Math No:

Full Name : KEY

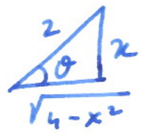
Math 104 – 3rd Midterm Exam
(25 January 2018, Time: 18:00-19:00)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones, smart watches and electronic devices are to be kept shut and out of sight. All cell phones and smart watches are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
6 pts	6 pts	6 pts	6 pts	24 pts

Q1. Evaluate the following improper integral $\int_{-2}^{0} \frac{dx}{\sqrt{4-x^2}}$ $= \lim_{b \to -2} \int_{b}^{0} \frac{dx}{\sqrt{4-x^2}}$



$x = 2\sin\theta \Rightarrow \sin\theta = x/2$

$dx = 2\cos\theta\, d\theta$

$$\int \frac{2\cos\theta\, d\theta}{\sqrt{4-4\sin^2\theta}} = \int \frac{\cancel{2\cos\theta}\, d\theta}{\cancel{2\cos\theta}} = \int d\theta = \theta = \operatorname{ArcSin}\frac{x}{2}\Big|_{-2}^{0}$$

$$= \operatorname{ArcSin}\frac{x}{2}\Big|_{-2}^{0}$$

$$= 0 - \operatorname{ArcSin}(-1)$$

$$= \operatorname{ArcSin}(1)$$

$$= \pi/2$$

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Q2. Determine if the following series converges or diverges. $\sum_{n=1}^{\infty}(-1)^n \frac{10^n}{(n+1)!}$

$$\rho = \lim_{n \to \infty} \frac{a_{n+1}}{a_n}$$

$$= \lim_{n \to \infty} \frac{10^{n+1}}{(n+1+1)!} \cdot \frac{(n+1)!}{10^n}$$

$$= \lim_{n \to \infty} \frac{10^n \cdot 10}{(n+2)!} \frac{(n+1)!}{10^n}$$

$$= 10 \lim_{n \to \infty} \frac{\cancel{10^n}}{(n+2)\cancel{(n+1)!}} \cdot \frac{\cancel{(n+1)!}}{\cancel{10^n}}$$

$$= 10 \lim_{n \to \infty} \frac{1}{n+2}$$

$= 0 < 1$, it converges by Ratio Test

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Q3. For what values of x does the following series converge? $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-3)^n}{n}$

$$\rho = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x-3)\cancel{(x-3)^n}}{n+1}\cdot\frac{n}{\cancel{(x-3)^n}}\right|$$

$$= |x-3| \underbrace{\lim_{n\to\infty}\frac{n}{n+1}}_{1}$$

$\rho = |x-3| < 1$, to converge.

$x-3<1 \Rightarrow x<4$

$x-3>-1 \Rightarrow x>2$

Now, we need to test the end values:

For $x=2$: $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2-3)^n}{n} = \sum(-1)(-1)^n\cdot(-1)^n/n$

$= -\sum 1/n$, diverges, $p=1$, series.

For $x=4$: $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(4-3)^n}{n} = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$, converges conditionally, it is an alternating series.

$\therefore \quad \underline{2 < x \leq 4}$

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Q4. (a) Expand the following integral function in Binomial series: $f(x)=\sqrt{1-x}$. First 4 terms are sufficient to calculate. (*Hint*: The Binomial Series is described as $(1+x)^m \approx 1+\sum_{k=1}^{\infty}\binom{m}{k}x^k$

$$(1-x)^{1/2} \simeq 1+\sum_{k=1}^{\infty}\binom{1/2}{k}(-x)^k$$

$$\simeq 1+\binom{1/2}{1}(-x)+\binom{1/2}{2}(-x)^2+\binom{1/2}{3}(-x)^3+\dots$$

$$\simeq 1+\frac{\cancel{\tfrac{1}{2}!}}{1!\cancel{(\tfrac{1}{2}-1)!}}(-x)+\frac{\tfrac{1}{2}!}{2!(\tfrac{1}{2}-\cancel{2})!}x^2+\frac{\tfrac{1}{2}!}{3!(\tfrac{1}{2}-3)!}(-x)^3+\dots$$

$$\simeq 1-\frac{x}{2}+\frac{\tfrac{1}{2}(\tfrac{1}{2}-1)\cancel{(\tfrac{1}{2}-2)!}}{2!\cancel{(\tfrac{1}{2}-2)!}}x^2-\frac{\tfrac{1}{2}(\tfrac{1}{2}-1)(\tfrac{1}{2}-2)\cancel{(\tfrac{1}{2}-3)!}}{3!\cancel{(\tfrac{1}{2}-3)!}}x^3+\dots$$

$$(1-x)^{1/2} \simeq 1-\frac{x}{2}-\frac{x^2}{8}-\frac{x^3}{16}-\dots$$