Student: Huseyin Kerem Mican **Date:** 5/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 1.4 Homework

1. Compute the product using (a) the definition where A**x** is the linear combination of the columns of A using the corresponding entries in **x** as weights, and (b) the row-vector rule for computing A**x**. If a product is undefined, explain why.

$$\begin{bmatrix} -3 & 2 \\ 10 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ -9 \\ 2 \end{bmatrix}$$

(a) Compute the product using the definition where A**x** is the linear combination of the columns of A using the corresponding entries in **x** as weights. If the product is undefined, explain why. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

- **A.** A**x** =
- **B.** The matrix-vector A**x** is not defined because the number of rows in matrix A does not match the number of entries in the vector **x**.
- ✔ **C.** The matrix-vector A**x** is not defined because the number of columns in matrix A does not match the number of entries in the vector **x**.

(b) Compute the product using the row-vector rule for computing A**x**. If the product is undefined, explain why. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

- **A.** A**x** =
- **B.** The matrix-vector A**x** is not defined because the row-vector rule states that the number of rows in matrix A must match the number of entries in the vector **x**.
- ✔ **C.** The matrix-vector A**x** is not defined because the row-vector rule states that the number of columns in matrix A must match the number of entries in the vector **x**.

2. Compute the product using the methods below. If a product is undefined, explain why.
a. The definition where A**x** is the linear combination of the columns of A using the corresponding entries in **x** as weights.
b. The row-vector rule for computing A**x**.

$$\begin{bmatrix} 6 & 4 \\ -2 & -3 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

a. Set up the linear combination of the columns of A using the corresponding entries in **x** as weights. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

A. $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = (x_1)\mathbf{a}_1 + (x_2)\mathbf{a}_2 + (x_3)\mathbf{a}_3 = (\quad)\,\_\_ + (\quad)\,\_\_$
(Type the terms of your expression in the same order as they appear in the original expression.)

B. $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = (x_1)\mathbf{a}_1 + (x_2)\mathbf{a}_2 = (5)\begin{bmatrix} 6 \\ -2 \\ 5 \end{bmatrix} + (-4)\begin{bmatrix} 4 \\ -3 \\ 3 \end{bmatrix}$
(Type the terms of your expression in the same order as they appear in the original expression.)

C. The matrix-vector A**x** is not defined because the number of columns in matrix A does not match the number of entries in the vector **x**.

D. The matrix-vector A**x** is not defined because the number of rows in matrix A does not match the number of entries in the vector **x**.

b. Set up the product A**x** using the row-vector rule. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. $\begin{bmatrix} 6 \cdot 5 + 4 \cdot -4 \\ -2 \cdot 5 + (-3 \cdot -4) \\ 5 \cdot 5 + 3 \cdot -4 \end{bmatrix}$ (Do not simplify.)

B. $\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$ (Do not simplify.)

C. The matrix-vector A**x** is not defined because the row-vector rule states that the number of columns in matrix A must match the number of entries in the vector **x**.

D. The matrix-vector A**x** is not defined because the row-vector rule states that the number of rows in matrix A must match the number of entries in the vector **x**.

Evaluate the expressions found in the previous steps. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $A\mathbf{x} = \begin{bmatrix} 14 \\ 2 \\ 13 \end{bmatrix}$ (Simplify your answer.)

B. The matrix-vector A**x** is not defined.

3. Use the definition of A**x** to write the matrix equation as a vector equation.

$$\begin{bmatrix} -6 & -5 \\ -8 & -2 \\ -1 & -1 \\ -5 & 7 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 16 \\ -1 \\ 43 \end{bmatrix}$$

The matrix equation written as a vector equation is $-3\begin{bmatrix} -6 \\ -8 \\ -1 \\ -5 \end{bmatrix} + 4\begin{bmatrix} -5 \\ -2 \\ -1 \\ 7 \end{bmatrix} = \begin{bmatrix} -2 \\ 16 \\ -1 \\ 43 \end{bmatrix}$.

4. Use the definition of A**x** to write the vector equation as a matrix equation.

$$x_1\begin{bmatrix} 4 \\ 7 \\ 6 \\ -2 \end{bmatrix} + x_2\begin{bmatrix} -2 \\ -7 \\ -4 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} 5 \\ 2 \\ -6 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 7 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 & 5 \\ 7 & -7 & 2 \\ 6 & -4 & -6 \\ -2 & 0 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 7 \\ 9 \end{bmatrix}$$

(Type an integer or simplified fraction for each matrix element.)

5. Write the system first as a vector equation and then as a matrix equation.

$$\begin{aligned} 3x_1 + x_2 - 3x_3 &= 4 \\ 3x_2 + 6x_3 &= 0 \end{aligned}$$

Write the system as a vector equation. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

A. $\underline{\quad} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underline{\quad}$

B. $x_1 \begin{bmatrix} 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

C. $\underline{\quad} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \underline{\quad}$

Write the system as a matrix equation. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

A. $\begin{bmatrix} 3 & 1 & -3 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

B. $\underline{\quad} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \underline{\quad}$

C. $x_1 \underline{\quad} + x_2 \underline{\quad} + x_3 \underline{\quad} = \underline{\quad}$

6. Given A and **b** to the right, write the augmented matrix for the linear system that corresponds to the matrix equation A**x** = **b**. Then solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & 4 \\ -2 & -5 & 6 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -7 \\ 2 \\ 26 \end{bmatrix}$$

Write the augmented matrix for the linear system that corresponds to the matrix equation A**x** = **b**. Select the correct choice below and fill in any answer boxes within your choice.

A. $\begin{bmatrix} \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} & \underline{\quad} \end{bmatrix}$

B. $\begin{bmatrix} 1 & 3 & -2 & \underline{\quad} \\ 2 & 2 & 4 & \underline{\quad} \\ -2 & -5 & 6 & \underline{\quad} \end{bmatrix}$

Solve the system and write the solution as a vector. Select the correct choice below and fill in any answer boxes within your choice.

A. $\mathbf{x} = \begin{bmatrix} \underline{\quad} \\ \underline{\quad} \end{bmatrix}$

B. $\mathbf{x} = \begin{bmatrix} -11 \\ 4 \\ 4 \end{bmatrix}$

7. Given A and **b** to the right, write the augmented matrix for the linear system that corresponds to the matrix equation A**x** = **b**. Then solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 7 & -5 \\ -2 & -3 & -1 \\ 5 & 2 & 6 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 16 \\ 12 \\ -52 \end{bmatrix}$$

Write the augmented matrix for the linear system that corresponds to the matrix equation A**x** = **b**. Select the correct choice below and fill in any answer boxes within your choice.

A. $\begin{bmatrix} & 1 & \\ & & -3 \\ & & \end{bmatrix}$

✔ B. $\begin{bmatrix} 1 & 7 & -5 & \\ -2 & -3 & -1 & \\ 6\ 5 & 2 & 6 & - \end{bmatrix}$

Solve the system and write the solution as a vector. Select the correct choice below and fill in any answer boxes within your choice.

✔ A. $\mathbf{x} = \begin{bmatrix} -12 \\ 4 \\ 0 \end{bmatrix}$

B. $\mathbf{x} = \begin{bmatrix} \\ \end{bmatrix}$

8. Let $A = \begin{bmatrix} 3 & -3 \\ -6 & 6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Show that the equation A**x** = **b** does not have a solution for some choices of **b**, and describe the set of all **b** for which A**x** = **b** does have a solution.

How can it be shown that the equation A**x** = **b** does not have a solution for some choices of **b**?

A. Row reduce the matrix A to demonstrate that A has a pivot position in every row.

B. Find a vector **x** for which A**x** = **b** is the identity vector.

✔ C. Row reduce the matrix A to demonstrate that A does not have a pivot position in every row.

D. Find a vector **b** for which the solution to A**x** = **b** is the identity vector.

E. Row reduce the augmented matrix $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ to demonstrate that $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ has a pivot position in every row.

Describe the set of all **b** for which A**x** = **b** does have a solution.

The set of all **b** for which A**x** = **b** does have a solution is the set of solutions to the equation 0 = 2 $b_1 + b_2$.
(Type an integer or a decimal.)

9. Let $\mathbf{u} = \begin{bmatrix} 12 \\ -6 \\ 12 \end{bmatrix}$ and $A = \begin{bmatrix} 4 & -2 \\ -5 & 7 \\ 2 & 2 \end{bmatrix}$. Is **u** in the plane in $\mathbb{R}^3$ spanned by the columns of A? Why or why not?

Select the correct choice below and fill in the answer box to complete your choice.
(Type an integer or decimal for each matrix element.)

✔ A. Yes, multiplying A by the vector $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ writes **u** as a linear combination of the columns of A.

B. No, the reduced row echelon form of the augmented matrix is ______, which is an inconsistent system.

10. Do the columns of A span $\mathbb{R}^4$? Does the equation A**x** = **b** have a solution for each **b** in $\mathbb{R}^4$?

$$A = \begin{bmatrix} 3 & 4 & -7 & 4 \\ 0 & 1 & 2 & -2 \\ 1 & 4 & 3 & -4 \\ -1 & -8 & -11 & 13 \end{bmatrix}$$

Do the columns of A span $\mathbb{R}^4$? Select the correct choice below and fill in the answer box to complete your choice. (Type an integer or decimal for each matrix element.)

A. No, because the reduced row echelon form of A is $\begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

B. Yes, because the reduced row echelon form of A is ______.

Does the equation A**x** = **b** have a solution for each **b** in $\mathbb{R}^4$?

A. Yes, because A does not have a pivot position in every row.

B. No, because A has a pivot position in every row.

C. No, because the columns of A do not span $\mathbb{R}^4$.

D. Yes, because the columns of A span $\mathbb{R}^4$.

11. Let $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$. Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span $\mathbb{R}^4$? Why or why not?

Choose the correct answer below.

A. No. The set of given vectors spans a plane in $\mathbb{R}^4$. Any of the three vectors can be written as a linear combination of the other two.

B. Yes. When the given vectors are written as the columns of a matrix A, A has a pivot position in every row.

C. No. When the given vectors are written as the columns of a matrix A, A has a pivot position in only three rows.

D. Yes. Any vector in $\mathbb{R}^4$ except the zero vector can be written as a linear combination of these three vectors.

12. Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ when n is less than m?

Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. Choose the correct answer below.

- **A.** Yes. Any number of vectors in $\mathbb{R}^4$ will span all of $\mathbb{R}^4$.
- **B.** Yes. A set of n vectors in $\mathbb{R}^m$ can span $\mathbb{R}^m$ when $n < m$. There is a sufficient number of rows in the matrix A formed by the vectors to have enough pivot points to show that the vectors span $\mathbb{R}^m$.
- **C.** No. The matrix A whose columns are the three vectors has four rows. To have a pivot in each row, A would have to have at least four columns (one for each pivot.)
- **D.** No. There is no way for any number of vectors in $\mathbb{R}^4$ to span all of $\mathbb{R}^4$.

Could a set of n vectors in $\mathbb{R}^m$ span all of $\mathbb{R}^m$ when n is less than m? Explain. Choose the correct answer below.

- **A.** No. The matrix A whose columns are the n vectors has m rows. To have a pivot in each row, A would have to have at least m columns (one for each pivot.)
- **B.** Yes. A set of n vectors in $\mathbb{R}^m$ can span $\mathbb{R}^m$ if $n < m$. There is a sufficient number of rows in the matrix A formed by the vectors to have enough pivot points to show that the vectors span $\mathbb{R}^m$.
- **C.** No. Without knowing values of n and m, there is no way to determine if n vectors in $\mathbb{R}^m$ will span all of $\mathbb{R}^m$.
- **D.** Yes. Any number of vectors in $\mathbb{R}^m$ will span all of $\mathbb{R}^m$.

13. How many rows of A contain a pivot position? Does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for each $\mathbf{b}$ in $\mathbb{R}^4$?

$$A = \begin{bmatrix} 1 & 3 & -6 & 13 \\ -2 & -2 & 2 & -6 \\ 0 & -2 & 5 & -10 \\ 2 & 0 & 3 & -3 \end{bmatrix}$$

How many rows of A contain a pivot position?

A has 3 rows which contain a pivot position.

Does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for each $\mathbf{b}$ in $\mathbb{R}^4$?

- **A.** No, because A does not have a pivot position in every row.
- **B.** No, because each b in $\mathbb{R}^4$ is a linear combination of the columns of A.
- **C.** Yes, because the columns of A do not span $\mathbb{R}^4$.
- **D.** Yes, because the reduced echelon form of A does not have a row of the form $\begin{bmatrix} 0 & \dots & 0 & b \end{bmatrix}$ with b nonzero.