Istanbul Şehir University

Math 104

Date: 15 March 2014	Full Name:
Time: 11:00-12:30	
	Student ID:
Fall 2014 First Exam	

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. You may continue your solutions on the back of the sheets. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cellphones and electronic devices are to be kept shut and out of sight.

Q1	Q2	Q3	Q4	TOTAL
20 pts	30 pts	20 pts	30 pts	100 pts

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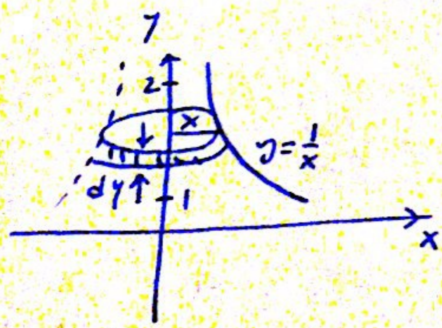
1) Find the volume of the solid generated by revolving the region between the curve $y = 1/x$ and the lines $y = 1, y = 2$ and $x = 0$ about the y-axis.



$V = \int_1^2 \pi x^2 dy = \int_1^2 \pi \left(\frac{1}{y}\right)^2 dy = \pi \left| -\frac{1}{y} \right|_1^2 = \pi/2$

2) Evaluate each of the following limits, if it exists:

(a) $\lim_{x\to\infty} x^{\tan(1/x)}$

$y = x^{\tan\frac{1}{x}}$, $\ln y = \tan\frac{1}{x}\ln x = \dfrac{\ln x}{\frac{1}{\tan\frac{1}{x}}} = \dfrac{\ln x}{\cot\frac{1}{x}}$

$\lim\limits_{x\to\infty} \ln y = \lim\limits_{x\to\infty} \dfrac{(\ln x)'}{(\cot\frac{1}{x})'} = \lim\limits_{x\to\infty} \dfrac{1/x}{\frac{1}{x^2}\cdot \csc^2 x\frac{1}{x}}$ (L'Hospital)

$= \lim\limits_{x\to\infty} \dfrac{1}{\frac{1}{x}\cdot\frac{1}{\sin^2\frac{1}{x}}} = \lim\limits_{x\to\infty} \dfrac{\sin^2\frac{1}{x}}{\frac{1}{x}} = \lim\limits_{x\to\infty} \underbrace{\sin\frac{1}{x}}_{0}\cdot \underbrace{\dfrac{\sin\frac{1}{x}}{\frac{1}{x}}}_{1} = 0$

$\lim\limits_{x\to\infty} \ln y = 0 \Rightarrow \lim\limits_{x\to\infty} y = e^0 = 1 \Rightarrow \lim\limits_{x\to\infty} x^{\tan\frac{1}{x}} = 1$ "

(b) $\lim_{x\to 0^+} \left(\ln\frac{1}{x}\right)^x$ $\quad y = \left(\ln\frac{1}{x}\right)^x \Rightarrow \ln y = x\ln\left(\ln\frac{1}{x}\right)$ $\quad \theta = \frac{1}{x}$ $\quad x\to\infty$, $\theta\to 0$ or $x\to 0$, $\theta\to\infty$

$\lim\limits_{\theta\to\infty} \ln y = \lim\limits_{\theta\to\infty} \dfrac{\ln(\ln\theta)}{\theta} = \lim\limits_{\theta\to\infty} \dfrac{\frac{1/\theta}{\ln\theta}}{1}$ (L'Hosp.)

$= \lim\limits_{\theta\to\infty} \dfrac{-\frac{1}{\theta^2}}{\frac{1}{\theta}}$ 2nd time L'Hosp.

$= \lim\limits_{\theta\to\infty} -\frac{1}{\theta} = 0$

$\lim\limits_{x\to 0^+} \ln y = 0$

$\lim\limits_{x\to 0^+} y = e^0 = 1$

$\lim\limits_{x\to 0^+} \left(\ln\frac{1}{x}\right)^x = 1$

NAME:

1) Differentiate each of the following functions:

(a) $y = \ln \tan^{-1} x$

$$\frac{d}{dx} \ln \underbrace{\tan^{-1} x}_{y} = \frac{d}{dx} \ln y = \frac{1}{y} \frac{dy}{dx}$$

$y = \tan^{-1} x$

$x = \tan y$

$\frac{dx}{dy} = \sec^2 y = \frac{1}{\cos^2 y}$

$\underbrace{\quad}_{1/(1+x^2)}$

$$\frac{d}{dx}\left[\ln(\tan^{-1} x)\right] = \frac{1}{y}\frac{dy}{dx} = \underbrace{\frac{1}{y}}_{\tan^{-1} x} \frac{1}{\frac{dx}{dy}}_{1+x^2} = \frac{1}{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

(b) $y = \underbrace{Sin^{-1}(e^{-x})}_{y}$

$y = \sin^{-1} e^{-x}$

$e^{-x} = \sin y$

$\frac{d}{dx} e^{-x} = \frac{d}{dx} \sin y$

$-e^{-x} = \cos y \cdot \frac{dy}{dx}$

$\frac{dx}{dy} = -\cos y \cdot e^{x}$

$\frac{d}{dx} \sin^{-1}(e^{-x}) = \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$= \frac{-e^{-x}}{\cos y}$$

$$= -\frac{e^{-x}}{\sqrt{1-\sin^2 y}}$$

$$= -\frac{e^{-x}}{\sqrt{1-e^{-2x}}} \quad //$$

NAME:

4) Evaluate the following integrals:

(a) $\int \frac{x\,dx}{\sqrt{16-x^4}}$

$x^2 = t$

$2x\,dx = dt$

$= \frac{1}{2}\int \frac{dt}{\sqrt{4^2-t^2}} = \frac{1}{2}\sin^{-1}\left(\frac{t}{4}\right) + C = \frac{1}{2}\sin^{-1}\left(\frac{x^2}{4}\right) + C$

(b) $\int e^{-x}\tan(e^{-x})dx$

$u = e^{-x}$

$-du = e^{-x}dx$

$= -\int \tan u\,du$

$= -\int \frac{\sin u}{\cos u}\,du$

$s = \cos u$

$ds = -\sin u\,du$

$= \int \frac{ds}{s}$

$= \ln|s| + C$

$= \ln|\cos e^{-x}| + C$