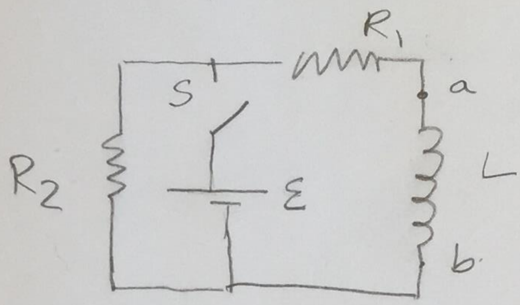
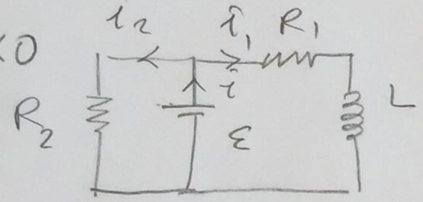
Ch. 32, P. 80



S is closed for a long time so that we have steady state. at $t=0$ S is opened. Find EMF across L immidiately after $t=0$.

$t<0$



$i = i_1 + i_3$

$\mathcal{E} - i_2 R_2 = 0 \qquad \mathcal{E} - i_1 R_1 = 0$

$i_2 = \dfrac{\mathcal{E}}{R_2} \qquad i_1 = \dfrac{\mathcal{E}}{R_1}$

a) when $t=0$ S is closed, so we have $-iR_1 + \mathcal{E}_{in} - iR_2 = 0$

$$-iR_1 - L\frac{di}{dt} - iR_2 = 0$$

$$i(R_1+R_2) = -L\frac{di}{dt} \qquad \frac{di}{i} = -\frac{L}{(R_1+R_2)}dt$$

$$\int_{i_1}^{i}\frac{di}{i} = -\int_0^t \frac{L\,dt}{R_1+R_2} \qquad \ln\frac{i}{i_1} = -\frac{L}{R_1+R_2}dt$$

$$i(t) = \frac{\mathcal{E}}{R_1}e^{-L(t)/R_1+R_2}$$

$$\mathcal{E}_L = -L\frac{di}{dt} \qquad \mathcal{E} = i(R_1+R_2)$$

$$\mathcal{E}_L = \frac{\mathcal{E}}{R}(R_1+R_2)e^{-L(t)/R_1+R_2} \qquad t=0 \quad \mathcal{E}_L = \frac{\mathcal{E}}{R}(R_1+R_2)$$

at the initial instant

b) after $t>0$, since the flux and current is decreasing, the inductor will induce an EMF to reinforce the current, hence it will act like a battery with lower end (b) as positive. The current through R_2 changes direction

FINAL ...

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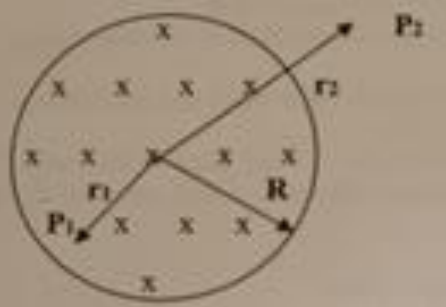
(10 points)

1. Magnetic field directed into the page changes with time according to $B=(0.3t^2+1.4)$ T where t is in seconds. The field has a cross section of radius $R = 0.05$ m.



a) What is the direction and magnitude of E at point P_1 when $t = 3$ s and $r_1 = 0.02$ m.

b) What is the direction and magnitude of the force exerted on an electron ($q = 1.6 \times 10^{-19}$ C) located at point $P_2 = 0.1$ m when $t = 2$ s.

a) $\oint \vec{E} \cdot \vec{ds} = \left| \frac{d\phi}{dt} \right|$ $\quad \phi = (\pi r_1^2)(0.3t^2 + 1.4)$

$$\frac{d\phi}{dt} = (\pi r_1^2)(0.6t)$$

$E(2\pi r_1) = \pi r_1^2 (0.6t)$ $\quad E = \frac{r_1}{2}(0.6t)$

$E = \frac{0.02}{2}(0.6(3)) = (0.01)(1.8) = 18 \times 10^{-3}$ V/m

Since B is increasing inward, the induced current is counterclockwise and E at any point is tangent to the counterclockwise circle.

b) $(E)(2\pi r_2) = \pi R^2 (0.6t)$ $\quad E = \frac{R^2}{2r_2}(0.6t)$

$E = \frac{5 \times 10^{-2}}{2(0.1)}(1.2) = (2.5)(0.6) \times 10^{-3} = 1.5 \times 10^{-3}$ V/m

$F = qE = (1.6 \times 1.5) \times 10^{-22}$ N (Tangent to the ccw ci

$F = 2.4 \times 10^{-22}$ N.

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FINAL EXAM

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(10 points)

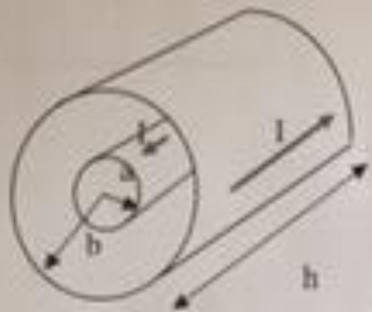
2. A very long coaxial cable consists of a small inner solid conductor of radius **a** and an outer conducting thin cylindrical shell of radius **b**. The inner and outer conductors carry equal currents I in opposite directions.

a) Use Ampere's law to find B at any point between the conductors.

b) Write the expression for the flux $d\Phi$ through a narrow strip of length h parallel to the axis, of width dr at a distance r from the center.

c) Find the total flux.

d) Determine the self inductance L of a length L of the cable.



a. Cross sectional view :

ampere loop

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I \qquad B = \frac{\mu_0 I}{2\pi r}$

tangent to ccw circle.

b.

Area

$d\Phi = BdA = \frac{\mu_0 I}{2\pi r}(dr)L$

dA

c. $\Phi = \int_a^b BdA = \frac{\mu_0 I L}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 I L}{2\pi} \ln \frac{b}{a}$

d. $L = \frac{N\Phi}{I} = \frac{\mu_0 L}{2\pi} \ln \frac{b}{a}$.

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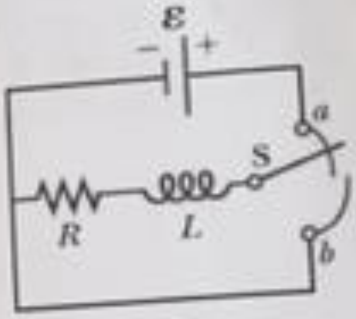
(10 points)

3. In the circuit shown in the figure, the switch S has been at position "*a*" for a long time, and at $t = 0$ is brought to position "*b*" where it stays thereafter.

(a) What is the current in the circuit before t=0 (when S was at a for *a* long time.)

(b) Find $I(t)$ for $t > 0$ (*after S is brought to b*). Derive the working equations.



a. $I = \mathcal{E}/R$

b. $$-RI - L\frac{dI}{dt} = 0$$

$$-RI = L\frac{dI}{dt} \qquad \frac{dI}{I} = -\frac{R}{L}\,dt$$

$$\int_{\mathcal{E}/R}^{I(t)} \frac{dI}{I} = -\int_0^t \frac{R}{L}\,dt \qquad \ln\frac{I(t)}{\mathcal{E}/R} = -\frac{R}{L}t$$

$$I(t) = \frac{\mathcal{E}}{R}\,e^{-\frac{R}{L}t}$$

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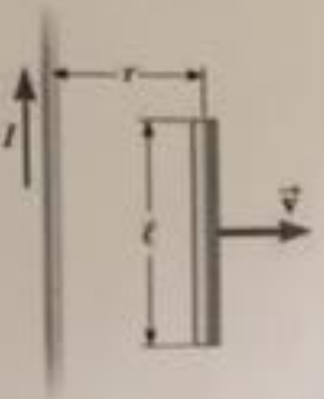
(10 points)

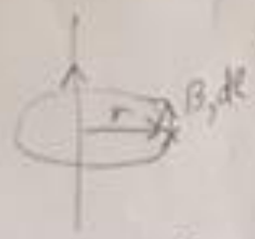
4. The long, straight wire shown in the figure carries constant current I. A metal bar with length l is moving at constant velocity v as shown in the figure. Calculate the emf induced in the bar. Determine the direction of emf.



When the bar moves a distance x, the flux covered is

$$\Phi = B l \, dx$$

where $B = \dfrac{\mu_0 I}{2\pi r}$ (from ampere's law)

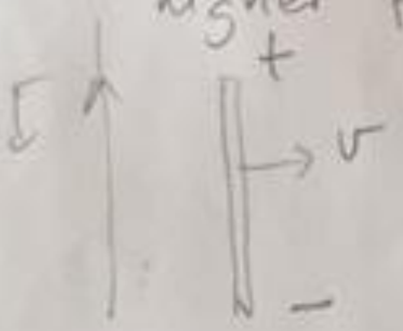


$$\oint B \cdot dl = \mu_0 I \qquad B(2\pi r) = \mu_0 I \qquad B = \frac{\mu_0 I}{2\pi r}$$

$$\mathcal{E} = \left|\frac{d\Phi}{dt}\right| = \left|\frac{d}{dt}\left(\frac{\mu_0 I}{2\pi r} l \, dx\right)\right| = \left|\frac{\mu_0 I}{2\pi r} l \frac{dx}{dt}\right|$$

$$\mathcal{E} = \left|\frac{\mu_0 I}{2\pi r} l v\right|$$

The upper part of the bar will be at a higher potential.



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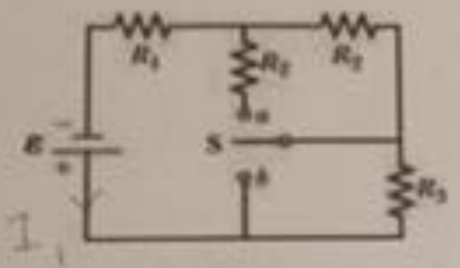
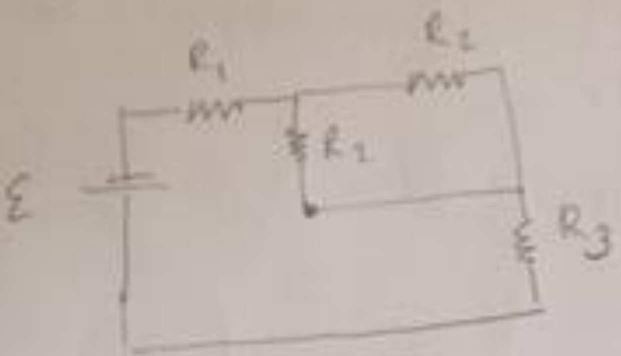
(10 points)

5. A battery of ε and no internal resistance supplies current to the circuit shown in Figure below. When the switch S is open as shown in the figure, the current in the battery is I_1. When the switch is closed in position a, the current in the battery is I_2. When the switch is at position b, the current in the battery is I_3. Determine resistences R_1, R_2, and R_3.

(You need 3 equations for three unkowns! Write the equations and DO NOT SOLVE.)



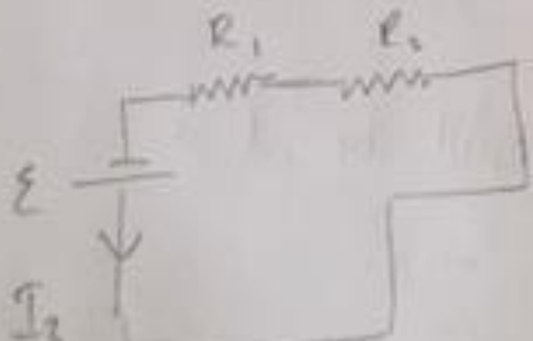
$$\boxed{\varepsilon - I_1 R_3 - I_1 R_2 - I_1 R_1 = 0}$$



R_2 and R_2 are in parallel

$$R_e = \frac{R_2^2}{2R_2} = \frac{R_2}{2}$$

$$\boxed{\varepsilon - I_2 R_1 - I_2 \frac{R_2}{2} - I_2 R_3 = 0}$$

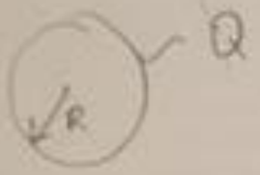


$$\boxed{\varepsilon - I_3 R_2 - I_3 R_1 = 0.}$$

(10 points)

6. A spherical conductor has a radius of R and a charge of Q. Determine the electric field and the electric potential at,

(a) r<R
(b) r>R



a) $r<R$ $\quad E=0 \quad V=\frac{kQ}{R}$

b) $r>R$ $\quad \oint E \cdot dA = \frac{Q}{\varepsilon_0} \qquad E=\frac{Q}{4\pi r^2 \varepsilon_0}$

$$E=\frac{kQ}{r^2}$$

$$\Delta V = -\int_{r_1}^{r_2} \vec{E} \cdot d\ell \ldots - kQ \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$\Delta V = -kQ\left[-\frac{1}{r}\right]_{r_1}^{r_2} = kQ\left[\frac{1}{r_2}-\frac{1}{r_1}\right]$$

$$V_{r_2}-V_{r_1} = -kQ\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

Take r_2 to be infinity at $V(r=\infty)=0$
and let $r_1=r$

$$V(r)=\frac{kQ}{r}$$

So when $r=R$ $\quad V(r)=\frac{kQ}{R}$ and the potential inside the conductor is constant with this value. (in part a)

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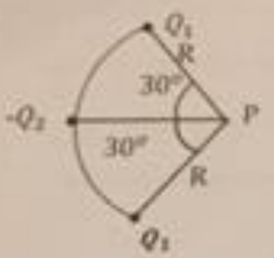
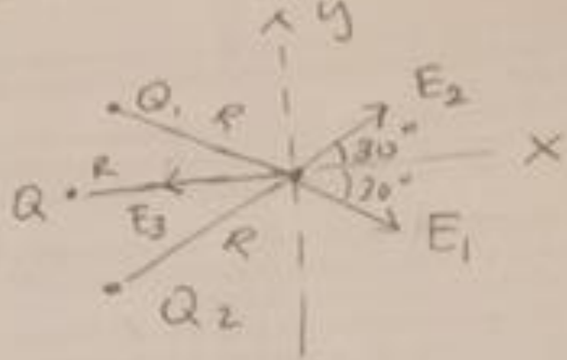
(10 points)

1. Three point charges are located on a circular arc as shown in the figure below. Find the total electric field at point P, the center of the circular arc.





$$E_1 = \frac{kQ_1}{R^2} = E_2 \qquad E_3 = \frac{kQ_2}{R^2}$$

The y components of E_2 and E_1 cancel each other.

$$E_{1x} = E_{2x} = E_1 \cos 30^\circ = \frac{kQ_1}{R^2}\frac{\sqrt{3}}{2}$$

$$\vec{E}_{Total} = (-E_3 + 2E_{1x})\hat{\imath}$$

$$\boxed{\vec{E}_{total} = \left(-\frac{kQ_2}{R^2} + \frac{kQ_1}{R^2}\sqrt{3}\right)\hat{\imath}}$$

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b)

(10 points)

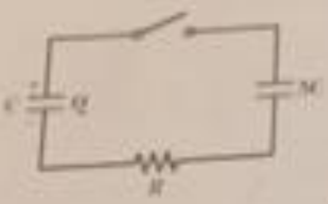
2. A charge Q is placed on a capacitor of capacitance C. The capacitor is connected into the circuit below with an open switch, a resistor and an initially uncharged capacitor of capacitance 3C. The switch is then closed and the circuit comes to equilibrium. Find,

a. the final potential difference between the plates of each capacitor

b. the charge on each capacitor

c. the final energy in each capacitor

d. the energy dissipated on the resistor



after the switch is closed and the circuit comes to equilibrium the current in the circuit is 0. and the potential across each capacitor is the same V.

$$V = \frac{Q_1}{C} = \frac{Q_2}{3C} \qquad Q_1 + Q_2 = Q \qquad Q_2 = Q - Q_1$$

$$3Q_1 = Q - Q_1$$

b) $\boxed{Q_1 = \frac{Q}{4} \qquad Q_2 = \frac{3Q}{4}}$

a) $\boxed{V = \frac{Q}{4C}}$

c) $u_1 = \frac{Q^2/16}{2C} = \boxed{\frac{Q^2}{32C}} \qquad u_2 = \frac{9Q^2/16}{2(3C)} = \boxed{\frac{3Q^2}{32C}}$

d) Total energy stored $u_1 + u_2 = \frac{4Q^2}{32C} = \frac{Q^2}{8C} \qquad u_{initial} = \frac{Q^2}{2C}$

the difference is the energy dissipated on R : $\Delta u = \frac{Q^2}{2C} - \frac{Q^2}{8C} = \boxed{\frac{3Q^2}{8C}}$

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(10 points)

3. A proton (q) moves with a velocity of $\vec{v} = (2\hat{i} - 4\hat{j} + \hat{k})$ m/s in a region in which the magnetic field is $\vec{B} = (\hat{i} + 2\hat{j} - \hat{k})$ tesla. What is the magnitude of the magnetic force this particle experiences?

$$\vec{F} = q\vec{v} \times \vec{B} \qquad q = 1.6 \times 10^{-19}\,C$$

$$\vec{v} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & -4 & 1 \\ 1 & 2 & -1 \end{vmatrix} = i(4-2) - j(-2-1) + k(4-(-4))$$

$$= 2\hat{i} + 3\hat{j} + 8\hat{k}$$

$$\boxed{\vec{F} = (1.6 \times 10^{-19})(2\hat{i} + 3\hat{j} + 8\hat{k})}$$

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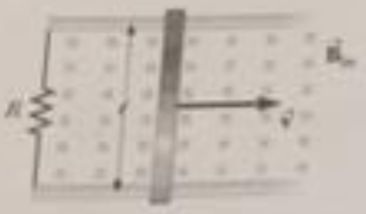
(10 points)

4. A conducting bar of length ℓ moves on two horizontal frictionless rails as shown in the figure below. A constant force of 1 N moves the bar at a constant speed of 2 m/s to the right through a magnetic field $\vec{B}$ that is directed into the page.

a. What is the current through the 8 Ω resister R?

b. What is the direction of the current?



a) $\mathcal{E} = B\ell v$ $\quad \mathcal{E} = iR$

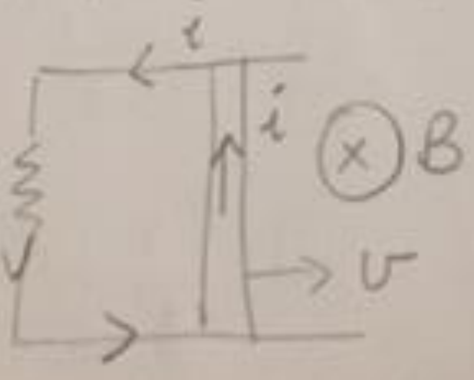
$$i = \frac{B\ell v}{R}$$

$$F = i\ell B \qquad B = \frac{F}{i\ell}$$

$$i = \frac{F}{i\ell}\frac{\ell v}{R} = \frac{Fv}{iR} \qquad i = \sqrt{\frac{Fv}{R}}$$

$$i = \sqrt{\frac{(1)(2)}{8}} = \sqrt{\frac{1}{4}} = \frac{1}{2} = 0.5\ \text{A}.$$

b) Φ is incresing, so induced i in the direction to create an induced B outwards. So i in the counterclockwise direction.



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(10 points)

5. A circular loop of radius R carries a current I. Determine the direction and the magnitude of the magnetic field at the center of the circle. Show your work!

$$d\vec{B} = \frac{\mu_0 I \, d\vec{s} \times \hat{r}}{4\pi r^2}$$

$r = R$ constant, ds is $\perp$ to R

$$dB = \frac{\mu_0 I \, ds}{4\pi R^2}$$

$$B = \frac{\mu_0 I}{4\pi R^2} \oint ds = \frac{\mu_0 I}{\cancel{4}_{2}\cancel{\pi} R^{\cancel{2}}} \cancel{2\pi} \cancel{R}$$

$$B = \frac{\mu_0 I}{2R} \quad \otimes \quad \text{for clockwise } I.$$

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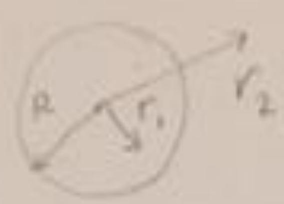
(10 points)

6. A long, cylindrical conductor of radius R carries a current I as shown in the figure. The current density is uniform. Find an expression for the magnetic field at r_1 and r_2.

front view



Use ampere's Law : $\oint \vec{B} \cdot \vec{ds} = \mu_0 I_{in}$

a) Draw an ampere loop at r_2

B is parallel to ds (chosen)

$\oint \vec{B} \cdot \vec{ds} = \oint B\,ds$

B should be constant on the ampere loop :

$B \oint ds = B(2\pi r_2) = \mu_0 I$ $\quad \boxed{B = \frac{\mu_0 I}{2\pi r_2}}$

B's direction is tangent to the counterclockwise circle.

Similarly $(B)(2\pi r_1) = \mu_0 I_i$

$\frac{I}{\pi R^2} = \frac{I_i}{\pi r_1^2}$ $\quad I_{in} = \frac{r_1^2}{R^2} I$

$B = \frac{\mu_0}{2\pi \cancel{r_1}} \frac{r_1^{\cancel{2}}}{R^2} I$ $\quad \boxed{B = \frac{\mu_0 I}{2\pi R^2} r_1}$ again, it is tangent to the counterclockwise circle.