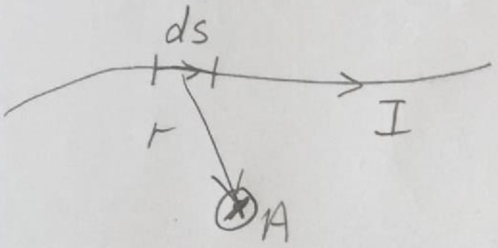
Chapter 30 - Sources of Magnetic Field

Magnetic Field (B) created by a current carrying wire is given by the BIOT-SAVART LAW:



The segment $\vec{ds}$ creates a magnetic field $\vec{dB}$ at point A:

$$\boxed{\vec{dB} = \frac{\mu_0 I \, \vec{ds} \times \hat{r}}{4\pi \, r^2}}$$

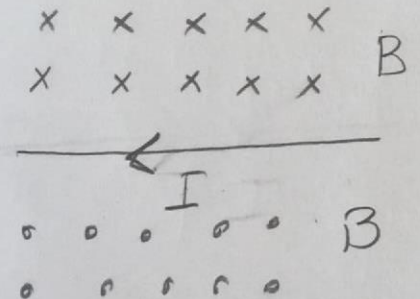
where μ_0 is the permeability of free space. $= 4\pi \times 10^{-7}$ T.m/A

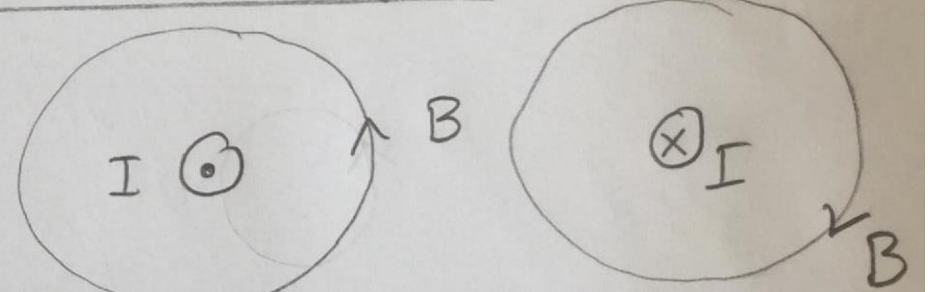
Using the right hand rule; $\vec{dB}$ is in $\otimes$ at A.

r is the distance from ds to point A.

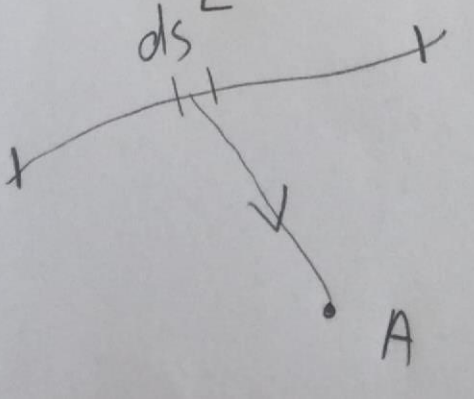
Examples for the direction of B (P.2 Ch.30)

a)



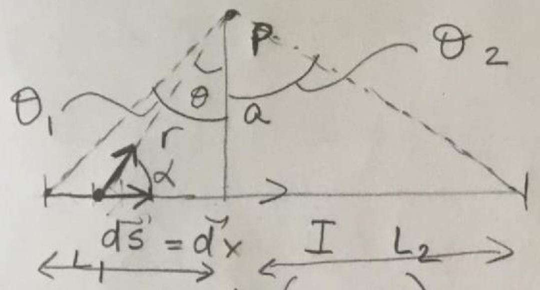


If we want to find $\vec{B}$ at point A created by a current carrying wire (I) of length L



$$B = \frac{\mu_0 I}{4\pi} \int_0^L \frac{\vec{ds} \times \hat{r}}{r^2}$$

If we have a straight wire of Length L
current I : Find B at P.



The direction of B at P is out $\odot$

$L = L_1 + L_2$ (2)

$$dB = \frac{\mu_0 I}{4\pi} \frac{dx\,(\sin\alpha)}{r^2}$$

$$\cos\theta = \frac{a}{r} \qquad \boxed{r = \frac{a}{\cos\theta}}$$

1 $\boxed{\sin\alpha = \cos\theta}$

$$\tan\theta = \frac{-x}{a}$$

$$x = -a\tan\theta \qquad dx = -a\frac{d}{d\theta}(\tan\theta)\,d\theta$$

$$\frac{dx}{d\theta} = -a\frac{d}{d\theta}\left(\frac{\sin\theta}{\cos\theta}\right) = -a\left[\frac{\cos^2\theta + (-\sin\theta)\sin\theta}{\cos^2\theta}\right]$$

3 $\boxed{dx = -a\frac{1}{\cos^2\theta}\,d\theta.}$

Substitute 1, 2, 3 into the integral :

$$B = \frac{\mu_0 I}{4\pi}\int\left(-a\frac{1}{\cos^2\theta}\,d\theta\right)\frac{\cos\theta\,(\cos^2\theta)}{a^2}$$

$$B = -\frac{\mu_0 I}{4\pi}\frac{1}{a}\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta = -\frac{\mu_0 I}{4\pi a}\sin\theta\Big|_{\theta_1}^{\theta_2}$$

$$\boxed{B = \frac{\mu_0 I}{4\pi a}\left[\sin\theta_1 - \sin\theta_2\right]}$$

$$\sin\theta_2 = \frac{L_2}{(L_2^2 + a^2)^{1/2}}$$

$$\sin\theta_1 = \frac{-L_1}{(L_1^2 + a^2)^{1/2}}$$

$$B = -\frac{\mu_0 I}{4\pi a}\left[\frac{L_1}{(L_1^2+a^2)^{1/2}} + \frac{L_2}{(-L_2^2+a^2)^{1/2}}\right]$$

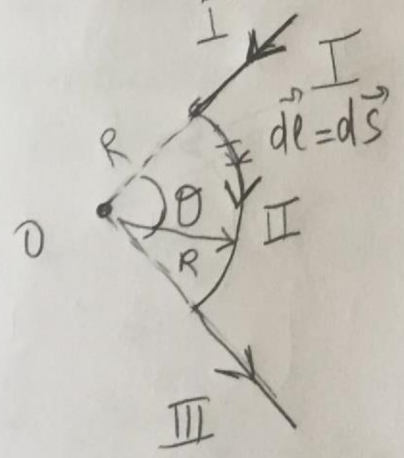
Now assume that the wire is infinitely long. In this case $\theta_1 = \frac{\pi}{2}$ $\theta_2 = -\frac{\pi}{2}$

$\sin\frac{\pi}{2} = 1$ $\sin\left(-\frac{\pi}{2}\right) = -1$

So: $B = \frac{\mu_0 I}{4\pi a}\left[1-(-1)\right] = \frac{\mu_0 I}{4\pi a}(2)$

$$\boxed{B = \frac{\mu_0 I}{2\pi a}}$$ for an infinitely long wire carrying current I.

B due to a curved Wire Segment



Find B at O:

We consider the wire in three segments: I, II, III

$$\vec B_0 = \vec B_I + \vec B_{II} + \vec B_{III}$$

$$B_I = \frac{\mu_0 I}{4\pi}\int \frac{d\vec l \times \hat r}{r^2}$$

Because dl and r are parallel for segments I + III, $\theta = 0$ $\sin\theta = 0$

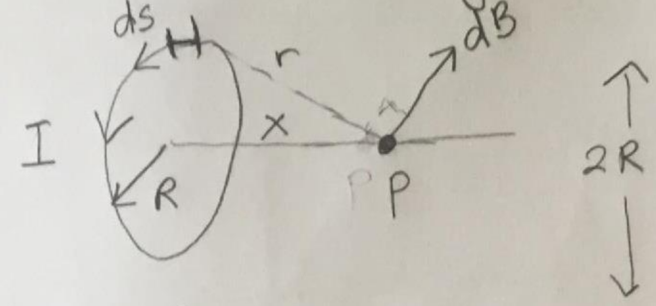
$$B_I = B_{III} = 0$$

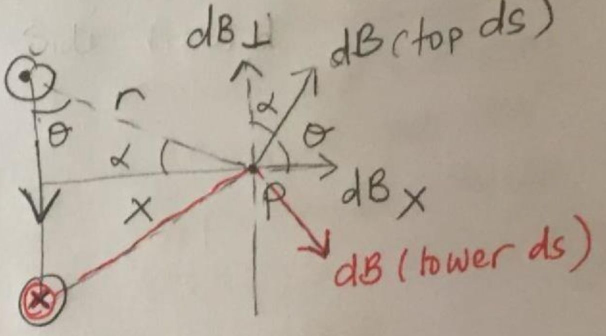
$$B_0 = B_{II} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec s}{R^2} = \frac{\mu_0 I}{4\pi R^2}\int_0^\theta R\,d\theta$$

$$\vec B_0 = \frac{\mu_0 I}{4\pi R}\,\theta \otimes \text{ in.}$$

④

B on the axis of a Circular Current Loop:





On the figures dB created by the top segment ds is shown. (and bottom ds by "red" line)

Side View

$$dB = \frac{\mu_0 I}{4\pi}\int \frac{ds}{r^2} \qquad r = (R^2+x^2)^{1/2} \qquad \cos\theta = \frac{R}{r}$$

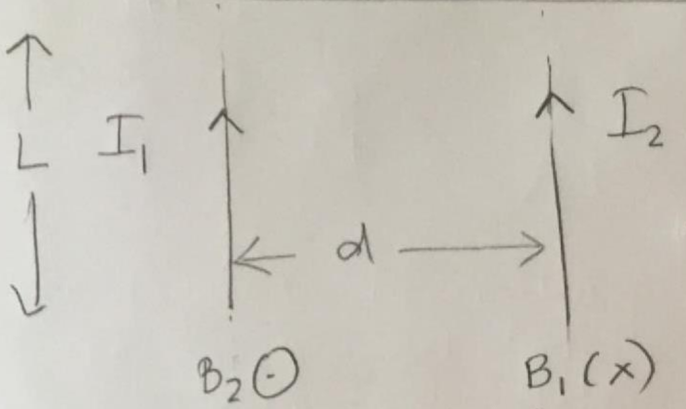
If you consider all ds around the circular current loop $\vec{dB}$ lines form a "cone". The ⊥ (perpendicular) components of all dB's cancel out. The parallel or X components of dB add.

$$B = \int dB_x = \int dB\cos\theta = \frac{\mu_0 I}{4\pi}\int \frac{ds}{(R^2+x^2)}$$ [redacted] $$\frac{R}{r}$$

$$B = \frac{\mu_0 I R}{4\pi (R^2+x^2)^{3/2}}\int ds \qquad \int ds = 2\pi R$$

$$\vec{B} = \frac{\mu_0 I R (2\cancel{\pi} R)}{\underset{2}{\cancel{4}}\cancel{\pi}(R^2+x^2)^{3/2}} \uparrow = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} \uparrow$$

Force between Current Carrying Wires



We assume L is very long, so we can approximate these wires as infinitely long wires.

So we have two wires carryin currents I_1 and I_2. B due to an infinitely long wire at d:

$$B = \frac{\mu_0 I}{2\pi d} \quad (x) \text{ in}$$

B due to I_1 at a distance d (where I_2 is)

$$\boxed{B_1 = \frac{\mu_0 I_1}{2\pi d}}$$

Force on the second wire: $F_2 = I_2 L \times B_1$

since $L \perp B_1$ $\quad F_2 = I_2 L \frac{\mu_0 I_1}{2\pi d}$

direction of F_2 is $-\hat{i}$: $\boxed{\vec{F}_2 = \frac{\mu_0}{2\pi} \frac{L I_1 I_2}{d} (-\hat{i})}$

B due to I_2 at the location of wire 1:

$$B_2 = \frac{\mu_0 I_2}{2\pi d} \odot \text{ out} \quad F_1 = I_1 L \times B_2 \quad L \perp B_2$$

$$F_1 = I_1 L \frac{\mu_0 I_2}{2\pi d}$$

direction of F_1 is $+\hat{i}$

$$\vec{F}_1 = \frac{\mu_0}{2\pi} \frac{L I_1 I_2}{d} (\hat{i})$$

So, the forces are equal in magnitude and opposite in direction (attractive)