Istanbul Şehir University

Math 104

Date: 6 November 2013	Full Name:
Time: 09:00-10:15	
	Student ID:
Fall 2013 Midterm 1	

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 5 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. You may continue your solutions on the back of the sheets. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cellphones and electronic devices are to be kept shut and out of sight.

Q1	Q2	Q3	Q4	Q5	TOTAL
20 pts	10 pts	30 pts	20 pts	20 pts	100 pts

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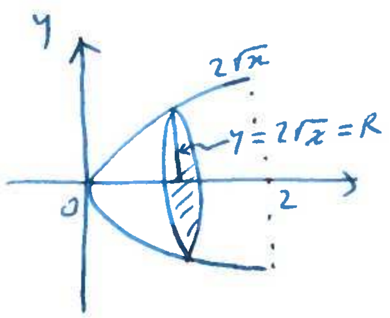
1) The region between the curve $y = 2\sqrt{x}$, $0 \le x \le 2$, and the x-axis is revolved about the x-axis to generate a solid. Using the disk method, find its volume.



$$V = \int_0^2 A(y)\,dx$$

$$V = \int \pi R(y)\,dx$$

$$= \int \pi y^2\,dx$$

$$= \int \pi (2\sqrt{x})^2\,dx$$

$$= 4\pi \int_0^2 x\,dx$$

$$= 4\pi \frac{x^2}{2}\Big|_0^2$$

$$= 8\pi$$

2) Use logarithmic differentiation to find the derivative of y with respect to the given independent variable.

$$y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$$

$$\ln y = \ln x + \frac{1}{2}\ln(x^2+1) - \frac{2}{3}\ln(x+1)$$

$$\frac{y'}{y} = \frac{1}{x} + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}x}{x^2+1} - \frac{2}{3} \cdot \frac{1}{x+1}$$

$$y' = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}} \left[\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)} \right]$$

3) Evaluate the integrals

(a) $\int_0^{\pi/3} \frac{4\sin\theta}{1-4\cos\theta} d\theta =$

$u = 1 - 4\cos\theta$ | $u(0) = -3$

$du = 4\sin\theta \, d\theta$ | $u(\pi/3) = -1$

$\int \frac{du}{u} = \ln|u| \Big|_{u(0)}^{u(\pi/3)}$

$= -\ln 3$ //

(b) $\int_2^{16} \frac{dx}{2x\sqrt{\ln x}} =$

$u = \ln x$; $du = \frac{dx}{x}$ | $u(2) = \ln 2$

$u(16) = \ln 16 = \ln 2^4 = 4\ln 2$

$\frac{1}{2}\int u^{-\frac{1}{2}} du =$

$\cancel{\frac{1}{2}} \frac{\sqrt{u}}{\cancel{1/2}} \Big|_{u(2)}^{u(16)} = \sqrt{4\ln 2} - \sqrt{\ln 2} = \sqrt{\ln 2}$ //

(c) $\int_1^4 \frac{\ln 2 \log_2 x}{x} dx = \int_1^4 \frac{\cancel{\ln 2}}{x} \frac{\ln x}{\cancel{\ln 2}} dx$

$= \int \frac{\ln x}{x} dx$

$u = \ln x$, $du = \frac{dx}{x}$

$u(1) = \ln 1 = 0$

$u(4) = \ln 2^2 = 2\ln 2$

$= \int u \, du$

$= \frac{u^2}{2} \Big|_{u(1)}^{u(4)}$

$= \frac{1}{2}(2\ln 2)^2$

$= 2(\ln 2)^2$ //

4) Evaluate the integral $\int (x^2 - 2x + 1)e^{2x}\,dx$

$x^2 - 2x + 1$		e^{2x}
		e^{2x}
$x^2 - 2x + 1$	$+$	$\frac{1}{2}e^{2x}$
$2x - 2$	$-$	$\frac{1}{4}e^{2x}$
2	$+$	$\frac{1}{8}e^{2x}$
0		

$$\int (x^2 - 2x + 1)e^{2x}\,dx = (x^2 - 2x + 1)\cdot\frac{1}{2}e^{2x} - (2x - 2)\frac{1}{4}e^{2x} + \frac{1}{16}e^{2x} + C$$

$$= \cancel{(8x^2 - 4x + 4 - 8x + 8 + 16)e^{2x} + C}$$

$$= \cancel{(12x^2 - 12x + 28)e^{2x} + C}$$

5) Evaluate the integrals

(a) $\int 8\cos^3 2x \sin 2x \, dx =$

$u = \cos 2x$
$du = 2\sin 2x \, dx$

$-4\int u^3 \, du =$

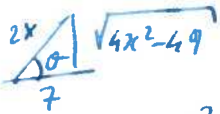
$\cancel{-4\frac{u^4}{4}} + C =$

$-u^4 + C =$

$-(\cos 2x)^4 + C$ //

(b) $\int \frac{dx}{\sqrt{4x^2-49}}, \; x > \frac{7}{2}$

$\frac{2}{7}x = \sec\theta$



$\sec\theta = \frac{2x}{7}$

$x = \frac{7}{2}\sec\theta$

$\tan\theta = \frac{\sqrt{4x^2-49}}{7}$

$dx = \frac{7}{2}\sec\theta\tan\theta \, d\theta$

$= \frac{1}{7}\int \frac{dx}{\sqrt{(\frac{2}{7}x)^2 - 1}}$

$= \cancel{\frac{1}{7}} \cdot \frac{\cancel{7}}{2}\int \frac{\sec\theta \cancel{\tan\theta} \, d\theta}{\underbrace{\sqrt{\sec^2\theta - 1}}_{\cancel{\tan\theta}}} = \frac{1}{2}\int \sec\theta \, d\theta = \frac{1}{2}\int \frac{\sec\theta(\sec\theta + \tan\theta) \, d\theta}{\sec\theta + \tan\theta}$

$u = \sec\theta + \tan\theta$
$du = (\sec\theta\tan\theta + \sec^2\theta) \, d\theta$

$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

$= \frac{1}{2}\ln\left|\frac{2x - \sqrt{4x^2-49}}{7}\right| + C$