## CSE2023 - ASSIGNMENT #1

Solve the following questions from course book. (Discrete Mathematics and Discrete Mathematics and Its Applications, 7th Ed. by Kenneth Rosen)

Note that we may grade selected questions from HWs.

- **1-** Let *p* and *q* be the propositions
  - p: You drive over 70 miles per hour.
  - q: You get a speeding ticket.

Write these propositions using p and q and logical connectives (including negations).

- a) You do not drive over 70 miles per hour.
- b) You drive over 70 miles per hour, but you do not get a speeding ticket.
- c) You will get a speeding ticket if you drive over 70 miles per hour.
- d) If you do not drive over 70 miles per hour, then you will not get a speeding ticket.
- e) Driving over 70 miles per hour is sufficient for getting a speeding ticket.
- f) You get a speeding ticket, but you do not drive over 70 miles per hour.
- g) Whenever you get a speeding ticket, you are driving over 70 miles per hour.
- 2- You are eligible to be President of the U.S.A. only if you are at least 35 years old, were born in the U.S.A, or at the time of your birth both of your parents were citizens, and you have lived at least 14 years in the country. Express your answer in terms of e: "You are eligible to be President of the U.S.A.," a: "You are at least 35 years old," b: "You were born in the U.S.A," p: "At the time of your birth, both of your parents where citizens," and r: "You have lived at least 14 years in the U.S.A."
- **3-** Show that  $(p \to q) \lor (p \to r)$  and  $p \to (q \lor r)$  are logically equivalent.
- **4-** Suppose that the domain of the propositional function P(x) consists of -5, -3, -1, 1, 3, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

a)  $\exists x P(x)$ 

- **b**)  $\forall x P(x)$
- c)  $\forall x((x \neq 1) \rightarrow P(x))$
- **d)**  $\exists x((x > 0) \land P(x))$
- e)  $\exists x (\neg P(x)) \land \forall x ((x < 0) \rightarrow P(x))$
- 5- Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

  - **a)**  $\forall x \exists y (x^2 = y)$  **b)**  $\forall x \exists y (x = y^2)$

  - c)  $\exists x \forall y (xy = 0)$  d)  $\exists x \exists y (x + y \neq y + x)$
  - e)  $\forall x (x \neq 0 \rightarrow \exists y (xy = 1))$
  - f)  $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$
  - g)  $\forall x \exists y (x + y = 1)$
  - **h)**  $\exists x \exists y (x + 2y = 2 \land 2x + 4y = 5)$
  - i)  $\forall x \exists y (x + y = 2 \land 2x y = 1)$
  - j)  $\forall x \forall y \exists z (z = (x + y)/2)$
- **6-** Determine whether these are valid arguments.
  - a) If x is a positive real number, then  $x^2$  is a positive real number. Therefore, if  $a^2$  is positive, where a is a real number, then a is a positive real number. **b)** If  $x^2 \neq 0$ , where x is a real number, then  $x \neq 0$ . Let
  - a be a real number with  $a^2 \neq 0$ ; then  $a \neq 0$ .
- **7-** Use resolution to show that the compound proposition  $(p \lor q) \land (\neg p \lor q) \land (p \lor \neg q) \land (\neg p \lor \neg q)$  is not satisfiable.
- 8- Prove that if n is a perfect square, then n + 2 is not a perfect square.
- 9- Show that if r is an irrational number, there is a unique integer n such that the distance between r and n is less than 1/2.
- **10-** Prove or disprove that if you have an 8-gallon jug of water and two empty jugs with capacities of 5 gallons and 3 gallons, respectively, then you can measure 4 gallons by successively pouring some of or all of the water in a jug into another jug.

## Submission Instruction

Please zip and submit all your files using filename YourNumberHW1.zip (ex: 150629573HW1.zip) to Canvas system (under Assignments tab).