5. Convert the following context-free grammar to an equivalent grammar in Chomsky Normal Form.

$S \to ASA \mid A \mid \varepsilon$

$A \to 11 \mid \varepsilon$

1. Add new start variable S_0

$S_0 \to S$

$S \to ASA \mid A \mid \varepsilon$

$A \to 11 \mid \varepsilon$

2. Remove all rules that contains ε!

Removing $S \to \varepsilon$, $A \to \varepsilon$

$S_0 \to S \mid \varepsilon$ → The rule $S_0 \to \varepsilon$ is accepted since S_0 is start variable

$S \to ASA \mid A \mid AS \mid SA \mid S \mid AA$

$A \to 11$

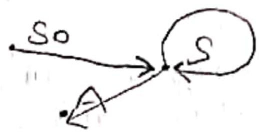
3. Remove unit rules.

Removing $S \to S$

$S_0 \to S \mid \varepsilon$

$S \to ASA \mid A \mid AS \mid SA \mid AA$

$A \to 11$



Removing $S \to A$

$S_0 \to S \mid \varepsilon$

$S \to ASA \mid 11 \mid AS \mid SA \mid AA$

$A \to 11$

Removing $S_0 \to S$

$S_0 \to ASA \mid 11 \mid AS \mid SA \mid AA \mid \varepsilon$

$S \to ASA \mid 11 \mid AS \mid SA \mid AA$

$A \to 11$

4. Create new var. for terminals

$T_a \to 1$

$S_0 \to ASA \mid T_aT_a \mid AS \mid SA \mid AA \mid \varepsilon$

$S \to ASA \mid 11 \mid AS \mid SA \mid AA$

$A \to T_a T_a$

5. Create intermediate var.

$T_a \to 1$

$S_0 \to AV_1 \mid T_aT_a \mid AS \mid SA \mid AA \mid \varepsilon$

$S \to AV_1 \mid T_aT_a \mid AS \mid SA \mid AA$

$A \to T_a T_a$

$V_1 \to SA$

} Final form

3. Prove or disprove the following statement:

The class of context free languages are closed under the intersection operation. Hint: Consider the following two languages:

$L_1 = \{a^m b^n c^n \mid m,n \geq 0\}$

$L_2 = \{a^n b^n c^m \mid m,n \geq 0\}$

$L_1 \cap L_2 = a^n b^n c^n \Rightarrow$ which is not CFL

4. Given the following context free grammar:

$S \to XY \mid \varepsilon$

$X \to xY$

$Y \to Sy$

a. What is the language generated by this grammar?

$S \to XY$ $\qquad S \to \varepsilon$ (1) $\qquad L = \{\varepsilon, xyy, xxyyyy \ldots$

$S \to \varepsilon$ $\qquad S \to$

$X \to xY$

$Y \to Sy$ $\qquad L = \{x^n y^{2n}, n \geq 0\}$

b. Draw the parse tree for the string xxyyyy

$S \to XY$

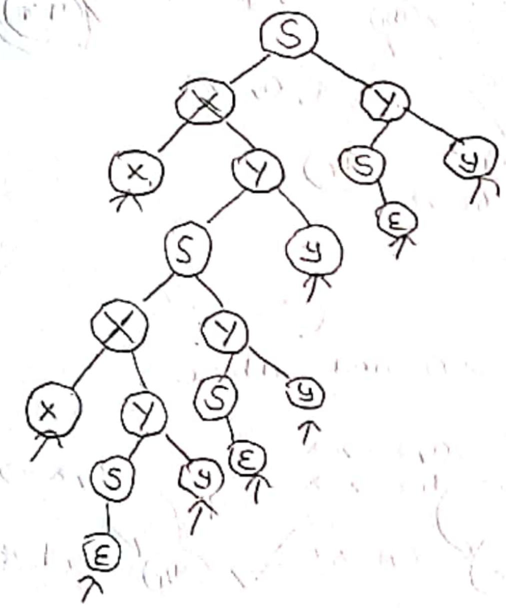
$XY \to xYY$

$xYY \to xSyY$

$xSyY \to xXYyY$

$xXYyY \to xxYYyY$

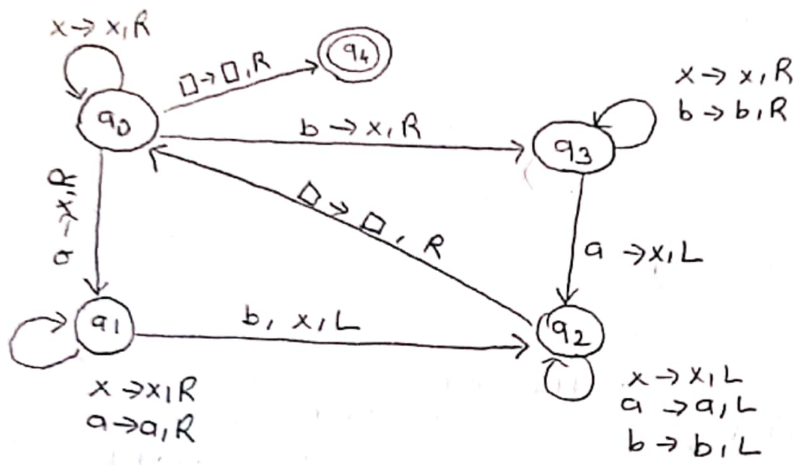
$xxYYyY \to xxSySyySy$



$\Rightarrow xx\varepsilon y\varepsilon yy\varepsilon y$

$= \boxed{xxyyyy}$

6. (What is the language on {a, b}? recognized by the following Turing Machine (a, b, x and box are the tape symbols where box denotes the empty cell)?



1. if b comes first → b will be x go right, move right until a comes. when a comes change it to x and move left until starting of string.

2. if a comes first → a will be x go right, move right until b comes. when b comes then change it to x and move left until start of string.

Repeat step 1 and step 2 until all string becomes sequence of x's if the string contains only x's then accept.

So, for each a one b should be there, for each b one a should be there.

So, Given Turing Machine accepts the strings with equal number of a's and b's.

- CSE 3064 Final Exam Study Questions -

1. Consider the context free grammar $S \to ySx \mid yySx \mid \varepsilon$

a. Show that the grammar is ambigous.

Using leftmost derivation

$S \to ySx \to yyySxx \to yyyxx$

$S \to yySx \to yyySxx \to yyyxx$

two same string with different rules Therefore language is ambigous.

b. Derive an equivalent unambiguous grammar.

$S \to ySx \mid T \mid \varepsilon$

$T \to yySx$

This is unambigous grammar.

$S \to T \to yySx \to yyyxx$

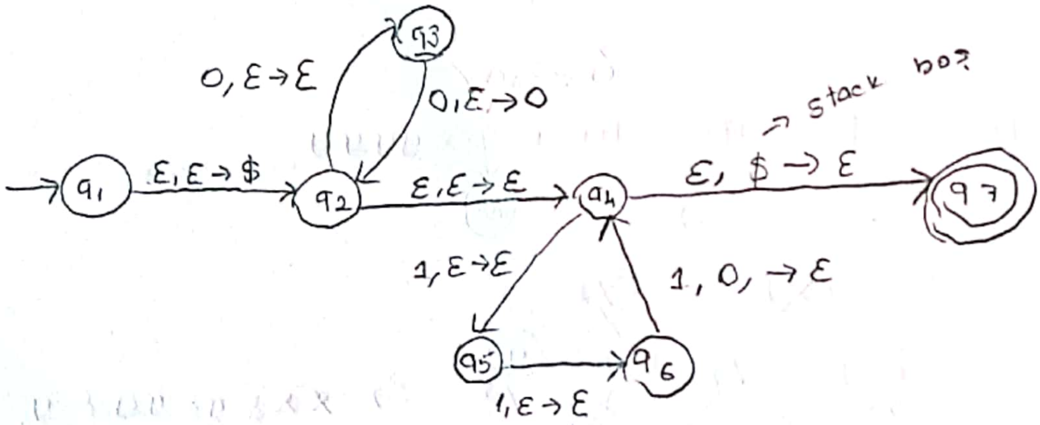
$S \to ySx \to yySxx \to yyySxxx$

can't derive same string

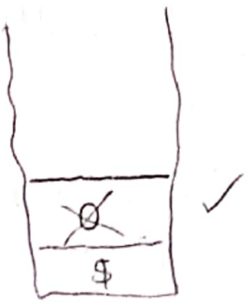
2. Design a PDA for the following languages.

a. $L_1 = \{0^{2k} 1^{3k} \mid k \geq 0\}$

$\to \varepsilon,\ 00111,\ 0000111111,\ 000000\ 111111111 \ldots$



0 0 1 1 1



b. $L_2 = \{0^a 1^b 2^c \mid a, b, c \geq 0 \text{ and } a+b=c\}$

0 1 1 2 2 2