Antiderivatives

DEFINITION A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

If F is antiderivative of f, so is $F+c$, since $(F+\underbrace{c}_{\text{a const}})' = F' = f$

The indefinite integral $\int f(x)\,dx$ is the set of all antiderivatives of f.

Ex: $\int x\,dx = \dfrac{x^{1+1}}{1+1} + c = \dfrac{x^2}{2} + c$ ← integral constant

Rules of Antiderivatives

$$\int a\,f(x)\,dx = a\int f(x)\,dx$$

$$\int \big(f(x) \pm g(x)\big)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \qquad n \neq -1$$

Ex Find the sln of diff. eqn. $\frac{dy}{dx} = \sqrt{x}(x+1)$ which passes through $(1,2)$. "initial-value problem".

$$\frac{dy}{dx} = \sqrt{x}\,(x+1) = x^{3/2} + x^{1/2}$$

$$\int dy = \int x^{3/2}\,dx + \int x^{1/2}\,dx$$

$$y = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C$$

$$y(x) = \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C$$

$$2 \; y(1) = \frac{2}{5}(1)^{5/2} + \frac{2}{3}(1)^{3/2} + C$$

$$2 = \frac{2}{5} + \frac{2}{3} + C \Rightarrow C = \frac{14}{15} \;//$$

$$y(x) = \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + \frac{14}{15} \;//$$

Some useful formulas

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

Ex $$\int \left(\frac{3}{\sqrt{x}} + \sin 2x\right) dx = 3\int x^{-\frac{1}{2}}dx + \int \sin 2x\,dx$$

$$= 3 \cdot \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - \frac{1}{2}\cos 2x + C$$

$$= 6x^{1/2} - \frac{1}{2}\cos 2x + C$$

$$\frac{d}{dx}\left[6x^{1/2} - \frac{1}{2}\cos 2x + C\right] = 3x^{-\frac{1}{2}} + \sin 2x \;//$$

Ex $\int\left(\frac{1}{x^2}-x^2-\frac{1}{3}\right)dx=\int x^{-2}dx-\int x^2dx-\frac{1}{3}\int dx$

$$=\frac{x^{-2+1}}{-2+1}-\frac{x^{2+1}}{2+1}-\frac{1}{3}\cdot\frac{x^{0+1}}{0+1}+C$$

$$=-\frac{1}{x}-\frac{1}{3}x^3-\frac{1}{3}x+C$$

Ex

$$\int\left(x^{-\frac{1}{3}}+\sqrt[3]{x}\right)dx=\frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}+\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}+C$$

$$=\frac{3}{2}x^{2/3}+\frac{3}{4}x^{4/3}+C$$