Clusters.Top Students Association

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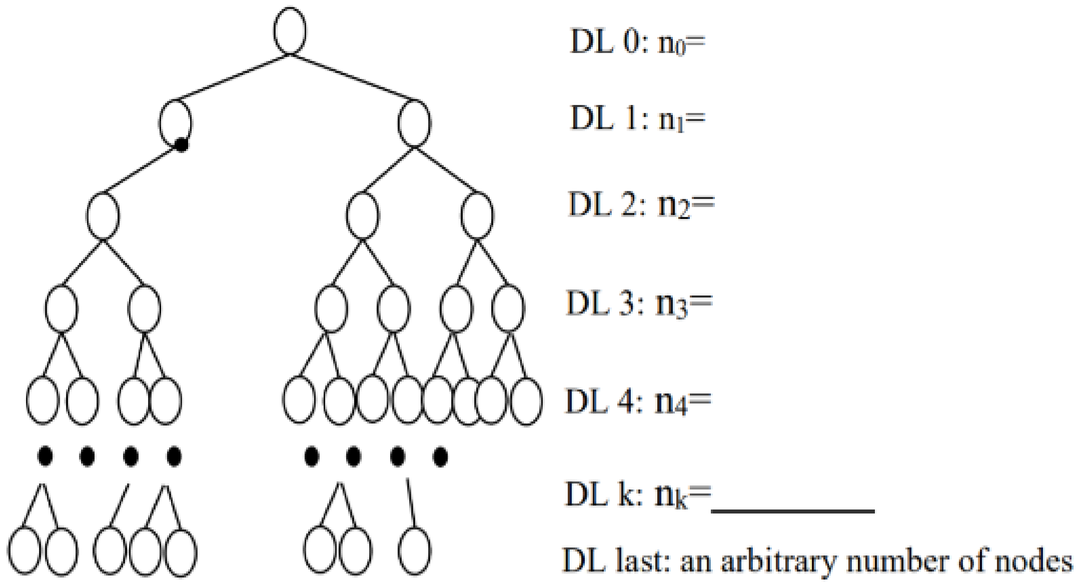
Question:

Assume that you have a BST with n nodes as shown in the figure below. This is ***the best*** possible BST topology with the missing right subtree of the root's left child. The dots over the last depth level (DL) (• • •) indicate an arbitrary number of depth levels all filled up. The only nodes that are missing compared with a complete binary tree are those that are a member of the right subtree of the root's left child.



a) What is the number of nodes at the DL $k \geq 2$?
Hint: Show the general relationship between the DL number k and the number of nodes n_k in DL2, DL3, DL4, …, DLk by writing the number of nodes n_k besides each depth level in the figure.

b) In terms of n, how many depth levels d are there in this BST topology?

c) As you see in the figure there is an arbitrary number of nodes at the last DL. Express this

c) As you see in the figure there is an arbitrary number of nodes at the last DL. Express this
number of nodes n_{last} in terms of n, number of all nodes in the BST?

Answer:

$a)$

DL 0: $n_0 = 1$

DL 1: $n_1 = 2$

DL 2: $n_2 = 3$

DL 3: $n_3 = 6$

DL 4: $n_4 = 12$

DL k: $n_k = 2^k - 2^{k-2} = 3 \cdot 2^{k-2}$

So, number of nodes at DL $k \geq 2$ is $3 \cdot 2^{k-2}$

$b)$

Number of nodes in level $0 = 1$

Number of nodes in level $1 = 2$

Number of nodes in level $2 = 3$

Number of nodes in level $3 = 6$

Similarly, Number of nodes in level $d = 3 \cdot 2^{d-2}$

Now, sum of nodes at all levels equals n

$1 + 2 + 3 + 6 + ... + 3 \cdot 2^{d-2} = n$

$1 + 2 + 3(1 + 2 + ... + \cdot 2^{d-2}) = n$

$3 + 3(1 + 2 + ... + \cdot 2^{d-2}) = n$

$3 + 3 \cdot (2^{d-1} - 1) = n$

$3 + 3 \cdot 2^{d-1} - 3 = n$

$3 \cdot 2^{d-1} = n$

$2^{d-1} = \frac{n}{3}$

$d - 1 = \log_2 \left(\frac{n}{3}\right)$

$d = \log_2 \left(\frac{n}{3}\right) + 1$

$c)$

Let n_{last} represent the number of nodes in the $last$ level

$last = \log_2 \left(\frac{n}{3}\right) + 1$

$n_{last} = 2^{last} - 2^{last-2}$

$n_{last} = 2^{last-2}(2^2 - 1)$

$n_{last} = 3 \cdot 2^{last-2}$

Substitute $last = \log_2 \left(\frac{n}{3}\right) + 1$:

$$n_{last} = 3 \cdot 2^{\log_2\left(\frac{n}{3}\right)+1-2}$$

$$n_{last} = 3 \cdot 2^{\log_2\left(\frac{n}{3}\right)-1}$$

$$n_{last} = 3 \cdot \frac{2^{\log_2\left(\frac{n}{3}\right)}}{2}$$

$$n_{last} = 3 \cdot \frac{\left(\frac{n}{3}\right)}{2}$$

$$n_{last} = \frac{n}{2}$$