Oxford-Man Institute of Quantitative Finance

Event-Driven Finance

Lecture 1: Introduction.

The Market (Reality).

Mike Lipkin Columbia University (IEOR)



What is event-driven finance?

A first, naïve, answer is this: Event-driven finance concerns the pricing of (derivative) securities concomitant to some temporal event.

This first answer is somewhat tautological. And in any case, events happen all the time. So why might we wish to introduce this new category of finance?

To answer this question we need to reexamine our preexisting ideas about derivatives pricing.



- In the course of doing so we shall see that standard approaches to pricing involve assumptions of equilibrium.
- These assumptions include the notion that many events may be averaged over; the events form a heat-bath in whose presence the expected stock behavior may be calculated.
- BUT what if we are not interested in the average behavior of a stock, but only its behavior in the temporal vicinity of ONE event.
- We should expect the pricing of the derivative securities to have a prominent time dependence- and it does.



So the story is two-fold:

Events are typically discrete changes in some characteristic at a fixed time;

And event-driven finance means that we are interested in the time-dependent price of securities *near* that time.

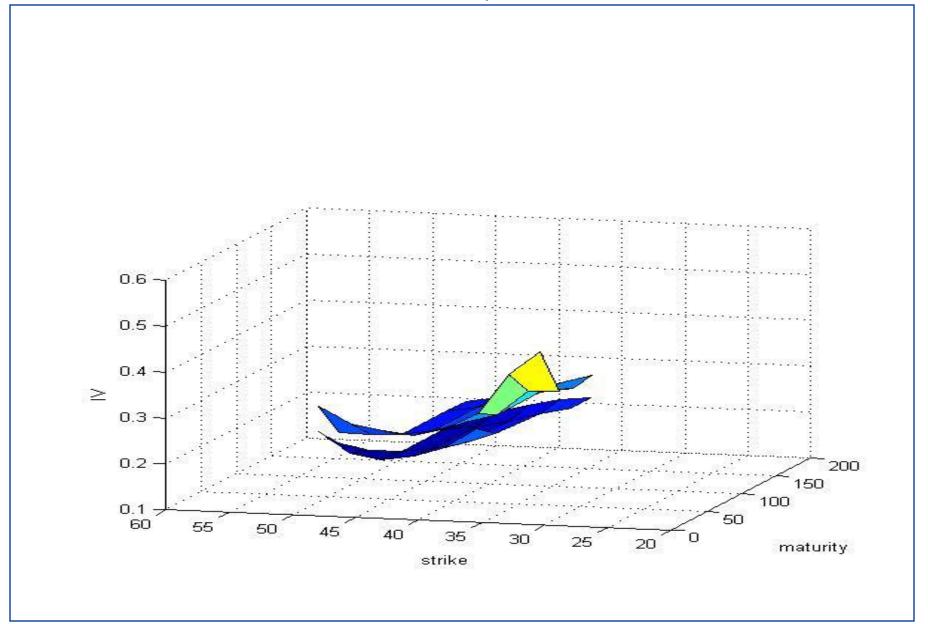
Let's look at some pictures:



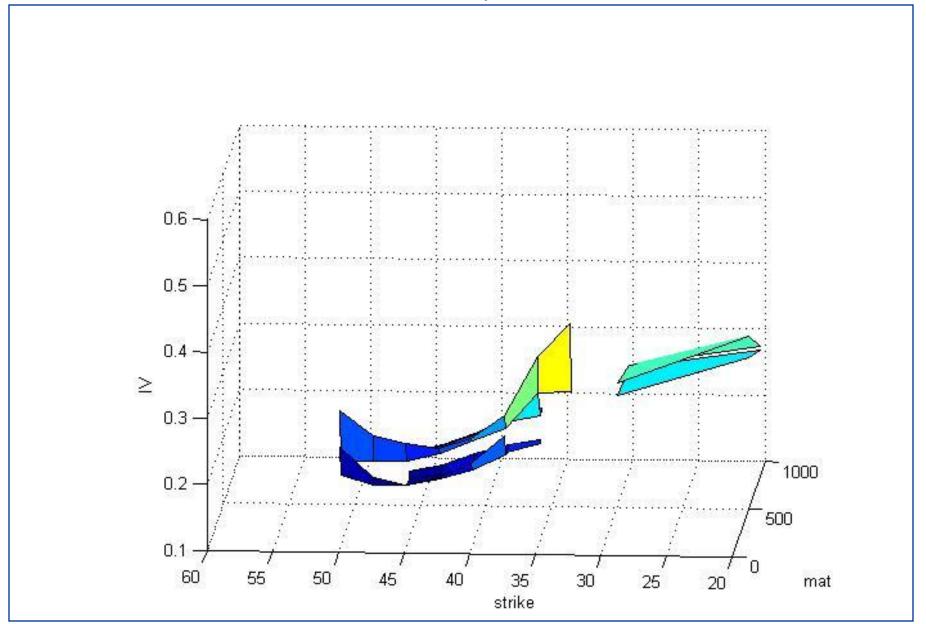
 The following three plots show the volatility surface for the stock, FDC, at the close of trading, September 15, 2005, (upper surface)

 And below it, the lower surface shows the same stock 1 day later:

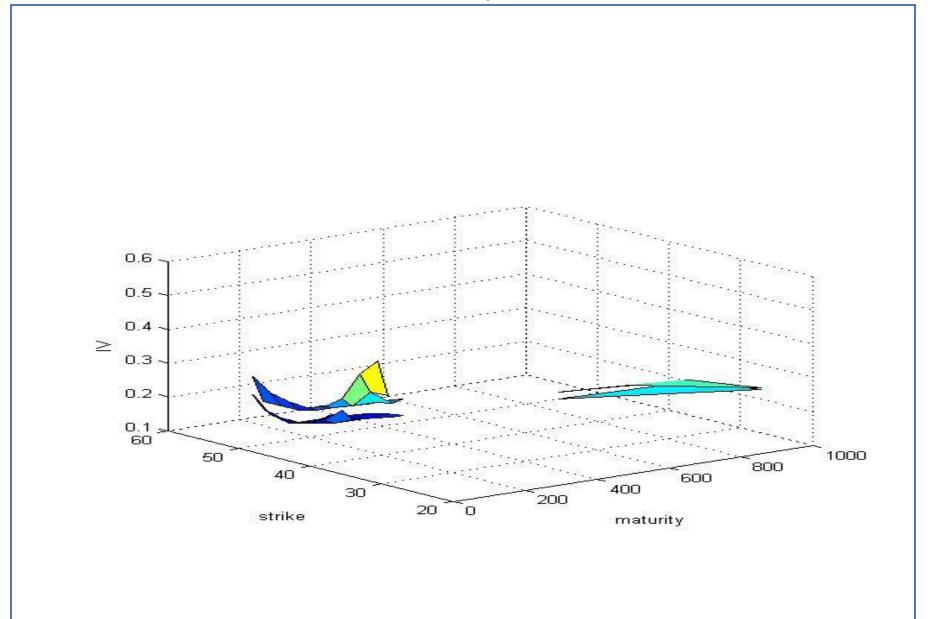














- Clearly some event had occurred to lower the implied volatilities across all expiries.
- This means that theoretical pricing of securities required a discrete change of input parameters.
- We will discuss what happened later, but you may be surprised to note that classical stochastic models do not include a parameter which directly encompasses this change.
- Some more pictures:

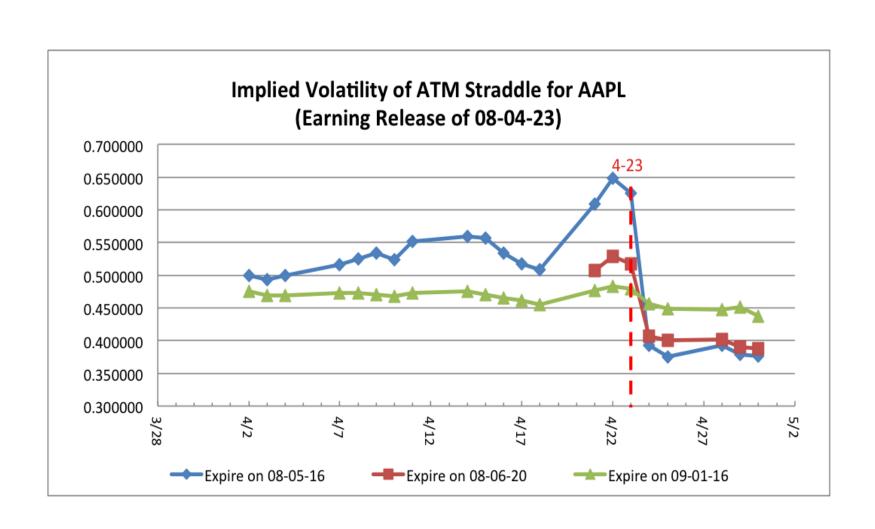


 Here is a graph of implied volatility for a period of four weeks in April, 2008 in the stock, AAPL

 For three of those weeks the implied volatility was steadily rising; after a crash, the volatility appears to flatten

After that, a similar fitted plot in MSFT

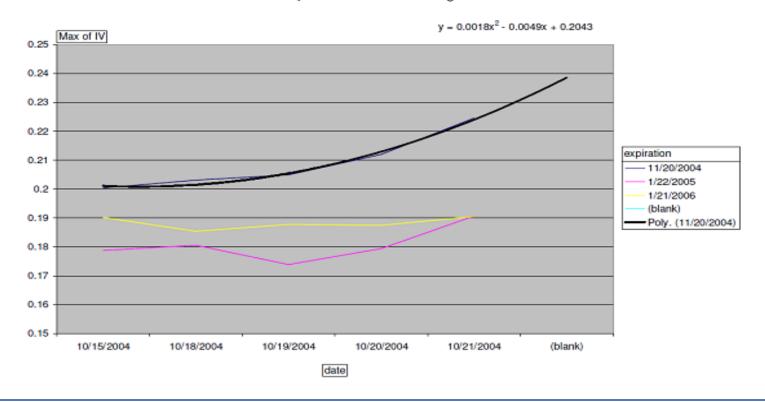






 Here is the rising portion of a similar graph for MSFT in October 2004

MSFT implied vols around earnings 10-21-04



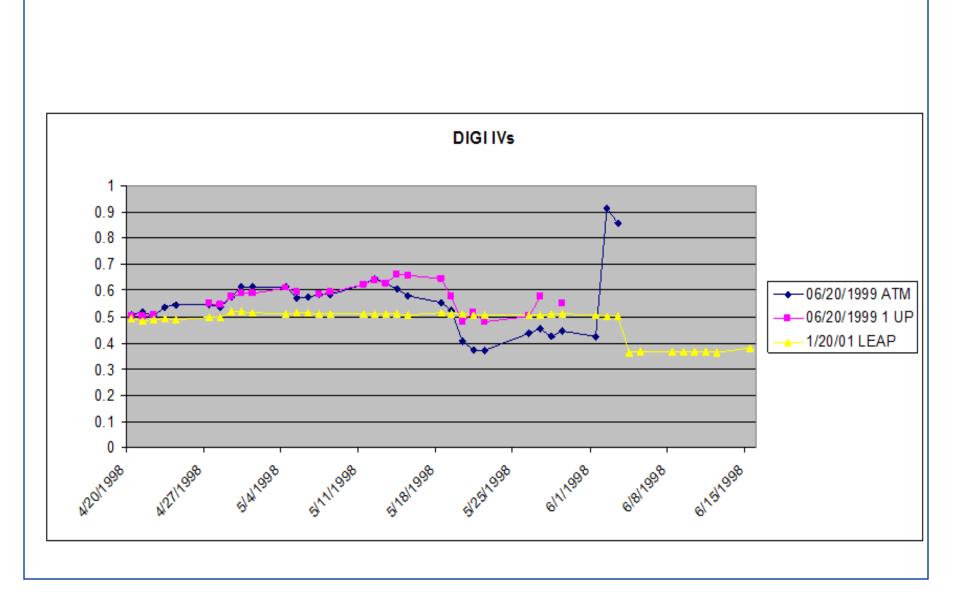


- For the previous two images, it is clear that while there
 appears to be an event date, the impact of the event is
 spread out over several earlier weeks broadly.
- This is typical of a certain class of events which we shall revisit in Lecture 3; they are clearly anticipatory in that we see effects in the volatility surface in advance of the event.



- The following is a graph of implied volatilities for several strikes in the stock, DIGI, for three months in 1998.
- At a certain date (ca. May 14) the volatility surface
 pleats- the front month at-the-money implied volatility
 dropping below the volatility of the next higher strike on a
 relative basis.







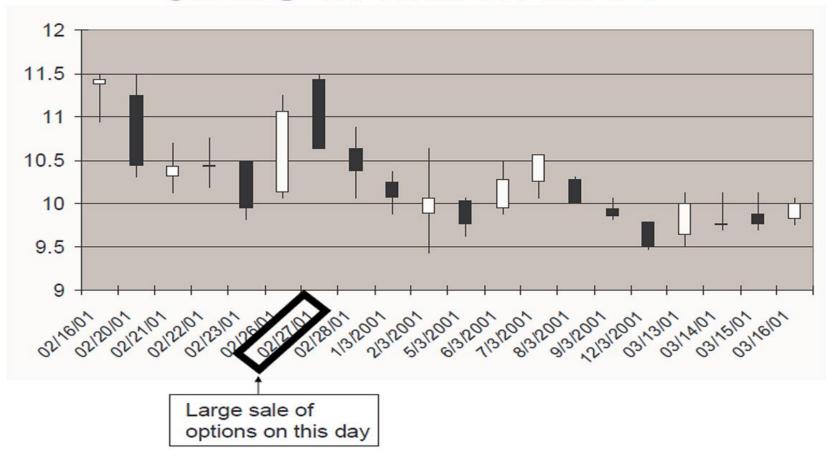
- In Lecture 4 we will come back to this example and discuss what happens here in more detail. This is a complex event in that it has multiple parts.
- Looking carefully at the long-term volatility, one sees that it drops abruptly in the first week of June.
- This sudden drop in the long-term volatility is, in fact, what most people would identify as the event.
- But while the volatility pleating of mid-May is consistent with the June occurrence it is not pre-ordained by it- nor the reverse!



- Here is a plot of stock price for the stock JDEC for a month (February - March) in 2001.
- The Japanese candlesticks indicate a large drop in daily volatility for the stock after Feb 27, and the stock zeroes in on the price of \$10.



JDEC in March 2001

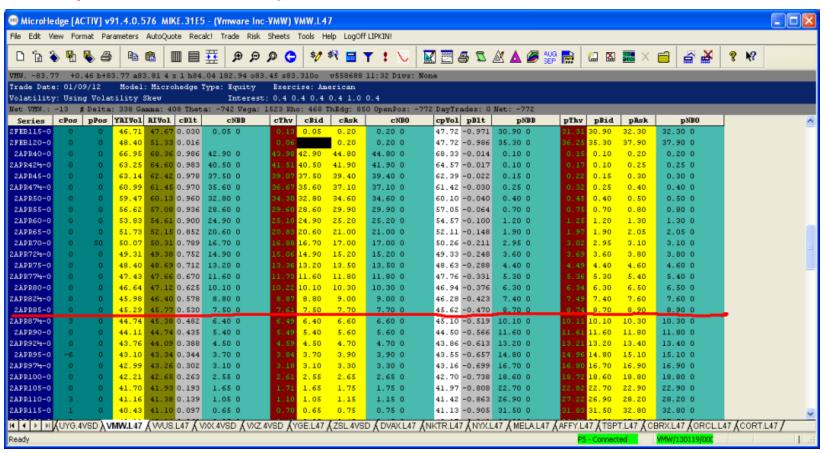




- In case 1, an event on Sept 16 in FDC produced a discrete immediate response in the volatility surface.
- In case 2, an event at a later date caused an anticipatory change in the volatility surface over several weeks.
- In case 3, a complex event stretches over several months and has variable temporal effects on the volatility surface.
- In case 4, -contrast with case 2- the event in JDEC can be associated with the date, Feb 27, but the effect on the volatility surface and stock price stretches forward in time. We will discuss this case in detail next Lecture.
- Let's jump in with a real world problem:

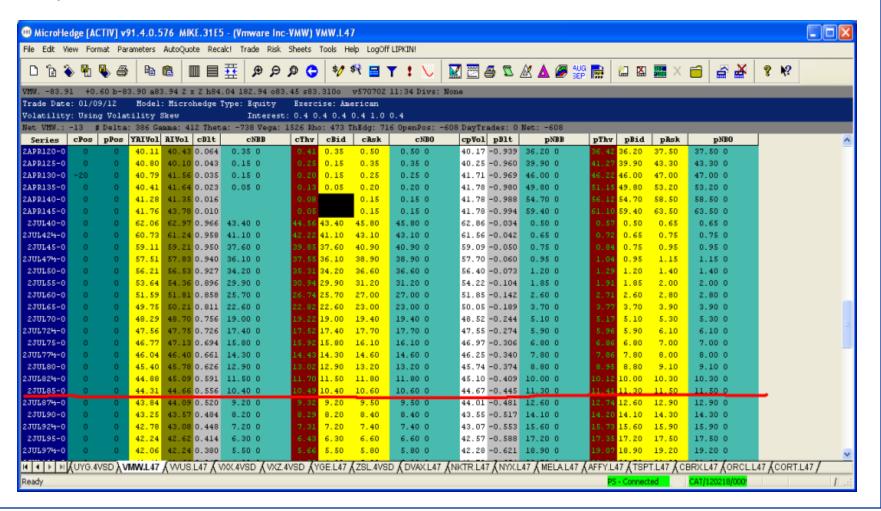


 Suppose you are working at a desk and running a variant of Black-Scholes, as sophisticated as you care to make it, and a hedge fund shows you 15000 contracts \$0.15 through your theoretical value: "I can sell you 15000 VMW Apr 85 calls for \$7.46."





Here is another page of VMW quotes:





EMC to maintain 80% VMware stake

EMC Corp., which specializes in high-end computer storage systems, is based in Hopkinton. (Neal Hamberg/Bloomberg News/ File 2004)

Bloomberg News / March 3, 2010





- Do you buy them?
 - What considerations do we need make?
 - What if the hedge fund wanted to sell 500 options only?
- Volatility/Vega
- Risk
- The above is an example of a volatility depression (spike). After the trade there will be a new volatility profile.
- What will that profile look like?
- Would it surprise you to know that there is no existing, accepted theory of the *dynamics* of pricing?
 - What we are interested in having at our disposal is not a static (or thermodynamic) model which allows stochastic volatility, but a way of learning about the "response function" of a real market.
- In a sophisticated theory, the following kind of mathematical object would be calculable: $<\!\!\Delta\sigma(K_1,t_1)\!\!\Delta\sigma(K_2,t_2)\!\!>$.



- As you can imagine. If we do decide to buy the Apr 85 calls we will have greatly increased our Vega. From the discussion it is clear that in any case, prices will decline in other strikes and series.
 - By how much?
 - No one knows. There is (almost) a complete absence of theory.
- If the Apr 85 calls decline by 1.5 (implied) vol points,
 - how many points will the Apr 90 calls come in by?
- The market there is \$5.40-\$5.60.
 - Does it make sense to hit the bid? (What does hit mean?)
- The July 85 calls are \$10.40-\$10.60.
 - Should you sell the calls at \$10.40 as a hedge?
 - Is this better than the \$5.40 sale?
 - What if there are earnings between April and July?



- Should you sell EMC volatility instead?!?
- Suppose that the hedge fund "informs" you that the calls will trade.
 - Should you be *leaning* short?
 - What does this say about the assumption that the stock process is independent of option trading?
 - Is there a flaw in the Martingale assumption?
- Later (Lecture 2) we will see that option volume can affect stock prices.
- Here are some Real World examples:

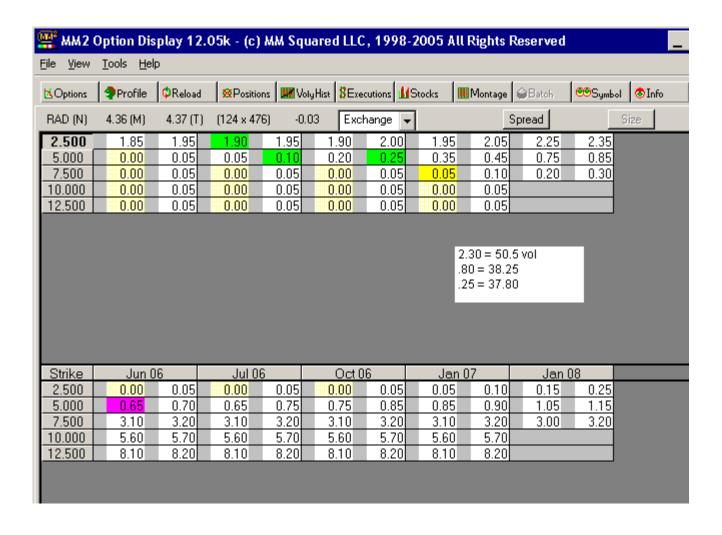


- On September 16, 2005, a BA customer sold 150,000
 FDC Jan 40 calls to market-makers, mostly within a two-hour window.
- The implied volatility of at-the-money options went from 23 to 19 in January and from 28 to 20 in November.

this was case 1 above

 On Tuesday, May 23, 2006, market-makers were told "133,000 RAD Jan '08 2½ calls will trade at 2.35 vs. 4.38 stock. How much would you like to sell?"

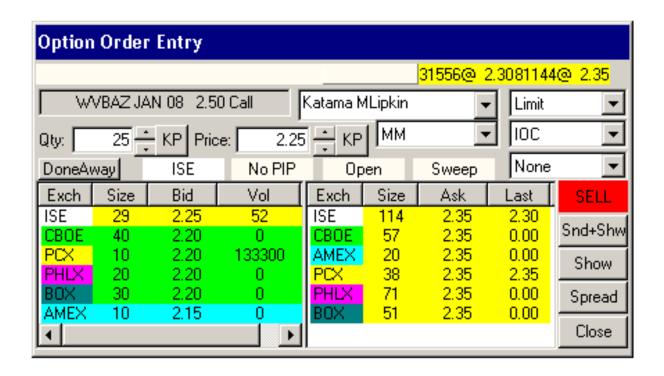




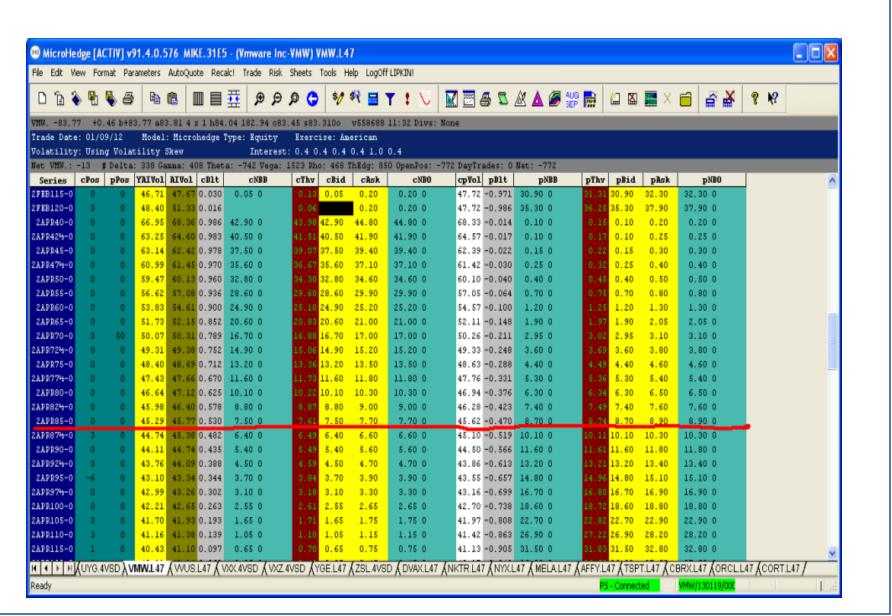


RAD = \$4.38 133,300 CONTRACTS TRADE 2.35

DO YOU SELL?









- Let's take the previous slide of VMW as a template.
- The standard approach to market pricing is calibration. All market models take input data from the actual prices out there. Suppose that the resultant model now "fits" the market, in the sense that no theoretical prices lie outside the bid-offer spreads.
 - Does this mean that the market is correctly priced?
- Suppose that over the next week, buyers show up for all the VMW 87.5 line options (previous slide S₀=83.77). As a result,
 - what will happen to the normal skew?
- If the skew "inverts", does this mean that the prices are wrong?
- We will see, (Lecture 4), that under certain circumstances such as take-overs the skew can take a strange but characteristic shape.

Static finance



- The main point is this: if all our (derivatives) prices are fit by calibrating an initial model- and then the prices no longer fit- we...
- cannot know if our model is now wrong
- or if profitable trading is now possible
- This is because events create a phase change in the system we are studying/trading
- Case 2: earnings dates in AAPL and MSFT
- Case 3: anticipation of, and then take-over of DSC (DIGI) by Alcatel
- Case 4: the expiration pinning of JDEC



- Let's try to summarize some of the ideas we have discussed.
- The **size** of a trade matters. The **time scale for** the *relaxation* of the market subsequent to a trade matters. A quant analyzing the *thermodynamics* of the market will not see many of the time scales needed to understand market dynamics.
- It is important to pay strict attention to time scales.
- Ex.: Optionmetrics IVY database closing prices
- This time scale suffices to look at earnings, drug announcements, take-overs and mini-crashes (Lectures 3 and 4). It does not allow us to look at the response to size trades.
 - What kind of database would you need for that?
 - Would such a database be useful for a trading house?
 - Do you think the *elasticity* of the response is a function of the individual stock? the open interest? the illiquidity of the stock? Anything else?



- Let's conclude this introductory talk by considering a typical problem about which there is a lack of theoretical understanding. The objective will be to abstract the nature of the problem, consider the time scales involved, and finally to propose a database experiment to search for market behavior.
- Let's take the VMW, EMC example. These are two related companies. Suppose we run a book with positions in VMW and EMC. When we are offered a large trade in VMW, we would like to know if we need to be hedging in EMC. Notice that this is not asking if stock prices are correlated (although they may be), but rather if volatility surfaces are correlated.
- For example, suppose that we are short 5000 Vega in VMW and long 5000 Vega in EMC. If we buy VMW premium we will become flat, say.
 - Do we need to sell some amount of EMC volatility?
 - If that is true, what would that tell us and how would we quantify it?
 - What time scale would the vol changes occur on?



 To begin with we need to locate significant volatility changes in the histories of VMW and EMC. We need these changes to occur over a characteristic time scale, say one or two days, and then we need to see if there is a subsequent change in the volatility of the partner stock. The following quantities may be relevant:

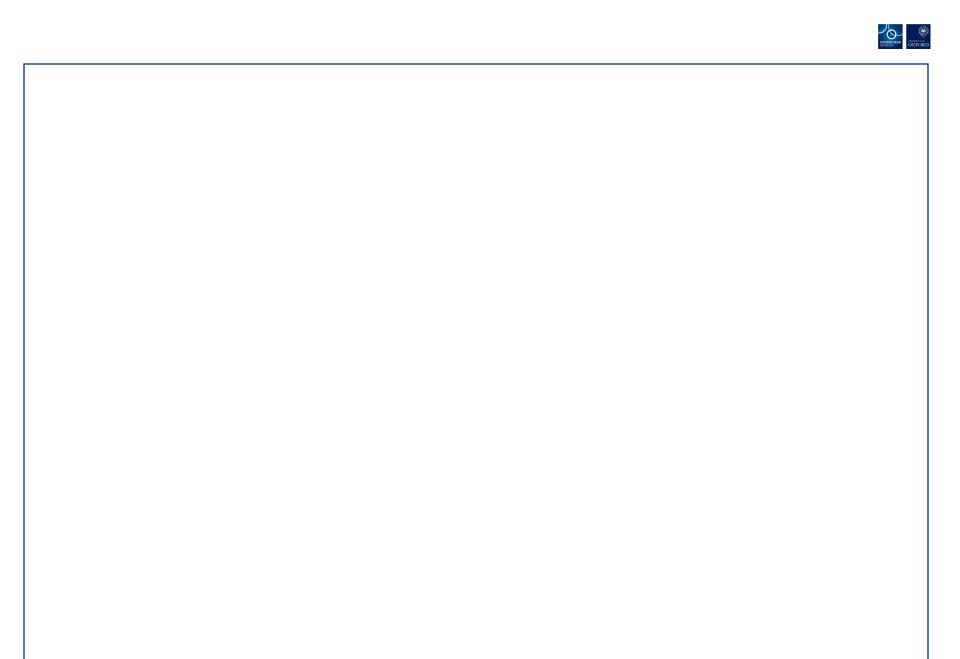
$$<\Delta\sigma_{VMW}(t,K_{\Delta 1})\Delta\sigma_{EMC}(t+\tau,K_{\Delta 1})>$$
 (1)

• What is this object? $\Delta\sigma$ is the change in vol, τ is the lag time (unknown but possibly very short) between the change in VMW vol and the subsequent change in EMC vol, $\tau > 0$ assumed. $K_{\Delta 1}$ is the strike corresponding to similar deltas in both products. (Notice how the assumptions are multiplying!!) From the physics of dynamical systems, this quantity is called a response function— for obvious reasons.



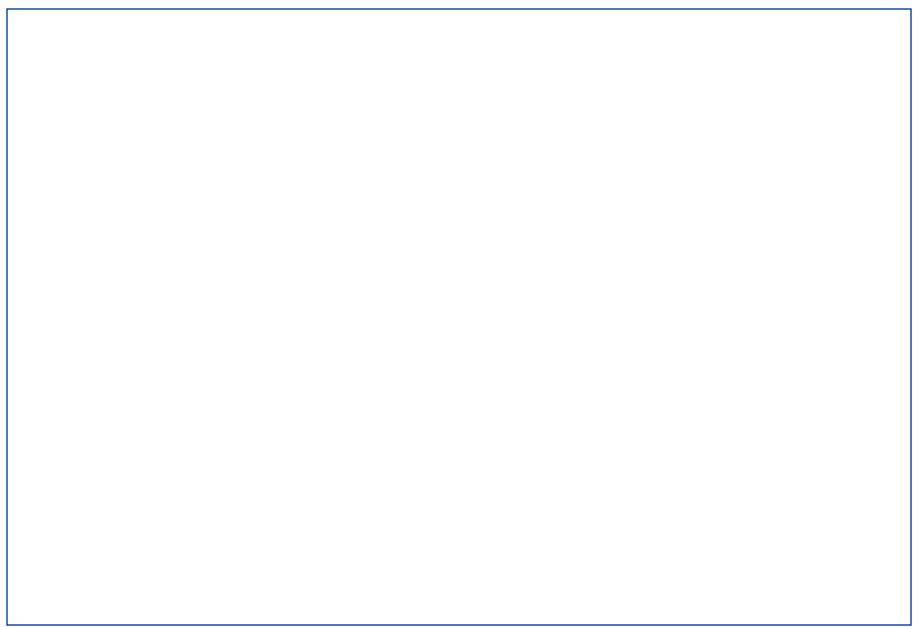
- Impact is frustrating (for me) in that it exposes the lack of theory.
- Given some set of parameters involving market cap, supply/demand, initial volatility surface, etc., a complete theory would explicitly yield the *new* volatility surface which results, given a large instantaneous trade of size, Q.
- This is far away, however:
- A "complete" solution exists for stock pinning (Lec. 2)
- "Partial" solutions exists for earnings and take-overs (Lecs. 3 and 4)
- A "complete" (hard) solution exists for hard-to-borrowness (another mini-course)
- The general technical approach is to identify *slow* variables in which reformulated static modeling approximately holds.
- We will see this next time...











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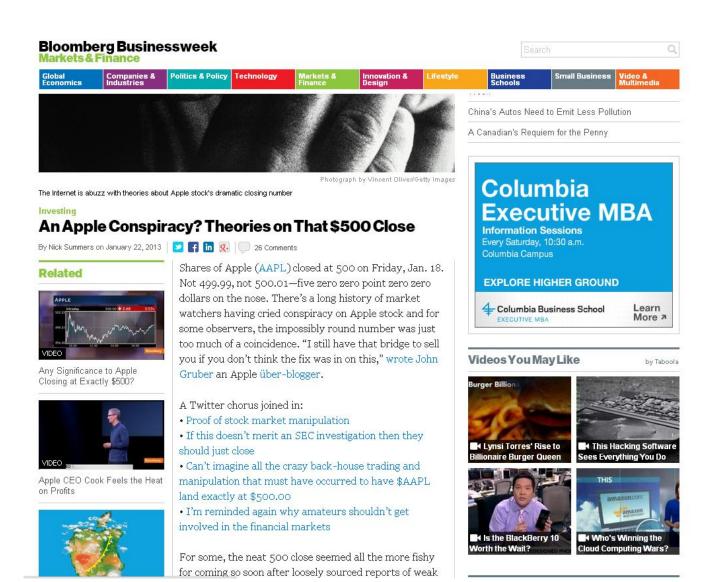
Event-Driven Finance

Lecture 2: Pinning.

Mike Lipkin Columbia University (IEOR)

Pinning

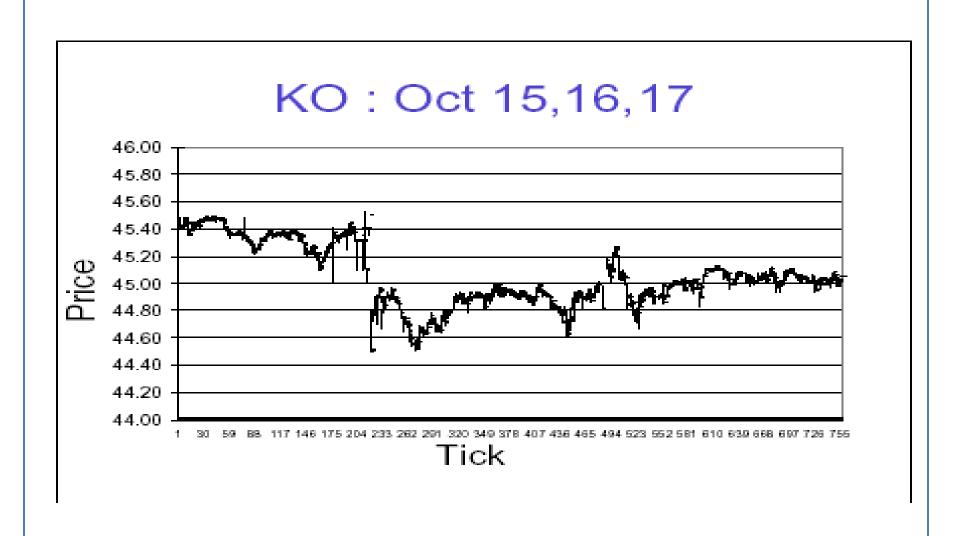














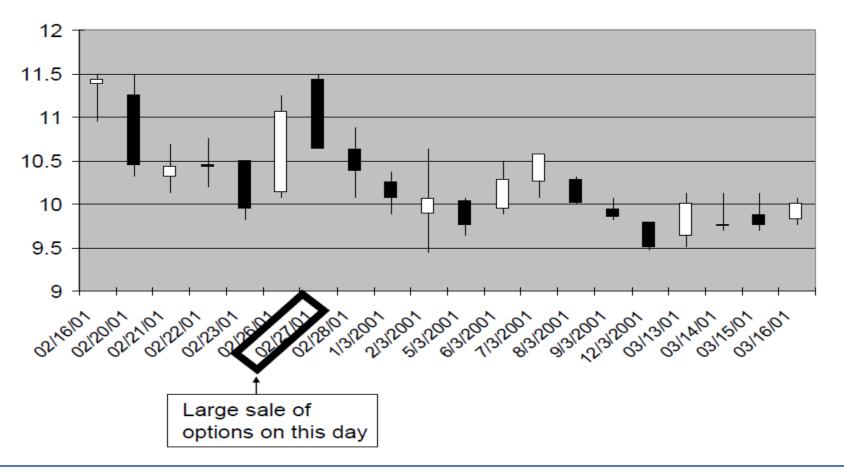
KO pinning to 67.50 (weeklies)





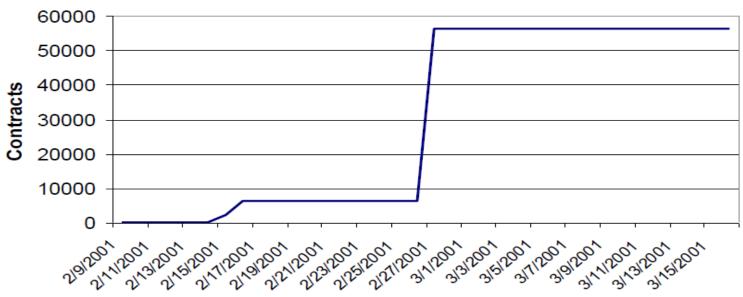
• (JDEC pin)

JDEC in March 2001





JDEC 2001 Mar 10 Put & Call Open Interest



Date

Average traded vol in stocks = 1MM shares

Notional number of shares corresponding to OI = 5.6 MM shares



- Today we want to look at a static property of the option markets.
- Not all phenomena which appear to violate "standard" option theory are dynamic. As you know, there are many assumptions made in standard classical finance which we know, or suspect, cannot hold in the real markets.
- Suppose you see the following market:

```
XYZ Jun 40 C 8.50 - 8.80 (100 x 450) (Underlying) 48.46 - 48.52 (650 x 75) Expiration day.
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- First of all, what does this mean? What is the fair value of the calls?
- Classical theory says that the Jun 40 calls are overpriced. By how much? Why haven't they traded?



- Costs are an obvious area typically ignored in order to price options.
- A more subtle idea is the assumption of a stock process. This is a stochastic process for the stock, independent of the presence of options trading.
- Suppose someone bids for 25000 calls all at once. (On Friday, April 28, 2006 this happened in MSFT May 25 (at-the-\$) calls.) Do you suspect that the stock would move in a correlated fashion? Which way? (In MSFT the stock price moved from 24.05 to 24.17 in 15 minutes from the origin of the order.)
- This means that on certain time scales a demand for (supply of) stock moves the stock. Quantifying this effect theoretically means identifying an *Impact Function*.
- What about the very presence of outstanding option open interest?
 Typically it would seem not, because undoubtedly positions are hedged. And yet, sometimes option positions lead to *changing* deltas.



- Suppose you hold an XYZ Jun 40 C; it is expiration day and the stock is at 40.35 at 10:30. You calculate the delta and find it is 58.
- At 1:30, three hours later, the stock is still at 40.35. What has
 happened to the delta of the call? When you recalculate the option
 delta, it is now 66. Why?
- To stay delta-neutral you must sell an additional 8 shares.
- Now couple this to the assumption that supply (demand) of the stock pushes the stock down (up) and the changing deltas of the option lead to long option holders selling the stock.
- An analogous argument applies with the stock below the strike; now buyers push the stock up toward the strike.
- In the Black-Scholes, classical world, there are an equal number of short option holders doing the exact opposite thing. The net effect should be zero.



- But is this an accurate assumption? Market makers are generally active hedgers. When they are long a strike they aggressively hedge, especially close to expiration. But when they are short a strike and since they cannot continuously hedge, they avoid hedging as long as possible.
- Consider the region over which the delta is changing most rapidly. This is also the region where $\theta = -(\partial C/\partial t)$ is largest. So there is an incentive for a trader to avoid hedging his short option, as long as the possibility of pinning remains high. On the other hand, the long option holder risks losing all the option value to pinning.
- So unlike the Black-Scholes world, real hedging strategies are asymmetric.
 Coupled with an additional non-classical assumption of stock price
 movement to supply/demand, there is the possibility of **pinning** the stock at
 expiry, that is a non-zero probability of the stock exactly closing at a strike
 price.

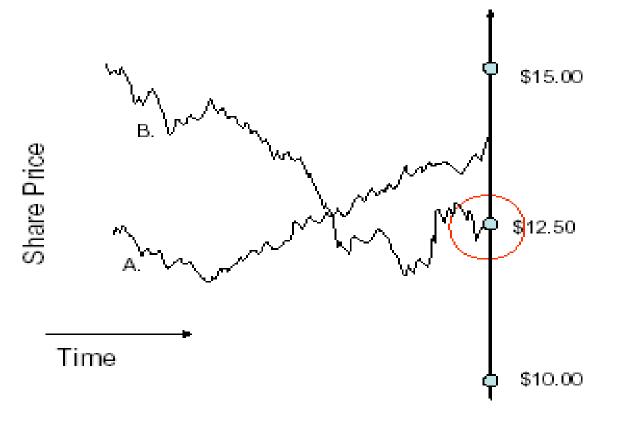


What is stock pinning?

- At the expiration of options, the close of trading on the third
 Friday of each month, a stock is **pinned** if it closes *exactly* at a
 strike price.
- For practical reasons, pinning can be considered to have occurred if the closing price is *close* to a strike (±\$0.25, say)
- Mathematically: P{|K-S|< ε} > 0 at expiration for all ε>0.







Stock B pinned

Stock A did not



Ul Urbana Study: Optionable vs. Non-Optionable Stocks

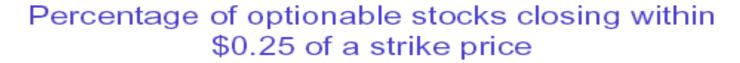
At least 80 expiration dates

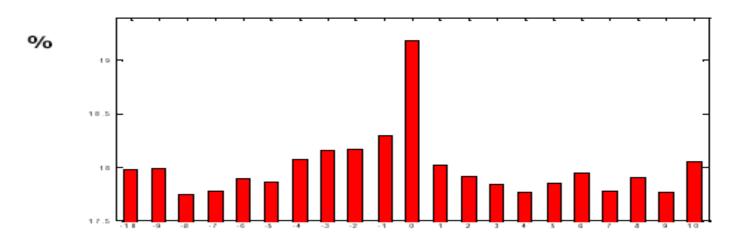
- 4,395 optionable stocks on at least one date
- 184,449 optionable stock-expiration pairs

- 12,001 non-optionable stocks on at least one date
- 417,007 non-optionable stock-expiration pairs



Several results from the UI group. Data from January 1996 through September 2002

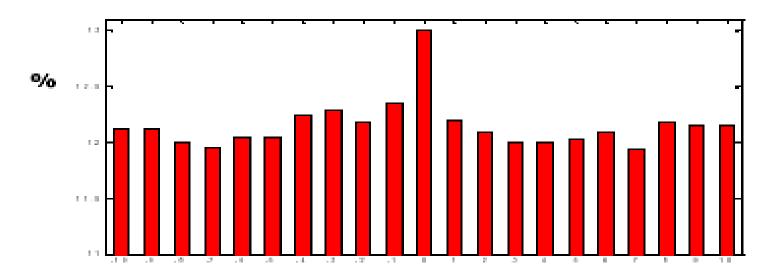




Relative Trading Date from Option Expiration Date



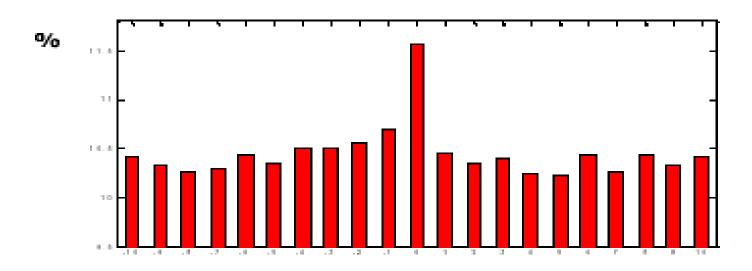
Percentage of <u>optionable</u> stocks closing within \$0.25 of an integer multiple of \$5



Relative Trading Date from Option Expiration Date



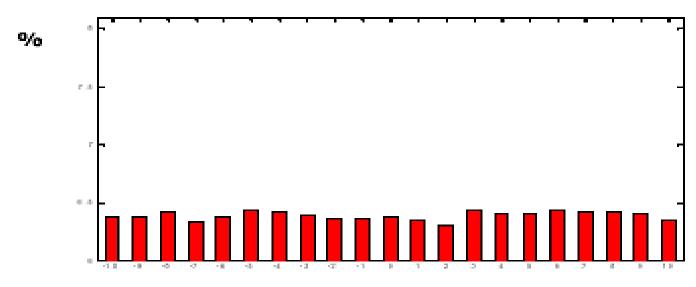
Percentage of optionable stocks closing within \$0.125 of a strike price



Relative Trading Date from Option Expiration Date



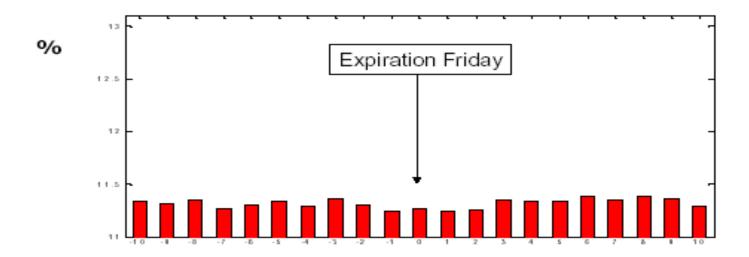




Relative Trading Date from Option Expiration Date



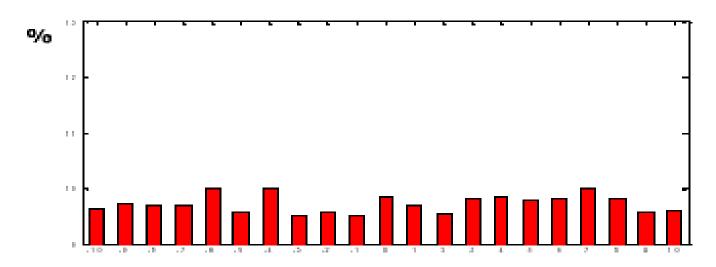
Percentage of <u>non-optionable</u> stocks closing within \$0.25 of an integer multiple of \$5



Relative Trading Date from Option Expiration Date



Non-optionable stocks that were previously optionable closing within \$0.125 of an integer multiple of \$2.50



Relative Trading Date from Option Expiration Date

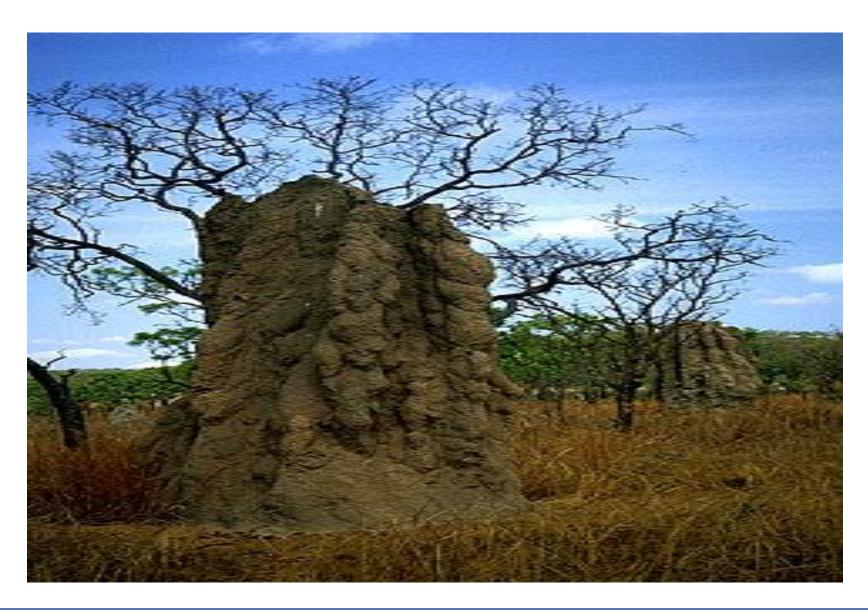


- So there is plenty of evidence for pinning, but only in optionable stocks. What models might suffice to explain the effect?
- Krishnan and Nelkin attack the problem of pinning by assuming that there exists an a priori mixture of pinning paths and independent random walks for the stock price. This model can get any desired probability of pinning, but leaves unanswered how actual option data and parameters, and stock price, may affect the probabilities. Also, once the KN mixture is fixed, the price of the straddle cannot be accurate for all eventual stock paths.
- Ni, Poteshman, Pearson originally suspected **collusion** on the part of market participants. (Post our work, somewhat less so.)



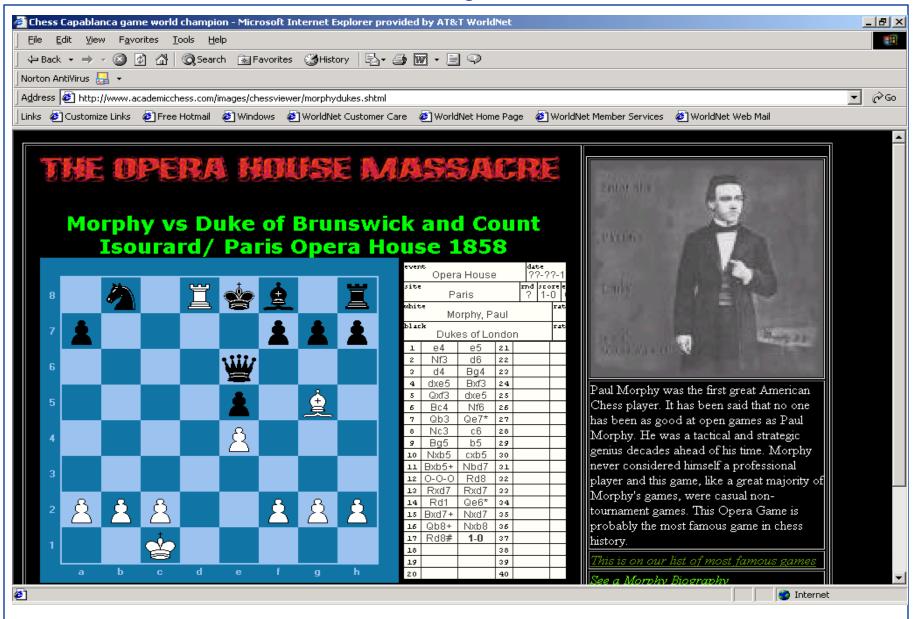
- Which of the following three slides doesn't belong?
- (And what are they?!)



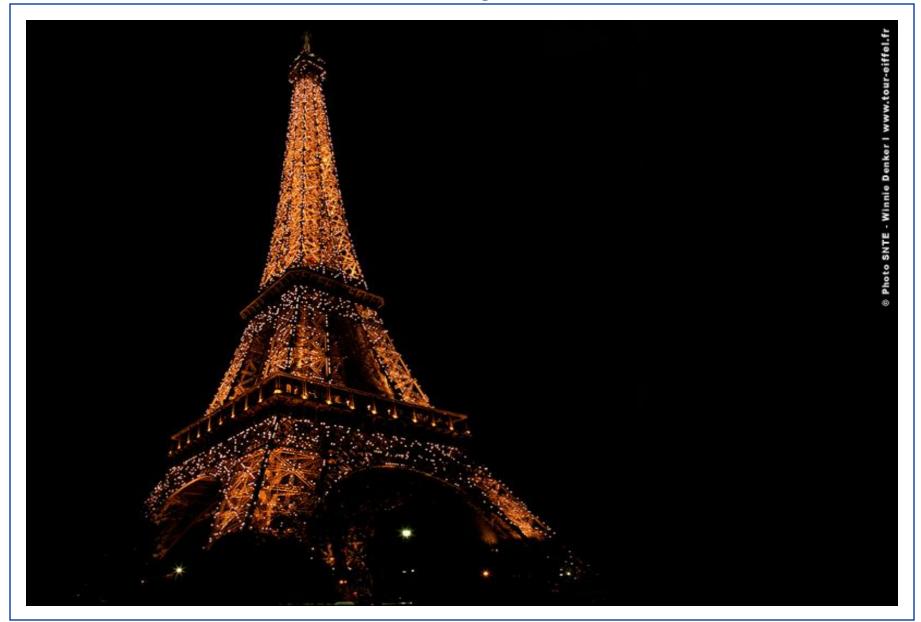


Pinning











- The answer is: the Eiffel tower. Both the termite mounds and the chess game are constructs of independent agents. In other words, although both those slides show a very specific final ordered result, they are the consequence of two or many agents playing out a game. NO MASTER ARCHITECT exists.
- In the game of options trading, individual market-makers play at HEDGING their positions. They do not collude to maintain unbalanced positions.



- All possible models cannot be known, but one which involves market-makers acting independently to maintain approximately delta-neutral positions satisfies Occam's razor. It requires the fewest assumptions about the outside world. A kind of greatest entropy model.
- It should be noted that there are two distinctions which may be drawn between market participants. Some, market-makers and desk proprietary traders among them, are active hedgers. Others, investors and positional traders, put on positions (often but not always long delta), and let them play out.
- This asymmetry will be important.



- A number of groups have examined the response of markets to orders entering an order book.
- One group is associated with J D Farmer:

Lillo, Farmer, Montegna: *Nature* **421(**2003) pp 129-130, Daniels, Farmer, Guillemot, Iori, Smith: cond-mat/0112422, a Los Alamos National Lab preprint.

Another group is associated with JP Bouchaud (CFM).



- These groups all agree on the common sense notion that BUYING stock raises the market price, and SELLING stock lowers the market price.
- Curiously they all disagree on the functional way in which the changing market varies with S/D. (This will be a subject for discussion later.)
- Δ S/S = f(Q) = EQ + E₂Q² + E₃Q³ + ... = EQ + g(Q), g analytic. This is a simple Taylor's expansion for market price change as a function of the demand for (supply of) stock. For simplicity, we throw out g(Q) and simply assume a linear form.

Estimating the Demand for Deltas using Black-Scholes

$$\Delta \delta = \frac{\partial \delta}{\partial t} dt, \qquad \tau = T - t$$

$$\delta = 2N(d_1),$$

$$\delta = 2N(d_1), \qquad d_1 = \frac{1}{\sigma\sqrt{\tau}} \left(\ln\left(\frac{S}{K}\right) + \left(\mu + \frac{\sigma^2}{2}\right) \frac{\sqrt{\tau}}{2\sigma} \right)$$

From **Black-Scholes**

$$y = \ln\left(\frac{S}{K}\right), \qquad a = \mu + \frac{\sigma^2}{2},$$

$$\frac{\partial \delta}{\partial t} = -\frac{1}{\sqrt{2\pi}} \frac{y - a\tau}{\sigma \tau^{3/2}} e^{-\frac{(y + a\tau)^2}{2\sigma^2 \tau}}$$



Taking into account demand for stock: Price-Impact Functions

$$\frac{dS}{S} \propto E \left(\frac{D}{\langle V \rangle}\right)^p \qquad \frac{D}{\langle V \rangle} >> 1$$

p=0.22 Farmer, Lillo, Mantegna

p=0.5 X. Gabaix

p=1 linear model, (A. & Lipkin)

p=1.5 convex model (Bouchaud, ...)



Dimensionless Model for Power-Law Price-Impact Function

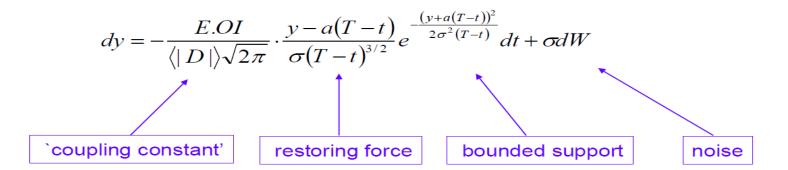
Price change= Price impact+ noise

$$\frac{dS}{S} \propto -const. \left| \frac{\partial \delta}{\partial t} \right|^p sign \left(\frac{\partial \delta}{\partial t} \right) dt + \sigma dW$$



Dynamics for Stock Price

$$\frac{dS}{S} = \frac{E.OI}{\langle |D| \rangle} \frac{\partial \delta}{\partial t} dt + \sigma dW \qquad y = \ln \left(\frac{S}{K} \right)$$





Dimensionless Variables

$$z = \frac{y}{\sigma\sqrt{T}}, \qquad s = \frac{t}{T}, \qquad z_0 = \frac{y_0}{\sigma\sqrt{T}} = \frac{1}{\sigma\sqrt{T}}\ln\left(\frac{S_0}{K}\right)$$

$$\alpha = \frac{a\sqrt{T}}{\sigma}, \qquad \beta = \frac{E.OI}{\left\langle \mid D\mid \right\rangle\sqrt{2\pi\sigma^2T}}$$

$$dz = -\frac{\beta(z - \alpha(1 - s))}{(1 - s)^{3/2}} e^{\frac{(z + \alpha(1 - s))^2}{2(1 - s)}} ds + d\overline{W}$$



- z represents the dimensionless (logarithmic) distance to the strike; it's presence in the formulation insures that the likelihood of pinning is subject to a feedback of the stock price itself
- β describes the strength of the pinning force. It is proportional to the open interest, OI, and the unknown elasticity constant, E, and inversely proportional to the stock volatility, σ
- β represents the strength of the coupling to the "pinning field"
 - You can think of OI as *charge*, E as the dimensionful *coupling constant*, and $\sigma\sqrt{T}$ as a *temperature*
- α the drift term we will arbitrarily set to 0

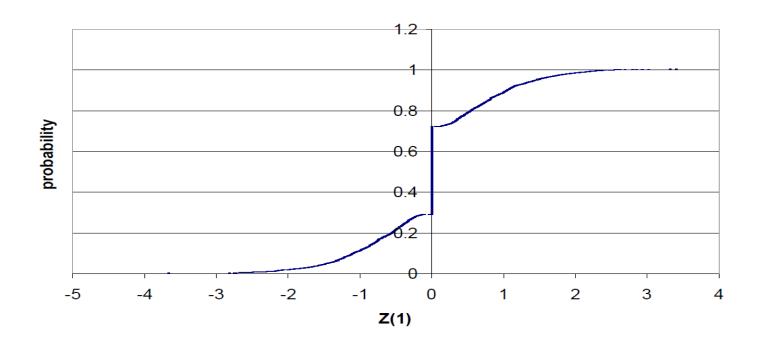
Dimensionless Model (alpha=0) for Linear Price-Impact Function

$$dz = -\frac{\beta \cdot z}{(1-s)^{3/2}} e^{-\frac{z^2}{2(1-s)}} ds + d\overline{W}$$

Linear restoring force with increasing coupling with time and compact support.



Cumulative PDF for price at expiration date (Beta=0.1)



Solving the linear response model (p=1)

Assume Alpha=0

Forward Fokker-Planck equation:

$$\frac{\partial F}{\partial s} + \frac{1}{2} \frac{\partial^2 F}{\partial z^2} - \frac{\beta z}{\tau^{3/2}} e^{-\frac{z^2}{2\tau}} \frac{\partial F}{\partial z} = 0, \qquad \tau = 1 - s$$

Look for solution of the form:

$$F(z,s) = \exp\left(\frac{1}{\sqrt{\tau}}\phi\left(\frac{z}{\sqrt{\tau}}\right)\right), \qquad \phi(\varsigma) \text{ unknown}, \quad \varsigma = \frac{z}{\sqrt{\tau}}$$

ODE for the `Phase Function' (WKB)

$$\frac{\phi + \varsigma \phi' + \phi''}{2\tau^{3/2}} + \frac{(\phi')^2 - 2\beta \varsigma \phi' e^{-\frac{\varsigma^2}{2}}}{2\tau^2} = 0$$

$$O(\tau^{-2})$$
 $(\phi')^2 - 2\beta\varsigma\phi'e^{-\frac{\varsigma^2}{2}} = 0$ Eikonal Equation

$$\therefore \qquad \phi(\varsigma) = -2\beta e^{\frac{-\varsigma^2}{2}} + c$$

$$O(\tau^{-3/2}) \qquad \phi + \varsigma \phi' + \phi'' = c \qquad c = 0$$

$$F(z,s) = \exp\left[-\frac{2\beta}{\sqrt{1-s}}e^{-\frac{z^2}{2(1-s)}}\right]$$

Exact solution of the FFP Equation!

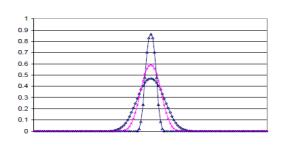


A Formula for the Pinning Probability

$$P(z,s) = 1 - \exp\left[-\frac{2\beta}{\sqrt{1-s}}e^{-\frac{z^2}{2(1-s)}}\right]$$

Satisfies:

$$\begin{cases} \lim_{s \to 1^+} P(z, s) = 0\\ \lim_{s \to 1^+} P(0, s) = 1 \end{cases}$$

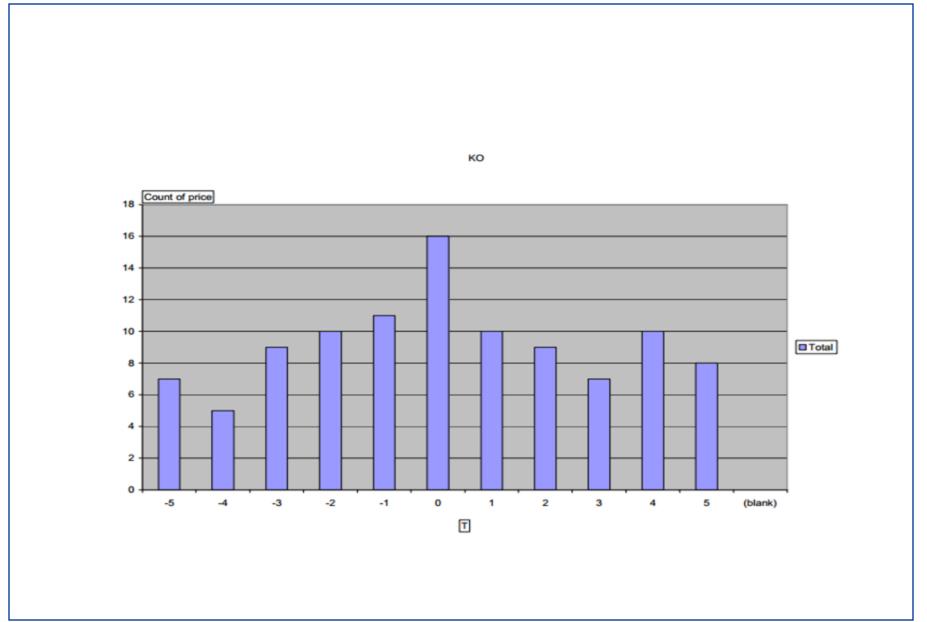


$$Prob(z(1) = 0 \mid z(0) = z_0) = 1 - e^{-2\beta e^{-\frac{z_0^2}{2}}}$$

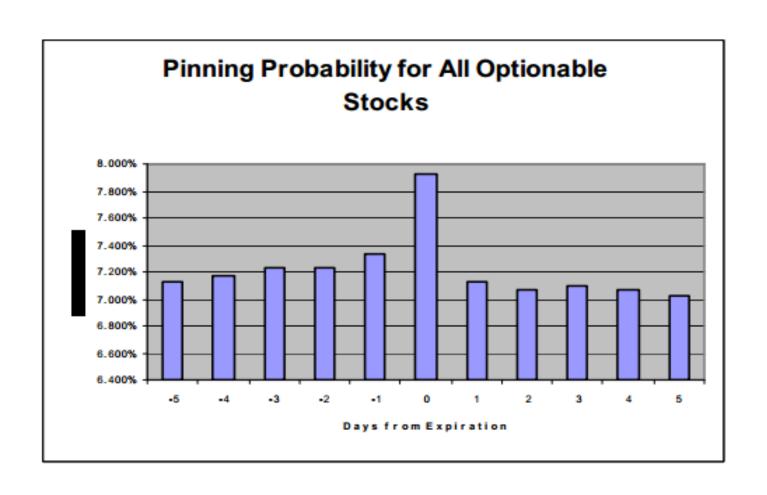


- From the solution (last slide), we see that to first order, the pinning probability should increase linearly in β essentially the OI/σ
- However as β increases the pinning probability should saturate
- As z increases the pinning probability should fall off quadratically to lowest order
- The following show unpublished work of my students- actually their PS solutions for the Event-Driven Finance class

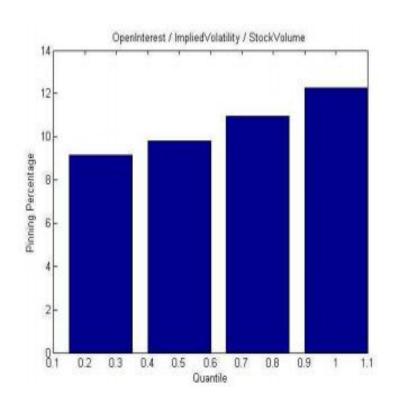






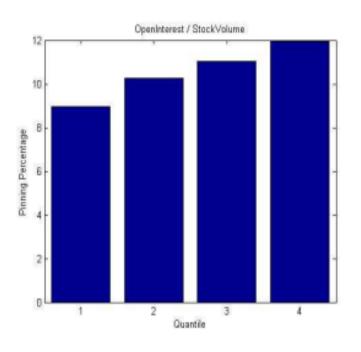


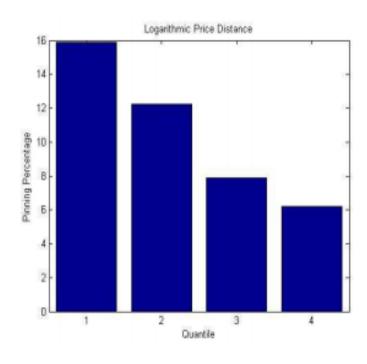




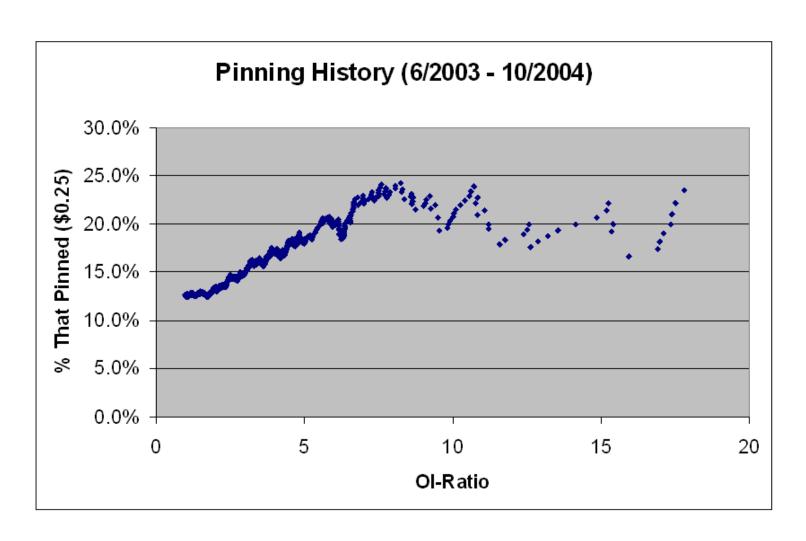


All stocks 2002-2003 (log distance with 1 week to expiry in 2^d graph)



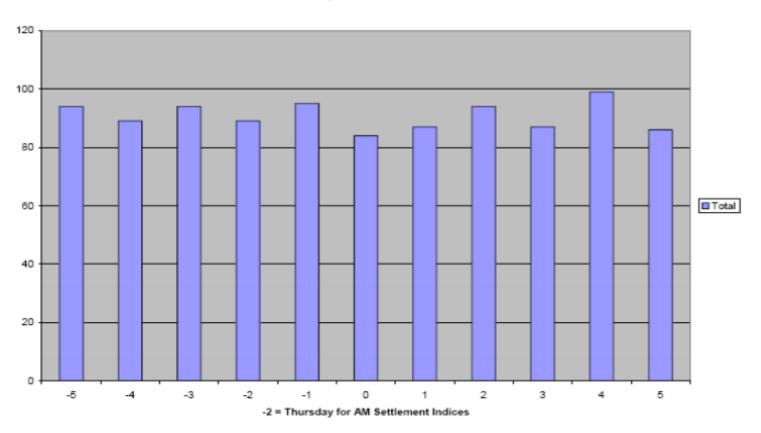


Cumulative likelihood of pinning with 1 week to go to expiry (T. MacFarland)



Indices do not pin

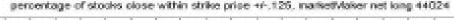
Count of Close Price within \$0.15 of Strike for 25 AM Settlement Indices

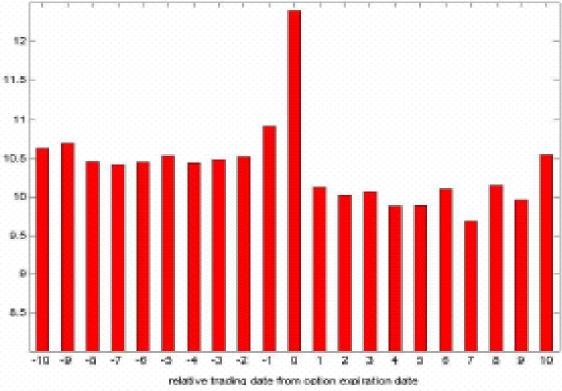


Pinning



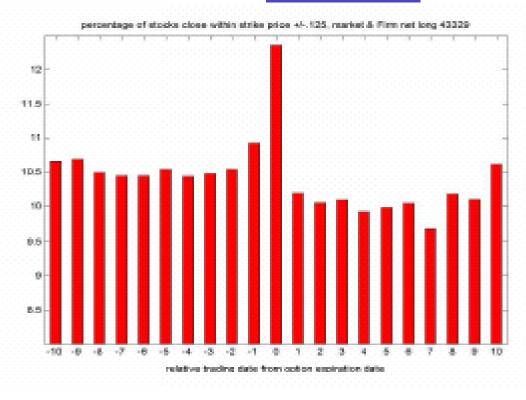
Observations with market-makers net long (~\$0.125)





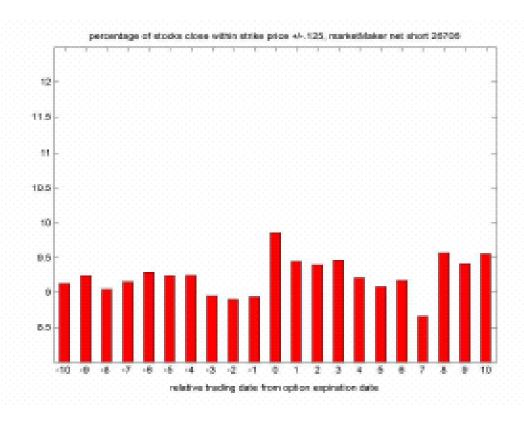


Market-makers + firm proprietary traders <u>net long</u>



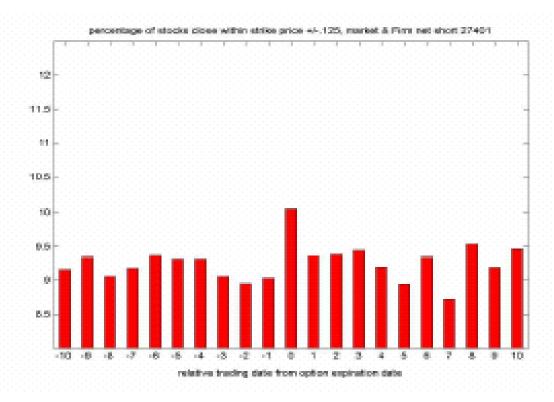








Market-makers + firm proprietary traders <u>net short</u>





Dimensionless Model for Power-Law Price-Impact Function (p>0)

Price change= Price impact+ noise

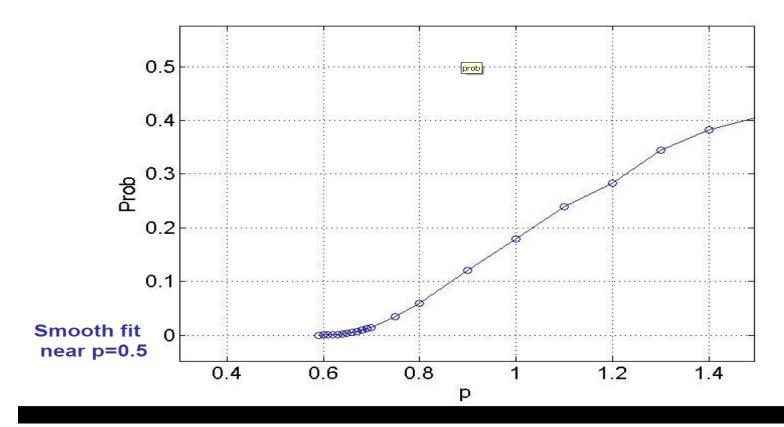
$$\frac{dS}{S} \propto -const \left| \frac{\partial \mathcal{S}}{\partial t} \right|^p sign \left(\frac{\partial \mathcal{S}}{\partial t} \right) dt + \sigma dW$$

$$dz = -\frac{\beta \cdot |z|^p \ sign(z)}{(1-s)^{3p/2}} e^{-\frac{pz^2}{2(1-s)}} ds + d\overline{W}$$

Dimensionless eq. without <u>irrelevant drift terms</u> (alpha=0).



Calculation of Pinning Probabilities by MC Simulation (Gennady Kasyan)





Pinning under non-linear priceimpact models

- (i) If $p \le 1/2$, there is <u>no pinning</u>, i.e. P[z(1)=0|z(0)=z]=0, for all z.
- (ii) If p>1/2 pinning occurs with finite probability (<1) and

ln P(z(1)=0|z(0)=z)
$$\propto -\frac{C(\beta,z)}{2p-1}$$

$$P_{pin} \propto e^{-\frac{C}{2p-1}}, \qquad p > 1/2$$

Impact functions



- The power, p, in the previous slides is included to suggest the possibility of a spectrum of (non-analytic) impact functions
- Recent work by R. Cont supports the value 1.0 for p
- p may be thought of as a measure of the competition between diffusion and pinning pressure- as p decreases, the impact of hedging becomes less and less
- Viewing this as a physicist would, we should typically expect a phase transition in the p- parameter space from pinning to non-pinning as p declines
- If this is the case (we shall see it is), then the experimental fact of pinning should constrain the possible impact models

Real world extensions



- As OI changes with time:
 - Integrate this model
- As other strikes compete:
 - Sum over strikes
- Should work for other instruments that are singly hedged (interest rate, commodity, etc.) but not necessarily indices depending on indirect hedging over multiple instruments



- Complex pricing may result from feedback situations
- Here, independent agents (traders) drive the stock price, which in turn alters their hedging behavior, etc., etc.
- Nevertheless simple models work, as long as they are constrained by appropriate boundary conditions
- Allowing the price impact to be a variable leads to the expected result of a phase transition
- Impact functions weaker than square root are suspectthey cannot explain pinning via our mechanism; if they hold for a class of stocks, those stocks will not pin

