İstanbul Şehir University

Math 104

Date: 15 April 2015	Full Name:
Time: 18:00-19:00	
	Student ID:
Spring 2015 3rd Exam	Math number:

IMPORTANT

1. Write down your name and surname on top of each sheet. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cellphones and electronic devices are to be kept shut and out of sight.

Q1	Q2	Q3	Q4	TOTAL
15 pts	25 pts	30 pts	30 pts	100 pts

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1) Given the repeating decimal $0.\overline{12} = 0.121212\ldots$

(a) Write a geometric series which is equal to this repeating decimal. (10 pts)

(b) Find the sum of the geometric series found in part (a) and use it to express this repeating decimal as a fraction. (5 pts)

(a) $0.1212\ldots = 0.12 + 0.0012 + \ldots$

$$= \frac{12}{100} + \frac{12}{10\,000} + \ldots$$

$$= \frac{12}{100}\left(1 + \frac{1}{100} + \frac{1}{100^2} + \ldots\right)$$

$$= \boxed{\frac{12}{100}\sum_{n=0}^{\infty}\left(\frac{1}{100}\right)^n}$$

(b) $0.1212\ldots = \frac{12}{100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{12}{1\cancel{00}}\cdot\frac{1}{\frac{99}{1\cancel{00}}}$

$$= \frac{12}{99} = \boxed{\frac{4}{33}}$$

2) Determine whether the given sequence converges or diverges. If it converges, find the limit:

(13 pts)

(a) $\{n^2e^{-2n}\}$

$$\lim_{x\to\infty} x^2e^{-2x} = \lim_{x\to\infty} \frac{x^2}{e^{2x}} \overset{\infty/\infty}{\underset{\text{L'Hospital}}{=}} \lim_{x\to\infty} \frac{\cancel{2}x}{\cancel{2}e^{2x}} \quad \infty/\infty$$

$$\underset{\text{L'Hospital}}{=} \lim_{x\to\infty} \frac{1}{2e^{2x}} = \frac{1}{\infty} = 0$$

$\therefore$ $\boxed{\text{The sequence converges to } 0.}$

(b) $\left\{\left(2+\frac{1}{n}\right)^{n^2}\right\}$ (12 pts)

$$y = \left(2+\frac{1}{x}\right)^{x^2}$$

$$\ln y = x^2 \ln\left(2+\frac{1}{x}\right)$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} x^2 \ln\left(2+\frac{1}{x}\right) = \infty$$

since $\ln\left(2+\frac{1}{x}\right) \to \ln 2$

$\therefore$ $y \to \infty$

$\boxed{\text{The sequence diverges to } +\infty.}$

3) Determine whether the following series converge or diverge:

(a) $\sum_{n=0}^{\infty} \frac{1+10^n}{1+5^n}$

∞/∞

$$\lim_{n\to\infty} \frac{1+10^n}{1+5^n} = \lim_{n\to\infty} \frac{10^n \ln 10}{5^n \ln 5}$$

L'Hospital

$$= \lim_{n\to\infty} \frac{\ln 10}{\ln 5}\left(\frac{10}{5}\right)^n = \frac{\ln 10}{\ln 5} \lim_{n\to\infty} 2^n = \infty$$

$\lim a_n \neq 0 \Rightarrow$ $\boxed{\text{The series diverges by the } n^{th} \text{ term test}}$

(b) $\sum_{n=1}^{\infty} \frac{1}{3+e^n}$

Limit Comparison Test, with $\sum_{n=1}^{\infty} \frac{1}{e^n} = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n$.

This is a geometric series, $r = \frac{1}{e} < 1$, so it converges.

$$\lim_{n\to\infty} \frac{\frac{1}{3+e^n}}{\frac{1}{e^n}} = \lim_{n\to\infty} \frac{e^n}{3+e^n} = \lim_{n\to\infty} \frac{1}{3e^{-n}+1} = 1$$

$\neq 0, \infty$

$\therefore$ $\boxed{\text{The series converges}}$

(Both questions can be solved by other methods)

4) Determine whether the following series converge or diverge:

(a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}(n+1)}$

Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$,

p-series, $p = 4/3 > 1$, convergent.

$$\lim_{n\to\infty} \frac{\frac{1}{n^{1/3}(n+1)}}{1/n^{4/3}} = \lim_{n\to\infty} \frac{\cancel{n^{1/3}} \cdot n}{\cancel{n^{1/3}}(n+1)} = \lim_{n\to\infty} \frac{1}{1+1/n} = 1$$

$\neq 0, \infty$

$\therefore$ $\boxed{\text{The series converges.}}$

(b) $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$

$\ln e = 1$, $y = \ln x$ is increasing $\Rightarrow \ln n > 1$ for $n \geq 3$

$\Rightarrow (\ln n)^2 > 1$ for $n \geq 3$

$\therefore \frac{(\ln n)^2}{n} > \frac{1}{n}$

$\sum \frac{1}{n}$ is the harmonic series, divergent

$\therefore$ $\boxed{\text{The given series diverges by the Comparison Test}}$

(There are other methods for solving these questions)