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Hedging local volume risk using forward markets: Nordic case ☆

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ABSTRACT

With focus on the Nordic electricity market, this paper develops hedging strategies for an electricity distributor who manages price and volume risk from fixed price agreements on stochastic electricity load. Whereas the distributor trades in the spot market at area prices, the financial contracts used for hedging are settled against the system price. Area and system prices are correlated with electricity load, as are price differences. In practice, however, this is often disregarded. Here, we develop a joint model for the area price, the system price and the load, accounting for correlations, and we suggest various strategies for hedging in the presence of local volume risk. We benchmark against a strategy that ignores correlation and hedges at expected load, as is common practice in the industry. Using data from 2013 and 2014 for two Danish bidding areas, we show that our best hedging strategy reduces gross loss by 5.8% and 13.6% and increases gross profit by 3.8% and 9.5%, respectively. Although this is partly due to the inclusion of correlation, we show that performance improvement is mainly driven by the choice of risk measure.

1. Introduction

The Nordic electricity market was liberalized in the late 90's to increase competition and create incentive to invest in new generation capacity and modernize existing production. At the same time, the liberalization reduced the barriers on import and export between countries, allowing for more efficient use of many power production technologies.

Currently, the Nordic market covers the countries in the Nordic and Baltic regions, i.e. Denmark, Norway, Sweden, Finland, Estonia, Latvia and Lithuania. It is divided into 17 bidding areas with individual area prices based on local supply and demand. Furthermore, an overall market price for electricity, referred to as the system price, is determined for contractual purposes. This price is based on aggregated supply and demand and disregards transmission constraints between bidding areas. In contrast, the bidding areas are established to avoid congestion in the system. The area price and the local load are, therefore, highly correlated. It is important to account for this market design in the derivation of hedging strategies.

In this paper we study the hedging problem of a Nordic distribution company that has agreed to deliver electricity to customers at a fixed price. The company has to buy electricity in the spot market, but knows neither the future electricity demand of the customers nor the future market price of electricity. As trades in the spot market are settled at the area price, the distributor is thereby exposed to both volume risk and area price risk. To mitigate risk, the company can lock in part of its profit by buying financial contracts on electricity, in advance and at a fixed price. In the Nordic electricity market, however, financial contracts are settled against

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the system price and not the area price. With significant differences between the system price and the area price, especially for periods with high load, this introduces considerable basis risk. Basis risk may be managed using forward contracts on the price difference. Nevertheless, only monthly contracts are available. As a result, the distribution company cannot completely eliminate risk from fixed price agreements. In spite of this, in 2010 more than 50% of contracts for electricity were based on fixed price agreements in the Nordic market and in EU 60% of contracts were fixed price agreements.[1] This paper contributes to the literature by developing a joint model for the area price, the system price and the load, accounting for both cross-correlations and auto-correlations, and by suggesting strategies for hedging in the presence of local volume risk. In addition to using base load and peak load contracts for hedging, we study the impact of including contracts for difference. Furthermore, since the profit distribution is asymmetrical, we complement the traditional variance-based approach by using a one-sided measure of risk in the hedging problem. We benchmark against the strategy that ignores correlation and hedges at the expected load, as is common practice in the industry.

The importance of accounting for correlation between electricity price and load has already been demonstrated in the existing literature. As an example, Bessembinder and Lemmon (2002) develop an equilibrium-based market model and find that correlation has a substantial impact on the optimal hedging strategies in a forward market. Closer to our work is Oum et al. (2006), who consider a load serving entity and study the influence of correlation on the residual risk following optimal hedging. The authors derive analytical solutions to the hedging problem for specific utility functions and approximate these solutions by call options to compensate for the lack of contracts to hedge volume risk. Their results likewise show that correlation has a significant impact on the payoff structure as well as on the hedging strategy. Whereas these references use a single-period setting, we include multiple periods and thereby capture the basis risk that arises as contracts cover an entire month. This makes our hedging strategies applicable to the Nordic Market.

An example of using a more advanced electricity price model for hedging is provided by Coulon et al. (2013), who develop a three-factor model with load-based regime switching to model the electricity market of Texas. The authors study variations of daily payoffs, using spark spreads or call options and considering a single day and one-dimensional hedging. The inclusion of load-based regime switching makes calibration and estimation much more difficult on longer time horizons, and, therefore, is not considered in this paper. For further electricity price modeling, Erlwein et al. (2010) and Weron et al. (2004) develop advanced reduced-form models that involve jumps and regime switching and present algorithms to calibrate their models to price data. In addition to such single-factor models, multi-factor models with jumps and regime switching have also been used by Deng (1999) and Schwartz and Smith (2000), capturing both short-term and long-term dynamics of electricity prices. Moreover, their approach is extended in Burger et al. (2004) to include a demand component in the pricing of derivatives. For a thorough review of electricity price models, see also Carmona and Coulon (2014), covering both structural and reduced-form models. In contrast to these references, our price model is specifically tailored to the Nordic market by including both load, area and system prices, whereas the modeling of each component is restricted to a single factor and does not involve jumps. The inclusion of area and system prices makes it possible to use contracts for difference when hedging. To the best of our knowledge, the literature has not previously addressed hedging strategies to manage differences between the area and system prices in the Nordic market.

[1] ECME (2010).

The paper is organized as follows. The spot and forward markets are described in Section 2. This includes the dynamics of the system price, the area price and the load as well as the financial contracts used to manage the uncertainty of payoffs. Section 3 covers the various sources of risk faced by a company trading in the spot and forward markets and offering fixed price agreements, whereas we formally introduce the accompanying hedging problem in Section 4. Section 5.1 analyzes the load and price data, defines seasonal components and describes calibration and Section 5.2 develops the joint model for the system price, the area price and the load. When calibrated to data from 2012 and applied to data from 2013 and 2014, we analyze the corresponding hedging strategies in Section 6. We study the effect of using another risk measure, the impact of including the contracts for difference and the implications of improved forecast of average prices. Finally, in Section 7, we summarize our findings and discuss future work.

2. The trading of electricity

In this section we describe the market dynamics of the Nordic electricity market and the financial instruments that will be used for hedging. We focus on the Nordic spot market, *Nord Pool Spot*, and the corresponding forward market at *Nasdaq Commodities*.

2.1. Area price and system price

In the Nordic and Baltic region, the system price provides an overall market price for electricity and is determined by an equilibrium that disregards the grid. In contrast, the area prices should ensure that electricity is produced in the least expensive way, aiming at a market equilibrium that accounts for transmission. In the absence of transmission congestion, all area prices coincide with the system price. In its presence, area prices are determined on the basis of the system price by adjusting for transmission. By raising the area price, local supply will increase and local demand will decrease. Similarly, by reducing the area price, local supply will decrease whereas local demand will increase. Thus, by raising the area price in bidding areas that would ideally be importing beyond their transmission limits, import is reduced. Likewise, by reducing the area price in bidding areas that would be exporting beyond their transmission limits, export is reduced. Thus, in equilibrium, bidding areas with low marginal cost will be exporting at full transmission capacity and bidding areas with high marginal cost will be importing at full capacity, and so, electricity is produced at minimal costs.

The load on the grid varies significantly throughout the day, which produces variations in both area price and system price. Differences between the area price and the system price, however, often occur in periods with high load. The reason is that capacity limits on transmission lines between bidding areas are met more often in hours with high load than hours with low load. Here, we focus on two large portfolios of fixed price contracts in DK1 and DK2, respectively. The load of the DK1 portfolio is shown in Fig. 1. This figure confirms the occurrence of price differences in hours of high load. Moreover, market prices suggest that the bidding area DK1 is importing throughout most of August (the area price exceeds the system price), but is exporting during a few hours in the beginning of February (the system price exceeds the area price). Other factors, such as changes in demand in other bidding areas and varying supply of wind power, may create differences in periods with low load.

2.2. Financial contracts on electricity

In the Nordic region, financial contracts on electricity prices are traded at *Nasdaq Commodities*. Here, we consider three types of contracts. The most simple type is a base load contract on the system price that covers every hour of a given month. It is not related to

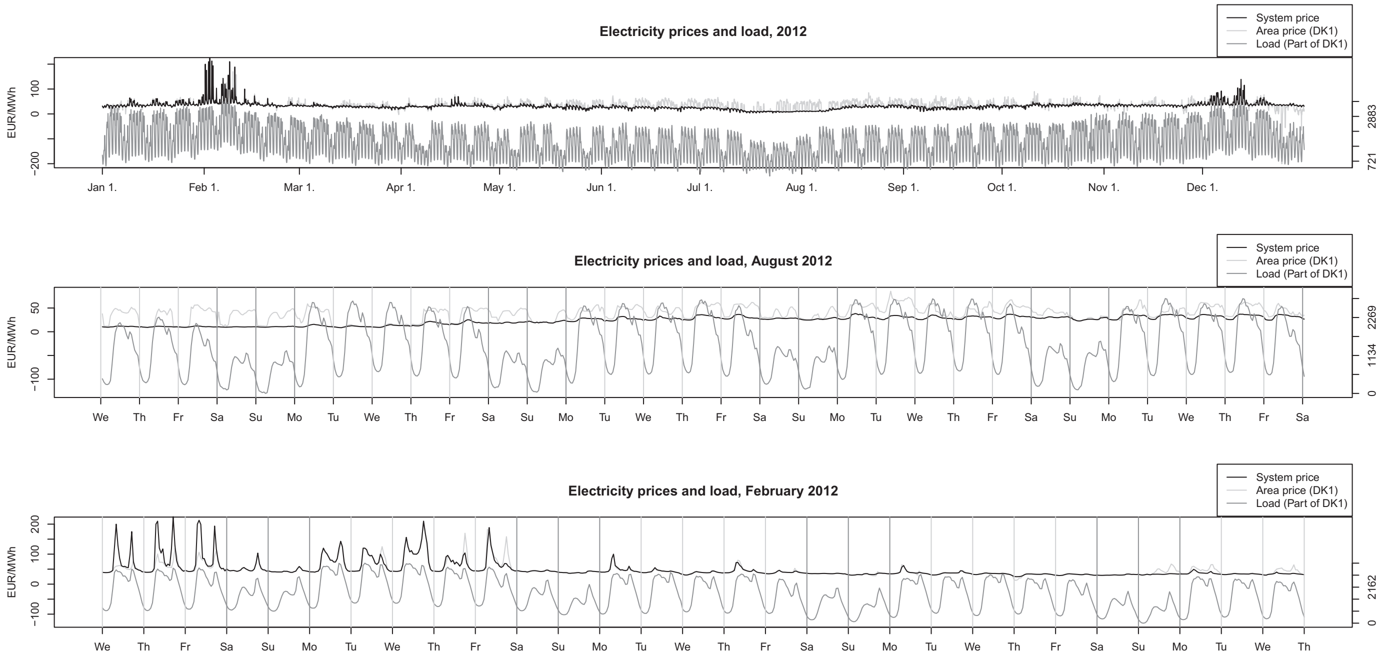


Fig. 1. Electricity prices and load for 2012 in West Denmark (DK1).

physical delivery of electricity, but is a purely financial contract that pays the difference between the system price and the forward price for every hour of the month. Load typically varies between a peak level and an off-peak level, as seen in Fig. 2. To manage these variations the market also includes peak load contracts that pay the difference between the system price and the forward price in peak hours, 8–20, during weekdays. A portfolio of base load and peak load contracts can to some extend replicate the load profile.

Base load and peak load contracts are both settled against the system price and not the area price that is the basis for physical trading. To handle the risk related to differences between area and system prices, we include contracts for difference (CfD). This type of contract pays the difference between the area price and the system price minus the cost of the CfD and covers the entire month. In spite of including the CfD, however, it remains impossible to completely eliminate the risk related to delivering an uncertain quantity, i.e. the volume risk.

3. Hedging volume risk

We start by assuming that the area price and the system price coincide and study hedging strategies when facing volume risk in a single-period setting.

When planning to buy a fixed load L_T at an uncertain price S_T at time T and resell it at a fixed price F, risk can be completely eliminated by buying L_T futures contracts with maturity T at time t, for $t < T$. The contracts pay the difference between the uncertain

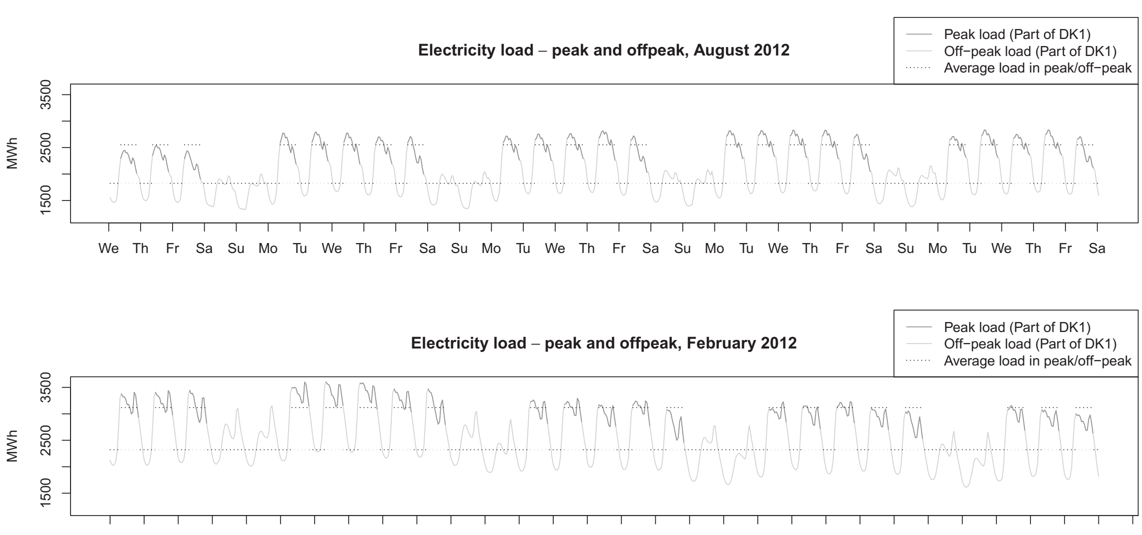


Fig. 2. Peak and off-peak load for February and August 2012 in Western Denmark (DK1).

price S_T and a fixed forward price $q_t(T)$. Thus, at time T we have the payoff

$$(F - S_T)L_T + (S_T - q_t(T))L_T = (F - q_t(T))L_T. \tag{1}$$

As a result, the purchase price is locked at $q_t(T)$, eliminating the price risk.

In contrast, when planning to buy an uncertain load L_T at an uncertain price S_T and reselling it at a fixed price F, it is impossible to completely eliminate the risk using only futures contracts. By buying V futures contracts at time t, the payoff at time T will be

$$(F - S_T)L_T + (S_T - q_t(T))V = (F - q_t(T))V + (F - S_T)(L_T - V).$$

If we could choose $V = L_T$, the risk would be eliminated. The problem is that L_T is stochastic whereas V has to be fixed at time t, for $t < T$. For this reason, we are interested in the quality of a hedge, which introduces the need for risk measures. See Artzner et al. (1999) for a detailed analysis of risk measures.

3.1. Variance as a measure of risk

A classical measure of risk is the variance of the payoff, i.e.

$$Var\left[(F - S_T)L_T + (S_T - q_t(T))V\right], \tag{2}$$

which is minimized by

$$V^* = \frac{Cov(S_T, S_T L_T)}{Var(S_T)} - F\frac{Cov(L_T, S_T)}{Var(S_T)}, \tag{3}$$

as shown in Lemma A1 of Appendix A. We refer to the minimizer as the minimum variance (Min Var) hedge. We note that V* is independent of the forward price $q_t(T)$, but not the fixed price F. We can rewrite Eq. (3) to

$$V^* = E(L_T) - (F - E(S_T))\frac{Cov(S_T, L_T)}{Var(S_T)} + \frac{Cov\left((S_T - E(S_T))^2, L_T\right)}{Var(S_T)}, \tag{4}$$

cf. Lemma A2 of Appendix A. Hence, for any distribution, it is optimal to hedge the expected load and compensate for expected unhedged payoff, depending on the covariance between price and load, and for the covariance between the quadratic deviation from the expected price and the load. If S_T and L_T are independent, $V^* = E(L_T)$ and the optimal strategy is to hedge the expected load. This is the straightforward extension of the case with fixed load and we refer to this strategy as the mean hedge.

Example 3.1. Assume S_T and L_T are jointly Normal with correlation ρ and standard deviations σ_S and σ_L, respectively. Then, the minimal variance hedge simplifies to

$$V^* = E(L_T) - (F - E(S_T))\rho\frac{\sigma_L}{\sigma_S}. \tag{5}$$

as shown in Lemma A3 in Appendix A. We note that if L_T and S_T are positively correlated, the minimal variance hedge satisfies $V^* < E(L_T)$ for $F - E(S_T) > 0$ and $V^* > E(L_T)$ for $F - E(S_T) < 0$. Finally, if $F - E(S_T) = 0$ or L_T and S_T are uncorrelated (and hence independent, as they are jointly Normal), the optimal strategy is again to hedge the expected load. Since the correlation between load and electricity price is typically significant, the mean hedge is suboptimal unless $F - E(S_T)$ is small.

The variance measures expected quadratic deviations from the mean and is a symmetrical risk measure. It is useful as it often allows for closed-form minimizers. Moreover, for symmetrical payoff distributions minimizing the two-sided risk is equivalent to minimizing the one-sided risk. In general, however, using the variance may not only reduce the downside but also the upside. Because of this, and as payoffs distributions are not necessarily symmetrical, we consider another classical measure of risk, namely the expected loss.

3.2. Expected loss as a measure of risk

We define expected loss as

$$-E\left[\min\left((F - S_T)L_T + (S_T - q_t(T))V, 0\right)\right]$$

which is the absolute value of expected payoff, conditional on the payoff being negative. We refer to the minimizer as the minimum loss (Min Loss) hedge. When facing price risk only, i.e. load is fixed, and provided $F > q_t(T)$, both the variance and the expected loss are minimized by $V^* = L_T$ with minimum 0. However, in the presence of volume risk, the two risk measures may result in different hedging strategies as demonstrated by the following example.

Example 3.2. Assume again that S_T and L_T are jointly Normal with $E(S_T) = 35$, $E(L_T) = 0.5$, $\sigma_S = 10$, $\sigma_Q = 0.1$, $\rho = 0.5$ and $q_t(T) = 29.75$. We compare the two strategies that minimize the expected loss and the variance, respectively, and further include the mean hedge for comparison. The strategy minimizing the expected loss is determined numerically. In the first plot of Fig. 3 the fixed price is $F = 40$ and the expected payoff per unit electricity is positive ($F - E(S_T) > 0$), whereas this is not the case in the second plot with $F = 30$. In both cases the forward price for electricity is below the expected price ($q_t(T) < E(S_T)$), which is known as backwardation. With expected loss, this makes the expected payoff increase linearly with the hedging volume. The variance, however, is always quadratic with a global minimum. We note that both the minimal variance hedge and the minimal loss hedge are below the mean load in the case of positive expected payoff and above the mean hedge in the case of negative expected payoff. Moreover, the hedged payoff with minimum loss has a lighter tail for negative payoffs than the minimum variance hedge in the case with negative expected payoff. It likewise has a heavier tail for positive payoffs. Thus, the skewness of the payoff density is affected when using expected loss.

For Normal distributions and positive correlation, the minimal variance hedge is always below the mean load in the case of positive expected payoff and above the mean load in the case of negative expected payoff, as observed from Eq. (5). This may not always be the case for the minimal loss hedge. For instance, if the forward price is higher than the expected price, known as contango, the minimum loss hedge deviates significantly from the mean hedge in the opposite direction of the minimum variance hedge, see Appendix B.

4. Hedging in the Nordic market

We proceed to introduce the specific problem of hedging in the Nordic market and discuss its relation to the analysis of volume risk in the previous section.

As prices are fixed for every hour, we let S_t and S_t^{sys} denote the area price and the system price, respectively, in hour t. Moreover, we let L_t denote the percentage of the maximal load delivered to the local customers in hour t. As a result, payoffs are scaled by the maximal load. Letting F_j be the fixed price for electricity in month j, the sales revenue for a given hour t in month j are

$$(F_j - S_t)L_t.$$

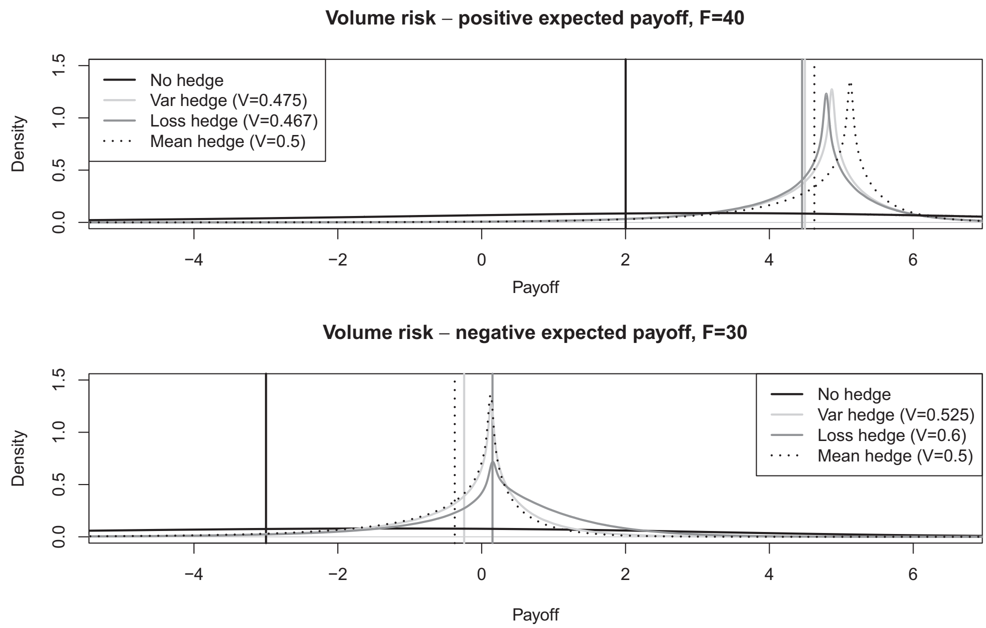


Fig. 3. Payoff densities and means (vertical lines) with parameters from Example 3.2 (Backwardation).

For risk mitigation, we consider three types of contracts, that is, base load contracts, peak load contracts and contracts for difference. We let q_j^b denote the forward price of the base load contract and V_j^b the percentage of maximal load that is covered by base load contracts in month j. For every hour of month j, the following cash flow is obtained by buying base load contracts

$$\left(S_t^{sys} - q_j^b\right) V_j^b.$$

Similarly, for the peak contracts we let q_j^p denote the forward price and V_j^p the percentage of the maximal load that is covered by the peak load contracts in month j. For every hour covered by peak load contracts in month j, the following cash flow is obtained

$$\left(S_t^{sys} - q_j^p\right) V_j^p.$$

We let m_j be the set of all hours in month j, $peak_j$ be the subset of m_j that are peak hours, and off_j be the subset of m_j that are offpeak hours. Finally, we let q_j^d denote the forward price and V_j^d denote the percentage of maximal load that is covered by CfDs. For every hour in month j, the following cash flow is obtained by buying CfD contracts

$$\left(S_t - S_t^{sys} - q_j^d\right) V_j^d.$$

Thus, the total cash flow in hour t of month j is given by

$$\begin{aligned}&(F_j - S_t)L_t + \left(S_t^{sys} - q_j^b\right) V_j^b + 1_{(t \in peak_j)} \left(S_t^{sys} - q_j^p\right) V_j^p \\ &\quad + \left(S_t - S_t^{sys} - q_j^d\right) V_j^d. \end{aligned} \tag{6}$$

where $1_{(t \in peak_j)}$ is 1 if $t \in peak_j$ and 0 otherwise. By introducing the effective hedging volume in peak hours, $V_j^e = V_j^b + V_j^p$, the effective forward price in peak hours, $q_j^e = q_j^b V_j^b / V_j^e + q_j^p V_j^p / V_j^e$, and rewriting Eq. (6), we obtain

$$\begin{aligned}&-q_j^d V_j^d + \left(S_t^{sys} - S_t\right)\left(L_t - V_j^d\right) \\ &\quad + 1_{(t \in off_j)}\left[\left(F_j - q_j^b\right) V_j^b + \left(F_j - S_t^{sys}\right)\left(L_t - V_j^b\right)\right] \\ &\quad + 1_{(t \in peak_j)}\left[\left(F_j - q_j^e\right) V_j^e + \left(F_j - S_t^{sys}\right)\left(L_t - V_j^e\right)\right]. \end{aligned} \tag{7}$$

This formulation shows how variation in the cash-flow originates from only two random terms for both peak or offpeak hours. The first term is the difference between the area price and the system price times the deviations from the hedging volume of the CfDs. Thus, if $L_t - V_j^d$ is small at a time when the system price and the area price differ, it barely has an impact on the payoff. The second random term is the difference between the system price and the fixed price times the deviations from the hedging volume of the base load and peak load contracts. As before, we note that if $L_t - V_j^b$ or $L_t - V_j^e$ is small when the system price deviates from F_j, it barely affects the payoff. This reveals that to minimize variations it is most important to replicate the load in periods of volatile prices.

We immediately recognize the payoff structure in the presence of volume risk, although with a sum of two components, $S_t^{sys} - S_t$ and $F_j - S_t^{sys}$, times the corresponding differences between the hedging volume and the load. As the system prices in peak hours are typically above the fixed price and the system prices in off-peak hours are typically below the fixed price, the results of Example 3.1 suggest hedging above the mean load in peak hours and below the mean load in off-peak hours. Unfortunately, the two terms cannot be handled separately as both include the system price and the load.

If we could perfectly predict L_t and adjust V_j^d, V_j^b and V_j^p every hour, price risk could be completely eliminated. This could be done by setting $V_j^d = L_t$ for all hours, $V_j^b = L_t$ for offpeak hours and $V_j^e = L_t$ for peak hours. This would result in the following cash flow

$$\left(F - q_j^d - 1_{(t \in off_j)} q_j^b - 1_{(t \in peak_j)} q_j^e\right) L_t. \tag{8}$$

Thus, q_j^d becomes the cost of hedging the difference between the area price and the system price and $F - q_j^b$ or $F - q_j^e$ becomes the payoff

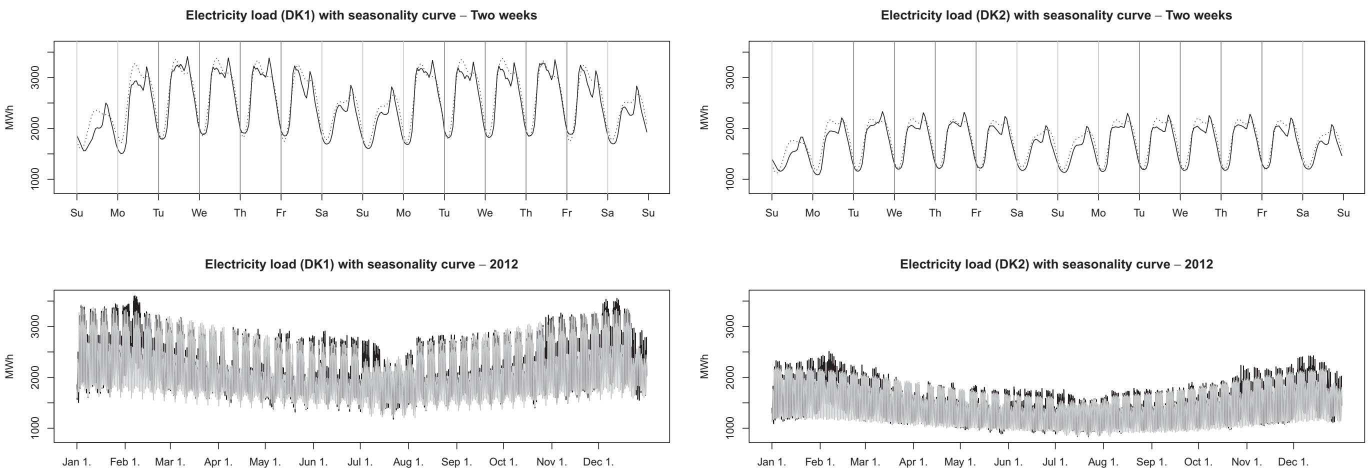


Fig. 4. Seasonality curves (gray) calibrated to historical electricity load (black) in 2012.

that is locked when hedging. The problem is that L_t is stochastic and varies from throughout each month, whereas V_j^b, V_j^p and V_j^d have to be fixed for month j, which creates the need for risk measures to determine the optimal hedge.

We proceed to modeling load and prices, with special emphasis on seasonality and correlation.

5. Modeling load and prices

This section introduces three models for the stochastic evolution of the area price, the system price and the load. The three models differ mainly by the modeling of the correlation structure. Appendix C shows correlation plots for the price data. As observed, the area and system prices are highly correlated. In the first two models we use a simple correlation structure, assuming independence between the system price and the differences between area and system prices. The plots reveals that this assumption does not entirely fit to data. Therefore, in the third model, we directly model correlation between the area and system prices. As further observed, both area and system prices are correlated over time, which is likewise captured by the third model. For reasons of confidentiality of the data, we cannot show correlation plots for price and load.

For the first and second models we obtain analytical solutions to the minimal variance hedging problem, for which the objective is the sum of variances of hourly cash flows

$$\sum_{t\in \mathrm{m}_j} Var\left((F_j - S_t)L_t + \left(S_t^{sys} - q_j^b\right)V_j^b + \left(S_t - S_t^{sys} - q_j^d\right)V_j^d + 1_{(t\in peak_j)}\left(S_t^{sys} - q_j^p\right)V_j^p\right).$$

For the third model we solve the hedging problem numerically and use the minimal loss, the objective of which is

$$-\sum_{t\in \mathrm{m}_j} E\left[\min\left((F_j - S_t)L_t + \left(S_t^{sys} - q_j^b\right)V_j^b + \left(S_t - S_t^{sys} - q_j^d\right)V_j^d + 1_{(t\in peak_j)}\left(S_t^{sys} - q_j^p\right)V_j^p, 0\right)\right].$$

This risk measure focuses on the expected hourly losses. Whereas we expect the payoffs from the minimal variance hedge to vary very little, we expect those of the minimal loss hedge to decrease very little over time.

5.1. Seasonality

To capture seasonality in load and prices, we calibrate seasonality curves to data from 2012 and use these to predict seasonality curves for 2013 and 2014. Furthermore, we describe how to calibrate expected monthly prices using base load contracts and peak load contracts. Finally, we determine a fixed price for 2013 and 2014 on the basis of 2012 data.

5.1.1. Seasonality in load

The load data is from two portfolios of customers on fixed price contracts from the bidding areas West Denmark (DK1) and East Denmark (DK2). The price data includes area prices for the two bidding areas as well as the system price for 2012–2014. The bidding areas have very different load characteristics and are therefore modeled separately. In particular, the load portfolio of DK1 is strongly affected by weekends and holidays, whereas the portfolio in DK2 is primarily affected by yearly variations in demand.

We let θ_t be the periodic function

$$\theta_t = \alpha + \left(1 + A_0\cos\left(\frac{2\pi}{\tau_0}t + B_0\right)^2\right)\sum_{i=1}^{p} A_i \sin\left(\frac{2\pi}{\tau_i}t + B_i\right), \tag{9}$$

with p periods $\tau_0, \ldots \tau_p$, amplitudes $A_0, \ldots, A_p$ and phases $B_0, \ldots B_p$. A_0, τ_0 and B_0 serve to capture seasonal behavior in the amplitude that occurs for the load of DK2 and we set $A_0 = 0$ in DK1. For calibration the load data is split into three subsets; weekdays, weekends and holidays. The function θ_t is calibrated to data from each of the subsets, numerically minimizing the sum of quadratic deviations, and combined to the dotted curve shown in Fig. 4. The periods are based on peaks of autocorrelation functions for 2012 data, with $\tau_0 = 2\cdot24\cdot365$, $\tau_1 = 12$, $\tau_2 = 24$, $\tau_3 = 24\cdot7$, $\tau_4 = 24\cdot365$, $\tau_5 = 24\cdot365$.[2]

Using the load for 2012 we predict the seasonality curves for 2013 and 2014 based on holidays, weekends and day-light savings. To reflect the long-term increase of load, α is adjusted to match the yearly average, which can usually be predicted with high accuracy by electricity companies. Fig. 5 shows that the load is extremely accurately predicted, i.e. the behavior of the data is very close to that of the function θ_t. This is also confirmed by coefficients of determination for out-of-sample data of 0.823 and 0.923 for DK1 and DK2, respectively.

5.1.2. Seasonality in prices

We apply the same approach for calibration and prediction of seasonality in prices. In the periodic function, we let $A_0 = 0$. The calibration results are shown in Fig. 6 with $\tau_1 = 12, \tau_2 = 24, \tau_3 = 24\cdot7$.

[2] We use two curves with yearly frequency to capture the yearly patterns.

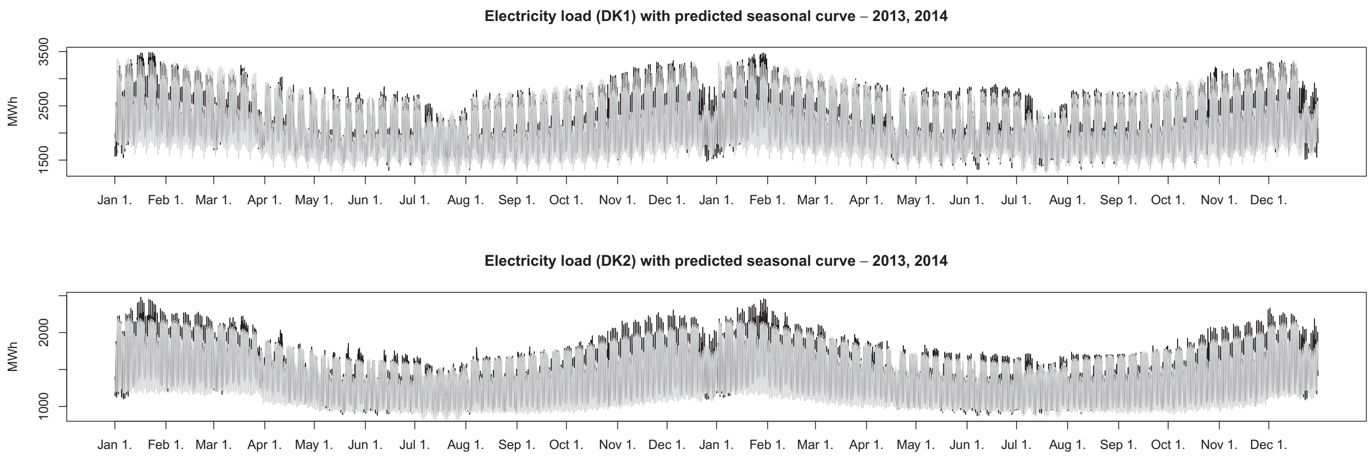


Fig. 5. Predicted seasonality curves (gray) for 2013 and 2014 and historical electricity load (black).

To adjust for more long-term variations in the system price, the forward prices of base load contracts and peak load contracts are used to adjust the monthly mean of the seasonality curves for the system price in peak and off-peak periods such that

$$\frac{1}{|peak_j|}\sum_{t\in peak_j}\theta_t^{sys} = q_j^p,$$
$$\frac{1}{|\mathrm{m}_j|}\sum_{t\in \mathrm{m}_j}\theta_t^{sys} = q_j^b.$$

To simplify results, we ignore the market price of risk as well as discounting. Furthermore, due to risk premium and seasonal bias in forward prices for base load contracts and CfDs, see Bessembinder and Lemmon (2002) and Kristiansen (2004), we do not use them to adjust the seasonality curves for the area prices. Fig. 7 shows that for prices randomness dominates seasonality and predictability is low, which is reflected by coefficients of determination for out-of-sample data of 0.213, 0.211 and 0.363 for the DK1 area price, the DK2 area price and system price, respectively.[3]

5.1.3. Fixed price of electricity

The fixed price F_j for each month in 2013 and 2014 is determined as

$$F_j = \frac{\sum_{t\in \mathrm{m}_j} S_t L_t}{\sum_{t\in \mathrm{m}_j} L_t},$$

using the data from 2012. This implies that

$$\sum_{t\in \mathrm{m}_j} F_j L_t = \sum_{t\in \mathrm{m}_j} S_t L_t$$

and that the company would break even in 2012. With this construction F_j will typically be higher than the average off-peak price and lower than the average peak price, indicating expected profit in off-peak hours and expected loss in peak hours. In practice F_j is increased with a margin to increase profitability of the contract and compensate for the risk, but we start by studying the problem without a margin .

[3] The coefficient of determination for DK1 has been computed without including a five hour price spike with prices over 1900 Euro/MWh in June 2013.

5.2. Stochastic modeling

We consider three different models for modeling the deviations from the seasonality curve, all with the underlying assumption that

$$S_t = \theta_t^S + \tilde{S}_t, \tag{10}$$

$$S_t^{sys} = \theta_t^{sys} + \tilde{S}_t^{sys}, \tag{11}$$

$$L_t = \theta_t^L + \tilde{L}_t, \tag{12}$$

where θ_t^S, θ_t^{sys} and θ_t^L are seasonal components of the area price, the system price and the load and $\tilde{S}_t$, $\tilde{S}_t^{sys}$ and $\tilde{L}_t$ are the deseasonalized components. The deseasonalized data inherits correlations over time as well as cross-correlations between the area price, system price and load. In the first two models, however, we disregard auto-correlations and assume a simple cross-correlation structure, obtained by formulating the models in terms of the difference between the area price and the system price. The first model assumes independence between load and price, whereas the second allow for correlation between the two. The third model incorporates temporal correlation, mean reversion as well a more advanced structure for cross-correlations by a direct modeling of the area price.

5.2.1. Model 1: independence of load, system price and price difference, independence over time

In this model we let $\tilde{\epsilon}_t = \tilde{S}_t - \tilde{S}_t^{sys}$ be the difference between the deseasonalized area and system prices such that the area price is the system price plus some noise caused by congestion. We assume that

$$\begin{pmatrix} \tilde{S}_t^{sys} \\ \tilde{\epsilon}_t \\ \tilde{L}_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{sys}^2 & 0 & 0 \\ 0 & \nu^2 & 0 \\ 0 & 0 & \sigma_L^2 \end{pmatrix} \right),$$

i.e. $\tilde{S}_t^{sys}, \tilde{\epsilon}_t, \tilde{L}_t$ are independent, and that $\left(\tilde{S}_t^{sys}, \tilde{\epsilon}_t, \tilde{L}_t\right)$ are independent of $\left(\tilde{S}_{t'}^{sys}, \tilde{\epsilon}_{t'}, \tilde{L}_{t'}\right)$ for $t \neq t'$. This is equivalent to

$$\begin{pmatrix} S_t \\ S_t^{sys} \\ L_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \theta_t^S \\ \theta_t^{sys} \\ \theta_t^L \end{pmatrix}, \begin{pmatrix} \sigma_{sys}^2 + \nu^2 & \sigma_{sys}^2 & 0 \\ \sigma_{sys}^2 & \sigma_{sys}^2 & 0 \\ 0 & 0 & \sigma_L^2 \end{pmatrix} \right),$$

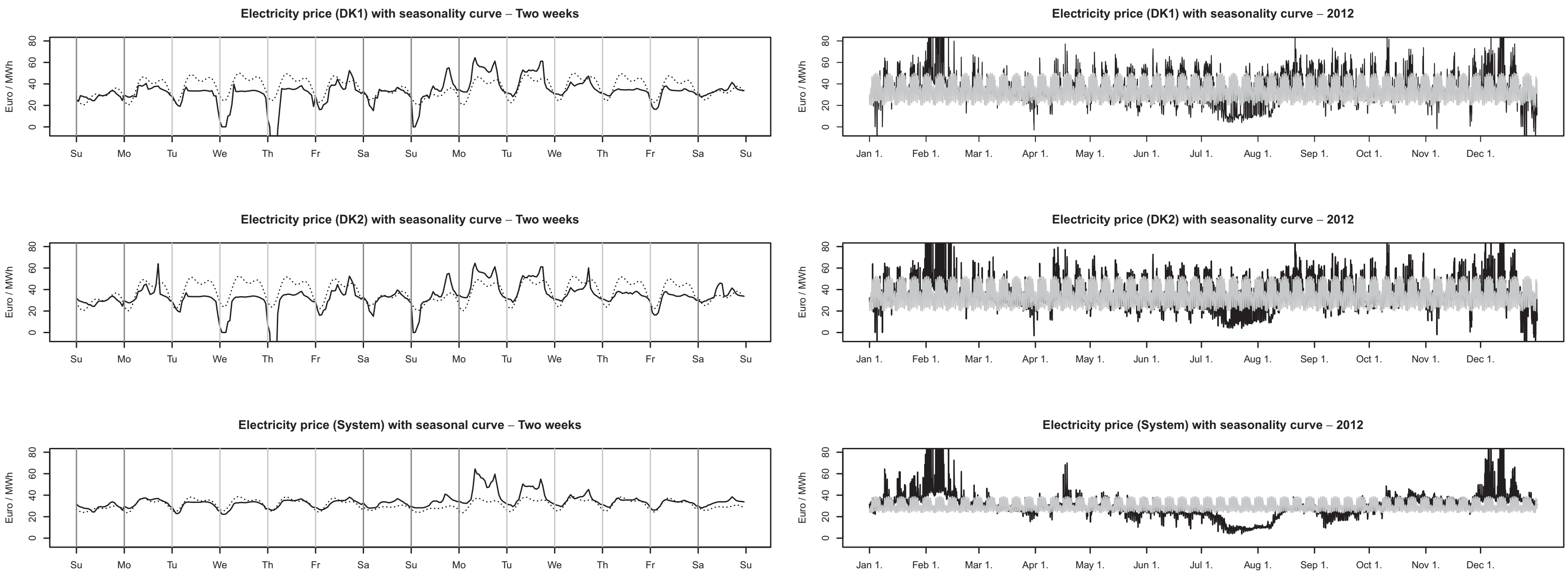


Fig. 6. Calibrated seasonality curves (gray) and historical electricity prices (black) for 2012.

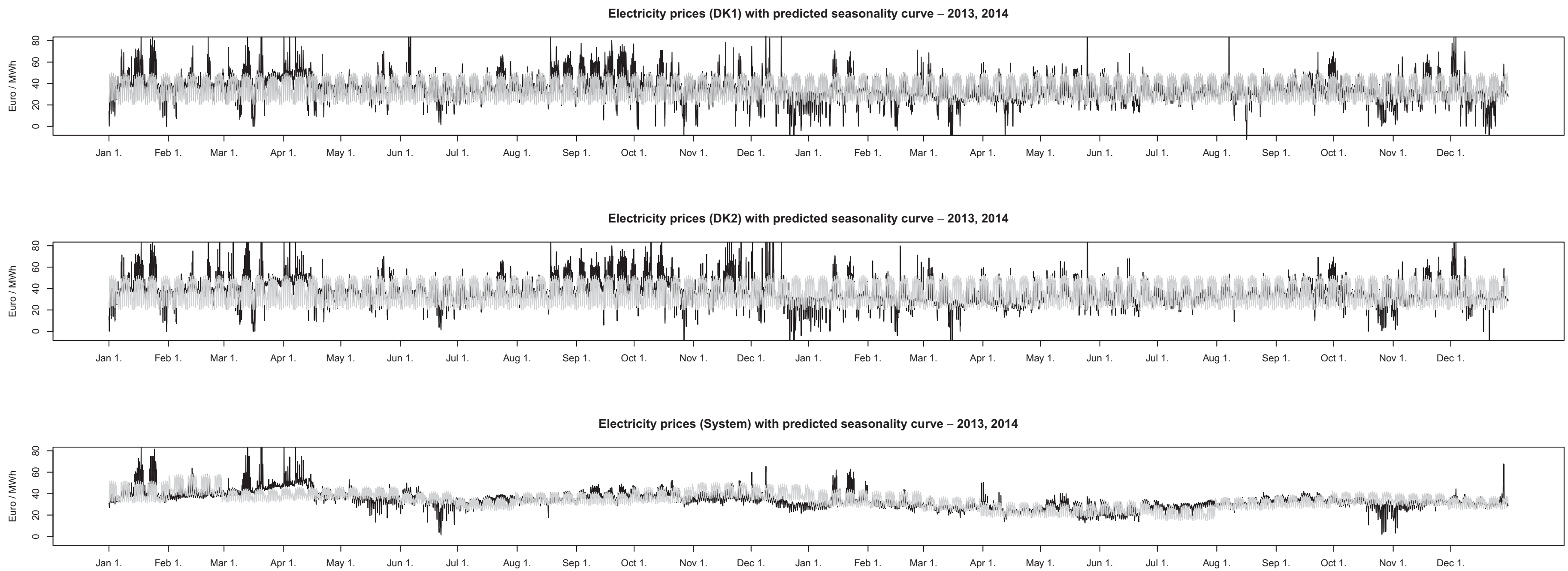


Fig. 7. Predicted seasonality curves (gray) and historical electricity prices (black) for 2013 and 2014. Extreme price spikes are not displayed in the plots.

with independence over time. With these assumptions, we obtain the following analytical expressions for the minimal variance hedge

$$V_j^b = \frac{1}{|off_j|} \sum_{t \in off_j} \theta_t^L, \tag{13}$$

$$V_j^p = \frac{1}{|peak_j|} \sum_{t \in peak_j} \theta_t^L - \frac{1}{|off_j|} \sum_{t \in off_j} \theta_t^L, \tag{14}$$

$$V_j^d = \frac{1}{|m_j|} \sum_{t \in m_j} \theta_t^L, \tag{15}$$

as shown in Appendix D. This hedging strategy only depends on the prediction of the load, which is one of the reasons it is widely used by electricity companies. As an extension of the previously introduced terminology, we refer to this as the mean hedge.

5.2.2. Model 2: correlation between load and system price, independence over time

In the second model we include correlation between the deseasonalized system price and load. The motivation is that the system price reflects the equilibrium between aggregated supply and demand. Thus, if the load is above its expectation, the system price is likely to be above its expectation, and similarly if the load is below its expectation. Our model is

$$\begin{pmatrix} \tilde{S}_t^{sys} \\ \tilde{\epsilon}_t \\ \tilde{L}_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{sys}^2 & 0 & \rho\sigma_{sys}\sigma_L \\ 0 & \nu^2 & 0 \\ \rho\sigma_{sys}\sigma_L & 0 & \sigma_L^2 \end{pmatrix} \right),$$

where $\tilde{S}_t^{sys}, \tilde{L}_t$ are correlated, whereas $\tilde{S}_t^{sys}, \tilde{\epsilon}_t$ and $\tilde{S}_t^{sys}, \tilde{\epsilon}_t$ are independent. As for Model 1, $\left(\tilde{S}_t^{sys}, \tilde{\epsilon}_t, \tilde{L}_t\right)$ are independent of $\left(\tilde{S}_u^{sys}, \tilde{\epsilon}_u, \tilde{L}_u\right)$ for $t \neq u$. This is equivalent to

$$\begin{pmatrix} S_t \\ S_t^{sys} \\ L_t \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \theta_t^S \\ \theta_t^{sys} \\ \theta_t^L \end{pmatrix}, \begin{pmatrix} \sigma_{sys}^2 + \nu^2 & \sigma_{sys}^2 & \rho\sigma_{sys}\sigma_L \\ \sigma_{sys}^2 & \sigma_{sys}^2 & \rho\sigma_{sys}\sigma_L \\ \rho\sigma_{sys}\sigma_L & \rho\sigma_{sys}\sigma_L & \sigma_L^2 \end{pmatrix} \right),$$

with independence over time. In terms of an adjusted load

$$\tilde{\theta}_t^L = \theta_t^L + \left(\theta_t^S - F\right)\rho\frac{\sigma_L}{\sigma_{sys}} \tag{16}$$

the minimal variance can be expressed analytically as follows

$$V_j^b = \frac{1}{|off_j|} \sum_{t \in off_j} \tilde{\theta}_t^L, \tag{17}$$

$$V_j^p = \frac{1}{|peak_j|} \sum_{t \in peak_j} \tilde{\theta}_t^L - \frac{1}{|off_j|} \sum_{t \in off_j} \tilde{\theta}_t^L, \tag{18}$$

$$V_j^d = \frac{1}{|m_j|} \sum_{t \in m_j} \theta_t^L, \tag{19}$$

cf. Appendix D. This is the natural extension of Model 1. With the positive correlation between load and price, the strategy is to hedge slightly above expected load for high prices and slightly below expected load for low prices. Note that the hedging volume for contracts for difference remains the same and that the peak load hedge, but not the effective peak hedge, is independent of F.

5.2.3. Model 3: correlation between load, area price and system price, correlation over time

In the third model we include temporal correlation in the deseasonalized components and assume $\left(\tilde{S}_t, \tilde{S}_t^{sys}, \tilde{L}_t\right)$ follow a three-dimensional Ornstein-Uhlenbeck process given by

$$d\tilde{S}_t = -\kappa_S \tilde{S}_t\, dt + \tilde{\sigma}_S\, dZ_t^S, \tag{20}$$

$$d\tilde{S}_t^{sys} = -\kappa_{sys} \tilde{S}_t^{sys}\, dt + \tilde{\sigma}_{sys}\, dZ_t^{sys}, \tag{21}$$

$$d\tilde{L}_t = -\kappa_L \tilde{L}_t\, dt + \tilde{\sigma}_L\, dZ_t^L. \tag{22}$$

Here, Z_t^S, Z_t^{sys} and Z_t^L are correlated Brownian motions with cross-correlation coefficients $\rho_{S,sys}, \rho_{S,L}, \rho_{sys,L}$. Conditional on $\left(\tilde{S}_u, \tilde{S}_u^{sys}, \tilde{L}_u\right)$ for $u < t$, the explicit solutions to Eqs. (20), (21) and (22) are

$$\tilde{S}_t = \tilde{S}_u e^{-\kappa_S(t-u)} + \tilde{\sigma}_S \int_u^t e^{-\kappa_S(t-v)}\, dZ_v^S,$$

$$\tilde{S}_t^{sys} = \tilde{S}_u^{sys} e^{-\kappa_{sys}(t-u)} + \tilde{\sigma}_{sys} \int_u^t e^{-\kappa_{sys}(t-v)}\, dZ_v^{sys},$$

$$\tilde{L}_t = \tilde{L}_u e^{-\kappa_L(t-u)} + \tilde{\sigma}_L \int_u^t e^{-\kappa_L(t-v)}\, dZ_v^L,$$

and, hence, for $t > u$

$$\begin{pmatrix} S_t \\ S_t^{sys} \\ L_t \end{pmatrix} \middle| \begin{pmatrix} S_u \\ S_u^{sys} \\ L_u \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \tilde{S}_u e^{-\kappa_S(t-u)} + \theta_t^S \\ \tilde{S}_u^{sys} e^{-\kappa_{sys}(t-u)} + \theta_t^{sys} \\ \tilde{L}_u e^{-\kappa_L(t-u)} + \theta_t^L \end{pmatrix}, \begin{pmatrix} \Sigma_S(u,t) & \Sigma_{S,sys}(u,t) & \Sigma_{S,L}(u,t) \\ \Sigma_{S,sys}(u,t) & \Sigma_{sys}(u,t) & \Sigma_{sys,L}(u,t) \\ \Sigma_{S,L}(u,t) & \Sigma_{sys,L}(u,t) & \Sigma_L(u,t) \end{pmatrix} \right),$$

with

$$\Sigma_S(u,t) = \frac{\tilde{\sigma}_S^2 \left(1 - e^{-2\kappa_S(t-u)}\right)}{2\kappa_S},$$

$$\Sigma_{sys}(u,t) = \frac{\tilde{\sigma}_{sys}^2 \left(1 - e^{-2\kappa_{sys}(t-u)}\right)}{2\kappa_{sys}},$$

$$\Sigma_L(u,t) = \frac{\tilde{\sigma}_L^2 \left(1 - e^{-2\kappa_L(t-u)}\right)}{2\kappa_L},$$

and

$$\Sigma_{S,sys}(u,t) = \rho_{S,sys}\tilde{\sigma}_S\tilde{\sigma}_{sys} \frac{1 - e^{-(\kappa_S + \kappa_{sys})(t-u)}}{\kappa_S + \kappa_{sys}},$$

$$\Sigma_{S,L}(u,t) = \rho_{S,L}\tilde{\sigma}_S\tilde{\sigma}_L \frac{1 - e^{-(\kappa_S + \kappa_L)(t-u)}}{\kappa_S + \kappa_L},$$

$$\Sigma_{sys,L}(u,t) = \rho_{sys,L}\tilde{\sigma}_{sys}\tilde{\sigma}_L \frac{1 - e^{-(\kappa_{sys} + \kappa_L)(t-u)}}{\kappa_{sys} + \kappa_L}.$$

Estimation procedures for the parameters in Models 1–3 can be found in Appendix E. The corresponding parameter estimates are found in Tables 1 and 2.

Table 1
Parameter estimates for Model 1 and Model 2.

Parameters	$\hat{\sigma}^2_{sys}$	$\hat{\nu}^2$	$\hat{\sigma}^2_L$	$\hat{\rho}$	Parameters	$\hat{\sigma}^2_{sys}$	$\hat{\nu}^2$	$\hat{\sigma}^2_L$	$\hat{\rho}$
DK1 - Model 1	171.87	176.24	0.00179	–	DK1 - Model 2	171.87	176.24	0.00179	0.16559
DK2 - Model 1	171.87	157.40	0.00126	–	DK2 - Model 2	171.87	157.40	0.00126	0.27545

Table 2
Parameter estimates for Model 3.

Parameters	κ_S	κ_{sys}	κ_L	$\hat{\hat{\sigma}}^2_S$	$\hat{\hat{\sigma}}^2_{sys}$	$\hat{\hat{\sigma}}^2_L$	$\rho_{S,sys}$	$\rho_{S,L}$	$\rho_{sys,L}$
DK1 - Model 3	0.10076	0.08604	0.09951	6.37	5.44	0.01887	0.52951	0.22932	0.16559
DK2 - Model 3	0.14235	0.08604	0.25760	8.43	8.43	0.02553	0.63801	0.31171	0.27578

5.2.4. Monte Carlo simulation

To determine the optimal hedging strategies of Model 3, we let P_t^k for $t \in m_j$ denote a sample of the stochastic hourly payoff P_t in month j, given by

$$P_t^k = \left(F_j - S_t^k\right) L_t^k + \left(S_t^{sys,k} - q_j^b\right) V_j^b + \left(S_t^k - S_t^{sys,k} - q_j^d\right) V_j^d + 1_{(t \in peak_j)} \left(S_t^{sys,k} - q_j^p\right) V_j^p,$$

where $\left(S_t^k\right)_{t \in m_j}$, $\left(S_t^{sys,k}\right)_{t \in m_j}$ and $\left(L_t^k\right)_{t \in m_j}$ for $k = 1, \ldots, K$ are sample paths obtained by simulating from Model 3. We let $\bar{P}_t$ be the sample average of the payoff in hour t, i.e.

$$\bar{P}_t = \frac{1}{K} \sum_{k=1}^{K} P_t^k. \tag{23}$$

We now determine the hedging strategy, Min Var, that minimizes the sum of sample variances of payoffs for hours in month j, defined as

$$\sum_{t \in m_j} \left(\frac{1}{K-1} \sum_{k=1}^{K} \left[P_t^k - \bar{P}_t\right]^2 \right) \approx \sum_{t \in m_j} Var(P_t). \tag{24}$$

Likewise, we determine the hedging strategy, Min Loss, that minimizes the sum of sample averages of hourly losses in month j, i.e.

$$-\sum_{t \in m_j} \left(\frac{1}{K} \sum_{k=1}^{K} \min\left(P_t^k, 0\right) \right) \approx -\sum_{t \in m_j} E\left[\min(P_t, 0)\right]. \tag{25}$$

6. Results

In this section we assess the performance of the optimal hedging strategies from Models 2 and 3 and benchmark against the mean hedge strategy derived from Model 1. All hedging strategies can be determined 14 days prior to the start of the month and do not use any other information than historical data from 2012, yearly predicted load as well as forward prices for base load contracts, peak load contracts and CfDs. Furthermore, all contracts are available at *Nasdaq Commodities* and our market structure closely reflects the real market. We study the influence of correlation, the choice of risk measure, the inclusion of CfDs, the effect of improved price forecast as well as the impact of margins on the fixed price.

To compare the payoff streams from implementing the optimal hedging strategies in 2013 and 2014, we let P_t denote the payoff in hour t for $t \in m_j$ and $j \in \{1, \ldots, 24\}$ and define the following quantities. The profit and loss (P&L) is

$$\sum_{j=1}^{24} \sum_{t \in m_j} P_t,$$

the gross loss is given by

$$-\sum_{j=1}^{24} \sum_{t \in m_j} \min(P_t, 0),$$

and the gross profit is

$$\sum_{j=1}^{24} \sum_{t \in m_j} \max(P_t, 0).$$

Finally, using the average monthly payoff,

$$\hat{P}_j = \frac{1}{m_j} \sum_{t \in m_j} P_t,$$

we define the realized variance

$$\sum_{j=1}^{24} \frac{1}{|m_j| - 1} \sum_{t \in m_j} \left(P_t - \hat{P}_j\right)^2.$$

The realized variance measures the stability of the payoffs throughout each month, but differs from the sum of hourly variances defined in Eq. (24). Realized monthly variance can be estimated from actual data as opposed to the sum of hourly variances. Minimizing the deviations from the hourly mean, however, creates a more stable cash flow than minimizing the deviations from the monthly mean.[4] For reasons of confidentiality, the load data has been anonymized by scaling with the 1/(maximal load). Thus, the P&L, gross loss and gross profit are measured in Euro/(maximal load). The sum of realized variances is likewise scaled by 1/(maximal load)^2.

6.1. Comparing hedging strategies

Table 3 contains the total P&L, gross loss, gross profit and realized variance of the payoffs without hedging (No hedge) and with the

[4] The realized monthly variance for DK1 has been computed without including payoffs from June 2013 (due to a 5 hour price spike, with prices over 1900 Euro/MWh) as this would blur the comparison significantly.

Table 3
Performance of hedging strategies in DK1 and DK2. Mean hedge and comp. mean hedge refer to variance minimizing strategies based on Model 1 and Model 2, respectively. Relative changes from the mean hedge are provided in parentheses and 95% confidence bands are market with stars.

	P&L		Gross loss		Gross profit		Realized variance	
West Denmark (DK1)								
No hedge	(100.6%)	24,630.07	(129.2%)	39,673.46	(117.3%)	64,303.52	(2367.7%)	844.41
Mean hedge	(0.0%)	12,278.02	(0.0%)	17,309.71	(0.0%)	29,587.73	(0.0%)	34.22
Comp. mean hedge	(0.9%)	12,385.57	(−0.3%)	17,260.62	(0.2%)	29,646.20	(−0.4%)	34.09
Min Var	(2.2%)	12,553.24	(0.1%)	17,328.72	(1.0%)	29,881.95	(4.3%)	35.69
	(12,519.66,12,574.80)*		(17,315.84,17,354.60)*		(29835.49,29,929.40)*		(35.62,35.75)*	
Min Loss	(17.2%)	14,389.79	(−5.8%)	16,313.42	(3.8%)	30,703.22	(81.3%)	62.03
	(14,298.30,14,499.95)*		(16,187.75,16,388.77)*		(30,486.05,30,888.71)*		(61.19,62.54)*	
East Denmark (DK2)								
No hedge	(12,069.4%)	21,874.97	(50.0%)	31,766.36	(151.2%)	53,641.33	(1299.7%)	657.72
Mean hedge	(0.0%)	179.75	(0.0%)	21,172.65	(0.0%)	21,352.41	(0.0%)	46.99
Comp. mean hedge	(93.0%)	346.85	(−0.2%)	21,125.01	(0.6%)	21,471.85	(−0.9%)	46.55
Min Var	(175.1%)	494.59	(−0.5%)	21,066.39	(1.0%)	21,560.98	(0.6%)	47.29
	(470.34,504.70)*		(21,061.51,21,087.03)*		(21,531.85,21,591.73)*		(47.26,47.33)*	
Min Loss	(2731.7%)	5090.01	(−13.6%)	18,293.28	(9.5%)	23,383.29	(133.1%)	109.55
	(4934.37,5177.18)*		(18,194.50,18,400.45)*		(23,128.87,23,577.63)*		(108.36,110.80)*	

variance minimizing strategy based on Model 1 (Mean hedge), the variance minimizing strategy based on Model 2 (Comp. mean hedge), the variance minimizing strategy based on Model 3 (Min Var) and the loss minimizing strategy based on Model 3 (Min Loss).[5] Furthermore, the monthly hedging volumes, P&L, gross loss and realized variance are shown in Appendix F.

We first observe that, as expected, the hedged cash-flows have lower P&L and gross profit than the unhedged, but the gross loss and especially the realized variance are also substantially lower.

6.1.1. Best practice

When comparing the Min Loss hedge to the mean hedge strategy, we find that the gross loss is 5.8% and 13.6% lower in DK1 and DK2, respectively. Furthermore, the gross profit is 3.8% and 9.5% higher and the P&L is 2111.77 Euro and 4910.26 Euro higher (times maximal load), respectively.[6] At the same time, the realized variance increases, but as this measure includes positive deviations from the monthly mean, it is of less importance than the gross loss. We note from the monthly P&L in Appendix F that the largest difference between the Min Loss hedge and the mean hedge strategy are in the months with a negative P&L. In these months the Min Loss hedge incurs much smaller losses, resulting in a larger accumulated P&L over the two years. With the superiority of the Min Loss hedge over the mean hedge strategy, we refer to best practice as opposed to common practice.

6.1.2. The inclusion of correlation

The plots of Appendix F show that the inclusion of correlation between price and load increases the base load hedging volume and reduces the peak load hedging volume, especially when comparing Model 3 to Models 1 or 2. These differences are reflected in the performance measures of Table 3, although to a lesser extend. When comparing the variance minimizing strategy from Model 2 to Model 1, we find that the gross profit slightly increases (by 0.2% and 0.6% for DK1 and DK2, respectively), whereas the gross loss and the realized variance slightly decrease (in the order of 0.2–0.9%). P&L likewise increases, although not much in absolute terms (by 107.55 Euro and 167.10 Euro, respectively). This suggests a moderate but valuable effect of including the correlation between the system price and load and thereby also the simple correlation structure between area price and load. When comparing the variance minimizing strategy from Model 3 to Model 1, gross profit increases by 1.0% for both DK1 and DK2, gross loss changes by 0.1% and −0.5% in DK1 and DK2, respectively, realized variance increases by 4.3% and 0.6%, and P&L increases by 167.67 Euro and 147.74 Euro. Thus, the direct modeling of the correlation between load, system price and area price generates higher profits at the expense of higher variances.

6.1.3. The choice of risk measure

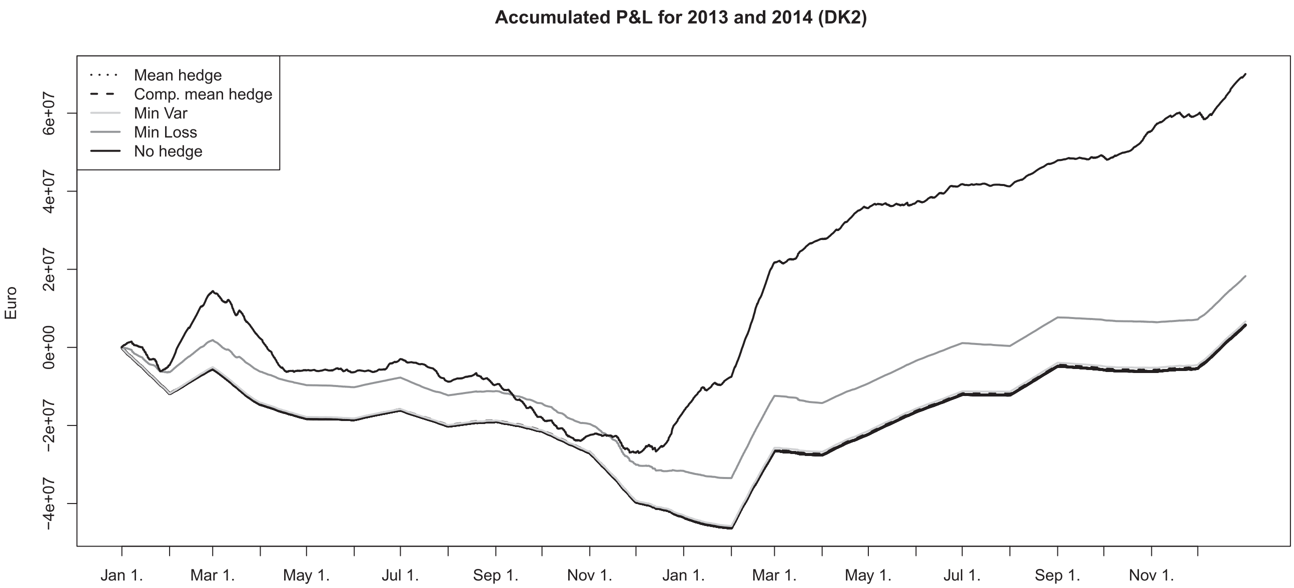
Appendix F and Table 3 show that hedging volumes differ considerably with the choice of risk measure and so does the performance measures. When comparing the loss minimizing strategy to the variance minimizing strategy using Model 3, as expected, the Min Loss hedge has lower gross loss than the Min Var hedge. As a side effect of minimizing the loss, the gross profit increases, which creates a P&L that is much higher (at least 13% and 90% in DK1 and DK2, respectively) than for the variance minimizing strategies. The realized variance is approximately double, indicating that the cash flows shows larger variations throughout each month. More importantly, however, the accumulated P&L does not decrease as much from month to month for the Min Loss strategy as for the Min Var strategy. Thus, in spite of variations on short time horizons, the Min Loss hedge generates a relatively stable cash flow on longer time horizons, while outperforming the other strategies in terms of P&L. This is confirmed for both bidding areas in Fig. 8.

6.1.4. The inclusion of CfDs

In this section we quantify the impact of including CfDs by repeating the analysis from Section 6.1 assuming that the CfD contracts are not available. Table 4 illustrates that for DK1, the inclusion of CfDs reduces the gross loss by 39.3% to 47.7%, whereas the gross profit decreases by 37.6% to 41.1%. Thus, gross loss is reduced significantly by introducing the CfDs for the three strategies, but at the expense of a decrease in profit. For DK2, the inclusion of CfDs reduces the gross loss by only 2.3% to 19.5%, whereas the gross profit is reduced by 42.8% to 40.9%, indicating that the benefits of including CfDs in DK2 are smaller than for DK1. A plausible explanation is that the risk premium for CfDs is larger in DK2 than in DK1, which could be due to more risk averse market participants in East Denmark than in West Denmark. The impact on the accumulated payoff of including CfDs is shown in Fig. 9. We note that the accumulated payoffs are more volatile without the CfDs and that the price spike in June in DK1 barely affects the accumulated payoff when the CfDs are included. In

5 *95% confidence interval based on 16 simulations, each with 1000 paths. The actual value is based on a single simulation with 1000 paths.

6 The relative change of the P&L is not always well defined as the numerator can be both positive and negative, which results in changes of 17.2% and 2731.7 % in DK1 and DK2, respectively.



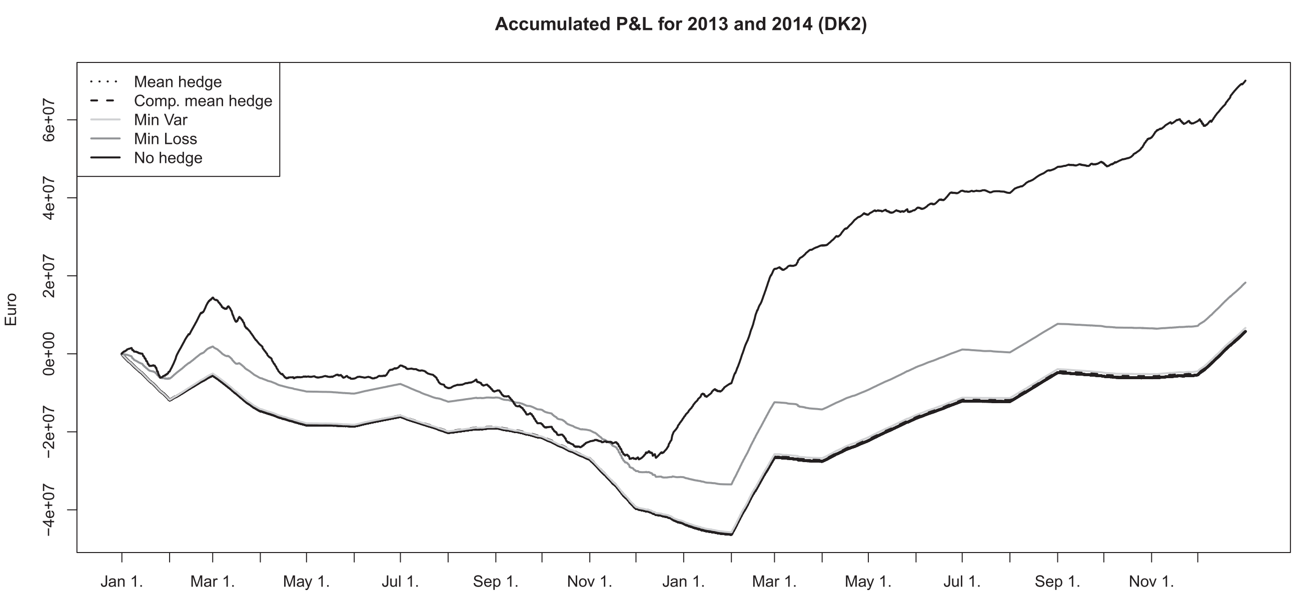
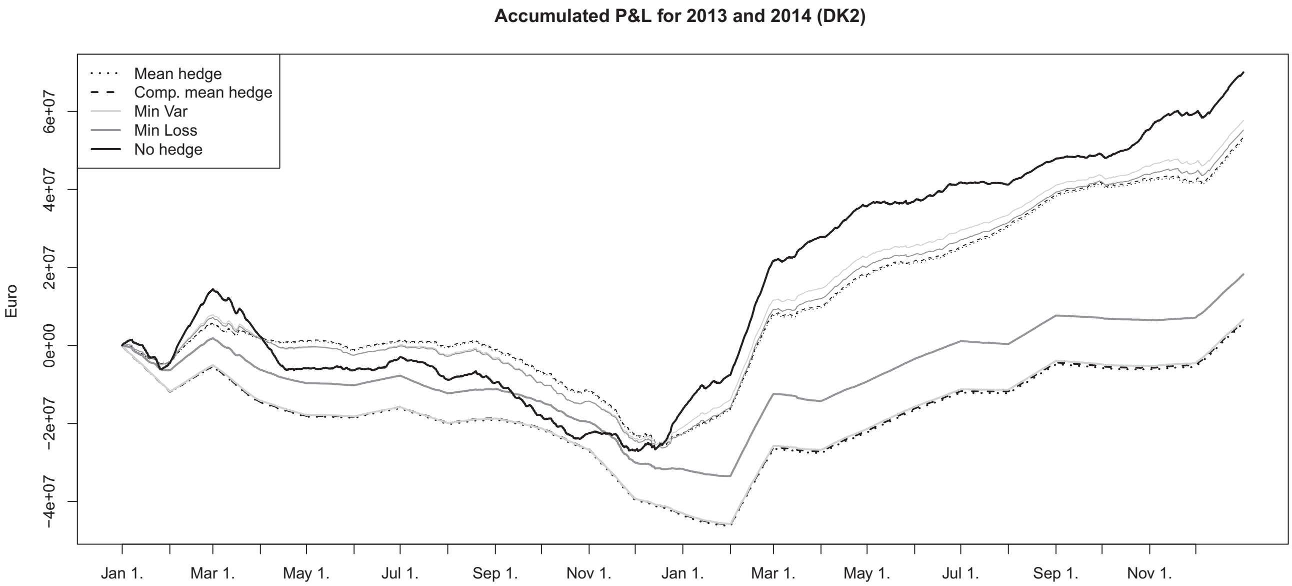


Fig. 8. Accumulated P&L in DK1 and DK2 with the different hedging strategies. Note that the variance minimizing strategies have very similar accumulated payoffs and the corresponding lines almost collapse.

Table 4
Comparison of hedging strategies with no CfDs available. Relative changes of including CfDs in parenthesis.

	P&L		Gross loss		Gross profit		Realized variance	
West Denmark (DK1) - no CfDs available								
No hedge	(0.0%)	24,630.07	(0.0%)	39,673.46	(0.0%)	64,303.52	(0.0%)	844.41
Mean hedge	(−35.0%)	18,895.39	(−39.3%)	28,496.69	(−37.6%)	47,392.09	(−93.1%)	497.40
Comp. mean hedge	(−34.8%)	19,002.95	(−39.4%)	28,483.75	(−37.6%)	47,486.70	(−93.2%)	498.34
Min Var	(−41.5%)	21,469.43	(−45.9%)	32,051.91	(−44.2%)	53,521.34	(−94.1%)	603.46
	(21,419.13,21,524.87)*		(32,003.20,32,074.13)*		(53,422.33,53,599.00)*		(602.39,604.09)*	
Min Loss	(−31.0%)	20,866.83	(−47.7%)	31,217.89	(−41.1%)	52,084.71	(−89.1%)	571.10
	(20,663.33,20,979.08)*		(31,060.72,31,334.22)*		(51,724.05,52,313.31)*		(569.28,572.29)*	
East Denmark (DK2) - no CfDs available								
No hedge	(0.0%)	21,874.97	(0.0%)	31,766.36	(0.0%)	53,641.33	(0.0%)	657.72
Mean hedge	(−98.9%)	15,645.24	(−2.3%)	21,670.22	(−42.8%)	37,315.46	(−86.7%)	353.81
Comp. mean hedge	(−97.8%)	15,812.33	(−2.4%)	21,650.83	(−42.7%)	37,463.16	(−86.9%)	355.05
Min Var	(−97.2%)	17,391.40	(−9.5%)	23,280.18	(−47.0%)	40,671.59	(−88.3%)	403.36
	(17,327.58,17,432.99)*		(23,228.10,23,302.88)*		(40,555.68,40,735.88)*		(402.33,403.67)*	
Min Loss	(−69.7%)	16,807.93	(−19.5%)	22,730.92	(−40.9%)	39,538.86	(−71.4%)	382.52
	(16,601.22,16,876.88)*		(22,618.61,22,834.60)*		(39,219.83,39,711.47)*		(380.98,383.53)*	

* 95% confidence intervals.



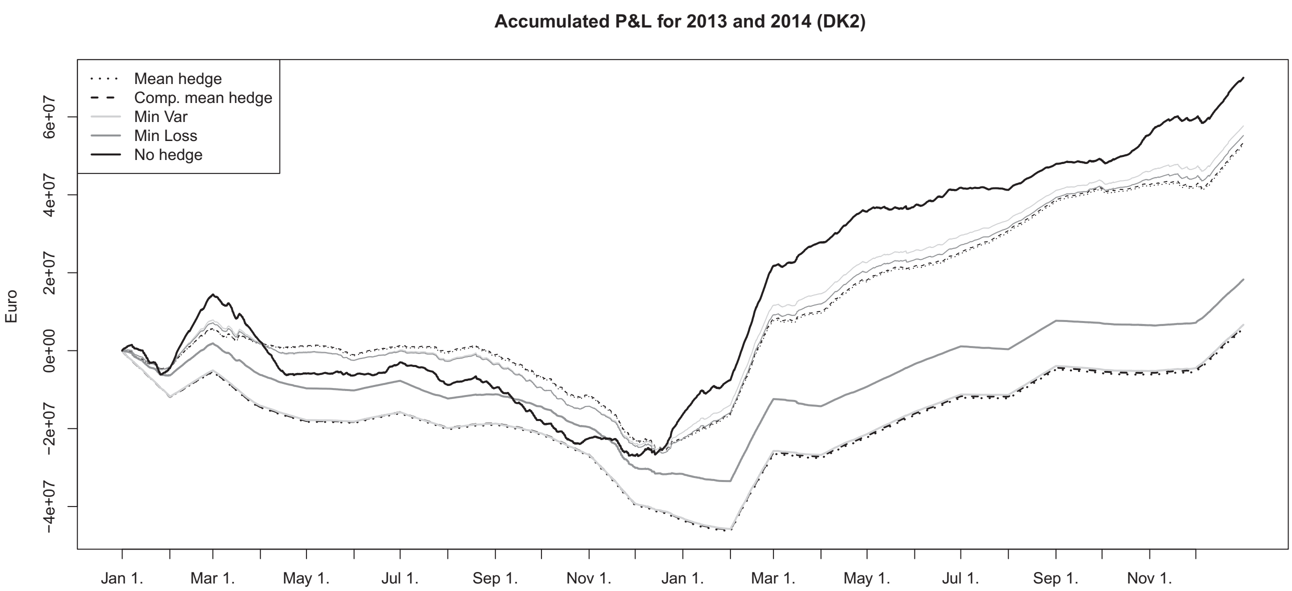


Fig. 9. Accumulated P&L with (thick lines) and without (thin lines) CfDs in East Denmark (DK1) and West Denmark (DK2).

Table 5
Comparison of hedging strategies with a perfect forecast of expected prices. Relative change from imperfect forecast.* 95% confidence intervals.

	P&L		Gross loss		Gross profit		Realized variance	
West Denmark (DK1) - perfect expected price forecast								
No hedge	(0.0%)	24,630.07	(0.0%)	39,673.46	(0.0%)	64,303.52	(0.0%)	844.41
Mean hedge	(0.0%)	12,278.02	(0.0%)	17,309.71	(0.0%)	29,587.73	(0.0%)	34.22
Comp. mean hedge	(0.0%)	12,385.57	(0.0%)	17,260.62	(0.0%)	29,646.20	(0.0%)	34.09
Min Var	(2.0%)	12,806.32	(−0.8%)	17,196.75	(0.4%)	30,003.07	(−1.5%)	35.17
	(12,771.81,12,827.93)*		(17,183.96,17,223.73)*		(29,955.77,30,051.67)*		(35.09,35.23)*	
Min Loss	(66.9%)	24,018.18	(−39.3%)	9896.73	(10.5%)	33,914.91	(−19.4%)	50.00
	(23,957.11,24,073.28)*		(9848.63,9921.47)*		(33,805.75,33,994.75)*		(49.80,50.99)*	
East Denmark (DK2) - perfect expected price forecast								
No hedge	(0.0%)	21,874.97	(0.0%)	31,766.36	(0.0%)	53,641.33	(0.0%)	657.72
Mean hedge	(0.0%)	179.75	(0.0%)	21,172.65	(0.0%)	21,352.41	(0.0%)	46.99
Comp. mean hedge	(0.0%)	346.85	(0.0%)	21,125.01	(0.0%)	21,471.85	(0.0%)	46.55
Min Var	(35.4%)	669.57	(−0.6%)	20,947.69	(0.3%)	21,617.26	(−0.9%)	46.87
	(645.97,679.30)*		(20,944.07,20,967.71)*		(21,590.04,21,647.01)*		(46.84,46.91)*	
Min Loss	(216.4%)	16,104.58	(−37.7%)	11,404.15	(17.6%)	27,508.72	(−21.2%)	86.33
	(16,020.23,16,147.65)*		(11,344.87,11,428.31)*		(27,365.10,27,575.96)*		(85.69,87.87)*	

* 95% confidence intervals.

general, the CfDs are effective in minimizing risk, but the costs are high.

6.1.5. Perfect forecast of average prices

The differences between the hedging strategies quantified in Sections 6.1 and 6.1.4 may be due to model assumptions such as the inclusion of auto-correlations and cross-correlations, the choice of risk measure, the availability of hedging instruments, but also the ability of the price model to predict prices. Whereas the mean hedge is only based on the prediction of expected load, the more advanced hedging strategies also depend on the predictions of price parameters. We therefore quantify the impact of being able to more accurately predict average prices. In particular, we assume a perfect forecast of monthly average prices in peak and off-peak periods. This does not impact Model 1 as the hedging strategy is independent of the predicted price. Moreover, even though Model 2 depends on the predicted price, the hedging strategies are not very sensitive to changes in predicted prices and its performance does not change. The results in Table 5 show that the impact on the hedging strategy Min Var is likewise very limited. This is not the case for the Min Loss hedge, for which the gross loss is reduced by 39.3% and 37.7% in DK1 and DK2, respectively, while P&L are increased by 66.9% and 216.4%. The result suggests that the superiority of the advanced hedging strategies is limited by the ability to predict average prices, and therefore, that an improved price forecast can significantly improve the Min Loss hedge. Some of these improvements could be obtained by modeling the seasonal bias on base load contracts, peak load contracts and CfDs, but due to unavailability of data, this has not been further pursued in this paper.

6.1.6. Margin

By changing the fixed price to $\tilde{F} = F + 2$, we obtain an increase in the expected payoffs by approximately the total scaled load times the margin of 2. DK1 and DK2 have a total scaled load of 9758.8 and 8409.5 over the two years, resulting in an increase of approximately 20.000 and 17.000 Euro times the maximal load. For the compensated mean hedge the base load volume is reduced by less than 1% and for the Min Var strategy the hedging volumes are reduced by less than 1%, suggesting that small changes in margin to the fixed price only have moderate impact on the variance minimizing hedging strategies. In contrast, the Min Loss hedge changes significantly, but the P&L increases by the margin times the scaled load and the gross loss remains significantly lower than for the variance minimizing strategies.

7. Conclusion and extensions

7.1. Conclusion

In this paper, we develop hedging strategies for an electricity distributor in the Nordic electricity market who manages price and volume risk from fixed price agreements on stochastic electricity load.

We analyze the market dynamics in the two bidding areas of West Denmark and East Denmark, with special emphasis on the correlation structure between system price, area price and load and we quantify the impact of including auto- and cross-correlations.

We benchmark against hedging at expected load, which is common industry practice. Our results show that the inclusion of correlation increases expected payoffs and reduces variance, although moderately. This can typically be achieved by hedging above the mean in peak periods and below the mean in off-peak periods.

We further improve performance of the hedging strategy, using expected loss as a risk measure instead of variance. In one area, this reduces the gross loss by 5.8% and increases the gross profit by 3.8%. In the other area, gross loss is reduced by 13.6% and gross profit is increased by 9.5%. The inclusion of CfDs in addition to peak load and base load contracts can likewise reduce risk, but this may be at the expense of a high risk premium. Finally, we demonstrate how improved forecasts of average prices have substantial potential to continue the improvement of performance.

We conclude that for companies currently using the mean hedge strategy, accumulated payoffs can be significantly increased, while at the same time reducing the loss from hours with negative payoffs. This can be achieved by the implementation of a more advanced price model and a hedging strategy that accounts for the asymmetry of payoffs. Although the inclusion of correlation has a beneficial impact on performance, however, we show that the choice of risk measure is of highest importance.

7.2. Improvements and extensions

We leave the improvement of price forecasts as future research but discuss various directions. An extensive survey on the topic is provided by Weron (2014), who categorizes contributions to the literature by forecasting methods and stress the importance of seasonality and fundamentals.

Our method falls into the category of reduced-form models. In general, such models serve to capture the main statistical properties of prices rather than generating accurate predictions, but often allow for analytical solutions to risk management problems. Within this framework, our price model may be improved by including jumps or regime switching, cf. Deng (1999), Erlwein et al. (2010), Schwartz and Smith (2000) and Weron et al. (2004). For example, we model the differences between area and system prices by a diffusion process with a fixed low volatility. In reality, the behavior of such differences may closer resemble that of a jump process, as the congestion problems creating the differences are usually quickly resolved. Moreover, the price process itself could be extended to include jumps. For both prices and price differences, the spikes created by congestion may be better captured by including demand in local and neighbouring bidding areas as exogenous factors to the processes. The modeling of spikes, however, is significantly more difficult. The inclusion of temporary and extreme behavior requires long stationary time series, and may even be infeasible due to very slow changes on the demand side as well as on the supply side. Moreover, the forecasting performance of mean-reverting jump-diffusions or regime switching may be poor, cf. Weron (2014). Besides jumps, empirical evidence suggests that electricity prices exhibit heteroskedasticity and one may investigate time-dependent volatility as Garcia et al. (2005).

The main drawback of spot price modeling is the problem of pricing derivatives, e.g. the consistency between spot and forward prices. One could compensate for a potential forward price bias, cf. Redl et al. (2009), to obtain better predictions of the system and area prices. For the direct modeling of the forward curve, see for example Fleten and Lemming (2003). Although the forward curve provide readily available forecasts, these may likewise contain bias.

In addition to exogenous local demand, relevant physical and economic factors include predictions of system load, fuel prices, weather variables, see Gonzalez et al. (2012), Karakatsani (2008) and Kristiansen (2012), who all use fundamental models for price modeling. Along the same lines, a reduced-form model for demand could be used in combination with a modeling of the supply curve, see Kanamura and Ōhashi (2008). In general, fundamental models allow for a better description of the market dynamics, but at the expense of increased complexity in analytical solutions and calibration procedures. To account for other market characteristics, equilibrium and game theoretic models serve to model, optimize or simulate the strategic behavior and interactions of generation companies, see the survey by Ventosa et al. (2005). A disadvantage is that such methods usually produce qualitative conclusions rather than quantitative results.

The discrete-time counterparts of the continuous-time reduced-form models are econometric models such as regressions and time series models, e.g. Conejo et al. (2005), Contreras et al. (2003) and Nogales et al. (2002). A newer paper by Raviv et al. (2015) likewise employs time series analysis to account for the simultaneous formation of hourly spot market prices for a whole day. Many recent contributions to electricity price modeling extend such point forecasts to probabilistic forecasts, see Nowotarski and Weron (2017). It is not entirely clear, however, how to handle spikes by econometric methods.

Finally, Nowotarski and Weron (2016) stress the importance of a long-term seasonal component in day-ahead electricity price forecasting. Although we already adjust for weekly and yearly seasonality, the forecast may be significantly improved by monthly recalibration of the seasonal components.

Appendix A

Lemma A1. *The hedge V that minimizes*

$$Var\left((F - S_T)L_T + (S_T - q_t(T))V)\right) \tag{A.1}$$

is

$$V^* = \frac{Cov(S_T, S_TL_T)}{Var(S_T)} - F\frac{Cov(L_T, S_T)}{Var(S_T)}. \tag{A.2}$$

Proof. *Observe that*

$$\begin{aligned} &Var\left((F - S_T)L_T + (S_T - q_t(T))V\right) \\ &\quad = Var(FL_T + S_TV - S_TL_T) \\ &\quad = Var(FL_T - S_TL_T) + V^2Var(S_T) \\ &\qquad +2FVCov(L_T, S_T) - 2VCov(S_TL_T, S_T). \end{aligned}$$

The first order condition for optimality implies that

$$2V^*Var(S_T) + 2FCov(L_T, S_T) - 2Cov(S_T, S_TL_T) = 0$$

and with the second order condition that $2Var(S_T) \geq 0$, we find that the optimal hedge is (A.2). □

Lemma A2. *V^* from Lemma A1 is equivalent to*

$$V^* = E(L_T) - (F - E(S_T))\frac{Cov(S_T, L_T)}{Var(S_T)} + \frac{Cov\left((S_T - E(S_T))^2, L_T\right)}{Var(S_T)}. \tag{A.3}$$

Proof. *Using that $E(XY) = E(X)E(Y) + Cov(X, Y)$ and $Cov(X, Y) = Cov(X + a, Y)$ for a constant, it follows that*

$$\begin{aligned} Cov(S_T, S_TL_T) &= E\left(S_T^2L_T\right) - E(S_T)E(S_TL_T) \\ &= E\left(S_T^2\right)E(L_T) + Cov\left(S_T^2, L_T\right) \\ &\quad - E(S_T)\left(E(S_T)E(L_T) + Cov(S_T, L_T)\right) \\ &= \left(E\left(S_T^2\right) - E(S_T)^2\right)E(L_T) \\ &\quad + Cov\left(S_T^2, L_T\right) - E(S_T)Cov(S_T, L_T) \\ &= Var(S_T)E(L_T) \\ &\quad + Cov\left(\left(S_T^2 - E(S_T)\right)^2, L_T\right) \\ &\quad + E(S_T)Cov(S_T, L_T). \end{aligned}$$

By inserting $Cov(S_T, S_TL_T)$ in (A.2) *we obtain* (A.3). □

Lemma A3. *Assume that*

$$\begin{pmatrix} L_T \\ S_T \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_L \\ \mu_S \end{pmatrix}, \begin{pmatrix} \sigma_S^2 & \rho\sigma_S\sigma_L \\ \rho\sigma_S\sigma_L & \sigma_L^2 \end{pmatrix}\right).$$

The hedge that minimizes

$$Var\left((F - S_T)L_T + V(S_T - q_t(T))\right) \tag{A.4}$$

is

$$V^* = \mu_L - (F - \mu_S)\rho\frac{\sigma_L}{\sigma_S}. \tag{A.5}$$

Proof. *We consider* V^* from Lemma A2 and have that $Cov(L_T, S_T) = \rho\sigma_S\sigma_L$. Let *X and Y be independent with $X, Y \sim \mathcal{N}(0, 1)$. Then,*

$$\begin{pmatrix} S_T \\ L_T \end{pmatrix} \stackrel{d}{=} \begin{pmatrix} \mu_S + \sigma_SX \\ \mu_L + \sigma_L\left(\rho X + \sqrt{1 - \rho^2}Y\right) \end{pmatrix}$$

where $\stackrel{d}{=}$ denotes equality in distribution. Using independence of X and Y as well as $E(Y) = E(X^3) = 0$, we find that

$$\begin{aligned} &Cov\left((S_T - E(S_T))^2, L_T\right) \\ &\quad = E\left[\left((S_T - E(S_T))^2 - Var(S_T)\right)(L_T - E(L_T))\right] \\ &\quad = E\left[(\sigma_SX)^2\sigma_L\left(\rho X + \sqrt{1 - \rho^2}Y\right)\right] \\ &\quad = \rho\sigma_L\sigma_S^2E(X^3) + \sqrt{1 - \rho^2}\sigma_L\sigma_S^2E(X^2Y) \\ &\quad = 0. \end{aligned}$$

Finally, by inserting this in (A.4) we obtain (A.5). □

Appendix B

Example 3.3. In Example 3.2 the forward price is below the expected price, which is known as backwardation. For commodities the opposite situation may also occur. Consider the same parameters as in Example 3.2, but with $q_t(T) = 36.75$. This situation ($q_t(T) > E(S_T)$) is known as contango. In the first plot of Fig. B.1, where $F = 40$, the optimal strategies are similar to those of Example 3.2. In the second plot of Fig. B.1, where $F = 30$, the Min Loss hedge ($V = 0.226$) deviates significantly from the mean hedge ($V = 0.5$) in the opposite direction of the Min Var hedge ($V = 0.525$).

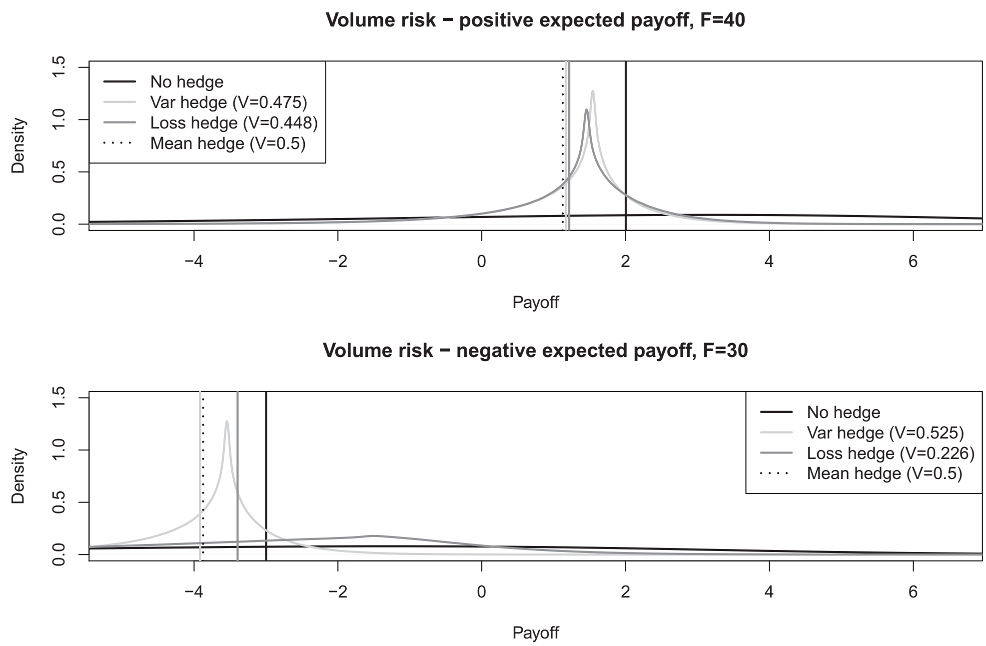


Fig. B.1. Payoff densities and their means (vertical lines) with parameters from Example 3.3 (Contango).

Appendix C. Correlation plots

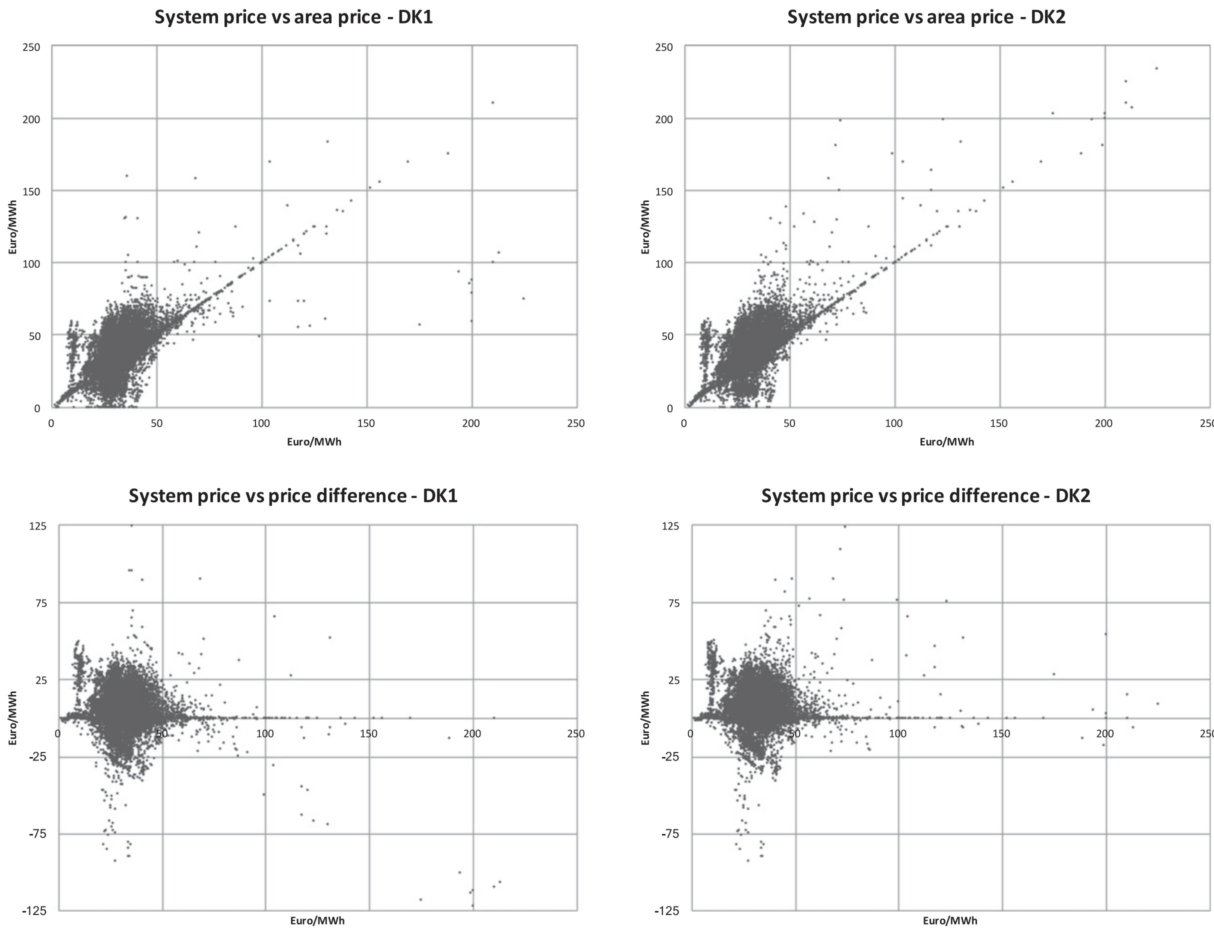


Fig. C.2. Cross-correlations between system and area prices and between system prices and price differences for West Denmark (DK1) and East Denmark (DK2).

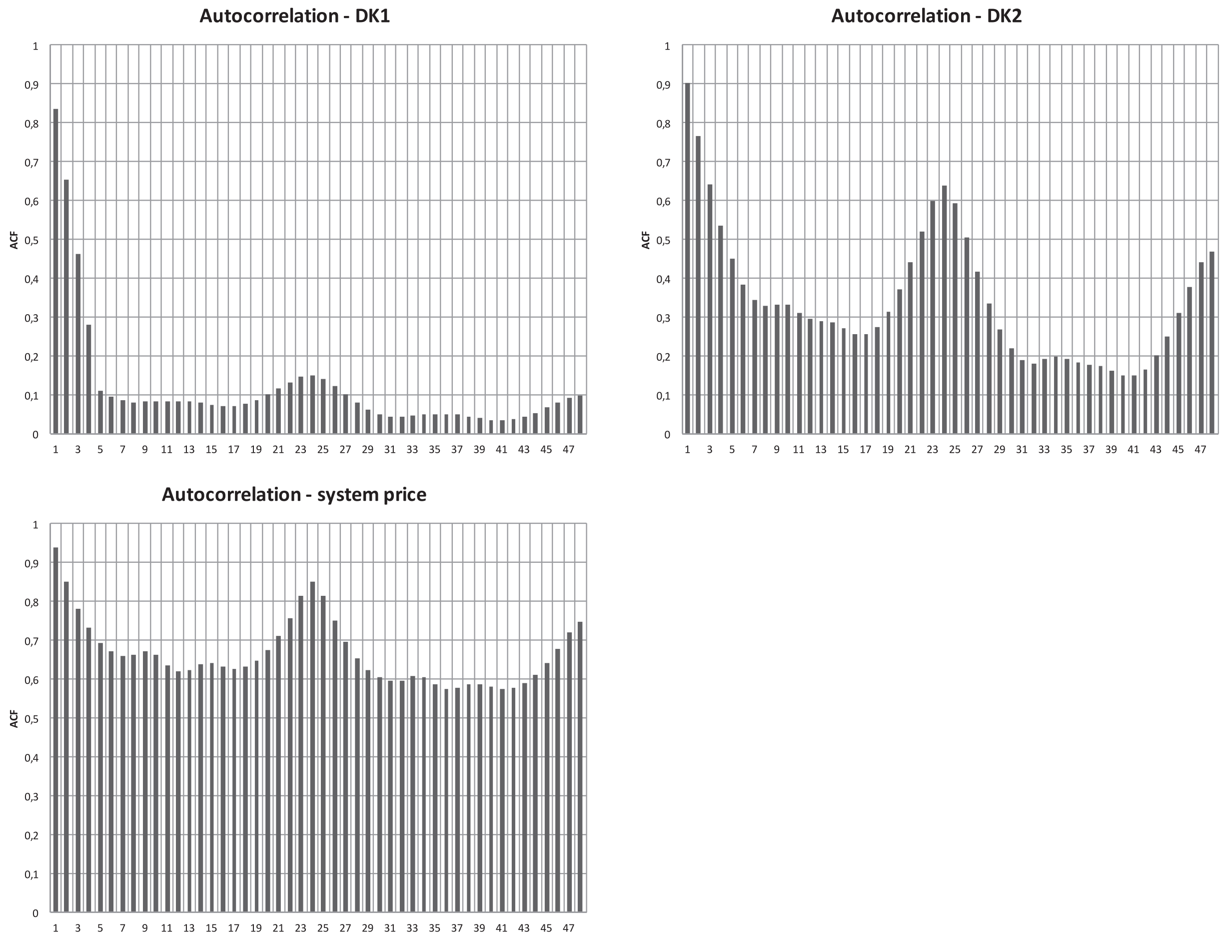


Fig. C.3. Auto-correlations for area prices of West Denmark (DK1) and East Denmark (DK2) and for system prices.

Appendix D. Variance analysis

When hourly payoffs are independent, as assumed in Models 1 and 2, the sum of hourly variances of payoffs equals the variance of the sum of hourly payoffs. To simplify notation, we therefore minimize the variance of the sum. The variance of the payoffs for month j is

$$
\begin{aligned}
&Var\left(\sum_{t\in \mathrm{m}_j}\left(S_t^{sys}-q_j^b\right)V_j^b+\sum_{t\in \mathrm{m}_j}\left(S_t-S_t^{sys}-q_j^d\right)V_j^d+\sum_{t\in \mathrm{m}_j}(F_j-S_t)L_t+\sum_{t\in peak_j}\left(S_t^{sys}-q_j^p\right)V_j^p\right)\\
&=Var\left(\sum_{t\in \mathrm{m}_j}S_t^{sys}V_j^b+\sum_{t\in \mathrm{m}_j}\left(S_t-S_t^{sys}\right)V_j^d+\sum_{t\in \mathrm{m}_j}(F_j-S_t)L_t+\sum_{t\in peak_j}S_t^{sys}V_j^p\right)\\
&=\left(V_j^b\right)^2Var\left(\sum_{t\in \mathrm{m}_j}S_t^{sys}\right)+\left(V_j^d\right)^2Var\left(\sum_{t\in \mathrm{m}_j}\left(S_t-S_t^{sys}\right)\right)+Var\left(\sum_{t\in \mathrm{m}_j}(F_j-S_t)L_t\right)+\left(V_j^p\right)^2Var\left(\sum_{t\in peak_j}S_t^{sys}\right)\\
&\quad+2V_j^bCov\left(\sum_{t\in \mathrm{m}_j}S_t^{sys},\sum_{t\in \mathrm{m}_j}\left(S_t-S_t^{sys}\right)V_j^d+\sum_{t\in \mathrm{m}_j}(F_j-S_t)L_t+\sum_{t\in peak_j}S_t^{sys}V_j^p\right)+2V_j^dCov\left(\sum_{t\in \mathrm{m}_j}\left(S_t-S_t^{sys}\right)V_j^d,\sum_{t\in \mathrm{m}_j}(F_j-S_t)L_t+\sum_{t\in peak_j}S_t^{sys}V_j^p\right)\\
&\quad+2V_j^pCov\left(\sum_{t\in peak_j}S_t^{sys},\sum_{t\in \mathrm{m}_j}(F_j-S_t)L_t\right).
\end{aligned}
$$

The variance is minimized as a function of V_j^p, V_j^b and V_j^d. The first order conditions imply that

$$
\begin{aligned}
V_j^b&=\frac{Cov\left(\sum_{t\in \mathrm{m}_j}S_t^{sys},\sum_{t\in \mathrm{m}_j}(S_t-F_j)L_t\right)}{Var\left(\sum_{t\in \mathrm{m}_j}S_t^{sys}\right)}-\frac{Cov\left(\sum_{t\in \mathrm{m}_j}S_t^{sys},V_j^d\sum_{t\in \mathrm{m}_j}\left(S_t-S_t^{sys}\right)+V_j^p\sum_{t\in peak_j}S_t^{sys}\right)}{Var\left(\sum_{t\in \mathrm{m}_j}S_t^{sys}\right)}\\
V_j^d&=\frac{Cov\left(\sum_{t\in \mathrm{m}_j}\left(S_t-S_t^{sys}\right),\sum_{t\in \mathrm{m}_j}(S_t-F_j)L_t\right)}{Var\left(\sum_{t\in \mathrm{m}_j}\left(S_t-S_t^{sys}\right)\right)}-\frac{Cov\left(\sum_{t\in \mathrm{m}_j}\left(S_t-S_t^{sys}\right),V_j^b\sum_{t\in \mathrm{m}_j}S_t^{sys}+V_j^p\sum_{t\in peak_j}S_t^{sys}\right)}{Var\left(\sum_{t\in \mathrm{m}_j}\left(S_t-S_t^{sys}\right)\right)}\\
V_j^p&=\frac{Cov\left(\sum_{t\in peak_j}S_t^{sys},\sum_{t\in \mathrm{m}_j}(S_t-F_j)L_t\right)}{Var\left(\sum_{t\in peak_j}S_t^{sys}\right)}-\frac{Cov\left(\sum_{t\in peak_j}S_t^{sys},V_j^b\sum_{t\in \mathrm{m}_j}S_t^{sys}+V_j^d\sum_{t\in \mathrm{m}_j}\left(S_t-S_t^{sys}\right)\right)}{Var\left(\sum_{t\in peak_j}S_t^{sys}\right)}.
\end{aligned}
$$

The second order conditions follows from convexity. With $f=|peak_j|/|\mathrm{m}_j|$, the conditions of Model 1 simplify to

$$
\begin{aligned}
V_j^b&=\frac{1}{|\mathrm{m}_j|}\sum_{t\in \mathrm{m}_j}\theta_t^L-fV_j^p\\
V_j^d&=\frac{1}{|\mathrm{m}_j|}\sum_{t\in \mathrm{m}_j}\theta_t^L\\
V_j^p&=\frac{1}{|peak_j|}\sum_{t\in peak_j}\theta_t^L-V_j^b
\end{aligned}
$$

and those of Model 2 to

$$
\begin{aligned}
V_j^b&=\frac{1}{|\mathrm{m}_j|}\sum_{t\in \mathrm{m}_j}\left(\theta_t^L-\left(F-\theta_t^S\right)\frac{\rho\sigma_L}{\sigma_{sys}}\right)-fV_j^p\\
V_j^d&=\frac{1}{|\mathrm{m}_j|}\sum_{t\in \mathrm{m}_j}\theta_t^L\\
V_j^p&=\frac{1}{|peak_j|}\sum_{t\in peak_j}\left(\theta_t^L-\left(F-\theta_t^S\right)\frac{\rho\sigma_L}{\sigma_{sys}}\right)-V_j^b.
\end{aligned}
$$

Appendix E. Calibration of the models

Let $(s_i)_{i=1}^N$, $\left(s_i^{sys}\right)_{i=1}^N$ and $(l_i)_{i=1}^N$ denote the observed area prices, system prices and loads in 2012, where N is the total number of hours.

E.1. Model 1

Model 1 only requires estimates of average load for each month as well as for peak and off-peak hours of each month.

E.2. Model 2

We estimate σ_L, σ_{sys} and ρ using the estimators

$$\hat{\sigma}_L^2 = \frac{1}{N}\sum_{i=1}^{N}\left(l_i - \theta_i^L\right)^2,$$
$$\hat{\sigma}_{sys}^2 = \frac{1}{N}\sum_{i=1}^{N}\left(s_i^{sys} - \theta_i^{sys}\right)^2,$$
$$\hat{\rho} = \frac{\sum_{i=1}^{N}\left(s_i^{sys} - \theta_i^{sys}\right)\left(l_i - \theta_i^L\right)}{\sqrt{\sum_{i=1}^{N}\left(s_i^{sys} - \theta_i^{sys}\right)^2 \sum_{i=1}^{N}\left(l_i - \theta_i^L\right)^2}}.$$

E.3. Model 3

An Ornstein-Uhlenbeck process U_t satisfying the following equation

$$dU_t = -\kappa U_t\, dt + \sigma\, dZ_t$$

has the solution

$$U_u = e^{-\kappa dt}U_t + \sigma\int_t^u e^{-\kappa(t-v)}\, dZ_v$$

for $u > t$. Now, let $t_i = t_1 + (i-1)\Delta$ for $i = 1, \ldots, N$. Then,

$$U_{t_{i+1}} = aU_{t_i} + bX_i, \quad i = 1, \ldots, N-1$$

with X_i independent and $X_i \sim \mathcal{N}(0,1)$, $a = e^{-\kappa\Delta}$ and $b^2 = \sigma^2(1 - e^{-2\kappa\Delta})/2\kappa$. The estimator for a that minimize

$$\sum_{i=1}^{N-1}(U_{t_{i+1}} - aU_{t_i})^2$$

is given by

$$\hat{a} = \frac{\sum_{i=1}^{N-1} U_{t_i}U_{t_{i+1}}}{\sum_{i=1}^{N-1} U_{t_i}^2}.$$

Therefore, κ can be estimated as

$$\hat{\kappa} = \frac{-\log(\hat{a})}{\Delta},$$

b^2 can be estimated as

$$\hat{b}^2 = \frac{1}{N-1}\sum_{i=1}^{N-1}(U_{t_{i+1}} - \hat{a}U_{t_i})^2$$

and the estimator of σ is given by

$$\hat{\sigma} = \sqrt{2\kappa/(1-e^{-2\kappa\Delta})\hat{b}^2}.$$

Finally, for two Ornstein-Uhlenbeck processes U_t and V_t satisfying the equations,

$$\begin{aligned} dU_t &= -\kappa U_t\ dt + \sigma\ dZ_t \\ dV_t &= -\lambda V_t\ dt + \nu\ dW_t \end{aligned}$$

with $dW_t\ dZ_t = \rho dt$, we have that

$$Cor(U_{t_i+\Delta} - e^{-\kappa\Delta}U_{t_i}, V_{t_i+\Delta} - e^{-\lambda\Delta}V_{t_i}) = \rho\frac{2\sqrt{\kappa\lambda}}{\sqrt{1-e^{-2\kappa\Delta}}\sqrt{1-e^{-2\lambda\Delta}}}\frac{1-e^{-(\kappa+\lambda)\Delta}}{\kappa+\lambda}.$$

The empirical correlation between the pairs of differences, $U_{t_i+\Delta} - \hat{a}_U U_{t_i}$ and $V_{t_i+\Delta} - \hat{a}_V V_{t_i}$ is given by

$$\hat{r}_{UV} = \frac{\sum_{i=1}^{N-1}(U_{t_i+\Delta} - \hat{a}_U U_{t_i})(V_{t_i+\Delta} - \hat{a}_V V_{t_i})}{\sqrt{\sum\limits_{i=1}^{N-1}(U_{t_i+\Delta} - \hat{a}_U U_{t_i})^2 \sum\limits_{i=1}^{N-1}(V_{t_i+\Delta} - \hat{a}_V V_{t_i})^2}}.$$

Therefore, ρ can be estimated by

$$\hat{\rho} = \hat{r}_{UV}\frac{\sqrt{1-e^{-2\hat{\kappa}\Delta}}\sqrt{1-e^{-2\hat{\lambda}\Delta}}}{2\sqrt{\hat{\kappa}\hat{\lambda}}}\frac{\hat{\kappa}+\hat{\lambda}}{1-e^{-(\hat{\kappa}+\hat{\lambda})\Delta}}.$$

Appendix F. Hedging strategies

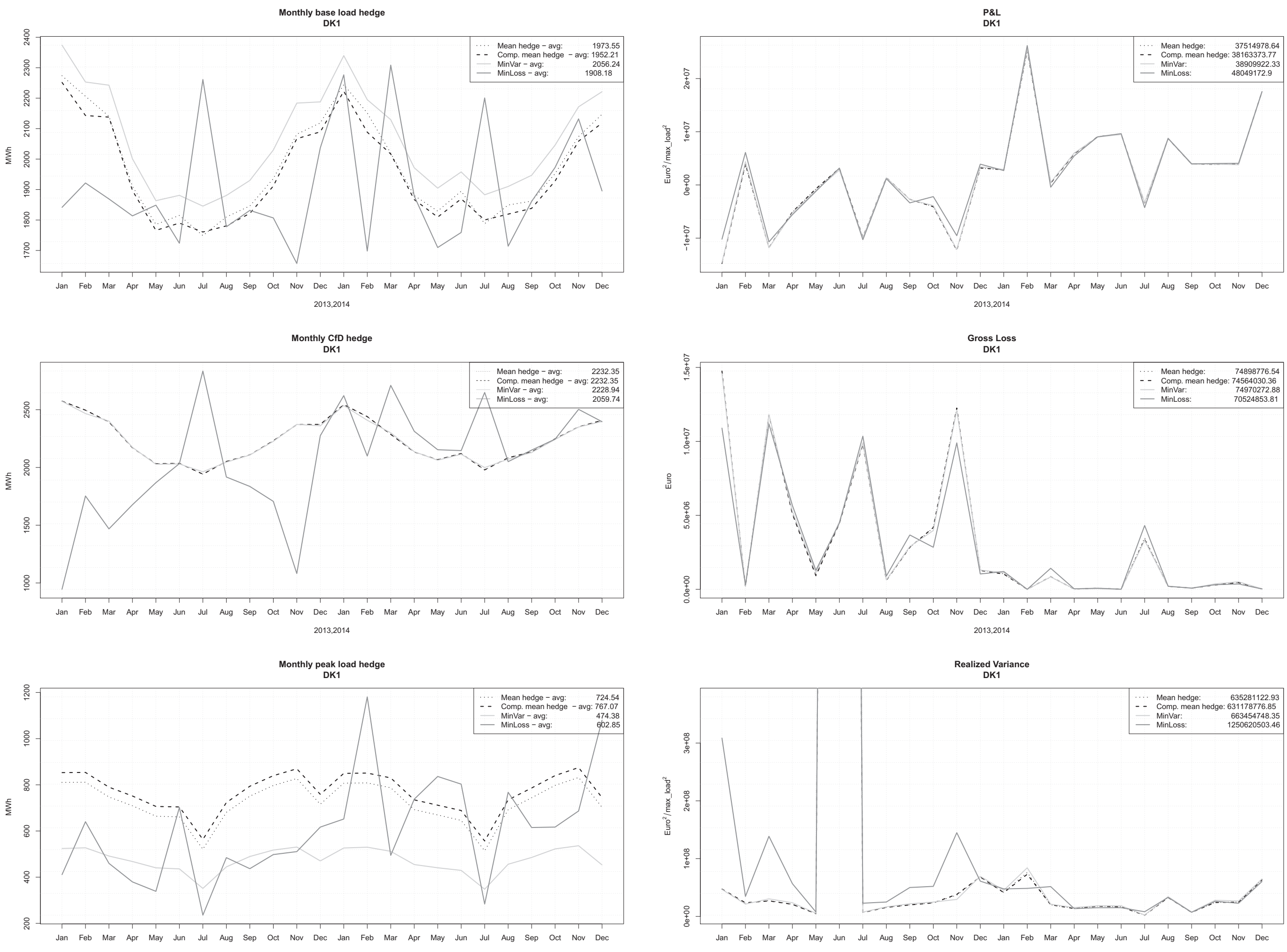


Fig. F.4. Monthly hedging volumes, P&L, gross loss and realized variance for West Denmark (DK1).

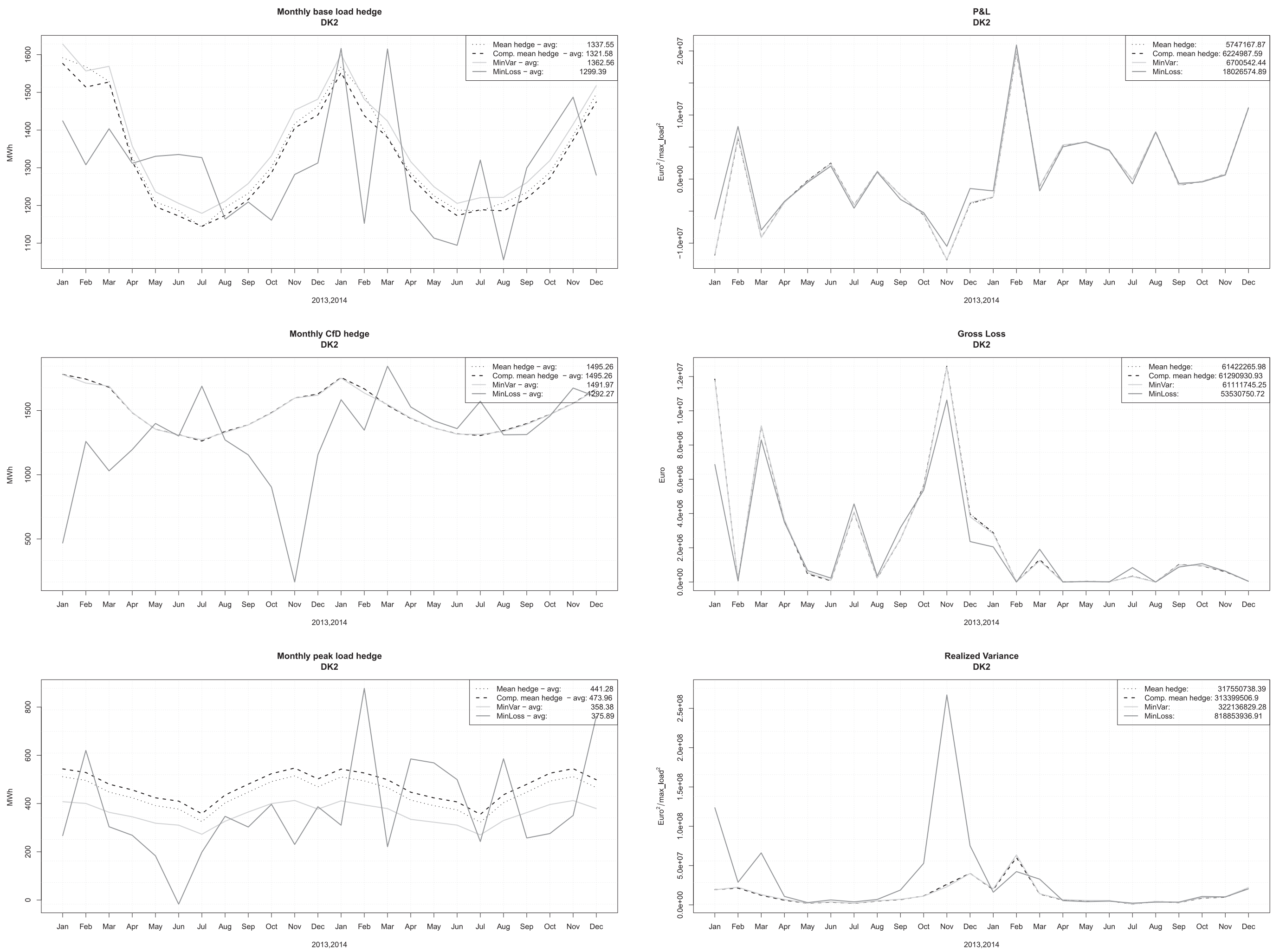


Fig. F.5. Monthly hedging volumes, P&L, gross loss and realized variance for East Denmark (DK2).

Appendix G. Supplementary material

Supplementary material to this article can be found online at https://doi.org/10.1016/j.eneco.2017.10.017.

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