Math No:

Full Name :

Math 104 Final Exam
(20 May 2017, 10:30-11:45)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
20 pts	30 pts	20 pts	40 pts	110 pts

Q1. Evaluate the following limit, if it exists:

$$\lim_{x\to 0^+} (sinx)^{3/lnx}$$

0^0

$y = (\sin x)^{3/\ln x} \Rightarrow \ln y = \frac{3}{\ln x} \ln \sin x$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{3 \ln \sin x}{\ln x}$ $0/0$

$\underset{\text{L'Hospital}}{=} 3 \lim_{x\to 0^+} \frac{\frac{\cos x}{\sin x}}{\frac{1}{x}}$

$= 3 \lim_{x\to 0^+} \frac{x}{\tan x}$ $0/0$

$\underset{\text{L'Hospital}}{=} 3 \lim_{x\to 0^+} \frac{1}{\sec^2 x} = 3$

$\ln y \to 3 \Rightarrow \lim y = \boxed{e^3}$

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Q2.

a) Determine whether the series given below converges or diverges:

$$\sum_{n=1}^{\infty} \frac{4+\cos n}{n^3}$$

$$0 \leq \frac{4+\cos n}{n^3} \leq \frac{4+1}{n^3} = 5 \cdot \frac{1}{n^3}$$

$\sum_{n=1}^{\infty} \frac{1}{n^3}$ is a p-series, $p=3>1$, so it converges

$\therefore$ The given series converges, by the Comparison Test.

b) Find the MacLaurin series of the function

$$f(x) = x\cos\left(\frac{x}{2}\right)$$

(Hint: To solve this question, you may use Taylor or MacLaurin series that you know.)

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\Rightarrow \cos\frac{x}{2} = \sum_{n=0}^{\infty} \frac{(-1)^n (x/2)^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(2n)!}$$

$$\Rightarrow f(x) = x\cos\frac{x}{2} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2^{2n}(2n)!}}$$

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Q3. Given the power series

$$\sum_{n=1}^{\infty} \frac{n^{\pi}(x-2)^n}{(2n+1)!}$$

a) Find the radius of convergence.
b) Find the interval of convergence.

Generalized Ratio Test:

$$\rho = \lim_{n\to\infty} \left| \frac{\dfrac{(n+1)^{\pi}(x-2)^{n+1}}{(2n+3)!}}{\dfrac{n^{\pi}(x-2)^n}{(2n+1)!}} \right| = |x-2| \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{\pi} \frac{(2n+1)!}{(2n+3)!}$$

$$= |x-2| \lim_{x\to\infty} \underbrace{\left(\frac{n+1}{n}\right)^{\pi}}_{\downarrow\, 1} \underbrace{\frac{1}{(2n+3)(2n+2)}}_{\downarrow\, 0}$$

$$= |x-2| \cdot 0 < 1 \text{ for all } x$$

$\therefore$ Converges everywhere

$$\boxed{\begin{array}{l} \text{Radius} = \infty \\ \text{Interval} = (-\infty, \infty) \end{array}}$$

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Q4. Evaluate the following integrals:

a) $\int \ln(x^2+4)\,dx$

Integration by Parts:

$u = \ln(x^2+4)$ $dv = dx$

$du = \dfrac{2x}{x^2+4}$ $v = x$

$= uv - \int v\,du$

$= x\ln(x^2+4) - 2\int \dfrac{x^2dx}{x^2+4}$

$$\begin{array}{r|l} x^2 & x^2+4 \\ \cline{2-2} -x^2+4 & 1 \\ \hline -4 & \end{array}$$

$= x\ln(x^2+4) - 2\int\left(1 - \dfrac{4}{x^2+4}\right)dx$

$= x\ln(x^2+4) - 2x + 8\int \dfrac{dx}{4\left(x^2/4+1\right)}$

$t = x/2 \Rightarrow dt = dx/2$

$= x\ln(x^2+4) - 2x + \dfrac{8}{4}\cdot 2\int \dfrac{dt}{t^2+1}$

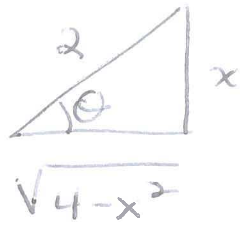
$= \boxed{x\ln(x^2+4) - 2x + 4\operatorname{Arctan}\dfrac{x}{2} + C}$

b) $\int \dfrac{dx}{x^2\sqrt{4-x^2}}$

(You may continue your work on the next page.)

(b)

Trigonometric substitution



$$\sin\theta = \frac{x}{2}$$

$$2\cos\theta\, d\theta = dx$$

$$\cos\theta = \frac{\sqrt{4-x^2}}{2}$$

$$\int \frac{dx}{x^2\sqrt{4-x^2}} = \int \frac{2\cos\theta\, d\theta}{4\sin^2\theta \cdot 2\cos\theta}$$

$$= \frac{1}{4}\int \frac{d\theta}{\sin^2\theta}$$

$$= \frac{1}{4}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{4}\cot\theta + C$$

$$= \boxed{-\frac{1}{4}\cdot\frac{\sqrt{4-x^2}}{x} + C}$$