Math No:

Full Name : KEY

ISTANBUL ŞEHİR UNIVERSITY

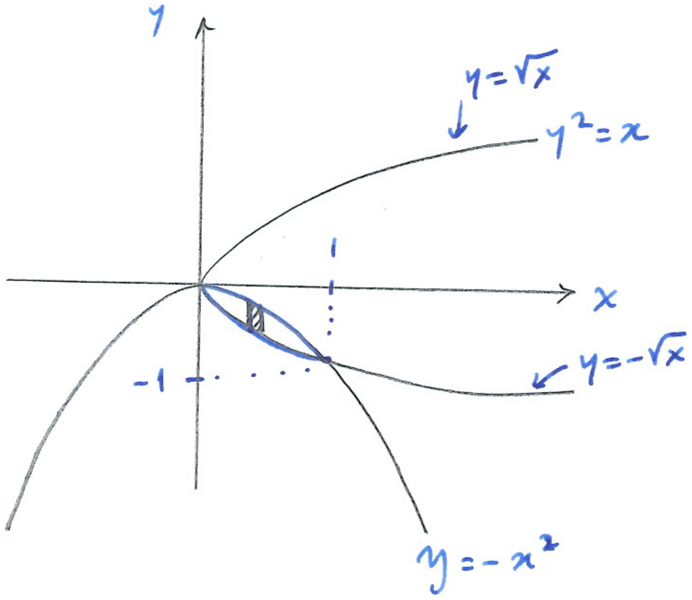
Math 104 Final Exam
(4 February 2018, 11:00-12:00)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones, smart watches and electronic devices are to be kept shut and out of sight. All cell phones and smart watches are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	Q5	TOT
5 pts	5 pts	5 pts	5 pts	5 pts	25 pts

Q1. Calculate the area between $y^2 = x$ and $y = -x^2$.



$\Big\}\ -x^2+\sqrt{x}$

dx

$dA = (-x^2+\sqrt{x})\,dx$

$y^2 = x$
$y = -x^2$

$y^2 = x^4$

$x = x^4$

$x(x^3-1) = 0$

$x = 0$
$x^3 = 1 \Rightarrow x = 1$

$$A = \int_0^1 (-x^2+\sqrt{x})\,dx$$

$$= -\frac{1}{3}x^3 + \frac{2}{3}x^{3/2}\Big|_0^1$$

$$= -\frac{1}{3} + \frac{2}{3}$$

$$= 1/3$$

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Q2. Evaluate the following integral $\int \frac{3^{-x}}{1+3^{-2x}}dx$

$u = 3^{-x} = e^{-x \ln 3}$

$du = -\ln 3 \underbrace{e^{-x\ln 3}}_{3^{-x}} dx$

$-\frac{du}{\ln 3} = 3^{-x} dx$

$\int \frac{3^{-x} dx}{1+(3^{-x})^2} = -\frac{1}{\ln 3} \int \frac{du}{1+u^2} = -\frac{1}{\ln 3} \text{Arc}\tan u + C$

$= -\frac{1}{\ln 3} \text{Arc}\tan(3^{-x}) + C$

Q3. Determine the following limit $\lim_{x\to 0} (1-\sin x)^{\frac{1}{\sin x}}$ $= 1^{\infty}$, indeterminate power

$y = (1-\sin x)^{\frac{1}{\sin x}}$

$\ln y = \frac{1}{\sin x} \ln(1-\sin x)$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\ln(1-\sin x)}{\sin x} = \lim_{x\to 0} \frac{\frac{-\cancel{\cos x}}{1-\sin x}}{\cancel{\cos x}} = -1$ L'Hosp. Rule

$\lim_{x\to 0} y = e^{-1} = 1/e$

$\lim_{x\to 0} (1-\sin x)^{\frac{1}{\sin x}} = 1/e$

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Q4. Find the following series' radius of convergence.

$$\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2} x^n$$

Applying n-root test

$$\rho = \lim_{n\to\infty} \left| \sqrt[n]{\left(\frac{n}{n+1}\right)^{n^2} x^n} \right| < 1$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n |x| < 1$$

$$\rho = |x| \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n < 1$$

$$\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^x = 1^\infty, \text{ indeterminate power!}$$

$$y = \left(\frac{x}{x+1}\right)^x$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln\left(x/(x+1)\right)}{1/x} = \lim_{x\to\infty} \frac{\dfrac{\dfrac{1}{(x+1)^{\cancel{2}}}}{\cancel{\dfrac{x}{x+1}}}}{-\dfrac{1}{x^{\cancel{2}}}}$$

$$\lim_{x\to\infty} \ln y = -\lim_{x\to\infty} \frac{x}{x+1} = -1$$

$$\lim_{x\to\infty} y = e^{-1} = \frac{1}{e}$$

$\rho = |x| \frac{1}{e} < 1 \Rightarrow |x| < e$, Radius of conv. is e.

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Q5. Find the Maclaurin series for the following function: $f(x)=e^{1-x}$.

[*Hint: The Taylor Series is given by* $\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$]

$$f(x) = e^{1-x} = e\cdot e^{-x}$$

Maclaurin series is $\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$

$$f(x) = e\left\{ f(0) + f'(0)\,x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + f^{(4)}(0)\frac{x^4}{4!} + f^{(5)}(0)\frac{x^5}{5!} + \dots \right\}$$

$$= e\left\{ 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \dots \right\}$$

$$= e\cdot\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$$