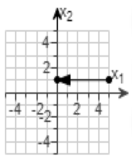
1.8.15

Use a rectangular coordinate system to plot $\mathbf{u} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, and their images under the given transformation T. Describe geometrically what T does to each vector $\mathbf{x}$ in $\mathbb{R}^2$.

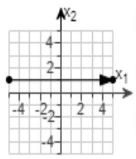
$$T(\mathbf{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Which graph below shows **u** and its image under the given transformation?

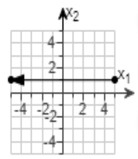
A.



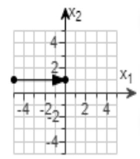
B.



C.

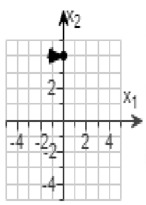


D.

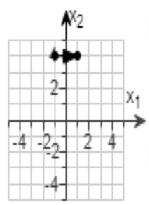


Which graph below shows **v** and its image under the given transformation?

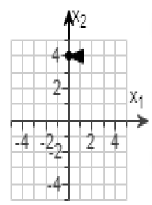
A.



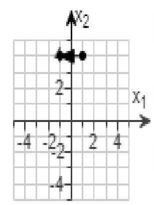
B.



C.



D.



What does T do geometrically to each vector **x** in $\mathbb{R}^2$?

- A. A shear transformation
- B. A rotation over the x-axis
- C. A projection onto the y-axis
- D. A reflection through the origin

Score: 1 of 1 pt

Test Score

1.8.5

If T is defined by $T(\mathbf{x}) = A\mathbf{x}$, find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$, and determine whether $\mathbf{x}$ is unique. Let $A = \begin{bmatrix} 1 & -6 & -9 \\ -4 & 15 & 18 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$.

Find a single vector $\mathbf{x}$ whose image under T is $\mathbf{b}$.

$\mathbf{x} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

Is the vector $\mathbf{x}$ found in the previous step unique?

- A. Yes, because there are no free variables in the system of equations.
- B. No, because there are no free variables in the system of equations.
- C. No, because there is a free variable in the system of equations.
- D. Yes, because there is a free variable in the system of equations.

Score: 1 of 1 pt

1.7.14

Find the value(s) of h for which the vectors are linearly dependent. Justify your answer.

$\begin{bmatrix} 1 \\ -3 \\ -6 \end{bmatrix}, \begin{bmatrix} -4 \\ 13 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ h \end{bmatrix}$

The value(s) of h which makes the vectors linearly dependent is(are) -178 because this will cause x_3 to be a free variable.
(Use a comma to separate answers as needed.)

Score: 1 of 1 pt

1.6.9

Balance the chemical equation.

$MnO_2 + K_2CO_3 + KNO_3 \rightarrow K_2MnO_4 + KNO_2 + CO_2$

Assume the coefficient of CO_2 is 1. What is the balanced equation?

$1\,MnO_2 + 1\,K_2CO_3 + 1\,KNO_3 \rightarrow 1\,K_2MnO_4 + 1\,KNO_2 + 1\,CO_2$

Score: 1 of 1 pt | 5 of 9 | Test Score:

1.9.19

Show that T is a linear transformation by finding a matrix that implements the mapping. Note that $x_1, x_2, \ldots$ are not vectors but are entries in vectors.

$$T(x_1,x_2,x_3) = (x_1 - 3x_2 + 6x_3,\ x_2 - 9x_3)$$

$A = \begin{bmatrix} 1 & -3 & 6 \\ 0 & 1 & -9 \end{bmatrix}$ (Type an integer or decimal for each matrix element.)

Score: 1 of 1 pt | 6 of 9 | Test Score: 100%, 9

1.8.36

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation that projects each vector $\mathbf{x} = (x_1, x_2, x_3)$ onto the plane $x_2 = 0$, so $T(\mathbf{x}) = (x_1, 0, x_3)$. Show that T is a linear transformation.

The first property for T to be linear is $T(\mathbf{0}) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. (Type a column vector.)

Check if this property is satisfied for T.

$$T(x_1, x_2, x_3) = (x_1, 0, x_3)$$
$$T(0,0,0) = (0, 0, 0)$$

So, is the first property satisfied?

- Yes
- No

The second property for T to be linear is $T(c\mathbf{u}+d\mathbf{v}) = cT(\mathbf{u}) + dT(\mathbf{v})$ for all vectors **u**, **v** in the domain of T and all scalars c, d.

Check if this property is satisfied for T. Let $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$.

$$T(c\mathbf{u} + d\mathbf{v}) = (cu_1 + dv_1, 0, cu_3 + dv_3)$$
$$= (cu_1, 0, cu_3) + (dv_1, 0, dv_3)$$

Factor out the scalar in each ordered triple.

$$T(c\mathbf{u} + d\mathbf{v}) = c(u_1, 0, u_3) + d(v_1, 0, v_3)$$

Further simplify the previous equation.

$$T(c\mathbf{u} + d\mathbf{v}) = c\,T(\mathbf{u}) + d\,T(\mathbf{v})$$

So, is the second property satisfied?

- No
- Yes

Thus, T is linear.

1.5.7

Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form, where A is row equivalent to the given matrix.

$$\begin{bmatrix} 1 & 4 & -3 & 9 \\ 0 & 1 & -2 & 8 \end{bmatrix}$$

$$\mathbf{x} = x_3 \begin{bmatrix} -5 \\ 2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 23 \\ -8 \\ 0 \\ 1 \end{bmatrix}$$

(Type an integer or fraction for each matrix element.)

1.8.19

Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{y}_1 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, and $\mathbf{y}_2 = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$, and let T: $\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{e}_1$ into $\mathbf{y}_1$ and maps $\mathbf{e}_2$ into $\mathbf{y}_2$. Find the images of $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

Which is the correct image of $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$?

A. $\begin{bmatrix} 7 \\ 18 \end{bmatrix}$

B. $\begin{bmatrix} 18 \\ 7 \end{bmatrix}$ ✔

C. $\begin{bmatrix} 7 \\ -18 \end{bmatrix}$

D. $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$

Which is the correct image of $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$?

A. $\begin{bmatrix} 3x_1 - x_2 \\ 5x_1 + 6x_2 \end{bmatrix}$ ✔

B. $\begin{bmatrix} 5x_1 - 6x_2 \\ 3x_1 + x_2 \end{bmatrix}$

C. $\begin{bmatrix} 5x_1 - x_2 \\ 3x_1 + 6x_2 \end{bmatrix}$

D. $\begin{bmatrix} 3x_1 + x_2 \\ 5x_1 - 6x_2 \end{bmatrix}$

1.7.32

Given $A = \begin{bmatrix} 1 & 11 & 5 \\ -7 & -19 & -6 \\ -4 & -18 & -7 \end{bmatrix}$, observe that the first column plus twice the third column equals the second column. Find a nontrivial solution of $A\mathbf{x} = 0$ without performing row operations. [Hint: Write $A\mathbf{x} = \mathbf{0}$ as a vector equation.]

$$\mathbf{x} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}$$