4.3.10

Find a basis for the null space of the matrix given below.

$$\begin{bmatrix} 1 & 1 & -3 & -1 & 7 \\ 0 & 1 & 0 & -3 & -3 \\ 0 & 0 & -8 & 0 & 24 \end{bmatrix}$$

A basis for the null space is $\left\{ \begin{bmatrix} -2 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate answers as needed.)

4.5.3

For the subspace below, (a) find a basis for the subspace, and (b) state the dimension.

$$\left\{ \begin{bmatrix} p-9q \\ 6p+3r \\ -9q+6r \\ -3p+6r \end{bmatrix} : p, q, r \text{ in } \mathbb{R} \right\}$$

(a) Find a basis for the subspace.

A basis for the subspace is $\left\{ \begin{bmatrix} 1 \\ 6 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -9 \\ 0 \\ -9 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 6 \\ 6 \end{bmatrix} \right\}$.

(Use a comma to separate answers as needed.)

(b) State the dimension.

The dimension is 3.

4.6.2

Assume that the matrix A is row equivalent to B. Without calculations, list rank A and dim Nul A. Then find bases for Col A, Row A, and Nul A.

$$A = \begin{bmatrix} 1 & 3 & 5 & -7 & -4 \\ 2 & 6 & 12 & -16 & -8 \\ -3 & -9 & -10 & 16 & 7 \\ -2 & -6 & -1 & 5 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & 5 & -7 & -4 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rank A = 3

dim Nul A = 2

A basis for Col A is $\left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 12 \\ -10 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ -8 \\ 7 \\ 0 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

A basis for Row A is $\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

4.1.18

Let W be the set of all vectors of the form shown on the right, where a, b, and c represent arbitrary real numbers. Find a set S of vectors that spans W or give an example or an explanation to show that W is not a vector space.

$$\begin{bmatrix} 3a + 4b \\ 0 \\ a + b + c \\ c - 2a \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The set W is a vector space and a spanning set is $S = \left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

B. The set W is not a vector space because W is not closed under vector addition.

C. The set W is not a vector space because W is not closed under scalar multiplication.

D. The set W is not a vector space because W does not contain the zero vector.

4.5.14

Determine the dimensions of Nul A and Col A for the matrix shown below.

$$A = \begin{bmatrix} 1 & 3 & -5 & 5 & -4 & 4 & -1 \\ 0 & 0 & 1 & -4 & 3 & 2 & -6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The dimension of Nul A is 5, and the dimension of Col A is 2.

4.4.13

The set $B = \{1 - t^2, 2t + t^2, 1 - t - t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 3 + 9t + 2t^2$ relative to B.

$$[\mathbf{p}]_B = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$$

(Simplify your answers.)