Part 4: Capital Structure

- Static capital structure:
  - Leland, H., 1994, "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, " JF.
- Dynamic capital structure:
  - Goldstein, R., Ju, N. and H. Leland, 2001, "An EBIT-Based Model of Dynamic Capital Structure," JB.
- Finite maturity debt:
  - Leland, H. and K. Toft, 1996, "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," JF.
  - Hilberink, B. and L. C. G. Rogers, 2002, "Optimal Capital Structure and Endogenous Default," F&S.
- Dynamic trade-off theory and empirics:
  - Strebulaev, I. A., 2007, "Do Tests of Capital Structure Theory Mean What They Say?," JF.

Notation:

- $r =$ constant risk-free rate.
- $X =$ cash flow (EBIT), GBM with coefficients $\mu < r$ and σ under RNP.
- $C =$ constant coupon on consol.
- $\alpha =$ proportional deadweight costs of bankruptcy.
- $\tau_i =$ tax rate on interest payments.
- $\tau_{\text{eff}} =$ effective tax rate on equity income:
  - $\tau_c =$ corporate tax rate.
  - $\tau_d =$ tax rate on dividends.
  - $1 - \tau_{\text{eff}} \stackrel{\text{def}}{=} (1 - \tau_c)(1 - \tau_d)$.
- $x^* =$ default boundary. Firm defaults at

$$H \stackrel{\text{def}}{=} \min\{t \mid X_t \leq x^*\}.$$

Distribution of Cash Flows and Present Values

	Cash Flow Before H	Present Value At H
Shareholders	$(1-\tau_{\text{eff}})(X-C)$	0
Deadweight Costs	0	$\alpha x^*/(r-\mu)$
Bondholders	$(1-\tau_i)C$	$(1-\tau_{\text{eff}})(1-\alpha)x^*/(r-\mu)$
Government	$\tau_i C + \tau_{\text{eff}}(X-C)$	$\tau_{\text{eff}}(1-\alpha)x^*/(r-\mu)$
Total	X	$x^*/(r-\mu)$

- Note: The firm has pre-tax losses when $X < C$. Above formula assumes full tax loss offsets.
- Restrictions on deducting losses implies strictly convex tax schedule, which could be incorporated.

Valuation

Need to value:

- Value of receiving 1 at hitting time
- Value of receiving 1 until hitting time
  - Equals value of receiving 1 forever minus value of receiving 1 after hitting time
  - Equals $1/r$ minus value of receiving $1/r$ at hitting time
- Value of receiving X until hitting time
  - Equals value of receiving X forever minus value of receiving X after hitting time
  - Equals $X_t/(r-\mu)$ minus value of receiving $x^*/(r-\mu)$ at hitting time

Valuation cont.

- Value V of receiving 1 at hitting time satisfies

$$x\mu V'(x) + \frac{1}{2}x^2\sigma^2 V''(x) = rV(x)$$

for $x > x^*$, $V(x^*) = 1$, and $\lim_{x\to\infty} V(x) = 0$.

- Solution is $V(x) = (x^*/x)^\gamma$, where γ is the absolute value of the negative root of

$$\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta = r\,.$$

Debt and Equity Values

- Equity value is

$$E(x, x^*, C) = (1-\tau_{\text{eff}})\left[\frac{x}{r-\mu} - \frac{x^*}{r-\mu}\left(\frac{x^*}{x}\right)^\gamma - \frac{C}{r} + \frac{C}{r}\left(\frac{x^*}{x}\right)^\gamma\right].$$

- Debt value is

$$\begin{aligned} D(x, x^*, C) = (1-\tau_i)&\left[\frac{C}{r} - \frac{C}{r}\left(\frac{x^*}{x}\right)^\gamma\right] \\ &+ (1-\tau_{\text{eff}})(1-\alpha)\left(\frac{x^*}{r-\mu}\right)\left(\frac{x^*}{x}\right)^\gamma. \end{aligned}$$

Optimal Default Boundary

- The optimal default boundary x^* is determined by $E_{x^*}(x, x^*, C) = 0$.
  - We always have value matching: $E(x^*, x^*, C) = 0$.
  - Therefore,

$$E_x(x, x^*, C)\Big|_{x=x^*} + E_{x^*}(x, x^*, C)\Big|_{x=x^*} = 0\,.$$

  - Therefore,

$$E_{x^*}(x, x^*, C) = 0\Big|_{x=x^*} = 0 \quad \Rightarrow \quad E_x(x, x^*, C)\Big|_{x=x^*} = 0\,.$$

  - The last condition is smooth pasting.
- Using either $E_{x^*}(x, x^*, C) = 0$ or smooth pasting, we find

$$x^* = (r-\mu)\frac{C}{r}\left(\frac{\gamma}{1+\gamma}\right)\,.$$

Optimal Leverage

- Assume there are proportional flotation costs q. Let $x^*(C)$ denote the optimal default boundary.
- Given the initial value x_0, the firm chooses C to maximize

$$E(x_0, x^*(C), C) + (1-q)D(x_0, x^*(C), C)\,.$$

- Agency problem:
  - x^* is chosen to maximize the equity value, not the equity cum debt value.
  - It is ex-post optimal (after debt is issued) not ex-ante optimal (before debt is issued).

Scaling

- The optimal coupon is

$$C^* = x_0 \left(\frac{r}{r-\mu}\right)\left(\frac{1+\gamma}{\gamma}\right)\left[\left(\frac{1}{1+\gamma}\right)\left(\frac{A}{A+B}\right)\right]^{1/\gamma},$$

where

$$A = (1-q)(1-\tau_i) - (1-\tau_{\text{eff}}),$$
$$B = \frac{\gamma}{1+\gamma}(1-\tau_{\text{eff}})[1-(1-q)(1-\alpha)].$$

- An all-equity firm ($C^* = 0$) is optimal if $A < 0$.
- Note that C^* and $x^*(C^*)$ are proportional to x_0.

Re-Levering

- Suppose the firm can issue new debt whenever it wants.
  - Assume existing debt must be retired (called at face value) before new debt can be issued, due to covenants.
  - Re-issuing existing debt is a fixed cost. Option will be exercised at discretely spaced times.
- The firm chooses two boundaries $x_L < x_U$ with x_L the default boundary and x_U the refinance boundary.
- When the firm refinances, it chooses new coupon C' and new default and refinance boundaries x_L' and x_U'.
- Because of scaling,

$$\frac{C'}{x_U} = \frac{C}{x_0}, \qquad \frac{x_L'}{x_U} = \frac{x_L}{x_0}, \qquad \frac{x_U'}{x_U} = \frac{x_U}{x_0}.$$

- So, $C' = \lambda C$, $x_L' = \lambda x_L$, $x_U' = \lambda x_U$, where $\lambda = x_U/x_0$.

Valuation

- Can reduce to valuing:
  - Receive 1 at hitting time of x_L if x_L is hit before x_U,
  - Receive 1 at hitting time of x_U if x_U is hit before x_L.
- Prior to hitting times, values satisfy

$$x\mu V'(x) + \frac{1}{2}x^2\sigma^2 V''(x) = rV(x)\,.$$

- Values are of form

$$V(x) = A_1 x^{\beta} + A_2 x^{-\gamma}\,,$$

  where β is positive root and γ is the absolute value of the negative root of the quadratic equation.
- Constants determined from
  - $V(x_L) = 1$ and $V(x_U) = 0$.
  - $V(x_L) = 0$ and $V(x_U) = 1$.

Implications

- Absent the option to relever, the optimal leverage ratio

$$\frac{D(x_0, x^*, C^*)}{D(x_0, x^*, C^*) + E(x_0, x^*, C^*)}$$

  is much higher than observed empirically.
- Adding the option to relever reduces the optimal leverage ratio to levels observed empirically.
- Adding the option to relever also reduces the bankruptcy threshold, because it increases the value of keeping the firm alive.

Finite Maturity Debt

- In $(t, t + \mathrm{d}t)$, firm issues new debt with face value $p\,\mathrm{d}t$ and maturity profile ϕ, where $\phi \geq 0$ and $\int_0^\infty \phi(s)\,\mathrm{d}s = 1$.
- The face value of debt outstanding at time t that matures in $(s, s + \mathrm{d}s)$ for $s \geq t$ is therefore

$$\left(\int_{-\infty}^{t} p\phi(s - v)\,\mathrm{d}v\right)\,\mathrm{d}s = p\Phi(s - t)\,\mathrm{d}s\,,$$

where

$$\Phi(s) \stackrel{\text{def}}{=} \int_s^\infty \phi(y)\,\mathrm{d}y\,.$$

- The total face value outstanding is constant and equal to

$$P = p\int_0^\infty \Phi(s)\,\mathrm{d}s\,.$$

- Leland-Toft: ϕ is delta function at T.
- Leland (1994 working paper): $\phi(t) = m\mathrm{e}^{-mt}$.

Valuing Debt

- Assume the coupon on all debt is the same constant c.
- Value of debt with face value 1 and maturity T:
  - Coupon $c\,\mathrm{d}t$ until $T \wedge H$, where

$$H \stackrel{\text{def}}{=} \inf\{t \mid X_t \leq x^*\}\,.$$

  - Face 1 at T if $T < H$,
  - $1/P$ times $(1 - \tau_{\text{eff}})(1 - \alpha)x^*/(r - \mu)$ at H if $H \leq T$.
- Can reduce everything to valuing
  - Receiving 1 at T if $T < H$,
  - Receiving 1 at H if $H \leq T$.
- Given c, p, and ϕ, calculate issue price $d(x)$ of new debt.

Optimal Default and Rollover Risk

- Total coupons paid in $(t, t + \mathrm{d}t)$ equal $C\,\mathrm{d}t$, where

$$C = c \int_0^\infty \Phi(s)\,\mathrm{d}s\,.$$

- Cash flows to shareholders equal

$$(1 - \tau_i)[d(X) - p + (1 - \tau_c)(X - C)]$$

  until default.
- Can calculate optimal default boundary similar to before.
- When x is small, $d(x) - p < 0$ is a cash outflow due to debt rollover. This induces earlier default.
- Deducting illiquidity premium in $d(x)$ accelerates default.
  - He, Z. and W. Xiong, 2012, “Rollover Risk and Credit Risk,” JF.
  - He, Z. and K. Milbradt, 2013, “Endogenous Liquidity and Defaultable Debt.”

Credit Spread Puzzle

- Model implies credit spread goes to zero as maturity approaches zero, which is counter-factual.
- Jump risk:
  - Hilberink, B. and L. C. G. Rogers, 2002, “Optimal Capital Structure and Endogenous Default,” F&S.
- Incomplete information:
  - Duffie, D. and D. Lando, 2001, “Term Structures of Credit Spreads with Incomplete Accounting Information,” Econometrica.
- Illiquidity

Strebulaev, JF, 2007

- Consol debt.
- Relevering option
- Strictly convex tax schedule: τ_{eff} takes two values, being lower when taxable income is small (negative)
- Proportional cost of negative dividends (raising equity): $(1 + q_E)z$ is paid by shareholders to cover a cash flow deficit of z
- Asset sales to repay debt when in distress

Asset Sales

- Strebulaev (JF, 2007) assumes firms can sell assets to retire debt when in distress.
  - Sell fraction $1 - k$ of assets to all-equity firm.
  - Value to all equity firm is

$$(1-k)(1-\tau_{\text{eff}})\frac{x}{r-\mu}\,.$$

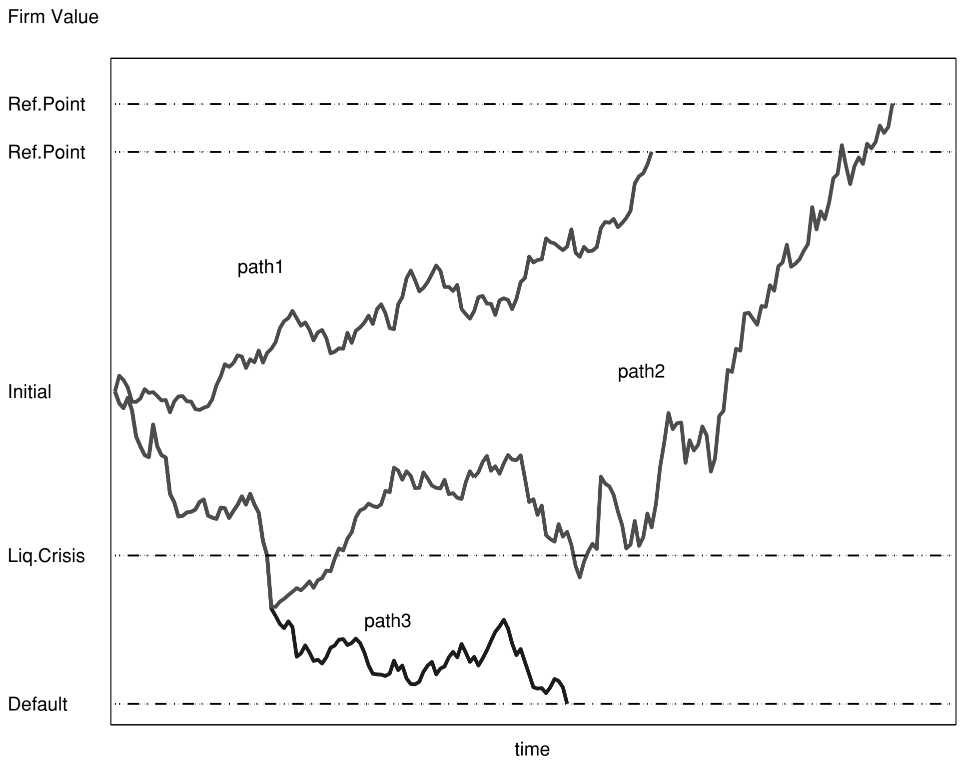
  - Firm sells at a discount (fire sale) and receives

$$(1-q_A)(1-k)(1-\tau_{\text{eff}})\frac{x}{r-\mu}\,.$$

  - Funds received are used to retire debt. Costly to retire debt, so only

$$(1-q_R)(1-q_A)(1-k)(1-\tau_{\text{eff}})\frac{x}{r-\mu}$$

    in debt is retired.
- Firm chooses x_U at which to relever, x_L at which to sell assets, and x_B at which to default.



Source: Strebulaev, I. A., 2007, "Do Tests of Capital Structure Theory Mean What They Say?," *Journal of Finance* 62, 1747–1787.

Trade-Off Theory and Implications

Trade-off theory: firms choose leverage to balance the tax advantage of debt with deadweight costs of distress. Some implications:

1. Firms should have high leverage
   - Deadweight costs of distress and bankruptcy are small compared to taxes
2. More profitable firms should have higher leverage
   - More profitable firms have more income in need of tax sheltering.
   - More profitable firms are less likely to experience distress and deadweight bankruptcy costs.
3. Firms should issue debt when the market value of equity increases
4. Leverage ratios should mean revert

Empirics Re Trade-Off Theory

1. Average quasi-market leverage ratio around 30%, which seems small
2. More profitable firms have lower leverage
3. Debt levels do not change in response to one-year changes in market equity
4. Leverage mean reverts “at a snail’s pace” (Fama and French, 2002)

Strebulaev’s Simulation

- Calibrate model.
- Generate 300 quarters of data for 3,000 firms.
  - Normal (Brownian) shock simulated as sum of common shock and idiosyncratic shock
- Discard first 148 quarters, leaving 38 years of “data”
- Calculate sample statistics including panel regressions
- Repeat 1,000 times to obtain sampling distribution of statistics

Results

1. Average leverage ratios are small
   - Due to relevering option
2. More profitable firms have lower leverage
   - Firms refinance infrequently
   - More profitable firms experience increases in market equity
   - Book debt responds with a delay (costly refinancing)
   - Market debt responds slightly but mostly with a delay
3. Correlation between one-year changes in market equity and debt issues are small
   - Refinancing is driven by long-term changes in profitability and market equity
4. Mean reversion of leverage is slow

Some Additional References

- Hennessy, C. A. and T. M. Whited, 2005, “Debt Dynamics,” *Journal of Finance* 60, 1129–1165.
- Strebulaev, I. A. and T. M. Whited, 2012, “Dynamic Models and Structural Estimation in Corporate Finance,” *Foundations and Trends in Finance* 6, 1–163.
- Welch, I., 2013, “A Critique of Recent Quantitative and Deep-Structure Modeling in Capital Structure and Beyond,” *Critical Finance Review*.
- Strebulaev, I. A. and T. M. Whited, 2013, “Dynamic Corporate Finance is Useful: A Comment on Welch (2013),” Working Paper.
- Hugonnier, J., S. Malamud, and E. Morellec, 2012, “Credit Market Frictions and Capital Structure Dynamics,” Working Paper.