Math

No:

Full Name :

Math 104 3rd Midterm Exam
(23 May 2018, 17:30-18:30)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones, smart watches and electronic devices are to be kept shut and out of sight. All cell phones and smart watches are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
20 pts	20 pts	40 pts	20 pts	100 pts

Q1. Determine whether the given sequence converges or diverges. If it converges, find the limit:

$\left\{\frac{\pi^n}{1+2^{2n}}\right\}$ ∞/∞

$$\lim_{n\to\infty} \frac{\pi^n}{1+2^{2n}} = \lim_{x\to\infty} \frac{\pi^x}{1+4^x}$$ using a continuous variable

$$\underset{\text{L'Hospital}}{=} \lim_{x\to\infty} \frac{\pi^x \ln \pi}{4^x \ln 4}$$

$$= \frac{\ln \pi}{\ln 4} \underbrace{\lim_{x\to\infty} \left(\frac{\pi}{4}\right)^x}_{0 \text{ because } \frac{\pi}{4}<1}$$

$\therefore$ The sequence $\boxed{\text{converges to } 0}$.

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Q2. Evaluate the following integral:

$$\int \frac{dx}{x(x-1)^2}$$

Partial fractions:

$$\frac{1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$1 = A(x-1)^2 + Bx(x-1) + Cx$$

$x=0$ $\quad 1 = A$

$x=1$ $\quad 1 = C$

$x=-1$ $\quad 1 = 4A + 2B - C$

$$\Downarrow$$

$$4 + 2B - 1 = 1 \Rightarrow B = -1$$

$$\therefore \int \frac{dx}{x(x-1)^2} = \int \frac{dx}{x} - \int \frac{dx}{x-1} + \int \frac{dx}{(x-1)^2}$$

$t = x-1$

$dt = dx$

$$= \ln|x| - \ln|x-1| + \int t^{-2}\,dt$$

$$= \ln\left|\frac{x}{x-1}\right| + \frac{t^{-1}}{-1} + C$$

$$= \boxed{\ln\left|\frac{x}{x-1}\right| - \frac{1}{x-1} + C}$$

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Q3. Determine whether the following series converge or diverge:

a) $\sum_{n=1}^{\infty} \frac{4^{n+1}}{n\,5^{n-1}}$

Ratio Test:

$$\rho = \lim_{n\to\infty} \frac{\frac{4^{n+2}}{(n+1)5^n}}{\frac{4^{n+1}}{n5^{n-1}}} = \lim_{n\to\infty} \frac{n}{n+1} \cdot 4 \cdot \frac{1}{5}$$

$$= \frac{4}{5} \underbrace{\lim_{n\to\infty} \frac{n}{n+1}}_{1} = \frac{4}{5} < 1$$

$\therefore$ The series converges

(There are other ways to solve this question)

b) $\sum_{n=1}^{\infty} \frac{1}{n(1+ln^2 n)}$

Integral Test

$$\int \frac{dx}{x(1+\ln^2 x)} = \int \frac{du}{1+u^2} = \operatorname{Arctan} u + c$$

$\ln x = u$

$\frac{dx}{x} = du$

$$= \operatorname{Arctan}(\ln x) + c$$

$$\therefore \int_1^{\infty} \frac{dx}{x(1+\ln^2 x)} = \lim_{b\to\infty} \operatorname{Arctan}(\ln b) - \operatorname{Arctan}(\ln 1)$$

$$= \operatorname{Arctan} \infty - \operatorname{Arctan} 0$$

$$= \frac{\pi}{2}$$

$\therefore$ The series converges.

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Q4. Determine whether the following series converges absolutely, converges conditionally, or diverges:

$$\sum_{n=1}^{\infty} \frac{(-1)^n cos2n}{n^2}$$

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n \cos 2n}{n^2} \right| = \sum_{n=1}^{\infty} \frac{|\cos 2n|}{n^2}$$

$$|\cos 2n| \leq 1$$

$$\therefore \quad \frac{|\cos 2n|}{n^2} \leq \frac{1}{n^2}$$

$\sum \frac{1}{n^2}$ is a p-series, $p = 2 > 1$,

so it converges

$\therefore \quad \sum \left| \frac{(-1)^n \cos 2n}{n^2} \right|$ converges by the Comparison Test

$\therefore$ The series $\sum \frac{(-1)^n \cos 2n}{n^2}$

converges absolutely.