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Course: Linear Algebra

Assignment: Midterm

1. Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation that maps $\mathbf{u} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and maps $\mathbf{v} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$ into $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$. Use the fact that T is linear to find the images under T of $4\mathbf{u}$, $2\mathbf{v}$, and $4\mathbf{u} + 2\mathbf{v}$.

What is the image of $4\mathbf{u}$?

- ☐ A. $\begin{bmatrix} -2 \\ 8 \end{bmatrix}$
- ☐ B. $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$
- ☒ C. $\begin{bmatrix} 16 \\ 4 \end{bmatrix}$
- ☐ D. $\begin{bmatrix} 4 \\ 16 \end{bmatrix}$

What is the image of $2\mathbf{v}$?

- ☒ A. $\begin{bmatrix} -2 \\ 8 \end{bmatrix}$
- ☐ B. $\begin{bmatrix} 16 \\ 4 \end{bmatrix}$
- ☐ C. $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$
- ☐ D. $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$

What is the image of $4\mathbf{u} + 2\mathbf{v}$?

- ☐ A. $\begin{bmatrix} -14 \\ -12 \end{bmatrix}$
- ☐ B. $\begin{bmatrix} 12 \\ 14 \end{bmatrix}$
- ☒ C. $\begin{bmatrix} 14 \\ 12 \end{bmatrix}$
- ☐ D. $\begin{bmatrix} -12 \\ 14 \end{bmatrix}$

2. If T is defined by $T(\mathbf{x}) = A\mathbf{x}$, find a vector \mathbf{x} whose image under T is \mathbf{b} , and determine whether \mathbf{x} is unique. Let

$$A = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -5 \\ 3 & -7 & 12 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -6 \\ -8 \\ -5 \end{bmatrix}.$$

Find a single vector \mathbf{x} whose image under T is \mathbf{b} .

$$\mathbf{x} = \begin{bmatrix} -28 \\ -13 \\ -1 \end{bmatrix}$$

Is the vector \mathbf{x} found in the previous step unique?

- ☐ A. No, because there are no free variables in the system of equations.
- ☐ B. Yes, because there is a free variable in the system of equations.
- ☒ C. Yes, because there are no free variables in the system of equations.
- ☐ D. No, because there is a free variable in the system of equations.

3. The vector \mathbf{x} is in a subspace H with a basis $B = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find the B -coordinate vector of \mathbf{x} .

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}$$

$$[\mathbf{x}]_B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

4. A matrix A and an echelon form of A are shown below. Find a basis for $\text{Col } A$ and a basis for $\text{Nul } A$.

$$A = \begin{bmatrix} 3 & 4 & 0 & -4 \\ 6 & 5 & -9 & 4 \\ 3 & 5 & 3 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 2 & -4 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for $\text{Col } A$.

$$\left\{ \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix} \right\}$$

(Simplify your answer. Use a comma to separate answers as needed.)

Find a basis for $\text{Nul } A$.

$$\left\{ \begin{bmatrix} 4 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 4 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(Simplify your answer. Use a comma to separate answers as needed.)

5. Let $A = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 \\ -3 & k \end{bmatrix}$. What value(s) of k , if any, will make $AB = BA$?

Select the correct choice below and, if necessary, fill in the answer box within your choice.

- ☒ A. $k =$ -1 (Use a comma to separate answers as needed.)
- ☐ B. No value of k will make $AB = BA$

6. Use the given inverse of the coefficient matrix to solve the following system.

$$\begin{aligned} 7x_1 + 2x_2 &= 6 \\ -6x_1 - 2x_2 &= -3 \end{aligned} \quad A^{-1} = \begin{bmatrix} 1 & 1 \\ -3 & -\frac{7}{2} \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- ☒ A. $x_1 =$ 3 and $x_2 =$ $-\frac{15}{2}$ (Simplify your answers.)
- ☐ B. There is no solution.

7. Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & h & 2 \\ -5 & 15 & -12 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The matrix is the augmented matrix of a consistent linear system if $h =$.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- ☒ B. The matrix is the augmented matrix of a consistent linear system if $h \neq$ -3 .
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- ☐ C. The matrix is the augmented matrix of a consistent linear system for every value of h .
- ☐ D. The matrix is not the augmented matrix of a consistent linear system for any value of h .

8. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -5 \\ -21 \\ 2 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 5 \\ 13 \\ h \end{bmatrix}$. For what value(s) of h is \mathbf{b} in the plane spanned by \mathbf{a}_1 and \mathbf{a}_2 ?

The value(s) of h is(are) 4 . (Use a comma to separate answers as needed.)