Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 4.1 Homework

1. Let V be the set of vectors shown below.

$$V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x < 0,\ y \geq 0 \right\}$$

a. If **u** and **v** are in V, is **u** + **v** in V? Why?
b. Find a specific vector **u** in V and a specific scalar c such that c**u** is not in V.

a. If **u** and **v** are in V, is **u** + **v** in V?

A. The vector **u + v** must be in V because the x-coordinate of **u + v** is the sum of two negative numbers, which must also be negative, and the y-coordinate of **u** + **v** is the sum of nonnegative numbers, which must also be nonnegative.

B. The vector **u + v** may or may not be in V depending on the values of x and y.

C. The vector **u + v** must be in V because V is a subset of the vector space $\mathbb{R}^2$.

D. The vector **u + v** is never in V because the entries of the vectors in V are scalars and not sums of scalars.

b. Find a specific vector **u** in V and a specific scalar c such that c**u** is not in V. Choose the correct answer below.

A. $\mathbf{u} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, c = 4

B. $\mathbf{u} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$, c = − 1

C. $\mathbf{u} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$, c = 4

D. $\mathbf{u} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$, c = − 1

2. Let $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : 6x^2 + 3y^2 \leq 1 \right\}$, which represents the set of points on and inside an ellipse in the xy-plane. Find two specific examples—two vectors, and a vector and a scalar—to show that H is not a subspace of $\mathbb{R}^2$.

H is not a subspace of $\mathbb{R}^2$ because the two vectors $\begin{bmatrix} \frac{1}{\sqrt{6}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{3}} \end{bmatrix}$ show that H is not closed under addition.
(Use a comma to separate vectors as needed.)

H is not a subspace of $\mathbb{R}^2$ because the scalar 3 and the vector $\begin{bmatrix} 0.27 \\ -0.38 \end{bmatrix}$ show that H is not closed under scalar multiplication.

YOU ANSWERED: $\begin{bmatrix} 0.27 \\ -0.47 \end{bmatrix}$

3. Determine if the given set is a subspace of $\mathbb{P}_5$. Justify your answer.

The set of all polynomials of the form $\mathbf{p}(t) = at^5$, where a is in $\mathbb{R}$.

Choose the correct answer below.

- **A.** The set is a subspace of $\mathbb{P}_5$. The set contains the zero vector of $\mathbb{P}_5$, the set is closed under vector addition, and the set is closed under multiplication by scalars.
- **B.** The set is not a subspace of $\mathbb{P}_5$. The set is not closed under multiplication by scalars when the scalar is not an integer.
- **C.** The set is not a subspace of $\mathbb{P}_5$. The set does not contain the zero vector of $\mathbb{P}_5$.
- **D.** The set is a subspace of $\mathbb{P}_5$. The set contains the zero vector of $\mathbb{P}_5$, the set is closed under vector addition, and the set is closed under multiplication on the left by $m \times 5$ matrices where m is any positive integer.

4. Determine if the given set is a subspace of $\mathbb{P}_6$. Justify your answer.

All polynomials of degree at most 6, with rational numbers as coefficients.

Complete each statement below.

The zero vector of $\mathbb{P}_6$ is in the set because zero is a rational number.

The set is closed under vector addition because the sum of two rational numbers is a rational number.

The set is not closed under multiplication by scalars because the product of a scalar and a rational number is not necessarily a rational number.

Is the set a subspace of $\mathbb{P}_6$?

- No
- Yes

5. Determine if the given set is a subspace of $\mathbb{P}_n$. Justify your answer.

The set of all polynomials in $\mathbb{P}_n$ such that $\mathbf{p}(0) = 0$

Choose the correct answer below.

- **A.** The set is not a subspace of $\mathbb{P}_n$ because the set is not closed under multiplication by scalars.
- **B.** The set is a subspace of $\mathbb{P}_n$ because $\mathbb{P}_n$ is a vector space spanned by the given set.
- **C.** The set is a subspace of $\mathbb{P}_n$ because the set contains the zero vector of $\mathbb{P}_n$, the set is closed under vector addition, and the set is closed under multiplication by scalars.
- **D.** The set is not a subspace of $\mathbb{P}_n$ because the set does not contain the zero vector of $\mathbb{P}_n$.
- **E.** The set is not a subspace of $\mathbb{P}_n$ because the set is not closed under vector addition.

6.

Let H be the set of all vectors of the form $\begin{bmatrix} -2t \\ t \\ 5t \end{bmatrix}$. Find a vector **v** in $\mathbb{R}^3$ such that H = Span{**v**}. Why does this show that H is a subspace of $\mathbb{R}^3$?

H = Span{**v**} for **v** = $\begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$

Why does this show that H is a subspace of $\mathbb{R}^3$?

- **A.** Since **v** is in $\mathbb{R}^3$, H = Span{**v**} is a subspace of $\mathbb{R}^3$.
- **B.** Since the zero vector of $\mathbb{R}^3$ is not in Span{**v**}, H is a subspace of $\mathbb{R}^3$.
- **C.** Since **v** spans $\mathbb{R}^3$ and **v** spans H, H spans $\mathbb{R}^3$.
- **D.** Since $\mathbb{R}^3$ = Span{**v**}, H = Span{**v**} is a subspace of $\mathbb{R}^3$.

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7. Let W be the set of all vectors of the form shown on the right, where b and c are arbitrary. Find vectors **u** and **v** such that W = Span{**u**, **v**}. Why does this show that W is a subspace of $\mathbb{R}^3$?

$$\begin{bmatrix} 8b - 3c \\ -b \\ 2c \end{bmatrix}$$

Using the given vector space, write vectors **u** and **v** such that W = Span{**u**, **v**}.

$$\{\mathbf{u}, \mathbf{v}\} = \left\{ \begin{bmatrix} 8 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} \right\}$$

(Use a comma to separate answers as needed.)

Choose the correct theorem that indicates why these vectors show that W is a subspace of $\mathbb{R}^3$.

- **A.** An indexed set $\{\mathbf{v}_1,\ldots,\mathbf{v}_p\}$ of two or more vectors in a vector space V, with $\mathbf{v}_1 \neq \mathbf{0}$ is a subspace of V if and only if some $\mathbf{v}_j$ is in Span $\{\mathbf{v}_1,\ldots,\mathbf{v}_{j-1},\mathbf{v}_j\}$.
- **B.** The null space of an m × n matrix is a subspace of $\mathbb{R}^n$. Equivalently, the set of all solutions to a system A**x** = **0** of m homogeneous linear equations in n unknowns is a subspace of $\mathbb{R}^n$.
- **C.** The column space of an m × n matrix A is a subspace of $\mathbb{R}^m$.
- **D.** If $\mathbf{v}_1,\ldots,\mathbf{v}_p$ are in a vector space V, then Span $\{\mathbf{v}_1,\ldots,\mathbf{v}_p\}$ is a subspace of V.

8. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 9 \\ 2 \\ 1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$.

a. Is $\mathbf{w}$ in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

b. How many vectors are in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

c. Is $\mathbf{w}$ in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Why?

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a. Is $\mathbf{w}$ in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

- **A.** Vector $\mathbf{w}$ is not in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is not $\mathbf{v}_1$, $\mathbf{v}_2$, or $\mathbf{v}_3$.
- **B.** Vector $\mathbf{w}$ is in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.
- **C.** Vector $\mathbf{w}$ is in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the subspace generated by $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ is $\mathbb{R}^3$.
- **D.** Vector $\mathbf{w}$ is not in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is not a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The number of vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is 3 .
- **B.** There are infinitely many vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

b. How many vectors are in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The number of vectors in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is ______ .
- **B.** There are infinitely many vectors in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

c. Is $\mathbf{w}$ in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

- **A.** Vector $\mathbf{w}$ is in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the subspace generated by $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ is $\mathbb{R}^3$.
- **B.** Vector $\mathbf{w}$ is in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because $\mathbf{w}$ is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$, which can be seen because any echelon form of the augmented matrix of the system has no row of the form [0 • • • 0 b] with $b \neq 0$.
- **C.** Vector $\mathbf{w}$ is not in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the rightmost column of the augmented matrix of the system $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{w}$ is a pivot column.
- **D.** Vector $\mathbf{w}$ is not in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because $\mathbf{w}$ is not a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

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9. Let W be the set of all vectors of the form shown on the right, where a and b represent arbitrary real numbers. Find a set S of vectors that spans W, or give an example or an explanation showing why W is not a vector space.

$$\begin{bmatrix} 2a+7b \\ -5 \\ 2a-5b \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** A spanning set is S = { }.
  (Use a comma to separate vectors as needed.)
- **B.** W is not a vector space because the zero vector and most sums and scalar multiples of vectors in W are not in W, because their second (middle) value is not equal to -5.
- **C.** W is not a vector space because not all vectors **u**, **v**, and **w** in W have the property that **u** + **v** = **v** + **u** and (**u** + **v**) + **w** = **u** + (**v** + **w**).

10. Let W be the set of all vectors of the form shown on the right, where a, b, and c represent arbitrary real numbers. Find a set S of vectors that spans W or give an example or an explanation to show that W is not a vector space.

$$\begin{bmatrix} 6a-5b \\ 7b-7c \\ 6c+9a \\ 8b \end{bmatrix}$$

Select the correct choice and fill in the answer box as needed to complete your choice.

- **A.** A spanning set is $S = \left\{ \begin{bmatrix} 6 \\ 0 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ -7 \\ 6 \\ 0 \end{bmatrix} \right\}$.
  (Use a comma to separate answers as needed.)
- **B.** There is no spanning set of W because W does not contain the zero vector.
- **C.** There is no spanning set of W because W is not closed under scalar multiplication.
- **D.** There is no spanning set of W because W is not closed under vector addition.

11. The set $M_{2\times2}$ of all 2×2 matrices is a vector space, under the usual operations of addition of matrices and multiplication by real scalars. Determine if the set H of all matrices of the form $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ is a subspace of $M_{2\times2}$.

Choose the correct answer below.

- **A.** The set H is a subspace of $M_{2\times2}$ because Span{H} = $M_{2\times2}$.
- **B.** The set H is a subspace of $M_{2\times2}$ because H contains the zero vector of $M_{2\times2}$, H is closed under vector addition, and H is closed under multiplication by scalars.
- **C.** The set H is not a subspace of $M_{2\times2}$ because the product of two matrices in H is not in H.
- **D.** The set H is not a subspace of $M_{2\times2}$ because H is not closed under vector addition.
- **E.** The set H is not a subspace of $M_{2\times2}$ because H is not closed under multiplication by scalars.
- **F.** The set H is not a subspace of $M_{2\times2}$ because H does not contain the zero vector of $M_{2\times2}$.