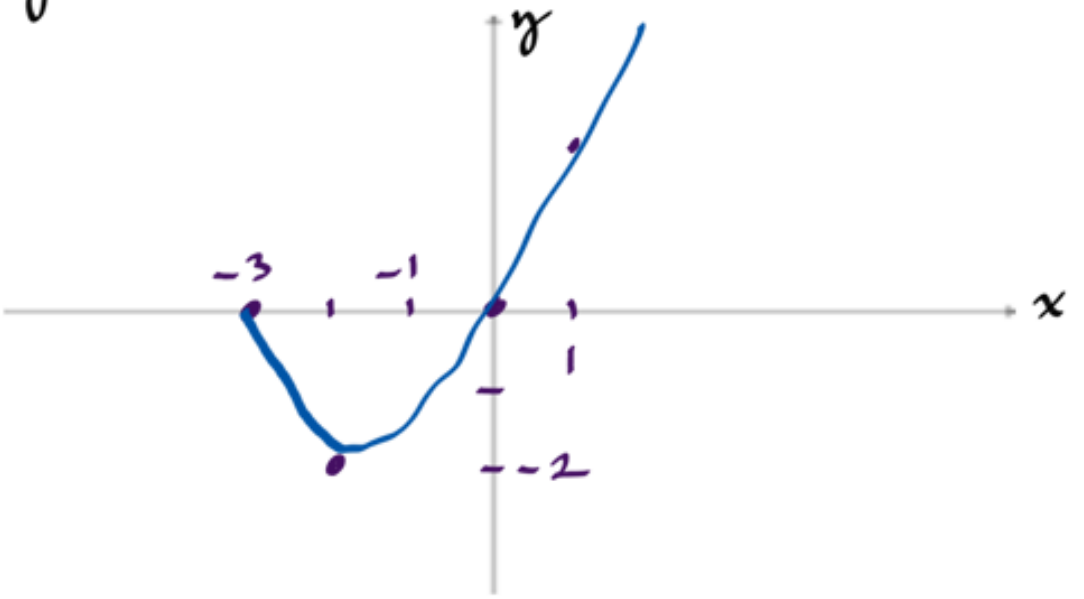
Ex $y = x\sqrt{x+3}$ $\quad y = x\sqrt{x+3} \Rightarrow x = 0, -3$

$y' = \sqrt{x+3} + x\frac{1}{2}(x+3)^{-\frac{1}{2}} = \frac{3(x+2)}{2\sqrt{x+3}} \Rightarrow y' = 0 \Rightarrow x+2 = 0 \Rightarrow x = -2$, crit. pt.

$y'' = \frac{1}{2}(x+3)^{-\frac{1}{2}} + \frac{1}{2}(x+3)^{-\frac{1}{2}} - \frac{x}{2}(-\frac{1}{2})(x+3)^{-\frac{3}{2}} = \frac{3}{4}\frac{x+4}{(x+3)^{3/2}}$

Watch out! y'' exists but pt. of inf. does not exist!

x	-3	-2 ← local min	-1	0	1
y	0	-2	$-\sqrt{2}$	0	2
y'	$-$	0	$+$	$+$	$+$
y''	$+$	$+$	$+$	$+$	$+$



Ex $y = \dfrac{x^2-2x+1}{x} = x-2+\dfrac{1}{x}$ $\lim\limits_{x\to\infty} y = x-2$, $y=x-2$ slant or oblique asymp.

$x=0$ is the vertical asymptote

$y = x-2+\frac{1}{x}$

$y=0 \Rightarrow x^2-2x+1=0$

$(x-1)^2=0 \Rightarrow x=1$, an intercept pt.

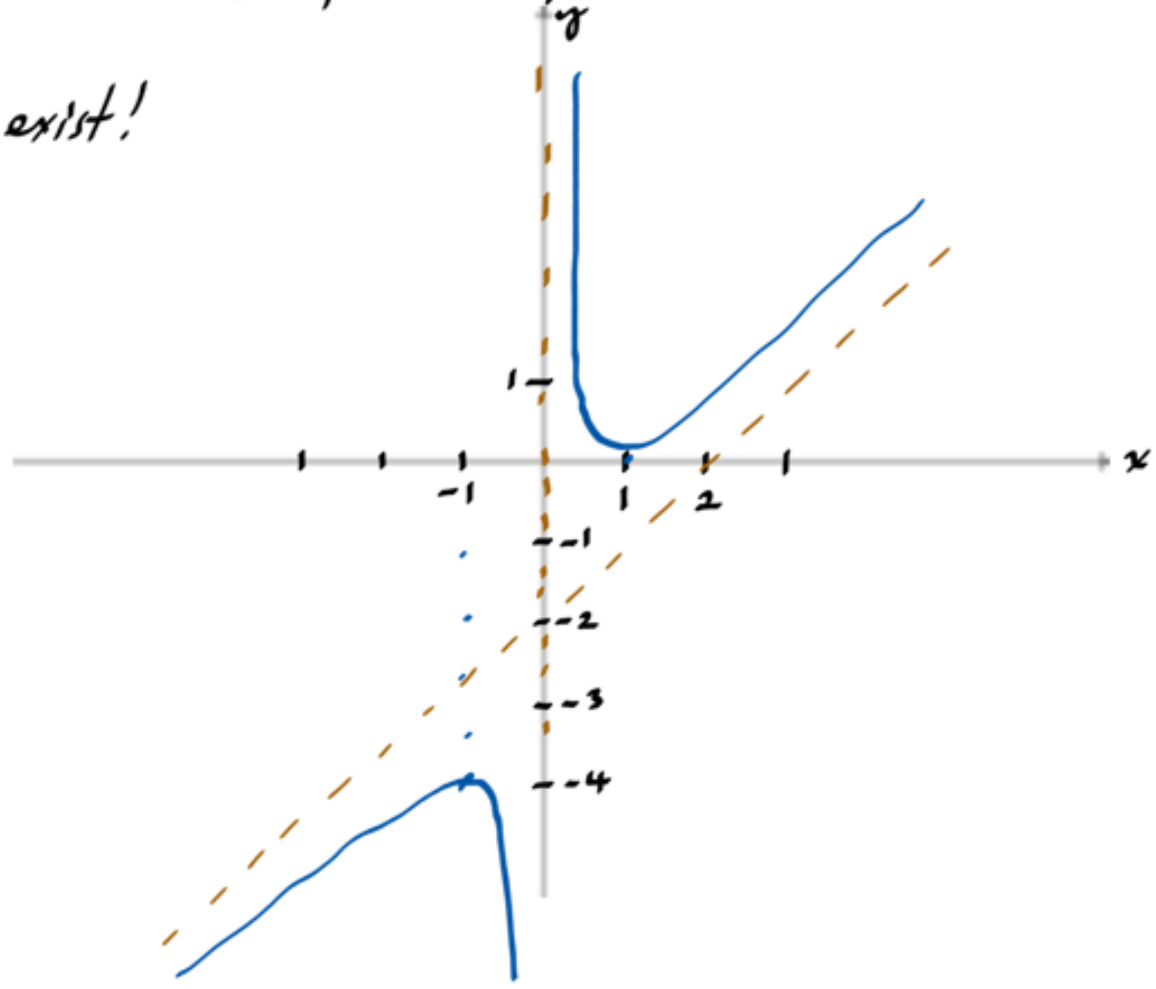
$y' = 1-\frac{1}{x^2} \Rightarrow y'=0 \Rightarrow x^2=1 \Rightarrow x=\pm 1$, critical pts.

$y'' = \frac{2}{x^3}$, y'' does not exist!

		← local max				← local min	
x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	$-4\frac{1}{2}$	-4	$-4\frac{1}{2}$	undefined	$\frac{1}{2}$	0	$\frac{1}{2}$
y'	$+$	0	$-$		$-$	0	$+$
y''	$-$	$-$	$-$		$+$	$+$	$+$

$\lim\limits_{x\to 0^+} y = +\infty$

$\lim\limits_{x\to 0^-} y = -\infty$



Ex $y = \dfrac{x}{x^2-1}$ odd function $\lim_{x\to\pm\infty} y = 0 \Rightarrow y = 0$ horizontal asympt.

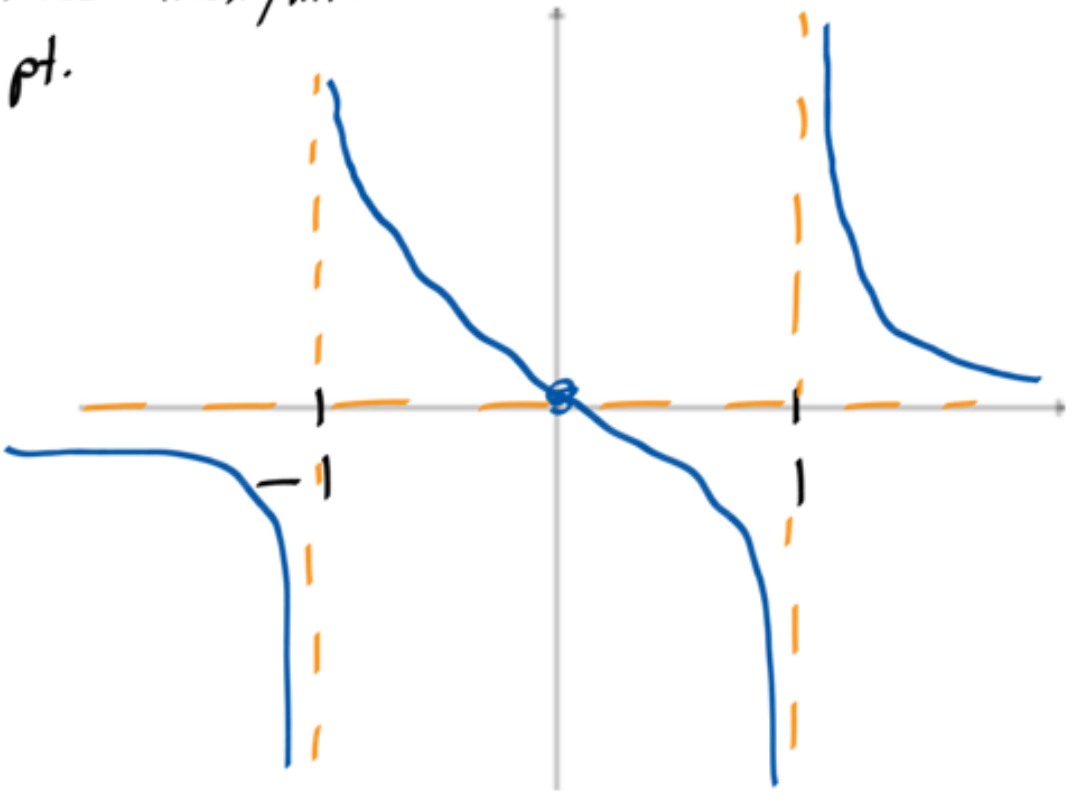
$y' = -\dfrac{1+x^2}{(x^2-1)^2}$, $y'' = \dfrac{2x(x^2+3)}{(x^2-1)^3}$ | $\lim_{x\to 1^+} y = \infty$, $\lim_{x\to 1^-} y = -\infty$

$x = \pm 1$, vertical asymptotes

No critical pts $\Rightarrow$ No local max/min

$y'' = 0 \Rightarrow x = 0$, inf. pt.

x	-2	-1	0	$1/2$	1	2
y	$-2/3$	undefined	0		undefined	$2/3$
y'	$-$		und.	$-$		$-$
y''	$-$		0	$-$		$+$



Homework: $y = x^{5/3} + 5x^{2/3}$