Math

No:

Full Name :

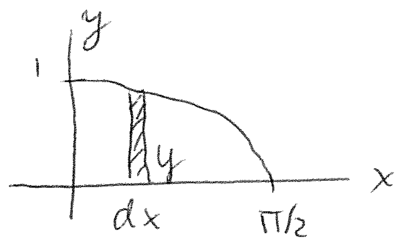
KEY

Math 104 Final Exam
(13 June 2015, Time: 12:00-13:30)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 6 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	Q5	Q6	TOT
10 pts	25 pts	30 pts	10 pts	20 pts	15 pts	110 pts

Q1. Find the volume of the solid of revolution obtained by rotating the region bounded between the curve $y = \sqrt{cosx}$, the x-axis and the y-axis, for $0 \leq x \leq \pi/2$, about the x-axis.



Disc method:

$$V = \pi \int_0^{\pi/2} y^2 dx$$

$$= \pi \int_0^{\pi/2} \cos x dx$$

$$= \pi \sin x \Big|_0^{\pi/2}$$

$$= \boxed{\pi}$$

Math

No:

Full Name :

Q2. Evaluate the following limits, if they exist:

a) $\lim\limits_{x\to 0}\frac{4^x-1}{3^x-1}$ $\overset{0/0}{\underset{\text{L'Hospital}}{=}}$ $\lim\limits_{x\to 0}\frac{4^x\ln 4}{3^x\ln 3} = \boxed{\frac{\ln 4}{\ln 3}}$

b) $\lim\limits_{x\to 0^+}(tanx)^x$ $\quad 0^0$

$y=(\tan x)^x \Rightarrow \ln y = x\ln\tan x$

$\lim\limits_{x\to 0^+}\ln y \overset{0\cdot(-\infty)}{=} \lim\limits_{x\to 0^+}\frac{\ln\tan x}{1/x} \overset{-\infty/\infty}{\underset{\text{L'Hospital}}{=}}$

$\lim\limits_{x\to 0^+}\frac{\frac{1}{\tan x}\sec^2 x}{-1/x^2} = \lim\limits_{x\to 0^+} -\frac{x^2\cancel{\cos x}}{\sin x\cdot\cos^{\cancel{2}} x}$

$= \lim\limits_{x\to 0^+}\left(-\underbrace{\frac{x}{\sin x}}_{1}\cdot\underbrace{\frac{x}{\cos x}}_{0}\right) = 0$

$\ln y\to 0 \Rightarrow y\to e^0 = \boxed{1}$

Math

No:

Full Name :

Q3. Evaluate the integrals given below:

(a) $\int_2^{16} \frac{dx}{2x\sqrt{lnx}}$ $= \frac{1}{2}\int_{\ln 2}^{\ln 16} u^{-1/2}\,du = \frac{1}{2}\left.\frac{u^{1/2}}{1/2}\right|_{\ln 2}^{\ln 16}$

$u = \ln x \Rightarrow du = \frac{dx}{x}$

$$= \sqrt{\underbrace{\ln 16}_{\ln 2^4 = 4\ln 2}} - \sqrt{\ln 2} = \sqrt{4\ln 2} - \sqrt{\ln 2}$$

$$= 2\sqrt{\ln 2} - \sqrt{\ln 2}$$

$$= \boxed{\sqrt{\ln 2}}$$

(b) $\int \frac{e^t dt}{e^{2t}+4e^t+3}$ $x = e^t \Rightarrow dx = e^t\,dt$

$$= \int \frac{dx}{x^2+4x+3} = \int \frac{dx}{(x+3)(x+1)}$$

$$\frac{1}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1}$$ Partial Fractions

$$1 = A(x+1) + B(x+3)$$

$x = -1$ $\quad 1 = 2B$ $\qquad A = -\frac{1}{2}, \; B = \frac{1}{2}$

$x = -3$ $\quad 1 = -2A$

$$\therefore -\frac{1}{2}\int \frac{dx}{x+3} + \frac{1}{2}\int \frac{dx}{x+1} = -\frac{1}{2}\ln|x+3| + \frac{1}{2}\ln|x+1| + C$$

$$= \frac{1}{2}\ln\left|\frac{x+1}{x+3}\right| + C = \boxed{\frac{1}{2}\ln\left(\frac{e^t+1}{e^t+3}\right) + C}$$

Math No:

Full Name :

Integration by parts

(c) $\int \theta sec^2\theta d\theta$ $= \theta \tan\theta - \int \tan\theta d\theta$

$u = \theta \quad dv = \sec^2\theta d\theta$

$du = d\theta \quad v = \tan\theta$

$$= \theta \tan\theta + \int \frac{-\sin\theta}{\cos\theta} d\theta = \boxed{\theta \tan\theta + \ln|\cos\theta| + C}$$

$u = \cos\theta$

$du = -\sin\theta d\theta$

Q4. Determine whether the following series converges or diverges:

$\sum_{n=1}^{\infty} \frac{Arctann}{1+n^2}$ Integral Test

$$\int_1^{\infty} \frac{\text{Arctan}x}{1+x^2} = \int_{\pi/4}^{\pi/2} u\,du = \left.\frac{u^2}{2}\right|_{\pi/4}^{\pi/2}$$

$u = \text{Arctan}x$

$du = \frac{dx}{1+x^2}$

converges

$y = \frac{\text{Arctan}x}{1+x^2}$ is an increasing function

$\therefore$ The integral $\boxed{\text{converges}}$, by the integral test.

Math

No:

Full Name :

Q5. Given the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^3 4^n}$$

(a) Find the radius of convergence.
(b) Find the interval of convergence.

Generalized Ratio Test

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{(x-1)^{n+1}}{(n+1)^3 4^{n+1}}}{\frac{(x-1)^n}{n^3 4^n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x-1)^{n+1}}{(x-1)^n} \cdot \frac{n^3}{(n+1)^3} \cdot \frac{4^n}{4^{n+1}} \right|$$

$$= \frac{|x-1|}{4} \underbrace{\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^3}_{1} = \frac{|x-1|}{4}$$

Converges absolutely when $\frac{|x-1|}{4} < 1$

$\Rightarrow -4 < x-1 < 4 \Rightarrow -3 < x < 5$

Diverges when $x > 5$ or $x < -3$.

$x = 5 \Rightarrow \sum_{n=1}^{\infty} \frac{4^n}{n^3 \cdot 4^n} = \sum_{n=1}^{\infty} \frac{1}{n^3}$ p-series, $p = 3 > 1$ $\therefore$ converges

$x = -3 \Rightarrow \sum_{n=1}^{\infty} \frac{(-4)^n}{n^3 4^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ converges absolutely by above.

$$\boxed{\begin{array}{ll} \text{Interval} & -3 \le x \le 5 \\ \text{radius} & = 4. \end{array}}$$

Full Name :

Q6. Let $w(x,y) = xe^{xy} + \cos xy$. Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ at the point $(1,\pi)$.

$$\frac{\partial w}{\partial x} = xye^{xy} + e^{xy} - y\sin xy$$

$$\left(\frac{\partial w}{\partial x}\right)_{(1,\pi)} = \pi e^{\pi} + e^{\pi} - \pi \underbrace{\cancel{\sin\pi}}_{0} = \boxed{(\pi+1)e^{\pi}}$$

$$\frac{\partial w}{\partial y} = x^2 e^{xy} - x\sin xy$$

$$\left(\frac{\partial w}{\partial y}\right)_{(1,\pi)} = e^{\pi} - \underbrace{\cancel{\sin\pi}}_{0} = \boxed{e^{\pi}}$$

Math No:

Full Name :