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1.

Compute each matrix sum or product if it is defined. If an expression is undefined, explain why. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 4 \end{bmatrix}$,

$B = \begin{bmatrix} 7 & -4 & 1 \\ 2 & -3 & -4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$, and $D = \begin{bmatrix} 3 & 5 \\ -2 & 5 \end{bmatrix}$.

$-2A$, $B-2A$, AC, CD

Compute the matrix product $-2A$. Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. $-2A = \begin{bmatrix} -2 & 0 & 2 \\ -8 & 8 & -8 \end{bmatrix}$
(Simplify your answer.)

B. The expression $-2A$ is undefined because A is not a square matrix.

C. The expression $-2A$ is undefined because matrices cannot be multiplied by numbers.

D. The expression $-2A$ is undefined because matrices cannot have negative coefficients.

Compute the martrix sum $B-2A$. Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. $B-2A = \begin{bmatrix} 5 & -4 & 3 \\ -6 & 5 & -12 \end{bmatrix}$
(Simplify your answer.)

B. The expression $B-2A$ is undefined because B and A have different sizes.

C. The expression $B-2A$ is undefined because B and $-2A$ have different sizes.

D. The expression $B-2A$ is undefined because A is not a square matrix.

Compute the matrix product AC. Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. AC =
(Simplify your answer.)

B. The expression AC is undefined because the number of rows in A is not equal to the number of rows in C.

C. The expression AC is undefined because the number of rows in A is not equal to the number of columns in C.

D. The expression AC is undefined because the number of columns in A is not equal to the number of rows in C.

Compute the matrix product CD. Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. $CD = \begin{bmatrix} 0 & 25 \\ -13 & -5 \end{bmatrix}$
(Simplify your answer.)

B. The expression CD is undefined because the corresponding entries in C and D are not equal.

C. The expression CD is undefined because matrices with negative entries cannot be multiplied.

D. The expression CD is undefined because square matrices cannot be multiplied.

2.

Compute each matrix sum or product if it is defined. If an expression is undefined, explain why. Let $A = \begin{bmatrix} 2 & 0 & -2 \\ 3 & -6 & 3 \end{bmatrix}$,

$B = \begin{bmatrix} 7 & -4 & 2 \\ 2 & -3 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 6 \\ -2 & 5 \end{bmatrix}$, and $E = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$.

$A + 2B$, $3C - 4E$, DB, EB

Compute the matrix sum $A + 2B$. Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** $A + 2B = \begin{bmatrix} 16 & -8 & 2 \\ 7 & -12 & -3 \end{bmatrix}$
  (Simplify your answer.)
- **B.** The expression $A + 2B$ is undefined because A is not a square matrix.
- **C.** The expression $A + 2B$ is undefined because B is not a square matrix.
- **D.** The expression $A + 2B$ is undefined because A and 2B have different sizes.

Compute the matrix sum $3C - 4E$. Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** $3C - 4E =$
  (Simplify your answer.)
- **B.** The expression $3C - 4E$ is undefined because E is not a square matrix.
- **C.** The expression $3C - 4E$ is undefined because 3C and 4E have different sizes.
- **D.** The expression $3C - 4E$ is undefined because the number of rows in C is not equal to the number of columns in E.

Compute the matrix product DB. Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** $DB = \begin{bmatrix} 26 & -26 & -14 \\ -4 & -7 & -19 \end{bmatrix}$
  (Simplify your answer.)
- **B.** The expression DB is undefined because the number of columns in D is not equal to the number of rows in B.
- **C.** The expression DB is undefined because the number of columns in D is not equal to the number of columns in B.
- **D.** The expression DB is undefined because the number of rows in D is not equal to the number of columns in B.

Compute the matrix product EB. Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** $EB =$
  (Simplify your answer.)
- **B.** The expression EB is undefined because the number of columns in E is not equal to the number of columns in B.
- **C.** The expression EB is undefined because the number of columns in E is not equal to the number of rows in B.
- **D.** The expression EB is undefined because the number of rows in E is not equal to the number of columns in B.

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3. Let $A = \begin{bmatrix} 3 & -2 \\ 5 & -1 \end{bmatrix}$. Compute $3I_2 - A$ and $(3I_2)A$.

$3I_2 - A = \begin{bmatrix} 0 & 2 \\ -5 & 4 \end{bmatrix}$

(Type an integer or decimal for each matrix element.)

$(3I_2)A = \begin{bmatrix} 9 & -6 \\ 15 & -3 \end{bmatrix}$

(Type an integer or decimal for each matrix element.)

4. Compute $A - 4I_3$ and $(4I_3)A$, where $A = \begin{bmatrix} 5 & -1 & 2 \\ -3 & 2 & -7 \\ -2 & 2 & 2 \end{bmatrix}$.

$A - 4I_3 = \begin{bmatrix} 1 & -1 & 2 \\ -3 & -2 & -7 \\ -2 & 2 & -2 \end{bmatrix}$

$(4I_3)A = \begin{bmatrix} 20 & -4 & 8 \\ -12 & 8 & -28 \\ -8 & 8 & 8 \end{bmatrix}$

5. Compute the product AB by the definition of the product of matrices, where $A\mathbf{b}_1$ and $A\mathbf{b}_2$ are computed separately, and by the row-column rule for computing AB.

$$A = \begin{bmatrix} -2 & 4 \\ 2 & 5 \\ 6 & -2 \end{bmatrix}, B = \begin{bmatrix} 5 & -3 \\ -1 & 4 \end{bmatrix}$$

Set up the product $A\mathbf{b}_1$, where $\mathbf{b}_1$ is the first column of B.

$$A\mathbf{b}_1 = \begin{bmatrix} -2 & 4 \\ 2 & 5 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

(Use one answer box for A and use the other answer box for $\mathbf{b}_1$.)

Calculate $A\mathbf{b}_1$, where $\mathbf{b}_1$ is the first column of B.

$$A\mathbf{b}_1 = \begin{bmatrix} -14 \\ 5 \\ 32 \end{bmatrix}$$

(Type an integer or decimal for each matrix element.)

Set up the product $A\mathbf{b}_2$, where $\mathbf{b}_2$ is the second column of B.

$$A\mathbf{b}_2 = \begin{bmatrix} -2 & 4 \\ 2 & 5 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

(Use one answer box for A and use the other answer box for $\mathbf{b}_2$.)

Calculate $A\mathbf{b}_2$, where $\mathbf{b}_2$ is the second column of B.

$$A\mathbf{b}_2 = \begin{bmatrix} 22 \\ 14 \\ -26 \end{bmatrix}$$

(Type an integer or decimal for each matrix element.)

Determine the numerical expression for the first entry in the first column of AB using the row-column rule. Choose the correct answer below.

- **A.** $-2(5) + 4(-1)$
- **B.** $-2(5) - 4(-1)$
- **C.** $((-2) + (5)) \cdot ((4) + (-1))$
- **D.** $((-2) - (5)) \cdot ((4) - (-1))$

Determine the product AB.

$$AB = \begin{bmatrix} -14 & 22 \\ 5 & 14 \\ 32 & -26 \end{bmatrix}$$

(Use integers or decimals for any numbers in the expression.)

6. If a matrix A is 6×7 and the product AB is 6×5, what is the size of B?

The size of B is 7 $\times$ 5 .

7. Let $A = \begin{bmatrix} 4 & 3 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ -4 & k \end{bmatrix}$. What value(s) of k, if any, will make AB = BA?

Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. k = -3 (Use a comma to separate answers as needed.)

B. No value of k will make AB = BA

8. Let $A = \begin{bmatrix} -4 & 6 \\ 8 & -12 \end{bmatrix}$, $B = \begin{bmatrix} 14 & -2 \\ 10 & 8 \end{bmatrix}$, and $C = \begin{bmatrix} 5 & -11 \\ 4 & 2 \end{bmatrix}$. Verify that AB = AC and yet $B \neq C$.

Show the calculations that are used to find the entries for matrix AB. Choose the correct answer below.

A. $\begin{bmatrix} 8(14)+6(10) & 8(-2)+6(8) \\ -4(14)+(-12)(10) & -4(-2)+(-12)(8) \end{bmatrix}$

B. $\begin{bmatrix} -4(14)+6(10) & -4(-2)+6(8) \\ 8(14)+(-12)(10) & 8(-2)+(-12)(8) \end{bmatrix}$

C. $\begin{bmatrix} -4(14)+(-12)(8) & -4(-2)+(-12)(10) \\ 8(14)+6(8) & 8(-2)+6(10) \end{bmatrix}$

D. $\begin{bmatrix} -4(-2)+6(10) & -4(14)+6(8) \\ 8(-2)+(-12)(10) & 8(14)+(-12)(8) \end{bmatrix}$

Show the calculations that are used to find the entries for matrix AC. Choose the correct answer below.

A. $\begin{bmatrix} -4(5)+6(4) & -4(-11)+6(2) \\ 8(5)+(-12)(4) & 8(-11)+(-12)(2) \end{bmatrix}$

B. $\begin{bmatrix} -4(-11)+6(4) & -4(5)+6(2) \\ 8(-11)+(-12)(4) & 8(5)+(-12)(2) \end{bmatrix}$

C. $\begin{bmatrix} 8(5)+6(4) & 8(-11)+6(2) \\ -4(5)+(-12)(4) & -4(-11)+(-12)(2) \end{bmatrix}$

D. $\begin{bmatrix} -4(5)+(-12)(2) & -4(-11)+(-12)(4) \\ 8(5)+6(2) & 8(-11)+6(4) \end{bmatrix}$

Verify that AB = AC by simplifying.

$AB = AC = \begin{bmatrix} 4 & 56 \\ -8 & -112 \end{bmatrix}$

(Type an integer or decimal for each matrix element.)

9. Let $A = \begin{bmatrix} 4 & -12 \\ -5 & 15 \end{bmatrix}$. Construct a 2×2 matrix B such that AB is the zero matrix. Use two different nonzero columns for B.

$B = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$

10. Let $\mathbf{r}_1,\ldots,\mathbf{r}_p$ be vectors in $\mathbb{R}^n$, and let Q be an $m \times n$ matrix. Write the matrix $\begin{bmatrix} Q\mathbf{r}_1 \ldots Q\mathbf{r}_p \end{bmatrix}$ as a product of two matrices (neither of which is an identity matrix).

If the matrix R is defined as $\begin{bmatrix} \mathbf{r}_1 & \ldots & \mathbf{r}_p \end{bmatrix}$, then the matrix $\begin{bmatrix} Q\mathbf{r}_1 \ldots Q\mathbf{r}_p \end{bmatrix}$ can be written as QR .

11. If $A = \begin{bmatrix} 1 & -4 \\ -4 & 3 \end{bmatrix}$ and $AB = \begin{bmatrix} -2 & -16 & 7 \\ -5 & -1 & 3 \end{bmatrix}$, determine the first and second columns of B. Let $\mathbf{b}_1$ be column 1 of B and $\mathbf{b}_2$ be column 2 of B.

$\mathbf{b}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\mathbf{b}_2 = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

12. View vectors in $\mathbb{R}^n$ as $n \times 1$ matrices. For $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, the matrix product $\mathbf{u}^T\mathbf{v}$ is a 1×1 matrix, called the scalar product, or inner product, of $\mathbf{u}$ and $\mathbf{v}$. It is usually written as a single real number without brackets. The matrix product $\mathbf{u}\mathbf{v}^T$ is an $n \times n$ matrix, called the outer product of $\mathbf{u}$ and $\mathbf{v}$. Let $\mathbf{u} = \begin{bmatrix} -3 \\ 2 \\ -8 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Compute $\mathbf{u}^T\mathbf{v}$, $\mathbf{v}^T\mathbf{u}$, $\mathbf{u}\mathbf{v}^T$, and $\mathbf{v}\mathbf{u}^T$.

$\mathbf{u}^T\mathbf{v} = -3a + 2b - 8c$
(Do not factor.)

$\mathbf{v}^T\mathbf{u} = -3a + 2b - 8c$
(Do not factor.)

$$\mathbf{u}\mathbf{v}^T = \begin{bmatrix} -3a & -3b & -3c \\ 2a & 2b & 2c \\ -8a & -8b & -8c \end{bmatrix}$$

$$\mathbf{v}\mathbf{u}^T = \begin{bmatrix} -3a & 2a & -8a \\ -3b & 2b & -8b \\ -3c & 2c & -8c \end{bmatrix}$$

13. Give a formula for $(AB\mathbf{x})^T$, where $\mathbf{x}$ is a vector and A and B are matrices of appropriate size.

Choose the correct answer below.

- **A.** $(AB\mathbf{x})^T = A^T B^T \mathbf{x}^T$, because $(AB\mathbf{x})^T = (AB)^T \mathbf{x}^T = A^T B^T \mathbf{x}^T$
- **B.** $(AB\mathbf{x})^T = B^T A^T \mathbf{x}^T$, because $(AB\mathbf{x})^T = (AB)^T \mathbf{x}^T = B^T A^T \mathbf{x}^T$
- **C.** $(AB\mathbf{x})^T = \mathbf{x}^T B^T A^T$, because $(AB\mathbf{x})^T = \mathbf{x}^T (AB)^T = \mathbf{x}^T B^T A^T$
- **D.** $(AB\mathbf{x})^T = \mathbf{x}^T A^T B^T$, because $(AB\mathbf{x})^T = \mathbf{x}^T (AB)^T = \mathbf{x}^T A^T B^T$

14. Let $S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Compute S^k for $k = 2, \ldots, 6$.

$$S^2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S^3 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S^4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S^5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S^6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$