FT

Continuous

$$X(\omega)=\int_{-\infty}^{\infty} x(t).e^{-j\omega t}.dt$$

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega).e^{+j\omega t}.d\omega$$

Discrete

$$X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

$$x(n)=\frac{1}{2\pi}\int_{<2\pi>} X(\Omega).e^{+j\Omega n}d\Omega$$

SORULAR

① $FT[e^{jft}]$

$$\int_{-\infty}^{\infty} e^{jft}.e^{-j\omega t}.dt=\int_{-\infty}^{\infty} e^{jt(f-\omega)}.dt$$

$$=\frac{e^{jt(f-\omega)}}{j(f-\omega)}\Big|_{-\infty}^{\infty}=\lim_{T\to\infty}\frac{e^{jt(f-\omega)}}{j(f-\omega)}\Big|_{-T}^{T}$$

$$=\lim_{T\to\infty}\frac{2.\left[e^{jT(f-\omega)}-e^{-jT(f-\omega)}\right]}{2.j(f-\omega)} \rightarrow \sin(T(f-\omega))$$

$$=\frac{2\sin((f-\omega)T)}{f-\omega}=2\pi.\delta(f-\omega)$$

② Inverse FT of $X_{(jf)}\{\omega \le f\}\,f(\omega)$

$$f(t)=\frac{1}{2\pi}\int_{-f}^{f} e^{j\omega t}d\omega \rightarrow \sin(ft)$$

$$\frac{1}{2\pi}\frac{e^{j\omega t}}{jt}\Big|_{-f}^{f} \quad \frac{2.1}{2\pi}\,\frac{e^{+jft}-e^{-jft}}{2.jt}$$

$$=\frac{\sin(ft)}{\pi t}$$

$$FT\left[\frac{\sin(ft)}{\pi t}\right]=X_{(-jf)}^{\omega}$$

SORULAR

① $x[n]=\underline{0}\ 2\ 5$

$X(\Omega)=?$

$$X(\Omega)=\sum_{-\infty}^{\infty} x[n].e^{-j\Omega n}$$

$$=x[0]e^{-j\Omega.0}+x[1].e^{-j.\Omega.1}+x[2]e^{-j.\Omega.2}$$

$$=2e^{-j\Omega}+5e^{-2j\Omega}$$

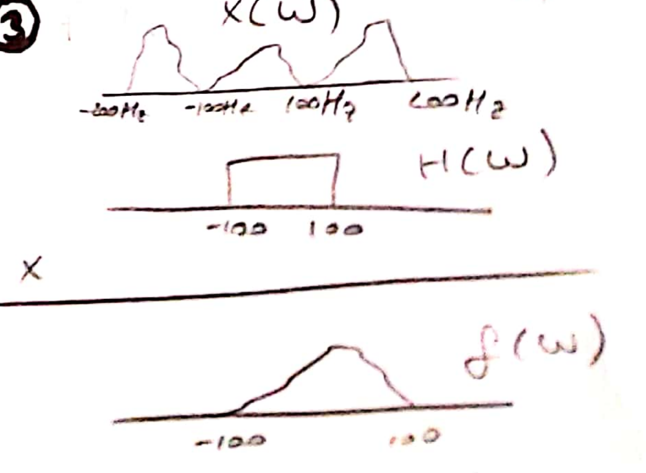
② $x[n]=e^{-\alpha n}u[n]$

DFT of $x[n]=?$

$$X(\Omega)=\sum_{n=0}^{\infty} e^{-\alpha n}.e^{-j\Omega n}$$

$$=\sum_{n=0}^{\infty} e^{-(\alpha+j\Omega)}=\frac{1}{1+\alpha+j\Omega}$$

③



$$f(\omega)=X(\omega).H(\omega)$$

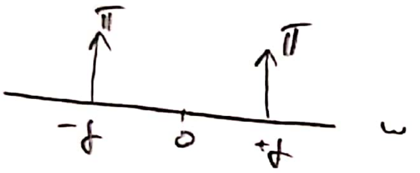
③ FT[cos(ft)]

sorude $\dfrac{e^{ift}+e^{-ift}}{2}$

$$FT\left[\frac{e^{ift}}{2}\right]+FT\left[\frac{e^{-ift}}{2}\right]$$

$$\frac{\cancel{2}\pi\delta(f-\omega)}{\cancel{2}}+\frac{\cancel{2}\pi\delta(\omega+f)}{\cancel{2}}$$

$$\pi\delta(f-\omega)+\pi\delta(\omega+f)$$



④ FT[$5e^{2it}$]

$$\int_{-\infty}^{\infty} 5e^{2it}\cdot e^{-i\omega t}\cdot dt$$

$$5\int_{-\infty}^{\infty} e^{i(2-\omega)t}\cdot dt=\frac{5e^{i(2-\omega)t}}{i(2-\omega)}\Big|_{-\infty}^{\infty}$$

$$\lim_{T\to\infty} 2.5\left[\frac{e^{i(2-\omega)T}-e^{-i(2-\omega)T}}{2.\,i(2-\omega)}\right] \quad \sin((2-\omega)T)$$

$$\lim_{T\to\infty}\frac{10.\sin(T(2-\omega))}{(2-\omega)}=$$

$$\lim_{T\to\infty}\frac{10.\sin(T(\omega-2))}{(\omega-2)}$$

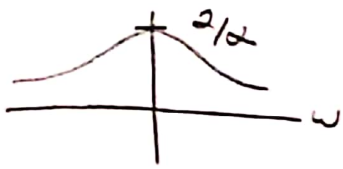
$$=10\pi.\delta(\omega-2)$$

⑤ FT[$e^{-a|t|}$]

$$\int_{-\infty}^{\infty} e^{-a|t|}\cdot e^{-j\omega t}\cdot dt$$

$$\int_{\infty}^{0} e^{at}e^{-j\omega t}dt+\int_{0}^{+\infty} e^{-at}\cdot e^{-j\omega t}dt$$

$$=\frac{2\alpha}{\alpha^2+\omega^2}$$



⑥ FT[$\delta(t-c)$]

$$=e^{-j\omega c}$$

What is amount of 5Hz in yukarıdaki

$$\frac{2\alpha}{\alpha^2+(2\pi 5)^2}=\frac{2\alpha}{\alpha^2+(10\pi)^2}$$

$$\omega=2\pi f$$

FT Türev

$$x(t) \longrightarrow X(\omega)$$

$$\frac{dx(t)}{dt} \longrightarrow j\omega X(\omega) \qquad \text{1. türev}$$

$$\frac{d^2x(t)}{d^2t} \longrightarrow (j\omega)(j\omega)X(\omega) = -\omega^2 X(\omega) \qquad \text{2. türev}$$

SORULAR

$$x(t) \longrightarrow \square \longrightarrow y(t)$$
$$\delta(t) \longrightarrow \qquad \longrightarrow h(t)$$

① $$\left.\begin{array}{l} \dfrac{dy(t)}{dt} + 5y(t) = x(t) \\[2ex] \dfrac{dh(t)}{dt} + 5h(t) = \delta(t) \end{array}\right\} h(\omega) = ?$$

$$\downarrow FT \qquad \downarrow FT \qquad \downarrow FT$$

$$j\omega h(\omega) + 5h(\omega) = 1$$

$$h(\omega)(j\omega + 1) = 1$$

$$\boxed{h(\omega) = \frac{1}{j\omega + 1} \Longrightarrow e^{-t} \cdot u(t) = h(t)}$$

② FT of following?

$$\frac{d^2y}{d^2t} + 5\frac{dy}{dt} + 6y = \delta(t)$$

$$\downarrow FT \qquad \downarrow FT \qquad \downarrow FT \qquad \downarrow FT$$

$$-\omega^2 Y(\omega) + 5j\omega Y(\omega) + 6Y(\omega) = 1$$

$$Y(\omega)(-\omega^2 + 5j\omega + 6) = 1$$

$$Y(\omega) = \frac{1}{(j\omega)^2 + 5j\omega + 6} = \frac{1}{(j\omega+2)(j\omega+3)}$$

$$= \frac{A}{j\omega+2} + \frac{B}{j\omega+3} \Longrightarrow A = 1, \; B = -1$$

$$\boxed{\frac{1}{j\omega+2} - \frac{1}{j\omega+3}}$$

$$e^{-2t}u(t) - e^{-3t}u(t)$$

Convolution

1 Convolve 1 2 3 with 1 0 -1

$$\begin{array}{r} 1\ 2\ 3 \\ \times\ -1\ 0\ 1 \\ \hline -1\ 0\ 3 \end{array} = 2\ (S_0)$$

$$\begin{array}{r} 1\ 2\ 3 \\ \times\ -1\ 0\ 1\ 0 \\ \hline 0\ 0\ 2\ 0 \end{array} = 2\ (S_{-1})$$

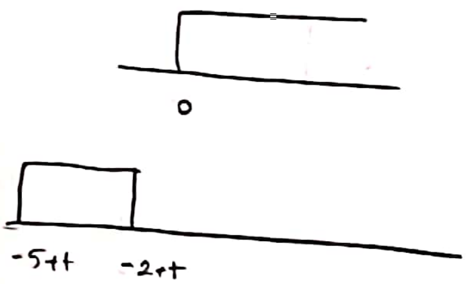
$$\begin{array}{r} 1\ 2\ 3 \\ \times\ 0\ -1\ 0\ 1 \\ \hline 0\ -2\ 0\ 0 \end{array} = -2\ (S_1)$$

$$\begin{array}{r} 1\ 2\ 3 \\ \times\ -1\ 0\ 1\ 0\ 0 \\ \hline 0\ 0\ 1\ 0\ 0 \end{array} = 1\ (S_{-2})$$

$$\begin{array}{r} 1\ 2\ 3 \\ \times\ 0\ 0\ -1\ 0\ 1 \\ \hline 0\ 0\ -3\ 0\ 0 \end{array} = -3\ (S_2)$$

$$\underset{S_{-2}}{1}\quad \underset{S_{-1}}{2}\quad \underset{S_0}{\underline{2}}\quad \underset{S_1}{-2}\quad \underset{S_2}{-3}$$

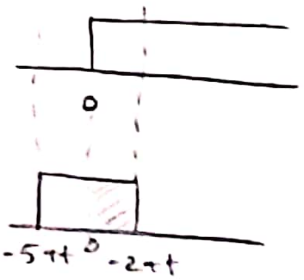
2 Convolve $\chi_{(2,5)}(t)$ and $u(t)$



$-2+t<0$

$y(t)=\int u(s).\chi_{(2,5)}(t-s)\,ds$

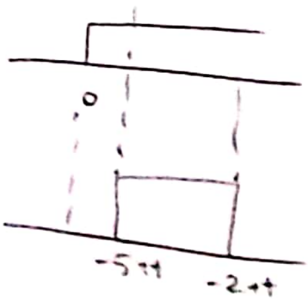
$t<2\ ,\ y(t)=0$



$-2+t>0$ $\quad -5+t<0$

$t>0$ $\quad t<5$

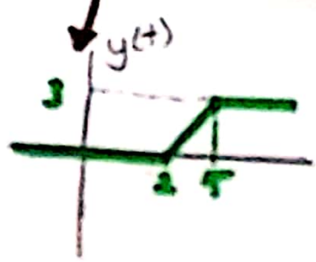
$0<t<5\ ,\ y(t)=-2+t$



$-5+t>0$

$t>5\ ,\ y(t)=3$

$-2+\cancel{t}+5-\cancel{t}=3=\text{area}$

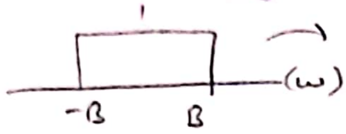


plotting of convolution

CamScanner ile tarandı

Filtering

① Derivation of LPF?

LP =  $\rightarrow X_{(-B,B)}(\omega)$

$$f(t) = \frac{1}{2\pi}\int_{-B}^{B} e^{j\omega t}\, d\omega$$

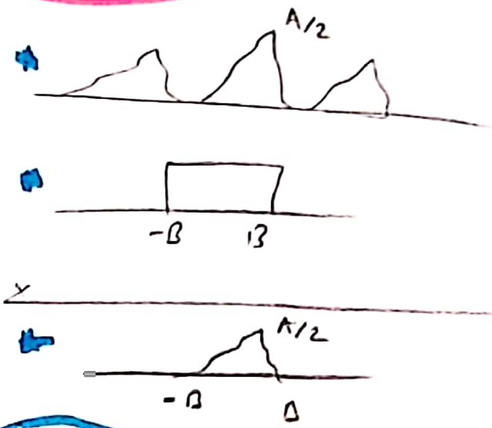
$$= \frac{1\cdot\cancel{2}}{\cancel{2}\pi}\cdot\frac{e^{jBt} - e^{-jBt}}{2\cdot jt} \rightarrow \sin Bt$$

$$= \frac{\sin Bt}{\pi t}$$

To get rid of noise

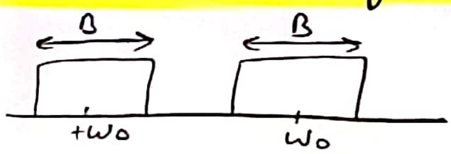
$$S_k(t)\cdot\cos^2\omega_k t * \frac{\sin Bt}{\pi t}$$

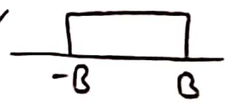
in time



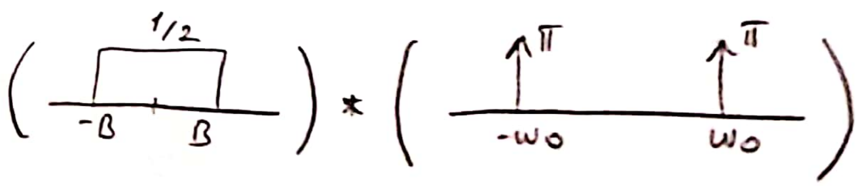
in freq

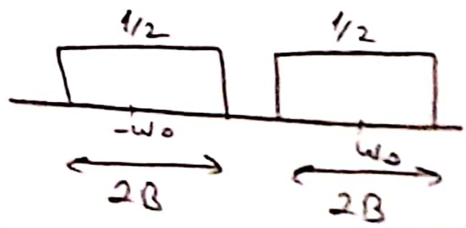
② Implement Band Pass Filter to derivation of LP



$\frac{\sin Bt}{\pi t}\cdot\cos\omega_0 t = \frac{1}{2\pi}\cdot\Big($  $\Big) * \big(\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)\big)$

mult in time — convolution in frequency

$=$ 

$=$ 

1

$x_1(n)$ $\qquad\qquad$ y_1

$-1 \quad 3 \quad 2 \quad 4 \longrightarrow 0 \quad -2 \quad 7 \quad 1 \quad 6 \quad -4 \quad 0 \quad 0$

$0, -2, 7, 1, 6, -4, 0, 0 \longrightarrow ?$

$$
\begin{array}{c|cccc}
x_1 n \rightarrow & -1 & 3 & 2 & 4 \\
\hline
a & -a & 3a & 2a & 4a \\
b & -b & 3b & 2b & 4b \\
c & -c & 3c & 2c & 4c \\
d & -d & 3d & 2d & 4d \\
e & -e & 3e & 2e & 4e
\end{array}
$$

$\downarrow h(n)$

$$(\underbrace{-a}_{0}, \underbrace{3a-b}_{-2}, \underbrace{2a+3b-c}_{7}, \underbrace{4a+2b+3c-d}_{1}, \overbrace{4b+2c+3d-e}^{6}, \overbrace{4c+2d+3e}^{-4}, \underbrace{4d+2e}_{0}, \underbrace{4e}_{0}) = y_1(n)$$

$-a = 0$ $\qquad$ $\boxed{a=0}$

$3a - b = -2$ $\qquad$ $-b = -2$ $\qquad$ $\boxed{b=2}$

$2a + 3b - c = 7$ $\qquad$ $6 - c = 7$ $\qquad$ $\boxed{c=-1}$

$4a + 2b + 3c - d = 1$ $\qquad$ $4 - 3 - d = 1$ $\qquad$ $\boxed{d=0}$

$4b + 2c + 3d - e = 6$ $\qquad$ $8 - 2 - e = 6$ $\qquad$ $\boxed{e=0}$

$h(n) = [0, 2, -1, 0, 0]$

2

$x_2(n) \rightarrow$	4	$\underline{0}$	-2	-7	1	0	2
$h(n) \downarrow$ 0	0	$\underline{0}$	0	0	0	0	0
2	8	$\underline{0}$	-4	-14	2	0	4
-1	-4	$\underline{0}$	+2	7	-1	0	-2
0	0	$\underline{0}$	0	0	0	0	0
0	0	$\underline{0}$	0	0	0	0	0

0, $\underline{8}$, -4, -4, -12, 9, -1, 4, -2, 0, 0 $= y_2(n)$

output

cos - sin

$$\boxed{\cos\theta} = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\boxed{\cos ft} = \frac{e^{ift} + e^{-ift}}{2}$$

$$\boxed{\sin\theta} = \frac{e^{i\theta} - e^{-i\theta}}{2}$$

$$\boxed{\sin ft} = \frac{e^{ift} - e^{-ift}}{2}$$

$$\boxed{e^{J\theta}} = \cos\theta + i\sin\theta$$

$$\boxed{e^{-J\theta}} = \cos\theta - i\sin\theta$$

CamScanner ile tarandı

$$\boxed{\delta(t)}$$

$$\boxed{\lim_{a \to \infty} \frac{\sin(at)}{t} = \pi\,\delta(t)}$$

Ex

$$\lim_{T \to \infty} 2 \cdot \frac{\sin(T \cdot (f-\omega))}{(f-\omega)} = 2\pi\,\delta(f-\omega)$$

CamScanner ile tarandı

L_1

$$L_1(f(t)) = \int_{-\infty}^{\infty} |f(t)|\,dt$$

L_2

$$L_2(f(t)) = \sqrt{\int_{-\infty}^{\infty} |f(t)|^2\,dt}$$

Riemann Lebesgue

If $f(t)$ has L_1 norm:

$$\lim_{\omega \to \infty} \int_{-\infty}^{\infty} f(t)\sin(\omega t)\,dt = 0$$

Euler's Law

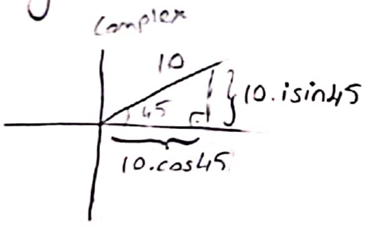
$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

Polar Rep

Polar Rep $\rightarrow$ $r.e^{i\theta}$

eg: $10e^{i\frac{\pi}{4}} = \sqrt{50} + \sqrt{50}\,i$



$= 5\sqrt{2} + 5\sqrt{2}\,i$ $\leftarrow$ Cartesion Rep

Magnitude of $e^{i\theta} = ?$

$$|e^{i\theta}| = \sqrt{e^{i\theta}.e^{-i\theta}}$$

$$= \sqrt{e^{0}} = 1$$

$\cos(i)$ when $i = \sqrt{-1}$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$+$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\cos i = \frac{e^{i^2} + e^{-i^2}}{2} = \frac{1/e + e}{2}$$

$\cos(x) = y$

$a.\cos(y) = x$