Energy Economics 68 (2017) 423–439

Contents lists available at ScienceDirect

Energy Economics

journal homepage: www.elsevier.com/locate/eneco

Risk-minimisation in electricity markets: Fixed price, unknown consumption☆

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ARTICLE INFO

Article history:
Received 13 January 2016
Received in revised form 6 October 2017
Accepted 16 October 2017
Available online 19 October 2017

JEL classification:
G13
G29
Q4

Keywords:
Quantity risk
Electricity markets
Hedging
Fixed price contracts

ABSTRACT

This paper analyses risk management of fixed price, unspecified consumption contracts in energy markets. We model the joint dynamics of the spot-price and the consumption of electricity, study expected loss minimisation for different loss measures, and derive optimal static hedge strategies based on forward contracts. The strategies are implemented empirically and compared to a benchmark strategy widely used by the industry. On 2012–2014 Nordic market data, the suggested hedges significantly outperform the benchmark: The realised cumulative profit-and-losses are greater for almost every single one-month period and the hourly realised payoffs result in an approximate 65% out-performance probability. Hedges based on asymmetric loss measures yield markedly higher reward-to-risk ratios than the benchmark, which can be exploited to release a premium from the contract in the financially significant order of 1.5% of the fixed price.

1. Introduction

1.1. The problem

A popular product sold by energy companies to medium-to-large customers is one where an unspecified amount of power can be bought at a fixed price during a certain time-interval. We call this a fixed price, unspecified consumption contract or simply a *fixed price agreement*. The business-to-business electricity market in Denmark in 2014 was 23.1 TWh, about MEUR 800 million in monetary terms; 60% was sold as fixed price agreements in some form.

Having entered into the fixed price agreement, the energy company must asses and manage the risks associated with the obligations of the contract. This is challenging for several reasons: If the seller faced only the risk from price-fluctuations, he could simply buy forward contracts to set up a full, perfect hedge. However, with a criterion function that takes into account both (expected) gains and losses, this is expensive if forward-prices are typically above expected future spot-prices. This so-called *contango* is the norm in Nordic power markets, as demonstrated by both Botterud et al. (2002) and our analysis in Section 5.1.

Another complicating factor is quantity risk: If the consumer's quantity choice were all-or-nothing, the contract would have payoff akin to call options. Handling that would require modelling the price randomness of the electricity market which has several characteristics that are different from standard financial markets such as stocks and exchange rates. But the consumer has more flexibility and because energy demand is created by physical use, her choice will not be all-or-nothing. Demand will be positively correlated with the price as high demand drives up the price. For the contract seller that creates what in other areas of finance would be known as

☆ We thank Antje Mahayni, Nina Lange, and Mads Schreiner, as well as the editor and two anonymous referees for valuable comments and suggestions. Tegnér and Poulsen gratefully acknowledge support from the Danish Strategic Research Council, Program Committee for Strategic Growth Technologies, via the research center HIPERFIT: Functional High Performance Computing for Financial Information Technology (hiperfit.dk) under contract number 10-092299 Ernstsen gratefully acknowledges support from the Danish Strategic Research Council via the 5s - Future Electricity Markets project, contract number 12132636/DSF.
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https://doi.org/10.1016/j.eneco.2017.10.014
0140-9883/

wrong-way risk: He will have to deliver a lot of energy for a fixed price when buying it in the spot market is expensive.

In this paper, we will deal with these risks with a joint model of spot-price and consumption load.

1.2. Previous work, related literature

The non-storability of electricity is a key complicating factor in energy markets. It makes physical hedging inoperable and it causes highly volatile spot-markets with structural price jumps, see for instance Benth et al. (2008). As a consequence, the problem of joint quantity and price risk on electricity markets is commonly studied in the literature and it stresses the importance of incorporating price–quantity correlation into the models. For an example, Bessembinder and Lemmon (2002) consider an equilibrium market model for which the correlation has a significant impact on optimal hedging strategies in forward markets. Their model has been extended by, for instance, Willems and Morbee (2010) who study the effect of including options onto the market. From the retailer's perspective with an exogenously given spot and forward market, Oum et al. (2006) use utility maximisation to derive optimal hedge strategies that exploit the correlation to manage price–quantity risk in a single period setting. As another example, Boroumand et al. (2015) address the problem of price–quantity risk on an intra-day scale and demonstrate with a simulation approach that intra-day hedging outperforms daily and longer term portfolios. We also take the retailer's perspective and model the spot-price/consumption-load correlation with a bivariate model where both the spot and load have temporal dependency. Based on that model, we set out to hedge the price–quantity risk of a fixed price agreement with forward contracts. In relation to our approach, Coulon et al. (2013) develop an intricate three-factor model and consider hedging strategies based on options that are priced with closed form expressions. In contrast, we do not consider the problem of pricing derivatives under our model, but concentrate on a operational hedging approach based on exogenously given market forward-price data.

1.3. Structure, contributions, and findings

The rest of the paper is organised as follows. In Section 2, we formulate a dynamic model for electricity spot-price and consumption-load — a two-dimensional Gaussian Ornstein-Uhlenbeck model with a seasonality component. In Section 3, we analyse expected loss minimisation. Looking specifically at static hedging strategies with forward contracts, a main contribution of the paper is the derivation of explicit formulas for optimal hedge portfolios based on three different loss functions: the square, the exponential, and the "hockey stick"; the two latter based on asymmetric risk measures appear new. Section 4 is an empirical study of the hedging performance based on publicly available consumption, spot- and forward-prices from the Nordic region. The empirical results demonstrate substantial benefits for the derived asymmetric hedges over the common industry benchmark that uses a position in forward contracts equal to the expected load. It should be noted that unlike many sources in the literature, the hedge experiments in this paper truly test out-of-sample performance: The hedge portfolio constructed at, say, time t uses only information about state-variables and model parameters that was truly available at time t. Section 5 investigates the robustness of the results. First, we test if forward-prices add information that can be easily exploited in the hedge constructions. We find this not to be the case. Second, we consider a model extension with jumps (or spikes) in which we derive optimal hedges and implement these. We find little to no empirical improvement over the original hedge performance. Section 6 concludes and describes directions for future research.

2. The electricity price and consumption model

Our approach will be to model jointly the electricity spot price and the consumption load with a continuous stochastic process. Both the price and the load exhibit seasonal patterns – the time of the day, day of the week and climatological season naturally effect the spot price and consumption, see Benth et al. (2008) and Haugom (2011) – and we will fit deterministic periodic functions to capture this seasonality. The residual price and load will then be modelled with a stationary process and we conveniently choose the continuous Ornstein-Uhlenbeck process for this purpose.[1]

2.1. The bivariate Ornstein-Uhlenbeck process

The bivariate mean-reverting Ornstein-Uhlenbeck process $(X_t, Y_t)_{t\geq 0}$ is given by the solution to the stochastic differential equation

$$
\begin{aligned}
dX_t &= \kappa_x(\theta_x - X_t)dt + \sigma_x dW_t^{(1)},\\
dY_t &= \kappa_y(\theta_y - Y_t)dt + \sigma_y dW_t^{(2)},
\end{aligned}
$$

where $\kappa_x, \kappa_y, \sigma_x, \sigma_y$ are positive parameters, $\theta_x, \theta_y \in \mathbb{R}$ and the Brownian motions $W^{(1)}$, $W^{(2)}$ are correlated with $\rho_w \in [-1, 1]$. The standard trick of multiplying by integrating factors, i.e. using the Ito formula on $e^{-\kappa_x t}X(t)$ and $e^{-\kappa_y t}Y(t)$, gives us the more explicit expressions

$$
\begin{aligned}
X_t &= X_0 e^{-\kappa_x t} + \sigma_x \int_0^t e^{-\kappa_x(t-u)} dW_u^{(1)} + \theta_x(1 - e^{-\kappa_x t}),\\
Y_t &= Y_0 e^{-\kappa_y t} + \sigma_y \int_0^t e^{-\kappa_y(t-u)} dW_u^{(2)} + \theta_y(1 - e^{-\kappa_y t}),
\end{aligned} \tag{1}
$$

from which we see that the univariate covariance functions are

$$
\mathrm{Cov}(X_{t+\Delta}, X_t) = \Sigma_x\, e^{-\kappa_x|\Delta|}, \quad \mathrm{Cov}(Y_{t+\Delta}, Y_t) = \Sigma_y\, e^{-\kappa_y|\Delta|},
$$

and that the cross-covariance function is

$$
\begin{aligned}
\mathrm{Cov}(X_{t+\Delta}, Y_t) &= \Sigma_{xy}\, e^{-\kappa_x|\Delta|}, \quad \Delta \geq 0,\\
\mathrm{Cov}(X_{t+\Delta}, Y_t) &= \Sigma_{xy}\, e^{-\kappa_y|\Delta|}, \quad \Delta < 0,
\end{aligned}
$$

where we have defined the parameters

$$
\Sigma_x = \frac{\sigma_x^2}{2\kappa_x}, \quad \Sigma_y = \frac{\sigma_y^2}{2\kappa_y} \quad \text{and} \quad \Sigma_{xy} = \rho_w \frac{\sigma_x \sigma_y}{\kappa_x + \kappa_y}.
$$

From Eq. (1) it also follow that (X, Y) is a Gaussian process, in particular the conditional distribution is bivariate normal (BVN) and given by

$$
\begin{pmatrix} X_{t+\Delta} \\ Y_{t+\Delta} \end{pmatrix} \Bigg| \begin{pmatrix} X_t \\ Y_t \end{pmatrix} = \begin{pmatrix} x_t \\ y_t \end{pmatrix} \sim BVN\left(\boldsymbol{\mu}_{t+\Delta|t}, \boldsymbol{\Sigma}_{t+\Delta|t}\right) \tag{2}
$$

with mean vector and covariance matrix

$$
\begin{aligned}
\boldsymbol{\mu}_{t+\Delta|t} &= \begin{bmatrix} \theta_x + (x_t - \theta_x)e^{-\kappa_x|\Delta|} \\ \theta_y + (y_t - \theta_y)e^{-\kappa_y|\Delta|} \end{bmatrix},\\
\boldsymbol{\Sigma}_{t+\Delta|t} &= \begin{bmatrix} \Sigma_x(1 - e^{-2\kappa_x|\Delta|}) & \Sigma_{xy}(1 - e^{-(\kappa_x+\kappa_y)|\Delta|}) \\ \Sigma_{xy}(1 - e^{-(\kappa_x+\kappa_y)|\Delta|}) & \Sigma_y(1 - e^{-2\kappa_y|\Delta|}) \end{bmatrix}.
\end{aligned}
$$

[1] Weron (2007) and Benth et al. (2008) give comprehensive treatments of electricity-price modelling.

2.2. Parameter estimation

With observed data we refer to a set of observations made at $n+1$ time-points $t_0, \dots, t_n$ with $\Delta_i = t_i - t_{i-1}$ denoting the spacings of the time grid. An observed path $x = (x_0, \dots, x_n)$ of the process $(X_t)_{t\geq 0}$ is a set of observations where x_i is the observed value of X_{t_i} for $i = 0, \dots, n$. From Eq. (2) we write the log-likelihood for the observed path x of the univariate Ornstein-Uhlenbeck process as a function of the parameters $\psi = (\kappa, \theta, \sigma)$,

$$l(\psi; x) = \sum_{i=0}^{n-1} \log\phi\left(x_{i+1}; \mu_{i+1|i}, \sigma^2_{i+1|i}\right) + \log\phi\left(x_0; \theta, \sigma^2/(2\kappa)\right)$$

where ϕ is the normal density function, here with (conditional) mean $\mu_{i+1|i} = x_i e^{-\kappa\Delta_{i+1}} + \theta(1 - e^{-\kappa\Delta_{i+1}})$ and variance $\sigma^2_{t+1|i} = \sigma^2(1 - e^{-2\kappa\Delta_{i+1}})/(2\kappa)$. The decomposition of the density comes from the fact that X is a Markov process: since the Markov property gives that $f(x_{i+1}|x_i, \dots, x_0) = f(x_{i+1}|x_i)$ for the conditional density, we can write

$$f(x_0, \dots, x_n) = f(x_n|x_{n-1}) \dots f(x_1|x_0) f(x_0).$$

The maximum likelihood estimate $\hat{\psi}$ is the argument that maximizes the log-likelihood

$$\hat{\psi} = \arg\max_{\psi} l(\psi, x)$$

which may be obtained by a numerical optimiser.[2] Calculating the observed information matrix

$$I_o = -\left.\frac{\partial^2 l}{\partial\psi^T \partial\psi}\right|_{\psi=\hat{\psi}}$$

by numerical differentiation at estimated values gives an estimated standard error of the j'th parameter as $\sqrt{\left(I_o^{-1}\right)_{jj}}$ where $(A^{-1})_{ij}$ denotes element i,j of the inverse matrix of A. Further, if $\hat{\psi}_x$ and $\hat{\psi}_y$ are univariate estimates from two observed paths x and y, then

$$\hat{\rho}_w = \frac{\hat{\kappa}_x + \hat{\kappa}_y}{\hat{\sigma}_x \hat{\sigma}_y} \cdot \hat{\rho}(x, y)$$

gives an estimator of ρ_w where $\hat{\rho}(x, y)$ is the sample correlation coefficient between x and y.

A bivariate observation (x, y) from the bivariate Ornstein-Uhlenbeck process with parameters $\psi_{xy} = (\kappa_x, \theta_x, \sigma_x, \kappa_y, \theta_y, \sigma_y, \rho_w)$ yields the log-likelihood from Eq. (2)

$$l(\psi_{xy}; x, y) = \sum_{i=0}^{n-1} \log\phi_2\left(x_{i+1}, y_{i+1}; \boldsymbol{\mu}_{i+1|i}, \boldsymbol{\Sigma}_{i+1|i}\right) + \log\phi_2(x_0, y_0; \boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0) \tag{3}$$

where $\boldsymbol{\mu}_{i+1|i} \equiv \boldsymbol{\mu}_{t_i+\Delta_i|t_i}$ and $\boldsymbol{\Sigma}_{i+1|i} \equiv \boldsymbol{\Sigma}_{t_i+\Delta_i|t_i}$ are the conditional mean vector and covariance matrix given in the previous section. Here ϕ_2 denotes the bivariate normal density and the maximum likelihood estimate $\hat{\psi}_{xy}$ may be obtained by numerical optimisation along with estimated standard errors from the numerically calculated information matrix.

[2] For equidistant observations estimators can be found in closed, but slightly lengthy, form.

2.3. The seasonal Ornstein-Uhlenbeck model

Since the price and consumption of electricity exhibit seasonal behaviours, we will work with a model for the spot-price S and consumption-load L as given by so-called seasonal Ornstein-Uhlenbeck model:

$$S_t = \tilde{S}_t + \Theta_S(t),$$
$$L_t = \tilde{L}_t + \Theta_L(t),$$

where $(\tilde{S}, \tilde{L})$ follows a bivariate Ornstein-Uhlenbeck process while $\Theta_S(t)$ and $\Theta_L(t)$ are deterministic seasonality functions of time. For this purpose, we let $\Theta(t)$ be a periodic function defined by

$$\Theta(t) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \cdot \sin\left(\frac{2\pi}{\tau_i} t + \phi_i\right) \tag{4}$$

with p periods $\tau_1, \dots \tau_p$ with amplitudes $\alpha_1, \dots, \alpha_p$ and phases $\phi = (\phi_1, \dots, \phi_p)$. To estimate the parameters we write Eq. (4) as a regression

$$\mathbf{y} = \mathbf{A}(\phi)\mathbf{a}$$

where $\mathbf{y} = (y_1, \dots, y_n)^T$, $\mathbf{a} = (\alpha_0, \dots, \alpha_p)^T$ and the regression matrix $\mathbf{A}(\phi)$ of dimension $[n \times p + 1]$ is a function of the phase vector ϕ

$$\mathbf{A}(\phi) = \begin{pmatrix} 1 & \sin\left(\frac{2\pi}{\tau_1} t_1 + \phi_1\right) & \dots & \sin\left(\frac{2\pi}{\tau_p} t_1 + \phi_p\right) \\ 1 & \sin\left(\frac{2\pi}{\tau_1} t_2 + \phi_1\right) & \dots & \sin\left(\frac{2\pi}{\tau_p} t_2 + \phi_p\right) \\ \vdots & \vdots & & \vdots \\ 1 & \sin\left(\frac{2\pi}{\tau_1} t_n + \phi_1\right) & \dots & \sin\left(\frac{2\pi}{\tau_p} t_n + \phi_p\right) \end{pmatrix}.$$

If, for a moment, we consider the phases to be fixed, then the estimation of the amplitudes is a standard least-squares optimisation problem

$$\mathbf{a}^* = \arg\min_{\mathbf{a}} \|\mathbf{y} - \mathbf{A}(\phi)\mathbf{a}\|^2$$

which has the solution

$$\mathbf{a}^* = (\mathbf{A}(\phi)^T \mathbf{A}(\phi))^{-1} \mathbf{A}(\phi)^T \mathbf{y}$$

ie. a function of the phase vector $\mathbf{a}^* = \mathbf{a}^*(\phi)$. This lead us to perform the estimation in two steps: First, we minimise

$$\hat{\phi} = \arg\min_{\boldsymbol{\phi}} \|\mathbf{y} - \mathbf{A}(\phi)\mathbf{a}^*(\phi)\|^2$$

by numerical optimisation. Then we perform the regression

$$\hat{\mathbf{a}} = \mathbf{a}^*(\hat{\phi}) = (\mathbf{A}(\hat{\phi})^T \mathbf{A}(\hat{\phi}))^{-1} \mathbf{A}(\hat{\phi})^T \mathbf{y}$$

to obtain parameter estimates $(\hat{\mathbf{a}}, \hat{\phi})$ of the periodic function. Our estimation approach fits the seasonal functions to price and load in the first step and an Ornstein-Uhlenbeck process to the de-seasonalised price and load in the second step.

Remark 1. As an alternative to a two-step estimation of the seasonal Ornstein-Uhlenbeck process, note that we may write the likelihood of the full model. Following the notational convention that we introduce in Section 3.2 and denoting the respective amplitude- and phase vectors with $\alpha_S, \phi_S, \alpha_L, \phi_L$, the $4p+9$ parameter vector

$$(\alpha_S, \alpha_L, \phi_S, \phi_L, \kappa_S, \kappa_L, \theta_S, \theta_L, \sigma_S, \sigma_L, \rho_W) \tag{5}$$

goes into the Gaussian likelihood of the same form as Eq. (3) with conditional mean being a function of amplitudes and phases as well, see Eqs. (8)–(9). Numerical optimisation of the likelihood is, however, unstable due to the large number of parameters. For example, in the empirical study of Section 4 we use five periods for the seasonal components which yields 29 parameters in total. We found the numerical optimisation of the likelihood on market data with respect to the full vector Eq. (5) to be unstable. For this reason, we employ the robust two-step approach instead.

3. Risk-minimising static hedging

The hedging problem we are facing is to replicate a future uncertain financial obligation. That is, one tries to replicate the future payment with other financial products, typically liquid contracts that are traded in the market, and in our case we will use forward contracts. We design a static hedging strategy: a buy-and-hold position in the forward contract that aims at hedging our financial obligation. Our approach to determine the forward position will be the method of expected loss minimisation.

To this end, let π_T denote the payoff at time T of the hedged position, that is, the financial obligation we want to hedge together with a portfolio of hedging instruments. Then $-\pi_T$ gives the loss of the position and

$$\mathbb{E}[u(-\pi_T)] \tag{6}$$

gives a measure of risk for some specified loss function u (see Artzner et al. (1999) for details of risk measures). The problem at hand is to choose the hedging portfolio such that this risk is minimised. Hence, the task is to choose both type of hedging instruments and positions to buy/sell of these instruments at initiation. Notice that if we were to set up a dynamic hedging strategy instead, then we would have to choose the time-points or events at which to rebalance the hedging portfolio.[3]

3.1. Hedging a fixed price agreement

Assume that we want to hedge a fixed price agreement that expires at a specified future time $T > t_0$ where t_0 is the initiation time. This is a financial contract with payoff $(S_T - F^{\text{fpa}})L_T$ at time T for the long position and its features can be explained by the following situation: At time t_0, we enter a contract with a counterparty where we obligate ourselves to deliver an unspecified quantity L_T of electricity at time T for the agreed price F^{fpa}. The quantity will be the unknown consumption of the counterparty. At the expiry time T, we will have to cover our contracted delivery of electricity by purchasing the same quantity from the spot market at a price S_T. Our cash-flow for the position at time T then amounts to

$$F^{\text{fpa}}L_T - S_T L_T$$

which is the short position of the fixed price agreement.

To set up a static, buy-and-hold, hedge at time t_0 for the naked position (notice that S_T and L_T are unknown at t_0), we enter a forward contract with expiry time T and forward price F. This is a contract with payoff $S_T - F$ at time T and we enter the forward with a position V to be determined at initiation. If we assume that the fixed price agreement has a price relative to the forward $F^{\text{fpa}} = F + m$, where m is some non-negative margin, then the total payoff of our hedged position is

$$\pi_T = (S_T - F)(V - L_T) + mL_T.$$

The hedging problem is to specify, at initiation time t_0, the forward position V in an optimal way. For this purpose, we employ the approach of minimising the risk measure Eq. (6) specified with three different loss functions: (i) the positive loss $u(x) = \max(x, 0)$, (ii) the quadratic loss $u(x) = x^2$ and (iii) the exponential loss $u(x) = \exp(x)$. Our aim is thus to minimise the risk of the hedged portfolio with respect to the forward position, that is,

$$f(V) = \mathbb{E}[u\left(-(S_T - F)(V - L_T) + mL_T\right)] \tag{7}$$

where the minimising argument $V^* = \arg\min f(V)$ forms the optimal position of the hedging strategy.

Common to all three loss functions is that they are convex which makes the corresponding risk measures convex as well, see Föllmer and Schied (2011). A second property which holds for the positive loss and exponential loss is monotonicity: if the outcome of position A is certainly smaller than that of position B, then position A will have greater risk. This means that these measures take downside risk into account while, in contrast, the quadratic-loss measure is symmetric: both positive and negative variations of the loss are equal contributors to the risk. The expected positive loss also satisfies the sub-additivity property part of the coherent risk measure definition. A monetary interpretation of this property means that one will benefit from diversification: the risk of a portfolio of assets is less or equal to the risk of the individual assets.

Financial contracts traded at the exchange for power derivatives are typically not specified with a single expiry-time as in the previous example. A power forward obligate the counterparties to settle the difference between spot and fixed price at a set of time-points during a specified period, for example a month, quarter or a year. There is typically a base-load and a peak-load version of the contract as well, where the latter has settlements on peak-hours only, that is weekdays 08:00 to 20:00 for the Nordic market, while the former has settlements on every full hour during the expiry period.

We consider fixed price agreements and forwards that are specified with an expiry month M and we let $M(p)$ denote peak-hours during this month and $M(op)$ the remaining off-peak hours. The payoff at time T_i for the hedged position during off-peak is thus $(S_{T_i} - F^{\text{b}})\left(V^{\text{b}} - L_{T_i}\right)$ for all $T_i \in M(op)$, where V^{b} is the position in the base-load forward with price F^{b}. Notice that we assume the pricing $F^{\text{b-fpa}} = F^{\text{b}} + m$ with margin $m \geq 0$ for delivery during off-peak of the fixed price agreement, in particular, we let $m = 0$. For a time-point during peak periods we have the choice to include in our hedge portfolio a position V^{p} in the peak-load forward with price $F^{\text{p}} \geq F^{\text{b}}$. This gives a payoff of the hedged position $\left(S_{T_i} - F^{\text{b}}\right)V^{\text{b}} + \left(S_{T_i} - F^{\text{p}}\right)V^{\text{p}} + \left(F^{\text{p-fpa}} - S_{T_i}\right)L_{T_i}$ for $T_i \in M(p)$. If we define $\tilde{F} = F^{\text{p}} - \frac{V^{\text{b}}}{(V^{\text{b}}+V^{\text{p}})}(F^{\text{p}} - F^{\text{b}})$ such that $F^{\text{b}} \leq \tilde{F} \leq F^{\text{p}}$, we obtain

$$\pi_{T_i} = \left(S_{T_i} - \tilde{F}\right)\left(V^{\text{b}} + V^{\text{p}} - L_{T_i}\right), \quad T_i \in M(p)$$

[3] For an overview of hedging in incomplete markets, and the case of dynamic hedging based on quadratic risk minimisation, see e.g. Bingham and Kiesel (2004), Chapter 7.2 and references therein.

where we assume a pricing $F^{\text{p-fpa}} \geq \tilde{F} + m$ with margin $m \geq 0$ and we set $m = 0$ here as well. To determine the optimal hedge for the entire month M with the expected loss measure, we minimise the function

$$f(V^{\mathrm{b}}, V^{\mathrm{p}}) = \sum_{T_i \in M(op)} \mathbb{E}\left[u\left(-\left(S_{T_i} - F^{\mathrm{b}}\right)\left(V^{\mathrm{b}} - L_{T_i}\right)\right)\right] + \sum_{T_i \in M(p)} \mathbb{E}\left[u\left(-\left(S_{T_i} - \tilde{F}\right)\left(V^{\mathrm{b}} + V^{\mathrm{p}} - L_{T_i}\right)\right)\right]$$

such that the minimising argument $\left(V^{\mathrm{b}^*}, V^{\mathrm{p}^*}\right)$ is the base- and peak-load forward positions of our hedge.

Remark 2. Note that, as the loss functions are convex, and convex functions are invariant under affine transformations, we have that $V \mapsto u(-(S-F)(V-L))$ is a convex map, as well as the total risk $V \mapsto \sum_i u(-(S_i - F)(V - L_i))$. Thus, we have that any local minimum is indeed a global minimum and the same holds for minimising $f(V^{\mathrm{b}}, V^{\mathrm{p}})$ jointly with respect to $(V^{\mathrm{b}}, V^{\mathrm{p}})$ due to linearity.

3.2. Hedging with the seasonal Ornstein-Uhlenbeck model

To be able to minimise the total expected loss of a monthly contract and find the optimal hedge we will derive an expression for the expectation in Eq. (7) with the margin set to zero. To simplify the notation, assume a current time $t_0 = 0$ with given spot-price $S_0 = s$ and load $L_0 = l$ such that $\tilde{S}_0 = \tilde{s} = s - \Theta_S(0)$ and $\tilde{L}_0 = \tilde{l} = l - \Theta_L(0)$ are the current values of the de-seasonalised spot and load. Let $(\kappa_S, \kappa_L, \theta_S, \theta_L, \sigma_S, \sigma_L, \rho_W)$ be the parameters of the Ornstein-Uhlenbeck process $(\tilde{S}, \tilde{L})$. The distribution of (S, L) at a future time T is then given by

$$\begin{pmatrix} S_T \\ L_T \end{pmatrix} \Bigg| \begin{pmatrix} S_0 \\ L_0 \end{pmatrix} \stackrel{d}{=} \begin{pmatrix} \mu_S(T,T) + \eta_S(T) Z \\ \mu_L(T,T) + \eta_L(T)\left(\rho_{SL}(T) Z + \sqrt{1 - \rho_{SL}^2(T)} Z^\perp\right) \end{pmatrix} \quad (8)$$

where $Z, Z^\perp$ are independent standard normal variables and we have defined the time-dependent moments

$$\begin{aligned}
\mu_S(t,\Delta) &= \Theta_S(t) + \theta_S + (\tilde{s} - \theta_S) e^{-\kappa_S \Delta}, \\
\mu_L(t,\Delta) &= \Theta_L(t) + \theta_L + (\tilde{l} - \theta_L) e^{-\kappa_L \Delta}, \\
\eta_S(\Delta) &= \sqrt{\frac{\sigma_S^2}{2\kappa_S}(1 - e^{-2\kappa_S \Delta})}, \\
\eta_L(\Delta) &= \sqrt{\frac{\sigma_L^2}{2\kappa_L}(1 - e^{-2\kappa_L \Delta})}, \\
\rho_{SL}(\Delta) &= 2\rho_W \frac{\sqrt{\kappa_S \kappa_L}}{(\kappa_S + \kappa_L)} \frac{(1 - e^{-(\kappa_S + \kappa_L)\Delta})}{\sqrt{(1 - e^{-2\kappa_S \Delta})(1 - e^{-2\kappa_L \Delta})}}.
\end{aligned} \quad (9)$$

Note that the first moments depend on both the time t and time-lag Δ while the second moments depend on the time-lag only. In effect we may write the payoff at time T of the hedged position as the product of two linear functions of Z and $Z^\perp$

$$(S_T - F)(V - L_T) \stackrel{d}{=} \underbrace{(\mu_S(T,T) - F + \eta_S(T) Z)}_{=f(Z)} \times \underbrace{\left(V - \mu_L(T,T) - \eta_L(T)\left(\rho_{SL}(T) Z + \sqrt{1 - \rho_{SL}^2(T)} Z^\perp\right)\right)}_{=g(Z,Z^\perp)}$$

that is, the payoff is a product of two correlated normal variables dependent on F, V, where F, V will play the role of $F^{\mathrm{b}}, V^{\mathrm{b}}$ and $\tilde{F}, V^{\mathrm{b}} + V^{\mathrm{p}}$ for the off-peak and peak contract respectively. Starting with the positive loss function $u(x) = max(x, 0)$, the expected loss may be written as

$$\mathbb{E}\left[-\min(f(Z) g(Z, Z^\perp), 0)\right] = -\mathbb{E}\left[f(Z) g(Z, Z^\perp)\left(\mathbf{1}_{f(Z)<0} \mathbf{1}_{g(Z,Z^\perp)>0} + \mathbf{1}_{f(Z)>0} \mathbf{1}_{g(Z,Z^\perp)<0}\right)\right].$$

To calculate the terms $\mathbb{E}\left[fg\mathbf{1}_{f<0}\mathbf{1}_{g>0}\right]$ and $\mathbb{E}[fg\mathbf{1}_{f>0}\mathbf{1}_{g<0}]$ we use the conditional expectations

$$\begin{aligned}
h_1(z) &= \mathbb{E}\left[g(Z, Z^\perp)\mathbf{1}_{g(Z,Z^\perp)>0} | Z = z\right], \\
h_2(z) &= \mathbb{E}\left[g(Z, Z^\perp)\mathbf{1}_{g(Z,Z^\perp)<0} | Z = z\right],
\end{aligned}$$

such that we can write the expected positive loss as

$$\begin{aligned}
&-\mathbb{E}\left[fg\mathbf{1}_{f<0}\mathbf{1}_{g>0}\right] - \mathbb{E}\left[fg\mathbf{1}_{f>0}\mathbf{1}_{g<0}\right] \\
&\quad = -\mathbb{E}\left[f(Z) h_1(Z)\mathbf{1}_{f<0}\right] - \mathbb{E}\left[f(Z) h_2(Z)\mathbf{1}_{f>0}\right] \\
&\quad = -\mathbb{E}\left[f(Z)\left(h_1(Z)\mathbf{1}_{f<0} + h_2(Z)\mathbf{1}_{f>0}\right)\right].
\end{aligned}$$

For a general Gaussian variable, $X \sim \mathcal{N}(\mu, \sigma^2)$, straightforward calculations give

$$\begin{aligned}
\mathbb{E}[X\mathbf{1}_{X>0}] &= \mu\Phi\left(\frac{\mu}{\sigma}\right) + \sigma\phi\left(\frac{\mu}{\sigma}\right), \\
\mathbb{E}[X\mathbf{1}_{X<0}] &= \mu\Phi\left(-\frac{\mu}{\sigma}\right) - \sigma\phi\left(\frac{\mu}{\sigma}\right),
\end{aligned}$$

where Φ and ϕ are the standard normal distribution and density respectively. These expressions may readily be used for h_1 and h_2 since

$$g(Z, Z^\perp) | Z = z \sim \mathcal{N}\Big(\underbrace{V - \mu_L(T,T) - \eta_L(T)\rho_{SL}(T) z}_{=\mu_g(z)}, \underbrace{\eta_L^2(T)\left(1 - \rho_{SL}^2(T)\right)}_{=\sigma_g^2}\Big). \quad (10)$$

This leads us to define

$$\psi(z) = -f(z)\left(\left\{\mu_g(z)\Phi\left(\frac{\mu_g(z)}{\sigma_g}\right) + \sigma_g\phi\left(\frac{\mu_g(z)}{\sigma_g}\right)\right\}\mathbf{1}_{f(z)<0} + \left\{\mu_g(z)\Phi\left(-\frac{\mu_g(z)}{\sigma_g}\right) - \sigma_g\phi\left(\frac{\mu_g(z)}{\sigma_g}\right)\right\}\mathbf{1}_{f(z)>0}\right)$$

which gives us a final expression for the expected positive loss

$$\mathbb{E}[\max(-(S_T - F)(V - L_T), 0)] = \mathbb{E}[\psi(Z)]$$

where Z is a standard normal variable.

For the special case when $\tilde{S}$ and $\tilde{L}$ are uncorrelated, and thus independent, i.e. $\rho_W = 0$ and $\rho_{SL}(\Delta) = 0$, we obtain

$$(S_T - F)(V - L_T) \stackrel{d}{=} \underbrace{(\mu_S(T,T) - F + \eta_S(T) Z)}_{=f(Z)}\underbrace{(V - \mu_L(T,T) - \eta_L(T) Z^\perp)}_{=g(Z^\perp)}$$

this simplifies the expected positive loss to a closed-form expression

$$
\begin{aligned}
&\mathbb{E}[-\min(f(Z)g(Z^\perp),0)]\\
&\quad= -\mathbb{E}\left[f(Z)g(Z^\perp)\left(\mathbf{1}_{f(Z)<0}\mathbf{1}_{g(Z^\perp)>0}+\mathbf{1}_{f(Z)>0}\mathbf{1}_{g(Z^\perp)<0}\right)\right]\\
&\quad= -\mathbb{E}\left[f(Z)\mathbf{1}_{f(Z)<0}\right]\cdot\mathbb{E}\left[g(Z^\perp)\mathbf{1}_{g(Z^\perp)>0}\right]-\mathbb{E}\left[f(Z)\mathbf{1}_{f(Z)>0}\right]\cdot\mathbb{E}\left[g(Z^\perp)\mathbf{1}_{g(Z^\perp)<0}\right]\\
&\quad= \left\{(F-\mu_S(T,T))\Phi\left(\frac{F-\mu_S(T,T)}{\eta_S(T)}\right)+\eta_S(T)\phi\left(\frac{\mu_S(T,T)-F}{\eta_S(T)}\right)\right\}\\
&\qquad\cdot\left\{(V-\mu_L(T,T))\Phi\left(\frac{V-\mu_L(T,T)}{\eta_L(T)}\right)+\eta_L(T)\phi\left(\frac{V-\mu_L(T,T)}{\eta_L(T)}\right)\right\}\\
&\qquad+\left\{(\mu_S(T,T)-F)\Phi\left(\frac{\mu_S(T,T)-F}{\eta_S(T)}\right)+\eta_S(T)\phi\left(\frac{\mu_S(T,T)-F}{\eta_S(T)}\right)\right\}\\
&\qquad\cdot\left\{(\mu_L(T,T)-V)\Phi\left(\frac{\mu_L(T,T)-V}{\eta_L(T)}\right)+\eta_L(T)\phi\left(\frac{V-\mu_L(T,T)}{\eta_L(T)}\right)\right\}.
\end{aligned}
$$

The expected loss calculated with dependency between the de-seasonalised spot and load[4] is shown in Fig. 1 along with the expected loss calculated from the independent specification of the model. The difference between the two expected loss functions is small. In particular, the minimum expected loss from the two models is obtained at the same argument, that is, the zero-correlation case yields the same optimal hedge position as the dependent case.

For the exponential loss function $u(x) = exp(x)$ we again have by conditioning

$$
\begin{aligned}
\mathbb{E}\left[\exp\left\{-(S_T-F)(V-L_T)\right\}\right] &= \mathbb{E}\left[\mathbb{E}\left[\exp\left\{-f(Z)g(Z,Z^\perp)\right\}\middle|\,Z=z\right]\right]\\
&= \mathbb{E}\left[\exp\left\{-\mu_g(Z)f(Z)+\frac{1}{2}\sigma_g^2 f^2(Z)\right\}\right]
\end{aligned}
$$

where the moment generating function of $g(Z,Z^\perp)|Z=z$ being normally distributed as in Eq. (10) is used for the second equality. Expanding the square, we may write the exponent as a quadratic function of Z

$$
\begin{aligned}
&\underbrace{\frac{1}{2}(\mu_S-F)^2\eta_L^2\left(1-\rho_{SL}^2\right)-(\mu_S-F)(V-\mu_L)}_{=a}\\
&\quad+\underbrace{\left((\mu_S-F)^2\eta_S\eta_L^2\left(1-\rho_{SL}^2\right)-(V-\mu_L)\eta_S-(\mu_S-F)\eta_L\rho_{SL}\right)}_{=b}Z\\
&\quad+\underbrace{\left(\frac{1}{2}\eta_S^2\eta_L^2\left(1-\rho_{SL}^2\right)-\eta_S\eta_L\rho_{SL}\right)}_{=c}Z^2
\end{aligned}
$$

and hence, with the coefficients a, b, c, defined as above,

$$
\mathbb{E}\left[\exp\left\{-(S_T-F)(V-L_T)\right\}\right] = \mathbb{E}\left[\exp\left\{c\left(Z+\frac{b}{2c}\right)^2\right\}\right]\exp\left\{a-\frac{b^2}{4c}\right\}
$$

where $\left(Z+\frac{b}{2c}\right)^2$ is chi-squared distributed with one degree of freedom and non-centrality parameter $b^2/(4c^2)$. Using the moment generating function of the non-central chi-squared distribution

$$
\mathbb{E}\left[\exp\left\{c\left(Z+\frac{b}{2c}\right)^2\right\}\right] = \exp\left\{\frac{b^2}{4c(1-2c)}\right\}\frac{1}{\sqrt{1-2c}}
$$

[4] The is calculated by approximating $\mathbb{E}[\psi(Z)]$ with $\frac{1}{n}\sum\psi(z_i)$ where z_i are 1000 generated outcomes from the standard normal distribution.

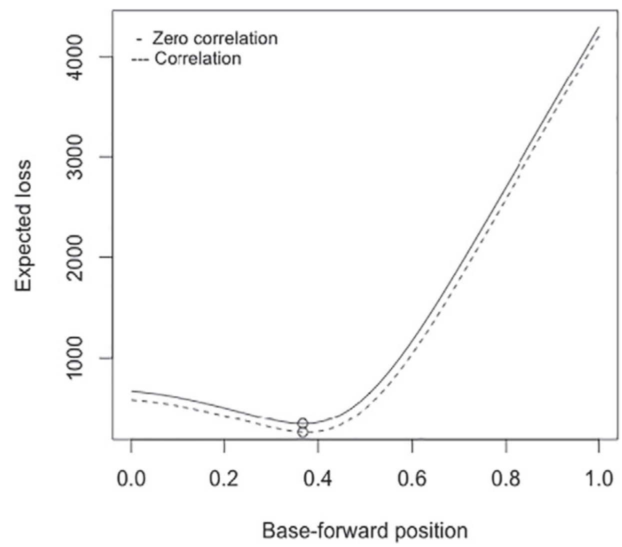

Fig. 1. Expected positive loss calculated from the dependent (solid line) and independent (dashed line) model for the de-seasonalised data with parameters in Table 2 and the January-13 contract in Table 1. Notice that both models yield the same minimising argument (circles), i.e. the same optimal hedging position in the forward contract.

we obtain a final expression for the expected exponential loss

$$
\mathbb{E}\left[\exp\left\{-(S_T-F)(V-L_T)\right\}\right] = \exp\left\{a+\frac{b^2}{2-4c}\right\}\frac{1}{\sqrt{1-2c}}.
$$

For the quadratic loss function $u(x) = x^2$ we have the expected squared loss measure

$$
\begin{aligned}
\mathbb{E}[(S_T-F)^2(V-L_T)^2] &= \mathbb{E}[f(Z)^2 g(Z,Z^\perp)^2]\\
&= \mathbb{E}\left[f(Z)^2\cdot\mathbb{E}[g(Z,Z^\perp)^2|Z=z]\right]
\end{aligned}
\tag{11}
$$

where the conditional expectation is

$$
\begin{aligned}
\mathbb{E}[g(Z,Z^\perp)^2|Z=z] &= \eta_L^2(T)\left(1-\rho_{SL}^2(T)\right)+(V-\mu_L(T,T))^2\\
&\quad+\eta_L^2(T)\rho_{SL}^2 z^2-2(V-\mu_L(T,T))\eta_L(T)\rho_{SL}(T)z.
\end{aligned}
$$

Substituting this back into Eq. (11) gives, after a few simplifications, the quadratic loss measure

$$
\begin{aligned}
&\mathbb{E}\left[(S_T-F)^2(V-L_T)^2\right]\\
&\quad= \left((\mu_S(T,T)-F)^2+\eta_S^2(T)\right)\left((V-\mu_L(T,T))^2+\eta_L^2(T)\right)\\
&\qquad+2\eta_S^2(T)\eta_L^2(T)\rho_{SL}^2(T)-4(\mu_S(T,T)-F)(V-\mu_L(T,T))\eta_S(T)\eta_L(T)\rho_{SL}(T)
\end{aligned}
$$

where we have employed the fact that $\mathbb{E}[Z] = \mathbb{E}[Z^3] = 0$ while $\mathbb{E}[Z^2] = 1$ and $\mathbb{E}[Z^4] = 3$ for a standard normal variable. Differentiation with respect to V and equating to zero yields

$$
V^* = \mu_L(T,T)+2\frac{(\mu_S(T,T)-F)\eta_S(T)\eta_L(T)\rho_{SL}(T)}{(\mu_S(T,T)-F)^2+\eta_S^2(T)}
\tag{12}
$$

which is the forward position of the optimal hedge obtained from minimising the expected quadratic loss. Notice here that quadratic hedging suggests a forward position equal to the expectation of the load in the case when the de-seasonalised spot and load are assumed

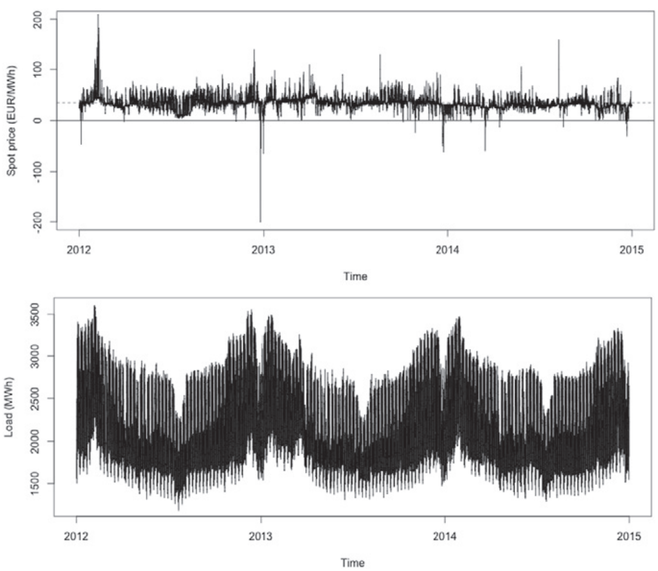

Fig. 2. Spot price (top) and consumption (bottom) from the DK1 price area of the electricity market in Denmark. The dashed line marks the average spot price at 34.96 EUR/MWh.

Table 1
Forward prices (in EUR) at time t_0 for contracts with expiry months during 2013–2014.

Initiation t_0	Expiry month M	Base forward F^b	Peak forward F^p
2012-12-17	January-13	44.55	53.25
2013-01-17	February-13	45.90	52.95
2013-02-14	March-13	37.90	38.80
2013-03-18	April-13	37.15	39.65
2013-04-16	May-13	36.60	36.05
2013-05-16	June-13	33.55	34.85
2013-06-17	July-13	32.90	35.30
2013-07-17	August-13	38.90	41.95
2013-08-16	September-13	40.20	43.10
2013-09-16	October-13	40.20	45.30
2013-10-17	November-13	40.30	46.30
2013-11-18	December-13	37.40	39.45
2013-12-17	January-14	36.00	41.10
2014-01-17	February-14	36.02	41.02
2014-02-17	March-14	29.90	33.95
2014-03-17	April-14	31.15	35.18
2014-04-16	May-14	30.35	34.35
2014-05-16	June-14	30.10	34.15
2014-06-17	July-14	30.79	33.60
2014-07-17	August-14	34.25	37.15
2014-08-18	September-14	35.50	38.50
2014-09-17	October-14	34.60	37.15
2014-10-17	November-14	32.60	34.65
2014-11-17	December-14	30.25	32.05

uncorrelated. The hedging strategy is hence completely independent of our expectation of the future spot price. I particular, the loss $(S_T - F)(L_T - V)$ will potentially be substantial for the hedger when the load is at high levels (above expectation) since then the spot-price is typically high as well, due to the actual positive correlation between the two (the wrong-way risk). On the other hand if one assumes a non-zero positive correlation, the hedge position Eq. (12) will be increased if the spot is expected to be high, $\mu_S > F$, which in turn decreases the potential large losses in high-spot-high-load scenarios.

4. Empirical study

In this section, we present an empirical study of the performance of our hedging approach. We base the study partly on publicly available market data, partly on a set of proprietary load data obtained from DONG Energy who is a major player in the Danish power market. The data is presented in the next section along with fitted seasonal functions and estimated Ornstein-Uhlenbeck parameters from the residual processes. Next, we report the results from our hedging approach based on expected loss minimisation and compare these with a benchmark average-load strategy which is widely used by the industry. Finally, we present a simple extension of our model to account for spikes in the spot price and investigate how this affects the hedging results.

4.1. Data and fitted model

We test the empirical performance of the model-based loss-minimising hedges using spot-prices, forward-prices and consumption-load data from the price area DK1 of the Danish power market.[5] The price and load data are recorded with an hourly observation frequency during the period 1 January 2012 to 31 December 2014 and it is shown in Fig. 2. The spot price (top panel) is measured in EUR/MWh (euros per megawatt hour) while the load data (bottom panel) is measured in MWh. In addition to the publicly available consumption data in Fig. 2, we have access to a set of proprietary load data that has been made available to us from DONG Energy. This data has been anonymized by scaling with the maximum load and it is shown in Fig. 2 in the Appendix. We return to the data from DONG Energy in Section 4.3.

We use market quotes of peak and base forwards with monthly expiries for the DK1 price area in addition to the spot-price and load data. We initiate our hedge approximately two weeks prior to the expiry month of the forward contract and we have listed the corresponding forward prices in Table 1.

Remark 3. The data from the Nordic power market contains quotes of base and peak forwards noted on the Nord-Pool system spot-price and certificate of difference forwards (CDF) for the different Nordic price areas, see Table 5 in the Appendix. As the CDF forward for the DK1–system difference is quoted for a base version only, we are, in fact, subject to a basis risk that we do not hedge against. However, since the basis affects peak-hours only with the DK1-system spot-price difference (typically an order of magnitude smaller than the actual price) and with the peak-position (typically half the size of the base-position), we chose to ignore the basis and perform our hedging experiment as if the peak-forward was noted on the DK1 spot price as well. We ignore the same basis for the average-load hedge to be able to fairly compare the two strategies.

Upon inspection of Fig. 2, we see that both the spot-price and load series exhibit seasonal patterns, particularly evidently for the load: Power consumption inherently varies with the time of the day and day of the week. Due to electrical heating during winter (and air cooling during summer) there is a variety with the climatological season as well. The seasonality of consumption is transferred to the spot price as demand for electricity affects its price. Further, as the climatological season affects the production of hydroelectricity, there will also be seasonal dependent factors that affects supply, and thus the price. Finally, notice that the spot price may turn negative with some remarkably large negative values recorded around the new years of 2012, 2013 and 2014.[6]

[5] Spot- and forward-prices for the Nordic market are publicly available at http://www.nordpoolspot.com and http://www.nasdaqomx.com. We use prices from the Elspot market, which is the day-ahead spot price. Load data for Denmark is available at http://energinet.dk.

[6] An extreme price entry of 2000 EUR/MWh recorded on 2013-07-08 is treated as an outlier and removed from the data.

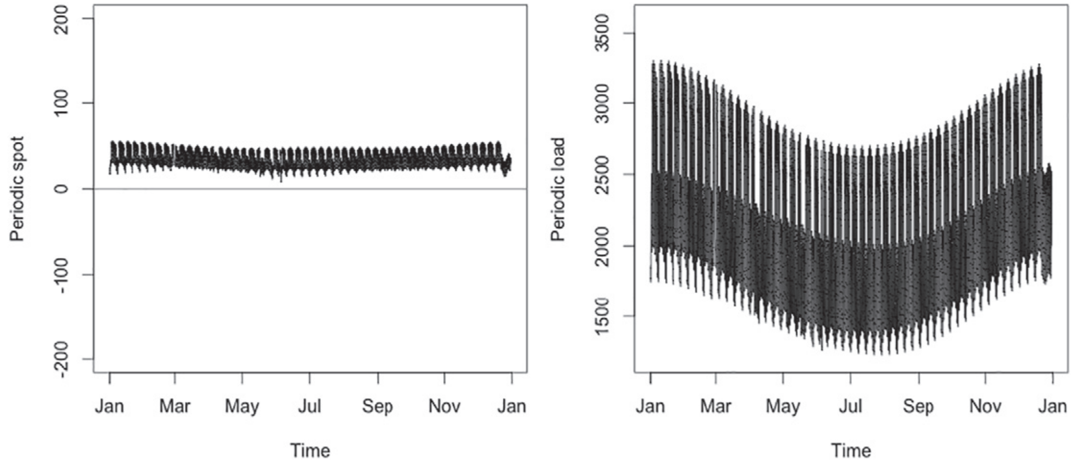

Fig. 3. The figures show seasonality functions fitted to the spot price (left) and consumption (right) of the period 2012-01-01 to 2012-12-31.

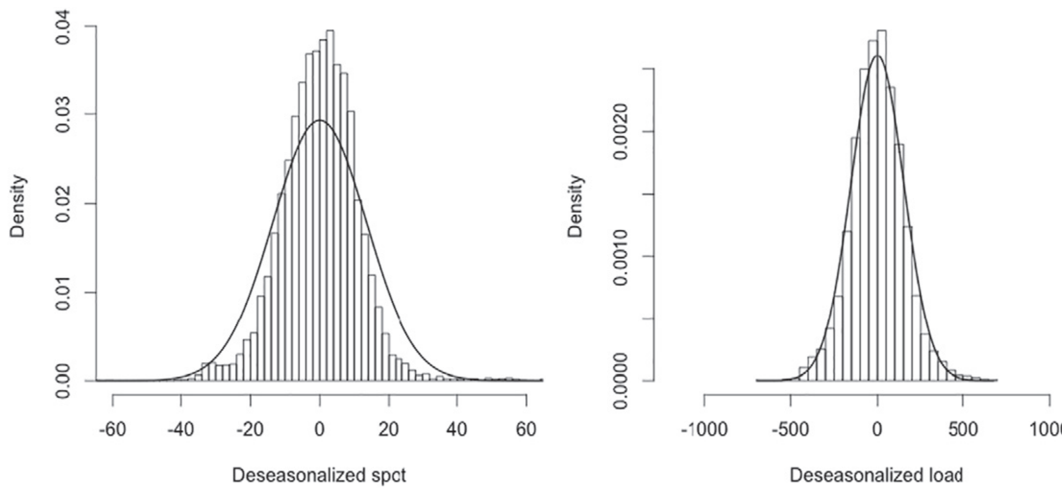

Fig. 4. Histograms of the de-seasonalised spot (left) and load (right) from the period 2012-01-01 to 2012-12-31. The solid lines are the fitted normal densities of the spot and load respectively.

To achieve a better fit for the seasonality functions, we separate weekends (including holidays) from weekdays (Monday to Friday) and fit two periodic functions separately to each of the two data sets.[7] The combined seasonality functions fitted to the load and spot price from the period 2012-01-01 to 2012-12-31 (Fig. 3) clearly capture a seasonal behaviour where the fitted function for the load is particularly reminiscent of the original data. We used five periods of 12, 24, 168 (one week), 4380 (half year) and 8760 (one year) hours for this purpose. Notice that the axes' scales of Fig. 3 are kept unchanged from Fig. 2 to highlight how the amplitude of fitted periodic functions compare to the amplitude of unfitted data. The residual processes (the de-seasonalised data shown in the middle panels of Fig. 5) are then fitted to a bivariate Ornstein-Uhlenbeck process with resulting estimated parameters and standard errors recorded in Table 2.

We do a rough investigation of the goodness-of-fit of our model before we continue with the hedging problem. The histogram of de-seasonalised spot-price data in Fig. 4 shows a decent fit to the normal distribution (the marginal distribution of the Ornstein-Uhlenbeck process) while the load data shows a strong fit. Next, if we compare simulated paths from the bivariate Ornstein-Uhlenbeck process (left panels of Fig. 5) with the de-seasonalised data (middle panels of Fig. 5), we find that the simulated processes are fairly reminiscent of the data we intend to model. Obviously, our model can not generate any jump discontinuities – the Ornstein-Uhlenbeck process and the periodic function are continuous functions – or jump-like extremes and thus the model fails to capture the jumps of the market data. This is particularly evident when we look at the simulated spot-spice (ie. the simulated Ornstein-Uhlenbeck process added to the fitted seasonality function, top right Fig. 5) compared to the original data (top Fig. 2), while it is not as evident for the load. However, as our ultimate objective is to find an efficient hedging strategy and not to find the best fitting time-series model, we are continue with our model while aware of some of its shortcomings.[8]

If we look at the sample autocorrelation functions of the de-seasonalised data and compare to the correlation function $e^{-\kappa\Delta}$ of the Ornstein-Uhlenbeck process, we find a mediocre match (left panel of Fig. 6). However, notice that the sample autocorrelation of the

[7] The market data is subject to wintertime changes (one "missing" hour) and summertime changes (one "additional" hour) that will incur an one-hour time shift of all time points during the summertime. To align the data with the 24-hour periodicity of the seasonal functions, we duplicate the last data-point of the winter-times and remove the last of summer-times.

[8] In Section Section 5.2 we will analyse how price spikes affect hedge performance.

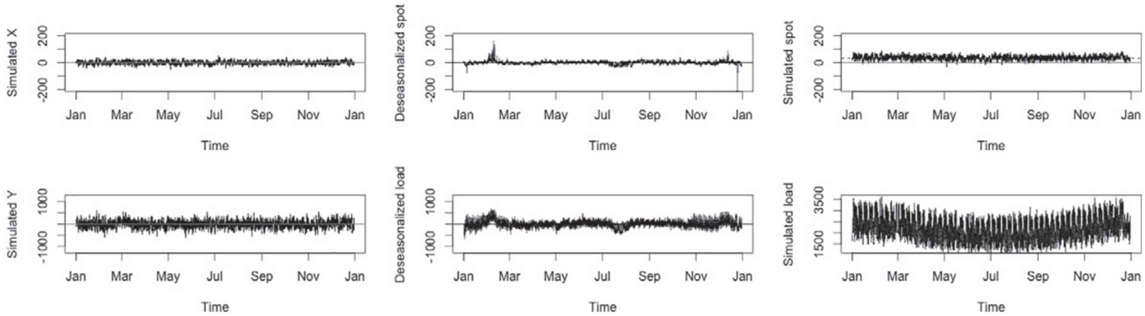

Fig. 5. Left figures show simulated sample paths from the bivariate Ornstein-Uhlenbeck processes which are used to model the de-seasonalised spot and load (middle figures). Right figures show simulated spot (top) and load (bottom) obtained by adding fitted seasonality functions to the simulated paths in the left figures.

spot and load (middle panel of Fig. 6) are quite similar to the sample autocorrelation functions of the simulated processes (right panel of Fig. 6). This is due to fact that the temporal dependency is partly captured by the seasonality functions.

Remark 4. When we derived an expression for the expected positive loss under the seasonal Ornstein-Uhlenbeck model (Section 3.2), we obtained a simpler, closed-form expression in the case when there is no cross-correlation between the de-seasonalised processes. We then compared to the dependent case with an example, and found that the two loss functions where close (in particular, their minimum was obtained at the same argument). An explanation for this is that the cross dependency is partly captured by the seasonality functions, partly by the Ornstein-Uhlenbeck process (just as for the temporal dependency). Indeed: for the 2012 data, the empirical cross-correlation is 0.37 of the de-seasonalised price and load, and 0.93 of the respective seasonality functions. This means that we might hedge well enough with the closed-form expression of the uncorrelated model specification, since much of the cross dependency is still captured by the seasonality functions.

4.2. Empirical hedging experiment

We set up an out-of-sample empirical experiment for the DK1 market to hedge 24 fixed-price agreements with monthly expires during 2013 and 2014. At initiation time t_0, set to be approx 15 days prior to the first day of the expiry month, we enter a fixed price agreement with reference load according to the consumption data presented in previous section and a fixed price with zero margin according to the corresponding t_0 market forward price of Table 1. To hedge the contract, we enter forwards (on the same expiry month) with hedge positions decided from either the minimum expected loss functions (the *model hedge strategies*) or from the benchmark *average-load strategy* which takes the average load of the contract month as the forward position. If the spot and load are uncorrelated, hedging at the expected load gives the minimal variance position — a defining property of expectation. Thus, it is a robust approach commonly used by the industry, but it does still leave the expected load to be specified by the user. A range of proprietary models are used by practitioners to estimate the expected load, some of which are probably more accurate than our Ornstein-Uhlenbeck model. In order not to stick our hands in a wasps' nest, we simply use realised average loads from the actual contract months. In effect, this also gives an upper bound on minimal-variance hedge performance.

For the experiment, we rescale the load data with the maximum load to improve stability of the numerical optimisation and to obtain a time-series that corresponds to the rescaled load data from DONG Energy. This just means that our hedging positions $(V^{\mathrm{b}}, V^{\mathrm{p}})$ are expressed as percentages of the maximum load. To begin with the January-13 contract, we employ six months of spot and rescaled load data up until t_0, that is 2012-06-18 to 2012-12-17, to estimate the seasonal Ornstein-Uhlenbeck model in the first step (parameters recorded in Table 6 in the Appendix). We use three periods of 12, 24 and 168 h: Including periods of a half/one year will improve the fit within the estimation set, while extrapolation from long periods will impair the out-of-sample fit, ie. the prediction of the periodic function throughout the expiry month.

To demonstrate with the positive loss measure, we calculate the expected loss based on estimated parameters as a function of peak- and base forward positions in the hedge portfolio (Fig. 7) and find the argument $(V^{\mathrm{b}^*}, V^{\mathrm{p}^*}) = (0.361, 0.340)$ that achieves the minimum-loss. Note that, as the load data is rescaled with the maximum load, V^{b^*}, V^{p^*}, are percentages of the maximum load as well. The realised payoff from the optimally hedged position is then calculated for each hour t_i of the expiry month M as

$$
\begin{aligned}
\pi_{t_i} &= (S_{t_i} - F^{\mathrm{b}})(V^{\mathrm{b}*} - L_{t_i}), \quad t_i \in M(op), \\
\pi_{t_i} &= (S_{t_i} - \tilde{F})(V^{\mathrm{b}*} + V^{\mathrm{p}*} - L_{t_i}), \quad t_i \in M(p),
\end{aligned}
$$

where S_{t_i} and L_{t_i} are the realised spot-price and (unscaled) consumption while the optimal forward positions have been scaled back with the maximum load. The results for the January-13 contract are shown in left Fig. 8.

To verify the results of the model hedge, we calculate the average-load, $(\bar{V}^{\mathrm{b}}, \bar{V}^{\mathrm{p}}) = (0.632, 0.222)$ from the contract month, (percentage of maximum load) and show the realised hourly payoff in the right-hand panel of Fig. 8. The average-load strategy means holding forward positions corresponding to the expected load as obtained by minimising the quadratic loss function under zero correlation.

The most notable feature from the hourly payoffs of the model hedge and average-load hedge (Fig. 8) is that the model hedge yields an average payoff per hour that is positive at 3208 EUR, while the average-load hedge yields a negative average hourly payoff of −2192 EUR. The negative average payoff inexorably leads to a negative accumulative payoff at expiry for the January-13 contract – see left panel of Fig. 9 – while the model hedge dispenses a positive net profit-and-loss at the expiry. On the other hand, the standard

Table 2
Estimated parameters of the Ornstein-Uhlenbeck process from de-seasonalised data 2012-01-01 to 2012-12-31 with standard errors in parenthesis. The parameters are estimated by numerical optimisation of the bivariate maximum likelihood function, where estimates from the univariate maximum likelihood are employed as starting values for the optimisation.

	κ	θ	σ	ρ_w
Spot	0.12 (0.005)	0.00 (0.59)	6.36 (0.051)	0.32 (0.010)
Load	0.19 (0.007)	0.00 (5.45)	94.2 (0.78)	

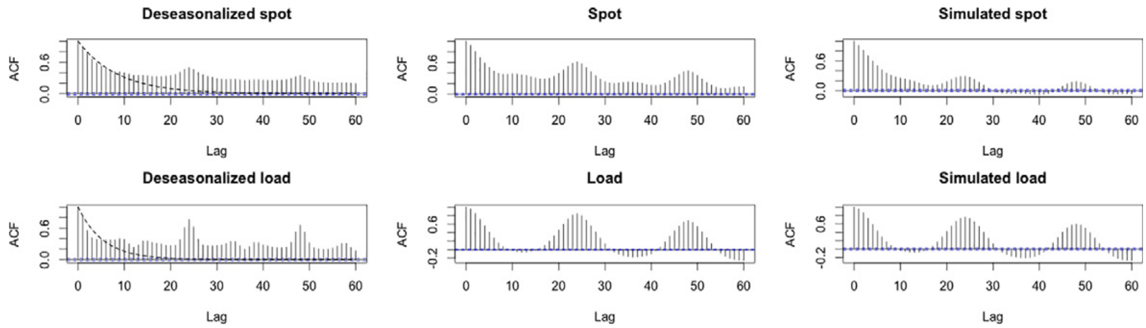

Fig. 6. Sample autocorrelation of de-seasonalised data (left figures). The dashed lines show the autocorrelation function of the Ornstein-Uhlenbeck process calculated with estimated parameters. Sample autocorrelation functions of spot and load (middle figures) and corresponding functions of the simulated spot and load (right figures).

deviation of hourly model-hedge payoff is 10,224 EUR while the average-load hedge yields a lower standard deviation of 4808 EUR. This means reducing the risk in classical terms with a lower variation of the outcome.

Further, if we calculate the optimal model hedge with the exponential loss measure we obtain an average hourly payoff 1859 EUR and standard deviation 8690 EUR. Compared to the linear positive loss, the exponential function penalises large losses more heavily which leads to a more risk-averse strategy with lower standard deviation and lower average of the realised payoff. Similarly, the quadratic loss measure realises an average payoff 525 EUR and standard deviation 7310 EUR per hour. Thus, we see a positive average realised payoff for the January-13 contract even if the quadratic measure is symmetric around zero. Still we have a higher variance than for the average-load: this is the trade-off we have to do for the benefit of higher average returns.

A second measure of efficiency for the hedging strategies is the probability of loss: 22.6% for the positive loss hedge, 23.1% for the exponential and 26.9% for the quadratic measure while it is considerable higher at 63.2% for the average-load hedge. In concrete terms, this means that we make a loss in 63.2% of the hourly payoff outcomes during January-13 with the average-load strategy while the model hedges yield losses for about 25% of the outcomes.

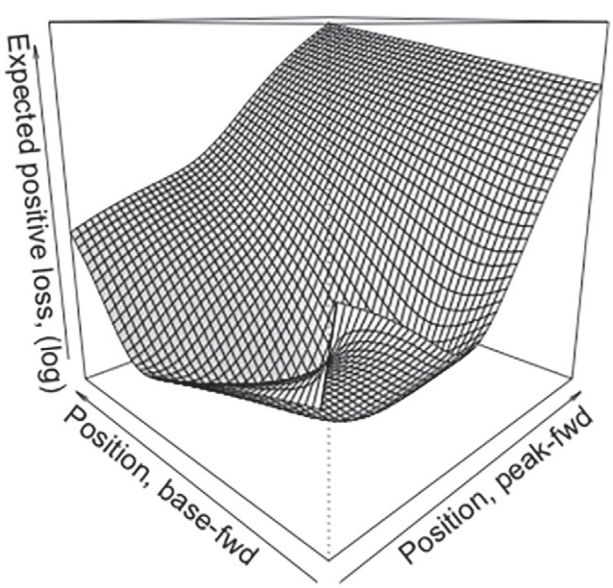

Fig. 7. Expected positive loss of the hedged position for the January-13 contract calculated from the seasonal Ornstein-Uhlenbeck model. The minimising argument $(V^{b^*}, V^{p^*}) = (0.361, 0.340)$ gives the optimal hedge (measured as percentages of maximum load).

4.3. Results

We now repeat the procedure above and determine the static hedges for all 24 contracts, thus obtaining the optimal peak/load forward positions reported in Table 3 for the model hedge with the positive loss measure and the average-load hedge, respectively. As for the January-13 contract, we see in Table 3 that monthly averages of hourly payoffs from the model hedge are positive for a majority of the contracts and higher (except for two contracts) than the average payoffs for the corresponding average-load hedge. This is natural as the purpose of the model hedge is to minimise the (expected) positive loss. We also observe that the standard deviation of hourly payoffs is higher for the model hedge for all but one contract (where the mean is still higher) when we compare to the average-load strategy. As we basically minimise the variance of the payoff (or the loss) with the average-load strategy, this should be expected as well.

The monetary effect of a relatively high hourly payoff with a higher standard deviation for the model hedge from the positive loss measure can be seen for the entire set of contracts in the top and middle panes of Fig. 10. The model hedge has a positive average hourly payoff of 821 EUR (when looking at the entire 24-month period) while the average-load has a negative average payoff per hour of −1107 EUR. In return, we observe from the quantiles that the model hedge yields a greater variation (a standard deviation of 5419 EUR while the average-load yields 3325 EUR). From the figures, it is notable that the negative part of the hourly payoff outcomes are fairly similar for the two strategies (with a few more large negative payoffs for the model hedge) while the set of positive outcomes is considerably reduced for the average-load hedge. If we compare the relative size of the hourly payoff from the strategies, we find that the model hedge yields the largest payout 66% of the time. This agrees with the probability of loss, which is lower for the model hedge for almost all contracts as recorded in Table 3. We can also observe from the hourly payoff distributions in the right panel of Fig. 9 that the model hedges yield positively skewed payoffs with fatter tails for the profitable part, while the average-load has a negativity skewed distribution with a relatively fatter tail for the losses.

Similar results are obtained with the model hedge based on the exponential measure: an average hourly payoff over all contracts of 559 EUR and standard deviation 4892 EUR. In terms of relative payoff sizes compared to the average-load strategy we have an out-performance probability of 63%. As for the January-13 contract, we thus see a somewhat lower average payoff and also a lower variation when comparing to the positive loss measure. With the quadratic loss we obtain a slightly negative average payoff of −34 EUR and standard deviation of 4258 EUR while the out-performance probability is 63% here as well. Hence we see a stronger effect from the symmetry of the measure with close to zero realised payoffs on the average but still a high out-performance rate when comparing to the

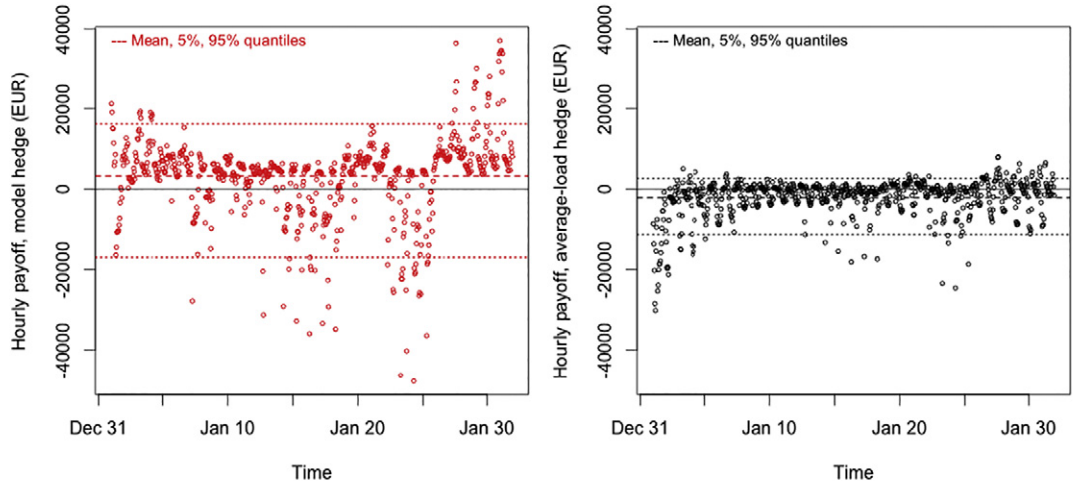

Fig. 8. Realised hourly payoff from the model hedge based on the positive loss measure (left-hand panel) calculated for the January-13 contract. The realised payoff from the average-load strategy (right-hand panel) has a lower (negative) average, while the variance of the average-load strategy's payoff is lower than the variance of the payoff from the model hedge.

average-load hedge. Optimal positions and payoff results for each of the 24 contracts are given in Table 4 for the exponential and quadratic measures.

In all, we have thus reduced the opportunity of large, positive payoff outcomes with the average-load strategy, for the benefit of a smaller variance. But this comes at a large cost in terms of profit. If we look at the month-end, accumulated profit-and-losses for all 24 contracts plotted in the bottom panel of Fig. 10, we find negative values for all contracts hedged with the average-load strategy, while in contrast the model hedge based on the positive loss yields a profit in the end of the month for most of the contracts (19 out of 24). The picture is similar for the exponential loss measure (profit for 18 out of 24 contracts) while the quadratic loss measure yields a profit at month-end for about every second contract (10 out of 24).

We now try to put the observations above in quantitative financial terms. First, we calculate reward-to-risk ratios (mean over standard deviation) of the realised hourly profit-and-losses. For the model hedges, the positive loss realises the highest ratio 0.15, the exponential loss 0.11 and the quadratic loss −0.008 while the average-load hedge yields a (negative) ratio of −0.33. One way of compensating for poor reward-to-risk performance is to charge a safety margin (or mark-up) m to the contract price, such that the total realised payoff adds to

$$\sum_{t_i \in M(op)} \pi_{t_i} + \sum_{t_i \in M(p)} \pi_{t_i} + \sum_{t_i \in M} mL_{t_i}.$$

We can then calculate a margin certainty equivalent that makes two strategies have the same realised reward-to-risk ratio. A margin of $m = 0.72$ EUR/MWh must be added to the average-load strategy to obtain a reward-to-risk ratio of 0.15 — the ratio realised by the model hedge based on the positive loss measure. In financial terms, this gives roughly $0.72 \times 30 \cdot 24 \cdot 2240 = 1.16$ MEUR per month,

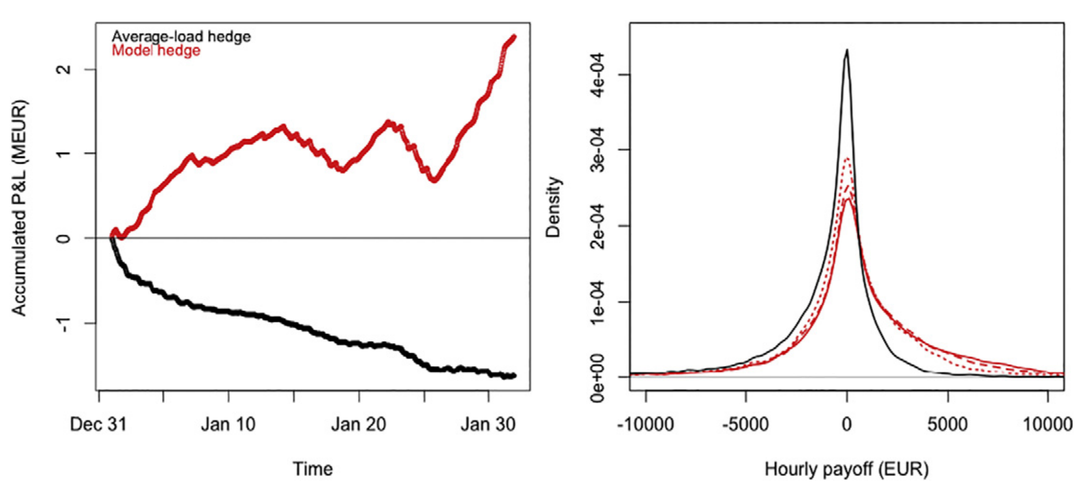

Fig. 9. Left: Accumulated profit-&-loss in million euros of the hedged position for the January-13 contract. The model hedge from positive-loss (red) yields a month-end profit of 2.39 MEUR while the average load-hedge (black) yields a loss of −1.63 MEUR. Right: Estimated density of the hourly payoff of the hedges from all 24 contracts. The black line shows the density from the average-load hedge (avg. −1107, sd. 3325) while the red lines show the model hedges: solid from the positive loss function (avg. 821, sd. 5419), dashed from the exponential loss (avg. 559, sd. 4892) and dotted from the quadratic loss measure (avg. −34, sd. 4258).

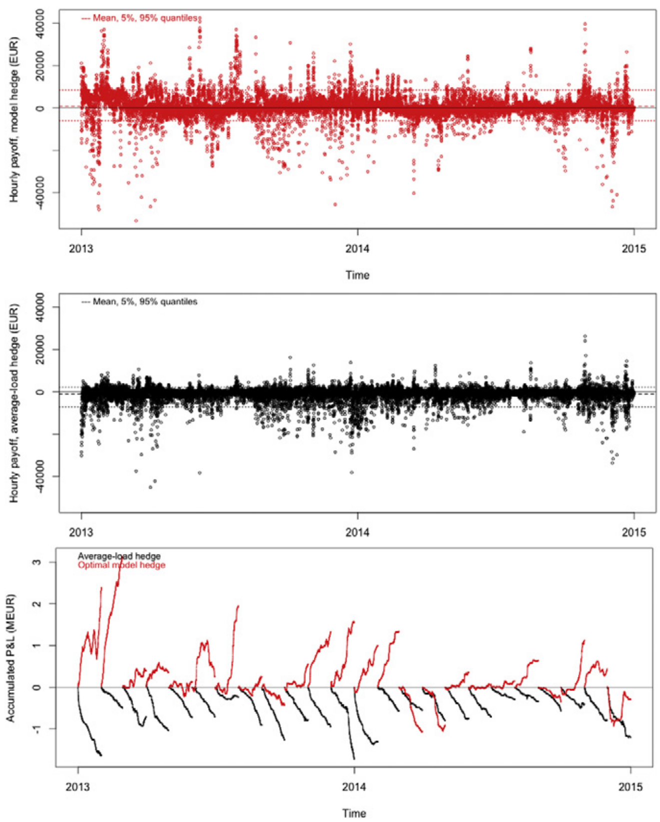

Fig. 10. Top panel: Realised hourly payoffs of the model hedge based on minimising the positive loss measure for the entire set of contracts. Middle panel: Realised hourly payoffs for the average-load strategy. Bottom panel: accumulated monthly profit-and-loss in million euros of the strategies for all 24 contracts. The model hedge (red) yields a higher month-end profit-and-loss for all but two contracts.

where 2240 MWh is the average load and 30·24 is the number of hours per month. Considering that the month-end profit-and-losses are in the range of −1–3 MEUR, (see bottom panel of Fig. 10), this is a substantial premium. Differently put, a margin of 0.72 would add about 1.7% to the price that the energy company charges the fixed price agreement customer. A similar analysis for the exponential loss function-based hedges leads to a mark-up of 1.3%.

Remark 5. In Remark 4, we noted that a specification with zero cross-dependency between the de-seasonalised processes could potentially perform as well as the original specification with the positive loss measure. If we repeat our hedging experiment with the independent specification (employing the closed-form expression for the expected positive loss), we obtain an average hourly payoff of EUR 742 and a standard deviation of 5259. This gives a reward-to-risk ratio of 0.14, compared with 0.15 for the dependent specification, while the out-performance probability is 65.5%.

Finally, when we repeat the empirical experiment with the proprietary set of consumption data from DONG Energy (shown in Fig. 10 in the Appendix) we obtain similar results as for the market data — see Table 4. For realised hourly payouts, the model hedge based on the positive loss measure out-performs with a 70% probability on an hourly basis and in every single one-month period, see Fig. 13 in the Appendix. For the reward-to-risk ratios, we have 0.14 for the model hedge and −0.30 for the average-load which corresponds to a margin certainty equivalent of 0.65 EUR/MWh.

5. Quantitative robustness checks

5.1. Is there extra information in forward-prices?

A reasonable question is: "Why do you not need, or why do you not use a model of the joint stochastic dynamic behaviour of spot- and forward-prices?" For the 'use' part: We only consider static hedges and use forward contracts whose expiries match the horizons in the risk-minimisations. On the dates where we initiate the hedges, we use the observed market forward-price — and ignore whence it came. At expiry dates, the forward contracts pay spot minus the forward-price from initiation date independently of any model. Thus we can set up operational hedges and empirical tests without reference to a dynamic model of forward-prices. If that beats the benchmark, *we're good*. Or, and here we get to the 'need' part, are we really? What we implicitly assume, or subtly hope, is that the forward-price at hedge initiation does not contain any information about the distribution of the future spot-price (for instance its mean) that is not already captured by the state variables. To investigate whether this "no extra information in forwards" -assumption is reasonable for our hedge purposes, we run some regression analyses. In the following 'spot' means average DK1 area base price, 'forward' means the system base forward-price adjusted by the certificate for difference-price. The data covers the period 2012–2014.[9]

Fig. 11 shows the results at a glance. The top left panel gives lagged de-seasonalised spot-prices (x) against their first differences (y). The depicted regression equation is $y = -0.62x - 0.09$ with a highly significant x-coefficient ($p = 0.001$) which, again, tells us that there is mean-reversion in spot-prices. In the top right panel we show spot-prices (x) against 1-month forward-prices (y). The black circles are (spot(t), forward(t)), the red circles are (spot(t), forward($t-1$)), i.e. realised spot against lagged forward. The dashed curve is a 45-degree line. The forward-on-spot regression equation is $y = 0.59x + 16.6$, which demonstrates the aforementioned contango. In the bottom left panel, we have plotted lagged forward-on-spot residuals from the top right panel (x) against unexplained-spot-changes residuals from the top left panel (y). The regression equation is $y = -0.04x - 0.35$, but neither coefficient is anywhere near significant (p-values 0.84 and 0.64). This shows when predicting the future spot-price, the current forward-price adds no information that isn't already in the current spot-price.[10] Finally, and arguably most importantly, the bottom right panel reports the lagged (spot, forward)-residuals (x) against accumulated profit-and-losses from the hedge positions (y from the bottom panel in Fig. 10 in MEUR; only 2013–2014 data due to the out-of-sample nature of the hedge experiment). The regression equation is

$$\underset{(0.30)}{y = 0.005x} - \underset{(0.03)}{0.05}.$$

The insignificance of the x-coefficient indicates that forward-prices add no extra information for the hedge portfolios we consider.

5.2. A simple approach for including price spikes

We mentioned in Section 4.1 that our continuous Gaussian model could not fully capture an apparent jump-like behaviour of the market price. Indeed, the "spikiness" of electricity spot-prices is a feature

[9] Similar results are obtained for peak prices. Also, for simplicity the load has been left out of the analyses in this subsection. Including it does not alter the results.
[10] Including squares of forward-prices or the like gives similar results.

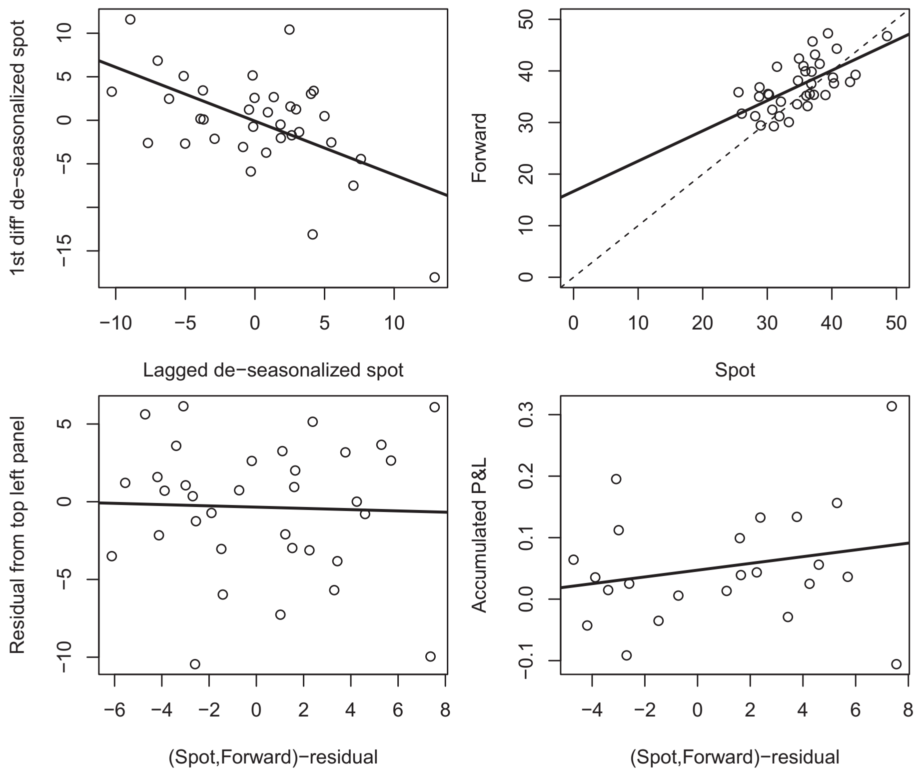

Fig. 11. Top panels: Scatter plots of spot- and forward-prices. Bottom panels: Using (spot, forward)-residuals for predicting spot-price changes and profit-and-losses from hedges.

commonly reported in the literature, see for instance Janczura and Weron (2010), Escribano et al. (2011) and Haugom (2011).

The get a rough idea of how our hedging approach conducts under a model that accommodates for spikes in the spot price, we make the following simple adjustment to the distribution of the seasonal model (cf. Eqs. (8)–(9))

$$S_T|S_0 \overset{d}{=} \mu_S(T,T) + \eta_S(T)Z + B\left(\mu_J + \sigma_J Z^J\right)$$

where B is an independent trinary random variable taking values {−1,1, 0} representing the occurrence of a negative/positive/no spike at time T while Z^J is an independent normal variable. In effect, the spot price follows a normal mixture distribution where a spike occurs independently with probability $p = P(B \neq 0)$ and with an independent spike size being normally distributed with mean $\pm\mu_J$ and variance σ_J^2. The merit of using this simple model is that the risk measure takes the form of a mixture

$$\begin{aligned}\mathbb{E}[u(-\pi_T)] = (1-p)\mathbb{E}[u(-\pi_T)|B=0] + p_+\mathbb{E}[u(-\pi_T)|B=1] \\ + p_-\mathbb{E}[u(-\pi_T)|B=-1]\end{aligned} \tag{13}$$

where p_+ and p_- are the probabilities of a positive and negative spike respectively, while the loss conditional on the occurrence of a spike, $-\pi_T|B$, is still a normal product. For simple and crude estimates of these parameters we identify a spike as a price change greater than four times their standard deviation. For the 2012–2014 market price, this yields 338 identified price spikes, that is, a probability $p = 1.29\%$ for the event with 60% chance of a positive and 40% of a negative spike respectively. The identified spikes have absolute sizes with mean $\mu_J = 28.0$ and $\sigma_J = 15.6$. Thus, about 10 spikes should have absolute sizes greater than ~60 which agrees with an inspection of Fig. 2.

When we repeat the hedging experiment for all 24 contracts we carry out the same steps as described in the previous section except that we now have an additional first step where we estimate probabilities and moments of spike events. Here, we use twelve months of spot data up until initiation t_0. We then remove all identified spikes from the estimation set used for estimating the seasonal Ornstein-Uhlenbeck model (six months of spot and load data up until t_0) before the risk measure including spikes (Eq. (13)) is minimised for the model hedge position. For the positive loss measure, we obtain an average hourly payoff 800 EUR, standard deviation 5418 EUR (reward-to-risk 0.15) and out-performance probability of 66% — thus the overall results are very similar to the continuous specification of the model (avg. 821 EUR, sd. 5419 EUR, ratio 0.15 and 66% out-performance). With the exponential loss measure we obtain slightly increased average payoff 610 EUR and standard deviation 4990 EUR which yields a ratio of 0.12 to be compared with 0.11 for the continuous model, while the out-performance probability is unchanged at 63%. Similarly, the quadratic measure yields an average −58 EUR, standard deviation 4251 EUR and ratio −0.014 which is slightly worse than before even if the out-performance probability is unchanged.

6. Conclusion and future research

In this paper, we derive model based optimal hedges for fixed price agreements in electricity markets and we test their empirical performance relative to an industry benchmark.

Two complicating factors are in play: contango and wrong-way risk. Our analysis shows that the former is the most important with the caveat that seasonality, which is easily confused with correlation, must be accounted for. Model based hedges come in several guises depending on the choice of loss function. In a period-by-period comparison, we find that the model hedges out-perform the benchmark every two out of three hours, and for almost every one-month period. Asymmetric loss functions, which punish losses but reward gains (to a smaller degree), lead to hedge portfolios with higher reward-to-risk ratios than the benchmark. For the benchmark to achieve similar performance, the energy company needs to mark up the fixed price by 1.5%. Or differently put, changing from the benchmark to a model hedge would enable the company to charge 1.5% less from its customers without affecting the quality of its risk-management. That may not sound like a lot, but as we show it makes the difference between earning and losing money in the long run. An interesting question for future research from a business perspective is thus to combine model based hedging with a choice of the fixed price (say, a half a percent mark-down) that is optimal given a risk-reward trade-off criterion.

6.1. Caveats, weaknesses and future research

We would be amiss not to point to some of the issues that this paper leaves unresolved.

6.1.1. Jumps/spikes

Although the asymmetric hedges based on the Gaussian Ornstein-Uhlenbeck model performed better than the benchmark, the left panel of Fig. 4 clearly showed that the empirical accuracy of the model left something to be desired. In Section 5.2, we considered an extension in the form of a normal mixture model. That did not improve hedge performance. But many variations of spot-price and derivative pricing models exist in the literature, see Carmona and Coulon (2014) for a survey. An interesting extension of our analysis would be to consider models that account for volatility clustering and for cross dependencies between the price and load discontinuities. One possible direction for this is to consider Ornstein-Uhlenbeck processes driven by Lévy processes, as suggested by for instance Benth (2011) and Goutte et al. (2014). However, leaving the Gaussian assumption will result in more complex expressions for the loss measures and their optimisation, and perhaps more importantly, a challenging estimation problem when fitting the model to data.

6.1.2. Information in forwards

The analysis in Section 5.1 revealed no easily exploitable relation between current forward-prices and expected future spot-prices. In our experiments, we set up the forward hedge position just 15 days prior to the start of the fixed price contract. This is industry standard, but sceptics would say that that is too short a horizon for forward-prices to add information over spot-prices; if anything, use weather forecasts! (Wind contributes significantly to Nordic power generation.) Over longer horizons, an explicitly forward-looking quantity might improve hedge performance; this is widely seen in financial derivative markets, with implied volatility being the forward-looking quantity; the literature ranges from Christensen and Prabhala (1998) to Ellersgaard et al. (2017).

6.1.3. Dynamic hedging and market frictions

A further extension would be to consider dynamic adjustments of the hedge strategies which would allow the energy company to adjust its position as new information arrives. That requires a joint dynamic model of spot-price, forward-prices and load, as well as some more intricate considerations about risk-measures (see Acciaio and Penner (2011) for a survey) — let alone how to optimise them. Finally, there could also be tactical reasons splitting up trading. If costs from market frictions — bid-ask spreads, transaction costs, or (here quite relevantly) adverse price effects from large trades — grow more rapidly than linearly in trade size, then it is advantageous to split one large trade – our static hedge as it were – into several smaller ones. Pedersen and Garleanu (2013) offer a framework for a deeper analysis for this matter.

Appendix A

Table 3
Positions for the peak- and base forwards for hedging of the fixed price agreements (measured in percentage of maximum load). The monthly average (standard deviation) of the hourly payoff (measured in EUR/maximum load) and the probability of loss.

	Positive loss minimisation				Average load			
Contract	V^b	V^p	Payoff	P (loss)	V^b	V^p	Payoff	P (loss)
Jan-13	0.361	0.34	0.89 (2.84)	0.23	0.632	0.222	−0.608 (1.33)	0.63
Feb-13	0.388	0.354	1.296 (1.33)	0.08	0.618	0.21	−0.204 (0.74)	0.56
Mar-13	0.512	0.363	0.146 (1.63)	0.41	0.607	0.169	−0.265 (1.11)	0.57
Apr-13	0.535	0.335	0.14 (1.6)	0.48	0.541	0.167	−0.403 (1.23)	0.61
May-13	0.585	0.301	0.05 (1.3)	0.57	0.504	0.161	−0.269 (0.51)	0.74
Jun-13	0.637	0.271	0.097 (2.08)	0.48	0.505	0.181	−0.271 (0.74)	0.67
Jul-13	0.641	0.26	0.729 (2.06)	0.39	0.484	0.149	−0.088 (0.47)	0.6
Aug-13	0.486	0.337	0.093 (1.23)	0.51	0.505	0.182	−0.338 (0.69)	0.7
Sep-13	0.438	0.31	−0.165 (1.57)	0.37	0.52	0.188	−0.488 (1.04)	0.72
Oct-13	0.443	0.294	0.209 (1.21)	0.34	0.544	0.201	−0.29 (0.95)	0.61
Nov-13	0.418	0.294	0.512 (1.33)	0.18	0.579	0.227	−0.365 (0.96)	0.63
Dec-13	0.441	0.3	0.584 (1.78)	0.25	0.587	0.202	−0.642 (1.51)	0.6
Jan-14	0.485	0.321	0.37 (1.15)	0.25	0.62	0.231	−0.487 (1.21)	0.56
Feb-14	0.486	0.345	0.553 (1.02)	0.27	0.605	0.201	−0.233 (0.87)	0.52
Mar-14	0.608	0.299	−0.394 (1.09)	0.76	0.567	0.199	−0.203 (0.76)	0.61
Apr-14	0.528	0.324	−0.352 (1.24)	0.66	0.526	0.179	−0.305 (0.79)	0.64
May-14	0.506	0.335	0.133 (1.1)	0.53	0.507	0.187	−0.278 (0.67)	0.68
Jun-14	0.493	0.338	0.023 (0.83)	0.51	0.519	0.199	−0.273 (0.62)	0.68
Jul-14	0.48	0.353	0.056 (0.56)	0.55	0.485	0.174	−0.108 (0.32)	0.61
Aug-14	0.407	0.351	0.24 (0.95)	0.3	0.51	0.202	−0.127 (0.61)	0.6
Sep-14	0.401	0.308	−0.135 (0.8)	0.44	0.519	0.201	−0.217 (0.49)	0.71
Oct-14	0.418	0.307	0.419 (1.32)	0.27	0.545	0.213	−0.151 (0.87)	0.59
Nov-14	0.453	0.319	0.168 (1.19)	0.29	0.582	0.213	−0.301 (0.93)	0.6
Dec-14	0.489	0.275	−0.108 (1.75)	0.61	0.599	0.187	−0.445 (1.27)	0.69

Table 4
Positions for the peak- and base forwards for hedging of the fixed price agreements (measured in percentage of maximum load). The monthly average (standard deviation) of the hourly payoff (measured in EUR/maximum load) and the probability of loss.

	Exponential loss minimisation				Quadratic loss minimisation			
Contract	V^b	V^p	Payoff	P (loss)	V^b	V^p	Payoff	P (loss)
Jan-13	0.421	0.294	0.516 (2.41)	0.23	0.482	0.248	0.146 (2.03)	0.27
Feb-13	0.472	0.282	0.763 (0.96)	0.10	0.504	0.259	0.558 (0.85)	0.15
Mar-13	0.553	0.295	0.022 (1.32)	0.45	0.553	0.257	−0.103 (1.29)	0.51
Apr-13	0.574	0.28	0.167 (1.44)	0.45	0.573	0.236	0.013 (1.25)	0.46
May-13	0.596	0.264	−0.025 (1.22)	0.56	0.594	0.221	−0.117 (1.04)	0.57
Jun-13	0.61	0.271	0.07 (1.82)	0.50	0.608	0.21	−0.091 (1.49)	0.5
Jul-13	0.604	0.263	0.622 (1.8)	0.40	0.603	0.194	0.42 (1.38)	0.4
Aug-13	0.51	0.302	0.017 (1.14)	0.59	0.532	0.239	−0.14 (0.89)	0.63
Sep-13	0.454	0.291	−0.207 (1.46)	0.45	0.498	0.225	−0.38 (1.16)	0.61
Oct-13	0.452	0.289	0.166 (1.16)	0.35	0.491	0.219	−0.028 (1)	0.49
Nov-13	0.42	0.295	0.502 (1.32)	0.19	0.474	0.223	0.197 (1.13)	0.27
Dec-13	0.457	0.269	0.461 (1.68)	0.25	0.484	0.2	0.263 (1.63)	0.27
Jan-14	0.505	0.299	0.245 (1.08)	0.27	0.516	0.229	0.172 (1.19)	0.27
Feb-14	0.516	0.303	0.367 (0.87)	0.36	0.525	0.243	0.353 (0.83)	0.3
Mar-14	0.586	0.313	−0.304 (0.97)	0.72	0.586	0.217	−0.287 (0.84)	0.69
Apr-14	0.556	0.282	−0.433 (1.16)	0.71	0.56	0.231	−0.429 (0.95)	0.69
May-14	0.542	0.291	0.106 (1.03)	0.54	0.553	0.24	−0.004 (0.8)	0.53
Jun-14	0.526	0.299	−0.022 (0.79)	0.54	0.544	0.24	−0.138 (0.67)	0.56
Jul-14	0.508	0.326	0.031 (0.59)	0.56	0.535	0.254	−0.032 (0.56)	0.57
Aug-14	0.406	0.348	0.239 (0.94)	0.29	0.508	0.248	−0.085 (0.62)	0.55
Sep -14	0.396	0.308	−0.139 (0.82)	0.44	0.492	0.226	−0.196 (0.53)	0.66
Oct-14	0.405	0.321	0.476 (1.4)	0.26	0.488	0.227	0.113 (0.99)	0.46
Nov-14	0.452	0.362	0.251 (1.17)	0.31	0.494	0.23	−0.088 (1.17)	0.43
Dec-14	0.493	0.269	−0.125 (1.73)	0.61	0.507	0.2	−0.285 (1.89)	0.64

Table 5
Nord-Pool system- and CFD forward prices (in EUR) for base- and peak contracts.

Notation time	Expiry	Base forward	Peak forward	Base CFD
2012-12-17	January-13	43.30	52.00	1.25
2013-01-17	February-13	45.30	52.35	0.60
2013-02-14	March-13	38.10	39.00	−0.20
2013-03-18	April-13	40.55	43.05	−3.40
2013-04-16	May-13	38.10	37.55	−1.50
2013-05-16	June-13	34.70	36.00	−1.15
2013-06-17	July-13	28.30	30.70	4.60
2013-07-17	August-13	36.95	40.00	1.95
2013-08-16	September-13	36.50	39.40	3.70
2013-09-16	October-13	38.90	44.00	1.30
2013-10-17	November-13	41.00	47.00	−0.70
2013-11-18	December-13	41.45	43.50	−4.05
2013-12-17	January-14	35.90	41.00	0.10
2014-01-17	February-14	36.30	41.30	−0.28
2014-02-17	March-14	29.00	33.05	0.90
2014-03-17	April-14	25.60	29.63	5.55
2014-04-16	May-14	24.50	28.50	5.85
2014-05-16	June-14	25.30	29.35	4.80
2014-06-17	July-14	22.99	25.80	7.80
2014-07-17	August-14	31.25	34.15	3.00
2014-08-18	September-14	32.70	35.70	2.80
2014-09-17	October-14	34.75	37.30	−0.15
2014-10-17	November-14	31.95	34.00	0.65
2014-11-17	December-14	33.00	34.80	−2.75

Table 6
Estimated parameters and standard errors of the Ornstein-Uhlenbeck process from rescaled de-seasonalised spot- and (scaled) load data.

Contract	κ_S	θ_S	σ_S	κ_L	θ_L	σ_L	ρ_W
Jan-13	0.103 (0.007)	0.000 (0.792)	5.42 (0.061)	0.065 (0.005)	0.000 (0.006)	0.025 (0.00027)	0.368 (0.013)
Feb-13	0.114 (0.007)	0.001 (0.896)	6.77 (0.076)	0.065 (0.005)	0.000 (0.006)	0.026 (0.00028)	0.291 (0.014)
Mar-13	0.121 (0.008)	0.000 (0.814)	6.53 (0.074)	0.093 (0.007)	0.000 (0.004)	0.027 (0.00004)	0.303 (0.014)
Apr-13	0.125 (0.008)	0.000 (0.79)	6.51 (0.073)	0.132 (0.008)	0.000 (0.003)	0.028 (0.00032)	0.31 (0.014)
May-13	0.107 (0.007)	0.000 (0.907)	6.42 (0.071)	0.133 (0.008)	0.000 (0.003)	0.028 (0.00032)	0.303 (0.014)
Jun-13	0.103 (0.007)	0.000 (0.917)	6.23 (0.069)	0.08 (0.006)	0.000 (0.005)	0.027 (0.00029)	0.311 (0.014)
Jul-13	0.099 (0.007)	0.000 (0.892)	5.82 (0.065)	0.06 (0.005)	−0.001 (0.006)	0.025 (0.00027)	0.259 (0.014)
Aug-13	0.086 (0.006)	0.000 (0.67)	3.82 (0.042)	0.053 (0.005)	−0.001 (0.007)	0.024 (0.00026)	0.348 (0.013)
Sep-13	0.085 (0.006)	0.000 (0.633)	3.56 (0.039)	0.055 (0.005)	0.000 (0.006)	0.021 (0.00022)	0.327 (0.014)
Oct-13	0.089 (0.006)	0.084 (0.608)	3.58 (0.04)	0.085 (0.006)	0.000 (0.003)	0.019 (0.00021)	0.285 (0.014)
Nov-13	0.091 (0.007)	0.000 (0.611)	3.69 (0.041)	0.148 (0.009)	0.000 (0.002)	0.02 (0.00023)	0.33 (0.014)
Dec-13	0.096 (0.007)	0.000 (0.638)	4.03 (0.045)	0.109 (0.007)	0.000 (0.003)	0.023 (0.00025)	0.324 (0.014)

(continued on next page)

Table 6 (*continued*)

Contract	κ_S	θ_S	σ_S	κ_L	θ_L	σ_L	ρ_W
Jan-14	0.110 (0.007)	−0.019 (0.616)	4.5 (0.051)	0.068 (0.006)	0.000 (0.005)	0.024 (0.00027)	0.32 (0.014)
Feb-14	0.103 (0.007)	−0.001 (0.716)	4.86 (0.055)	0.065 (0.006)	0.000 (0.006)	0.025 (0.00028)	0.28 (0.014)
Mar-14	0.096 (0.007)	−0.001 (0.774)	4.93 (0.056)	0.079 (0.007)	0.000 (0.005)	0.027 (0.00029)	0.262 (0.014)
Apr-14	0.106 (0.007)	0.000 (0.712)	4.98 (0.056)	0.113 (0.008)	0.000 (0.004)	0.028 (0.00031)	0.255 (0.014)
May-14	0.123 (0.008)	−0.001 (0.578)	4.69 (0.053)	0.143 (0.009)	0.000 (0.003)	0.028 (0.00031)	0.255 (0.014)
Jun-14	0.124 (0.008)	−0.001 (0.548)	4.49 (0.051)	0.098 (0.007)	0.000 (0.004)	0.026 (0.00029)	0.256 (0.014)
Jul-14	0.109 (0.007)	0.000 (0.558)	4.04 (0.046)	0.078 (0.006)	−0.001 (0.005)	0.024 (0.00027)	0.222 (0.015)
Aug-14	0.108 (0.007)	0.000 (0.492)	3.51 (0.04)	0.062 (0.006)	0.000 (0.006)	0.023 (0.00025)	0.255 (0.014)
Sep-14	0.109 (0.008)	0.000 (0.521)	3.75 (0.042)	0.091 (0.007)	0.000 (0.003)	0.02 (0.00022)	0.219 (0.015)
Oct-14	0.105 (0.007)	0.018 (0.475)	3.31 (0.037)	0.127 (0.008)	0.000 (0.002)	0.019 (0.00021)	0.207 (0.015)
Nov-14	0.123 (0.008)	0.000 (0.412)	3.36 (0.038)	0.16 (0.009)	0.000 (0.002)	0.019 (0.00022)	0.259 (0.014)
Dec-14	0.100 (0.007)	−0.001 (0.535)	3.54 (0.04)	0.12 (0.008)	0.000 (0.003)	0.023 (0.00025)	0.214 (0.015)

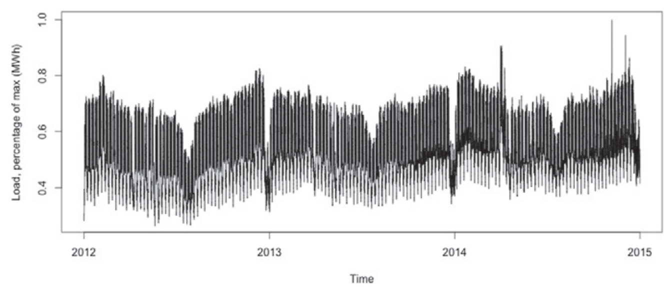

Fig. 12. The consumption data from DONG Energy. Note that the consumption has been anonymized and rescaled with the maximum load to lie between zero and one. The data is recorded with an hourly frequency and covers the period from 1 January 2012 to 31 December 2014.

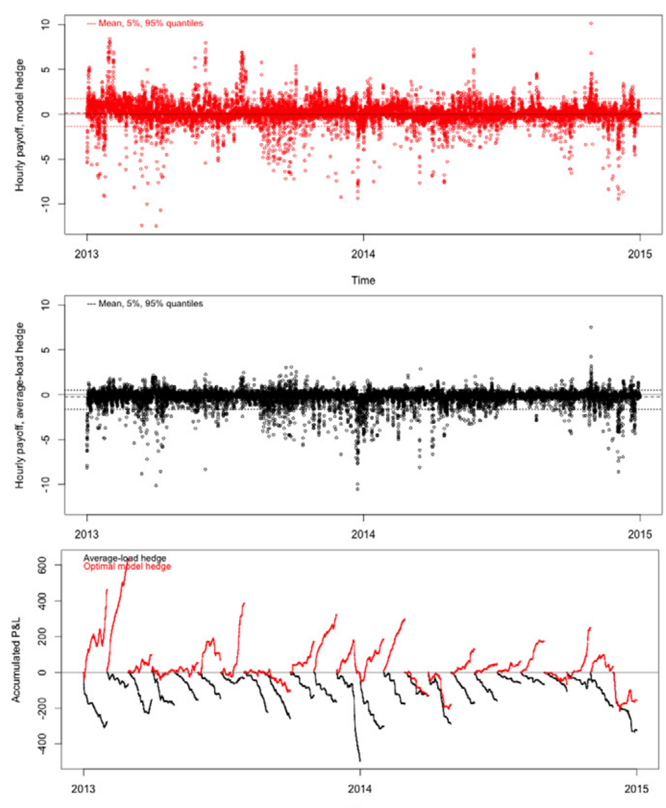

Fig. 13. Results for the consumption data from DONG Energy. Top panel: Realised hourly payoffs of the model hedge based on the positive loss measure. Middle panel: Realised hourly payoffs for the average-load strategy. Bottom panel: accumulated monthly profit-&-loss of the strategies for all 24 contract. The model hedge (red) yields a higher month-end P&L for every contract. All figures measured in EUR/maximum load.

Table 7
Results from the hedging experiment based on the consumption data from DONG Energy. Forward positions in percentage of maximum load while the monthly average (standard deviation) of the hourly payoff is measured in EUR/maximum load.

	Positive loss minimisation				Average load			
Contract	V^b	V^p	Payoff	P (loss)	V^b	V^p	Payoff	P (loss)
Jan-13	0.32	0.274	0.623 (1.98)	0.25	0.499	0.167	−0.371 (1.07)	0.6
Feb-13	0.349	0.279	0.949 (1.07)	0.1	0.512	0.169	−0.1 (0.56)	0.53
Mar-13	0.431	0.305	0.134 (1.32)	0.4	0.509	0.145	−0.206 (0.96)	0.55
Apr-13	0.452	0.281	0.015 (1.35)	0.46	0.492	0.142	−0.252 (1.04)	0.56
May-13	0.481	0.266	0.074 (0.84)	0.52	0.454	0.137	−0.208 (0.47)	0.66
Jun-13	0.509	0.249	0.094 (1.3)	0.51	0.458	0.156	−0.202 (0.68)	0.65
Jul-13	0.534	0.216	0.52 (1.45)	0.4	0.436	0.128	−0.044 (0.47)	0.51
Aug-13	0.436	0.266	−0.016 (0.92)	0.43	0.471	0.158	−0.301 (0.69)	0.65
Sep-13	0.402	0.259	−0.144 (1.39)	0.4	0.48	0.163	−0.355 (0.94)	0.65
Oct-13	0.411	0.256	0.234 (1)	0.32	0.493	0.163	−0.171 (0.76)	0.59
Nov-13	0.391	0.26	0.449 (0.94)	0.21	0.517	0.181	−0.232 (0.73)	0.59
Dec-13	0.412	0.259	−0.042 (1.38)	0.38	0.492	0.143	−0.666 (1.49)	0.57
Jan-14	0.45	0.272	0.25 (0.99)	0.26	0.555	0.18	−0.406 (1.06)	0.54
Feb-14	0.447	0.284	0.442 (0.71)	0.26	0.551	0.171	−0.258 (0.81)	0.57
Mar-14	0.526	0.267	−0.17 (0.8)	0.65	0.531	0.169	−0.177 (0.72)	0.60
Apr-14	0.475	0.273	−0.249 (1.21)	0.51	0.527	0.131	−0.398 (0.96)	0.65
May-14	0.472	0.279	0.172 (1.02)	0.51	0.48	0.131	−0.229 (0.57)	0.66
Jun-14	0.463	0.278	0.066 (0.73)	0.4	0.508	0.137	−0.218 (0.54)	0.65
Jul-14	0.466	0.286	0.081 (0.49)	0.52	0.471	0.118	−0.08 (0.26)	0.62
Aug-14	0.407	0.277	0.235 (0.68)	0.26	0.5	0.153	−0.092 (0.37)	0.61
Sep-14	0.406	0.239	−0.093 (0.73)	0.42	0.514	0.156	−0.145 (0.44)	0.65
Oct-14	0.435	0.236	0.338 (1.02)	0.29	0.523	0.158	−0.054 (0.7)	0.58
Nov-14	0.453	0.251	0.048 (0.91)	0.32	0.543	0.184	−0.265 (0.71)	0.65
Dec-14	0.485	0.214	−0.211 (1.31)	0.65	0.546	0.146	−0.435 (1.16)	0.70

Appendix B. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.eneco.2017.10.014.

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