# Part 3: Value, Investment, and SEO Puzzles

- Model of Zhang, L., 2005, "The Value Premium," JF.
  - Discrete time
  - Operating leverage
  - Asymmetric quadratic adjustment costs
  - Counter-cyclical price of risk
- Algorithm of Krusell, P. and A. A. Smith Jr., 1998, "Income and Wealth Heterogeneity in the Macroeconomy," JPE.
  - Solves for equilibrium with heterogeneous agents and aggregate risks.
  - Issue is how the cross-sectional distribution evolves in conjunction with the aggregate shock.
- Implications for the value premium.
- Implications for the capital investment and seasoned equity offering (SEO) puzzles.



# Zhang, JF, 2005:

- Analyze a single industry with price-taking firms.
- Constant elasticity demand: $P_t = Y_t^{-\eta}$.
- Output of firm $i \in [0, 1]$ is

$$Y_{it} = e^{X_t + Z_{it}} K_{it}^{\alpha}$$

  with $\alpha < 1$.
- $X$ represents systematic risk and $Z_i$ represents idiosyncratic risk.
- Operating leverage: operating cash flow is $P_t Y_{it} - C$ for a constant $C$.
- Asymmetric costly adjustment: investment cost is

$$h(I, K) = I + \begin{cases} K\theta^{+} \cdot (I/K)^2 & \text{if } I/K > 0\,, \\ K\theta^{-} \cdot (I/K)^2 & \text{if } I/K < 0\,, \end{cases}$$

  for constants $\theta^{-} > \theta^{+}$.

## Stochastic Discount Factor and Risks

- $M =$ SDF process, meaning $M_{t+1}/M_t$ is the date–$t$ SDF for pricing cash flows at $t+1$.
- Firm $i$ seeks to maximize

$$\mathsf{E} \sum_{t=0}^{\infty} M_t [\pi(X_t, Z_{it}, K_{it}) - h(I_t, K_t)] \,,$$

- Counter-cyclical price of risk: Assume

$$\Delta \log M_{t+1} = \log \beta - [\gamma_0 - \gamma_1 (X_t - \bar{X})] \Delta X_{t+1} \,,$$

  with $X$ being an AR(1) process and $\gamma_0, \gamma_1 > 0$.
- Assume $Z_i$ are independent AR(1) processes with long run means of zero.



## Operating Leverage

- Value of firm is value without constant cost $C$ minus value of consol bond paying $C$.
- Thus, firm value is levered, and leverage is higher when revenues $P_t Y_{it}$ are low.
- Zhang emphasizes asymmetric costly adjustment and counter-cyclical price of risk.
- Kogan and Papanikolaou argue that asymmetric costly adjustment does little without operating leverage.
  - Kogan, L. and D. Papanikolaou, 2012, "Economic Activity of Firms and Asset Prices," *Annual Review of Financial Economics* Vol. 2.

## State Variable

- The state of the economy at date $t$ is defined by $X_t$ and $\{(Z_{it}, K_{it}) \mid i \in [0,1]\}$.
- The names $i$ of the firms are unimportant, so we can replace
$$\{(K_{it}, Z_{it}) \mid i \in [0,1]\}$$
with the induced measure $\mu_t$ on $\mathbb{R} \times \mathbb{R}_+$:
$$\mu_t(A) \stackrel{\text{def}}{=} \text{Leb}\,\{i \in [0,1] \mid (Z_{it}, K_{it}) \in A\}\,.$$
- Each firm's operating cash flow depends on $(\mu, x, z_i, k_i)$:
$$\mathrm{e}^{x+z_i} k_i^{\alpha} \left( \int_{\mathbb{R}\times\mathbb{R}_+} \mathrm{e}^{x+z} k^{\alpha}\, \mathrm{d}\mu(z,k) \right)^{-\eta} - C\,.$$



## Fixed Point

- Each firm does dynamic programming, taking $(\mu, X)$ as an exogenous Markov process.
- Basic idea:
  - Conjecture dynamics for $\mu$:
  $$\mu_{t+1} = g(\mu_t, X_t)\,.$$
  - Solve dynamic programming problems to compute optimal
  $$(\mu_t, X_t, Z_{it}, K_{it}) \mapsto K_{i,t+1}\,.$$
  - The optimal policies yield a new map
  $$(\mu_t, X_t) \mapsto \mu_{t+1} \stackrel{\text{def}}{=} \hat{g}(\mu_t, X_t)\,.$$
  - Find a fixed point $\hat{g} = g$.

## Approximate State Variable

- In the fixed point map, replace $\mu$ with $\nu = (\nu_1, \ldots, \nu_n)$, where

$$\nu_i \stackrel{\text{def}}{=} \int_{\mathbb{R}\times\mathbb{R}_+} f_i(z,k)\,\mathrm{d}\mu(k,z)$$

  for functions $f_i$.
  - If we define the $f_i$ appropriately, then we can recover $\mu$ from a countable family $\nu_1, \nu_2, \ldots$.
  - For example, $f_1 = 1_{A_i}$ for a countable basis $A_1, A_2, \ldots$ of the Borel $\sigma$–field.
- We need operating cash flows to depend on $(\nu, x, z, k)$ in order to use $\nu$ in the dynamic programming.
  - Define $f_1(z,k) = \mathrm{e}^z k^\alpha$.
  - Operating cash flow of firm $i$ is

$$\mathrm{e}^{x+z_i} k_i^\alpha \mathrm{e}^{-\eta x} \nu_1^{-\eta}\,.$$



## Approximate Fixed Point

- Conjecture $\nu_{t+1} = g(\nu_t, X_t)$.
- Solve dynamic programming problems to compute optimal

$$(\nu_t, X_t, Z_{it}, K_{it}) \mapsto K_{i,t+1}\,.$$

- The optimal policies yield a new map

$$(\nu_t, X_t) \mapsto \nu_{t+1} \stackrel{\text{def}}{=} \hat{g}(\nu_t, X_t)\,.$$

- Find an approximate fixed point $\hat{g} \approx g$.
- Increase the dimension of $\nu$ to better approximate $\mu$ and repeat.

# Some More Details

- Look for the approximate fixed point within a parametric class, for example linear.
- Start with particular coefficients, defining $g$.
- To calculate $\hat{g}$:
  - Numerically solve the dynamic programming problems using a finite grid for the state variables.
  - Simulate the economy to produce a time series for $(\nu_t, X_t)$ (discarding initial draws to obtain stationarity).
  - Regress $\nu_{t+1}$ on $(\nu_t, X_t)$ to obtain new approximate linear function $\hat{g}$.
- An approximate fixed point is obtained if $\hat{g} \approx g$ and if the $R^2$ in the regression is large.



# Law of Large Numbers

- In equilibrium of the original (not approximated) economy, dynamic programming yields

$$(\mu_t, X_t, Z_{it}, K_{it}) \mapsto K_{i,t+1} \stackrel{\text{def}}{=} \kappa(\mu_t, X_t, Z_{it}, K_{it})\,.$$

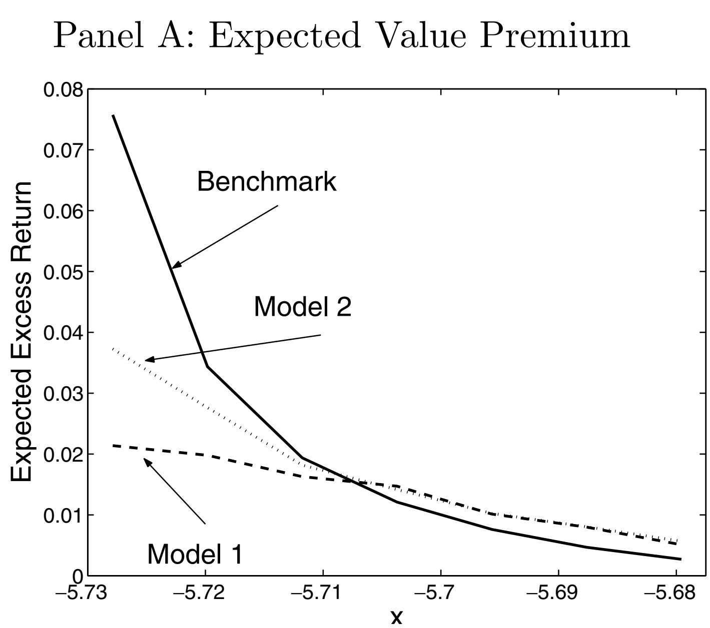
- By the AR(1) assumption, $Z_{i,t+1} = a + bZ_{it} + \sigma\varepsilon_{i,t+1}$ for independent standard normals $\varepsilon_{i,t+1}$.
- For $A = [z_0, z_1] \times [k_0, k_1]$, consider

$$\int_{\mathbb{R}\times\mathbb{R}_+} 1_{\{(z,k)|k_0 \le \kappa(\mu_t, X_t, z, k) \le k_1\}} \times \left[\mathrm{N}\left(\frac{z_1 - a - bz}{\sigma}\right) - \mathrm{N}\left(\frac{z_0 - a - bz}{\sigma}\right)\right] \mathrm{d}\mu_t(z,k)\,.$$

- This is $\mu_{t+1}(A) \stackrel{\text{def}}{=} g(\mu_t, X_t)(A)$ under some law of large numbers.

# References: Krusell-Smith and JEDC special issue


- den Haan, W.J., 2010. Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents. JEDC.
- den Haan, W.J., 2010. Comparison of solutions to the incomplete markets model with aggregate uncertainty. JEDC.
- den Haan, W.J., Rendahl, P., 2010. Solving the incomplete markets model with aggregate uncertainty using explicit aggregation. JEDC.
- Kim, S., Kollmann, R., Kim, J., 2010. Solving the incomplete markets model with aggregate uncertainty using a perturbation method. JEDC.
- Krusell, P., Smith Jr., A.A., 1998. Income and wealth heterogeneity in the macroeconomy. JPE.
- Maliar, L., Maliar, S., Valli, F., 2010. Solving the incomplete markets model with aggregate uncertainty using the Krusell-Smith algorithm. JEDC.
- Reiter, M., 2010a. Solving heterogeneous-agent models by projection and perturbation. JEDC.
- Reiter, M., 2010b. Solving the incomplete markets economy with aggregate uncertainty by backward induction. JEDC.
- Young, E.R., 2010. Solving the incomplete markets model with aggregate uncertainty using the Krussell-Smith algorithm and non-stochastic simulations. JEDC.


Source: Zhang, L., 2005, "The Value Premium," JF 60, 67–103. Horizontal axis: Aggregate productivity *x*. Vertical axis: Spread in expected returns between high B-to-M and low B-to-M stocks. Model 1: Symmetric adjustment costs and constant price of risk. Model 2: Asymmetric adjustment costs, constant price of risk. Benchmark: Asymmetric adjustment costs, counter-cyclical price of risk.

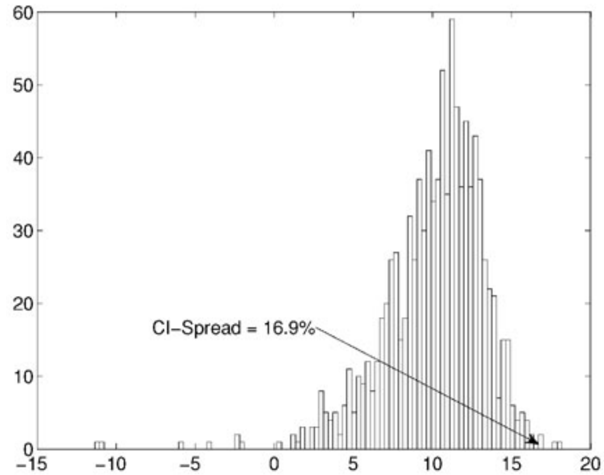
# Investment, New Issues and Risk

- Equity issues and capital investment are correlated.
- When a firm invests, it converts a growth option into assets in place, lowering risk. See
  - Carlson, M., Fisher, A., and R. Giammarino, 2004, "Corporate Investment and Asset Price Dynamics: Implications for the Cross-section of Returns," JF.
  - Carlson, M., Fisher, A., and R. Giammarino, 2006, "Corporate Investment and Asset Price Dynamics: Implications for SEO Event Studies and Long-Run Performance," JF.
- Other things equal, low-risk firms will invest more.
- So, high investment firms and firms that have made new issues should have lower average returns.
- Alternative story: market timing. Firms with over-priced stock issue it and subsequently have low returns.



# Li-Livdan-Zhang (RFS, 2009)

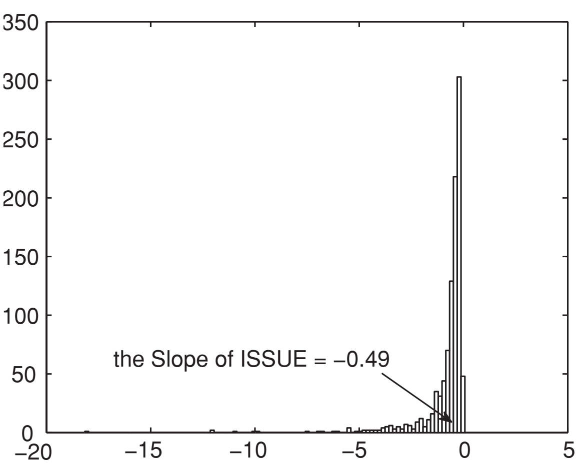
- Zhang (2005) model, but with costly external finance:
- When cash flow is positive, it is paid to shareholders.
- When cash flow is negative, it is raised with a fixed plus proportional cost.
- No cash holdings.




Source: Li, E. X. N., Livdan, D. and L. Zhang, 2009, “Anomalies,” RFS. Histogram (across simulations) of the mean return spread between the lowest quintile of capital investment (CI) firms and the highest quintile, where

$$\mathrm{CI}_t = \frac{3\mathrm{CE}_{t-1}}{\mathrm{CE}_{t-2} + \mathrm{CE}_{t-3} + \mathrm{CE}_{t-4}} - 1\,,$$

and CE is capital expenditure scaled by sales, following Titman, Wei, and Xie (2004).




Source: Li, E. X. N., Livdan, D. and L. Zhang, 2009, “Anomalies,” RFS. Histogram (across simulations) of the average slope coefficient in the cross-sectional (Fama-MacBeth) regression

$$r_{i,t+1} = a_t + b_t\ \mathrm{ISSUE}_{it} + \varepsilon_{i,t+1}\,,$$

where $\mathrm{ISSUE}_{it}$ is 1 if the firm conducted an SEO within the past 60 months and 0 otherwise.