Reading & Remarks

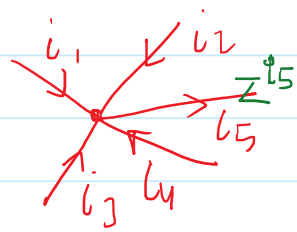
Charles K. Alexander and Matthew N. O. Sadiku, *Fundamentals of Electric Circuits 5th Edition*, McGrawHill, 2015.

- Chapter 3

Remarks:

- Homework - 1
- Quiz-1 — Q1 - ✓
  Q2 - ✓
  Q3 - ✓

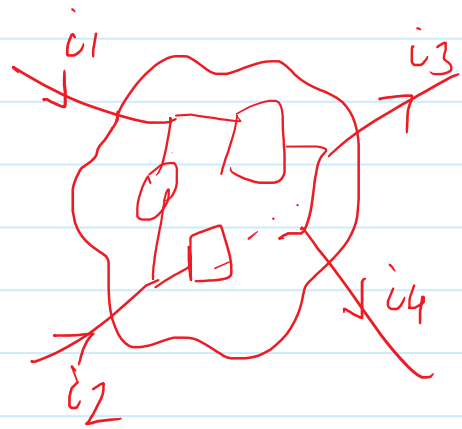
KCL



$i_1 + i_2 + i_3 + i_4 = i_5$

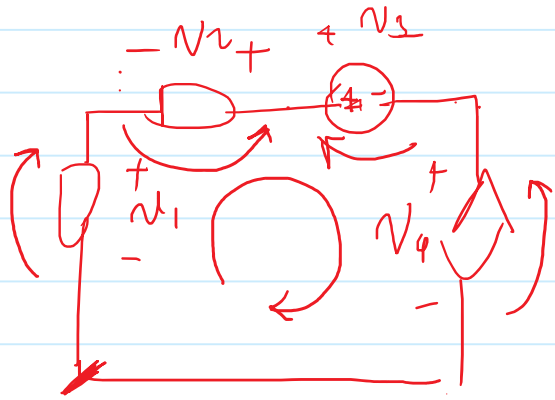
$i_1 + i_2 + i_3 + i_4 - i_5 = 0$

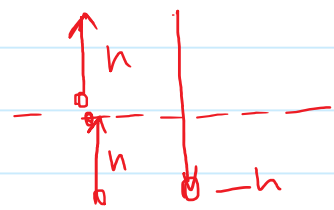
KCL



$i_1 + i_2 = i_3 + i_4$

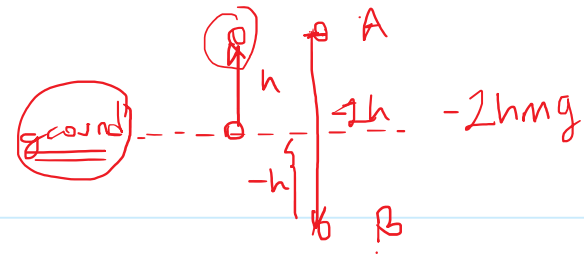
KVL





$$V_1 + V_2 - V_3 - V_4 = 0$$

Nodal Analysis



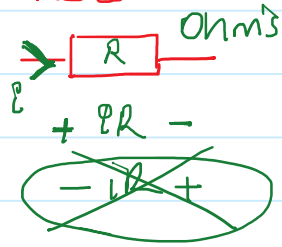
1. Select a node as the reference node. Assign v1, v2,…, vn-1.
2. Apply KCL to each n-1 non-reference node. Use Ohm's law to express branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

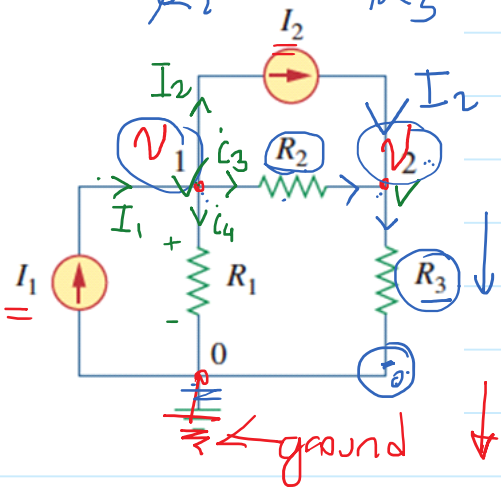
$$I_2 + \frac{V_1 - V_2}{R_2} = \frac{V_2 - 0}{R_3}$$

n-1

KCL

Ohm's Law





0V

KCL@node 1

$$I_1 = I_2 + i_3 + i_4$$

$$i_3 \cdot R_2 = V_1 - V_2 \Rightarrow i_3 = \frac{V_1 - V_2}{R_2}$$

$$i_4 \cdot R_1 = V_1 - 0 \Rightarrow i_4 = \frac{V_1}{R_1}$$

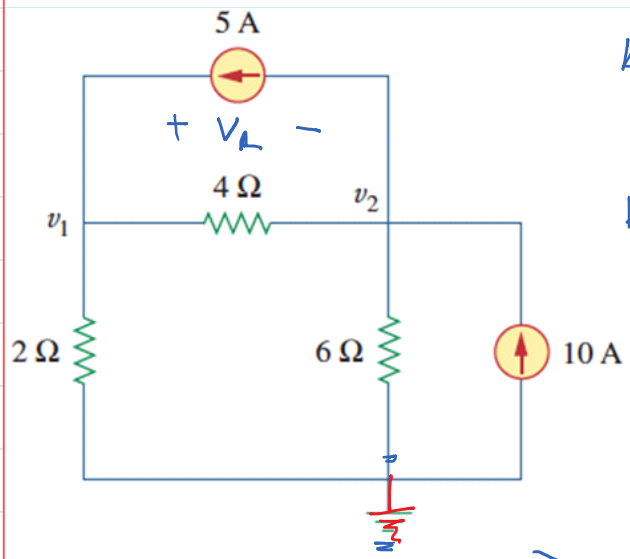
(1) $$I_1 = I_2 + \frac{V_1 - V_2}{R_2} + \frac{V_1}{R_1}$$

(2) $$I_2 + \frac{V_1 - V_2}{R_2} = \frac{V_2}{R_3}$$

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

Example



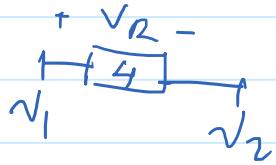
KCL 1: $5 + \frac{v_2 - v_1}{4} = \frac{v_1}{2}$ (4) (4) (2)

KCL 2: $10 = 5 + \frac{v_2 - v_1}{4} + \frac{v_2}{6}$ (12) (3) (2)

(1) $20 + v_2 - v_1 = 2v_1$

(2) $60 = 3v_2 - 3v_1 + 2v_2$

$V_R = v_1 - v_2 = \frac{40}{3} - 20 = -\frac{20}{3}$



$-3v_1 + v_2 = -20$

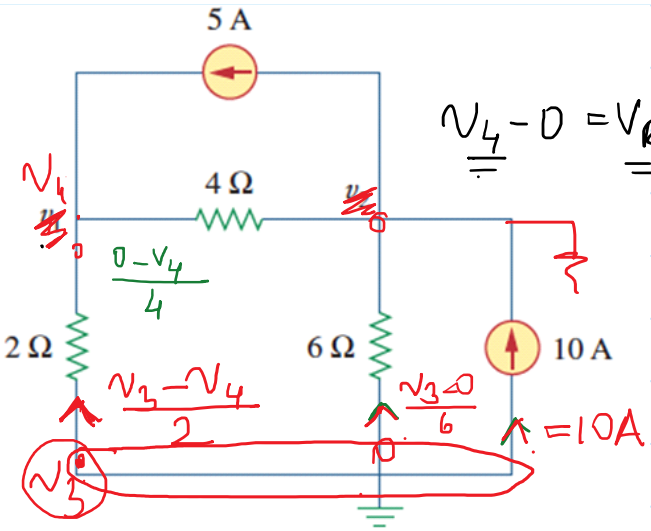
$\pm 3v_1 \mp 5v_2 = 60$

$-4v_2 = -80$ $\quad v_2 = 20V$

$v_1 = 40/3$

V1 = 40/3V, V2 = 20V

Example



$V_4 - 0 = V_R$

$V_R = -20/3$

$$5 - \frac{V_4}{4} + \frac{V_3 - V_4}{2} = 0$$

(4) (1) (2)

$$-10 + \frac{V_3}{6} + \frac{V_3 - V_4}{2} = 0$$

(6) (3)

$$20 - V_4 + 2V_3 - 2V_4 = 0$$

$$60 + V_3 + 3V_3 - 3V_4 = 0$$

$$-20 = -3V_4 + 2V_3$$

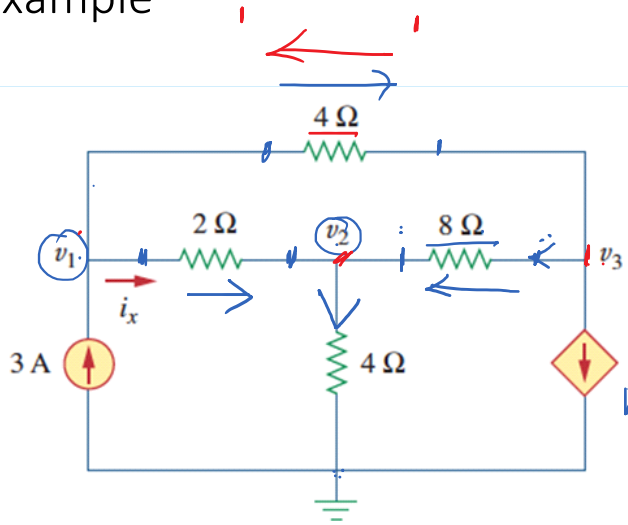
$$60 = +3V_4 - 4V_3$$

$$40 = -2V_3 \quad V_3 = -20$$

$$20 = +3V_4 + 40 \Rightarrow \frac{-20}{3} = V_4$$

V1 = 40/3V, V2 = 20V

Example



$$\frac{v_3 - v_1}{4} + \frac{v_3 - v_2}{8} = -2 \cdot \left(\frac{v_1 - v_2}{2}\right)$$

$$i_x = \frac{v_1 - v_2}{2}$$

KCL1: $3 + \frac{v_3 - v_1}{4} = \frac{v_1 - v_2}{2}$

(4) (1) (2)

KCL2: $\frac{v_1 - v_2}{2} + \frac{v_3 - v_2}{8} = \frac{v_2}{4}$

4 8 2

$$\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{(v_1 - v_2)}{2}$$

(2) (1) (2)

$$12 + v_3 - v_1 = 2v_1 - 2v_2$$

$$4v_1 - 4v_2 + v_3 - v_2 = 2v_2$$

$$2v_1 - 2v_3 + v_2 - v_3 = 8v_1 - 8v_2$$

$$Ax = b$$

$$\boxed{x = A^{-1} b}$$

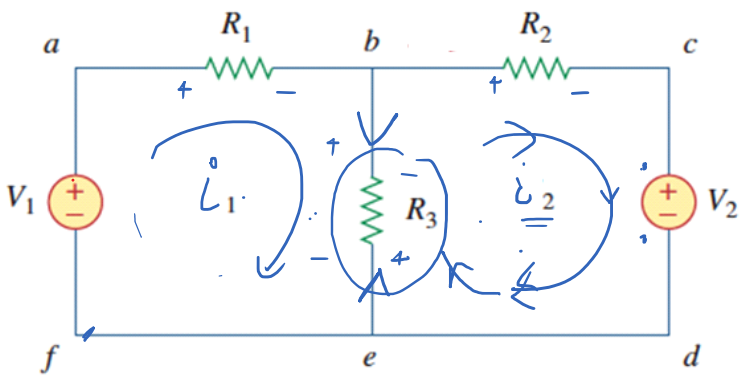
V1=4.8V, V2=2.4V, V3=-2.4V

Mesh Analysis

KCL

KVL

1. Assign mesh currents i1,i2,…,in to the n meshes
2. Apply KVL to all meshes. Use Ohm's law to express voltages in terms of mesh currents
3. Solve the resulting n simultaneous equations to get mesh currents



$$0 = V_1 - i_1 R_1 - (i_1 - i_2) R_3$$

$$0 = -V_2 + (i_1 - i_2) R_3 - i_2 R_2$$

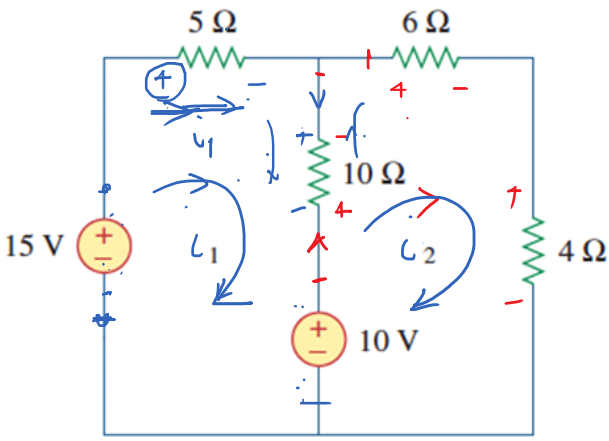
$$V_1 - i_1 R_1 - (i_1 - i_2) R_3 = 0$$

$$-V_2 - R_3(i_2 - i_1) - i_2 R_2 = 0$$

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

$$-R_3 i_1 + (R_2 + R_3) i_2 = -V_2$$

Example



$15 - 5i_1 - (i_1 - i_2)10 - 10 = 0$

$10V - 10(i_2 - i_1) - 6i_2 - 4i_2 = 0$

2/ $5 = 15i_1 - 10i_2$

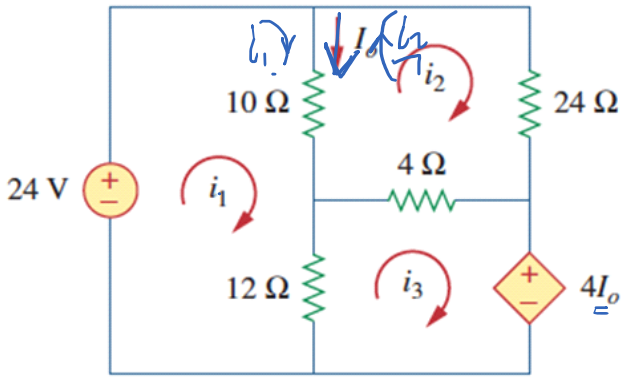
$10 = -10i_1 + 20i_2$

$20 = 20i_1 \Rightarrow i_1 = 1A$

$5 = 15 - 10i_2 \Rightarrow i_2 = 1A$

$i_1 = 1$ A. $i_2 = 1$ A

Example



$$24 - 10(i_1 - i_2) - 12(i_1 - i_3) = 0$$

$$-24 i_2 - 4(i_2 - i_3) - 10(i_2 - i_1) = 0$$

$$-4I_o - 12(i_3 - i_1) - 4(i_3 - i_2) = 0$$

$(i_1 - i_2)$

$$-22 i_1 + 10 i_2 + 12 i_3 = -24$$

$$10 i_1 - 38 i_2 + 4 i_3 = 0$$

$$8 i_1 + 8 i_2 - 16 i_3 = 0$$

$$\rightarrow \begin{bmatrix} -22 & 10 & 12 \\ 10 & -38 & 4 \\ 8 & 8 & -16 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -24 \\ 0 \\ 0 \end{bmatrix}$$

$$A^{-1} A \cdot x = A^{-1} b$$

$$I$$

$$x = A^{-1} b$$

$$\rightarrow \begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$