Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 4.4 Homework

1. Find the vector **x** determined by the given coordinate vector $[\mathbf{x}]_B$ and the given basis B.

$$B = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix} \right\}, [\mathbf{x}]_B = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} -10 \\ 9 \\ -10 \end{bmatrix}$$

(Simplify your answers.)

2. Find the coordinate vector $[\mathbf{x}]_B$ of **x** relative to the given basis $B = \{\mathbf{b}_1, \mathbf{b}_2\}$.

$$\mathbf{b}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -5 \\ -3 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} -6 \\ -2 \end{bmatrix}$$

$$[\mathbf{x}]_B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(Simplify your answers.)

3. Find the change-of-coordinates matrix from B to the standard basis in $\mathbb{R}^2$.

$$B = \left\{ \begin{bmatrix} -5 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\}$$

$$P_B = \begin{bmatrix} -5 & 3 \\ -4 & -1 \end{bmatrix}$$

4. The set $B = \{1 - t^2, 2t + t^2, 1 - t - t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 1 + 12t + 4t^2$ relative to B.

$$[\mathbf{p}]_B = \begin{bmatrix} 3 \\ 5 \\ -2 \end{bmatrix}$$

(Simplify your answers.)

5. If B is the standard basis of the space $\mathbb{P}_3$ of polynomials, then let $B = \{1, t, t^2, t^3\}$. Use coordinate vectors to test the linear independence of the set of polynomials below. Explain your work.

$1 + 9t^2 - t^3$, $t + 3t^3$, $1 + t + 9t^2$

Write the coordinate vector for the polynomial $1 + 9t^2 - t^3$.

$(1, 0, 9, -1)$

Write the coordinate vector for the polynomial $t + 3t^3$.

$(0, 1, 0, 3)$

Write the coordinate vector for the polynomial $1 + t + 9t^2$.

$(1, 1, 9, 0)$

To test the linear independence of the set of polynomials, row reduce the matrix which is formed by making each coordinate vector a column of the matrix. If possible, write the matrix in reduced echelon form.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 9 & 0 & 9 \\ -1 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Are the polynomials linearly independent?

- **A.** Since the matrix does not have a pivot in each column, its columns (and thus the given polynomials) are linearly independent.
- **B.** Since the matrix does not have a pivot in each column, its columns (and thus the given polynomials) are not linearly independent.
- **C.** Since the matrix has a pivot in each column, its columns (and thus the given polynomials) are linearly independent.
- **D.** Since the matrix has a pivot in each column, its columns (and thus the given polynomials) are not linearly independent.