Math No:

Full Name : KEY

İSTANBUL ŞEHİR UNIVERSITY

Math 104 - 3rd Midterm Exam

(5 December 2015, Time: 11:30-12:30)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which may have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
5 pts	4 pts	5 pts	5 pts	19 pts

Q1. Evaluate the following improper integral: $\int_{-\infty}^{-2} \frac{2dx}{x^2-1}$

$$\frac{2}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$

$2 = (x+1)A + (x-1)B$; for $x=1$, $A=1$; for $x=-1$, $B=-1$

$$\int_{-\infty}^{-2} \frac{2\,dx}{x^2-1} = \int_{-\infty}^{-2} \frac{dx}{x-1} - \int_{-\infty}^{-2} \frac{dx}{x+1}$$

$$= \ln|x-1| - \ln|x+1| \Big|_{-\infty}^{-2}$$

$$= \ln\left(\frac{x-1}{x+1}\right)\Big|_{-\infty}^{-2}$$

$$= \ln 3 - \lim_{x\to-\infty} \ln\left(\frac{x-1}{x+1}\right)$$

$$= \ln 3 - 0$$

$$= \ln 3$$

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Q2. Determine if the following sequences converge or diverge. If they converge, find their limits.

(a) $\left\{ \dfrac{1-n^3}{70-4n^2} \right\}_{n=1}^{\infty}$

$$\lim_{n\to\infty} \frac{1-n^3}{70-4n^2} = \lim_{n\to\infty} \frac{1/n^3 - 1}{70/n^3 - 4/n} = -\infty, \text{ diverges}$$

or

$$\lim_{n\to\infty} \frac{+3n^2}{+8n} = \infty, \text{ diverges} \quad \text{(L'Hop. Rule)}$$

(b) $\left\{ \left(\dfrac{3}{n}\right)^{1/n} \right\}_{n=1}^{\infty}$

$$y = \left(\frac{3}{x}\right)^{\frac{1}{x}}$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{1}{x} \ln(3/x)$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\frac{-3/x^2}{3/x}}{1} \quad \text{(L'Hop.)}$$

$$= -\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} y = e^0 = 1$$

$$\lim_{x\to\infty} (3/x)^{1/x} = 1$$

The sequence converges to 1.

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Q3. Determine whether the following series converge or diverge

(a) $\sum_{n=2}^{\infty} e^{-n/2}$

Using the Integral Test: $\int_2^{\infty} e^{-x/2}\,dx = -2\int e^{u}\,du = -e^{-x/2}\Big|_2^{\infty} = 1/e$

$u = -x/2$

$-2du = dx$

This is a geometric series with $|e^{-1/2}| < 1$

$$\sum_{n=2}^{\infty}(e^{-\frac{1}{2}})^n = \sum_{n=0}^{\infty}(e^{-\frac{1}{2}})^{n+2} = \frac{1}{e}\sum_{n=0}^{\infty}(e^{-\frac{1}{2}})^n = \frac{1}{e}\cdot\frac{1}{1-\frac{1}{\sqrt{e}}} = \frac{1}{e}\cdot\frac{\sqrt{e}}{\sqrt{e}-1}$$

From the root test:

$$\rho = \lim_{n\to\infty} \sqrt[n]{(e^{-1/2})^n} = \lim_{n\to\infty}\frac{1}{\sqrt{e}} = \frac{1}{\sqrt{e}} < 1, \text{ conv.}$$

(b) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

From the comparison test: Compare with $\sum_{n=2}^{\infty}\frac{1}{n^{1/2}}$

limit comparison Test:

$$\lim_{n\to\infty}\frac{1/n^{1/2}}{\frac{1}{1/n^{1/2}-1}} = \lim_{n\to\infty}\frac{\sqrt{n}-1}{\sqrt{n}} = \lim_{n\to\infty}(1-1/\sqrt{n})$$

$= 1$

Both series are comparable.

Since $\sum\frac{1}{n^{1/2}}$ is a divergent series (p-series with $p = 1/2$)

$\sum_{n=2}^{\infty}\frac{1}{\sqrt{n}-1}$ diverges.

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Q4. Determine whether the following series converge or diverge

(a) $\sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{5^n n!}$

Using the Ratio Test:

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\dfrac{(n+1)2^{n+1}(n+1+1)!}{5^{n+1}(n+1)!}}{\dfrac{n\,2^n(n+1)!}{5^n n!}}$$

$$\rho = \lim_{n\to\infty} \frac{\cancel{(n+1)}\cdot 2\cdot \cancel{2^n}(n+2)\cancel{(n+1)!}}{5\cdot \cancel{5^n}\,\cancel{(n+1)}\cancel{n!}} \cdot \frac{\cancel{5^n}\,\cancel{n!}}{n\,\cancel{2^n}\cancel{(n+1)!}}$$

$$= \frac{2}{5}\lim_{n\to\infty} \frac{n+2}{n} = 2/5 < 1, \text{ converges by the Ratio Test!}$$

(b) $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{\ln n}$

(i) $\frac{1}{\ln n}$ terms are all positive

(ii) $\frac{1}{\ln n}$ non-increasing.

(iii) $\lim_{n\to\infty} \frac{1}{\ln n} = 0$

satisfies the Alternating Series Thm. The series converges.