Math

ISTANBUL ŞEHİR UNIVERSITY

No:

Full Name :

KEY

Math 104 1st Midterm Exam
(26 July 2016, 9:30-11:00)

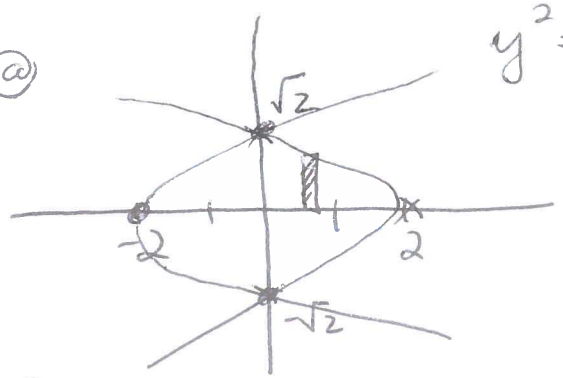
IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 5 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	Q5	TOT
20 pts	20 pts	20 pts	20 pts	20 pts	100 pts

Q1. Consider the region bounded between the parabolas $y^2 = x + 2$ and $y^2 = -x + 2$.

a) Sketch the region.
b) Set up an integral (or a sum of integrals) for the area of this region. DO NOT EVALUATE THE INTEGRAL.

(a)



$y^2 = -x + 2$

(b) By symmetry, $A = 4\int_0^2 y\,dx = \boxed{4\int_0^2 \sqrt{2-x}\,dx}$

OR

$A = 4\int_0^{\sqrt{2}} x\,dy = \boxed{4\int_0^{\sqrt{2}} (2-y^2)\,dy}$

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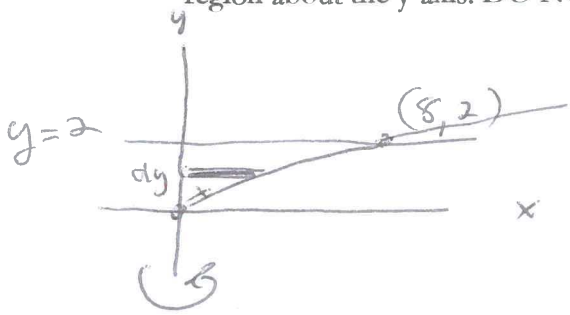
$x = y^3$

Q2. Consider the region bounded between the curve $y = x^{1/3}$, the line $y = 2$ and the y-axis, for $x \geq 0$.

a) Set up an integral for the volume of the solid of revolution obtained by rotating this region about the y-axis. DO NOT EVALUATE.



Disc method

$$V = \pi \int_0^2 x^2 dy = \pi \int_0^2 y^6 dy$$

OR Shell method

$$V = \int 2\pi x y \, dx = 2\pi \int_0^8 x\left(2 - x^{1/3}\right) dx$$

b) Set up an integral for the volume of the solid of revolution obtained by rotating this region about the x-axis. DO NOT EVALUATE.

Shell method

$$V = 2\pi \int_0^2 xy\,dy = 2\pi \int_0^2 y^4 dy$$

You can do this by the disc method too:

$$V = \pi \cdot 2^2 \cdot 8 - \pi \int_0^8 y^2 dx$$

$$= 32\pi - \pi \int_0^8 x^{2/3} dx$$

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Q3. Differentiate the function $y = \frac{1+lnx}{1-lnx}$.

(Remember that you must simplify your answers to get full credit)

Quotient Rule

$$y' = \frac{\frac{1-\ln x}{x} - \frac{1+\ln x}{-x}}{(1-\ln x)^2} = \frac{(1-\ln x) + (1+\ln x)}{x(1-\ln x)^2}$$

$$= \boxed{\frac{2}{x(1-\ln x)^2}}$$

OR logarithmic differentiation

$$\ln y = \ln(1+\ln x) - \ln(1-\ln x)$$

$$\frac{1}{y}y' = \frac{1/x}{1+\ln x} - \frac{-1/x}{1-\ln x}$$

$$= \frac{1 - \ln x + 1 + \ln x}{x(1+\ln x)(1-\ln x)} = \frac{2}{x(1+\ln x)(1-\ln x)}$$

$$\therefore y' = \frac{2}{x(1+\ln x)(1-\ln x)} \cdot \frac{1+\ln x}{1-\ln x}$$

$$= \boxed{\frac{2}{x(1-\ln x)^2}}$$

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Q4. Evaluate the following integral:

$$\int \frac{1}{\theta^2} cos\left(\frac{1}{\theta}\right) d\theta$$

Let $x = \frac{1}{\theta} \Rightarrow dx = -\frac{d\theta}{\theta^2}$

$$\int -\cos x \, dx = -\sin x + C$$

$$= -\sin\left(\frac{1}{\theta}\right) + C$$

Q5. Evaluate the following limit, if it exists:

$$\lim_{x\to e^+} (\ln x)^{\frac{1}{x-e}}$$

1^{∞}

y

$$\ln y = \frac{1}{x-e} \ln \ln x$$

$0/0$

$$\lim_{x\to e^+} \ln y = \lim_{x\to e^+} \frac{\ln \ln x}{x-e}$$

$$\overset{\text{L'Hospital}}{=} \lim_{x\to e^+} \frac{\frac{1}{x \ln x}}{1}$$

$$= \lim_{x\to e^+} \frac{1}{x \ln x} = \frac{1}{e}$$

$$\ln y \to 1/e$$

$$y \to \boxed{e^{1/e}}$$