MATH2059 Numerical Methods
Homework 2

1. Find the binary expansions of 1/5, 1/7, 1/9 and 1/11.
2. Compute the expression

$$Y = X\left(\left(\left(\frac{2}{X} + X\right) - X\right) - \frac{1}{X}\right)$$

For $X = 10^{-k}, k = 1,2, \ldots 10$. First compose the vector X with components $X(k) = 10^{-k}$ and then use the command

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>> Y = X.*(((2./X + X) - X) - 1./X)
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To compute the vector Y. Compare with the exact answer Y=[1,1,…1] and comment on the results.

3. (a) Evaluate the polynomial $y = x^3 - 7x^2 + 8x - 0.35$ at x = 1.37. Use 3-digit arithmetic with chopping. Evaluate the percent relative error.
(b) Repeat (a) but express y as $y = \big((x - 7)x + 8\big)x - 0.35$. Evaluate the error and compare with part (a).

4. Consider the function $f(x) = x^3 - 2x + 4$ on the interval [-2,2] with h=0.25. Use forward, backward, and centered finite difference approximations for the first and second derivatives so as to graphically illustrate which approximation is more accurate. Graph all three first-derivative finite difference approximations along with the theoretical, and do the same for the second derivative as well.

5. Determine the positive real root of $\ln(x^2) = 0.7$. (a) graphically, (b) using three iterations of the bisection method, with initial guesses of $x_l = 0.5$ and $x_u = 2$, and (c) using three iterations of the false-position method, with the same initial guesses as in (b).

6. The upward velocity of a rocket can be computed by the following formula:

$$v = u \ln\frac{m_0}{m_0 - qt} - gt$$

Where $v =$ upward velocity, $u =$ the velocity at which fuel is expelled relative to the rocket, $m_0 =$ the initial mass of the rocket at time t=0, $q =$ the fuel consumption rate, and $g =$ the downward acceleration of the gravity (assumed constant = 9.81m/s^2). If $u = 1800$ m/s, m_0= 160,000 kg, and , $q = 2600$ kg/s, compute the time at which $v = 750$ m/s. (Hint: t is somewhere between 10 and 50 s). Determine your result so that it is within 1% of the true value. Check your answer.

7. Employ fixed-point iteration to locate the root of $f(x) = \sin(\sqrt{x}) - x$. Use an initial guess of $x_0 = 0.5$ and iterate until $\varepsilon_a \leq 0.01\%$. Verify that the process is linearly convergent.

8. Determine the highest real root of $f(x) = x^3 - 6x^2 + 11x - 6.1$
   (a) Graphically.
   (b) Using the Newton-Raphson method (three iterations, $x_i = 3.5$).
   (c) Using the secant method (three iterations, $x_{i-1} = 2.5$ and $x_i = 3.5$).
   (d) Using the modified secant method (three iterations, $x_i = 3.5$, $\delta = 0.01$).
   (e) Determine all the roots with MATLAB.

How to Submit Your Homework:

1. Each student should submit his/her own homework. You can discuss the questions with your friends, but you must write your own code. Group work is not allowed.

2. Write a detailed report, which includes explanations about each part in each question. Explain how your scripts and functions work, i.e., which parts of your functions/scripts accomplish which task and how it is accomplished. Include the outputs of your functions to your report. You can save a figure as a *.jpg image file using "File —> Save as" in the Figure window. Then, you can include the jpg image to your Word document.

3. Don't forget to put detailed comments into your functions/scripts to explain what your code is doing. Also indicate the inputs and outputs in the comment section. (% sign is used to put comments in MATLAB)

4. Combine your report and MATLAB codes into a single file. Plots should go into the report. Name your zip file as "name_surname_studentnumber_hw_no.zip". For example, a student whose name is Ayşe Çalışkan and student number is 1234567 will name her file as: "ayse_caliskan_1234567_hw1.zip" for the first homework. Also, write your name, surname and student number as comments at the beginning of your codes.

5. Submit your homework via Google Classroom before the deadline.