Ex

$$y=\sin^2(\underbrace{\pi t-2}_{u}) \quad , \quad y'=?$$

$$y=\sin^2 u \qquad u=\pi t-2$$

$$du/dt=\pi$$

$$\frac{dy}{du}=2\sin u\cos u=2\sin(\pi t-2)\cos(\pi t-2)$$

the chain rule says

$$\frac{dy}{dt}=\frac{dy}{du}\cdot\frac{du}{dt}=2\sin(\pi t-2)\cos(\pi t-2).\pi=2\pi\sin(\pi t-2)\cos(\pi t-2)$$

Ex $y=(1+\cos 2t)^{-4}$ $\quad u=1+\cos 2t\ ,\quad du/dt=-2\sin 2t$

$$y=u^{-4} \qquad dy/du=-4u^{-5}$$

$$\frac{dy}{dt}=\frac{dy}{du}\cdot\frac{du}{dt}=-4u^{-5}.(-2\sin 2t)=8\sin 2t(1+\cos 2t)^{-5}$$

Ex

$$y = \left(1+\cos\left(\tfrac{2}{t}\right)\right)^{-2}$$

$$u = 1+\cos\left(\tfrac{2}{t}\right), \quad \frac{du}{dt} = -\sin\left(\tfrac{2}{t}\right)\frac{d}{dt}\left(\tfrac{2}{t}\right)$$

$$\frac{du}{dt} = -2\sin\left(\tfrac{2}{t}\right)\frac{d}{dt}\left(t^{-1}\right)$$

$$\frac{du}{dt} = -2\sin\left(\tfrac{2}{t}\right)(-1)t^{-2} = 2\sin\left(\tfrac{2}{t}\right)t^{-2}$$

$$y = u^{-2} \qquad \frac{dy}{du} = -2u^{-3}$$

$$\frac{dy}{dt} = \frac{dy}{du}\cdot\frac{du}{dt}$$

$$= -2u^{-3}\cdot 2\sin\left(\tfrac{2}{t}\right)t^{-2}$$

$$= -\frac{4}{t^2}\left(1+\cos\tfrac{2}{t}\right)^{-3}\sin\left(\tfrac{2}{t}\right)$$

Ex $y = \left(t^{-3/4}\sin\sqrt{t}\right)^{4/3}$

$$y' = \frac{4}{3}\left(t^{-3/4}\sin\sqrt{t}\right)^{\frac{4}{3}-1}\frac{d}{dt}\left(t^{-3/4}\sin\sqrt{t}\right)$$

$$y' = \frac{4}{3}\left(t^{-3/4}\sin\sqrt{t}\right)^{1/3}\left\{-\frac{3}{4}t^{-7/4}\sin\sqrt{t} + t^{-3/4}\cos\sqrt{t}\cdot\frac{1}{2\sqrt{t}}\right\}$$

Ex

$$y = (\theta\tan\theta)^{10}, \quad y' = 10(\theta\tan\theta)^{9}\cdot(\tan\theta + \theta\sec^2\theta)$$

Ex

$$y=\left(\frac{x^2}{x^3-4x}\right)^3 \qquad y'=3\left(\frac{x^2}{x^3-4x}\right)^2\frac{d}{dx}\left(\frac{x^2}{x^3-4x}\right)$$

$$y'=3\left(\frac{x^2}{x^3-4x}\right)^2\cdot\left[\frac{2x(x^3-4x)-(3x^2-4)x^2}{(x^3-4x)^2}\right]$$

Ex

$$y=\sqrt{1+\cos^3t^2} \qquad y'=\frac{1}{\cancel{2}}\left(1+\cos^3t^2\right)^{-\frac{1}{2}}\cdot\left(-3\cos^2t^2\sin t^2\cdot\cancel{2}t\right)$$

$$y'=\frac{-3t\cos^2t^2\sin t^2}{\sqrt{1+\cos^3t^2}}$$

Ex

$$y=\tan^2(\sin^3x) \qquad y'=2\tan(\sin^3x)\sec^2(\sin^3x)\cdot3\sin^2x\cos x$$

Ex

$$y=\cos^4(\sec^23t) \qquad y'=4\cos^3(\sec^23t)\frac{d}{dt}\left[\cos(\sec^23t)\right]$$

$$y'=-4\cos^3(\sec^23t)\sin(\sec^23t)\cdot2\sec3t\cdot3\sec3t\cdot\tan3t$$

Ex

$$y=3x(2x^2-5)^4 \qquad y'=3(2x^2-5)^4+3x\cdot4(2x^2-5)^3\cdot4x$$

Hw: $y=\sqrt{3t+\sqrt{2+\sqrt{1-t}}}$ $\quad y'=?$