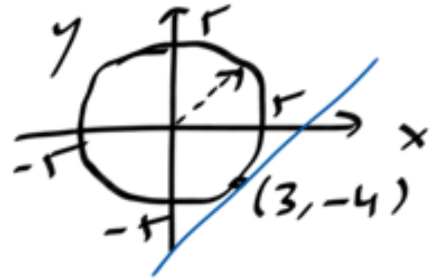
3.7 Implicit Differentiation

Ex Find the slope of the tangent line to the circle $x^2+y^2=25$ at pt $(3,-4)$.



Method I: $y=\sqrt{25-x^2}$, corresponds to the upper part of the circle.

$$y=-\sqrt{25-x^2}, \text{ for the lower part.}$$

$$y=-(25-x^2)^{1/2}$$

$$y' = \cancel{(-)}\frac{1}{\cancel{2}}\cdot(25-x^2)^{-\frac{1}{2}}(\cancel{-2}x) = \frac{x}{\sqrt{25-x^2}}$$

$$y'\Big|_{x=3} = \frac{x}{\sqrt{25-x^2}}\Bigg|_{x=3} = \frac{3}{\sqrt{25-9}} = \frac{3}{4}$$

The eqn. of the tangent line is

$$\frac{3}{4} = \frac{y-(-4)}{x-3} \Rightarrow \quad y=\frac{3}{4}x-\frac{25}{4}$$

Method II: Implicit diff.

$x^2+y^2=25$, $\quad \frac{dy}{dx}=y' \equiv$ slope at pt $(3,-4)$?

$$\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}(25)$$

$$\cancel{2}x\cancel{\frac{dx}{dx}}+\cancel{2}y\frac{dy}{dx}=0 \Rightarrow x+y\frac{dy}{dx} \Rightarrow \left.\frac{dy}{dx}\right|_{(3,-4)}=-\left.\frac{x}{y}\right|_{(3,-4)}=-\frac{3}{-4}=\frac{3}{4}$$

Ex Find the eqn. of the tangent line to the curve

$$x^3+y^3-9xy=0 \qquad \text{at pt } (1,2)$$

$$\frac{d}{dx}(x^3+y^3-9xy)=\frac{d}{dx}0$$

$$3x^2+3y^2\frac{dy}{dx}-9y-9x\frac{dy}{dx}=0$$

$$3x^2-9y+(3y^2-9x)\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{9y-3x^2}{3y^2-9x}=\frac{3y-x^2}{y^2-3x}$$

$$\left.\frac{dy}{dx}\right|_{(1,2)}=\left.\frac{3y-x^2}{y^2-3x}\right|_{(1,2)}=5$$

$$5=\frac{y-2}{x-1} \Rightarrow y=5x-3$$

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Ex

Find the eqn. of the tangent line to the curve

$$x^2+xy-y^2-1=0 \qquad \text{at pt } (2,3)$$

$$2x+y+x\frac{dy}{dx}-2y\frac{dy}{dx}=0$$

$$2x+y+(x-2y)\frac{dy}{dx}=0 \Rightarrow \left.\frac{dy}{dx}\right|_{(2,3)} = \left.\frac{2x+y}{2y-x}\right|_{(2,3)} = \frac{7}{4}$$

$$\frac{7}{4}=\frac{y-3}{x-2} \Rightarrow y=\frac{7}{4}x-\frac{1}{2}$$

Ex Find the pts on the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$, where the slope of the tangent line is -1.

$$x^{\frac{1}{2}}+y^{\frac{1}{2}}=a^{1/2} \Rightarrow \cancel{\tfrac{1}{2}}x^{-\frac{1}{2}}+\cancel{\tfrac{1}{2}}y^{-\frac{1}{2}}\frac{dy}{dx}=0$$

$$\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\sqrt{\frac{y}{x}}$$

$$\frac{dy}{dx}=-\sqrt{\frac{y}{x}}=-1 \Rightarrow y=x \Rightarrow$$

$$\sqrt{x} + \sqrt{y} = \sqrt{a} \qquad \text{since } y = x$$

$$\sqrt{x} + \sqrt{x} = \sqrt{a}$$

$$2\sqrt{x} = \sqrt{a} \Rightarrow \sqrt{x} = \sqrt{a}/2 \Rightarrow x = a/4 = y$$

$$\text{pts} \Rightarrow \left(\frac{a}{4}, \frac{a}{4}\right)$$

Ex $x^2(x-y)^2 = x^2 - y^2 \qquad y' = ?$

$$2x(x-y)^2 + x^2 \cdot 2(x-y) \cdot \left(1 - \frac{dy}{dx}\right) = 2x - 2y\frac{dy}{dx}$$

$$2x(x-y)^2 + 2x^2(x-y) - 2x^2(x-y)\frac{dy}{dx} + 2y\frac{dy}{dx} = 2x$$

$$\left[2x(x-y)^2 + 2x^2(x-y) - 2x\right] / \left[2x^2(x-y) - 2y\right] = \frac{dy}{dx}$$

Ex $y^2 = \dfrac{x-1}{x+1}$

$$2y\frac{dy}{dx} = \frac{\cancel{x}+1 - \cancel{x}+1}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{1}{(x+1)^2 y}$$

Ex

$$x^3 = \frac{2x-y}{x+3y} = (2x-y)(x+3y)^{-1}$$

$$3x^2 = \left(2-\frac{dy}{dx}\right)(x+3y)^{-1} + (2x-y)(-1)(x+3y)^{-2}\left(1+3\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = ?$$

Ex $\quad x = \tan y$

$$1 = \sec^2 y \cdot \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y$$

Ex $\quad xy = \cot(xy)$

$$y + x\frac{dy}{dx} = -\csc^2(xy)\cdot\left(y + x\frac{dy}{dx}\right)$$

$$y + x\frac{dy}{dx} = -y\csc^2 xy - x\frac{dy}{dx}\csc^2 xy$$

$$(x + x\csc^2 xy)\frac{dy}{dx} = -\frac{y}{x}\cdot\cancel{\frac{1+\csc^2 xy}{1+\csc^2 xy}} = -y/x$$