Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 4.2 Homework

1. Determine if **w** = $\begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$ is in Nul A, where A = $\begin{bmatrix} 2 & -1 & -3 \\ 4 & -3 & -11 \\ -5 & 3 & 10 \end{bmatrix}$.

Is **w** in Nul A? Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

A. No, because A**w** =

B. Yes, because A**w** = $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

2. Find an explicit description of Nul A by listing vectors that span the null space.

$$A = \begin{bmatrix} 1 & 2 & 5 & 0 \\ 0 & 1 & 4 & -3 \end{bmatrix}$$

A spanning set for Nul A is $\left\{ \begin{bmatrix} 3 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

3. Either use an appropriate theorem to show that the given set, W, is a vector space, or find a specific example to the contrary.

$$W = \left\{ \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix} : \begin{array}{r} -p-3q=5s \\ 2p=s-3r \end{array} \right\}$$

Rewrite the system of equations in the form $A\mathbf{x}=0$.

$$A\mathbf{x} = \begin{bmatrix} -1 & -3 & 0 & -5 \\ 2 & 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

What does the given set represent?

- **A.** The set of all solutions to the homogeneous system of equations.
- **B.** The set of solutions to one of the homogeneous equations.
- **C.** The set represents the values which are not solutions.

Therefore, the set W = Nul A.

The null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$. Equivalently, the set of all solutions to a system $A\mathbf{x}=0$ of m homogeneous linear equations in n unknowns is a subspace of $\mathbb{R}^n$.

Which of the following is a true statement?

- **A.** The proof is complete since W is a subspace of $\mathbb{R}^3$. The given set W must be a vector space because a subspace itself is a vector space.
- **B.** The proof is complete since W is a subspace of $\mathbb{R}^2$. The given set W must be a vector space because a subspace itself is a vector space.
- **C.** The proof is complete since W is a subspace of $\mathbb{R}$. The given set W must be a vector space because a subspace itself is a vector space.
- **D.** The proof is complete since W is a subspace of $\mathbb{R}^4$. The given set W must be a vector space because a subspace itself is a vector space.

4. Either use an appropriate theorem to show that the given set, W, is a vector space, or find a specific example to the contrary.

$$W = \left\{ \begin{bmatrix} s - 2t \\ 3 + 3s \\ 3s + t \\ 2s \end{bmatrix} : s,t \text{ real} \right\}$$

The set W is a subset of $\mathbb{R}^4$. If W were a vector space, what else would be true about it?

- **A.** The set W would be a subspace of $\mathbb{R}^2$.
- **B.** The set W would be a subspace of $\mathbb{R}^4$.
- **C.** The set W would be the null space of $\mathbb{R}^2$.
- **D.** The set W would be the null space of $\mathbb{R}^4$.

Determine whether the zero vector is in W. Find values for t and s such that $\begin{bmatrix} s - 2t \\ 3 + 3s \\ 3s + t \\ 2s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

- **A.** The zero vector is in W. The vector equation is satisfied when t = ______ and s = ______ .
- **B.** The zero vector is not in W. There is no t and s such that the vector equation is satisfied.

Which of the following is a true statement?

- **A.** Since the zero vector is not in W, W is not the null space of $\mathbb{R}^2$. Thus W is not a vector space.
- **B.** Since the zero vector is in W, W is the null space of $\mathbb{R}^4$. Thus W is a vector space.
- **C.** Since the zero vector is not in W, W is not a subspace of $\mathbb{R}^4$. Thus W is not a vector space.
- **D.** Since the zero vector is in W, W is a subspace of $\mathbb{R}^2$. Thus W is a vector space.

5. Find A such that the given set is Col A.

$$\left\{ \begin{bmatrix} -3r+2s+3t \\ 2r+3s-3t \\ -3s-2t \\ -r+s+2t \end{bmatrix} : r, s, t \text{ real} \right\}$$

Choose the correct answer below.

A. $A = \begin{bmatrix} 2 & -3 & 3 \\ 3 & 2 & -3 \\ -3 & 0 & -2 \\ -1 & 1 & 2 \end{bmatrix}$

B. $A = \begin{bmatrix} -3 & 2 & 3 \\ -3 & 3 & 2 \\ -2 & -3 & 0 \\ 2 & 1 & -1 \end{bmatrix}$

C. $A = \begin{bmatrix} -3 & 3 & 2 \\ 2 & -3 & 3 \\ 0 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix}$

D. $A = \begin{bmatrix} -3 & 2 & 3 \\ 2 & 3 & -3 \\ 0 & -3 & -2 \\ -1 & 1 & 2 \end{bmatrix}$

6. Complete parts (a) and (b) for the matrix below.

$$A = \begin{bmatrix} -3 & -8 & -4 \\ -3 & -4 & 5 \\ 6 & -5 & 6 \\ 9 & -1 & -7 \\ -8 & 0 & 2 \end{bmatrix}$$

(a) Find k such that Nul(A) is a subspace of $\mathbb{R}^k$.

k = 3

(b) Find k such that Col(A) is a subspace of $\mathbb{R}^k$.

k = 5

7. For the matrix A below, find a nonzero vector in Nul A and a nonzero vector in Col A.

$$A = \begin{bmatrix} 12 & -16 \\ 3 & -4 \\ -12 & 16 \\ -6 & 8 \end{bmatrix}$$

A nonzero vector in Nul A is $\begin{bmatrix} \frac{4}{3} \\ 1 \end{bmatrix}$.

A nonzero vector in Col A is $\begin{bmatrix} 12 \\ 3 \\ -12 \\ -6 \end{bmatrix}$.

8. Let $A = \begin{bmatrix} -12 & 36 \\ -4 & 12 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Determine if **w** is in Col(A). Is **w** in Nul(A)?

Determine if **w** is in Col(A). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The vector **w** is in Col(A) because A**x** = **w** is a consistent system. One solution is

$$\mathbf{x} = \begin{bmatrix} \frac{11}{4} \\ 1 \end{bmatrix}.$$

B. The vector **w** is not in Col(A) because **w** is a linear combination of the columns of A.

C. The vector **w** is in Col(A) because the columns of A span $\mathbb{R}^2$.

D. The vector **w** is not in Col(A) because A**x** = **w** is an inconsistent system. One row of the reduced row echelon form of the augmented matrix [A 0] has the form [0 0 b] where b = ______ .

Is **w** in Nul(A)? Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

A. The vector **w** is not in Nul(A) because A**w** = ______ .

B. The vector **w** is in Nul(A) because $A\mathbf{w} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

YOU ANSWERED: A.: $\begin{bmatrix} \frac{19}{5} \\ 1 \end{bmatrix}$

9. Determine whether **w** is in the column space of A, the null space of A, or both.

$$\mathbf{w} = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, A = \begin{bmatrix} -5 & 3 & 1 & 0 \\ -8 & 4 & 4 & 10 \\ 10 & -8 & 4 & 14 \\ 3 & -2 & 0 & 0 \end{bmatrix}$$

Is **w** in the column space of A, the null space of A, or both?

- Both
- Null Space
- Column Space

10.

Define a linear transformation $T: \mathbb{P}_2 \to \mathbb{R}^2$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(0) \end{bmatrix}$. Find polynomials $\mathbf{p}_1$ and $\mathbf{p}_2$ in $\mathbb{P}_2$ that span the kernel of T, and describe the range of T.

Find polynomials $\mathbf{p}_1$ and $\mathbf{p}_2$ in $\mathbb{P}_2$ that span the kernel of T. Choose the correct answer below.

- **A.** $\mathbf{p}_1(t) = t^2$ and $\mathbf{p}_2(t) = -t^2$
- **B.** $\mathbf{p}_1(t) = t$ and $\mathbf{p}_2(t) = t^2 - 1$
- **C.** $\mathbf{p}_1(t) = t$ and $\mathbf{p}_2(t) = t^2$ ✔
- **D.** $\mathbf{p}_1(t) = 1$ and $\mathbf{p}_2(t) = t^2$
- **E.** $\mathbf{p}_1(t) = t$ and $\mathbf{p}_2(t) = t^3$
- **F.** $\mathbf{p}_1(t) = t + 1$ and $\mathbf{p}_2(t) = t^2$
- **G.** $\mathbf{p}_1(t) = 3t^2 + 5t$ and $\mathbf{p}_2(t) = 3t^2 - 5t + 7$

Describe the range of T. Choose the correct answer below.

- **A.** $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- **B.** $\varnothing$
- **C.** $\left\{ \begin{bmatrix} a \\ 0 \end{bmatrix} : \text{a real} \right\}$
- **D.** $\left\{ \begin{bmatrix} a \\ a \end{bmatrix} : \text{a real, } a \neq 0 \right\}$
- **E.** $\left\{ \begin{bmatrix} a \\ a \end{bmatrix} : \text{a real} \right\}$ ✔
- **F.** $\left\{ \begin{bmatrix} a \\ a \end{bmatrix} : \text{a an integer} \right\}$
- **G.** $\left\{ \begin{bmatrix} 0 \\ a \end{bmatrix} : \text{a real} \right\}$
- **H.** $\left\{ \begin{bmatrix} a \\ a \end{bmatrix} : \text{a real, } a > 0 \right\}$