Full Name : KEY

Student ID:

İSTANBUL ŞEHİR UNIVERSITY

Math 104, Final Exam

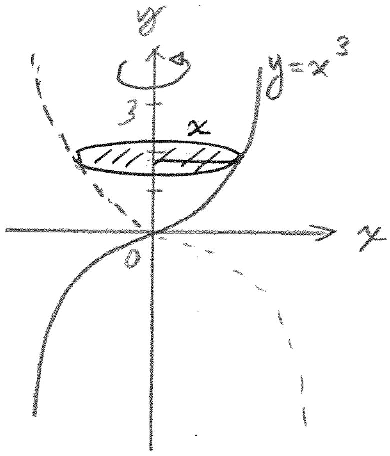
(12 January 2015, Time: 12:00-14:00)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 11 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	TOT
10pt	10pt	10pt	10pt	10pt	10pt	10pt	10pt	10pt	10pt	10pt	110p

Q1. Find the volume of the solid generated by revolving $y = x^3$ about y-axis for the region between y=0 and y=3.



$$V = \int_0^3 \pi x^2 \, dy$$

$$y = x^3$$
$$y^{1/3} = x$$

$$= \int_0^3 \pi \left(y^{1/3}\right)^2 dy$$

$$= \pi \int_0^3 y^{2/3} dy$$

$$= \frac{3}{5}\pi \, y^{5/3} \Big|_0^3$$

$$= \frac{3}{5}\pi \, 3^{5/3}$$

$$= \frac{3^{8/3}}{5}\pi \quad //$$

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Q2. Evaluate the following integral: $\int xe^{-x}\,dx$

$u = x \qquad dv = e^{-x}dx$

$du = dx \qquad v = -e^{-x}$

$$\int xe^{-x}dx = uv - \int v\,du = -xe^{-x} + \int e^{-x}dx$$

$$= -xe^{-x} - e^{-x} + C$$

$$= -e^{-x}(x+1) + C \quad //$$

Q3. Evaluate the following integral: $\int \frac{dx}{x^2+x-2}$

$$\frac{1}{x^2+x-2} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$1 = (x+2)A + (x-1)B$$

$x=1: \quad 1 = 3A \Rightarrow A = 1/3$

$x=-2: \quad 1 = -3B \Rightarrow B = -1/3$

$$\int \frac{dx}{x^2+x-2} = \frac{1}{3}\int\frac{dx}{x-1} - \frac{1}{3}\int\frac{dx}{x+2}$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C$$

$$= \frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C \quad //$$

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Q4. Determine the following integral: $\int \sin^2 x \cos^3 x \, dx$

$$\int \sin^2 x \cos^2 x \cos x \, dx = \int \sin^2 x (1 - \sin^2 x) \cos x \, dx$$

$$u = \sin x, \quad du = \cos x \, dx$$

$$= \int u^2 (1 - u^2) \, du$$

$$= \int (u^2 - u^4) \, du = \frac{u^3}{3} - \frac{u^5}{5} + c$$

$$= \frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + c \quad //$$

Q5. Determine the following integral: $\int \sin x \cos 3x \, dx$

$$\sin A \cos B = \frac{1}{2} \left[\sin(A+B) + \sin(A-B) \right]$$

$A = x$

$B = 3x$

$$\int \sin x \cos 3x \, dx = \frac{1}{2} \int \sin 4x \, dx + \frac{1}{2} \int \sin(-2x) \, dx$$

$$= \frac{1}{2} \left(\frac{-\cos 4x}{4} \right) + \frac{1}{2} \left(\frac{\cos 2x}{2} \right) + C$$

$$= -\frac{1}{8} \cos 4x + \frac{1}{4} \cos 2x + C \quad //$$

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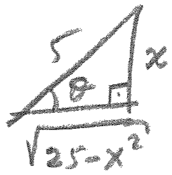
Q6. Determine the following integral:

$$\int \frac{x^2}{\sqrt{25-x^2}}dx$$



$x = 5\sin\theta$

$dx = 5\cos\theta \, d\theta$

$$\int \frac{25\sin^2\theta \cdot 5\cos\theta \, d\theta}{\sqrt{5^2-5^2\sin^2\theta}} = \int \frac{25\cdot\cancel{5}\cdot\sin^2\theta\cdot\cancel{\cos\theta}\, d\theta}{\cancel{5}\underbrace{\sqrt{1-\sin^2\theta}}_{\cancel{\cos\theta}}}$$

$$= 25\int \sin^2\theta \, d\theta$$

$$= 25\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$= \frac{25}{2}\left(\theta - \frac{1}{\cancel{2}}\underbrace{\sin 2\theta}_{\cancel{2}\sin\theta\cdot\cos\theta}\right) + C$$

$$= \frac{25}{2}\left(\theta - \sin\theta\cdot\cos\theta\right) + C$$

$$= \frac{25}{2}\left[\operatorname{ArcSin}\left(\frac{x}{5}\right) - \frac{x}{5}\cdot\frac{\sqrt{25-x^2}}{5}\right] + C$$

$$= \frac{25}{2}\operatorname{ArcSin}\left(\frac{x}{5}\right) - \frac{x}{2}\sqrt{25-x^2} + C \quad //$$

Q7. Find $\frac{\partial w}{\partial x}$ *and* $\frac{\partial w}{\partial y}$ for function $w = xy\sin(xy)$

$$\frac{\partial w}{\partial x} = y\sin(xy) + xy^2\cos(xy) \quad //$$

$$\frac{\partial w}{\partial y} = x\sin(xy) + x^2y\cos(xy) \quad //$$

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Q8. (a) Determine the limit of the following sequence: $\left\{\dfrac{n\cos(n!)}{n^3-1}\right\}_{n=1}^{\infty}$

By the Sandwich Thm:

$$-1 \leq \cos(n!) \leq 1$$

$$\lim_{n\to\infty} \quad -\frac{n}{n^3-1} \leq \frac{n}{n^3-1}\cos(n!) \leq +\frac{n}{n^3-1}$$

$$0 \leq \text{has to be zero} \leq 0$$

(b) Find the sum of the following series: $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{3}{3^n}$

This is a Geometric Series:

$$3\sum_{n=0}^{\infty}\left(-\frac{1}{3}\right)^n = \frac{3}{1+\frac{1}{3}} = \frac{9}{4} \,/\!/, \quad \text{since } \left|-\frac{1}{3}\right| < 1$$

Q9. Determine if the following series converges or diverges: $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt[3]{3n}}$

By the Integral Test:

$$\int_1^{\infty}\frac{dx}{(3x)^{1/3}} = \frac{1}{3}\int u^{-1/3}\,du = \frac{1}{3}\cdot\frac{3}{2}\,u^{2/3}\Big|_1^{\infty} = \infty \quad \text{diverges.}$$

$$u = 3x, \quad \frac{1}{3}du = dx$$

However, by the Thm on the Alternating Series:

(i) $\dfrac{1}{\sqrt[3]{3n}}$ are all positive

(ii) $\dfrac{1}{\sqrt[3]{3n}}$ is non-increasing

(iii) $\lim \dfrac{1}{\sqrt[3]{3n}} \to 0$ $\quad\therefore\quad \displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt[3]{3n}}$ <u>CONVERGES conditionally!</u>

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Q10. For what values of x does the following series converge? [*Hint: You may use the Generalized Ratio Theorem*]

$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1 \quad \text{to converge}$$

$$\rho = \lim_{n\to\infty} \left| \frac{x^{2(n+1)}}{[2(n+1)]!} \cdot \frac{(2n)!}{x^{2n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^2 \cdot \cancel{x^{2n}}}{(2n+2)(2n+1)\cancel{(2n)!}} \cdot \frac{\cancel{(2n)!}}{\cancel{x^{2n}}} \right|$$

$$= \frac{1}{2}|x^2| \underbrace{\lim_{n\to\infty} \frac{1}{(n+1)(2n+1)}}_{0} = 0$$

The series converges for <u>all</u> x-values

Q11. Find the Taylor series of $f(x) = e^{-x/2}$ at $x=0$.

$f(x) = e^{-x/2} \rightarrow f(0) = 1$

$f'(x) = -\frac{1}{2} e^{-x/2} \rightarrow f'(0) = -\frac{1}{2}$

$f''(x) = +\frac{1}{4} e^{-x/2} \rightarrow f''(0) = \frac{1}{4}$

$f'''(x) = -\frac{1}{8} e^{-x/2} \rightarrow f'''(0) = -\frac{1}{8}$

$f^{(4)}(x) = +\frac{1}{16} e^{-x/2} \rightarrow f^{(4)}(0) = \frac{1}{16}$

$f^{(5)}(x) = -\frac{1}{32} e^{-x/2} \rightarrow f^{(5)}(0) = \frac{-1}{32}$

$\vdots \qquad \vdots$

$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + f^{(4)}(0)\frac{x^4}{4!} + \cdots$

$f(x) = 1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{48} + \frac{x^4}{384} - \frac{x^5}{32.5!} + \cdots$