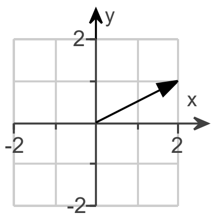
Student: Huseyin Kerem Mican **Date:** 5/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 1.3 Homework

1. Display **u**, **v**, − **v**, − 2**v**, **u** + **v**, **u** − **v**, and **u** − 2**v** using arrows on an xy-graph.

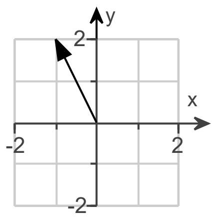
$$\mathbf{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -7 \\ -5 \end{bmatrix}$$

Choose the correct graph below that displays **u**.

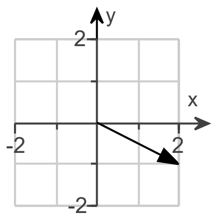
A.



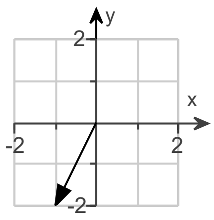
B.



C.

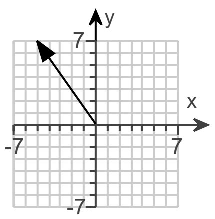


D.

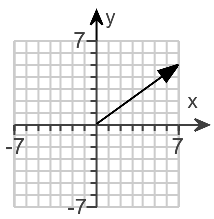


Choose the correct graph below that displays **v**.

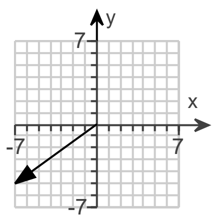
A.



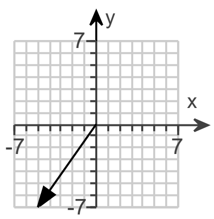
B.



C.

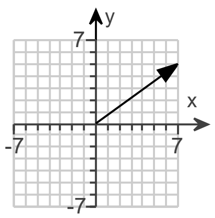


D.

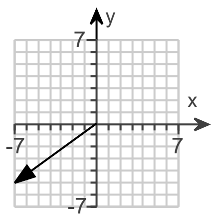


Choose the correct graph below that displays − **v**.

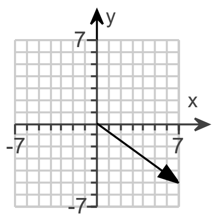
A.



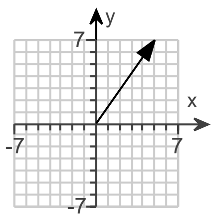
B.



C.

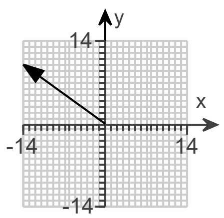


D.

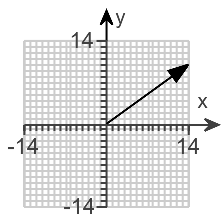


Choose the correct graph below that displays − 2**v**.

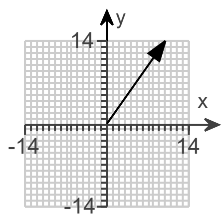
A.



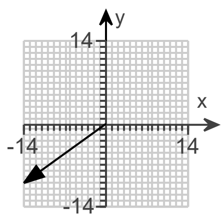
B.



C.

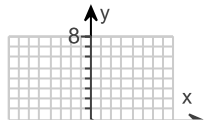


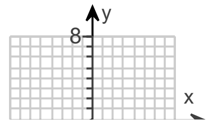
D.

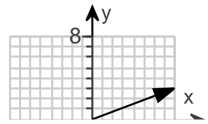


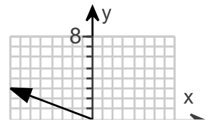
Choose the correct graph below that displays **u** + **v**.

A. B. C. D.



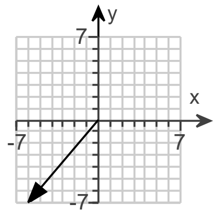




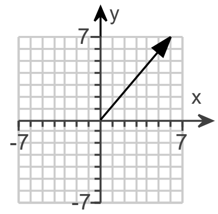


Choose the correct graph below that displays $\mathbf{u} - \mathbf{v}$.

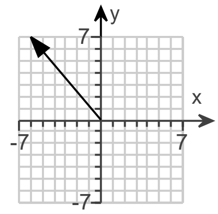
A.



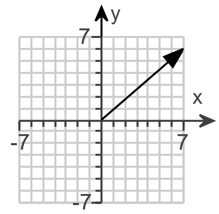
B.



C.

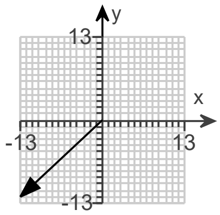


D.

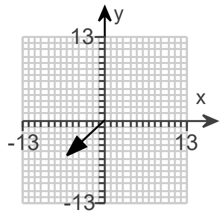


Choose the correct graph below that displays $\mathbf{u} - 2\mathbf{v}$.

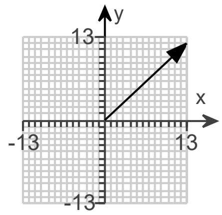
A.



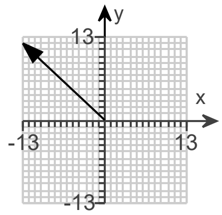
B.



C.



D.



What is the geometric relationship between $\mathbf{u}$, $-\mathbf{v}$, and $\mathbf{u} - \mathbf{v}$?

- **A.** The vectors $\mathbf{u}$, $-\mathbf{v}$, and $\mathbf{u} - \mathbf{v}$ form a parallelogram whose other vertex is at $\mathbf{u} + \mathbf{v}$.
- **B.** The vectors $\mathbf{u}$, $-\mathbf{v}$, and $\mathbf{u} - \mathbf{v}$ form a right triangle.
- **C.** The vectors $\mathbf{u}$, $-\mathbf{v}$, and $\mathbf{u} - \mathbf{v}$ form a parallelogram whose other vertex is at $\mathbf{0}$.
- **D.** The vectors $\mathbf{u}$, $-\mathbf{v}$, and $\mathbf{u} - \mathbf{v}$ form an equilateral triangle.

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2. Write a system of equations that is equivalent to the given vector equation.

$$x_1\begin{bmatrix} 4 \\ -3 \\ 9 \end{bmatrix} + x_2\begin{bmatrix} 5 \\ 0 \\ -6 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix}$$

Choose the correct answer below.

A.
$$\begin{aligned} 4x_1 + 5x_2 &= 2 \\ -3x_1 + x_2 &= -2 \\ 9x_1 - 6x_2 &= 6 \end{aligned}$$

B.
$$\begin{aligned} 4x_1 + 5x_2 &= 6 \\ -3x_1 &= -2 \\ 9x_1 - 6x_2 &= 2 \end{aligned}$$

C.
$$\begin{aligned} 4x_1 + 5x_2 &= -2 \\ -3x_1 &= 2 \\ 9x_1 - 6x_2 &= 6 \end{aligned}$$

D.
$$\begin{aligned} 4x_1 + 5x_2 &= 2 \\ -3x_1 &= -2 \\ 9x_1 - 6x_2 &= 6 \end{aligned}$$

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3. Write a vector equation that is equivalent to the given system of equations.

$$\begin{aligned} x_2 + 4x_3 &= 0 \\ 3x_1 + 7x_2 - x_3 &= 0 \\ -x_1 + 6x_2 - 7x_3 &= 0 \end{aligned}$$

Choose the correct answer below.

A. $x_1\begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix} + x_3\begin{bmatrix} 4 \\ -1 \\ -7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

B. $x_1\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} + x_3\begin{bmatrix} 4 \\ -1 \\ -7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

C. $x_1\begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix} + x_3\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ -7 \end{bmatrix}$

D. $x_1\begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} + x_2\begin{bmatrix} 4 \\ -1 \\ -7 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

4. Determine if b is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 4 \\ -4 \\ 16 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -2 \\ 8 \end{bmatrix}$$

Choose the correct answer below.

A. Vector b is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. The pivots in the corresponding echelon matrix are in the first entry in the first column and the second entry in the second column.

B. Vector b is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. The pivots in the corresponding echelon matrix are in the first entry in the first column, the second entry in the second column, and the third entry in the third column.

C. Vector b is not a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

D. Vector b is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. The pivots in the corresponding echelon matrix are in the first entry in the first column, the second entry in the second column, and the third entry in the fourth column.

5. Determine if **b** is a linear combination of the vectors formed from the columns of the matrix A.

$$A = \begin{bmatrix} 1 & -3 & 4 \\ 0 & 3 & 4 \\ -2 & 6 & -8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ -5 \\ -3 \end{bmatrix}$$

Choose the correct answer below.

- **A.** Vector **b** is a linear combination of the vectors formed from the columns of the matrix A. The pivots in the corresponding echelon matrix are in the first entry in the first column, the second entry in the second column, and the third entry in the fourth column.
- **B.** Vector **b** is not a linear combination of the vectors formed from the columns of the matrix A.
- **C.** Vector **b** is a linear combination of the vectors formed from the columns of the matrix A. The pivots in the corresponding echelon matrix are in the first entry in the first column and the third entry in the second column, and the third entry in the third column.
- **D.** Vector **b** is a linear combination of the vectors formed from the columns of the matrix A. The pivots in the corresponding echelon matrix are in the first entry in the first column, the second entry in the second column, and the third entry in the third column.

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6. Determine if **b** is a linear combination of the vectors formed from the columns of the matrix A.

$$A = \begin{bmatrix} 1 & -4 & -4 \\ 0 & 5 & 4 \\ 3 & -12 & 11 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 13 \\ -7 \\ 8 \end{bmatrix}$$

Choose the correct answer below.

- **A.** Vector **b** is not a linear combination of the vectors formed from the columns of the matrix A. The pivots in the corresponding echelon matrix are only in the first entry in the first column and the second entry in the second column. There are no pivots in the third and fourth columns.
- **B.** Vector **b** is a linear combination of the vectors formed from the columns of the matrix A. The pivots in the corresponding echelon matrix are in the first entry in the first column and the second entry in the second column. There are no pivots in the third and fourth columns.
- **C.** Vector **b** is not a linear combination of the vectors formed from the columns of the matrix A. There is no solution to the linear system corresponding to the matrix $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$.
- **D.** Vector **b** is a linear combination of the vectors formed from the columns of the matrix A. The pivots in the corresponding echelon matrix are in the first entry in the first column, the second entry in the second column, and the third entry in the third column.

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7. List five vectors in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$. Do not make a sketch.

$$\mathbf{v}_1 = \begin{bmatrix} 8 \\ 4 \\ -5 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix}$$

List five vectors in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix}\begin{bmatrix} 8 \\ 4 \\ -5 \end{bmatrix}\begin{bmatrix} 3 \\ 8 \\ -5 \end{bmatrix}\begin{bmatrix} 13 \\ 0 \\ -5 \end{bmatrix}$$

(Use the matrix template in the math palette. Use a comma to separate vectors as needed. Type an integer or a simplified fraction for each vector element. Type each answer only once.)

8. Give a geometric description of Span $\{\mathbf{v}_1,\mathbf{v}_2\}$ for the vectors $\mathbf{v}_1 = \begin{bmatrix} 24 \\ 12 \\ -30 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 16 \\ 8 \\ -20 \end{bmatrix}$.

Choose the correct answer below.

- **A.** Span $\{\mathbf{v}_1,\mathbf{v}_2\}$ is the set of points on the line through $\mathbf{v}_1$ and $\mathbf{0}$.
- **B.** Span $\{\mathbf{v}_1,\mathbf{v}_2\}$ is $\mathbb{R}^3$.
- **C.** Span $\{\mathbf{v}_1,\mathbf{v}_2\}$ cannot be determined with the given information.
- **D.** Span $\{\mathbf{v}_1,\mathbf{v}_2\}$ is the plane in $\mathbb{R}^3$ that contains $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{0}$.

9. Mark each statement True or False. Justify each answer. Complete parts a through e below.

a. When $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, Span$\{\mathbf{u},\mathbf{v}\}$ contains only the line through $\mathbf{u}$ and the line through $\mathbf{v}$ and the origin.

- **A.** True. Span$\{\mathbf{u},\mathbf{v}\}$ is the set of all scalar multiples of $\mathbf{u}$ and all scalar multiples of $\mathbf{v}$.
- **B.** False. Span$\{\mathbf{u},\mathbf{v}\}$ includes linear combinations of both $\mathbf{u}$ and $\mathbf{v}$.
- **C.** False. Span$\{\mathbf{u},\mathbf{v}\}$ will not contain the origin.

b. Any list of five real numbers is a vector in $\mathbb{R}^5$.

- **A.** True. $\mathbb{R}^5$ denotes the collection of all lists of five real numbers.
- **B.** False. A list of five real numbers is a vector in $\mathbb{R}^6$.
- **C.** False. A list of numbers is not enough to constitute a vector.
- **D.** False. A list of five real numbers is a vector in $\mathbb{R}^n$.

c. Asking whether the linear system corresponding to an augmented matrix $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{b} \end{bmatrix}$ has a solution amounts to asking whether $\mathbf{b}$ is in Span $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$.

- **A.** True. An augmented matrix has a solution when the last column can be written as a linear combination of the other columns. A linear system augmented has a solution when the last column of its augmented matrix can be written as a linear combination of the other columns.
- **B.** False. If $\mathbf{b}$ corresponds to the origin then it cannot be in Span $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$.
- **C.** False. An augmented matrix having a solution does not mean $\mathbf{b}$ is in Span $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$.

d. The vector $\mathbf{v}$ results when a vector $\mathbf{u}-\mathbf{v}$ is added to the vector $\mathbf{v}$.

- **A.** False. Adding $\mathbf{u}-\mathbf{v}$ to $\mathbf{v}$ results in $2\mathbf{v}$.
- **B.** False. Adding $\mathbf{u}-\mathbf{v}$ to $\mathbf{v}$ results in $\mathbf{u}-2\mathbf{v}$.
- **C.** False. Adding $\mathbf{u}-\mathbf{v}$ to $\mathbf{v}$ results in $\mathbf{u}$.
- **D.** True. Adding $\mathbf{u}-\mathbf{v}$ to $\mathbf{v}$ results in $\mathbf{v}$.

e. The weights $c_1,\ldots,c_p$ in a linear combination $c_1\mathbf{v}_1 + \ldots + c_p\mathbf{v}_p$ cannot all be zero.

- **A.** True. Setting all the weights equal to zero does not result in the vector $\mathbf{0}$.
- **B.** False. Setting all the weights equal to zero results in the vector $\mathbf{0}$.
- **C.** True. Setting all the weights equal to zero results in the vector $\mathbf{0}$.
- **D.** False. Setting all the weights equal to zero does not result in the vector $\mathbf{0}$.

10.

Let $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 2 & -2 \\ -2 & 4 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ -3 \end{bmatrix}$. Denote the columns of A by $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, and let $W = \text{Span}\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$.

a. Is **b** in $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$? How many vectors are in $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$?
b. Is **b** in W? How many vectors are in W?
c. Show that $\mathbf{a}_2$ is in W. [Hint: Row operations are unnecessary.]

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a. Is **b** in $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$?

- Yes
- ✔ No

How many vectors are in $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$?

- ✔ **A.** Three
- **B.** Two
- **C.** One
- **D.** Infinitely many

b. Set up the appropriate augmented matrix for determining if **b** is in W.

$$\begin{bmatrix} 1 & 0 & -4 & 3 \\ 0 & 2 & -2 & 1 \\ -2 & 4 & 2 & -3 \end{bmatrix}$$

(Simplify your answers.)

Is **b** in W?

- **A.** Yes, because the row-reduced form of the augmented matrix has a pivot in the rightmost column.
- **B.** No, because the row-reduced form of the augmented matrix does not have a pivot in the rightmost column.
- **C.** No, because the row-reduced form of the augmented matrix has a pivot in the rightmost column.
- ✔ **D.** Yes, because the row-reduced form of the augmented matrix does not have a pivot in the rightmost column.

How many vectors are in W?

- **A.** Three
- **B.** One
- **C.** Two
- ✔ **D.** Infinitely many

c. The vector $\mathbf{a}_2$ is in $W = \text{Span}\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$ because $\mathbf{a}_2$ can be written as a linear combination $c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + c_3\mathbf{a}_3$ where c_1, c_2, and c_3 are scalars.

Thus, $\mathbf{a}_2$ is in W because $\mathbf{a}_2 =$ 0 $\mathbf{a}_1 +$ 1 $\mathbf{a}_2 +$ 0 $\mathbf{a}_3$.
(Simplify your answers.)