m = (a + b) / 2
  if f(m) close to zero then return mu
  if f(m) * f(a) > 0
    return bisect(f, m, b)
  else
    return bisect(f, a, m)
end
```



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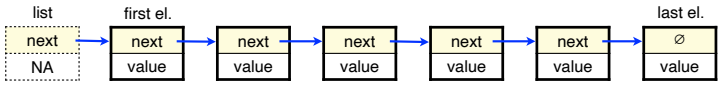
Linked lists

A limitation of arrays is that inserting an element into an arbitrary position is O(n).

This is because existing elements must be shifted (moved in memory) in order to make space for the a one.

Linked lists solve this problem by using pointers:



In C a list data type could be defined as follows:

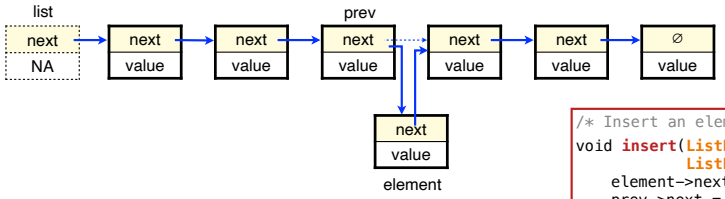
```
/* List element datatype */
typedef struct ListElement_ {
    struct ListElement_ *next ;
    double value ;
} ListElement ;

/* List datatype */
typedef ListElement List ;
```

The list could be defined as a pointer to its first element.

It is customary to use instead a fake list element (which contains such a pointer) to simplify coding functions using the list.

Inserting an element into a linked list

To **insert** an element into a list, use pointers to create a “bypass” at cost O(1).




```
/* Insert an element in a list */
void insert(ListElement *prev,
            ListElement *element) {
    element->next = prev->next ;
    prev->next = element ;
}
```

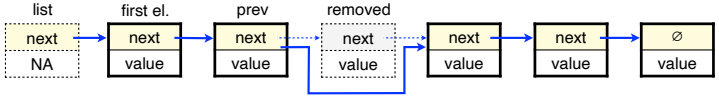
Example usage

```
/* Create an empty list */
List list ;
list->next = NULL ;

/* Create an element */
ListElement *element =
    malloc(sizeof(ListElement));
element->next= NULL ;
element->value = 42.0 ;
```

```
/* Insert at the beginning of the list
*/
insert(&list, element) ;

/* Insert after element */
insert(element, element2) ;
```

Removing an element from a linked list

To **insert** an element into a list, use pointers to create a “bypass” at cost O(1).




```
/* Remove an element */
ListElement *remove(ListElement *prev) {
    ListElement removed = prev->next ;
    if (removed != NULL) {
        prev->next = removed->next ;
    }
    return removed ;
}
```

Example usage

```
/* Remove the element after previous */
ListElement * removed ;
removed = remove(previous) ;

/* Do not forget to release the memory if needed */
if (removed != NULL) {
    free(removed) ;
}
```

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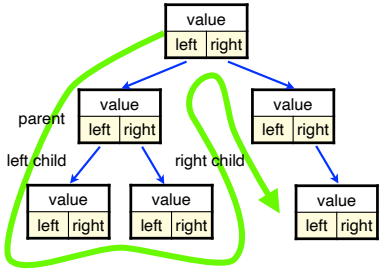
Binary tree

Each node in a **binary tree** has one **left child** and one **right child**.

There are no backward links (no cycles).



Example C data type
Similar to a linked list:

```
typedef struct Node_ {
    struct Node_ *left ;
    struct Node_ *right ;
    double value ;
} Node ;
```

Depth first traversal (post-order)
This algorithm visits recursively all the nodes in a tree:

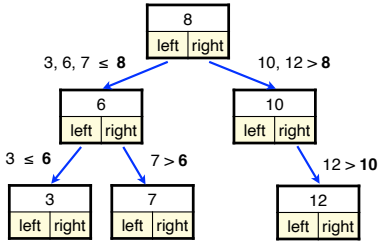
```
function visit(node)
    if node == NULL then return
    visit(node.left)
    visit(node.right)
    print(node.value)
end
```

Binary search tree

A **binary search tree** is a binary tree such that the value of each node is

- at least as large as the value of its left descendants
- smaller than the values of its right descendants

Its main purpose is to support the binary search algorithm.



Binary search algorithm

Problem: find a node with value x in a binary search tree.

The **binary search algorithm** searches for x recursively, using the *binary search tree property* to descend only into one branch every time.

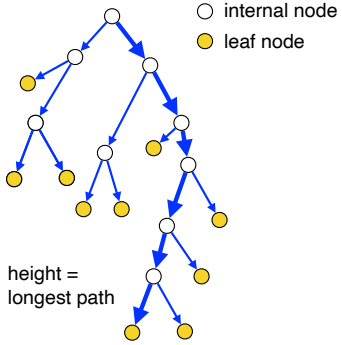
```
function node = binarySearch(node, x)
    if node == NULL return NULL
    if node.value == x return node
    if x > node.value
       return binarySearch(node.right, x)
    else
       return binarySearch(node.left, x)
    end
end
```

The **cost** is $O(h)$ where h is the **depth** of the binary tree.

Typically $h = O(\log n)$, where n is the number of nodes in the tree. Hence **the search cost is $O(\log n)$, sub-linear**.

Compare this with the $O(h)$ cost of searching in an array or a linked list.

Balanced binary trees

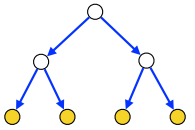


A binary tree of height h has at most $n = 2^h$ leaves.

Proof:

- Let $n(h)$ the maximum number of leaves of a binary tree of height h
- Split it at the root in two subtrees of height $h' \leq h - 1$
- Then at best $n(h) = 2n(h-1)$

For a given n, the shortest binary tree is *balanced:*

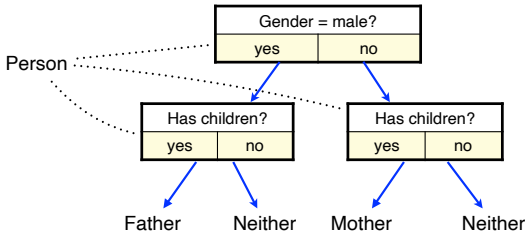


$4 = 2^2$

Decision tree

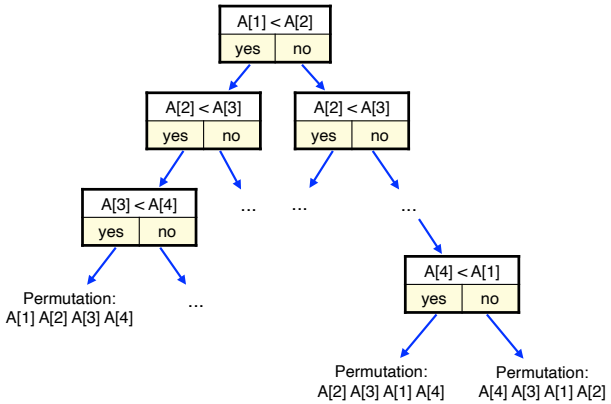
Many algorithms can be described as decision trees: at every step one changes the state based on a binary test applied to the input.

For example, the following algorithm decides whether a person is a father, a mother, or neither.

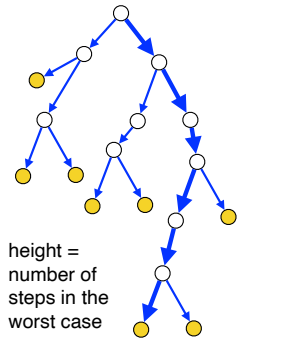


Sorting algorithms as decision trees

An algorithm that sorts an **array A** based on **pairwise comparisons** can be thought of as a decision tree.



Sorting algorithms as decision trees



For a sorting algorithm:

- Leaves = possible permutations of the array
- Path from root to leaf = comparisons in a run of the algorithm
- Tree height = maximum number of steps required

For an array of length n, there are $n!$ possible permutations

The tree height is at least $\log_2(n!)$ (achieved by a balanced tree)

Hence the best possible algorithm requires at least $t = \log_2(n!)$ steps in the worst case

Sorting algorithms as decision trees

$$t \geq \log_2 n! = \underbrace{\log_2 n \cdot (n-1) \cdot \ldots \cdot \frac{n}{2}}_{\geq \left(\frac{n}{2}\right)^{\frac{n}{2}}} \cdot \left(\frac{n-1}{2}\right) \cdot \ldots \cdot 2 \cdot 1 \geq \frac{n}{2} \log \frac{n}{2}$$

The best possible algorithm is $O(n \log n)$
Hence **merge sort** is optimal*!

*in the asymptotic worst case sense

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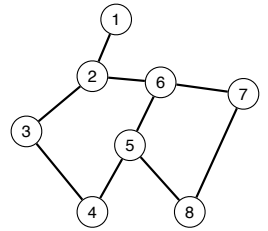
Graphs

A (**directed**) **graph** is a set of **vertices** V and **edges** $E \subset V \times V$ connecting the vertices. An **undirected graph** is a graph such that for each edge (u, v) there is an opposite edge (v, u).

```
% MATLAB representation
edges = [1 2 2 3 4 5 5 6 2 3 6 4 5 6 8 7
         2 3 6 4 5 6 8 7 1 2 2 3 4 5 5 6] ;
```

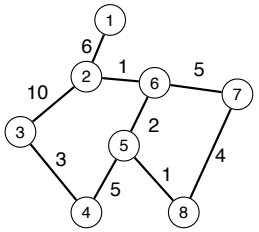
An alternative representation of a graph is the **adjacency matrix** A. A is a $n \times n$ matrix such that $A(u, v) = 1$ if, and only if, $(u, v) \in E$.

```
A = [0 1 0 0 0 0 0 0
     1 0 1 0 0 1 0 0
     0 1 0 1 0 0 0 0
     0 0 1 0 1 0 0 0
     0 0 0 1 0 1 0 1
     0 1 0 0 1 0 1 0
     0 0 0 0 0 1 0 1
     0 0 0 0 1 0 1 0] ;
```



Minimum spanning tree

Consider a **weighted undirected graph** with non-negative weights on the edges:



A **spanning tree** is a subset of the edges forming a tree including all the nodes.

A **minimum spanning tree** (MST) is a spanning tree such that the sum of the edge weights is minimal.

A famous algorithm to compute the MST is explored in the tutorial sheet.

Concept summary

Software engineering processes

- Specification, design & implementation, validation, evolution
- Waterfall and extreme programming

Software engineering tools

- Abstraction and modularity
- Procedures
- Variables, data type, scoping
- Dynamic memory allocation
- Pointers, references
- Recursion, stack, stack frames
- Pointers to functions
- Compound data types

Data structures and algorithms

- Complexity and correctness
- Arrays, lists, trees, graphs
- Sorting, searching, numerical problems

Exam questions

- See tutorial sheet

Acknowledgments

Thanks to Samuel Albanie for suggesting numerous improvements