KEY

Istanbul Şehir University
Math 104

Date: 10 May 2014	Full Name:
Time: 10:00-11:30	
	Student ID:
Spring 2014 Third Exam	

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. You may continue your solutions on the back of the sheets. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cellphones and electronic devices are to be kept shut and out of sight.

Q1	Q2	Q3	Q4	TOTAL
20 pts	25 pts	25 pts	30 pts	100 pts

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1) Determine whether the sequence given below converges or diverges. If it converges, find the limit.

(a) $\left\{\dfrac{\cos 2n}{n!}\right\}$

$$-1 \le \cos 2n \le 1$$

$$-\frac{1}{n!} \le \frac{\cos 2n}{n!} \le \frac{1}{n!}$$

$\lim_{n\to\infty}$: $-\frac{1}{n!} \to 0$, $\frac{1}{n!} \to 0$

$\therefore$ By the Sandwich thm, $\boxed{\lim_{n\to\infty} \frac{\cos 2n}{n!} = 0}$

(b) $\left\{(e^n+n)^{1/n}\right\}$ $\quad \infty^0$, indeterminate form

Let $y = (e^x + x)^{1/x}$

$\ln y = \frac{1}{x}\ln(e^x+x)$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(e^x+x)}{x} \quad \infty/\infty$$

$$\underset{\text{L'Hospital}}{=} \lim_{x\to\infty} \frac{\frac{e^x+1}{e^x+x}}{1} \quad \infty/\infty$$

$$\underset{\text{L'Hospital}}{=} \lim_{x\to\infty} \frac{e^x}{e^x+1}$$

$$= \lim_{x\to\infty} \frac{1}{1+e^{-x}} = 1$$

$\ln y \to 1 \Rightarrow y \to e^1 = e$

$$\therefore \boxed{\lim_{n\to\infty} (e^n+n)^{1/n} = e}$$

(There may be other ways of doing these problems)

2) Determine whether the series given below converges or diverges:

(a) $\sum_{n=1}^{\infty} \frac{4n}{n+2}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{4n}{n+2} = \lim_{n\to\infty} \frac{4}{1+2/n} = 4 \neq 0$$

$\therefore$ The series $\boxed{\text{diverges}}$ by the n^{th} term test.

(b) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$ The terms are comparable to

$$\frac{n}{n^4} = \frac{1}{n^3}$$

Limit Comparison Test:

$$\lim_{n\to\infty} \frac{\frac{2n+1}{n^2(n+1)^2}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{n^3(2n+1)}{n^2(n+1)^2}$$

$$= \lim_{n\to\infty} \frac{2n^4+n^3}{n^4+2n^3+n^2} = \lim_{n\to\infty} \frac{2+1/n}{1+2/n+1/n^2} = 2$$

$\neq 0, \infty$

Since $\sum \frac{1}{n^3}$ is a p-series, $p=3>1$, it converges.

$\therefore$ The given series $\boxed{\text{converges}}$ by the Limit Comparison Test.

3) Determine whether the series given below converges or diverges:

(a) $\sum_{n=2}^{\infty} \frac{3}{n(\ln n)^2}$ Integral Test

$$\int_2^{\infty} \frac{3\,dx}{x(\ln x)^2} = \int \frac{3\,du}{u^2} = -\frac{3}{u}\Big| = -\frac{3}{\ln x}\Big|_2^{\infty}$$

$u = \ln x \Rightarrow du = \frac{dx}{x}$

$$= 0 + \frac{3}{\ln 2}$$

Since the improper integral converges, the series $\boxed{\text{converges}}$ by the Integral Test

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2 2^n}{3^n}$ Generalized Ratio Test

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1}(n+1)^2 2^{n+1}}{3^{n+1}}}{\frac{(-1)^n n^2 2^n}{3^n}} \right|$$

$$= \lim_{n\to\infty} \frac{2}{3}\left(\frac{n}{n+1}\right)^2 = \frac{2}{3}\underbrace{\left(\lim \frac{n}{n+1}\right)^2}_{1} = \frac{2}{3} < 1$$

$\therefore$ The series $\boxed{\text{converges,}}$ by the Generalized Ratio Test.

4) Given the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} x^n$

(a) Find the radius of convergence.
(b) Find the interval of convergence.

Generalized Ratio Test:

$$\rho = \lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} x^{n+1}}{\sqrt{n+1}}}{\frac{(-1)^n x^n}{\sqrt{n}}} \right| = |x| \lim_{n\to\infty} \sqrt{\frac{n}{n+1}}$$

$$= |x| \underbrace{\sqrt{\lim_{n\to\infty} \frac{n}{n+1}}}_{1} = |x|$$

$\therefore$ The series converges absolutely if $|x|<1$, diverges if $|x|>1$.

$x = 1 \Rightarrow$ The series is $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

✓ This is an alternating series

✓ $\lim_{n\to\infty} \frac{(-1)^n}{\sqrt{n}} = 0$

✓ The sequence $\left\{ \frac{1}{\sqrt{n}} \right\}$ is decreasing

$\therefore$ It converges by the Alternating Series Test

$x = -1 \Rightarrow$ the series is $\sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

This is a p-series, $p = \frac{1}{2} < 1$, diverges.

$\therefore$ The interval of convergence is $-1 < x \leq 1$

The radius of convergence is 1