Score: 1 of 1 pt

2.1.9

Let $A = \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 \\ -2 & k \end{bmatrix}$. What value(s) of k, if any, will make AB = BA?

Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. $k = -2$ (Use a comma to separate answers as needed.)

B. No value of k will make AB = BA

Score: 1 of 1 pt

2.1.12

Let $A = \begin{bmatrix} 4 & -8 \\ -5 & 10 \end{bmatrix}$. Construct a 2×2 matrix B such that AB is the zero matrix. Use two different nonzero columns for B.

$B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

Score: 1 of 1 pt

2.2.4

Find the inverse of the matrix.

$$\begin{bmatrix} 5 & 8 \\ -3 & -5 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. $\begin{bmatrix} 5 & 8 \\ -3 & -5 \end{bmatrix}^{-1} = \begin{bmatrix} 5 & 8 \\ -3 & -5 \end{bmatrix}$ (Simplify your answers.)

B. The matrix is not invertible.

2.2.34

Use the algorithm for finding A^{-1} to find the inverses of the matrices shown to the right. Let A be the corresponding $n \times n$ matrix, and let B be its inverse. Guess the form of B, and then show that AB = I.

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 3 & 3 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 3 & 3 & 0 \\ 4 & 4 & 4 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 3 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 3 & 3 & 0 \\ 4 & 4 & 4 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} & \frac{1}{4} \end{bmatrix}$$

If A is the corresponding $n \times n$ matrix and B is its inverse, which of the following is B?

A.

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & -\frac{1}{2} & 0 & & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{3} & & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{n-1} & -\frac{1}{n} \end{bmatrix}$$

B.

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & \frac{1}{2} & 0 & & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{1}{3} & & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & -\frac{1}{n-1} & \frac{1}{n} \end{bmatrix}$$

For $j = 1,2,\ldots,n$, let $\mathbf{a}_j$, $\mathbf{b}_j$, and $\mathbf{e}_j$ denote the jth columns of A, B, and I, respectively. First let $j = 1,2,\ldots,n-1$ and evaluate $A\mathbf{b}_j$.

$$A\mathbf{b}_j = A\left[\frac{1}{j}\left(\mathbf{e}_j - \mathbf{e}_{j+1}\right)\right]$$

Distribute to remove the brackets.

$$A\mathbf{b}_j = \frac{1}{j}\left(\mathbf{a}_j - \mathbf{a}_{j+1}\right)$$

Rewrite $\mathbf{a}_j$ and $\mathbf{a}_{j+1}$ in terms of the columns of I.

$$A\mathbf{b}_j = \frac{1}{j}\left(\left[j\mathbf{e}_j + (j+1)\mathbf{e}_{j+1} + \cdots + n\mathbf{e}_n\right] - \left[(j+1)\mathbf{e}_{j+1} + \cdots + n\mathbf{e}_n\right]\right)$$

Simplify this expression.

$$A\mathbf{b}_j = \frac{1}{j}\left(j\mathbf{e}_j\right)$$

Because the result from the previous step is equal to e_j, it follows that AB = I for j = 1,2,...,n − 1.

Which of the following shows that AB = I for j = n, completing the proof?

A.
$$\begin{aligned} Ab_n &= A\left(\frac{1}{n-1}e_{n-1}\right) \\ &= \frac{1}{n-1}a_{n-1} \\ &= e_n \end{aligned}$$

B.
$$\begin{aligned} Ab_n &= A\left(\frac{1}{n}e_n\right) \\ &= \frac{1}{n}a_n \\ &= e_n \end{aligned}$$

C.
$$\begin{aligned} Ab_n &= A\left(\frac{1}{n+1}e_{n+1}\right) \\ &= \frac{1}{n+1}a_{n+1} \\ &= e_n \end{aligned}$$

D.
$$\begin{aligned} Ab_n &= A(ne_n) \\ &= e_n \end{aligned}$$

Score: 1 of 1 pt | 5 of 6 | Tes

2.3.1

Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 3 & 5 \\ -6 & -2 \end{bmatrix}$$

Choose the correct answer below.

- A. The matrix is invertible because its determinant is not zero.
- B. The matrix is invertible because its columns are multiples of each other. The columns of the matrix form a linearly dependent set.
- C. The matrix is not invertible because its determinant is zero.
- D. The matrix is not invertible because the matrix has 2 pivot positions.

Score: 1 of 1 pt | 6 of 6

2.3.6

Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 1 & -4 & -6 \\ 0 & 5 & 4 \\ -4 & 12 & 0 \end{bmatrix}$$

Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

- A. The matrix is not invertible. In the given matrix the columns do not form a linearly independent set.
- B. The matrix is invertible. The given matrix is not row equivalent to the $n \times n$ identity matrix.
- C. The matrix is not invertible. If the given matrix is A, the equation $A\mathbf{x} = \mathbf{b}$ has no solution for at least one $\mathbf{b}$ in $\mathbb{R}^3$.
- D. The matrix is invertible. The given matrix has 3 pivot positions.