Math

İSTANBUL ŞEHİR UNIVERSITY

No:

Full Name :

Math 104 2nd Midterm Exam
(27 March 2017, 18:30-19:30)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
20 pts	25 pts	35 pts	20 pts	100 pts

Q1. Find the derivative of the function

$$y = (\ln x)^{\sin x}$$

Logarithmic differentiation:

$$\ln y = \ln (\ln x)^{\sin x} = \sin x \ln(\ln x)$$

$$\frac{y'}{y} = \sin x \cdot \frac{1}{x \ln x} + \cos x \ln(\ln x)$$

$$y' = (\ln x)^{\sin x} \left\{ \frac{\sin x}{x \ln x} + \cos x \ln(\ln x) \right\}$$

Q2. Find the volume of the solid generated by revolving the region between the curve $xy = 4$ and the line $x + y = 5$ about the x-axis.

Points of intersection:

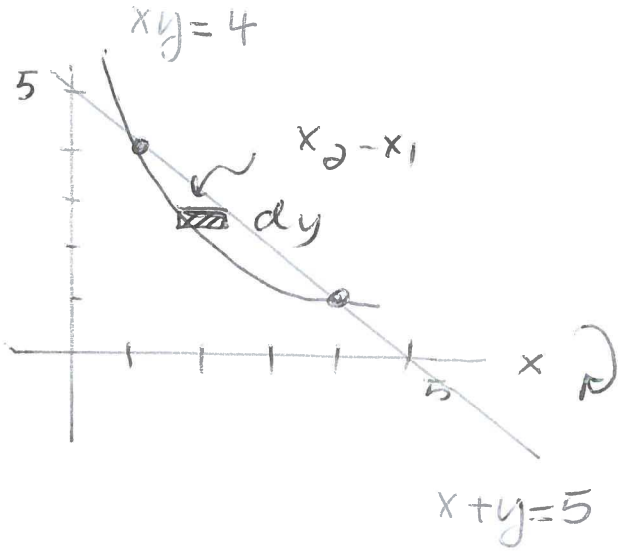
$x(5-x) = 4$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 1, 4$

$(1,4)$ and $(4,1)$



Shell method:

$$V = 2\pi \int_1^4 (x_2 - x_1)\, y\, dy$$

$$= 2\pi \int_1^4 \left(5 - y - \frac{4}{y}\right) y\, dy$$

$$= 2\pi \int_1^4 (5y - y^2 - 4)\, dy$$

$$= 2\pi \left(\frac{5y^2}{2} - \frac{y^3}{3} - 4y\right)\Big|_1^4$$

$$= 2\pi \left\{\left(\frac{5}{2}\cdot 16 - \frac{64}{3} - 16\right) - \left(\frac{5}{2} - \frac{1}{3} - 4\right)\right\}$$

$$= 2\pi \cdot \frac{9}{2} = \boxed{9\pi}$$

Math

İSTANBUL ŞEHİR UNIVERSITY

No:

Full Name :

Q3.

a) Evaluate the following limit, if it exists:

$$\lim_{x\to 0^+} \frac{(\ln x)^2}{\ln(\sin x)}$$

$\infty/-\infty$

$$= \text{L'Hospital} \lim_{x\to 0^+} \frac{\frac{2\ln x}{x}}{\frac{\cos x}{\sin x}}$$

$$= \lim_{x\to 0^+} 2\cdot \ln x \cdot \frac{\sin x}{x} \cdot \frac{1}{\cos x}$$

$\ln x \to -\infty$, $\frac{\sin x}{x} \to 1$, $\frac{1}{\cos x} \to 1$

$$= -\infty$$

Limit does not exist.

b) Evaluate the following integral:

$$\int \frac{dx}{(Arcsinx)\sqrt{1-x^2}}$$

$$= \int \frac{du}{u}$$

$u = Arcsinx$

$du = \frac{dx}{\sqrt{1-x^2}}$

$$= \ln|u| + C$$

$$= \ln|Arcsinx| + C$$

Q4. Evaluate the following integral:

$$\int x\tan^2 x\,dx = \int x(\sec^2 x - 1)\,dx$$

$$= \int x\sec^2 x - \frac{x^2}{2}$$

Integration by parts:

$u = x \qquad dv = \sec^2 x$

$du = dx \qquad v = \tan x$

$$\therefore \int x\tan^2 x\,dx = x\tan x - \int \tan x - \frac{x^2}{2}$$

$$= x\tan x + \int \frac{-\sin x\,dx}{\cos x} - \frac{x^2}{2}$$

$$= x\tan x + \ln|\cos x| - \frac{x^2}{2} + C$$