Math No:

Full Name : KEY

İSTANBUL ŞEHİR UNIVERSITY

Math 104 – Final Exam

(10 January 2019, Time: 11:00-12:00)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones, smart watches and electronic devices are to be kept shut and out of sight. All cell phones and smart watches are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
6 pts	6 pts	6 pts	6 pts	24 pts

Q1. Find the MacLaurin series of the function $f(x) = xsin(\sqrt{x})$. [*Hint*: *You should be able to write down the MacLaurin Series for sinx first and then adapt it to this problem. In case if you need to use the Taylor Series, it is given below*]

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad \text{converges for all } x.$$

$$\Rightarrow \sin\sqrt{x} = \sum_{n=0}^{\infty} \frac{(-1)^n (x^{1/2})^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1/2}}{(2n+1)!}$$

$$x \sin\sqrt{x} = \sum_{n=0}^{\infty} \frac{(-1)^n x . x^{n+1/2}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+3/2}}{(2n+1)!}$$

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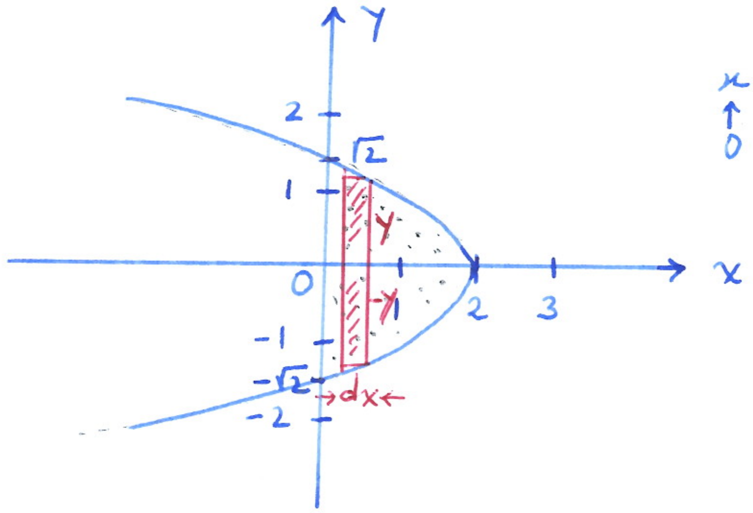
Q2. Consider the region bounded between the parabola $x = 2 - y^2$ and the vertical line $x = 0$. (a) Sketch the region, and (b) determine its area.



$x = 2 - y^2$

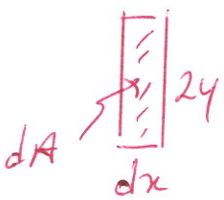
$0 = 2 - y^2$

$y = \pm\sqrt{2}$, y-intercepts

$x = 2 - y^2$

$y = 0$

$x = 2$, x-intercept.



$dA = 2y\,dx$

$dA = 2(2-x)^{1/2}dx$

$A = 2\int_0^2 (2-x)^{1/2}dx$

$u = 2 - x$ | $u(0) = 2$

$-du = dx$ | $u(2) = 0$

$A = -2\int_{u(0)}^{u(2)} u^{1/2}du$

$A = -2\, u^{3/2} \cdot \frac{2}{3}\Big|_2^0$

$A = -\frac{4}{3}\left[(0)^{3/2} - (2)^{3/2}\right]$

$= \frac{4}{3}\cdot\sqrt{2^3}$

$= \frac{8}{3}\sqrt{2}$ //

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Q3. Evaluate the following limit, if it exists: $\lim_{x\to\infty}\underbrace{\left(1-sin\frac{1}{x}\right)^{x}}_{y} = 1^{\infty}$, An indeterminate power!

$$y = \left(1-\sin\frac{1}{x}\right)^{x}$$

$$\ln y = x\ln\left(1-\sin\frac{1}{x}\right)$$

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty}\frac{\ln\left(1-\sin\frac{1}{x}\right)}{\frac{1}{x}}$$

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty}\frac{\frac{+\cancel{\frac{1}{x^2}}\cos\frac{1}{x}}{1-\sin\frac{1}{x}}}{-\cancel{\frac{1}{x^2}}}$$

$$\lim_{x\to\infty}\ln y = -\lim_{x\to\infty}\frac{\cos\frac{1}{x}}{1-\sin\frac{1}{x}} = -1$$

$$\lim_{x\to\infty} y = e^{-1} = 1/e$$

$$\therefore \lim_{x\to\infty}\left(1-\sin\frac{1}{x}\right)^{x} = 1/e \;//$$

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Q4. (a) Evaluate the following integral $\int cos^2 2x \, sin^3 2x dx$

$$\int \cos^2 2x \underbrace{\sin^2 2x}_{1-\cos^2 2x} \cdot \sin 2x \, dx = -\frac{1}{2}\int u^2(1-u^2)\,du$$

$u = \cos 2x$

$-\frac{1}{2}du = \sin 2x \, dx$

$$= -\frac{1}{2}\left[\frac{u^3}{3} - \frac{u^5}{5}\right] + C$$

$$= \frac{u^3}{2}\left[\frac{1}{3} - \frac{u^2}{5}\right] + C$$

$$= \frac{\cos^3 2x}{2}\left[\frac{1}{3} - \frac{\cos^2 2x}{5}\right] + C$$

(b) Evaluate the following integral $\int x \, e^x dx$

$u = x$ | $dv = e^x dx$

$du = dx$ | $v = e^x$

$$\int x e^x dx = uv - \int v \, du$$

$$= x e^x - \int e^x dx$$

$$= x e^x - e^x + C$$

$$= e^x(x-1) + C$$