Math

İSTANBUL ŞEHİR UNIVERSITY

No:

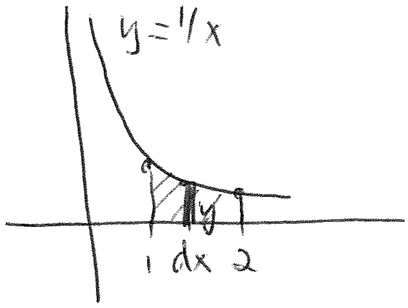
Full Name :

Math 104 Final Exam
(2 June 2016, 12:00-13:30)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 6 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	Q5	Q6	TOT
20 pts	20 pts	20 pts	20 pts	10 pts	20 pts	110 pts

Q1. The region between the curve $y = 1/x$ and the x-axis, for $1 \leq x \leq 2$, is rotated about the x-axis. Find the volume generated.



$$V = \int_1^2 \pi y^2 \, dx$$

$$= \int_1^2 \pi x^{-2} \, dx$$

$$= -\frac{\pi}{x} \Big|_1^2$$

$$= -\pi \left(\frac{1}{2} - 1\right)$$

$$= \boxed{\frac{\pi}{2}}$$

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Q2. Evaluate the following limit, if it exists:

$$\lim_{x\to 0}(1-2x)^{1/x}$$

1^{∞}

$$y=(1-2x)^{1/x} \Rightarrow \ln y = \frac{1}{x}\ln(1-2x)$$

$$\lim_{x\to 0}\ln y = \lim_{x\to 0}\frac{\ln(1-2x)}{x} \quad 0/0$$

$$\underset{\text{L'Hospital}}{=} \lim_{x\to 0}\frac{\frac{-2}{1-2x}}{1}$$

$$= -2$$

$$\ln y \to -2 \Rightarrow y \to e^{-2} = \boxed{\frac{1}{e^2}}$$

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Q3. Evaluate the following integral: Integration by paris

$$\int Arctan(1/x)\,dx$$

$u = Arctan(1/x)$ $\quad$ $dv = dx$

$du = \frac{1}{1+1/x^2}\left(-\frac{1}{x^2}\right)dx$ $\quad$ $v = x$

$= -\frac{1}{x^2+1}\,dx$

$= x\,Arctan\left(\frac{1}{x}\right) + \frac{1}{2}\int \frac{2x\,dx}{x^2+1}$

$= \boxed{x\,Arctan(1/x) + \frac{1}{2}\ln(x^2+1) + C}$

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Q4. Determine whether the following series converge or diverge:

a) $\sum_{n=1}^{\infty} \frac{n^2+5n}{n^3+n+1}$

Limit Comparison Test with
$\sum \frac{1}{n}$ (harmonic series, divergent)

$$\lim_{n\to\infty} \frac{\frac{n^2+5n}{n^3+n+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^3+5n^2}{n^3+n+1}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^3}(1+5/n)}{\cancel{n^3}(1+1/n^2+1/n^3)} = 1 \neq 0, \infty$$

$\therefore$ $\boxed{\text{The series diverges.}}$

b) $\sum_{n=0}^{\infty} \frac{3+\sin n}{10^n}$

Comparison Test with
$4\sum\left(\frac{1}{10}\right)^n$, geometric series, convergent

$$0 < \frac{3+\sin n}{10^n} < 4 \cdot \frac{1}{10^n}$$

$\therefore$ $\boxed{\text{The series converges}}$

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Q5. Find the MacLaurin series of the function $f(x) = e^x + e^{2x}$.

(Hint: To solve this question, you may use Taylor or MacLaurin series that you know.)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} + \sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{1+2^n}{n!} x^n}$$

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Q6. Let $w = f(s^3 + t^2)$, where $f'(x) = e^x$. Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ in terms of the variables s and t.

$(s,t) \longmapsto x \longmapsto w$ Chain Rule

$x = s^3 + t^2$

$\Downarrow$

$\frac{\partial x}{\partial s} = 3s^2$

$\frac{\partial x}{\partial t} = 2t$

$$\frac{\partial w}{\partial s} = f'(x)\frac{\partial x}{\partial s} = e^x \cdot 3s^2$$

$$\boxed{\frac{\partial w}{\partial s} = 3s^2 e^{s^3+t^2}}$$

$$\frac{\partial w}{\partial t} = f'(x)\frac{\partial x}{\partial t} = e^x\, 2t$$

$$\boxed{\frac{\partial w}{\partial t} = 2t\, e^{s^3+t^2}}$$