"On my honor, I have neither given nor received any unauthorized and/or inappropriate assistance for all sessions of this exam. The work done on this exam is totally my own. I understand that by the school code, violation of the principles will lead to a zero grade and is subject to harsh discipline issues.

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Q1) $f(x) = \frac{x^2}{20} - 5\sin(x)$ 3 iteration find minimum sın

a) from $x_l = 0$ (lower) to $x_u = 5$ (upper)

iteration	x_1	$f(x_1)$	x_2	$f(x_2)$	x_1	$f(x_1)$	x_u	$f(x_u)$	d
1	3.1	$\frac{9.61}{20} - 5\sin(3.1)$	1.9	$\frac{3.61}{20} - 5\sin(1.9)$	1.9	$\frac{3.61}{20} - 5\sin(1.9)$	3.1	$\frac{9.61}{20} - 5\sin(3.1)$	3.1
2	1.9	$\frac{3.61}{20} - 5\sin(1.9)$	0	0	0	0	1.9	$\frac{3.61}{20} - 5\sin(1.9)$	1.92
3	0	0	0	0					

$x_1 = x_l + d$ $\quad d = (\phi - 1)(x_u - x_l)$
0.618

$x_2 = x_u + d$

I can't calculate this?

Not: Asistanlara soru sordum hiçbiri cevap vermedi, el kaldırdığımı da görmedi. Normal sınav alma hakkı içinde keşke bir ipucu verseydiniz.

1st → $d = 0.62 \cdot (5) - 3.1$

$x_1 = 0 + 3.1 = 3.1$ $\quad f(x_1) = \frac{(3.1)^2}{20} - 5 \cdot \sin(3.1)$?

$x_2 = 5 - 3.1 = 1.9$ $\quad f(x_2) = \frac{(1.9)^2}{20} - 5 \cdot \sin(1.9)$

$x_2 = x_u - d$
$= 3.1 - 3.1 = 0$

$f(x_1) > f(x_2)$

$x_u = x_1 = 3.1$ $\quad f(x_1) = \frac{3.61}{20} - 5\sin(1.9)$

$x_1 = x_2 = 1.9$

2nd) $x_1 = 1.9$ $x_2 = 0$ $d = 0.62 \cdot 3.1 = 1.92$

$x_u = 3.1$ $x_l = 0$

$f(x_1) = \frac{3.61}{20} - 5\sin(1.9)$ $f(x_1) > f(x_2)$ I guess

$f(x_2) = 0$

$x_u = x_1 = 1.9$

$x_1 = x_2 = 0$ $f(x_1) = 0$

$x_2 = x - d = 1.9 - 1.9 = 0$

3rd) $x_1 = 0$ $x_2 = 0$

$x_u = 1.9$

$x_l = 0$

$f(x_1) = f(x_2)$?

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b) Parabolic $x_1 = 0$ $x_2 = 3$ $x_3 = 5$

1st) $f(x_1) = 0$ $f(x_2) = \frac{9}{20} - 5\sin(3)$ $f(x_3) = \frac{25}{20} - 5\sin(5)$

initial guess

$x_4 = 4$ $f(x_4) = \frac{16}{20} - 5\sin(4)$

	x_1	$f(x_1)$	x_2	$f(x_2)$	x_3	$f(x_3)$	x_4	$f(x_4)$
1	0	0	3	$\frac{9}{20} - 5\sin(3)$	5	$\frac{25}{20} - 5\sin(5)$	4	$\frac{16}{20} - 5\sin(4)$

2st)

①

$$\frac{l_1 + l_2}{l_1} = \frac{l_1}{l_2}$$ → euclid's golden ratio → 0.618

If $f(x_1) < f(x_2)$ → replace x_l with x_2, x_1 becomes x_2, $x_1 = x_l + d_{new}$

$f(x_2) < f(x_1)$ → tersi

$x_1 = x_l + d$, $x_2 = x_u - d$; $d = (\phi - 1)(x_u - x_l)$ → **Golden sec search** (0.618)

$$\varepsilon_a = (2 - \phi)\left|\frac{x_u - x_l}{x_{opt}}\right| \times 100\%$$ → golden sec search error.

approximate x_1, x_2, x_3 verince the slight opt

$[x_{min}, fval] = fminbnd(function, x_1, x_2)$

Gradient Methods

If slope $f > 0$, x will increase f(x)

$x = 0$, optimum, $f'(x) = 0$ → $f''(x) > 0$ a min; $f''(x) < 0$ a max

If slope < 0 decreasing x will increase f(x)

Gradient: $\nabla f = \frac{df}{dx} i + \frac{df}{dy} j$ or $\nabla f = \begin{bmatrix} df/dx \\ df/dy \end{bmatrix}$, $x = x_0 + \frac{df}{dx} h$

Newton's method (open)

$f(x) = 0$ $\quad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

find optimum of $f(x)$ $\quad g(x) = f'(x)$

$f'(x^*) = g(x^*) = 0$

$$x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)}$$

Hesian 2.pdf

Steepest Ascent method

$x = x_0 + \frac{df}{dx} \cdot h$

$y = y_0 + \frac{df}{dy} \cdot h$

Constrained pdf look

$Ax = b$ → $x = A^{-1}b = A \backslash b$, $^{-1}$ olsun

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $\quad A^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{11}} \cdot \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$

Beyond 3 dim's graphical methods are unuseful. 3 problem second pdf

Determinant 2. pdf look

→ bir sayı, matrix değil

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}, \ldots$$

Gauss

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b1 \\ a_{21} & a_{22} & a_{23} & b2 \\ a_{31} & a_{32} & a_{33} & b3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b1 \\ 0 & a_{22}' & a_{23}' & b2' \\ 0 & 0 & a_{33}'' & b3'' \end{array}\right]$$

2. satırın ilkini 0 yapmak için $\frac{a_{21}}{a_{11}}$ ile çarpıp sıkar

3. satır 1 için $\frac{a_{31}}{a_{11}}$

3. satır 2 için $\frac{a_{32}}{a_{22}}$

Flops → $\frac{2n^3}{3} + O(n^2) + n^2 + O(n)$ → $\frac{2n^3}{3} + O(n^2)$

bide pivoting var ona bakarsın

Cramers rule

→ bilinmeyenlerle eşitlediği sayılardan 1 column'luk matrix. x_1, x_2, x_3 bulurken yerine koyup D'ye böl.

$x_1 = \frac{|\ldots|}{D}$, $x_2 = \frac{|\ldots|}{D}$, ⋮

~ 'ler normal matrixteki elemanlar

LU

$Ax = b$, $A = LU$

$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ $\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

Gauss Seidel

① $x_1^j = \frac{b_1 - a_{12}x_2^{j-1} - a_{13}x_3^{j-1}}{a_{11}}$

② $x_2^j = \frac{b_2 - a_{21}x_1^j - a_{23}x_3^{j-1}}{a_{22}}$

③ $x_3^j = \frac{b_3 - a_{31}x_1^j - a_{32}x_2^j}{a_{33}}$

j, j-1 present and past iterations →

(2)

$$\varepsilon_{a,i} = \left|\frac{x_i^j - x_i^{j-1}}{x_i^j}\right| \times 100\% \le \varepsilon_s$$

jacobi var sipdf
convergence of giedel

Eigenvalues non-homo → $Ax = b$ nontrivial → $|A - \lambda I| = 0$ (↳ identity matrix)

homo → $Ax = 0$ (initial guess yap hepsi 1 de, eger deger vermediyse)

Linear Regression

median: küçükten büyüğe, ortadaki eleman aritmetik ortalaması.

$\bar{y} = \frac{y_i}{n}$ (arithmetic mean) mode: en çok gözüken value.

$$s_y^2 = \frac{\sum_i (y_i - \bar{y})^2}{n-1} = \frac{\sum_i y_i^2 - (\sum y_i)^2/n}{n-1}$$

large standart deviation: aritm. ort. üzerinden sayılar birbirinden uzak

small " " : yakın

coefficient of variation $= \frac{s_y}{\bar{y}} \times 100\%$ (normalize std with mean)

Regression'ın sonunda using MATLAB var

Not: MATLAB kodları pdfe bak

Sum of least squares → $S_r = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2$

$$a_1 = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{n\sum x_i^2 - (\sum x_i)^2} \qquad a_0 = \bar{y} - a_1\bar{x}$$

($\bar{y}$ ↓ mean of y, $\bar{x}$ ↓ mean of x)

Standard error of estimate $= \sqrt{\frac{S_r}{n-2}}$

x'in önündeki her slope.

fminsearch önemli optimize eder

Polynomial Interpolation

x yerine koy sonra yazıp matrise koy.

Newton Intr. Poly. → $f_1(x) = f(x_1) + \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1)$

Quadratic Intr. → $f_2(x) = b_1 + b_2(x - x_1) + b_3(x - x_1)(x - x_2)$

$x = x_1 \to f(x_1) = b_1$

$x = x_2 \to b_2 = \frac{f(x_2) - f(x_1)}{(x_2 - x_1)}$

$x = x_3$ ↓ $b_3 = \frac{f(x_3) - f(x_2)}{}$ pdften bakarsın

$(x_1, f(x_1))$, $(x_2, f(x_2))$, $(x_3, f(x_3))$ verir

Lagrange Intr. Poly

$$f(x) = \frac{x - x_2}{x_1 - x_2} f(x_1) + \frac{x - x_1}{x_2 - x_1} f(x_2)$$

↓ 3pt hali için pdf bak (second order)

spline aşağılarda

f easy to integrate → analytical sol

impossible " → numerical sol

unknown

Trapezoid → $I = (b-a)\frac{f(a) + f(b)}{2}$ its error → $E_t = -\frac{1}{12} f''(\xi)(b-a)^3$

Composite Trapezoid → $I = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]$ its error → $E_a = -\frac{(b-a)^3}{12n^2}\bar{f}''$ → $h = \frac{b-a}{n}$ (↳ artma aralığı)

Simpsons 1/3 rule → $I = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$ its error → $E_t = -\frac{1}{90} h^5 f^{(4)}(\xi)$

MATLAB'ler pdfte diff. ...

base için ... trapezoidden daha iyi ... Simpsons 3/8 var sonda ... MULTIPLE INTEGRALS VAR