Student: Huseyin Kerem Mican **Date:** 5/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 2.2 Homework

1. Find the inverse of the matrix. $\begin{bmatrix} 6 & 4 \\ 5 & 8 \end{bmatrix}$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. $\begin{bmatrix} 6 & 4 \\ 5 & 8 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{2}{7} & -\frac{1}{7} \\ -\frac{5}{28} & \frac{3}{14} \end{bmatrix}$ (Simplify your answers.)

B. The matrix is not invertible.

2. Find the inverse of the matrix.

$$\begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. $\begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix}$ (Simplify your answers.)

B. The matrix is not invertible.

3. Use the given inverse of the coefficient matrix to solve the following system.

$$\begin{aligned} 5x_1 + 2x_2 &= 8 \\ -6x_1 - 2x_2 &= -3 \end{aligned} \qquad A^{-1} = \begin{bmatrix} -1 & -1 \\ 3 & \frac{5}{2} \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. $x_1 = -5$ and $x_2 = \frac{33}{2}$ (Simplify your answers.)

B. There is no solution.

4. Let $A = \begin{bmatrix} 1 & 2 \\ 8 & 18 \end{bmatrix}$, $\mathbf{b}_1 = \begin{bmatrix} -5 \\ -36 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 5 \\ 34 \end{bmatrix}$, $\mathbf{b}_3 = \begin{bmatrix} 0 \\ -6 \end{bmatrix}$, and $\mathbf{b}_4 = \begin{bmatrix} 4 \\ 22 \end{bmatrix}$.

(a) Find A^{-1} and use it solve the four equations $A\mathbf{x} = \mathbf{b}_1$, $A\mathbf{x} = \mathbf{b}_2$,$A\mathbf{x} = \mathbf{b}_3$, and $A\mathbf{x} = \mathbf{b}_4$.

(b) The four equations in part (a) can be solved by the same set of operations, since the coefficient matrix is the same in each case. Solve the four equations in part (a) by row reducing the augmented matrix $[A\ \mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\ \mathbf{b}_4]$.

Find A^{-1}. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice

- **A.** $A^{-1} = \begin{bmatrix} 9 & -1 \\ -4 & \frac{1}{2} \end{bmatrix}$ (Simplify your answers.)
- **B.** The matrix is not invertible.

Solve $A\mathbf{x} = \mathbf{b}_1$.

$\mathbf{x} = \begin{bmatrix} -9 \\ 2 \end{bmatrix}$ (Simplify your answers.)

Solve $A\mathbf{x} = \mathbf{b}_2$.

$\mathbf{x} = \begin{bmatrix} 11 \\ -3 \end{bmatrix}$ (Simplify your answers.)

Solve $A\mathbf{x} = \mathbf{b}_3$.

$\mathbf{x} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$ (Simplify your answers.)

Solve $A\mathbf{x} = \mathbf{b}_4$.

$\mathbf{x} = \begin{bmatrix} 14 \\ -5 \end{bmatrix}$ (Simplify your answers.)

(b) Solve the four equations by row reducing the augmented matrix $[A\ \mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\ \mathbf{b}_4]$. Write the augmented matrix $[A\ \mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\ \mathbf{b}_4]$ in reduced echelon form.

$\begin{bmatrix} 1 & 0 & -9 & 11 & 6 & 14 \\ 0 & 1 & 2 & -3 & -3 & -5 \end{bmatrix}$ (Simplify your answers.)

Are the solutions the same in (a) and (b)?

- No
- Yes

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5. Use matrix algebra to show that if A is invertible and D satisfies $AD = I$, then $D = A^{-1}$.

Choose the correct answer below.

- **A.** Left-multiply each side of the equation $AD = I$ by A^{-1} to obtain $A^{-1}AD = A^{-1}I$, $ID = A^{-1}$, and $D = A^{-1}$.
- **B.** Add A^{-1} to both sides of the equation $AD = I$ to obtain $AD + A^{-1} = I + A^{-1}$, $DI = A^{-1}$, and $D = A^{-1}$.
- **C.** Right-multiply each side of the equation $AD = I$ by A^{-1} to obtain $ADA^{-1} = IA^{-1}$, $DI = A^{-1}$, and $D = A^{-1}$.
- **D.** Add A^{-1} to both sides of the equation $AD = I$ to obtain $A^{-1} + AD = A^{-1} + I$, $ID = A^{-1}$, and $D = A^{-1}$.

6. Explain why the columns of an $n \times n$ matrix A are linearly independent when A is invertible.

Choose the correct answer below.

- **A.** If A is invertible, then A has an inverse matrix A^{-1}. Since $AA^{-1} = I$, A must have linearly independent columns.
- **B.** If A is invertible, then for all $\mathbf{x}$ there is a $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$. Since $\mathbf{x} = \mathbf{0}$ is a solution of $A\mathbf{x} = \mathbf{0}$, the columns of A must be linearly independent.
- **C.** If A is invertible, then A has an inverse matrix A^{-1}. Since $AA^{-1} = A^{-1}A$, A must have linearly independent columns.
- **D.** If A is invertible, then the equation $A\mathbf{x} = \mathbf{0}$ has the unique solution $\mathbf{x} = \mathbf{0}$. Since $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, the columns of A must be linearly independent.

7. Find the inverse of the given matrix, if it exists.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 3 \\ -4 & 4 & 3 \end{bmatrix}$$

Find the inverse. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** $A^{-1} = \begin{bmatrix} -\frac{9}{7} & -\frac{8}{7} & \frac{2}{7} \\ \frac{-3}{7} & \frac{-5}{7} & \frac{3}{7} \\ \frac{-8}{7} & \frac{-4}{7} & \frac{1}{7} \end{bmatrix}$ (Type integers or simplified fractions.)
- **B.** The matrix A does not have an inverse.

8. Find the inverse of the given matrix, if it exists.

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 5 & -9 & 4 \\ 4 & 1 & -5 \end{bmatrix}$$

Find the inverse. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** $A^{-1} =$
  (Type an integer or decimal for each matrix element.)
- **B.** The matrix A does not have an inverse.