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1. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 6 & 5 \\ -4 & -5 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is invertible because its determinant is not zero. ✔
- **B.** The matrix is invertible because its columns are multiples of each other. The columns of the matrix form a linearly dependent set.
- **C.** The matrix is not invertible because the matrix has 2 pivot positions.
- **D.** The matrix is not invertible because its determinant is zero.

2. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 5 & 0 & 0 \\ -2 & -4 & 0 \\ 6 & 3 & -1 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is not invertible. The given matrix has two pivot positions.
- **B.** The matrix is invertible. If the given matrix is A, there is a 3×3 matrix C such that CI = A.
- **C.** The matrix is invertible. The given matrix has three pivot positions. ✔
- **D.** The matrix is not invertible. If the given matrix is A, the equation A**x** = **b** has no solution for some **b** in $\mathbb{R}^3$.

3. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 3 & 0 & -5 \\ 2 & 0 & 3 \\ -5 & 0 & 5 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is not invertible. If the given matrix is A, the columns of A do not form a linearly independent set. ✔
- **B.** The matrix is invertible. The columns of the given matrix span $\mathbb{R}^3$.
- **C.** The matrix is not invertible. If the given matrix is A, the equation A**x** = **0** has only the trivial solution.
- **D.** The matrix is invertible. The given matrix has 2 pivot positions.

4. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 0 & 1 & -4 \\ 1 & 0 & 2 \\ -3 & -2 & 2 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is invertible. The given matrix has 3 pivot positions.
- **B.** The matrix is not invertible. If the given matrix is A, the equation A**x** = **b** has at least one solution for each **b** in $\mathbb{R}^3$.
- **C.** The matrix is not invertible. If the given matrix is A, A is not row equivalent to the $n \times n$ identity matrix.
- **D.** The matrix is invertible. The columns of the given matrix span are linearly dependent.

5. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} -1 & -3 & 0 & 1 \\ 4 & 7 & 10 & -4 \\ -4 & -12 & 4 & 4 \\ 0 & -1 & 2 & 1 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is not invertible. In the given matrix the columns do not form a linearly independent set.
- **B.** The matrix is invertible. The given matrix is not row equivalent to the $n \times n$ identity matrix.
- **C.** The matrix is invertible. The given matrix has 4 pivot positions.
- **D.** The matrix is not invertible. If the given matrix is A, the equation A**x** = **b** has no solution for at least one **b** in $\mathbb{R}^4$.

6. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 3 & 4 & 7 & 4 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is not invertible. If the given matrix is A, the equation A**x** = **b** has no solution for at least one **b** in $\mathbb{R}^4$.
- **B.** The matrix is invertible. The given matrix is not row equivalent to the $n \times n$ identity matrix.
- **C.** The matrix is not invertible. In the given matrix the columns do not form a linearly independent set.
- **D.** The matrix is invertible. The given matrix has 4 pivot positions.

7. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 4 & 0 & -3 & -7 \\ -6 & 8 & 8 & 8 \\ 8 & -4 & 11 & 18 \\ -1 & 2 & 4 & -1 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is invertible. The given matrix has 4 pivot positions.
- **B.** The matrix is invertible. The given matrix is not row equivalent to the $n \times n$ identity matrix.
- **C.** The matrix is not invertible. In the given matrix the columns do not form a linearly independent set.
- **D.** The matrix is not invertible. If the given matrix is A, the equation $A\mathbf{x} = \mathbf{b}$ has no solution for at least one $\mathbf{b}$ in $\mathbb{R}^4$.

8. An $m \times n$ upper triangular matrix is one whose entries below the main diagonal are zeros, as is shown in the matrix to the right. When is a square upper triangular matrix invertible? Justify your answer.

$$\begin{bmatrix} 3 & 4 & 7 & 4 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Choose the correct answer below.

- **A.** A square upper triangular matrix is invertible when all entries above the main diagonal are zeros as well. This means that the matrix is row equivalent to the $n \times n$ identity matrix.
- **B.** A square upper triangular matrix is invertible when all entries on its main diagonal are nonzero. If all of the entries on its main diagonal are nonzero, then the $n \times n$ matrix has n pivot positions.
- **C.** A square upper triangular matrix is invertible when the matrix is equal to its own transpose. For such a matrix A, $A = A^T$ means that the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$.
- **D.** A square upper triangular matrix is invertible when all entries on the main diagonal are ones. If any entry on the main diagonal is not one, then the equation $A\mathbf{x} = \mathbf{b}$, where A is an $n \times n$ square upper triangular matrix, has no solution for at least one $\mathbf{b}$ in $\mathbb{R}^n$.

9. Can a square matrix with two identical columns be invertible? Why or why not?

Select the correct choice below.

- **A.** It depends on the values in the matrix. According to the Invertible Matrix Theorem, if the two columns are larger than any other columns the matrix will be invertible, otherwise it will not.
- **B.** The matrix is not invertible. According to the Invertible Matrix Theorem a square matrix can never be invertible.
- **C.** The matrix is invertible. If a matrix has two identical columns then its columns are linearly independent. According to the Invertible Matrix Theorem this makes the matrix invertible.
- **D.** The matrix is not invertible. If a matrix has two identical columns then its columns are linearly dependent. According to the Invertible Matrix Theorem this makes the matrix not invertible.

10. Is it possible for a 5×5 matrix to be invertible when its columns do not span $\mathbb{R}^5$? Why or why not?

Select the correct choice below.

- **A.** It is not possible; according to the Invertible Matrix Theorem an $n \times n$ matrix cannot be invertible when its columns do not span $\mathbb{R}^n$.
- **B.** It will depend on the values in the matrix. According to the Invertible Matrix Theorem, a square matrix is only invertible if it is row equivalent to the identity.
- **C.** It is possible; according to the Invertible Matrix Theorem all square matrices are always invertible.
- **D.** It is possible; according to the Invertible Matrix Theorem an $n \times n$ matrix can be invertible when its columns do not span $\mathbb{R}^n$.

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11. If the columns of a 7×7 matrix D are linearly independent, what can you say about the solutions of $D\mathbf{x} = \mathbf{b}$? Why?

Select the correct choice below.

- **A.** Equation $D\mathbf{x} = \mathbf{b}$ has a solution for each $\mathbf{b}$ in $\mathbb{R}^7$. According to the Invertible Matrix Theorem, a matrix is invertible if the columns of the matrix form a linearly independent set; this would mean that the equation $D\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$.
- **B.** Equation $D\mathbf{x} = \mathbf{b}$ has no solutions for each $\mathbf{b}$ in $\mathbb{R}^7$. According to the Invertible Matrix Theorem, the equation $D\mathbf{x} = \mathbf{0}$ has only the trivial solution.
- **C.** Equation $D\mathbf{x} = \mathbf{b}$ has many solutions for each $\mathbf{b}$ in $\mathbb{R}^7$. According to the Invertible Matrix Theorem, a matrix is not invertible if the columns of the matrix form a linearly independent set, and the equation $D\mathbf{x} = \mathbf{b}$ has many solutions for each $\mathbf{b}$ in $\mathbb{R}^n$.
- **D.** It will depend on the values in the matrix. If the diagonal of the matrix is zero, $D\mathbf{x} = \mathbf{b}$ has a solution for each $\mathbf{b}$ in $\mathbb{R}^7$. However, if the diagonal is all non-zero, equation $D\mathbf{x} = \mathbf{b}$ has many solutions for each $\mathbf{b}$ in $\mathbb{R}^7$.

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12. Suppose H is an $n \times n$ matrix. If the equation $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ in $\mathbb{R}^n$, what can you say about the equation $H\mathbf{x} = \mathbf{0}$? Why?

Select the correct choice below.

- **A.** The statement that $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ is equivalent to the statement that $H\mathbf{x} = \mathbf{c}$ has no solution for some $\mathbf{c}$. From this, all of the statements in the Invertible Matrix Theorem are false, including the statement that $H\mathbf{x} = \mathbf{0}$ has only the trivial solution. Thus, $H\mathbf{x} = \mathbf{0}$ has no solution.
- **B.** The statement that $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ is equivalent to the statement that $H\mathbf{x} = \mathbf{c}$ has a solution for every $\mathbf{c}$. From this, all of the statements in the Invertible Matrix Theorem are true, including the statement that the columns of H form a linearly independent set. Thus, $H\mathbf{x} = \mathbf{0}$ has an infinite number of solutions.
- **C.** The statement that $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ is equivalent to the statement that $H\mathbf{x} = \mathbf{c}$ has no solution for some $\mathbf{c}$. From this, all of the statements in the Invertible Matrix Theorem are false, including the statement that $H\mathbf{x} = \mathbf{0}$ has only the trivial solution. Thus, $H\mathbf{x} = \mathbf{0}$ has a nontrivial solution.
- **D.** The statement that $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ is equivalent to the statement that $H\mathbf{x} = \mathbf{c}$ has a solution for every $\mathbf{c}$. From this, all of the statements in the Invertible Matrix Theorem are true, including the statement that $H\mathbf{x} = \mathbf{0}$ has only the trivial solution.

13. If an $n \times n$ matrix K cannot be row reduced to I_n, what can you say about the columns of K? Why?

Select the correct choice below.

- A. The columns of K are linearly dependent and the columns span $\mathbb{R}^n$. According to the Invertible Matrix Theorem, if a matrix cannot be row reduced to I_n, the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.
- B. The columns of K are linearly independent and the columns span $\mathbb{R}^n$. According to the Invertible Matrix Theorem, if a matrix cannot be row reduced to I_n that matrix is invertible.
- C. The columns of K are linearly independent and the columns span $\mathbb{R}^n$. According to the Invertible Matrix Theorem, if a matrix cannot be row reduced to I_n, the equation $Ax = b$ has at least one solution for each b in $\mathbb{R}^n$.
- ✔ D. The columns of K are linearly dependent and the columns do not span $\mathbb{R}^n$. According to the Invertible Matrix Theorem, if a matrix cannot be row reduced to I_n that matrix is non invertible.

14. The given T is a linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$. Show that T is invertible and find a formula for T^{-1}.

$$T(x_1,x_2) = (5x_1 - 8x_2, -5x_1 + 6x_2)$$

To show that T is invertible, calculate the determinant of the standard matrix for T. The determinant of the standard matrix is -10.

(Simplify your answer.)

$$T^{-1}(x_1,x_2) = \left(-\frac{3}{5}x_1 + -\frac{4}{5}x_2, -\frac{1}{2}x_1 - \frac{1}{2}x_2\right)$$

(Type an ordered pair. Type an expression using x_1 and x_2 as the variables.)