Student: Huseyin Kerem Mican **Date:** 5/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 1.8 Homework

1. Let $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, and define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$. Find the images under T of $\mathbf{u} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} a \\ b \end{bmatrix}$.

$T(\mathbf{u}) = \begin{bmatrix} 3 \\ -15 \end{bmatrix}$

$T(\mathbf{v}) = \begin{bmatrix} 3a \\ 3b \end{bmatrix}$

2. If T is defined by $T(\mathbf{x}) = A\mathbf{x}$, find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$, and determine whether $\mathbf{x}$ is unique. Let $A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -3 \\ 3 & -10 & 6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -4 \\ -7 \\ -2 \end{bmatrix}$.

Find a single vector **x** whose image under T is **b**.

$\mathbf{x} = \begin{bmatrix} -32 \\ -10 \\ -1 \end{bmatrix}$

Is the vector **x** found in the previous step unique?

- **A.** No, because there are no free variables in the system of equations.
- **B.** No, because there is a free variable in the system of equations.
- **C.** Yes, because there is a free variable in the system of equations.
- **D.** Yes, because there are no free variables in the system of equations.

3. Find a vector **x** whose image under T, defined by $T(\mathbf{x}) = A\mathbf{x}$, is **b**, and determine whether **x** is unique. Let $A = \begin{bmatrix} 1 & 4 & 7 \\ 3 & 16 & 25 \\ 0 & 1 & 1 \\ -3 & -13 & -22 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 23 \\ 85 \\ 4 \\ -73 \end{bmatrix}$.

Find a single vector **x** whose image under T is **b**.

$\mathbf{x} = \begin{bmatrix} 7 \\ 4 \\ 0 \end{bmatrix}$

Is the vector **x** found in the previous step unique?

- **A.** Yes, because there are no free variables in the system of equations.
- **B.** No, because there are no free variables in the system of equations.
- **C.** Yes, because there is a free variable in the system of equations.
- **D.** No, because there is a free variable in the system of equations.

4. Let A be a 4×4 matrix. What must a and b be in order to define $T: \mathbb{R}^a \to \mathbb{R}^b$ by $T(\mathbf{x}) = A\mathbf{x}$?

a = 4
(Simplify your answer.)

b = 4
(Simplify your answer.)

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5. Find all $\mathbf{x}$ in $\mathbb{R}^4$ that are mapped into the zero vector by the transformation $\mathbf{x} \mapsto A\mathbf{x}$ for the given matrix A.

$$A = \begin{bmatrix} 1 & -5 & 6 & -12 \\ 0 & 1 & -3 & 4 \\ 2 & -8 & 6 & -16 \end{bmatrix}$$

Select the correct choice below and fill in the answer box(es) to complete your choice.

A. There is only one vector, which is $\mathbf{x}$ = ____ .

B. x_3 ____

C. x_1 ____ $+ x_2$ ____ $+ x_4$ ____

D. $x_3 \begin{bmatrix} 9 \\ 3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -8 \\ -4 \\ 0 \\ 1 \end{bmatrix}$

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6. Let $\mathbf{b} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$, and let A be the matrix $\begin{bmatrix} 1 & -4 & 6 & -6 \\ 0 & 1 & -3 & 6 \\ 3 & -10 & 12 & -5 \end{bmatrix}$. Is $\mathbf{b}$ in the range of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$? Why or why not?

Is **b** in the range of the linear transformation? Why or why not?

A. No, **b** is not in the range of the linear transformation because the system represented by the augmented matrix [A **b**] is inconsistent.

B. No, **b** is not in the range of the linear transformation because the system represented by the augmented matrix [A **b**] is consistent.

C. Yes, **b** is in the range of the linear transformation because the system represented by the augmented matrix [A **b**] is inconsistent.

D. Yes, **b** is in the range of the linear transformation because the system represented by the augmented matrix [A **b**] is consistent.

7. Let $\mathbf{b} = \begin{bmatrix} -8 \\ 1 \\ -3 \\ 7 \end{bmatrix}$, and let A be the matrix $\begin{bmatrix} 1 & 2 & 6 & 0 \\ 1 & 0 & 2 & -4 \\ 0 & 1 & 2 & 3 \\ -3 & 1 & -4 & 5 \end{bmatrix}$. Is **b** in the range of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$? Why or why not?

Is **b** in the range of the linear transformation? Why or why not?

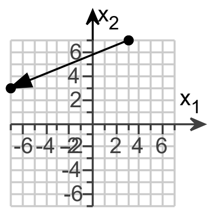
- **A.** Yes, **b** is in the range of the linear transformation because the system represented by the appropriate augmented matrix is inconsistent.
- **B.** No, **b** is not in the range of the linear transformation because the system represented by the appropriate augmented matrix is consistent.
- **C.** No, **b** is not in the range of the linear transformation because the system represented by the appropriate augmented matrix is inconsistent.
- **D.** Yes, **b** is in the range of the linear transformation because the system represented by the appropriate augmented matrix is consistent.

8. Use a rectangular coordinate system to plot $\mathbf{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$, and their images under the given transformation T. Describe geometrically what T does to each vector **x** in $\mathbb{R}^2$.

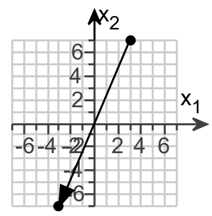
$$T(\mathbf{x}) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Which graph below shows **u** and its image under the given transformation?

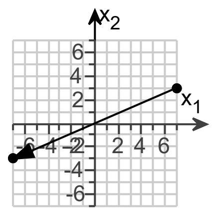
A.



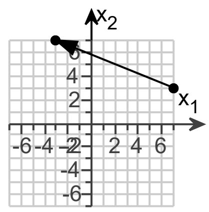
B.



C.

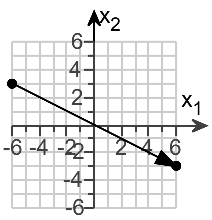


D.

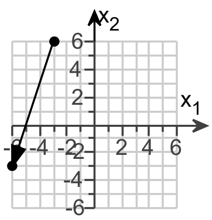


Which graph below shows **v** and its image under the given transformation?

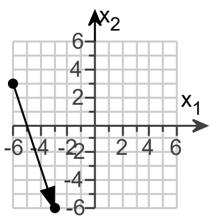
A.



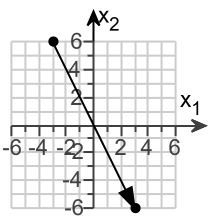
B.



C.



D.



What does T do geometrically to each vector **x** in $\mathbb{R}^2$?

- **A.** A reflection through the origin
- **B.** A dilation transformation over the x-axis
- **C.** A projection onto the x-axis
- **D.** A shear transformation

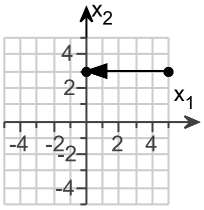
9. Use a rectangular coordinate system to plot $\mathbf{u} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$, and their images under the given transformation T.

Describe geometrically what T does to each vector $\mathbf{x}$ in $\mathbb{R}^2$.

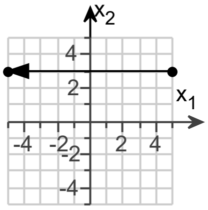
$$T(\mathbf{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Which graph below shows **u** and its image under the given transformation?

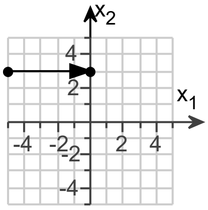
A.



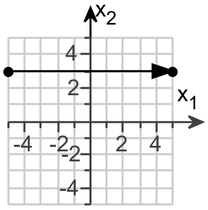
B.



C.

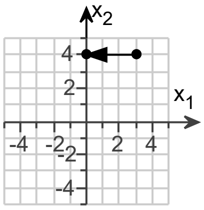


D.

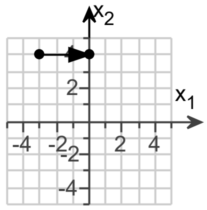


Which graph below shows **v** and its image under the given transformation?

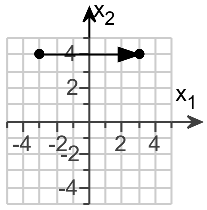
A.



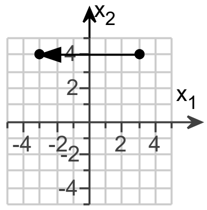
B.



C.



D.



What does T do geometrically to each vector $\mathbf{x}$ in $\mathbb{R}^2$?

- **A.** A reflection through the origin
- **B.** A projection onto the y-axis
- **C.** A shear transformation
- **D.** A rotation over the x-axis

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10. Let T: $\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{u} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ into $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and maps $\mathbf{v} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$. Use the fact that T is linear to find the images under T of $2\mathbf{u}$, $3\mathbf{v}$, and $2\mathbf{u} + 3\mathbf{v}$.

What is the image of $2\mathbf{u}$?

- A. $\begin{bmatrix} 2 \\ 10 \end{bmatrix}$
- B. $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$
- C. $\begin{bmatrix} -3 \\ 9 \end{bmatrix}$
- D. $\begin{bmatrix} 10 \\ 2 \end{bmatrix}$ ✔

What is the image of $3\mathbf{v}$?

- A. $\begin{bmatrix} -3 \\ 9 \end{bmatrix}$ ✔
- B. $\begin{bmatrix} 9 \\ -3 \end{bmatrix}$
- C. $\begin{bmatrix} -3 \\ 3 \end{bmatrix}$
- D. $\begin{bmatrix} 10 \\ 2 \end{bmatrix}$

What is the image of $2\mathbf{u} + 3\mathbf{v}$?

- A. $\begin{bmatrix} -11 \\ 7 \end{bmatrix}$
- B. $\begin{bmatrix} 11 \\ 7 \end{bmatrix}$
- C. $\begin{bmatrix} -7 \\ -11 \end{bmatrix}$
- D. $\begin{bmatrix} 7 \\ 11 \end{bmatrix}$ ✔

11.

Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{y}_1 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, and $\mathbf{y}_2 = \begin{bmatrix} -1 \\ 8 \end{bmatrix}$, and let T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation that maps $\mathbf{e}_1$ into $\mathbf{y}_1$ and maps $\mathbf{e}_2$ into $\mathbf{y}_2$. Find the images of $\begin{bmatrix} 4 \\ -4 \end{bmatrix}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

Which is the correct image of $\begin{bmatrix} 4 \\ -4 \end{bmatrix}$?

- **A.** $\begin{bmatrix} 16 \\ -12 \end{bmatrix}$
- **B.** $\begin{bmatrix} -12 \\ 16 \end{bmatrix}$
- **C.** $\begin{bmatrix} -12 \\ -16 \end{bmatrix}$
- **D.** $\begin{bmatrix} 4 \\ -4 \end{bmatrix}$

Which is the correct image of $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$?

- **A.** $\begin{bmatrix} 5x_1 - x_2 \\ 3x_1 + 8x_2 \end{bmatrix}$
- **B.** $\begin{bmatrix} 3x_1 - x_2 \\ 5x_1 + 8x_2 \end{bmatrix}$
- **C.** $\begin{bmatrix} 5x_1 - 8x_2 \\ 3x_1 + x_2 \end{bmatrix}$
- **D.** $\begin{bmatrix} 3x_1 + x_2 \\ 5x_1 - 8x_2 \end{bmatrix}$

12.

Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} -2 \\ -8 \end{bmatrix}$, and $\mathbf{v}_2 = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$, and let T : $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation that maps $\mathbf{x}$ into $x_1\mathbf{v}_1 + x_2\mathbf{v}_2$. Find a matrix A such that T($\mathbf{x}$) is A$\mathbf{x}$ for each $\mathbf{x}$.

$A = \begin{bmatrix} -2 & 6 \\ -8 & -5 \end{bmatrix}$

13. Determine whether each statement below is true or false. Justify each answer.

a. A linear transformation is a special type of function.

- **A.** False. A linear transformation is not a function because it maps one vector **x** to more than one vector T(**x**).
- **B.** False. A linear transformation is not a function because it maps more than one vector **x** to the same vector T(**x**).
- **C.** True. A linear transformation is a function from $\mathbb{R}$ to $\mathbb{R}$ that assigns to each vector **x** in $\mathbb{R}$ a vector T(**x**) in $\mathbb{R}$.
- ✓ **D.** True. A linear transformation is a function from $\mathbb{R}^n$ to $\mathbb{R}^m$ that assigns to each vector **x** in $\mathbb{R}^n$ a vector T(**x**) in $\mathbb{R}^m$.

b. If A is a 3×5 matrix and T is a transformation defined by T(**x**) = A**x**, then the domain of T is $\mathbb{R}^3$.

- **A.** True. The domain is $\mathbb{R}^3$ because A has 3 columns, because in the product A**x**, if A is an $m \times n$ matrix then **x** must be a vector in $\mathbb{R}^m$.
- **B.** False. The domain is actually $\mathbb{R}$, because in the product A**x**, if A is an $m \times n$ matrix then **x** must be a vector in $\mathbb{R}$.
- ✓ **C.** False. The domain is actually $\mathbb{R}^5$, because in the product A**x**, if A is an $m \times n$ matrix then **x** must be a vector in $\mathbb{R}^n$.
- **D.** True. The domain is $\mathbb{R}^3$ because A has 3 rows, because in the product A**x**, if A is an $m \times n$ matrix then **x** must be a vector in $\mathbb{R}^m$.

c. If A is an $m \times n$ matrix, then the range of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is $\mathbb{R}^m$.

- **A.** False. The range of the transformation is $\mathbb{R}^n$ because the domain of the transformation is $\mathbb{R}^m$.
- **B.** True. The range of the transformation is $\mathbb{R}^m$, because each vector in $\mathbb{R}^m$ is a linear combination of the columns of A.
- ✓ **C.** False. The range of the transformation is the set of all linear combinations of the columns of A, because each image of the transformation is of the form A**x**.
- **D.** True. The range of the transformation is $\mathbb{R}^m$, because each vector in $\mathbb{R}^m$ is a linear combination of the rows of A.

d. Every linear transformation is a matrix transformation.

- **A.** True. Every linear transformation T(**x**) can be expressed as a multiplication of a vector A by a matrix **x** such as A**x**.
- ✓ **B.** False. A matrix transformation is a special linear transformation of the form $\mathbf{x} \mapsto A\mathbf{x}$ where A is a matrix.
- **C.** True. Every linear transformation T(**x**) can be expressed as a multiplication of a matrix A by a vector **x** such as A**x**.
- **D.** False. A matrix transformation not a linear transformation because multiplication of a matrix A by a vector **x** is not linear.

e. A transformation T is linear if and only if $T\left(c_1\mathbf{v}_1 + c_2\mathbf{v}_2\right) = c_1T\left(\mathbf{v}_1\right) + c_2T\left(\mathbf{v}_2\right)$ for all $\mathbf{v}_1$ and $\mathbf{v}_2$ in the domain of T and for all scalars c_1 and c_2.

- ✓ **A.** True. This equation correctly summarizes the properties necessary for a transformation to be linear.
- **B.** False. A transformation T is linear if and only if T(c**u**) = cT(**u**) for all scalars c and all **u** in the domain of T.
- **C.** False. A transformation T is linear if and only if T(**0**) = **0**.

- **D.** False. A transformation T is linear if and only if T(**u** + **v**) = T(**u**) + T(**v**) for all **u**, **v** in the domain of T.

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14. Show that the transformation T defined by $T(x_1, x_2) = (4x_1 - 3x_2, x_1 + 5, 4x_2)$ is not linear.

If T is a linear transformation, then $T(\mathbf{0}) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $T(c\mathbf{u} + d\mathbf{v}) = cT(\mathbf{u}) + dT(\mathbf{v})$ for all vectors **u**, **v** in the domain of T and all scalars c, d.
(Type a column vector.)

Check if T(**0**) follows the correct property to be linear.

$T(0,0) = (4(0) - 3(0), (0) + 5, 4(0))$ Substitute.

$= (0 , 5 , 0)$ Simplify.

What is true about T(**0**)?

- **A.** $T(\mathbf{0}) = \mathbf{0}$
- **B.** $T(\mathbf{0}) \neq \mathbf{0}$
- **C.** $T(\mathbf{0}) = (1,1,1)$
- **D.** $T(\mathbf{0}) = 5$

Therefore, T is not linear.

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15. The given matrix determines a linear transformation T. Find all **x** such that T(**x**) = **0**.

$$\begin{bmatrix} 3 & -1 & 3 & -2 \\ -5 & 3 & -9 & 0 \\ -5 & 7 & 3 & -10 \\ 8 & -6 & 9 & 3 \end{bmatrix}$$

Select the correct choice below and fill in the answer box within your choice.

- **A.** $\mathbf{x} = x_1$ ______ $+ x_4$ ______
- **B.** $\mathbf{x} = x_2$ ______ $+ x_3$ ______ $+ x_4$ ______
- **C.** There is only one vector, which is **x** = ______ .
- **D.** $\mathbf{x} = x_4 \begin{bmatrix} \frac{3}{2} \\ \frac{5}{2} \\ 0 \\ 1 \end{bmatrix}$