On my honor, I have neither given nor received any unauthorized assistance on this examination. The work done on this exam is totally my own. I understand that by the school code, violation of these principles will lead to a zero grade and is subject to harsh discipline issues.

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1) a) Say $A = \{1,2\}$, $B = \{2,3\}$, $C = \{1,2,3\}$

$A \cap B = A \cap C$ Say $x \in B$ $x \in C$ $x \in A \cup B$ so $x \in A$

$A \cup B = A \cup C$ $x \in A \cap B$ so $\boxed{B \subseteq C,\ C \subseteq B \quad B = C}$

b) $A \cap B = A \cap C$

Say $B = \{1,2\}$, $C = \{1,3\}$, $A = \{1\}$ It doesn't make $B = C$

c) $f(g(x)) = g(f(x))$

Say $f(x) = 2x + 8$ $f(g(x)) = 6x + 8$ $6x+8 \neq 6x+24$

$g(x) = 3x$ $g(f(x)) = 3(2x+8) = 6x + 24$ not equal

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2) a) $f(m,n) = 2^m \cdot 3^n$ $2^m \cdot 3^n = y$, say $n=0$ $2^m = y$ $m = \log_2 y$

$f(\log_2 y, 0) = 2^{\log_2 y} \cdot 1 = y$ thus this func is not onto

$2^m \cdot 3^n$ does not give "0" so it's not one-to-one

b)

3) a) True b) false c) True d) False e) True

f) false g) False h) false i) True j) false

4)

| Step | Reason |
|---|---|
| 1. $\neg r \vee s$ | premise |
| 2. $\neg q \vee \neg s$ | premise |
| 3. $\neg r \vee \neg q$ | resolution 1 and 2 |
| 4. $\neg (q \wedge r)$ | de morgan's law 3 |
| 5. $p \to (q \wedge r)$ | premise |
| 6. $\neg p$ | Modus tollens 4 and 5 |

5)

| p | q | r | $((\neg p \wedge (\neg q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r)))$ |
|---|---|---|---|
| F | F | F | F |
| F | F | T | T |
| F | T | F | F |
| F | T | T | T |
| T | F | F | F |
| T | F | T | T |
| T | T | F | F |
| T | T | T | T |

result is r because values (results) are the same as you can see in truth table

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6) find inverse

$7x \equiv 13 \pmod{19}$

say $19 = 7.2 + 5$

$7 = 5.1 + 2$

$5 = 2.2 + 1 \rightarrow gcd(19,7) = 1$

$2 = 1.2 + 0$

$= 4.19 + 7(-4)$

we will multiply by this

$(-4).\ 7x \equiv 13.(-4) \pmod{19}$

therefore

$x \equiv 5 \bmod (19)$

7) lets say $5n + 3 = 7n + 4 = x$

$x \equiv 3 \pmod 5$

$x \equiv 4 \pmod 7$

they are relatively prime. we'll use chinese theorem.

$M_1 = M_2.1 = 7$ $M_2 = M_1.1 = 5$

$7 \equiv 2 \pmod 5 \rightarrow z.2 = 1 \pmod 5$ $z = 3$

$5 \equiv 5 \pmod 7 \rightarrow y.5 = 1 \pmod 7$ $y = 3$

$x = 3.M_1.z + 4.M_2.y = 3.7.3 + 4.3.5 = 123 \pmod{(M_1.M_2)}$ 35

$x = 123 \pmod{35}$ $123 - 35.3 = 18$ $x = 18 \pmod{35}$

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8) $x = 2 \pmod 4$

$x = 1 \pmod 5$ We will use chinese remainder

$x = 3 \pmod 7$ theorem again

$x = 2 \pmod 3$

$M_1 = M_2 . M_3 . M_4 = 5.7.3 = 105$

$M_2 = M_1 . M_3 . M_4 = 4.7.3 = 84$

$M_3 = M_1 . M_2 . M_4 = 4.5.3 = 60$

$M_4 = M_1 . M_2 . M_3 = 4.5.7 = 140$

$y_1 \rightarrow$ $105 \equiv 2 \pmod 4$ $y_1 . 1 \equiv 1 \pmod 4$ $y_1 = 1$

$y_2 \rightarrow$ $84 \equiv 1 \pmod 5$ $y_2 . 4 \equiv 1 \pmod 4$ $y_2 = 4$

$y_3 \rightarrow$ $60 \equiv 3 \pmod 7$ $y_3 . 4 \equiv 1 \pmod 7$ $y_3 = 2$

$y_4 \rightarrow$ $140 \equiv 2 \pmod 3$ $y_4 . 2 \equiv 1 \pmod 3$ $y_4 = 2$

$x = M_1 . y_1 . a_1 + M_2 . y_2 . a_2 + M_3 . y_3 . a_3 + M_4 . y_3 . a_3$

$= 105 . 1 . 2 + 84 . 4 . 1 + 60 . 2 . 3 + 140 . 2 . 2$

$= 1466 \pmod{420}$

$\boxed{x \equiv 206 \pmod{420}}$