

MATH 104 TUTORIAL 13 ANSWERS

1) a) $P_0(x) = 1,$

$$P_1(x) = 1 + 2\left(x - \frac{\pi}{4}\right), P_2(x) = 1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2, P_3(x) = 1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3$$

b) $P_0(x) = 1,$

$$P_1(x) = 1 - \frac{1}{2}x, P_2(x) = 1 - \frac{1}{2}x - \frac{1}{8}x^2, P_3(x) = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3$$

2) a) $2 + 3x + 3x^2 + 3x^3 + \dots = 2 + \sum_{n=1}^{\infty} 3x^n$

b) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

3) a) $-2 + 11(x-1) + 7(x-1)^2 + 2(x-1)^3$

b) $2^x = 2 + (2 \ln 2)(x-1) + \frac{2(\ln 2)^2}{2}(x-1)^2 + \frac{2(\ln 2)^3}{3!}(x-1)^3 + \dots = \sum_{n=0}^{\infty} \frac{2(\ln 2)^n (x-1)^n}{n!}$

4) a) $x - \frac{\pi^2 x^3}{2!} + \frac{\pi^4 x^5}{4!} - \frac{\pi^6 x^7}{6!} + \dots$

b) $\frac{d}{dx} \left(\frac{1}{1-x} \right) = \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + \dots = \sum_{n=1}^{\infty} nx^{n-1} = \sum_{n=0}^{\infty} (n+1)x^n$

c) $x \tan^{-1} x^2 = x \left(x^2 - \frac{1}{3}(x^2)^3 + \frac{1}{5}(x^2)^5 - \frac{1}{7}(x^2)^7 + \dots \right)$
 $= x^3 - \frac{1}{3}x^7 + \frac{1}{5}x^{11} - \frac{1}{7}x^{15} + \dots = \sum_{n=1}^{\infty} \frac{(-1)^n x^{4n-1}}{2n-1}$

5) a) $1 + 4x + 6x^2 + 4x^3 + x^4$

b) $1 - 2x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{16}x^4$

6) $= \lim_{x \rightarrow \infty} \left(-1 + \frac{1}{2x^2} - \frac{1}{6x^4} + \dots \right) = -1$