Part 2: Monopoly and Oligopoly Investment

- Irreversible investment and real options for a monopoly
- Risk of growth options versus assets in place
- Oligopoly: industry concentration, value versus growth, and risk
- Open loop investment equilibria
- Perfection and closed loop strategies

Review

- The objective function is

$$\mathsf{E}\int_0^\infty \mathrm{e}^{-rt}\pi(X_t,k_0)\,\mathrm{d}t+\mathsf{E}\int_0^\infty \mathrm{e}^{-rt}\left[\int_{k_0}^{K_t}\pi'(X_t,k)\,\mathrm{d}k\,\mathrm{d}t-\mathrm{d}I_t\right].$$

- The first term is the value of assets in place. The maximized value of the second term is the value of growth options.
- The value of growth options is

$$\int_{k_0}^\infty \sup_{\tau_k}\ \mathsf{E}\left[\mathrm{e}^{-r\tau_k}\{S(X_{\tau_k},k)-1\}\right]\,\mathrm{d}k\,,$$

where

$$S(x,k)=\mathsf{E}\left[\int_t^\infty \mathrm{e}^{-r(u-t)}\pi'(X_u,k)\,\mathrm{d}u\;\middle|\;X_t=x\right].$$

Review continued

- Define the value of option k:

$$V(x,k) = \sup_{\tau} \mathrm{E}\left[\mathrm{e}^{-r\tau}\{S(X_\tau,k)-1\} \;\middle|\; X_0 = x \right] .$$

- When it is optimal to invest, we must have value matching: $V(X_t,k) = S(X_t,k) - 1$.
- Given x, the largest capital stock such that it would be optimal to invest is

$$\kappa(x) \overset{\text{def}}{=} \sup\{k \mid V(x,k) = S(x,k) - 1\} .$$

- The optimal capital stock process is

$$K_t = k_0 \vee \sup_{0 \le s \le t} \kappa(X_s) .$$

Example

- Assume constant returns to scale and constant elasticity demand, so $\pi(x,k) = xk^{1-1/\gamma}$. Assume X is a GBM with coefficients μ and σ, with $\mu < r$.
- The underlying asset for each perpetual call is a GBM:

$$S(x,k) = \frac{1}{r-\mu}\left(\frac{\gamma-1}{\gamma}\right) k^{-1/\gamma} x .$$

- The option values are given by Merton (*BJE*, 1973) as

$$V(x,k) = \frac{1}{\beta-1}\left(\frac{\beta-1}{\beta}\right)^{\beta} S(k,x)^{\beta} ,$$

where β is the positive root of the previous quadratic equation. Assume $\beta > \gamma$. Otherwise, the integral of growth option values is infinite.

Monopoly Example cont.

- Value matching occurs when $S(k, x) = \beta/(\beta - 1)$.
- This is equivalent to

$$k = \kappa(x) \stackrel{\text{def}}{=} \left[\frac{1}{r-\mu}\left(\frac{\beta-1}{\beta}\right)\left(\frac{\gamma-1}{\gamma}\right)x\right]^{\gamma}.$$

- The optimal capital stock process is

$$K_t = k_0 \vee \left[\frac{1}{r-\mu}\left(\frac{\beta-1}{\beta}\right)\left(\frac{\gamma-1}{\gamma}\right)\right]^{\gamma} \sup_{0\le s\le t} X_s^{\gamma}.$$

Reflection and q

- The monopolist invests when $K_t = \kappa(X_t)$, which is equivalent to

$$X_t K_t^{-1/\gamma} = (r-\mu)\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\beta}{\beta-1}\right).$$

- The output price $P_t = X_t K_t^{-1/\gamma}$ is a GBM reflected at

$$p_m^* \stackrel{\text{def}}{=} (r-\mu)\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\beta}{\beta-1}\right).$$

- Similar to perfect competition, marginal q is

$$\frac{\beta}{\beta-1}\frac{P_t}{p_m^*} - \frac{1}{\beta-1}\left(\frac{P_t}{p_m^*}\right)^{\beta}.$$

- The risk of $\mathrm{d}q/q$ decreases as P_t increases towards p_m^*, vanishing at $P_t = p_m^*$, but $J/K > q$, so the risk of $\mathrm{d}J/J$ is different from the risk of $\mathrm{d}q/q$.

Growth is Riskier than Value

- The value of assets in place is proportional to $K_t P_t$: $K_t P_t/(r-\mu)$.
- The value of growth options is proportional to $K_t P_t^\beta$:

$$\left(\frac{\gamma}{(\beta-1)(\beta-\gamma)}\right) K_t \left(\frac{P_t}{p_m^*}\right)^\beta .$$

- Because $\beta > 1$, growth options are riskier.
- As P_t rises towards p_m^*, growth options become a larger part of the total firm value, so the firm becomes riskier.
- Here, assets in place are valued as if the firm never invests again. Erosion of the value of assets in place due to growth is a debit to the value of growth options.

What About Oligopoly?

- Natural question: How does risk vary with value/growth under imperfect competition?
- Related question: Are firms in competitive industries riskier than firms in concentrated industries?
- Hou-Robinson (JF, 2006) show firms in competitive industries have higher average returns.

Aguerrevere (JF, 2009)

Industry demand $P_t = X_t Y_t^{-1/\gamma}$, with X a GBM. Feasible output for firm i at date t is any $y_i \in [0, K_{it}]$. Variable cost cy_i. Firms play Cournot game.

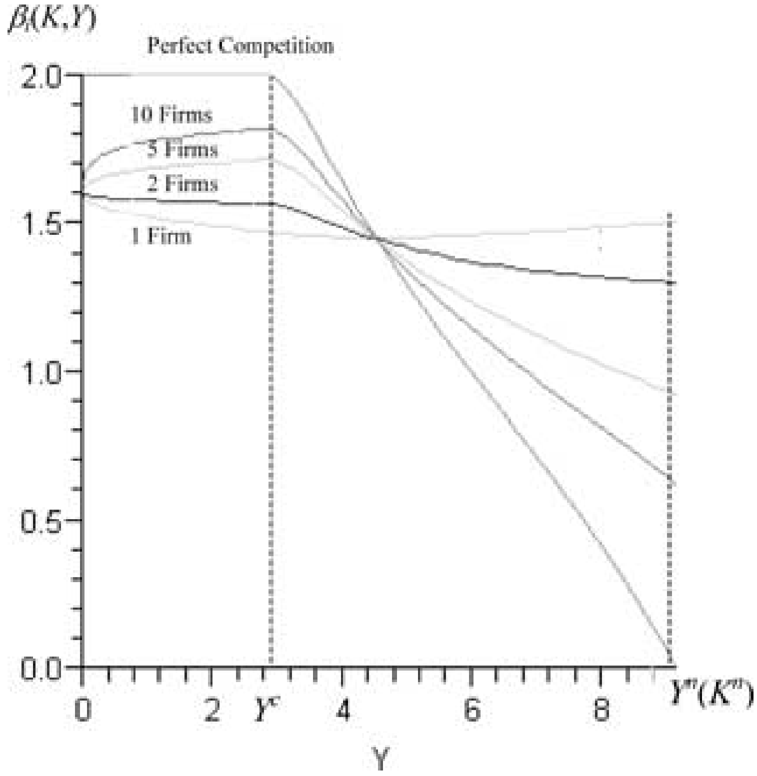
Assume symmetric capital $K_{it} = K_t$ for all i. For X_t above some critical level depending on K_t, firms produce at capacity. For lower X_t, the capacity constraints are not binding and operating cash flows are a constant multiple of X_t^γ.

Firms play dynamic game. Stage game payoffs are $\pi(X_t, K_{it})\,\mathrm{d}t - \mathrm{d}K_{it}$. No depreciation. Strategies are capital stock processes adapted to X. Solution concept is Nash.

Industry Concentration and Risk

Given K_t, there is a switch point x^* below the investment boundary such that:

- For $X_t < x^*$, perfect competition is riskier than oligopoly, which is riskier than monopoly.
- For $X_t > x^*$, monopoly is riskier than oligopoly, which is riskier than perfect competition.



Source: Aguerrevere, F., 2009, "Real Options, Product Market Competition, and Asset Returns, "*Journal of Finance* 64, 957–983.

Open vs. Closed Loop

Terminology comes from engineering.

Example:

1. Air conditioner at a cheap motel: You turn it on High Cool and that's it.
2. At a reasonable motel or hotel: There is a thermostat and you set the temperature you want. The AC then goes on and off based on the current room temperature.

The second system is closed loop. "Closed" means there is feedback from the output to the input. The first system is open loop.

Open Loop Investment Equilibria

Assume the operating cash flow of firm i is $\pi(x, k_i, k_{-i})$, where X is a diffusion and k_{-i} denotes the aggregate capital of firms $j \neq i$. Assume there is no depreciation and each firm has the same initial capital stock k_0.

Open-Loop Equilibrium: Each firm chooses a nondecreasing capital stock process K_{it} adapted to X to maximize

$$\mathsf{E} \int_0^\infty \mathrm{e}^{-rt}\{\pi(X_t, K_{it}, K_{-i,t})\,\mathrm{d}t - \mathrm{d}K_{it}\}\,,$$

taking the stochastic process $K_{-i} \stackrel{\text{def}}{=} \sum_{j \neq i} K_j$ as given.

Solving Symmetric Games

- Each firm i chooses y_i to maximize
$$\max_{y_i} \quad \pi(y_i, y_{-i}) - y_i\,,$$
taking y_{-i} as given.
- The FOC is
$$\pi_{y_i}(y_i, y_{-i}) = 1\,.$$
- Assuming concavity, we obtain a symmetric equilibrium $y_i = a$ by solving
$$\pi_{y_i}(a, (n-1)a) = 1\,. \tag{1}$$
- Define
$$\hat{\pi}(a) \stackrel{\text{def}}{=} \int_0^a \pi_{y_i}(b, (n-1)b)\,\mathrm{d}b\,.$$
- The equilibrium condition (1) is the first order condition for the problem : max $\quad \hat{\pi}(a) - a$.

Open Loop Investment Equilibria

We can find an open-loop equilibrium by maximizing

$$\mathsf{E}\int_0^\infty \mathrm{e}^{-rt}\{\hat{\pi}(X_t, K_t)\,\mathrm{d}t - \mathrm{d}K_t\}\,,$$

where

$$\hat{\pi}(x,k) = \int_{k_0}^{k} \pi_{k_i}(x, a, (n-1)a)\,\mathrm{d}a\,.$$

See Steg, J.-H., 2012, "Irreversible Investment in Oligopoly," *Finance & Stochastics* 16, 207–224.

An Example of Perfection

- Two firms choose quantities q_i to maximize pq_i, where $p = 1 - (q_1 + q_2)/2$.
  - Cournot game: simultaneous moves, equilibrium is $q_1 = q_2 = 2/3$
  - Stackelberg game: player 1 moves first, equilibrium is $q_1 = 1$, $q_2 = 1 - q_1/2$.
- Is the Cournot equilibrium Nash in the Stackelberg game?
- Is the Cournot equilibrium subgame perfect in the Stackelberg game?
- Player 1 is more aggressive in the subgame perfect equilibrium than in the open loop Nash equilibrium (benefit of commitment).

Another Example

Suppose your significant other is willing to get married. Should you?

- You have an American call on marriage.
- You should not exercise early unless there is a dividend.
- Commitment may be the source of the dividend.

Closed-Loop Continuous-Time Strategies

- Consider a game with one player who has two choices (U and D) at each time
- Consider strategy:
    - Play U at date 0.
    - At $t > 0$, play U if played U at all $s < t$ and play D if played D at any $s < t$.
    - Following this strategy, which direction will the player go?

- Pick any $\tau \geq 0$. Playing U for all $t \leq \tau$ and D for all $t > \tau$ is consistent with the strategy.
    - For any $\varepsilon > 0$, at $t = \tau + \varepsilon$ the strategy says to play D if D was played at $\tau + \varepsilon/2$.
    - The problem is that there is no smallest $t > \tau$.

Stochastic Differential Game

Players choose investment rates. Strategies are functions f_i defining investment rates as

$$\frac{\mathrm{d}K_{it}}{\mathrm{d}t} = f_i(t, K_{1t}, \ldots, K_{nt}, X_t)\,.$$

Markov perfect equilibrium: strategies form Nash equilibrium starting at any state $(t, k_1, \ldots, k_n, x)$.

Could assume quadratic adjustment costs, for example.