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1. Row reduce the matrix to reduced echelon form. Identify the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 6 & 7 & 8 & 9 \\ 8 & 9 & 10 & 11 \end{bmatrix}$$

Row reduce the matrix to reduced echelon form and identify the pivot positions in the final matrix. The pivot positions are indicated by bold values. Choose the correct answer below.

A. $\begin{bmatrix} \mathbf{1} & 2 & 0 & 0 \\ 0 & 0 & \mathbf{1} & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} \mathbf{1} & 0 & 0 & 1 \\ 0 & \mathbf{1} & 0 & 6 \\ 0 & 0 & \mathbf{1} & 8 \end{bmatrix}$

C. $\begin{bmatrix} \mathbf{1} & 0 & -1 & -2 \\ 0 & \mathbf{1} & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} \mathbf{1} & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 \\ 0 & 0 & \mathbf{1} & \mathbf{1} \end{bmatrix}$

Identify the pivot positions in the original matrix. The pivot positions are indicated by bold values. Choose the correct answer below.

A. $\begin{bmatrix} \mathbf{1} & 2 & 3 & 4 \\ 6 & 7 & \mathbf{8} & 9 \\ 8 & 9 & 10 & 11 \end{bmatrix}$

B. $\begin{bmatrix} \mathbf{1} & 2 & 3 & 4 \\ 6 & \mathbf{7} & 8 & 9 \\ 8 & 9 & \mathbf{10} & 11 \end{bmatrix}$

C. $\begin{bmatrix} \mathbf{1} & 2 & 3 & 4 \\ 6 & \mathbf{7} & 8 & 9 \\ 8 & 9 & \mathbf{10} & \mathbf{11} \end{bmatrix}$

D. $\begin{bmatrix} \mathbf{1} & 2 & 3 & 4 \\ 6 & \mathbf{7} & 8 & 9 \\ 8 & 9 & 10 & 11 \end{bmatrix}$

List the pivot columns. Select all that apply.

- **A.** Column 4
- **B.** Column 3
- **C.** Column 1
- **D.** Column 2

2. Describe the possible echelon forms of a nonzero 2×2 matrix.

Select all that apply. (Note that leading entries marked with a ■ may have any nonzero value and starred entries (*) may have any value including zero.)

A. $\begin{bmatrix} \blacksquare & * \\ * & \blacksquare \end{bmatrix}$

B. $\begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} \blacksquare & * \\ 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} \blacksquare & * \\ * & * \end{bmatrix}$

E. $\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \end{bmatrix}$

F. $\begin{bmatrix} * & \blacksquare \\ 0 & * \end{bmatrix}$

3. Find the general solution of the system whose augmented matrix is given below.

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 4 & 8 & 7 & -2 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in any answer boxes to complete your answer.

A. $x_1 =$ ____
$x_2 =$ ____
$x_3 =$ ____
(Use integers or fractions for any numbers in the equation.)

B. $x_1 = -4 - 2x_2$
x_2 is free
$x_3 = 2$
(Use integers or fractions for any numbers in the equation.)

C. $x_1 =$ ____
x_2 is free
x_3 is free
(Use integers or fractions for any numbers in the equation.)

D. The system has no solution.

4. Find the general solution of the system whose augmented matrix is given below.

$$\begin{bmatrix} 0 & 1 & -2 & 4 \\ 1 & -2 & -1 & -4 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in any answer boxes to complete your answer.

A. $x_1 =$ ____
x_2 is free
x_3 is free
(Use integers or fractions for any numbers in the equation.)

B. $x_1 = 4 + 5x_3$
$x_2 = 4 + 2x_3$
x_3 is free
(Use integers or fractions for any numbers in the equation.)

C. $x_1 =$ ____
$x_2 =$ ____
$x_3 =$ ____
(Use integers or fractions for any numbers in the equation.)

D. The system has no solution.

5. Find the general solution of the system whose augmented matrix is given below.

$$\begin{bmatrix} 5 & -2 & 3 & 0 \\ 15 & -6 & 9 & 0 \\ 20 & -8 & 12 & 0 \end{bmatrix}$$

Choose the correct answer below.

A.
$x_1 = -5x_2$
$x_2 = 2x_3$
x_3 is free

B. ✓
$x_1 = \frac{2}{5}x_2 - \frac{3}{5}x_3$
x_2 is free
x_3 is free

C.
$x_1 = 5$
$x_2 = -2$
$x_3 = 3$

D.
The system has no solutions.

6. Find the general solution of the system whose augmented matrix is given below.

$$\begin{bmatrix} 1 & -7 & 0 & -1 & 0 & -4 \\ 0 & 1 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your answer.

A.
$x_1 =$ ____
x_2 is free
$x_3 =$ ____
x_4 is free
x_5 is free

B.
$x_1 =$ ____
$x_2 =$ ____
x_3 is free
$x_4 =$ ____
$x_5 =$ ____

C. ✓
$x_1 = 6 + 12x_5$
$x_2 = 1 + 2x_5$
x_3 is free
$x_4 = 3 - 2x_5$
x_5 is free

D.
The system is inconsistent.

7. Suppose each matrix below represents the augmented matrix for a system of linear equations. For matrices (a) and (b), determine if the system is consistent. If the system is consistent, determine if the solution is unique. (Note that leading entries marked with an X may have any nonzero value and starred entries (*) may have any value including zero.)

a. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & 0 \end{bmatrix}$ b. $\begin{bmatrix} \blacksquare & * & * & * & * \\ 0 & \blacksquare & * & * & * \\ 0 & 0 & 0 & 0 & \blacksquare \end{bmatrix}$

a. Select the correct answer below.

- **A.** The consistency of the system and the number of solutions cannot be determined with the given information.
- **B.** The system is inconsistent.
- **C.** The system is consistent, with infinitely many solutions.
- **D.** The system is consistent, with a unique solution. ✔

b. Select the correct answer below.

- **A.** The system is consistent, with a unique solution.
- **B.** The system is inconsistent. ✔
- **C.** The system is consistent, with infinitely many solutions.
- **D.** The consistency of the system and the number of solutions cannot be determined with the given information.

8. Choose h and k such that the system has (a) no solution, (b) a unique solution, and (c) many solutions.

$$x_1 + hx_2 = 2$$
$$3x_1 + 9x_2 = k$$

a. Select the correct answer below and fill in the answer box(es) to complete your choice.
(Type an integer or simplified fraction.)

- **A.** The system has no solutions only when $h \neq$ ____ and $k \neq$ ____.
- **B.** The system has no solutions only when $h =$ ____ and $k =$ ____.
- **C.** The system has no solutions only when $h =$ ____ and k is any real number.
- **D.** The system has no solutions only when $k =$ ____ and h is any real number.
- **E.** The system has no solutions only when $k \neq$ ____ and h is any real number.
- **F.** The system has no solutions only when $h \neq$ ____ and k is any real number.
- **G.** The system has no solutions only when $h =$ 3 and $k \neq$ 6. ✔
- **H.** The system has no solutions only when $h \neq$ ____ and $k =$ ____.

b. Select the correct answer below and fill in the answer box(es) to complete your choice.
(Type an integer or simplified fraction.)

- **A.** The system has a unique solution only when $k =$ ____ and h is any real number.
- **B.** The system has a unique solution only when $h \neq$ ____ and $k \neq$ ____.
- **C.** The system has a unique solution only when $h =$ ____ and k is any real number.
- **D.** The system has a unique solution only when $k \neq$ ____ and h is any real number.
- **E.** The system has a unique solution only when $h =$ ____ and $k \neq$ ____.
- **F.** The system has a unique solution only when $h \neq$ ____ and $k =$ ____.
- **G.** The system has a unique solution only when $h \neq$ 3 and k is any real number. ✔
- **H.** The system has a unique solution only when $h =$ ____ and $k =$ ____.

c. Select the correct answer below and fill in the answer box(es) to complete your choice.
(Type an integer or simplified fraction.)

- **A.** The system has many solutions only when $k \neq$ ____ and h is any real number.
- **B.** The system has many solutions only when $k =$ ____ and h is any real number.
- **C.** The system has many solutions only when $h =$ ____ and $k \neq$ ____.
- **D.** The system has many solutions only when $h \neq$ ____ and $k =$ ____.
- **E.** The system has many solutions only when $h =$ 3 and $k =$ 6. ✔
- **F.** The system has many solutions only when $h \neq$ ____ and $k \neq$ ____.
- **G.** The system has many solutions only when $h =$ ____ and k is any real number.
- **H.** The system has many solutions only when $h \neq$ ____ and k is any real number.

9. In parts **(a)** through **(e)** below, mark the statement True or False. Justify each answer.

(a) In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations.

Is this statement true or false?

- **A.** The statement is false. For each matrix, there is only one sequence of row operations that row reduces it.
- **B.** The statement is false. Each matrix is row equivalent to one and only one reduced echelon matrix. ✔
- **C.** The statement is true. It is possible for there to be several different sequences of row operations that row reduces a matrix.
- **D.** The statement is true. The echelon form of a matrix is always unique, but the reduced echelon form of a matrix might not be unique.

(b) The row reduction algorithm applies only to augmented matrices for a linear system.

Is this statement true or false?

- **A.** The statement is false. It is possible to create a linear system such that the row reduction algorithm does not apply to the corresponding augmented matrix.
- **B.** The statement is false. The algorithm applies to any matrix, whether or not the matrix is viewed as an augmented matrix for a linear system. ✔
- **C.** The statement is true. Every matrix with at least two columns can be interpreted as the augmented matrix of a linear system.
- **D.** The statement is true. The row reduction algorithm is only useful when it is used to find the solution of a linear system.

(c) A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix.

Is this statement true or false?

- **A.** The statement is false. Not every linear system has basic variables.
- **B.** The statement is true. It is the definition of a basic variable. ✔
- **C.** The statement is true. If a linear system has both basic and free variables, then each basic variable can be expressed in terms of the free variables.
- **D.** The statement is false. A variable that corresponds to a pivot column in the coefficient matrix is called a free variable, not a basic variable.

(d) Finding a parametric description of the solution set of a linear system is the same as solving the system.

Is this statement true or false?

- **A.** The statement is false. The solution set of a linear system can only be expressed using a parametric description if the system has at least one solution. ✔
- **B.** The statement is true. Solving a linear system is the same as finding the solution set of the system. The solution set of a linear system can always be expressed using a parametric description.
- **C.** The statement is false. The solution set of a linear system can only be expressed using a parametric description if the system has no more than one solution.
- **D.** The statement is true. Regardless of whether a linear system has free variables, the solution set of the system can be expressed using a parametric description.

(e) If one row in an echelon form of an augmented matrix is $\begin{bmatrix} 0 & 0 & 0 & 5 & 0 \end{bmatrix}$, then the associated linear system is inconsistent.

Is this statement true or false?

- **A.** The statement is true. The indicated row corresponds to the equation $5x_4 = 0$. This equation is not a contradiction, so the linear system is inconsistent.
- **B.** The statement is true. The indicated row corresponds to the equation $5 = 0$. This equation is a contradiction, so the linear system is inconsistent.
- **C.** The statement is false. The indicated row corresponds to the equation $5x_4 = 0$, which means the system is consistent.
- **D.** The statement is false. The indicated row corresponds to the equation $5x_4 = 0$, which does not by itself make the system inconsistent.

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10. A system of linear equations with more equations than unknowns is sometimes called an overdetermined system. Can such a system be consistent? Illustrate your answer with a specific system of three equations in two unknowns.

Choose the correct answer below.

- **A.** Yes, overdetermined systems can be consistent. For example, the system of equations below is consistent because it has the solution _____.
(Type an ordered pair.)
$x_1 = 2, x_2 = 4, x_1 + x_2 = 8$
- **B.** Yes, overdetermined systems can be consistent. For example, the system of equations below is consistent because it has the solution (2,4).
(Type an ordered pair.)
$x_1 = 2, x_2 = 4, x_1 + x_2 = 6$
- **C.** No, overdetermined systems cannot be consistent because there are fewer free variables than equations. For example, the system of equations below has no solution.
$x_1 = 2, x_2 = 4, x_1 + x_2 = 12$
- **D.** No, overdetermined systems cannot be consistent because there are no free variables. For example, the system of equations below has no solution.
$x_1 = 2, x_2 = 4, x_1 + x_2 = 24$