Math

No:

Full Name :

KEY

Math 104 1st Midterm Exam
(28 March 2018, 17:30-18:30)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones, smart watches and electronic devices are to be kept shut and out of sight. All cell phones and smart watches are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
20 pts	30 pts	20 pts	30 pts	100 pts

Q1. Evaluate the definite integral

$$I = \int_0^8 \frac{\cos\sqrt{x+1}}{\sqrt{x+1}}\,dx$$

$u = \sqrt{x+1}$

$du = \frac{dx}{2\sqrt{x+1}} \Rightarrow \frac{dx}{\sqrt{x+1}} = 2\,du$

$x = 0 \Rightarrow u = 1, \quad x = 8 \Rightarrow u = 3$

$$\therefore\ I = 2\int_1^3 \cos u\,du = 2\sin u\Big|_1^3$$

$$= \boxed{2(\sin 3 - \sin 1)}$$

OR

$$\int \frac{\cos\sqrt{x+1}}{\sqrt{x+1}}\,dx = 2\int \cos u\,du = 2\sin u + C$$

$$= 2\sin\sqrt{x+1} + C$$

$$\therefore\ I = 2\sin\sqrt{x+1}\Big|_0^8 = 2(\sin 3 - \sin 1)$$

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Q2. Evaluate the following integrals:

(a) $\int tan^{-7/2}\,\theta sec^2\theta d\theta$ $u = \tan\theta \Rightarrow du = \sec^2\theta\, d\theta$

$$= \int u^{-7/2}\, du$$

$$= \frac{u^{-5/2}}{-5/2} + C$$

$$= \boxed{-\frac{2}{5}\tan^{-5/2}\theta + C}$$

(b) $\int x^4\,(1+3x^5)^{101} dx$

$u = 1 + 3x^5$

$du = 15x^4 dx$

$\therefore x^4 dx = \frac{1}{15} du$

$$= \frac{1}{15}\int u^{101}\, du$$

$$= \frac{1}{15}\cdot\frac{u^{102}}{102} + C$$

$$= \boxed{\frac{1}{15\cdot 102}(1+3x^5)^{102} + C}$$

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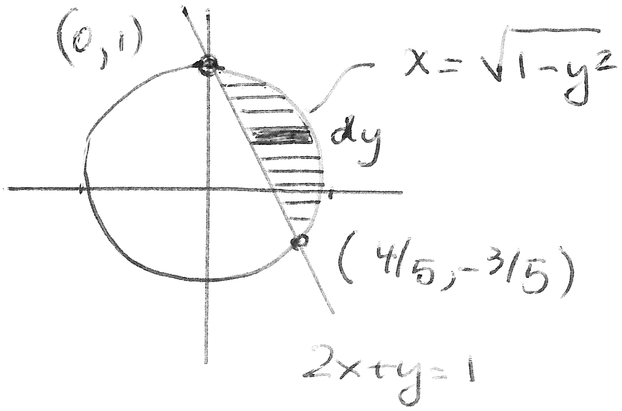
Q3. Consider the region between the semicircle $x = \sqrt{1-y^2}$ and the line $2x + y = 1$.

(a) Sketch the region.

(b) Set up an integral (or a sum of integrals) for the area of this region. DO NOT EVALUATE THE INTEGRAL.

(a)



(b) Pts of intersection

$x^2+y^2=1$

$y^2 = 1-x^2 = (1-2x)^2$

$1-x^2 = 1-4x+4x^2$

$5x^2-4x=0$

$x(5x-4)=0$

$x=0 \quad , \quad x=4/5$

$\Downarrow$

$y=1, \qquad y=-3/5$

$$A = \int_{-3/5}^{1} (x_2 - x_1)\,dy$$

$$= \boxed{\int_{-3/5}^{1} \left(\sqrt{1-y^2} - \frac{1-y}{2}\right) dy}$$

OR

$$A = \int_0^{4/5} (y_2 - y_1)\,dx + \int_{4/5}^{1} (y_2 - y_1)\,dx$$

$$= \int_0^{4/5} \left[\sqrt{1-x^2} - (1-2x)\right] dx + \int_{4/5}^{1} \left[\sqrt{1-x^2} - \left(-\sqrt{1-x^2}\right)\right] dx$$

$$= \boxed{\int_0^{4/5} \left(\sqrt{1-x^2} - 1 + 2x\right) dx + 2\int_{4/5}^{1} \sqrt{1-x^2}\,dx}$$

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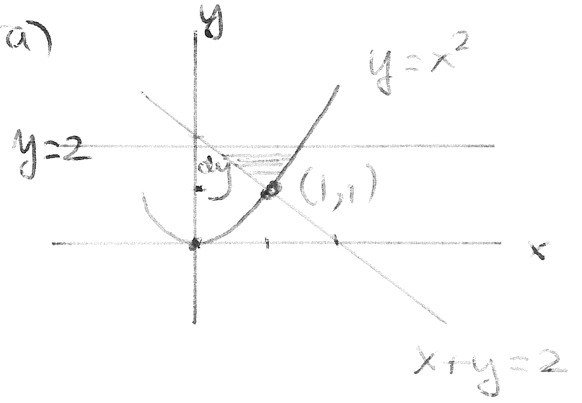
Q4. Consider the region bounded on the left by $x + y = 2$, on the right by $y = x^2$, and above by $y = 2$.

(a) Sketch the region.
(b) Find the area of the region.

(a)



(b) Pts of intersection

$$y = x^2 = 2 - x$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2, 1$$

$$\Downarrow$$

$$y = 1$$

$$A = \int_1^2 (x_2 - x_1)\,dy$$

$$= \int_1^2 \left[\sqrt{y} - (2-y)\right] dy$$

$$= \int_1^2 (y^{1/2} - 2 + y)\,dy$$

$$= \frac{y^{3/2}}{3/2} - 2y + \frac{y^2}{2} \Big|_1^2$$

$$= \frac{2}{3} \cdot 2\sqrt{2} - 4 + 2 - \left(\frac{2}{3} - 2 + \frac{1}{2}\right)$$

$$= \frac{4\sqrt{2}}{3} - \frac{7}{6}$$

$$= \boxed{\frac{8\sqrt{2} - 7}{6}}$$

(This can also be solved by integrating over x)