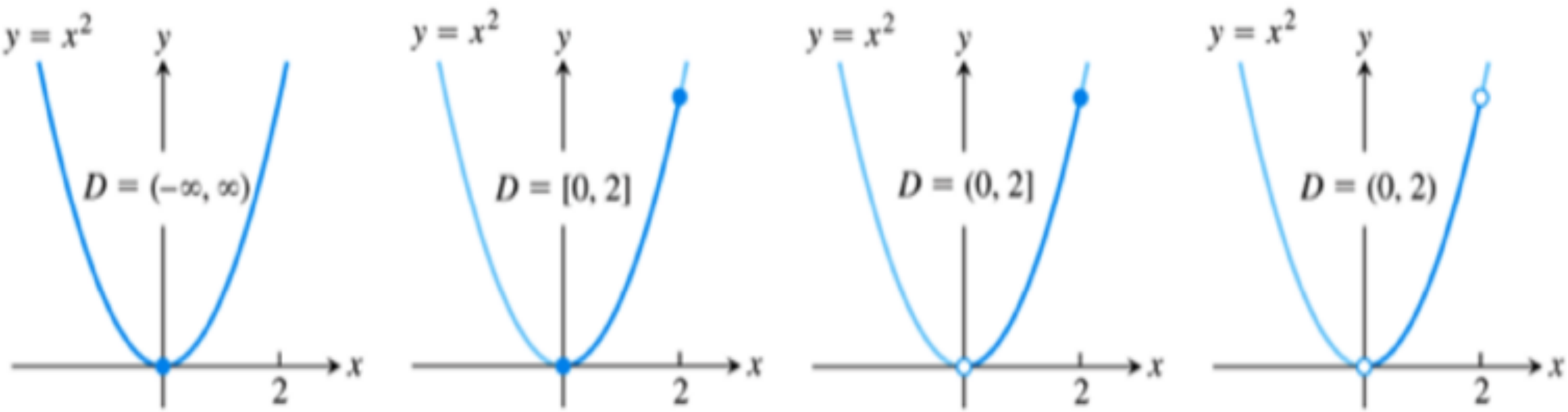
Ch4 Applications of differentiation: extreme values of functions

Def. f has an absolute max/min value at x_0

if $f(x_0) \underset{\leqslant}{\geqslant} f(x) \quad \forall x \in D_f$



(a) abs min only (b) abs max and min (c) abs max only (d) no max or min

Thm. A continuous function on a closed and bounded interval always has an absolute max value and an absolute min value on that interval

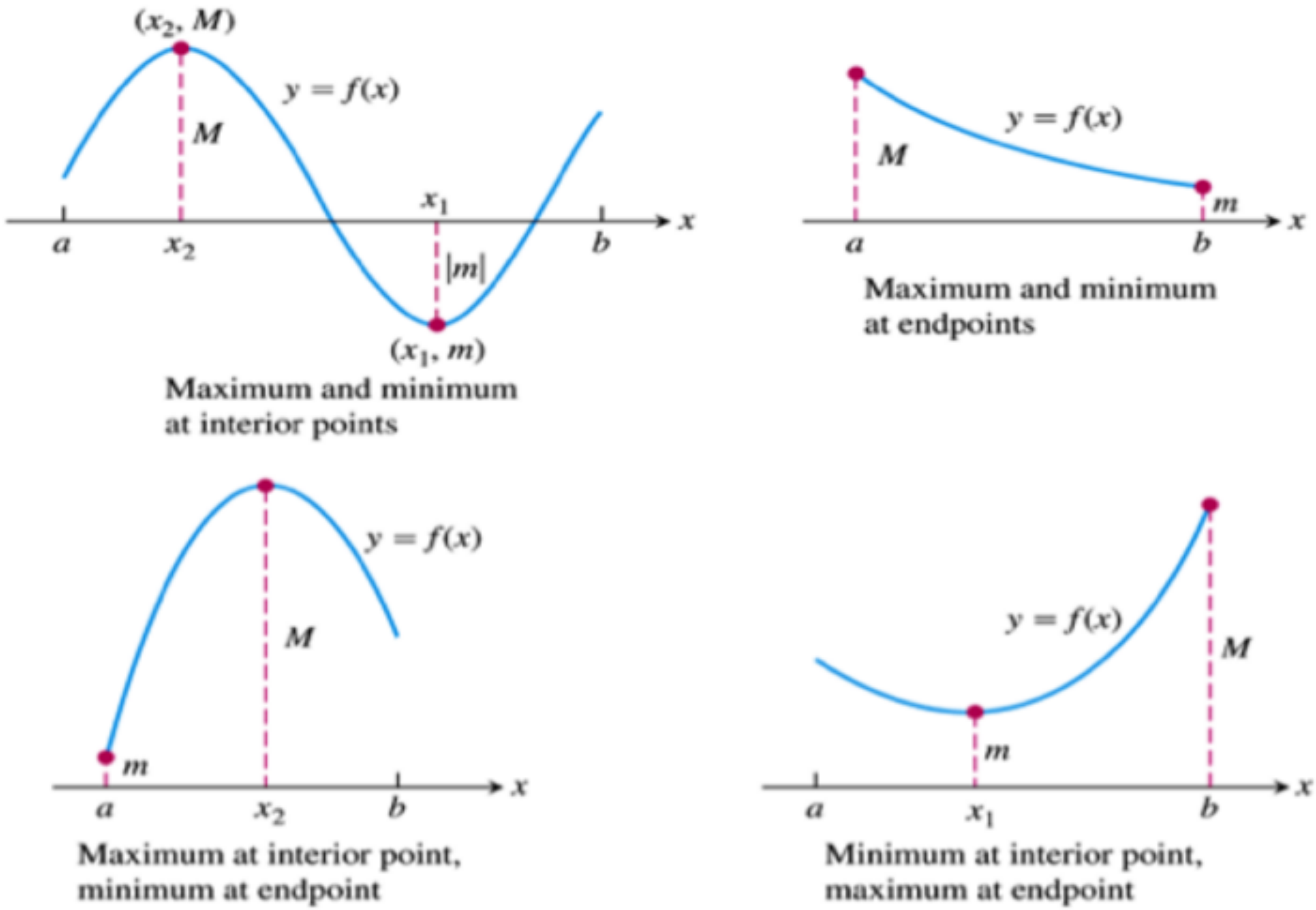


FIGURE Some possibilities for a continuous function's maximum and minimum on a closed interval $[a, b]$.

DEFINITIONS A function f has a **local maximum** value at a point c within its domain D if $f(x) \leq f(c)$ for all $x \in D$ lying in some open interval containing c.

A function f has a **local minimum** value at a point c within its domain D if $f(x) \geq f(c)$ for all $x \in D$ lying in some open interval containing c.

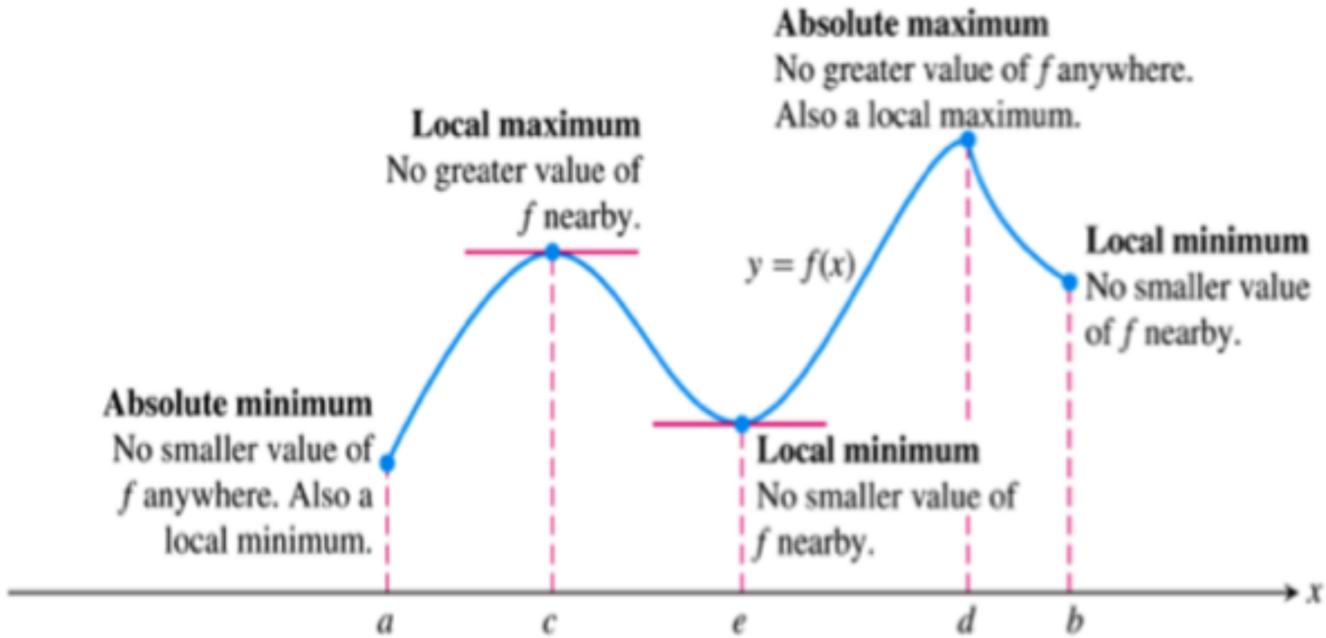
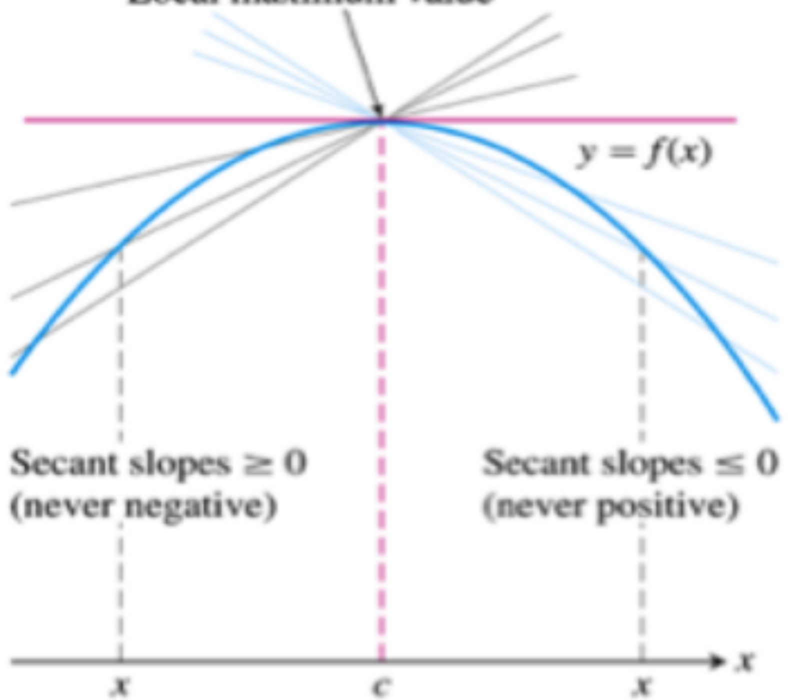


FIGURE How to identify types of maxima and minima for a function with domain $a \leq x \leq b$.

THEOREM – The First Derivative Theorem for Local Extreme Values If f has a local maximum or minimum value at an interior point c of its domain, and if f' is defined at c, then

$$f'(c) = 0.$$



A curve with a local maximum value. The slope at c, simultaneously the limit of nonpositive numbers and nonnegative numbers, is zero.

Ex

Find the absolute max and min values of $f(x)=x^{5/3}$ on $-1 \leq x \leq 8$

$$f'(x) = \frac{5}{3}x^{2/3} = 0 \Rightarrow x=0, \quad f(0)=0$$

$$f(-1) = -1, \quad f(8) = \sqrt[3]{8^5} = \sqrt[3]{(2^3)^5} = \cancel{2^{\frac{3\cdot5}{3}}} = 2^5$$

Def. A point $x_0 \in D_f$ is a critical point of f if $f'(x_0)=0$.

Absolute max (min) is either at a critical pt. or at an end pt.