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1. Determine if the system has a nontrivial solution. Try to use as few row operations as possible.

$$\begin{aligned} 4x_1 - 6x_2 + 13x_3 &= 0 \\ -4x_1 - 2x_2 - 7x_3 &= 0 \\ 8x_1 + 4x_2 + 14x_3 &= 0 \end{aligned}$$

Choose the correct answer below.

- **A.** The system has a nontrivial solution. ✔
- **B.** The system has only a trivial solution.
- **C.** It is impossible to determine.

2. Determine if the system has a nontrivial solution. Try to use as few row operations as possible.

$$\begin{aligned} -2x_1 + 3x_2 - 6x_3 &= 0 \\ -4x_1 + 7x_2 + 3x_3 &= 0 \end{aligned}$$

Choose the correct answer below.

- **A.** The system has a nontrivial solution. ✔
- **B.** The system has only a trivial solution.
- **C.** It is impossible to determine.

3. Write the solution set of the given homogeneous system in parametric vector form.

$$\begin{aligned} 2x_1 + 2x_2 + 4x_3 &= 0 \\ -4x_1 - 4x_2 - 8x_3 &= 0 \\ -7x_2 + 21x_3 &= 0 \end{aligned}$$

where the solution set is $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\mathbf{x} = x_3 \begin{bmatrix} -5 \\ 3 \\ 1 \end{bmatrix}$$

(Type an integer or simplified fraction for each matrix element.)

4. Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form, where A is row equivalent to the given matrix.

$$\begin{bmatrix} 1 & 2 & -3 & 6 \\ 0 & 1 & -3 & 5 \end{bmatrix}$$

$$\mathbf{x} = x_3 \begin{bmatrix} -3 \\ 3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 4 \\ -5 \\ 0 \\ 1 \end{bmatrix}$$

(Type an integer or fraction for each matrix element.)

5. Describe all solutions of A**x** = **0** in parametric vector form, where A is row equivalent to the given matrix.

$$\begin{bmatrix} 1 & 4 & 0 & -2 \\ 2 & 8 & 0 & -4 \end{bmatrix}$$

$$\mathbf{x} = x_2 \begin{bmatrix} -4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

(Type an integer or fraction for each matrix element.)

6. Describe all solutions of A**x = 0** in parametric vector form, where A is row equivalent to the given matrix.

$$\begin{bmatrix} 1 & -1 & -3 & 0 & -2 & 4 \\ 0 & 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{x} = x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -8 \\ 0 \\ -6 \\ 0 \\ 7 \\ 1 \end{bmatrix}$$

(Type an integer or fraction for each matrix element.)

7. Suppose the solution set of a certain system of linear equations can be described as $x_1 = 6 + 3x_3$, $x_2 = -5 - 8x_3$, with $x_3$ free. Use vectors to describe this set as a line in $\mathbb{R}^3$.

Geometrically, the solution set is a line through $\begin{bmatrix} 6 \\ -5 \\ 0 \end{bmatrix}$ parallel to $\begin{bmatrix} 3 \\ -8 \\ 1 \end{bmatrix}$.

8. Find the parametric equation of the line through **a** parallel to **b**, using t as the parameter.

$$\mathbf{a} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -8 \\ 3 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} -2 \\ 5 \end{bmatrix} + t \begin{bmatrix} -8 \\ 3 \end{bmatrix}$$

(Type an integer or a simplified fraction for each matrix element.)

9. Mark each statement True or False. Justify each answer.

a. A homogeneous equation is always consistent.

- **A.** True. A homogenous equation can be written in the form A**x** = **0**, where A is an $m \times n$ matrix and **0** is the zero vector in $\mathbb{R}^m$. Such a system A**x** = **0** always has at least one solution, namely, **x = 0**. Thus a homogenous equation is always consistent.
- **B.** False. A homogenous equation can be written in the form A**x** = **0**, where A is an $m \times n$ matrix and **0** is the zero vector in $\mathbb{R}^m$. Such a system A**x** = **0** always has at least one nontrivial solution. Thus a homogenous equation is always inconsistent.
- **C.** True. A homogenous equation can be written in the form A**x** = **0**, where A is an $m \times n$ matrix and **0** is the zero vector in $\mathbb{R}^m$. Such a system A**x** = **0** always has at least one nontrivial solution. Thus a homogenous equation is always consistent.
- **D.** False. A homogenous equation can be written in the form A**x** = **0**, where A is an $m \times n$ matrix and **0** is the zero vector in $\mathbb{R}^m$. Such a system A**x** = **0** always has at least one solution, namely, **x** = **0**. Thus a homogenous equation is always inconsistent.

b. The equation A**x** = **0** gives an explicit description of its solution set.

- **A.** True. Since the equation is solved, A**x** = **0** gives an explicit description of the solution set.
- **B.** True. The equation A**x** = **0** gives an explicit description of its solution set. Solving the equation amounts to finding an implicit description of its solution set.
- **C.** False. The equation A**x** = **0** gives an implicit description of its solution set. Solving the equation amounts to finding an explicit description of its solution set.
- **D.** False. Since the equation is solved, A**x** = **0** gives an implicit description of its solution set.

c. The homogenous equation A**x** = **0** has the trivial solution if and only if the equation has at least one free variable.

- **A.** False. The homogeneous equation A**x** = **0** never has the trivial solution.
- **B.** True. The homogeneous equation A**x** = **0** has the trivial solution if and only if the matrix A has a row of zeros which implies the equation has at least one free variable.
- **C.** True. The homogenous equation A**x** = **0** has the trivial solution if and only if the equation has at least one free variable which implies that the equation has a nontrivial solution.
- **D.** False. The homogeneous equation A**x** = **0** always has the trivial solution.

d. The equation **x** = **p** + t**v** describes a line through **v** parallel to **p**.

- **A.** True. The effect of adding **p** to **v** is to move **p** in a direction parallel to the line through **v** and **0**. So the equation **x = p** + t**v** describes a line through **v** parallel to **p**.
- **B.** False. The effect of adding **p** to **v** is to move **v** in a direction parallel to the plane through **p** and **0**. So the equation **x** = **p** + t**v** describes a plane through **p** parallel to **v**.
- **C.** False. The effect of adding **p** to **v** is to move **v** in a direction parallel to the line through **p** and **0**. So the equation **x** = **p** + t**v** describes a line through **p** parallel to **v**.
- **D.** False. The effect of adding **p** to **v** is to move **p** in a direction parallel to the plane through **v** and **0**. So the equation **x** = **p** + t**v** describes a plane through **v** parallel to **p**.

e. The solution set of A**x** = **b** is the set of all vectors of the form **w** = **p** + $\mathbf{v}_h$, where $\mathbf{v}_h$ is any solution of the equation A**x** = **0**.

- **A.** True. The equation A**x** = **b** is always consistent and there always exists a vector **p** that is a solution.
- **B.** False. The solution set could be the trivial solution. The statement is only true when the equation A**x** = **b** is inconsistent for some given **b**, and there exists a vector **p** such that **p** is a solution.
- **C.** False. The solution set could be empty. The statement is only true when the equation A**x** = **b** is

inconsistent for some given **b**, and there exists a vector **p** such that **p** is a solution.

**D.** False. The solution set could be empty. The statement is only true when the equation A**x** = **b** is consistent for some given **b**, and there exists a vector **p** such that **p** is a solution.

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10. A is a $3 \times 3$ matrix with three pivot positions.

    (a) Does the equation A**x** = **0** have a nontrivial solution?
    (b) Does the equation A**x** = **b** have at least one solution for every possible **b**?

(a) Does the equation A**x** = **0** have a nontrivial solution?

- [x] No
- [ ] Yes

(b) Does the equation A**x** = **b** have at least one solution for every possible **b**?

- [ ] No
- [x] Yes

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11. A is a $2 \times 5$ matrix with two pivot positions.

    (a) Does the equation A**x** = **0** have a nontrivial solution?
    (b) Does the equation A**x** = **b** have at least one solution for every possible **b**?

(a) Does the equation A**x** = **0** have a nontrivial solution?

- [ ] No
- [x] Yes

(b) Does the equation A**x** = **b** have at least one solution for every possible **b**?

- [ ] No
- [x] Yes

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12. Given $A = \begin{bmatrix} -3 & -6 \\ 5 & 10 \\ -4 & -8 \end{bmatrix}$, find one nontrivial solution of A**x** = **0** by inspection. [Hint: Think of the equation A**x** = **0** written as a vector equation.]

$\mathbf{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(Type an integer or simplified fraction for each matrix element.)

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13. Construct a $3 \times 3$ nonzero matrix A such that the vector $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ is a solution of A**x** = **0**.

$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$