1. $\int_0^{\ln 2}\int_1^{\ln 5} e^{x+y}\,dx\,dy = \int_0^{\ln 2}\left[\int_1^{\ln 5} e^{x+y}\,dx\right]dy = \int_0^{\ln 2}\left(\left|e^{x+y}\right|_1^{\ln 5}\right)dy$

$$= \int_0^{\ln 2}\left(e^{\ln 5+y} - e^{1+y}\right)dy = \left|e^{\ln 5+y} - e^{1+y}\right|_0^{\ln 2}$$

$$= e^{\ln 5+\ln 2} - e^{1+\ln 2} - e^{\ln 5}$$

$$= e^{\ln 2}\left(e^{\ln 5} - e\right) - e^{\ln 5}$$

$$= 2\cdot(5-e) - 5$$

$$= 5 - e \;//$$

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2. $\int_1^2\int_0^4 \frac{\sqrt{x}}{y^2}\,dx\,dy = \int_1^2\left[\frac{1}{y^2}\int_0^4 \sqrt{x}\,dx\right]dy = \int_1^2 \frac{1}{y^2}\cdot\frac{2}{3}\left|x^{3/2}\right|_0^4 dy = \frac{16}{3}\int_1^2 y^{-2}\,dy$

$$= \frac{16}{3}\left|-\frac{1}{y}\right|_1^2 = 8/3 \;//$$

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3. All (x, y, z) so that $xyz > 0$.

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4. $\lim_{P\to(1,-1,-1)} \frac{2xy+yz}{x^2+z^2} = \frac{2(1)(-1)+(-1)(-1)}{(1)^2+(-1)^2} = -1/2 \;//$

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5. $f(x,y) = ye^{x^2-y} \rightarrow \frac{\partial f}{\partial x} = 2xy\,e^{x^2-y}$, $\frac{\partial^2 f}{\partial x^2} = 4x^2y\,e^{x^2-y} + 2y\,e^{x^2-y} = (4x^2y+2y)\,e^{x^2-y}$

$\frac{\partial f}{\partial y} = e^{x^2-y} - ye^{x^2-y}$, $\frac{\partial^2 f}{\partial y^2} = (y-2)\,e^{x^2-y}$

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6. $V = abc \Rightarrow \frac{\partial V}{\partial t} = \frac{\partial V}{\partial a}\frac{\partial a}{\partial t} + \frac{\partial V}{\partial b}\frac{\partial b}{\partial t} + \frac{\partial V}{\partial c}\frac{\partial c}{\partial t}$

$\frac{\partial V}{\partial a} = bc$ $\qquad = (bc)\frac{da}{dt} + (ac)\frac{db}{dt} + (ab)\frac{dc}{dt}$

$\frac{\partial V}{\partial b} = ac$ $\qquad \left.\frac{dV}{dt}\right|_{a=1,b=2,c=3} = (2)(3)(1) + (1)(3)(1) + (1)(2)(-3) = 3\,m^3/s$

$\frac{\partial V}{\partial c} = ab$

the volume is increasing;

$S = 2ab + 2ac + 2bc$

$\frac{\partial S}{\partial a} = 2(b+c)$ $\qquad \frac{\partial S}{\partial t} = \frac{\partial S}{\partial a}\frac{da}{dt} + \frac{\partial S}{\partial b}\frac{db}{dt} + \frac{\partial S}{\partial c}\frac{dc}{dt}$

$\frac{\partial S}{\partial b} = 2(a+c)$ $\qquad = 2(b+c)\cdot 1 + 2(a+c)\cdot 1 + 2(a+b)(-3)$

$\frac{\partial S}{\partial c} = 2(a+b)$

$\left.\frac{dS}{dt}\right|_{a=1,b=2,c=3} = 0\ m^2/s$, and the surface area is not changing!

$D = \sqrt{a^2+b^2+c^2} \Rightarrow \frac{dD}{dt} = \frac{\partial D}{\partial a}\frac{da}{dt} + \frac{\partial D}{\partial b}\frac{db}{dt} + \frac{\partial D}{\partial c}\frac{dc}{dt}$

$\left.\frac{dD}{dt}\right|_{a=1,b=2,c=3} = \frac{1}{\sqrt{a^2+b^2+c^2}}\cdot\left(a\frac{da}{dt} + b\frac{db}{dt} + c\frac{dc}{dt}\right)$

$= -6/\sqrt{14}\ m/s < 0$, the diagonal are decreasing!

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7. $f(x) = f(0) + f'(0)(x) + \frac{1}{2!}f''(0)\cdot x^2 + \frac{1}{3!}f'''(0)x^3 + \dots$

$= 1 + \frac{1}{2}x + \frac{1}{2!}\left(\frac{1}{4}\right)x^2 + \frac{1}{3!}\left(-\frac{1}{8}\right)x^3 + \frac{1}{4!}\frac{1}{16}x^4 + \frac{1}{5!}\left(-\frac{1}{32}\right)x^5 + \dots$

$= 1 - \frac{x}{2} + \frac{x^2}{2^2 2!} - \frac{x^3}{2^3 3!} + \frac{x^4}{2^4 4!} - \frac{x^5}{2^5 5!} + \dots = \sum_{n=0}^{\infty}\frac{(-1)^n x^n}{2^n n!}$ //

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8. $\int_3^{\infty}\frac{\ln(x)}{x}\,dx = \lim_{b\to\infty}\int_3^b \frac{\ln x^2}{x}\,dx = \lim_{b\to\infty}[2\ln b - 2\ln 3] = \infty$

diverges $\quad \sum_{n=3}^{\infty}\frac{\ln n^2}{n}$ diverges

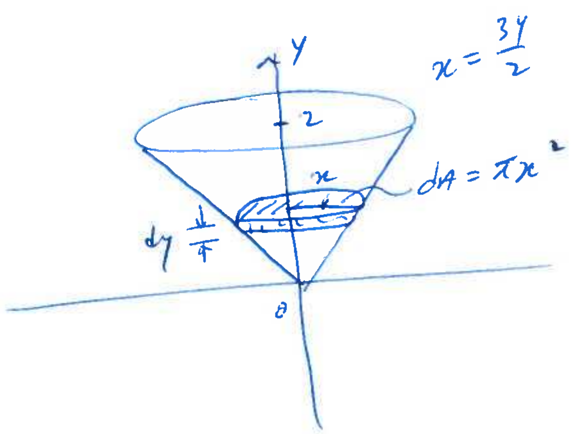
9. $\int \sin^3 x \cos^2 x \, dx = \int \sin x \cdot (1-\cos^2 x) \cdot \cos^2 x \, dx$; $u = \cos x$, $du = -\sin x \, dx$

$$= -\int (u^2 - u^4) \, du$$

$$= -\frac{1}{3}u^3 + \frac{1}{5}u^5 + c$$

$$= \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + c$$

10.



$$V = \int dA \, dy$$

$$= \int_0^2 \pi x^2 \, dy$$

$$= \pi \int_0^2 \left(\frac{3y}{2}\right)^2 dy$$

$$= \frac{9\pi}{4} \frac{y^3}{3} \Big|_0^2$$

$$= \frac{\cancel{9}^{3}\pi \cdot \cancel{8}^{2}}{\cancel{4} \cdot \cancel{3}} = 6\pi$$