Ex

$$\lim_{x \to 4} \frac{4x - x^2}{2 - \sqrt{x}} =$$

$$\lim_{x \to 4} \frac{(4-x)x}{2-\sqrt{x}} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}} =$$

$$\lim_{x \to 4} \frac{\cancel{(4-x)}x}{\cancel{4-x}} (2+\sqrt{x}) = 16$$

Ex

$$\lim_{x \to 0} \frac{x^2 + x}{x^5 + 2x^4 + x^3} =$$

$$\lim_{x \to 0} \frac{\cancel{x}(x+1)}{x^{\cancel{3}\,2}(x^2+2x+1)} =$$

$$\lim_{x \to 0} \frac{\cancel{x+1}}{x^2 (x+1)^{\cancel{2}}} = \infty$$

Ex

$$\lim_{x\to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x} =$$

$$\lim_{x\to 0} \frac{\frac{\cancel{2} - \cancel{2} - \cancel{x}}{2(2+x)}}{\cancel{x}} = \frac{-1}{4}$$

Ex

$$-2\sin^{\cancel{2}} x$$

$$\lim_{x\to 0} \frac{\cos 2x - 1}{\cancel{\sin x}} = -2\lim_{x\to 0} \sin x$$

$$= 0$$

$$\cos(2x) = \cos(x+x)$$

$$= \underbrace{\cos^2 x}_{1-\sin^2 x} - \sin^2 x$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\cos 2x - 1 = -2\sin^2 x$$

Ex

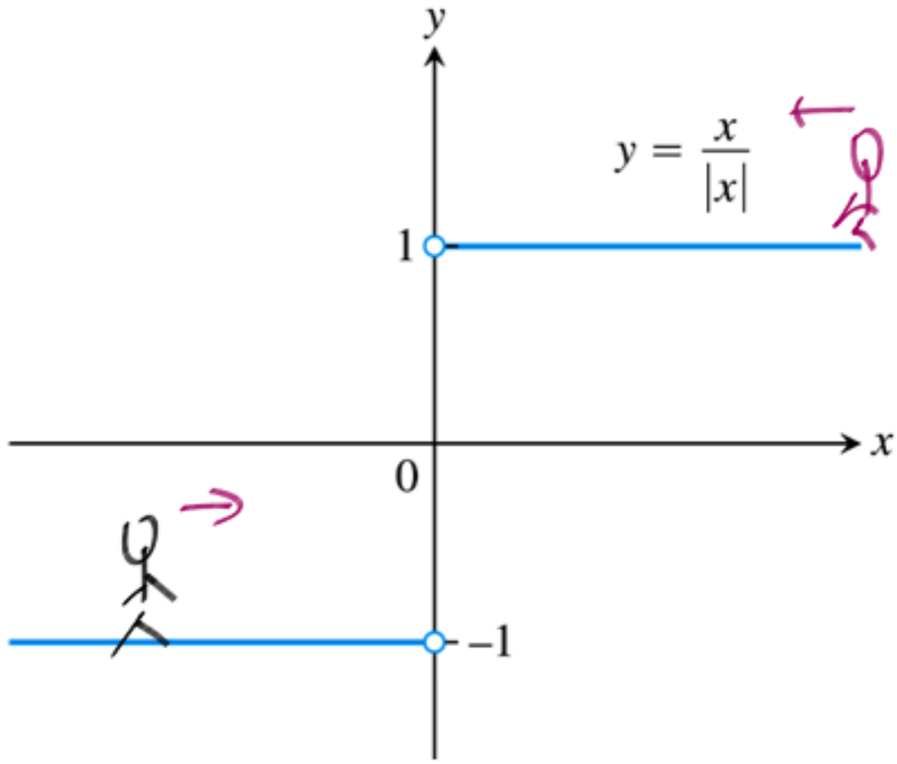
$$\lim_{x\to 0} \frac{8x\,/x}{\frac{3\sin x}{x} - \frac{x}{x}} =$$

$$8 \lim_{x\to 0} \frac{1}{3\underbrace{\frac{\sin x}{x}}_{1} - 1} = 8 \times \frac{1}{2} = 4$$

Ex

$$\lim_{x\to 64} \frac{x^{2/3} - 16}{\sqrt{x} - 8} = ?$$

Sec. 2.4 One-Sided Limit



$$f(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

$$\left.\begin{aligned} \lim_{x \to 0^+} f(x) &= 1 \\ \lim_{x \to 0^-} f(x) &= -1 \end{aligned}\right\} \text{ since } \lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$$

$\lim_{x \to 0} f(x)$ does <u>NOT</u> exist!

Theorem:

$$\lim_{x\to a} f(x) = L \quad \text{if}$$

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$$

In other words, if left and right limits are equal, then limit exists! Otherwise limit does <u>NOT</u> exist!

Ex

$$f(x) = \begin{cases} 2 & x \leq -1 \\ -x & -1 \leq x \leq 1 \\ -x^2 & 1 < x \end{cases}$$

$$\lim_{x \to +1} f(x) = ? \quad , \quad \lim_{x \to -1} f(x) = ?$$

$$\left.\begin{aligned} \lim_{x \to 1^+} -x^2 &= -1^2 = -1 \\ \lim_{x \to 1^-} -x &= -1 = -1 \end{aligned}\right\} \lim_{x \to 1} f(x) = -1$$

$$\left.\begin{aligned} \lim_{x \to -1^+} -x &= 1 \\ \lim_{x \to -1^-} 2 &= 2 \end{aligned}\right\} \begin{aligned} &\lim_{x \to -1^+} f(x) \neq \lim_{x \to -1^-} f(x) \\ &\lim_{x \to -1} f(x) \text{ does } \underline{\text{NOT}} \text{ exist!} \end{aligned}$$

Ex

Determine $a \in \mathbb{R}$ such that

$$\lim_{x \to -2} \frac{x^3 - ax + 2}{x+2} = a, \text{ find } a.$$

At $x = -2$, the denominator is 0.

For the limit to exist, this must be an indeterminate form, $0/0$.

$$
\begin{array}{l|l}
x^3 - ax + 2 & x+2 \\ \hline
\mp x^3 \mp 2x^2 & x^2 - 2x + 4 - a \\ \hline
\quad -2x^2 - ax + 2 & \\
\quad \pm 2x^2 \pm 4x & \\ \hline
\qquad (4-a)x + 2 & \\
\quad \mp (4-a)x \mp 2(4-a) & \\ \hline
\qquad\qquad 2 - 8 + 2a = 2a - 6 &
\end{array}
$$

$$\frac{x^3 - ax + 2}{x+2} = x^2 - 2x + 4 - a + \frac{2a-6}{x+2}$$

$$2a - 6 = 0$$

$$2a = 6$$

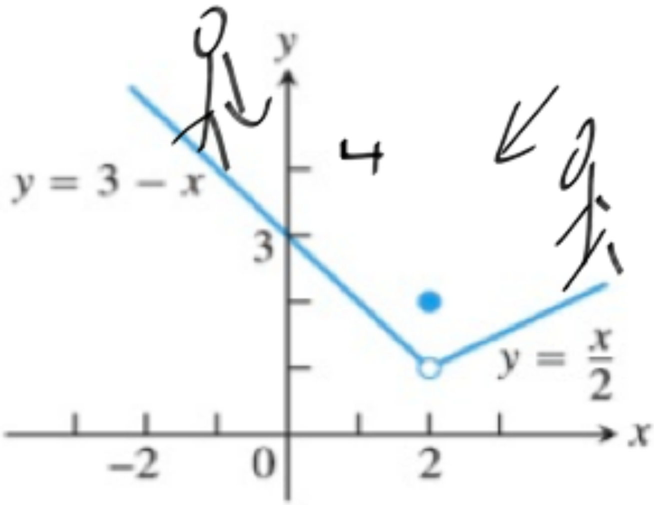
$$\underline{\underline{a = 3}}$$

Let $f(x) = \begin{cases} 3 - x, & x < 2 \\ 2, & x = 2 \\ \dfrac{x}{2}, & x > 2. \end{cases}$

$\lim_{x \to -1^-} f(x) = 4$

$\lim_{x \to -1^+} f(x) = 4$

$\lim_{x \to -1} f(x) = 4$



a. Find $\lim_{x\to 2^+} f(x)$, $\lim_{x\to 2^-} f(x)$, and $f(2)$.

b. Does $\lim_{x\to 2} f(x)$ exist? If so, what is it? If not, why not?

c. Find $\lim_{x\to -1^-} f(x)$ and $\lim_{x\to -1^+} f(x)$.

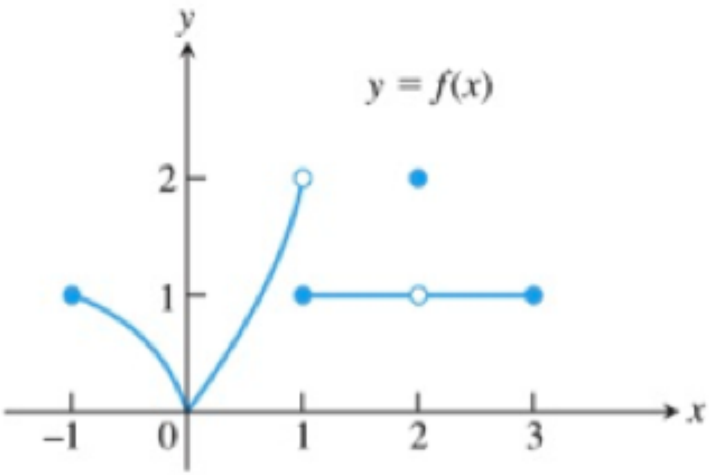
d. Does $\lim_{x\to -1} f(x)$ exist? If so, what is it? If not, why not?

$\lim_{x \to 2^+} f(x) = 1$, although $f(2) = 2$

$\lim_{x \to 2^-} f(x) = 1$

$\lim_{x \to 2} f(x) = 1$

2. Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?



T **a.** $\lim_{x\to -1^+} f(x) = 1$ F **b.** $\lim_{x\to 2} f(x)$ does not exist.

F **c.** $\lim_{x\to 2} f(x) = 2$ T **d.** $\lim_{x\to 1^-} f(x) = 2$

T **e.** $\lim_{x\to 1^+} f(x) = 1$ T **f.** $\lim_{x\to 1} f(x)$ does not exist.

T **g.** $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

T **h.** $\lim_{x\to c} f(x)$ exists at every c in the open interval $(-1, 1)$.

T **i.** $\lim_{x\to c} f(x)$ exists at every c in the open interval $(1, 3)$.

F **j.** $\lim_{x\to -1^-} f(x) = 0$ T **k.** $\lim_{x\to 3^+} f(x)$ does not exist.

Finding One-Sided Limits Algebraically

Find the limits in Exercises

$$\lim_{x \to -0.5^-} \sqrt{\frac{x+2}{x+1}}$$

$$\lim_{x \to 1^+} \sqrt{\frac{x-1}{x+2}}$$

$$\lim_{x \to -2^+} \left(\frac{x}{x+1}\right)\left(\frac{2x+5}{x^2+x}\right)$$

$$\lim_{x \to 1^-} \left(\frac{1}{x+1}\right)\left(\frac{x+6}{x}\right)\left(\frac{3-x}{7}\right)$$

$$\lim_{h \to 0^+} \frac{\sqrt{h^2+4h+5}-\sqrt{5}}{h}$$

$$\lim_{x \to -2^+} (x+3)\frac{|x+2|}{x+2}$$

$$\lim_{x \to -2^-} (x+3)\frac{|x+2|}{x+2}$$

Using $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

Find the limits in Exercises

$$\lim_{\theta \to 0} \frac{\sin\sqrt{2}\theta}{\sqrt{2}\theta}$$

$$\lim_{t \to 0} \frac{\sin kt}{t} \quad (k \text{ constant})$$

$$\lim_{y \to 0} \frac{\sin 3y}{4y}$$

$$\lim_{h \to 0^-} \frac{h}{\sin 3h}$$

$$\lim_{\theta \to 0} \frac{\sin \theta}{\sin 2\theta}$$

$$\lim_{x \to 0} \frac{\sin 5x}{\sin 4x}$$

$$\lim_{\theta \to 0} \theta \cos \theta$$

$$\lim_{\theta \to 0} \sin \theta \cot 2\theta$$

$$\lim_{x \to 0} \frac{\tan 3x}{\sin 8x}$$

$$\lim_{y \to 0} \frac{\sin 3y \cot 5y}{y \cot 4y}$$