Student: Huseyin Kerem Mican	**Instructor:** Taylan Sengul	**Assignment:** Section 1.1 Homework
Date: 5/1/21	**Course:** Linear Algebra	

1. Solve the system by using elementary row operations on the equations. Follow the systematic elimination procedure.

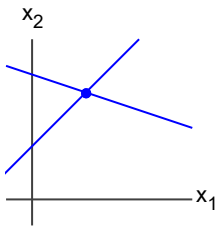
$$x_1 + 4x_2 = 7$$
$$2x_1 + 7x_2 = 11$$

Find the solution to the system of equations.

$(-5,3)$

(Simplify your answer. Type an ordered pair.)

2. Find the point (x_1,x_2) that lies on the line $x_1 + 3x_2 = 14$ and on the line $x_1 - x_2 = -2$. See the figure.



The point (x_1,x_2) that lies on the line $x_1 + 3x_2 = 14$ and on the line $x_1 - x_2 = -2$ is $(2,4)$.

(Simplify your answer. Type an ordered pair.)

3. Consider the accompanying matrix as the augmented matrix of a linear system. State in words the next two elementary row operations that should be performed in the process of solving the system.

$$\begin{bmatrix} 1 & -4 & 3 & 0 & -2 \\ 0 & 3 & -6 & 0 & 5 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 & 4 \end{bmatrix}$$

What should be the first elementary row operation performed? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** Interchange row 3 and row 2.
- **B.** Scale row 1 by _____.
  (Type an integer or a simplified fraction.)
- **C.** Replace row 4 by its sum with -2 times row 3.
  (Type an integer or a simplified fraction.)
- **D.** Replace row 2 by its sum with _____ times row 4.
  (Type an integer or a simplified fraction.)

What should be the second elementary row operation performed? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** Interchange row 1 and row 2.
- **B.** Replace row 1 by its sum with _____ times row 4.
  (Type an integer or a simplified fraction.)
- **C.** Scale row 4 by $-\frac{1}{2}$.
  (Type an integer or a simplified fraction.)
- **D.** Replace row 3 by its sum with _____ times row 2.
  (Type an integer or a simplified fraction.)

4. The augmented matrix of a linear system has been reduced by row operations to the form shown. Continue the appropriate row operations and describe the solution set of the original system.

$$\begin{bmatrix} 1 & 6 & 2 & -3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 4 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** The solution set has exactly one element, (, ,). (Type integers or simplified fractions.)
- **B.** The solution set has infintely many elements.
- **C.** The solution set is empty.

5. Solve the system.

$$\begin{aligned} x_2 + 5x_3 &= 6 \\ x_1 + 2x_2 + 4x_3 &= 4 \\ 3x_1 + 5x_2 + 7x_3 &= -4 \end{aligned}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** The unique solution of the system is (, ,). (Type integers or simplified fractions.)
- **B.** The system has infinitely many solutions.
- **C.** The system has no solution.

6. Solve the system.

$$\begin{aligned} x_1 - 6x_3 &= 15 \\ 2x_1 + 2x_2 + x_3 &= 8 \\ x_2 + 3x_3 &= -4 \end{aligned}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** The unique solution of the system is (3 , 2 , −2). (Type integers or simplified fractions.)
- **B.** The system has infinitely many solutions.
- **C.** The system has no solution.

7. Determine if the given system is consistent. Do not completely solve the system.

$$\begin{aligned} x_1 + 3x_3 &= 4 \\ x_2 - 3x_4 &= 3 \\ -5x_2 + 3x_3 + 2x_4 &= 2 \\ 3x_1 + 6x_4 &= -2 \end{aligned}$$

Choose the correct answer below.

- **A.** The system is inconsistent because the system can be reduced to a triangular form that contains a contradiction.
- **B.** The system is consistent because the system can be reduced to a triangular form that indicates that a solution ex
- **C.** The system is inconsistent because the system cannot be reduced to a triangular form.
- **D.** The system is consistent because the system can be reduced to a triangular form that indicates that no solutions

8. Do the three lines $4x_1 - 8x_2 = 16$, $8x_1 + 40x_2 = -136$, and $-4x_1 - 48x_2 = 152$ have a common point of intersection? Explain.

Choose the correct answer below.

- **A.** The three lines have at least one common point of intersection.
- **B.** The three lines do not have a common point of intersection.
- **C.** There is not enough information to determine whether the three lines have a common point of intersection.

9. Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & h & 2 \\ 3 & 12 & 4 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The matrix is the augmented matrix of a consistent linear system if $h \neq$ 4 .
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- **B.** The matrix is the augmented matrix of a consistent linear system if $h =$ _____ .
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- **C.** The matrix is the augmented matrix of a consistent linear system for every value of h.
- **D.** The matrix is not the augmented matrix of a consistent linear system for any value of h.

10. Indicate whether the statements given in parts (a) through (d) are true or false and justify the answer.

a. Is the statement "Every elementary row operation is reversible" true or false? Explain.

- **A.** False, because only interchanging is a reversible row operation.
- **B.** False, because only scaling and interchanging are reversible row operations.
- **C.** True, because interchanging can be reversed by scaling, and scaling can be reversed by replacement.
- **D.** True, because replacement, interchanging, and scaling are all reversible. ✔

b. Is the statement "A 5×6 matrix has six rows" true or false? Explain.

- **A.** True, because a 5×6 matrix has five columns and six rows.
- **B.** False, because a 5×6 matrix has five rows and six columns. ✔
- **C.** True, because a 5×6 matrix has six columns and six rows.
- **D.** False, because a 5×6 matrix has five rows and five columns.

c. Is the statement "The solution set of a linear system involving variables $x_1, ..., x_n$ is a list of numbers $(s_1, ..., s_n)$ that makes each equation in the system a true statement when the values $s_1, ..., s_n$ are substituted for $x_1, ..., x_n$, respectively" true or false? Explain.

- **A.** False, because the description applies to a single solution. The solution set consists of all possible solutions. ✔
- **B.** False, because the list of numbers $(s_1, ..., s_n)$ is the solution set for a linear system involving the variables x_1,
- **C.** True, because the solution set of a linear system will have the same number of elements as the list of the variab
- **D.** True, because the list of variables $(x_1, ..., x_n)$ and the list of numbers $(s_1, ..., s_n)$ have a one-to-one correspo

d. Is the statement "Two fundamental questions about a linear system involve existence and uniqueness" true or false? Explain.

- **A.** False, because two fundamental questions address whether it is possible to solve the system with row operatior
- **B.** True, because two fundamental questions address whether the equations of the linear system exist in n-dimens
- **C.** False, because two fundamental questions address the type of row operations that can be used on the system a
- **D.** True, because two fundamental questions address whether the solution exists and whether there is only one sol ✔

11. Suppose the system to the right is consistent for all possible values of f and g. What can you say about the coefficients c and d? Justify your answer.

$$\begin{aligned} x_1 + 12x_2 &= f \\ cx_1 + dx_2 &= g \end{aligned}$$

Select the correct answer below.

- **A.** The row reduction of $\begin{bmatrix} 1 & 12 & f \\ c & d & g \end{bmatrix}$ to $\begin{bmatrix} 1 & 12 & f \\ 0 & d-12c & g-cf \end{bmatrix}$ shows that $d - 12c = 0$ since f and g are arbitrary. Othe
- **B.** The row reduction of $\begin{bmatrix} 1 & 12 & f \\ c & d & g \end{bmatrix}$ to $\begin{bmatrix} 1 & 12 & f \\ 0 & d-12c & g-cf \end{bmatrix}$ shows that $d - 12c \neq 0$ since f and g are arbitrary. Othe ✔
- **C.** Since f and g are arbitrary, c and d can be any value and the system will be consistent.
- **D.** Since f and g are arbitrary and the system is consistent for all possible values of f and g, $c = 0$ and $d = 1$. Otherw

12. Find the elementary row operation that transforms the first matrix into the second, and then find the reverse row operation that transforms the second matrix into the first.

$$\begin{bmatrix} 1 & -3 & 3 & 0 \\ 2 & -2 & 2 & -6 \\ 0 & 2 & -2 & 4 \end{bmatrix}, \begin{bmatrix} 1 & -3 & 3 & 0 \\ 0 & 4 & -4 & -6 \\ 0 & 2 & -2 & 4 \end{bmatrix}$$

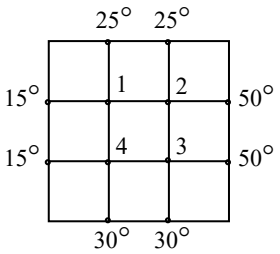
Find the elementary row operation that transforms the first matrix into the second. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** Replace row 2 by its sum with -2 times row 1.
  (Type an integer or a simplified fraction.)
- **B.** Interchange row 2 and row 3.
- **C.** Replace row 2 by its sum with ____ times row 3.
  (Type an integer or a simplified fraction.)
- **D.** Scale row 2 by ____.
  (Type an integer or a simplified fraction.)

Find the reverse operation that transforms the second matrix into the first. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** Replace row 2 by its sum with ____ times row 3.
  (Type an integer or a simplified fraction.)
- **B.** Replace row 2 by its sum with 2 times row 1.
  (Type an integer or a simplified fraction.)
- **C.** Scale row 2 by ____.
  (Type an integer or a simplified fraction.)
- **D.** Interchange row 2 and row 3.

13. An important concern in the study of heat transfer is to determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known. The figure represents a cross section of a metal beam, with negligible heat flow in the direction perpendicular to the plate. Let T_1, ..., T_4 denote the temperatures at the four interior nodes of the figure. The temperature at a node is approximately equal to the average of the four nearest nodes—to the left, above, to the right, and below. Solve the system of equations below to find the temperatures T_1, ..., T_4.



$$\begin{aligned} 4T_1 - T_2 - T_4 &= 40 \\ -T_1 + 4T_2 - T_3 &= 75 \\ -T_2 + 4T_3 - T_4 &= 80 \\ -T_1 - T_3 + 4T_4 &= 45 \end{aligned}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** The unique solution of the system is (25 , 33.75 , 35 , 26.25).
  (Type integers or decimals rounded to the nearest thousandth as needed.)
- **B.** The system has infinitely many solutions.
- **C.** The system has no solution.