Math

ISTANBUL SEHİR UNIVERSITY

No:

KEY

Full Name :

Math 104 2nd Midterm Exam
(26 March 2016, 11:30-12:30)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
20 pts	30 pts	25 pts	25 pts	100 pts

Q1. Evaluate the following integral:

$$\int \tan^4\theta\, d\theta = \int (\tan^2\theta)^2\, d\theta = \int (\sec^2\theta - 1)^2\, d\theta$$

$$= \int (\sec^4\theta - 2\sec^2\theta + 1)\, d\theta$$

$$\sec^2\theta \cdot \sec^2\theta$$

$$1 + \tan^2\theta$$

$$= \int (\sec^2\theta + \tan^2\theta\sec^2\theta - 2\sec^2\theta + 1)\, d\theta$$

$$= \int (\tan^2\theta\sec^2\theta - \sec^2\theta + 1)\, d\theta$$

$$= \boxed{\frac{\tan^3\theta}{3} - \tan\theta + \theta + C}$$

Q2. Evaluate the following limits, if they exist:

a) $\lim_{x\to 0} \frac{Arctan(3x)}{x^3}$ $\overset{0/0}{=}$ L'Hospital $\lim_{x\to 0} \frac{\frac{3}{1+9x^2}}{3x^2} = 1/0 = \infty$

b) $\lim_{x\to 0^+} (cosx)^{1/x^2}$ 1^{∞}

$y = (\cos x)^{1/x^2} \Rightarrow \ln y = \frac{1}{x^2}\ln\cos x$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln\cos x}{x^2}$ $0/0$

$\overset{\text{L'Hospital}}{=} \lim_{x\to 0^+} \frac{\frac{-\sin x}{\cos x}}{2x}$

$0/0$

$= -\frac{1}{2}\lim_{x\to 0^+} \frac{\tan x}{x}$

$\overset{\text{L'Hospital}}{=} -\frac{1}{2}\lim_{x\to 0^+} \frac{\sec^2 x}{1} = -\frac{1}{2}$

$\ln y \to -\frac{1}{2} \Rightarrow y \to e^{-1/2} = \boxed{\frac{1}{\sqrt{e}}}$

(You can evaluate $\lim_{x\to 0^+} \frac{\tan x}{x}$ by other methods also)

Q3. Evaluate the following integral:

$$\int sin\sqrt{x}dx$$

$t = \sqrt{x} \Rightarrow dt = \frac{dx}{2\sqrt{x}} \Rightarrow dx = 2t\,dt$

$\therefore \int \sin\sqrt{x}\,dx = \int 2t \sin t\,dt$

Integration by parts: $u = t$, $dv = \sin t\,dt$; $du = dt$, $v = -\cos t$

$\therefore \int \sin\sqrt{x}\,dx = -2t\cos t + 2\int \cos t\,dt$

$= -2t\cos t + 2\sin t + C$

$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$

Math

ISTANBUL ŞEHİR UNIVERSITY

No:

Full Name :

Q4. Evaluate the following integral:

$$I = \int \frac{x^4+x^2-1}{x^3+x}\,dx$$

Long division:

$$\begin{array}{r|l} x^4+x^2-1 & x^3+x \\ -x^4+x^2 & x \\ \hline -1 & \end{array}$$

$$I = \int \left(x - \frac{1}{x^3+x}\right) dx = \frac{x^2}{2} - \int \frac{dx}{x(x^2+1)}$$

Partial fractions:

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$1 = A(x^2+1) + Bx^2 + Cx$$

$$= (A+B)x^2 + Cx + A$$

$$A+B=0 \Rightarrow B=-1$$

$$C=0$$

$$A=1$$

$$\therefore I = \frac{x^2}{2} - \int \frac{dx}{x} + \frac{1}{2}\int \frac{2x\,dx}{x^2+1}$$

$$= \boxed{\frac{x^2}{2} - \ln|x| + \frac{1}{2}\ln(x^2+1) + C}$$

or

$$= \frac{x^2}{2} + \ln\frac{\sqrt{x^2+1}}{|x|} + C.$$