A Theoretical Analysis of Momentum and Value Strategies

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1. Introduction

- Two of the most common strategies in active management:
  - **Momentum:** Buy recent winners / sell recent losers.
  - **Value:** Buy assets that are cheap relative to measures of fundamental value / sell assets that are expensive.
- Vast empirical literature documents that momentum and value strategies are profitable.
  - Short-run momentum: Jegadeesh-Titman (1993)...
  - Long-run reversal: DeBondt-Thaler (1985)...
  - Value: Fama-French (1992)...

Practice vs. Theory

- Design of momentum and value strategies has mainly been data-driven with little theoretical guidance.
- Guidance should ideally be based on model that:
  - Explains momentum and value effects.
  - Allows for multiple assets.

This Paper

- Analyze momentum and value strategies within model of Vayanos-Woolley (2011).
  - Momentum and value effects arise from flows between investment funds.
  - Rational investors and fund managers.
  - Multiple assets.

Main Results

- Closed-form Sharpe ratios (SR) for many active strategies.
- Momentum and value returns negatively correlated...

  ... yet, overall optimal SR significantly higher than of optimal combination of momentum and value.
- Momentum more sensitive than value to its implementation.
- Attractiveness of momentum relative to value decreases as investment horizon increases.

Related Papers

- Theories of momentum and reversal.
  - Behavioral: Barberis-Shleifer-Vishny (1998), Daniel-Hirshleifer-Subrahmanyam (1998), Hong-Stein (1999), Barberis-Shleifer (2003).
  - Rational: Berk-Green-Naik (1999), Johnson (2002), Shin (2006), Albuquerque-Miao (2010), Cespa-Vives (2011), Dasgupta-Prat-Verardo (2011), Vayanos-Woolley (2011).
  - Multiple assets only in BS and VW. BS study portfolio optimization but not correlation, implementation sensitivity, or horizon.
- Portfolio optimization with exogenous momentum and value effects.
  - Koijen-Rodriguez-Sbuelz (2006). One asset.

2. Overview of VW

Basic Mechanism

- An asset's fundamental value $\downarrow$

  $\Rightarrow$ Funds holding asset realize poor returns

  $\Rightarrow$ Outflows by investors updating negatively on managers' efficiency.

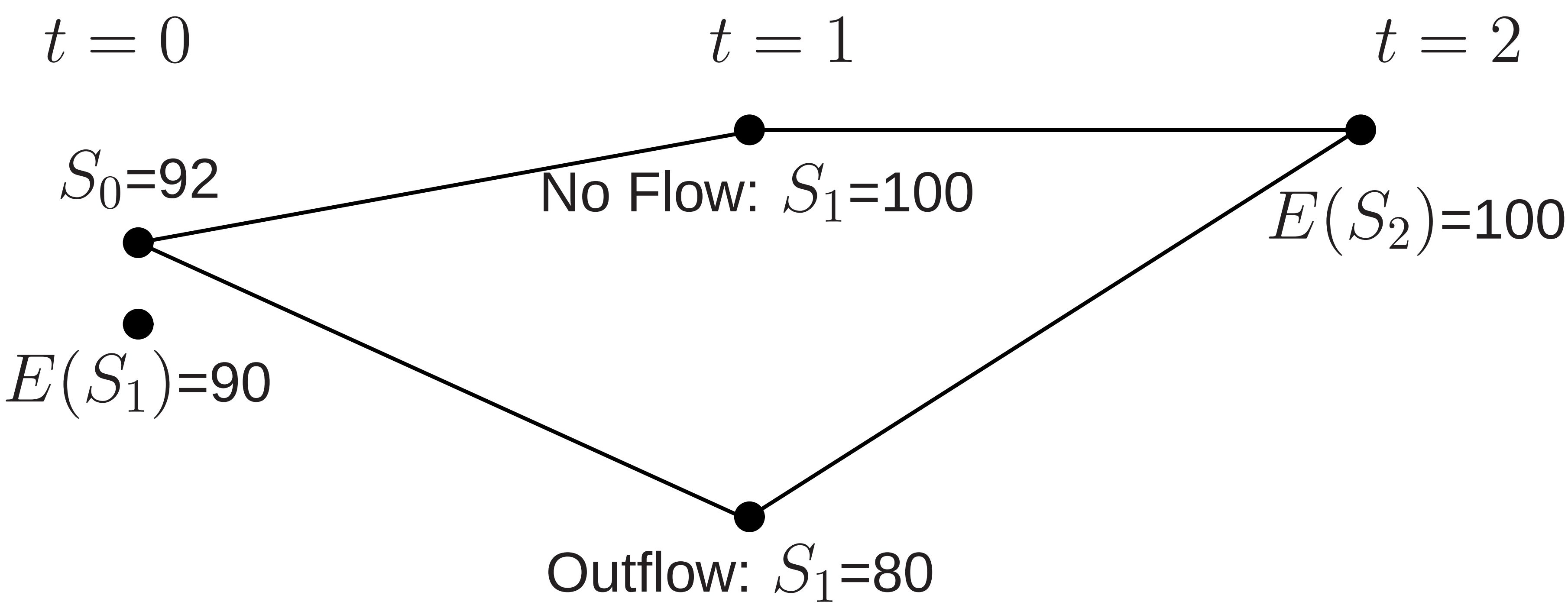
  $\Rightarrow$ Funds sell asset, further depressing price.

- If outflows are gradual, so is price drop $\Rightarrow$ Momentum.
- Price below fundamental value $\Rightarrow$ Reversal.

Bird-in-the-Hand Effect

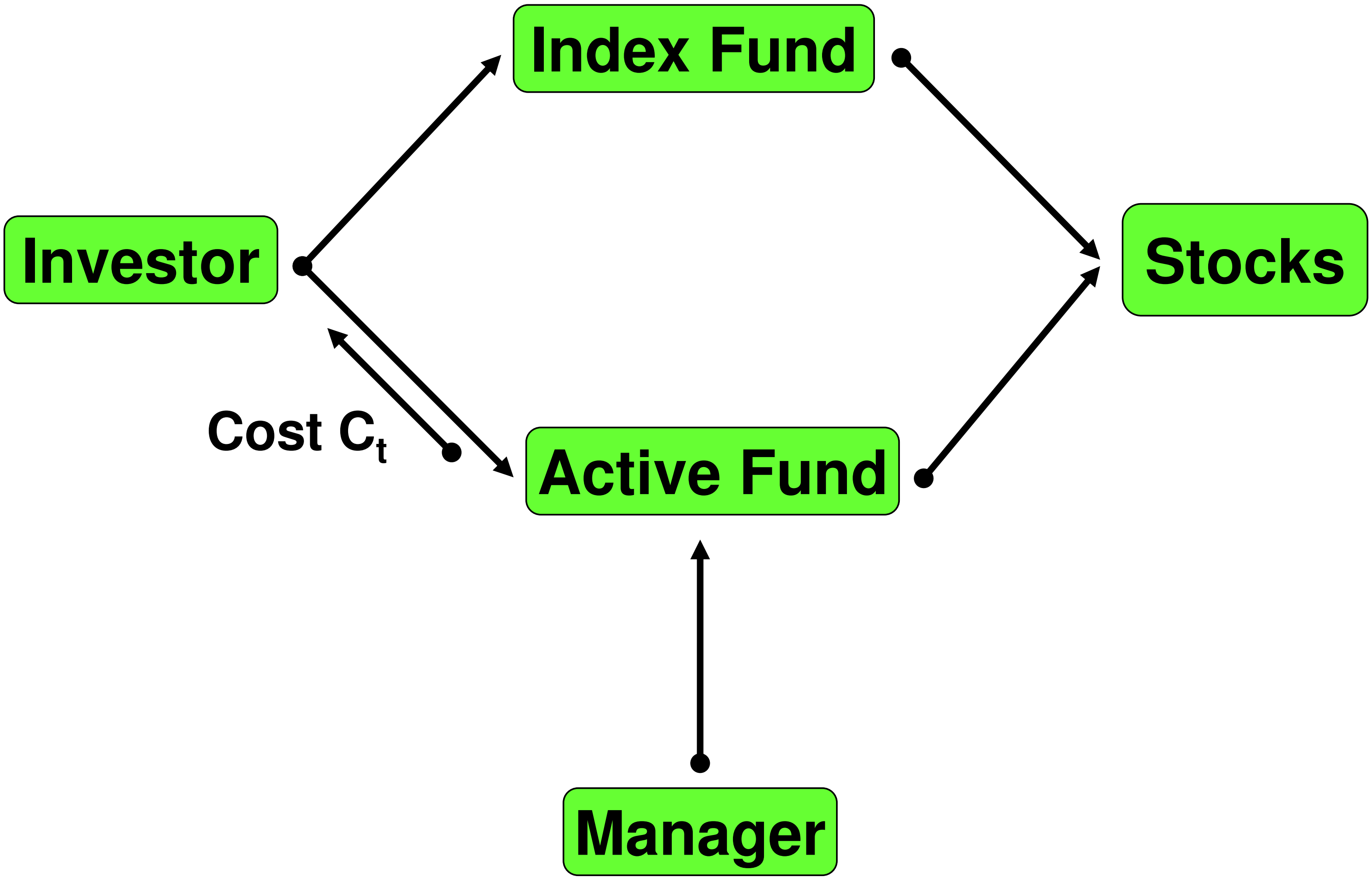
- Why do gradual outflows cause gradual price drop?
  - Why buy asset whose expected return has decreased?
- Buy asset now:
  - Attractive long-run return (because expectation of gradual outflows renders asset underpriced).
- Buy asset after outflows occur:
  - More attractive return, on average...
  - ... but risk that underpricing disappears.

Example



- Buy at $t = 0 \Rightarrow$ “Lock-in” expected return of 8.
- Buy at $t = 1 \Rightarrow$ “Gamble” for expected return of 20 or 0.
- Buying at $t = 0$ is attractive despite $E(S_1 - S_0) < 0$.

Flowchart



Assets

- Continuous time $t \in [0, \infty)$.
- Exogenous riskless rate r.
- N risky stocks (industry sectors, asset classes).
  - Supply $\pi \equiv (\pi_1, .., \pi_N)$ shares.
  - Endogenous prices S_t.

Cashflows

- Stocks' cumulative cashflows follow

$$dD_t = F_t dt + \sigma dB_t^D.$$

- Expected cashflows follow

$$dF_t = \kappa(\bar{F} - F_t)dt + \phi\sigma dB_t^F.$$

- Covariance matrix $\Sigma \equiv \sigma\sigma'$.

Index Fund

- Tracks index consisting of $\eta \equiv (\eta_1, .., \eta_N)$ shares.
- Index η differs from true market portfolio.
  - Differs from market portfolio π...
  - ... or exogenous buy-and-hold investors hold portfolio $\hat{\pi}$ different from π.
- Denoting by $\theta \equiv \pi - \hat{\pi}$ residual supply left over from buy-and-hold investors, η and θ are not collinear.

Investor

- Can invest in riskless asset, index fund and active fund.
- Holds x_t shares of index fund and fraction y_t of active fund.
- Maximizes expected utility of intertemporal consumption

$$-E\int_0^\infty \exp(-\alpha c_t - \beta t)dt.$$

Manager

- Chooses active portfolio.
- Can invest personal wealth in riskless asset and active fund.
  - Pins down manager's objective.
  - Manager acts as trading counterparty to investor.
- Holds fraction $\bar{y}_t$ of active fund.
- Maximizes expected utility of intertemporal consumption

$$-E \int_0^\infty \exp(-\bar{\alpha}\bar{c}_t - \beta t)dt.$$

Cost of Active Management

- Return of active fund to investor is net of a cost.
  - Return gap. (Grinblatt-Titman (1989), Wermers (2002), Kacperczyk-Sialm-Zhang (2008))
    - ∗ Managerial perk.
    - ∗ Reduced form for managerial ability.
- Flow cost is $C_t y_t$, where

$$dC_t = \kappa(\bar{C} - C_t)dt + s dB_t^C.$$

Equilibrium for $C_t = 0$

- Investor holds zero shares in index fund and

$$y_t = \frac{\bar{\alpha}}{\alpha + \bar{\alpha}}$$

shares in active fund.

- Expected returns are

$$E_t(dR_t) = \frac{r\alpha\bar{\alpha}}{\alpha + \bar{\alpha}} Cov_t(dR_t, \theta dR_t).$$

  - Constant over time.
  - Depend on covariance with true market portfolio.

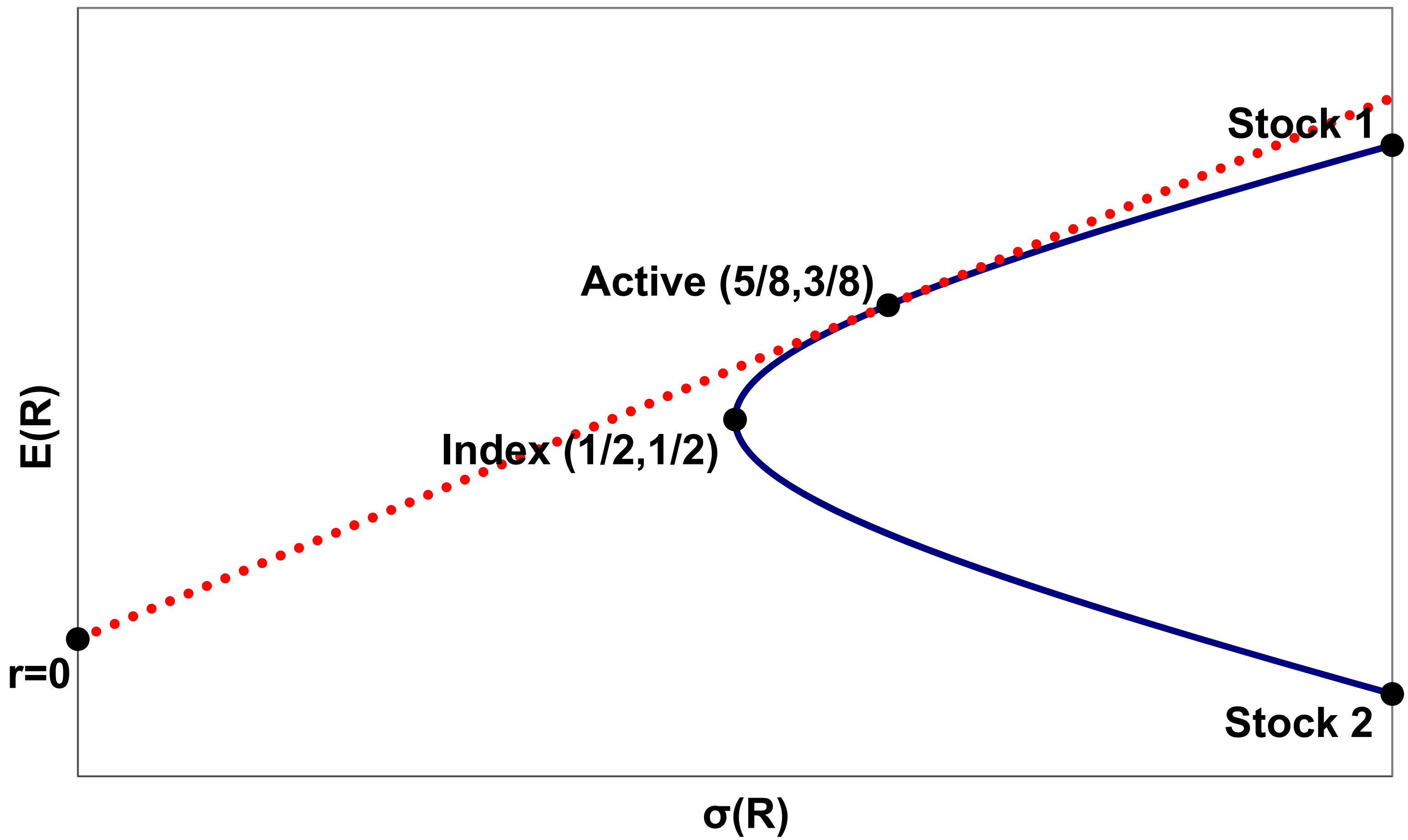
Time-Varying C_t: Benchmark Case

- Symmetric information: Investor and manager observe C_t.
- Instantaneous flows.

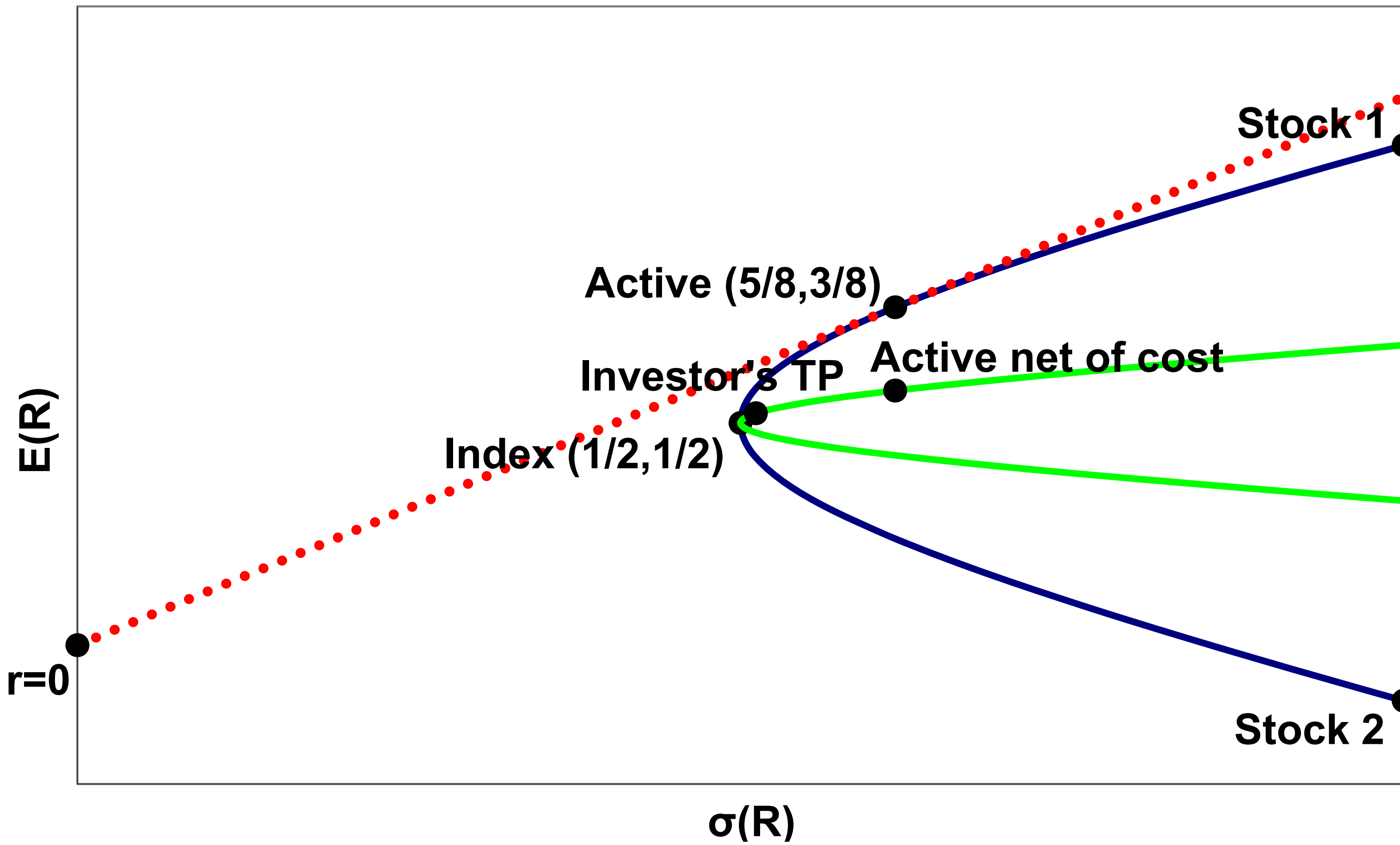
Example

- Two periods, two stocks, same variance.
- Index: (50,50) shares.
- True market portfolio: (50,30) shares.
  - Stock 1: Active overweight.
  - Stock 2: Active underweight.

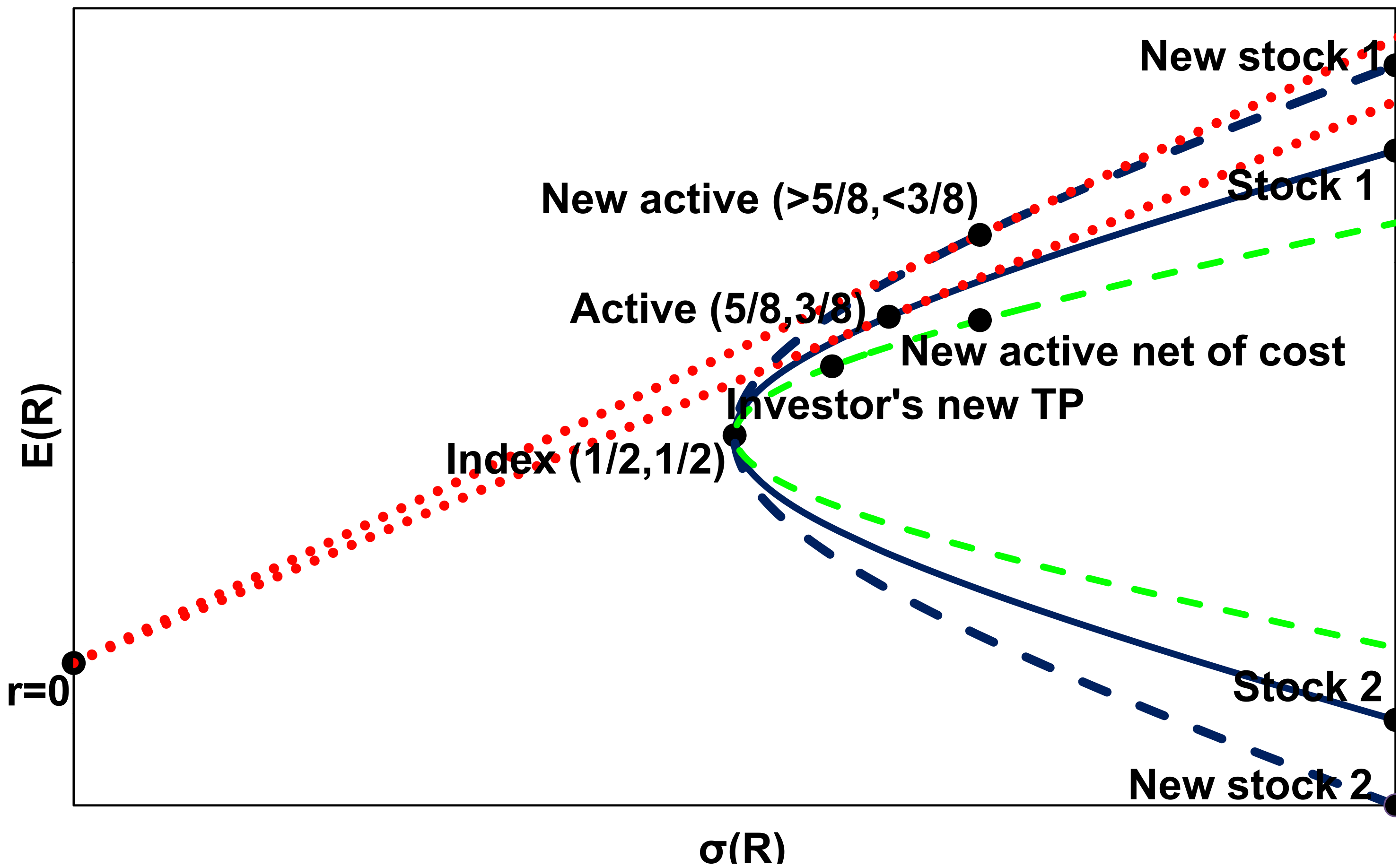
Equilibrium for $C_t = 0$



Increase in C_t



Price Adjustment



Flows

- Following increase in C_t, investor
  - Flows out of active fund.
  - Maintains constant overall index exposure.

  $\Rightarrow$ Sells slice of flow portfolio

$$p_f \equiv \theta - \frac{\eta\Sigma\theta'}{\eta\Sigma\eta'}\eta.$$

- Long positions in p_f: Active overweights.
- Short positions in p_f: Active underweights.

Prices and Expected Returns

- Following increase in C_t ($\Rightarrow$ outflows from active fund),
  - Stocks covarying positively with $p_f \downarrow$ and $E_t(dR_t) \uparrow$.
  - Stocks covarying negatively with $p_f \uparrow$ and $E_t(dR_t) \downarrow$.
- Reversal only.
- Comovement and lead-lag effects.

Time-Varying C_t: Enrichments

- Gradual flows.
  - Investor incurs flow cost $\frac{\psi}{2}\left(\frac{dy_t}{dt}\right)^2$ to adjust position in the active fund.
  - Institutional constraints or decision lags.
- Asymmetric information: Only manager observes C_t.

Equilibrium

- Investor believes that C_t is normal with mean $\hat{C}_t$.
- Equilibrium stock prices are

$$S_t = \frac{\bar{F}}{r} + \frac{F_t - \bar{F}}{r + \kappa} - (a_0 + a_1 \hat{C}_t + a_2 C_t + a_3 y_t).$$

- Investor's speed of adjustment is

$$\frac{dy_t}{dt} = b_0 - b_1 \hat{C}_t - b_2 y_t.$$

Prices and Expected Returns

- Following increase in $\hat{C}_t$ or C_t,
  - Stocks covarying positively with $p_f \downarrow$ and $E_t(dR_t) \downarrow$.
  - Stocks covarying negatively with $p_f \uparrow$ and $E_t(dR_t) \uparrow$.
- Following decrease in y_t,
  - Stocks covarying positively with $p_f \downarrow$ and $E_t(dR_t) \uparrow$.
  - Stocks covarying negatively with $p_f \uparrow$ and $E_t(dR_t) \downarrow$.
- Short-run momentum, long-run reversal.

Remainder of the Talk

- Calibrate model and use it to study performance of many active strategies.
- Evaluate performance at equilibrium prices.
  - Strategies are implemented by a “small” investor.

3. Trading Strategies and Sharpe Ratios

- Consider strategy consisting of $w_t \equiv (w_{1t}, .., w_{Nt})$ shares.
- Evaluate strategy w_t by annualized Sharpe ratio (SR)

$$SR_w \equiv \frac{E(\hat{w}_t dR_t)}{\sqrt{Var(\hat{w}_t dR_t)dt}}$$

of its index-adjusted version

$$\hat{w}_t \equiv w_t - \frac{Cov_t(w_t dR_t, \eta dR_t)}{Var_t(\eta dR_t)}\eta.$$

SR and Portfolio Optimization

- Consider investor with
  - Access to index η and strategy w_t.
  - Horizon dt and mean-variance preferences

$$E(dW_t) - \frac{a}{2} Var(dW_t).$$

- Investor can achieve maximum utility

$$\frac{E(\eta dR_t)^2}{2aVar(\eta dR_t)} + \frac{SR_w^2 dt}{2a}.$$

- Optimal strategy maximizes SR.

Per Share vs. Per Dollar Returns

- Per share and per dollar returns yield the same:
  - SR if conditional SR is constant over time.
  - Correlation if conditional correlation is constant over time.
- In the data:
  - Correlation for per share and per dollar returns differs by $\approx$ 2.5% for monthly stock returns and 10-year sample.

Computing SR

- Conditional expected returns given by two-factor model:

$$E_t(dR_t) = (\Psi \Sigma \eta' + \Lambda_t \Sigma p_f')dt.$$

  - Factor 1: Index η; constant premium $\Psi > 0$.
  - Factor 2: Flow portfolio p_f; premium affine in $(\hat{C}_t, C_t, y_t)$.

- Does w_t load highly on p_f when Λ_t is high?

Computing SR (cont'd)

- SR of w_t is

$$SR_w = \frac{\left(f + \frac{k\Delta}{\eta\Sigma\eta'}\right) E\left(\Lambda_t w_t \Sigma p_f'\right)}{\sqrt{f\left[E(w_t\Sigma w_t') - \frac{E[(w_t\Sigma\eta')^2]}{\eta\Sigma\eta'}\right] + kE[(w_t\Sigma p_f')^2]}},$$

where (f, k, Δ) are constants.

- SR is high if w_t loads highly on p_f when Λ_t is high.

Optimal Strategy

- Optimal strategy is

$$w_t = \Lambda_t p_f .$$

- How close to this strategy can momentum and value strategies get?

4. Calibration of Model Parameters

Assets and Portfolios

- Set $N = 10$.
  - Ten industry sectors.
- Set $\eta = \mathbf{1} \equiv (1, .., 1)$ and $\bar{\theta} \equiv \frac{\sum_{n=1}^{N} \theta_n}{N} = 1$.
  - Index includes one share of each stock.
  - True market portfolio includes an average of one share of each stock.
  - Normalizations: Redefine one share of each stock and of the index.
- Set $\sigma(\theta) \equiv \sqrt{\frac{\sum_{n=1}^{N}(\theta_n - \bar{\theta})^2}{N}} = 0.6$.
  - Active weight in typical industry sector differs from index weight by 60% (Kacperczyk-Sialm-Zheng 2005).

Cashflows

- Set $\Sigma = \hat{\sigma}^2(I + \omega \mathbf{1}'\mathbf{1})$.
  - Symmetric stocks.
- Set $\hat{\sigma} = 0.22$.
  - Market index η has SR=30%.
- Set $\omega = 7$.
  - Average correlation between industry sector and index returns is 87% (Ang-Chen 2002).
- Set $\phi = 0.3$.
  - Shocks to F_t relative to D_t. Small effect on calibration results.

Cost of Active Management

- Set $s^2 = 1.6$.
  - Top decile of mutual funds in terms of return gap earn monthly CAPM alpha 0.273%, while bottom decile earn -0.431% (KSZ 2008).
- Set $\kappa = 0.3$.
  - Shocks to return gap shrink to 1/3 of their size within 4 years.
- Set $\bar{C} = 0$.
  - Average return gap in cross section is 0.

Remaining Parameters

- Set $r = 4\%$.
- Set $\psi = 1.2$.
  - Impulse response of flows to performance peaks at one year (Coval-Stafford 2007).
- Set $\alpha = 1$.
  - Normalization: Redefine consumption units.
- Set $\bar{\alpha} = 30$.
  - Manager accounts for 3.2% =(1/(30+1)) of aggregate risk tolerance.
  - GDP share of the Finance and Insurance industry 5.5% on average during 1960-2007 in the US (Philippon 2008).

Independent Checks

- St. dev. of stock turnover generated by fund flows (s).
  - Lou (2011): 7% of assets managed by funds, quarterly.
  - Model: 7.8%.
- % of stock return variance generated by fund flows ($s, \bar{\alpha}$).
  - Greenwood-Thesmar (2011): 8% (sample includes $<$ 50% of funds).
  - Model: 16%.

5. Results

- Momentum strategies.
- Value strategies.
- Combining momentum and value.
- Long horizons.

Momentum Strategies

- Raw returns:

$$\left(w_t^M\right)' \equiv \int_{t-\tau}^{t} dR_u.$$

- Index-adjusted returns:

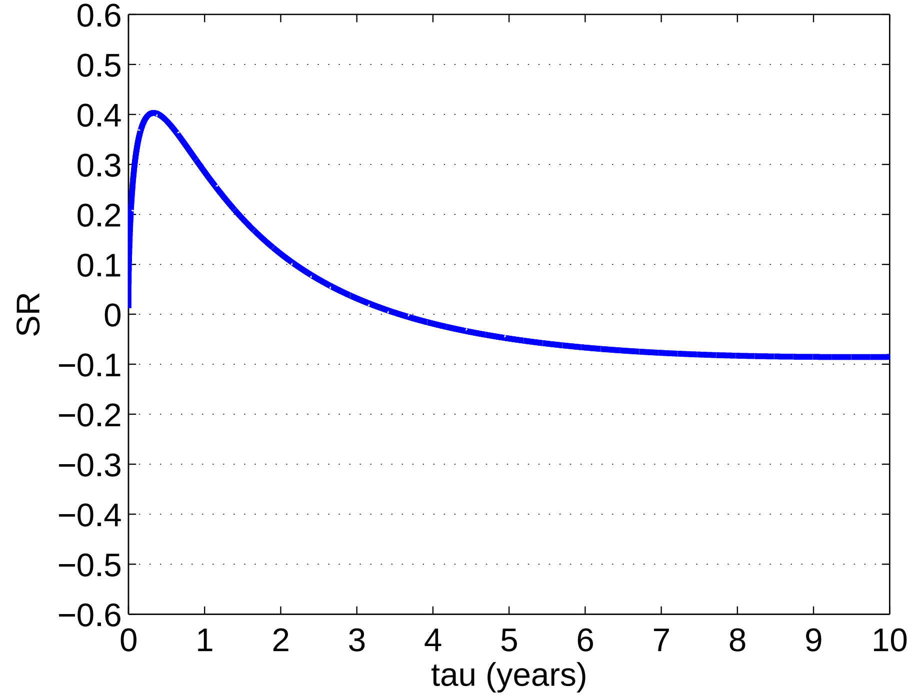
$$\left(w_t^{\hat{M}}\right)' \equiv \int_{t-\tau}^{t} d\hat{R}_u,$$

where $d\hat{R}_t \equiv dR_t - \frac{Cov_t(dR_t,\eta dR_t)}{Var_t(\eta dR_t)}\eta dR_t$.

- Equivalent for symmetric stocks.

- Lookback τ.

Momentum SR



- Maximum SR is 40% and is achieved for $\tau = 4$ months.
  - Empirically: 70% for individual stocks and 34% for country-level indices (Asness-Moskowitz-Pedersen 2009).

Sources of Momentum SR

- Conditional covariance.
  - A stock's higher-than-expected return predicts high future return.
- Covariance of conditional expectations.
  - A stock can have temporarily high expected return.
- Cross-sectional covariance of unconditional expectations.
  - Some stocks have permanently high expected return (Conrad-Kaul 1998).
- Calibration: 62%/36%/2%.

Value Strategies

- Raw prices:

$$\left(w_t^V\right)' \equiv \frac{\bar{F}}{r} + \frac{\epsilon(F_t - \bar{F})}{r+\kappa} - S_t.$$

  - $\epsilon = 1$: Can forecast cashflows as in model.
  - $\epsilon = 0$: Cannot forecast cashflows.

- Index adjusted prices:

$$\left(w_t^{\hat{V}}\right)' \equiv \frac{\bar{F}}{r} + \frac{\epsilon(F_t - \bar{F})}{r+\kappa} - \hat{S}_t,$$

where $\hat{S}_t \equiv S_t + E_t\left[\int_t^\infty \Psi Cov_{t'}(dR_{t'}, \eta dR_{t'}) e^{-r(t'-t)}\right]$.

Value SR

- 25.5% under optimal forecast; 26% under crude forecast.
  - Empirically: 36% for individual stocks and 34% for country-level indices (AMP).
- Crude forecast does not hurt SR!
  - Value less sensitive than momentum to its implementation.
  - Intuition: Forecast error helps predict expected returns. E.g., negative shock to expected dividends raises:
    - Weight under crude forecast.
    - Expected return.

Correlation Between Momentum and Value

- Momentum and value weights negatively correlated

  $\Longrightarrow$ Returns negatively correlated.

- Average momentum and value weights are identical (high for stocks with high unconditional expected returns)

  $\Longrightarrow$ Returns positively correlated.

- Which effect dominates?

Correlation Between Momentum and Value (cont'd)

- Calibration: Correlation=-3%.
  - Empirically: More negative (AMP).
- Diversification gains: SR of optimal combination is 48%.
- Yet, overall optimal SR is 61%, significantly higher.
  - Momentum and value strategies can be improved.
  - Use information on fund flows.
    - First principal component of flows?

Lagged Value

- Value strategy with lagged signal:

$$(w_t^V)' \equiv \frac{\bar{F}}{r} + \frac{\epsilon(F_{t-\tau} - \bar{F})}{r+\kappa} - S_{t-\tau}.$$

- Higher SR than with current signal:
  - Maximum for $\tau = 1$ year, and equal to 35%.
- Has element of momentum.
- When combined with momentum, SR same as with current signal.

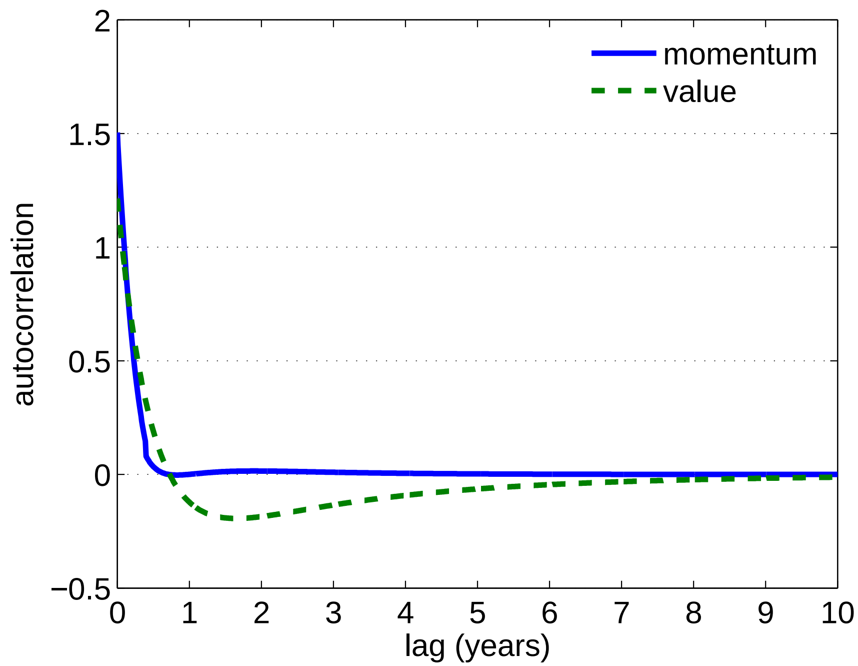
Longer Investment Horizons

- So far: evaluate risk and return for investor with infinitesimal horizon.
- Annualized SR for strategy w_t over horizon T is

$$SR_{w,T} \equiv \frac{E\left(\int_t^{t+T} \hat{w}_u dR_u\right)}{\sqrt{Var\left(\int_t^{t+T} \hat{w}_u dR_u\right) T}}.$$

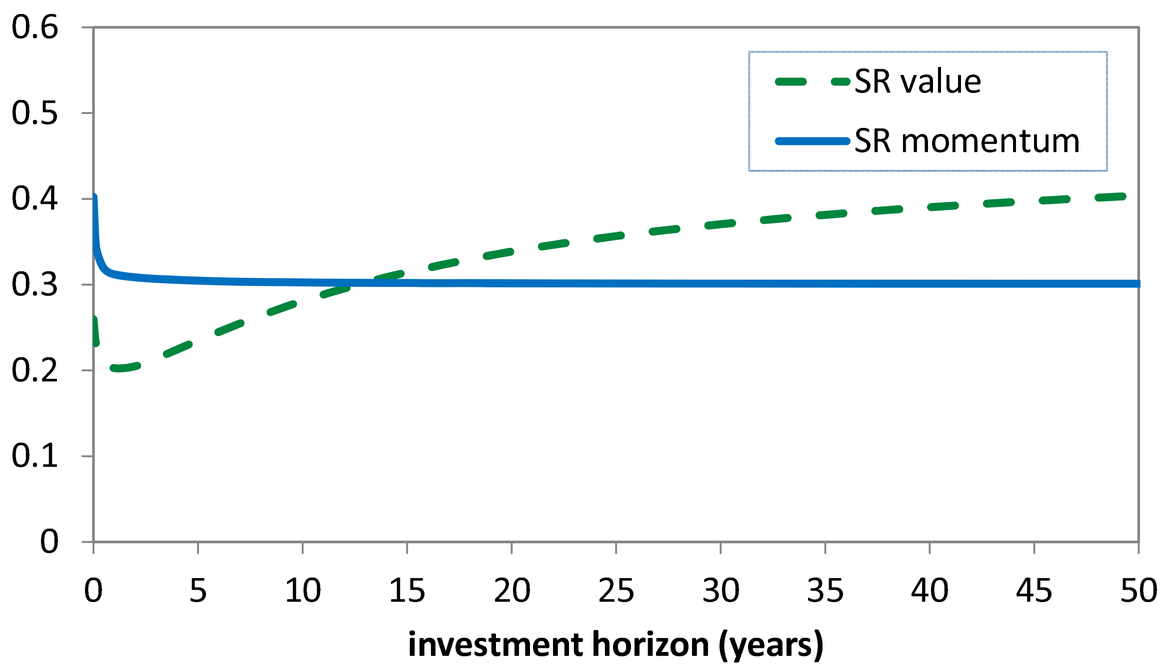
- Exceeds SR over infinitesimal horizon if autocovariances are negative, and vice-versa.

Autocovariance of Momentum and Value Strategies



- Momentum has small short-run momentum.
  - Weights change rapidly $\longrightarrow$ inherit only part of stock return momentum.
- Value has short-run momentum and long-run reversal.
  - Weights change slowly $\longrightarrow$ inherit both momentum and reversal.

Long-Horizon SR



- Long-run risk of momentum is sum of short-run risks.
  - Series of uncorrelated bets.
- Long-run risk of value is smaller than sum of short-run risks.
  - Expected return becomes higher following poor performance.

Other Results

- Lead-lag effects between momentum and value.
  - Small. Larger from value to momentum.
- Persistence of expected returns.
  - Larger for value.
- Continuous-rebalancing vs. buy and hold.
  - Not rebalancing for one year reduces:
    * Value SR by 1%.
    * Momentum SR by 30%.

6. Conclusion

- Momentum, reversal and value effects can arise from flows between investment funds.
- Tractable model to study these effects.
- Use model as laboratory to study performance of many active strategies.