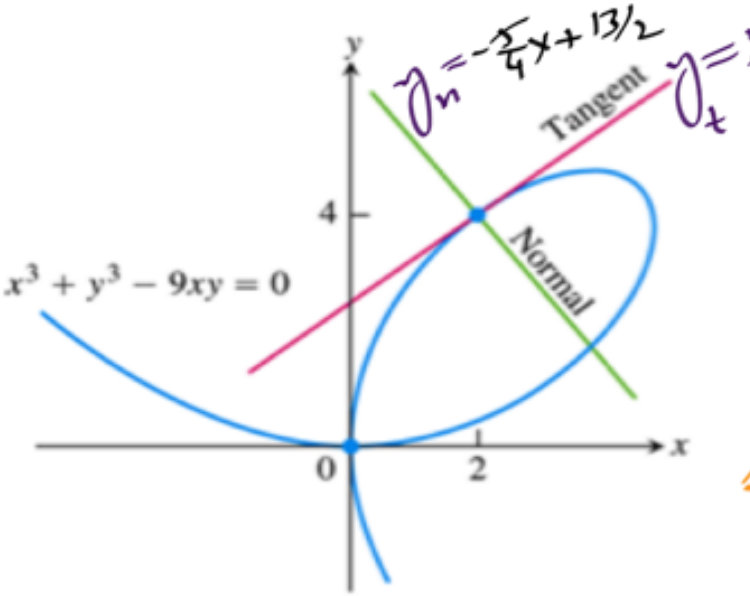


shows how to find equations for the tangent and normal to the folium of Descartes at (2, 4).

For tangent: $y' = \frac{dy}{dx} = \text{slope}$

$$x^3 + y^3 - 9xy = 0$$

$$\frac{\cancel{3}x^2}{\cancel{3}} + \frac{\cancel{3}y^2}{\cancel{3}}\frac{dy}{dx} - \frac{\overset{3}{\cancel{9}}y}{\cancel{3}} - \frac{\overset{3}{\cancel{9}}x}{\cancel{3}}\frac{dy}{dx} = 0$$

$$x^2 - 3y = (3x - y^2)\frac{dy}{dx}$$

$$\left.\frac{dy}{dx}\right|_{(2,4)} = \left.\frac{x^2 - 3y}{3x - y^2}\right|_{(2,4)} = \frac{4-12}{6-16} = \frac{4}{5}$$

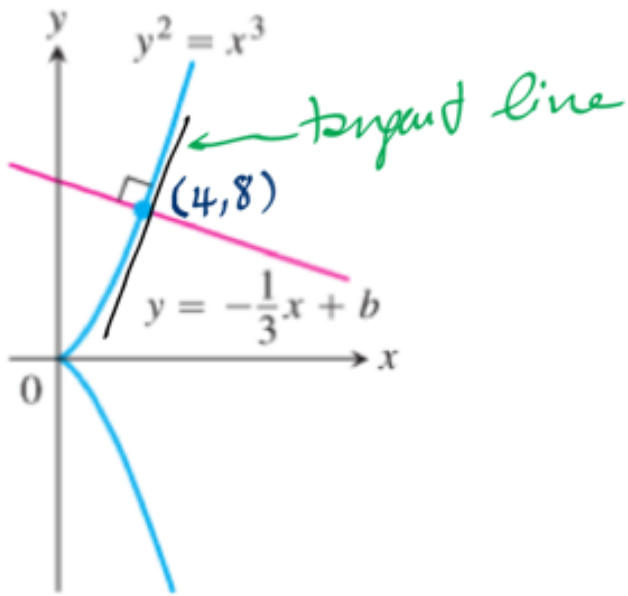
The eqn. of tan. line:

$$\frac{4}{5} = \frac{y-4}{x-2} \Rightarrow y = \frac{4}{5}x + \frac{12}{5}$$

$$\boxed{m_t \cdot m_n = -1} \qquad m_t = \frac{4}{5} \Rightarrow \frac{4}{5} \cdot m_n = -1 \Rightarrow m_n = -\frac{5}{4}$$

$$-\frac{5}{4} = \frac{y-4}{x-2} \Rightarrow y_n = -\frac{5}{4}x + \frac{13}{2}$$

The graph of $y^2 = x^3$ is called a **semicubical parabola** and is shown in the accompanying figure. Determine the constant b so that the line $y = -\frac{1}{3}x + b$ meets this graph orthogonally.



We shall find the slope of the tangent line.

$$y^2 = x^3 \Rightarrow y = x^{3/2}$$

$$2y\frac{dy}{dx} = 3x^2$$

$$\text{slope} = m = \frac{dy}{dx} = \frac{3}{2}\cdot\frac{x^2}{y}$$

$$\frac{dy}{dx} = \frac{3}{2}\cdot\frac{x^2}{x^{3/2}} = \frac{3}{2}\sqrt{x}$$

$$m_n \cdot m_t = -1$$

$$-\frac{1}{3}\cdot m_t = -1 \Rightarrow \underline{m_t = 3}$$

$$\cancel{3} = \frac{\cancel{3}}{2}\sqrt{x}$$

$$2 = \sqrt{x}$$

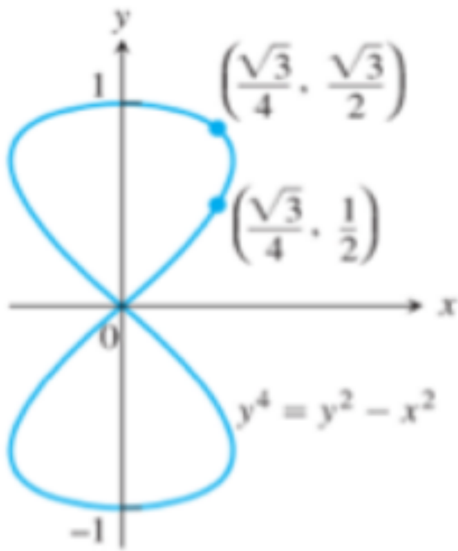
$$\boxed{4 = x} \qquad y = x^{3/2} = 4^{3/2} = 8$$

$$y = -\frac{1}{3}x + b$$

$$8 = -\frac{1}{3}4 + b \Rightarrow b = 28/3$$

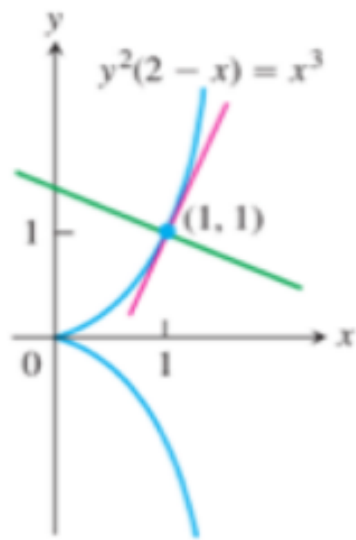
Homework- 1

The eight curve Find the slopes of the curve $y^4 = y^2 - x^2$ at the two points shown here.



Homework-2

The cissoid of Diocles (from about 200 B.C.) Find equations for the tangent and normal to the cissoid of Diocles $y^2(2 - x) = x^3$ at (1, 1).



Ex

$$x\cos(2x+3y) = y\sin x \qquad dy/dx = ?$$

$$\cos(2x+3y) - x\sin(2x+3y)\cdot\left(2+3\frac{dy}{dx}\right) = \frac{dy}{dx}\sin x + y\cos x$$

$dy/dx = \dots$

Ex $\quad x + \tan(xy) = 0$

$$1 + \sec^2(xy)\cdot\left(y + x\frac{dy}{dx}\right) = 0 \Rightarrow y\sec^2 xy + \sec^2 xy \cdot x\frac{dy}{dy} = -1$$

$$\frac{dy}{dx} = \frac{-(1+y\sec^2 xy)}{x\sec^2 xy}$$

Ex

$$y\sin y = 1 - xy$$

$$\frac{dy}{dx}\sin y + y\cos y\cdot\frac{dy}{dx} = -y - x\frac{dy}{dx}$$

$$-y/(\sin y + y\cos y + x) = \frac{dy}{dx}$$

Ex

$(x^2+y^2)^2=(x-2)^2$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $\left(\frac{2}{5},\frac{6}{5}\right)$

$$\cancel{2}(x^2+y^2)\cdot\left(2x+2y\frac{dy}{dx}\right)=\cancel{2}(x-2) \Rightarrow \left(\frac{4}{25}+\frac{36}{25}\right)\cdot\left(\frac{4}{5}+\frac{12}{5}y'\right)=\frac{2}{5}-2 \Rightarrow y'=-\frac{9}{12}$$

$$\left(2x+2y\frac{dy}{dx}\right)\left(2x+2y\frac{dy}{dx}\right)+(x^2+y^2)\left[2+2\left(\frac{dy}{dx}\right)^2+2y\frac{d^2y}{dx^2}\right]=1$$

$$\left[\frac{4}{5}+\frac{12}{5}\cdot\left(-\frac{9}{12}\right)\right]^2+\left(\frac{4}{25}+\frac{36}{25}\right)\cdot\left[2+2\left(-\frac{9}{12}\right)^2+2\cdot\frac{6}{5}y''\right]=1 \Rightarrow y''=-\frac{125}{96}<0$$