Math No:

Full Name : KEY

İSTANBUL ŞEHİR UNIVERSITY

Math 104 – 2nd Midterm Exam
(28 December 2017, Time: 18:00-19:00)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones, smart watches and electronic devices are to be kept shut and out of sight. All cell phones and smart watches are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
6 pts	6 pts	6 pts	6 pts	24 pts

Q1. Evaluate the following limit $\lim_{x\to 0}(1+\tan x)^{\frac{1}{x}}$ 1^{∞}, indeterminate form (power)

$y = (1+\tan x)^{\frac{1}{x}}$

$\ln y = \frac{1}{x}\ln(1+\tan x)$

$\lim_{x\to 0}\ln y = \lim_{x\to 0}\frac{\ln(1+\tan x)}{x}$

$= \lim_{x\to 0}\frac{\frac{\sec^2 x}{1+\tan x}}{1}$ (L'Hospt. Rule)

$\lim_{x\to 0}\ln y = 1$

$\lim_{x\to 0} y = e^1$

$\lim_{x\to 0}(1+\tan x)^{\frac{1}{x}} = e$ //

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Q2. (a) Evaluate the following integral $\int \frac{dx}{x(1+\ln^2 x)}$

$u = \ln x$

$du = \frac{dx}{x}$

$\int \frac{du}{1+u^2} = \text{Arctan}\, u + C$

$= \text{Arctan}(\ln|x|) + C$

(b) Evaluate the following integral $\int_0^1 \ln(1+x)\,dx$

$u = \ln(1+x)$ $\quad dv = dx$

$du = \frac{dx}{1+x}$ $\quad v = x$

$\int_0^1 \ln(1+x)\,dx = uv - \int v\,du = x\ln(1+x)\Big|_0^1 - \int_0^1 \frac{x\,dx}{1+x}$

$\frac{x}{1+x} = 1 - \frac{1}{1+x}$

$= x\ln(1+x)\Big|_0^1 - \int_0^1 \left(1 - \frac{1}{1+x}\right) dx$

$= \{x\ln(1+x) - x + \ln(1+x)\}\Big|_0^1$

$= \{\ln(1+x)\cdot(x+1) - x\}\Big|_0^1 = 2\ln 2 - 1$ //

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Q3. Evaluate the following integral $\int \frac{dx}{x^3 - x}$

$$\int \frac{dx}{x^3-x} = \int \frac{dx}{x(x^2-1)} = \int \frac{A}{x}dx + \int \frac{B}{x-1}dx + \int \frac{C}{x+1}dx$$

$$\frac{1}{x^3-x} = \frac{1}{x(x^2-1)} = \frac{1}{x(x-1)(x+1)}$$

$$\frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$1 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$$

$x = 0$: $A = -1$

$x = 1$: $B = 1/2$

$x = -1$: $C = 1/2$

$$\int \frac{dx}{x^3-x} = -\int \frac{dx}{x} + \frac{1}{2}\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{dx}{x+1}$$

$$= -\ln|x| + \frac{1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1| + C$$

$$= \ln\left|\frac{\sqrt{x^2-1}}{x}\right| + C$$

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Q4. (a) Evaluate the following integral $\int \sec^2 x \tan x \, dx$

$u = \tan x, \quad du = \sec^2 x \, dx$

$\int \sec^2 x \tan x \, dx = \int u \, du = \frac{u^2}{2} + C = \frac{1}{2} \tan^2 x + C$

(b) Evaluate the following integral $\int \sin^2 x \cos^3 x \, dx$

$\int \sin^2 x \cdot \cos^2 x \cdot \cos x \, dx =$

$\int \sin^2 x (1 - \sin^2 x) \cdot \cos x \, dx =$

$u = \sin x$

$du = \cos x \, dx$

$\int u^2 (1 - u^2) \, du =$

$\int (u^2 - u^4) \, du =$

$\frac{1}{3} u^3 - \frac{1}{5} u^5 + C =$

$\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$ //