Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 3.1 Homework

1. Compute the determinant using a cofactor expansion across the first row. Also compute the determinant by a cofactor expansion down the second column.

$$\begin{vmatrix} 2 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -2 \end{vmatrix}$$

Compute the determinant using a cofactor expansion across the first row. Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- **A.** Using this expansion, the determinant is $-(2)(-16)+(0)(-4)-(4)(10)=$ ______.
- **B.** Using this expansion, the determinant is $(0)(-4)-(3)(-4)+(5)(-4)=$ ______.
- **C.** Using this expansion, the determinant is $(2)(-16)-(0)(-4)+(4)(10)=$ 8.
- **D.** Using this expansion, the determinant is $-(0)(-4)+(3)(-4)-(5)(-4)=$ ______.

Compute the determinant using a cofactor expansion down the second column. Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- **A.** Using this expansion, the determinant is $-(2)(-16)+(0)(-4)-(4)(10)=$ ______.
- **B.** Using this expansion, the determinant is $(2)(-16)-(0)(-4)+(4)(10)=$ ______.
- **C.** Using this expansion, the determinant is $-(0)(-4)+(3)(-4)-(5)(-4)=$ 8.
- **D.** Using this expansion, the determinant is $(0)(-4)-(3)(-4)+(5)(-4)=$ ______.

YOU ANSWERED: C.: 0

2. Compute the determinant using a cofactor expansion across the first row. Also compute the determinant by a cofactor expansion down the second column.

$$\begin{vmatrix} 1 & -1 & 6 \\ 6 & 5 & 1 \\ 5 & 6 & -5 \end{vmatrix}$$

Write the expression for the determinant using a cofactor expansion across the first row. Choose the correct answer below.

- **A.** Using this expansion, the determinant is $(1)(-19)+(-1)(41)+(6)(61)$.
- **B.** Using this expansion, the determinant is $(1)(-31)+(-1)(-35)+(6)(11)$.
- **C.** Using this expansion, the determinant is $(1)(-31)-(-1)(-35)+(6)(11)$.
- **D.** Using this expansion, the determinant is $(1)(-19)-(-1)(41)+(6)(61)$.

Write the expression for the determinant using a cofactor expansion down the second column. Choose the correct answer below.

- **A.** Using this expansion, the determinant is $(-1)(-25)+(5)(25)+(6)(37)$.
- **B.** Using this expansion, the determinant is $-(-1)(-35)+(5)(-35)-(6)(-35)$.
- **C.** Using this expansion, the determinant is $-(-1)(-25)+(5)(25)-(6)(37)$.
- **D.** Using this expansion, the determinant is $(-1)(-35)+(5)(-35)+(6)(-35)$.

The determinant is 0 .
(Simplify your answer.)

3. Compute the determinant using a cofactor expansion down the first column.

$$A = \begin{bmatrix} 6 & -5 & 2 \\ 8 & 1 & 3 \\ 0 & 4 & -2 \end{bmatrix}$$

Determine the value of the first term in the cofactor expansion. Substitute the value for a_{11} and complete the matrix for C_{11} below.

$$a_{11}C_{11} = (6)\det\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$$

Determine the value of the second term in the cofactor expansion. Substitute the value for a_{21} and complete the matrix for C_{21} below.

$$a_{21}C_{21} = -(8)\det\begin{bmatrix} -5 & 2 \\ 4 & -2 \end{bmatrix}$$

Determine the value of the third term in the cofactor expansion. Substitute the value for a_{31} and complete the matrix for C_{31} below.

$$a_{31}C_{31} = (0)\det\begin{bmatrix} -5 & 2 \\ 1 & 3 \end{bmatrix}$$

Complete the cofactor expansion to compute the determinant.

det A = -100

4. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 1 & 0 & 0 & 5 \\ 4 & 7 & 3 & -7 \\ 3 & 0 & 0 & 0 \\ 7 & 3 & 1 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 & 5 \\ 4 & 7 & 3 & -7 \\ 3 & 0 & 0 & 0 \\ 7 & 3 & 1 & 3 \end{vmatrix} = -30$$ (Simplify your answer.)

5. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 4 & 8 & -6 & 3 \\ 0 & -3 & 1 & -1 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

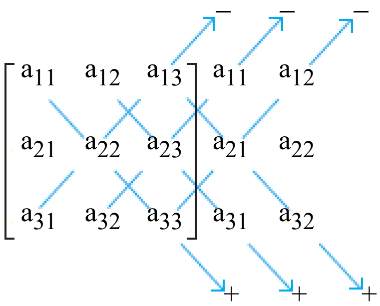
$$\begin{vmatrix} 4 & 8 & -6 & 3 \\ 0 & -3 & 1 & -1 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 2 \end{vmatrix} = -24$$ (Simplify your answer.)

6. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 3 & 0 & -7 & 3 & -5 \\ 0 & 0 & 3 & 0 & 0 \\ 8 & 4 & -6 & 5 & -9 \\ 4 & 0 & 6 & 2 & -4 \\ 0 & 0 & 7 & -2 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 0 & -7 & 3 & -5 \\ 0 & 0 & 3 & 0 & 0 \\ 8 & 4 & -6 & 5 & -9 \\ 4 & 0 & 6 & 2 & -4 \\ 0 & 0 & 7 & -2 & 2 \end{vmatrix} = -48$$ (Simplify your answer.)

7. The expansion of a 3×3 determinant can be remembered by this device. Write a second copy of the first two columns to the right of the matrix, and compute the determinant by multiplying entries on six diagonals. Add the downward diagonal products and subtract the upward products. Use this method to compute the following determinant.



$$\begin{vmatrix} 3 & 1 & 0 \\ -2 & 3 & 4 \\ 0 & -2 & -4 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 1 & 0 \\ -2 & 3 & 4 \\ 0 & -2 & -4 \end{vmatrix} = -20$$

8. Explore the effect of an elementary row operation on the determinant of a matrix. State the row operation and describe how it affects the determinant.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} a+kc & b+kd \\ c & d \end{bmatrix}$$

What is the elementary row operation?

- **A.** Row 2 is replaced with the sum of itself and k times row 1.
- **B.** Row 2 is multiplied by k.
- **C.** Rows 1 and 2 are interchanged.
- **D.** Row 1 is multiplied by k.
- **E.** Row 1 is replaced with the sum of itself and k times row 2.

How does the row operation affect the determinant?

- **A.** It changes the sign of the determinant.
- **B.** It increases the determinant by k.
- **C.** It multiplies the determinant by k.
- **D.** It does not change the determinant.

9. Explore the effects of an elementary row operation on the determinant of a matrix. State the row operation and describe how it affects the determinant.

$$\begin{bmatrix} 5 & 5 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 5 & 5 \\ 3+5k & 1+5k \end{bmatrix}$$

What is the elementary row operation?

- **A.** Replace row 2 with k times row 1.
- **B.** Replace row 2 with row 1 plus k times row 2.
- **C.** Replace row 2 with k times row 1 plus row 2.
- **D.** Replace row 2 with k times row 2.

How does the row operation affect the determinant?

- **A.** The determinant is increased by 50k.
- **B.** The determinant is decreased by 25k.
- **C.** The determinant is increased by 25k.
- **D.** The determinant does not change.

10. Explore the effect of an elementary row operation on the determinant of a matrix. State the row operation and describe how it affects the determinant.

$$\begin{bmatrix} 3 & 2 & 4 \\ a & b & c \\ 8 & 5 & 1 \end{bmatrix}, \begin{bmatrix} a & b & c \\ 3 & 2 & 4 \\ 8 & 5 & 1 \end{bmatrix}$$

What is the elementary row operation?

- **A.** Row 1 is replaced with the sum of rows 1 and 2.
- **B.** Row 1 is replaced with the sum of rows 1 and 3.
- **C.** Rows 1 and 2 are interchanged.
- **D.** Rows 1 and 3 are interchanged.
- **E.** Row 2 is replaced with the sum of rows 2 and 3.
- **F.** Rows 2 and 3 are interchanged.
- **G.** Row 2 is replaced with the sum of rows 1 and 2.

How does the row operation affect the determinant?

- **A.** It changes the sign of the determinant.
- **B.** It increases the determinant by 1.
- **C.** It multiplies the determinant by 2.
- **D.** It does not change the determinant.

11. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and let k be a scalar. Find a formula that relates det(kA) to k and det(A).

Find det(A).

det(A) = $ad - bc$ (Simplify your answer.)

Find det(kA).

det(kA) = $k^2ad - k^2bc$ (Simplify your answer.)

Use the preceding steps to find a formula for det(kA). Select the correct choice below and fill in the answer box(es) to complete your choice.
(Simplify your answer.)

- **A.** det(kA) = k^2 • det(A)
- **B.** det(kA) = ______ + det(A)
- **C.** det(kA) = ______ − det(A)
- **D.** det(kA) = ______ + ______ • det(A)

Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 3.2 Homework

1. Find the determinant by row reduction to echelon form.

$$\begin{vmatrix} 1 & 5 & -7 \\ -1 & -4 & -5 \\ 2 & 8 & 7 \end{vmatrix}$$

Use row operations to reduce the matrix to echelon form.

$$\begin{bmatrix} 1 & 5 & -7 \\ -1 & -4 & -5 \\ 2 & 8 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the determinant of the given matrix.

$$\begin{vmatrix} 1 & 5 & -7 \\ -1 & -4 & -5 \\ 2 & 8 & 7 \end{vmatrix} = -3$$ (Simplify your answer.)

2. Find the determinant by row reduction to echelon form.

$$\begin{vmatrix} 1 & -1 & -3 & 0 \\ 7 & -6 & 3 & 2 \\ 1 & 3 & -2 & 3 \\ -3 & 7 & 10 & 3 \end{vmatrix}$$

Use row operations to reduce the matrix to echelon form.

$$\begin{bmatrix} 1 & -1 & -3 & 0 \\ 7 & -6 & 3 & 2 \\ 1 & 3 & -2 & 3 \\ -3 & 7 & 10 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & \frac{17}{19} \\ 0 & 1 & 0 & \frac{14}{19} \\ 0 & 0 & 1 & \frac{1}{19} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find the determinant of the given matrix.

$$\begin{vmatrix} 1 & -1 & -3 & 0 \\ 7 & -6 & 3 & 2 \\ 1 & 3 & -2 & 3 \\ -3 & 7 & 10 & 3 \end{vmatrix} = 0$$ (Simplify your answer.)

3. Find the determinant by row reduction to echelon form.

$$\begin{vmatrix} 1 & -3 & 1 & 0 & -6 \\ 0 & 2 & -2 & 9 & 2 \\ -2 & 6 & -2 & 2 & 4 \\ 1 & -5 & 4 & 1 & 6 \\ 0 & 2 & -2 & 11 & 4 \end{vmatrix}$$

Use row operations to reduce the matrix to echelon form.

$$\begin{bmatrix} 1 & -3 & 1 & 0 & -6 \\ 0 & 2 & -2 & 9 & 2 \\ -2 & 6 & -2 & 2 & 4 \\ 1 & -5 & 4 & 1 & 6 \\ 0 & 2 & -2 & 11 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Find the determinant of the given matrix.

$$\begin{vmatrix} 1 & -3 & 1 & 0 & -6 \\ 0 & 2 & -2 & 9 & 2 \\ -2 & 6 & -2 & 2 & 4 \\ 1 & -5 & 4 & 1 & 6 \\ 0 & 2 & -2 & 11 & 4 \end{vmatrix} = -40$$ (Simplify your answer.)

4. Combine the methods of row reduction and cofactor expansion to compute the determinant.

$$\begin{vmatrix} -1 & 2 & 6 & 0 \\ 3 & 3 & 5 & 0 \\ 7 & 6 & 8 & 6 \\ 5 & 3 & 5 & 3 \end{vmatrix}$$

The determinant is 126 .
(Simplify your answer.)

5. Combine the methods of row reduction and cofactor expansion to compute the determinant.

$$\begin{vmatrix} 1 & 5 & 4 & 1 \\ 0 & -4 & -8 & 0 \\ 3 & 5 & 4 & 1 \\ -6 & 4 & 4 & 0 \end{vmatrix}$$

The determinant is 32 .
(Simplify your answer.)

6. Find the determinant below, where $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 8.$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 7g & 7h & 7i \end{vmatrix}$$

$\begin{vmatrix} a & b & c \\ d & e & f \\ 7g & 7h & 7i \end{vmatrix} =$ 56 (Simplify your answer.)

7. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$, find $\begin{vmatrix} a & b & c \\ 6d+g & 6e+h & 6f+i \\ g & h & i \end{vmatrix}$.

$\begin{vmatrix} a & b & c \\ 6d+g & 6e+h & 6f+i \\ g & h & i \end{vmatrix} =$ 18 (Simplify your answer.)

8. Use determinants to find out if the matrix is invertible.

$$\begin{bmatrix} -5 & 0 & 1 \\ 1 & -3 & -2 \\ 0 & -5 & -3 \end{bmatrix}$$

The determinant of the matrix is 0 . (Simplify your answer.)

Is the matrix invertible? Choose the correct answer below.

- The matrix is invertible.
- The matrix is not invertible. ✔

9. Use determinants to find out if the matrix is invertible.

$$\begin{bmatrix} 1 & -1 & -2 & 0 \\ 0 & 1 & 5 & 4 \\ 3 & -1 & -3 & 4 \\ -1 & 2 & 7 & 5 \end{bmatrix}$$

The determinant of the matrix is -7 . (Simplify your answer.)

Is the matrix invertible? Choose the correct answer below.

- **A.** The matrix is invertible. ✔
- **B.** The matrix is not invertible.

10. Use determinants to decide if the set of vectors is linearly independent.

$$\begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ 8 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -8 \end{bmatrix}$$

The determinant of the matrix whose columns are the given vectors is -120.
(Simplify your answer.)

Is the set of vectors linearly independent? Choose the correct answer below.

- **A.** The set of vectors is linearly dependent, because the determinant is not zero.
- **B.** The set of vectors is linearly independent, because the determinant is not zero.
- **C.** The set of vectors is linearly dependent, because the determinant exists.
- **D.** The set of vectors is linearly independent, because the determinant exists.

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11. Compute det B^4 where $B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 2 & 3 \\ 2 & 2 & 2 \end{bmatrix}$.

det B^4 = 16 (Simplify your answer.)

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12. Verify that det AB = (det A)(det B), where the matrices A and B are given below.

$$A = \begin{bmatrix} 5 & 0 \\ 10 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 \\ 5 & 6 \end{bmatrix}$$

Calculate det A and det B.

det A = 5, det B = 18 (Simplify your answers.)

Now calculate the product (det A)(det B).

(det A)(det B) = 90 (Simplify your answer.)

Calculate the product of matrices AB.

$$AB = \begin{bmatrix} 15 & 0 \\ 35 & 6 \end{bmatrix}$$ (Type an integer or decimal for each matrix element.)

Now calculate the determinant of the product of matrices A and B.

det (AB) = 90 (Simplify your answer.)

13. Let A and B be 3×3 matrices, with det A = 4 and det B = 3. Use properties of determinants to complete parts (a) through (e) below.

a. Compute det AB.

det AB = 12 (Type an integer or a fraction.)

b. Compute det 5A.

det 5A = 500 (Type an integer or a fraction.)

c. Compute det B^T.

det B^T = 3 (Type an integer or a fraction.)

d. Compute det A^{-1}.

det A^{-1} = $\frac{1}{4}$ (Type an integer or a simplified fraction.)

e. Compute det A^3.

det A^3 = 64 (Type an integer or a fraction.)

Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 3.3 Homework

1. Use Cramer's rule to compute the solutions of the system.

$$7x_1 + 7x_2 = 0$$

$$4x_1 + 8x_2 = -16$$

What is the solution of the system?

$x_1 =$ 4

$x_2 =$ -4

2. Use Cramer's rule to compute the solution of the system.

$$\begin{aligned} x_1 + x_2 &= 2 \\ 5x_1 + 2x_3 &= 0 \\ x_2 - 2x_3 &= 3 \end{aligned}$$

$x_1 = \frac{1}{4}$; $x_2 = \frac{7}{4}$; $x_3 = -\frac{5}{8}$

(Type integers or simplified fractions.)

3. Determine the values of the parameter s for which the system has a unique solution, and describe the solution.

$$5sx_1 + 4x_2 = 5$$

$$8x_1 + 4sx_2 = -2$$

Choose the correct answer below.

- **A.** $s \neq \pm 2\sqrt{\frac{2}{5}}$; $x_1 = \frac{5s+2}{5s^2-8}$; $x_2 = \frac{5(-s-4)}{2(5s^2-8)}$
- **B.** $s \neq \pm 2\sqrt{\frac{2}{5}}$; $x_1 = \frac{5(-s-4)}{2(5s^2-8)}$; $x_2 = \frac{5s+2}{5s^2-8}$
- **C.** $s \neq 0$; $x_1 = \frac{5s+2}{5s^2-8}$; $x_2 = \frac{5(-s-4)}{2(5s^2-8)}$
- **D.** $s \neq 0$; $x_1 = \frac{5(-s-4)}{2(5s^2-8)}$; $x_2 = \frac{5s+2}{5s^2-8}$

4. Compute the adjugate of the given matrix, and then use the Inverse Formula to give the inverse of the matrix.

$$A = \begin{bmatrix} 0 & -5 & -1 \\ 6 & 0 & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

The adjugate of the given matrix is $\text{adj } A = \begin{bmatrix} 0 & 4 & 0 \\ -6 & -2 & -6 \\ 6 & 10 & 30 \end{bmatrix}$.

The inverse of the given matrix is $A^{-1} = \begin{bmatrix} 0 & \frac{1}{6} & 0 \\ -\frac{1}{4} & -\frac{1}{12} & -\frac{1}{4} \\ \frac{1}{4} & \frac{5}{12} & \frac{5}{4} \end{bmatrix}$

(Simplify your answers.)

5. Find the area of the parallelogram whose vertices are listed.

$(-1,0), (0,6), (7,-4), (8,2)$

The area of the parallelogram is 52 square units.

6. Find the volume of the parallelepiped with one vertex at the origin and adjacent vertices at $(4,0,-5)$, $(1,2,5)$, and $(8,2,0)$.

The volume of the parallelepiped is 30 . (Type an integer or a decimal.)

Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 4.1 Homework

1. Let V be the set of vectors shown below.

$$V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x < 0,\ y \geq 0 \right\}$$

a. If **u** and **v** are in V, is **u** + **v** in V? Why?
b. Find a specific vector **u** in V and a specific scalar c such that c**u** is not in V.

a. If **u** and **v** are in V, is **u** + **v** in V?

- **A.** The vector **u + v** must be in V because the x-coordinate of **u + v** is the sum of two negative numbers, which must also be negative, and the y-coordinate of **u** + **v** is the sum of nonnegative numbers, which must also be nonnegative.
- **B.** The vector **u + v** may or may not be in V depending on the values of x and y.
- **C.** The vector **u + v** must be in V because V is a subset of the vector space $\mathbb{R}^2$.
- **D.** The vector **u + v** is never in V because the entries of the vectors in V are scalars and not sums of scalars.

b. Find a specific vector **u** in V and a specific scalar c such that c**u** is not in V. Choose the correct answer below.

- **A.** $\mathbf{u} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, c = 4$
- **B.** $\mathbf{u} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}, c = -1$
- **C.** $\mathbf{u} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}, c = 4$
- **D.** $\mathbf{u} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}, c = -1$

2. Let $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : 6x^2 + 3y^2 \leq 1 \right\}$, which represents the set of points on and inside an ellipse in the xy-plane. Find two specific examples—two vectors, and a vector and a scalar—to show that H is not a subspace of $\mathbb{R}^2$.

H is not a subspace of $\mathbb{R}^2$ because the two vectors $\begin{bmatrix} \frac{1}{\sqrt{6}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{3}} \end{bmatrix}$ show that H is not closed under addition.
(Use a comma to separate vectors as needed.)

H is not a subspace of $\mathbb{R}^2$ because the scalar 3 and the vector $\begin{bmatrix} 0.27 \\ -0.38 \end{bmatrix}$ show that H is not closed under scalar multiplication.

YOU ANSWERED: $\begin{bmatrix} 0.27 \\ -0.47 \end{bmatrix}$

3. Determine if the given set is a subspace of $\mathbb{P}_5$. Justify your answer.

The set of all polynomials of the form $\mathbf{p}(t) = at^5$, where a is in $\mathbb{R}$.

Choose the correct answer below.

- ✔ **A.** The set is a subspace of $\mathbb{P}_5$. The set contains the zero vector of $\mathbb{P}_5$, the set is closed under vector addition, and the set is closed under multiplication by scalars.
- **B.** The set is not a subspace of $\mathbb{P}_5$. The set is not closed under multiplication by scalars when the scalar is not an integer.
- **C.** The set is not a subspace of $\mathbb{P}_5$. The set does not contain the zero vector of $\mathbb{P}_5$.
- **D.** The set is a subspace of $\mathbb{P}_5$. The set contains the zero vector of $\mathbb{P}_5$, the set is closed under vector addition, and the set is closed under multiplication on the left by $m \times 5$ matrices where m is any positive integer.

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4. Determine if the given set is a subspace of $\mathbb{P}_6$. Justify your answer.

All polynomials of degree at most 6, with rational numbers as coefficients.

Complete each statement below.

The zero vector of $\mathbb{P}_6$ is in the set because zero is a rational number.

The set is closed under vector addition because the sum of two rational numbers is a rational number.

The set is not closed under multiplication by scalars because the product of a scalar and a rational number is not necessarily a rational number.

Is the set a subspace of $\mathbb{P}_6$?

- ✔ No
- Yes

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5. Determine if the given set is a subspace of $\mathbb{P}_n$. Justify your answer.

The set of all polynomials in $\mathbb{P}_n$ such that $\mathbf{p}(0) = 0$

Choose the correct answer below.

- **A.** The set is not a subspace of $\mathbb{P}_n$ because the set is not closed under multiplication by scalars.
- **B.** The set is a subspace of $\mathbb{P}_n$ because $\mathbb{P}_n$ is a vector space spanned by the given set.
- ✔ **C.** The set is a subspace of $\mathbb{P}_n$ because the set contains the zero vector of $\mathbb{P}_n$, the set is closed under vector addition, and the set is closed under multiplication by scalars.
- **D.** The set is not a subspace of $\mathbb{P}_n$ because the set does not contain the zero vector of $\mathbb{P}_n$.
- **E.** The set is not a subspace of $\mathbb{P}_n$ because the set is not closed under vector addition.

6. Let H be the set of all vectors of the form $\begin{bmatrix} -2t \\ t \\ 5t \end{bmatrix}$. Find a vector **v** in $\mathbb{R}^3$ such that H = Span{**v**}. Why does this show that H is a subspace of $\mathbb{R}^3$?

H = Span{**v**} for **v** = $\begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$

Why does this show that H is a subspace of $\mathbb{R}^3$?

- **A.** Since **v** is in $\mathbb{R}^3$, H = Span{**v**} is a subspace of $\mathbb{R}^3$.
- **B.** Since the zero vector of $\mathbb{R}^3$ is not in Span{**v**}, H is a subspace of $\mathbb{R}^3$.
- **C.** Since **v** spans $\mathbb{R}^3$ and **v** spans H, H spans $\mathbb{R}^3$.
- **D.** Since $\mathbb{R}^3$ = Span{**v**}, H = Span{**v**} is a subspace of $\mathbb{R}^3$.

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7. Let W be the set of all vectors of the form shown on the right, where b and c are arbitrary. Find vectors **u** and **v** such that W = Span{**u**, **v**}. Why does this show that W is a subspace of $\mathbb{R}^3$?

$$\begin{bmatrix} 8b-3c \\ -b \\ 2c \end{bmatrix}$$

Using the given vector space, write vectors **u** and **v** such that W = Span{**u**, **v**}.

$\{\mathbf{u}, \mathbf{v}\} = \left\{ \begin{bmatrix} 8 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} \right\}$

(Use a comma to separate answers as needed.)

Choose the correct theorem that indicates why these vectors show that W is a subspace of $\mathbb{R}^3$.

- **A.** An indexed set $\{\mathbf{v}_1,...,\mathbf{v}_p\}$ of two or more vectors in a vector space V, with $\mathbf{v}_1 \neq \mathbf{0}$ is a subspace of V if and only if some $\mathbf{v}_j$ is in Span$\{\mathbf{v}_1,...,\mathbf{v}_{j-1},\mathbf{v}_j\}$.
- **B.** The null space of an m × n matrix is a subspace of $\mathbb{R}^n$. Equivalently, the set of all solutions to a system A**x** = **0** of m homogeneous linear equations in n unknowns is a subspace of $\mathbb{R}^n$.
- **C.** The column space of an m × n matrix A is a subspace of $\mathbb{R}^m$.
- **D.** If $\mathbf{v}_1,...,\mathbf{v}_p$ are in a vector space V, then Span$\{\mathbf{v}_1,...,\mathbf{v}_p\}$ is a subspace of V.

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8. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 9 \\ 2 \\ 1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$.

a. Is $\mathbf{w}$ in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

b. How many vectors are in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

c. Is $\mathbf{w}$ in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Why?

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a. Is $\mathbf{w}$ in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

- **A.** Vector $\mathbf{w}$ is not in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is not $\mathbf{v}_1$, $\mathbf{v}_2$, or $\mathbf{v}_3$.
- **B.** Vector $\mathbf{w}$ is in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.
- **C.** Vector $\mathbf{w}$ is in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the subspace generated by $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ is $\mathbb{R}^3$.
- **D.** Vector $\mathbf{w}$ is not in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because it is not a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The number of vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is 3 .
- **B.** There are infinitely many vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

b. How many vectors are in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The number of vectors in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is ______ .
- **B.** There are infinitely many vectors in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

c. Is $\mathbf{w}$ in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

- **A.** Vector $\mathbf{w}$ is in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the subspace generated by $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ is $\mathbb{R}^3$.
- **B.** Vector $\mathbf{w}$ is in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because $\mathbf{w}$ is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$, which can be seen because any echelon form of the augmented matrix of the system has no row of the form $[0 \cdots 0 \ b]$ with $b \neq 0$.
- **C.** Vector $\mathbf{w}$ is not in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because the rightmost column of the augmented matrix of the system $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{w}$ is a pivot column.
- **D.** Vector $\mathbf{w}$ is not in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ because $\mathbf{w}$ is not a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

9. Let W be the set of all vectors of the form shown on the right, where a and b represent arbitrary real numbers. Find a set S of vectors that spans W, or give an example or an explanation showing why W is not a vector space.

$$\begin{bmatrix} 2a+7b \\ -5 \\ 2a-5b \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** A spanning set is S = { }.
  (Use a comma to separate vectors as needed.)
- **B.** W is not a vector space because the zero vector and most sums and scalar multiples of vectors in W are not in W, because their second (middle) value is not equal to -5.
- **C.** W is not a vector space because not all vectors **u**, **v**, and **w** in W have the property that $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$ and $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$.

10. Let W be the set of all vectors of the form shown on the right, where a, b, and c represent arbitrary real numbers. Find a set S of vectors that spans W or give an example or an explanation to show that W is not a vector space.

$$\begin{bmatrix} 6a-5b \\ 7b-7c \\ 6c+9a \\ 8b \end{bmatrix}$$

Select the correct choice and fill in the answer box as needed to complete your choice.

- **A.** A spanning set is $S=\left\{\begin{bmatrix} 6 \\ 0 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ -7 \\ 6 \\ 0 \end{bmatrix}\right\}$.
  (Use a comma to separate answers as needed.)
- **B.** There is no spanning set of W because W does not contain the zero vector.
- **C.** There is no spanning set of W because W is not closed under scalar multiplication.
- **D.** There is no spanning set of W because W is not closed under vector addition.

11. The set $M_{2\times2}$ of all 2×2 matrices is a vector space, under the usual operations of addition of matrices and multiplication by real scalars. Determine if the set H of all matrices of the form $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ is a subspace of $M_{2\times2}$.

Choose the correct answer below.

- **A.** The set H is a subspace of $M_{2\times2}$ because Span{H} = $M_{2\times2}$.
- **B.** The set H is a subspace of $M_{2\times2}$ because H contains the zero vector of $M_{2\times2}$, H is closed under vector addition, and H is closed under multiplication by scalars.
- **C.** The set H is not a subspace of $M_{2\times2}$ because the product of two matrices in H is not in H.
- **D.** The set H is not a subspace of $M_{2\times2}$ because H is not closed under vector addition.
- **E.** The set H is not a subspace of $M_{2\times2}$ because H is not closed under multiplication by scalars.
- **F.** The set H is not a subspace of $M_{2\times2}$ because H does not contain the zero vector of $M_{2\times2}$.

Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 4.2 Homework

1. Determine if $\mathbf{w} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$ is in Nul A, where $A = \begin{bmatrix} 2 & -1 & -3 \\ 4 & -3 & -11 \\ -5 & 3 & 10 \end{bmatrix}$.

Is **w** in Nul A? Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

A. No, because A**w** =

B. Yes, because A**w** = $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

2. Find an explicit description of Nul A by listing vectors that span the null space.

$$A = \begin{bmatrix} 1 & 2 & 5 & 0 \\ 0 & 1 & 4 & -3 \end{bmatrix}$$

A spanning set for Nul A is $\left\{ \begin{bmatrix} 3 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

3. Either use an appropriate theorem to show that the given set, W, is a vector space, or find a specific example to the contrary.

$$W = \left\{ \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix} : \begin{array}{r} -p-3q=5s \\ 2p=s-3r \end{array} \right\}$$

Rewrite the system of equations in the form A**x** = 0.

$$A\mathbf{x} = \begin{bmatrix} -1 & -3 & 0 & -5 \\ 2 & 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

What does the given set represent?

- **A.** The set of all solutions to the homogeneous system of equations.
- **B.** The set of solutions to one of the homogeneous equations.
- **C.** The set represents the values which are not solutions.

Therefore, the set W = Nul A.

The null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$. Equivalently, the set of all solutions to a system $A\mathbf{x} = 0$ of m homogeneous linear equations in n unknowns is a subspace of $\mathbb{R}^n$.

Which of the following is a true statement?

- **A.** The proof is complete since W is a subspace of $\mathbb{R}^3$. The given set W must be a vector space because a subspace itself is a vector space.
- **B.** The proof is complete since W is a subspace of $\mathbb{R}^2$. The given set W must be a vector space because a subspace itself is a vector space.
- **C.** The proof is complete since W is a subspace of $\mathbb{R}$. The given set W must be a vector space because a subspace itself is a vector space.
- **D.** The proof is complete since W is a subspace of $\mathbb{R}^4$. The given set W must be a vector space because a subspace itself is a vector space.

4. Either use an appropriate theorem to show that the given set, W, is a vector space, or find a specific example to the contrary.

$$W = \left\{ \begin{bmatrix} s-2t \\ 3+3s \\ 3s+t \\ 2s \end{bmatrix} : s,t \text{ real} \right\}$$

The set W is a subset of $\mathbb{R}^4$. If W were a vector space, what else would be true about it?

- **A.** The set W would be a subspace of $\mathbb{R}^2$.
- **B.** The set W would be a subspace of $\mathbb{R}^4$.
- **C.** The set W would be the null space of $\mathbb{R}^2$.
- **D.** The set W would be the null space of $\mathbb{R}^4$.

Determine whether the zero vector is in W. Find values for t and s such that $\begin{bmatrix} s-2t \\ 3+3s \\ 3s+t \\ 2s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

- **A.** The zero vector is in W. The vector equation is satisfied when t = ____ and s = ____.
- **B.** The zero vector is not in W. There is no t and s such that the vector equation is satisfied.

Which of the following is a true statement?

- **A.** Since the zero vector is not in W, W is not the null space of $\mathbb{R}^2$. Thus W is not a vector space.
- **B.** Since the zero vector is in W, W is the null space of $\mathbb{R}^4$. Thus W is a vector space.
- **C.** Since the zero vector is not in W, W is not a subspace of $\mathbb{R}^4$. Thus W is not a vector space.
- **D.** Since the zero vector is in W, W is a subspace of $\mathbb{R}^2$. Thus W is a vector space.

5. Find A such that the given set is Col A.

$$\left\{ \begin{bmatrix} -3r+2s+3t \\ 2r+3s-3t \\ -3s-2t \\ -r+s+2t \end{bmatrix} : r,\, s,\, t \text{ real} \right\}$$

Choose the correct answer below.

A. $A = \begin{bmatrix} 2 & -3 & 3 \\ 3 & 2 & -3 \\ -3 & 0 & -2 \\ -1 & 1 & 2 \end{bmatrix}$

B. $A = \begin{bmatrix} -3 & 2 & 3 \\ -3 & 3 & 2 \\ -2 & -3 & 0 \\ 2 & 1 & -1 \end{bmatrix}$

C. $A = \begin{bmatrix} -3 & 3 & 2 \\ 2 & -3 & 3 \\ 0 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix}$

D. $A = \begin{bmatrix} -3 & 2 & 3 \\ 2 & 3 & -3 \\ 0 & -3 & -2 \\ -1 & 1 & 2 \end{bmatrix}$

6. Complete parts (a) and (b) for the matrix below.

$$A = \begin{bmatrix} -3 & -8 & -4 \\ -3 & -4 & 5 \\ 6 & -5 & 6 \\ 9 & -1 & -7 \\ -8 & 0 & 2 \end{bmatrix}$$

(a) Find k such that Nul(A) is a subspace of $\mathbb{R}^k$.

k = 3

(b) Find k such that Col(A) is a subspace of $\mathbb{R}^k$.

k = 5

7. For the matrix A below, find a nonzero vector in Nul A and a nonzero vector in Col A.

$$A = \begin{bmatrix} 12 & -16 \\ 3 & -4 \\ -12 & 16 \\ -6 & 8 \end{bmatrix}$$

A nonzero vector in Nul A is $\begin{bmatrix} \frac{4}{3} \\ 1 \end{bmatrix}$.

A nonzero vector in Col A is $\begin{bmatrix} 12 \\ 3 \\ -12 \\ -6 \end{bmatrix}$.

8. Let $A = \begin{bmatrix} -12 & 36 \\ -4 & 12 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Determine if **w** is in Col(A). Is **w** in Nul(A)?

Determine if **w** is in Col(A). Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The vector **w** is in Col(A) because A**x** = **w** is a consistent system. One solution is

  $\mathbf{x} = \begin{bmatrix} \frac{11}{4} \\ 1 \end{bmatrix}$.

- **B.** The vector **w** is not in Col(A) because **w** is a linear combination of the columns of A.
- **C.** The vector **w** is in Col(A) because the columns of A span $\mathbb{R}^2$.
- **D.** The vector **w** is not in Col(A) because A**x** = **w** is an inconsistent system. One row of the reduced row echelon form of the augmented matrix [A 0] has the form [0 0 b] where b = ____.

Is **w** in Nul(A)? Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- **A.** The vector **w** is not in Nul(A) because A**w** = ____.
- **B.** The vector **w** is in Nul(A) because A**w** = $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

YOU ANSWERED: A.: $\begin{bmatrix} \frac{19}{5} \\ 1 \end{bmatrix}$

---

9. Determine whether **w** is in the column space of A, the null space of A, or both.

$$\mathbf{w} = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, A = \begin{bmatrix} -5 & 3 & 1 & 0 \\ -8 & 4 & 4 & 10 \\ 10 & -8 & 4 & 14 \\ 3 & -2 & 0 & 0 \end{bmatrix}$$

Is **w** in the column space of A, the null space of A, or both?

- Both
- Null Space
- Column Space

10. Define a linear transformation $T : \mathbb{P}_2 \to \mathbb{R}^2$ by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(0) \end{bmatrix}$. Find polynomials $\mathbf{p}_1$ and $\mathbf{p}_2$ in $\mathbb{P}_2$ that span the kernel of T, and describe the range of T.

Find polynomials $\mathbf{p}_1$ and $\mathbf{p}_2$ in $\mathbb{P}_2$ that span the kernel of T. Choose the correct answer below.

- **A.** $\mathbf{p}_1(t) = t^2$ and $\mathbf{p}_2(t) = -t^2$
- **B.** $\mathbf{p}_1(t) = t$ and $\mathbf{p}_2(t) = t^2 - 1$
- **C.** $\mathbf{p}_1(t) = t$ and $\mathbf{p}_2(t) = t^2$
- **D.** $\mathbf{p}_1(t) = 1$ and $\mathbf{p}_2(t) = t^2$
- **E.** $\mathbf{p}_1(t) = t$ and $\mathbf{p}_2(t) = t^3$
- **F.** $\mathbf{p}_1(t) = t + 1$ and $\mathbf{p}_2(t) = t^2$
- **G.** $\mathbf{p}_1(t) = 3t^2 + 5t$ and $\mathbf{p}_2(t) = 3t^2 - 5t + 7$

Describe the range of T. Choose the correct answer below.

- **A.** $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- **B.** $\varnothing$
- **C.** $\left\{ \begin{bmatrix} a \\ 0 \end{bmatrix} : \text{a real} \right\}$
- **D.** $\left\{ \begin{bmatrix} a \\ a \end{bmatrix} : \text{a real, } a \neq 0 \right\}$
- **E.** $\left\{ \begin{bmatrix} a \\ a \end{bmatrix} : \text{a real} \right\}$
- **F.** $\left\{ \begin{bmatrix} a \\ a \end{bmatrix} : \text{a an integer} \right\}$
- **G.** $\left\{ \begin{bmatrix} 0 \\ a \end{bmatrix} : \text{a real} \right\}$
- **H.** $\left\{ \begin{bmatrix} a \\ a \end{bmatrix} : \text{a real, } a > 0 \right\}$

Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 4.3 Homework

1. Determine whether the set $\left\{ \begin{bmatrix} 7 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 7 \\ 7 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$. If the set is not a basis, determine whether the set is linearly independent and whether the set spans $\mathbb{R}^3$.

Which of the following describe the set? Select all that apply.

- [x] **A.** The set spans $\mathbb{R}^3$.
- [x] **B.** The set is linearly independent.
- [x] **C.** The set is a basis for $\mathbb{R}^3$.
- [ ] **D.** None of the above are true.

2. Determine whether the set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -11 \end{bmatrix}, \begin{bmatrix} 8 \\ -2 \\ 28 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$. If the set is not a basis, determine whether the set is linearly independent and whether the set spans $\mathbb{R}^3$.

Which of the following describe the set? Select all that apply.

- [ ] **A.** The set is a basis for $\mathbb{R}^3$.
- [ ] **B.** The set is linearly independent.
- [ ] **C.** The set spans $\mathbb{R}^3$.
- [x] **D.** None of the above

3. Determine if the set of vectors shown to the right is a basis for $\mathbb{R}^3$. If the set of vectors is not a basis, determine whether it is linearly independent and whether the set spans $\mathbb{R}^3$.

$$\left\{ \begin{bmatrix} 1 \\ -5 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 8 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -8 \\ 6 \end{bmatrix} \right\}$$

Which of the following describe the set? Select all that apply.

- [ ] **A.** The set is linearly independent.
- [ ] **B.** The set is a basis for $\mathbb{R}^3$.
- [x] **C.** The set spans $\mathbb{R}^3$.
- [ ] **D.** None of the above are true.

4. Determine if the set of vectors shown to the right is a basis for $\mathbb{R}^3$. If the set of vectors is not a basis, determine whether it is linearly independent and whether the set spans $\mathbb{R}^3$.

$$\left\{ \begin{bmatrix} 2 \\ 3 \\ -12 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 8 \end{bmatrix} \right\}$$

Which of the following describe the set? Select all that apply.

- [x] **A.** The set is linearly independent.
- [ ] **B.** The set spans $\mathbb{R}^3$.
- [ ] **C.** The set is a basis for $\mathbb{R}^3$.
- [ ] **D.** None of the above

5. Find a basis for the null space of the matrix $\begin{bmatrix} 1 & 0 & -5 & 7 \\ 0 & 1 & -2 & 4 \\ 5 & -13 & 1 & -17 \end{bmatrix}$.

A basis for the null space is $\left\{ \begin{bmatrix} 5 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ -4 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

6. Find a basis for the set of vectors in $\mathbb{R}^2$ on the line $y = 7x$.

A basis for the set of vectors in $\mathbb{R}^2$ on the line $y = 7x$ is $\left\{ \begin{bmatrix} 1 \\ 7 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

7. Assume that A is row equivalent to B. Find bases for Nul A and Col A.

$$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -2 & 1 \\ -3 & 8 & 1 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 5 & 5 \\ 0 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A basis for Col A is $\left\{ \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

A basis for Nul A is $\left\{ \begin{bmatrix} -5 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -\frac{3}{2} \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

8. Find a basis for the space spanned by the given vectors.

$$\begin{bmatrix}1\\0\\0\\1\end{bmatrix},\begin{bmatrix}6\\0\\0\\-6\end{bmatrix},\begin{bmatrix}2\\-3\\1\\-1\end{bmatrix},\begin{bmatrix}8\\-9\\3\\-10\end{bmatrix},\begin{bmatrix}13\\-6\\2\\-11\end{bmatrix}$$

A basis for the space spanned by the given vectors is $\left\{\begin{bmatrix}1\\0\\0\\1\end{bmatrix},\begin{bmatrix}6\\0\\0\\-6\end{bmatrix},\begin{bmatrix}2\\-3\\1\\-1\end{bmatrix}\right\}$.

(Use a comma to separate answers as needed.)

9. Let $\mathbf{v}_1 = \begin{bmatrix}7\\-9\\4\end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix}4\\3\\-8\end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix}23\\-16\\-4\end{bmatrix}$, and $H = \text{Span}\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$. It can be verified that $7\mathbf{v}_1 + 5\mathbf{v}_2 - 3\mathbf{v}_3 = \mathbf{0}$. Use this information to find a basis for H.

A basis for H is $\left\{\begin{bmatrix}7\\-9\\4\end{bmatrix},\begin{bmatrix}4\\3\\-8\end{bmatrix}\right\}$.

(Type an integer or decimal for each matrix element. Use a comma to separate vectors as needed.)

10. Consider the polynomials $\mathbf{p}_1(t) = 4 + t^2$ and $\mathbf{p}_2(t) = 4 - t^2$. Is $\{\mathbf{p}_1, \mathbf{p}_2\}$ a linearly independent set in $\mathbb{P}_3$? Why or why not?

Choose the correct answer below.

- **A.** The set $\{\mathbf{p}_1, \mathbf{p}_2\}$ is a linearly dependent set because both polynomials have degree less than 3.
- **B.** The set $\{\mathbf{p}_1, \mathbf{p}_2\}$ is a linearly independent set because there are only two elements in this set but $\mathbb{P}_3$ has dimension 3.
- **C.** The set $\{\mathbf{p}_1, \mathbf{p}_2\}$ is a linearly independent set because neither polynomial is a multiple of the other polynomial.
- **D.** The set $\{\mathbf{p}_1, \mathbf{p}_2\}$ is a linearly dependent set because $\mathbf{p}_1(t) + \mathbf{p}_2(t)$ does not contain the variable t.

Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 4.4 Homework

1. Find the vector **x** determined by the given coordinate vector $[\mathbf{x}]_B$ and the given basis B.

$$B=\left\{\begin{bmatrix}-2\\1\\0\end{bmatrix},\begin{bmatrix}5\\0\\1\end{bmatrix},\begin{bmatrix}2\\-3\\5\end{bmatrix}\right\},\ [\mathbf{x}]_B=\begin{bmatrix}3\\0\\-2\end{bmatrix}$$

$$\mathbf{x}=\begin{bmatrix}-10\\9\\-10\end{bmatrix}$$

(Simplify your answers.)

2. Find the coordinate vector $[\mathbf{x}]_B$ of **x** relative to the given basis $B=\{\mathbf{b}_1,\mathbf{b}_2\}$.

$$\mathbf{b}_1=\begin{bmatrix}-1\\1\end{bmatrix},\ \mathbf{b}_2=\begin{bmatrix}-5\\-3\end{bmatrix},\ \mathbf{x}=\begin{bmatrix}-6\\-2\end{bmatrix}$$

$$[\mathbf{x}]_B=\begin{bmatrix}1\\1\end{bmatrix}$$

(Simplify your answers.)

3. Find the change-of-coordinates matrix from B to the standard basis in $\mathbb{R}^2$.

$$B=\left\{\begin{bmatrix}-5\\-4\end{bmatrix},\begin{bmatrix}3\\-1\end{bmatrix}\right\}$$

$$P_B=\begin{bmatrix}-5&3\\-4&-1\end{bmatrix}$$

4. The set $B=\{1-t^2, 2t+t^2, 1-t-t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t)=1+12t+4t^2$ relative to B.

$$[\mathbf{p}]_B=\begin{bmatrix}3\\5\\-2\end{bmatrix}$$

(Simplify your answers.)

5. If *B* is the standard basis of the space $\mathbb{P}_3$ of polynomials, then let $B = \{1,t,t^2,t^3\}$. Use coordinate vectors to test the linear independence of the set of polynomials below. Explain your work.

$1 + 9t^2 - t^3,\ t + 3t^3,\ 1 + t + 9t^2$

Write the coordinate vector for the polynomial $1 + 9t^2 - t^3$.

$(1, 0, 9, -1)$

Write the coordinate vector for the polynomial $t + 3t^3$.

$(0, 1, 0, 3)$

Write the coordinate vector for the polynomial $1 + t + 9t^2$.

$(1, 1, 9, 0)$

To test the linear independence of the set of polynomials, row reduce the matrix which is formed by making each coordinate vector a column of the matrix. If possible, write the matrix in reduced echelon form.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 9 & 0 & 9 \\ -1 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Are the polynomials linearly independent?

- **A.** Since the matrix does not have a pivot in each column, its columns (and thus the given polynomials) are linearly independent.
- **B.** Since the matrix does not have a pivot in each column, its columns (and thus the given polynomials) are not linearly independent.
- **C.** Since the matrix has a pivot in each column, its columns (and thus the given polynomials) are linearly independent.
- **D.** Since the matrix has a pivot in each column, its columns (and thus the given polynomials) are not linearly independent.

Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 4.5 Homework

1. For the subspace below, (a) find a basis for the subspace, and (b) state the dimension.

$$\left\{ \begin{bmatrix} s-7t \\ s+t \\ 8t \end{bmatrix} : s, t \text{ in } \mathbb{R} \right\}$$

(a) Find a basis for the subspace.

A basis for the subspace is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 1 \\ 8 \end{bmatrix} \right\}$.

(Use a comma to separate answers as needed.)

(b) State the dimension.

The dimension is 2 .

2. For the subspace below, (a) find a basis, and (b) state the dimension.

$$\left\{ \begin{bmatrix} 6a+12b-2c \\ 3a-b-c \\ -12a+5b+4c \\ -3a+b+c \end{bmatrix} : a, b, c \text{ in } \mathbb{R} \right\}$$

a. Find a basis for the subspace.

A basis for the subspace is $\left\{ \begin{bmatrix} 6 \\ 3 \\ -12 \\ -3 \end{bmatrix}, \begin{bmatrix} 12 \\ -1 \\ 5 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

b. State the dimension.

The dimension is 2 .

3. For the subspace below, (a) find a basis for the subspace, and (b) state the dimension.

$$\left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : a - 5b + 6c = 0 \right\}$$

(a) Find a basis for the subspace.

A basis for the subspace is $\left\{ \begin{bmatrix} 5 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate matrices as needed.)

(b) State the dimension.

The dimension is 3.

---

4. Find the dimension of the subspace of all vectors in $\mathbb{R}^6$ whose first and fifth entries are equal.

The dimension is 5.
(Type a whole number.)

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5. Find the dimension of the subspace spanned by the given vectors.

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -15 \\ 4 \\ -26 \end{bmatrix}, \begin{bmatrix} 11 \\ -3 \\ 19 \end{bmatrix}$$

The dimension of the subspace spanned by the given vectors is 2.

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6. Determine the dimensions of Nul A and Col A for the matrix shown below.

$$A = \begin{bmatrix} 1 & 9 & 9 & 1 & 4 \\ 0 & 1 & -1 & -1 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The dimension of Nul A is 3, and the dimension of Col A is 2.

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7. Determine the dimensions of Nul A and Col A for the matrix shown below.

$$A = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The dimension of Nul A is 0, and the dimension of Col A is 4.

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8. Let B be the basis of $\mathbb{P}_3$ consisting of the Hermite polynomials 1, 2t, $-2+4t^2$, and $-12t+8t^3$; and let $\mathbf{p}(t) = -2+4t^2-8t^3$. Find the coordinate vector of **p** relative to B.

$$[\mathbf{p}]_B = \begin{bmatrix} 0 \\ -6 \\ 1 \\ -1 \end{bmatrix}$$

Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 4.6 Homework

1. Assume that the matrix A is row equivalent to B. Without calculations, list rank A and dim Nul A. Then find bases for Col A, Row A, and Nul A.

$$A = \begin{bmatrix} 1 & -4 & 8 & -3 \\ -1 & 2 & -3 & -1 \\ 6 & -8 & 8 & 14 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 0 & -2 & 5 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rank A = 2

dim Nul A = 2

A basis for Col A is $\left\{ \begin{bmatrix} 1 \\ -1 \\ 6 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

A basis for Row A is $\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -5 \\ 4 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

A basis for Nul A is $\left\{ \begin{bmatrix} 2 \\ \frac{5}{2} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

2. Assume that the matrix A is row equivalent to B. Without calculations, list rank A and dim Nul A. Then find bases for Col A, Row A, and Nul A.

$$A = \begin{bmatrix} 1 & -2 & -2 & 1 & 2 \\ -2 & 4 & 6 & 0 & -14 \\ -3 & 6 & 2 & -7 & 6 \\ 2 & -4 & -10 & -4 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 & -2 & 1 & 2 \\ 0 & 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rank A = 3

dim Nul A = 2

A basis for Col A is $\left\{ \begin{bmatrix} 1 \\ -2 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \\ 2 \\ -10 \end{bmatrix}, \begin{bmatrix} 2 \\ -14 \\ 6 \\ 0 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

A basis for Row A is $\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

A basis for Nul A is $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

3. If a 7×6 matrix A has rank 2, find dim Nul A, dim Row A, and rank A^T.

dim Nul A = 4

dim Row A = 2

rank A^T = 2

4. Suppose a 7×9 matrix A has six pivot columns. What is dim Nul A? Is Col A = $\mathbb{R}^6$? Why or why not?

dim Nul A = 3 (Simplify your answer.)

Is Col A = $\mathbb{R}^6$? Why or why not?

- **A.** No, because Col A is a subspace of $\mathbb{R}^9$.
- **B.** No, because Col A is a subspace of $\mathbb{R}^7$.
- **C.** Yes, because the number of pivot positions in A is 6.
- **D.** Yes, because dim Col A = rank A = 6.

5. If the null space of a 9×12 matrix A is 6-dimensional, what is the dimension of the column space of A?

dim Col A = 6 (Simplify your answer.)

6. If the null space of a 3×9 matrix A is 7-dimensional, what is the dimension of the row space of A?

dim Row A = 2

7. If A is a 6×5 matrix, what is the largest possible rank of A? If A is a 5×6 matrix, what is the largest possible rank of A? Explain your answers.

Select the correct choice below and fill in the answer box(es) to complete your choice.

A. The rank of A is equal to the number of non-pivot columns in A. Since there are more rows than columns in a 6×5 matrix, the rank of a 6×5 matrix must be equal to ______. Since there are 5 rows in a 5×6 matrix, there are a maximum of 5 pivot positions in A. Thus, there is 1 non-pivot column. Therefore, the largest possible rank of a 5×6 matrix is ______.

B. The rank of A is equal to the number of pivot positions in A. Since there are only 5 columns in a 6×5 matrix, and there are only 5 rows in a 5×6 matrix, there can be at most 5 pivot positions for either matrix. Therefore, the largest possible rank of either matrix is 5.

C. The rank of A is equal to the number of columns of A. Since there are 5 columns in a 6×5 matrix, the largest possible rank of a 6×5 matrix is ______. Since there are 6 columns in a 5×6 matrix, the largest possible rank of a 5×6 matrix is ______.

8. Is it possible that all solutions of a homogeneous system of fourteen linear equations in seventeen variables are multiples of one fixed nonzero solution? Discuss.

Consider the system as $A\mathbf{x} = \mathbf{0}$, where A is a 14×17 matrix. Choose the correct answer below.

A. No. Since A has 14 pivot positions, rank A = 14. By the Rank Theorem, dim Nul A = 14 − rank A = 0. Since Nul A = 0, it is impossible to find a single vector in Nul A that spans Nul A.

B. Yes. Since A has at most 14 pivot positions, rank A ≤ 14. By the Rank Theorem, dim Nul A = 17 − rank A ≥ 3. Since there is at least one free variable in the system, all solutions are multiples of one fixed nonzero solution.

C. Yes. Since A has 14 pivot positions, rank A = 14. By the Rank Theorem, dim Nul A = 14 − rank A = 0. Thus, all solutions are multiples of one fixed nonzero solution.

D. No. Since A has at most 14 pivot positions, rank A ≤ 14. By the Rank Theorem, dim Nul A = 17 − rank A ≥ 3. Thus, it is impossible to find a single vector in Nul A that spans Nul A.

Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 5.1 5.2 Homework

1. Is $\mathbf{v} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$ an eigenvector of $A = \begin{bmatrix} -3 & 6 \\ -5 & 8 \end{bmatrix}$? If so, find the eigenvalue.

Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. Yes, **v** is an eigenvector of A. The eigenvalue is $\lambda =$ 2 .

B. No, **v** is not an eigenvector of A.

2. Is $\lambda = 2$ an eigenvalue of $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & -4 \\ -2 & 4 & -2 \end{bmatrix}$? If so, find one corresponding eigenvector.

Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. Yes, $\lambda = 2$ is an eigenvalue of $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & -4 \\ -2 & 4 & -2 \end{bmatrix}$. One corresponding eigenvector is

$\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$.

(Type a vector or list of vectors. Type an integer or simplified fraction for each matrix element.)

B. No, $\lambda = 2$ is not an eigenvalue of $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & -4 \\ -2 & 4 & -2 \end{bmatrix}$.

3. Find the eigenvalues of the matrix.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

The eigenvalue(s) of the matrix is/are $0, -3, 2$.
(Use a comma to separate answers as needed.)

4. Find the characteristic polynomial and the eigenvalues of the matrix.

$$\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

The characteristic polynomial is $\lambda^2 - 8\lambda + 12$.
(Type an expression using λ as the variable. Type an exact answer, using radicals as needed.)

Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. The real eigenvalue(s) of the matrix is/are 2,6.
(Type an exact answer, using radicals as needed. Use a comma to separate answers as needed. Type each answer only once.)

B. The matrix has no real eigenvalues.

5. Find the characteristic polynomial and the eigenvalues of the matrix.

$$\begin{bmatrix} -9 & -4 \\ 4 & -6 \end{bmatrix}$$

The characteristic polynomial is $\lambda^2 + 15\lambda + 70$.
(Type an expression using λ as the variable. Type an exact answer, using radicals as needed.)

Select the correct choice below and, if necessary, fill in the answer box within your choice.

A. The real eigenvalue(s) of the matrix is/are ______.
(Type an exact answer, using radicals as needed. Use a comma to separate answers as needed. Type each answer only once.)

B. The matrix has no real eigenvalues.

6. Find the characteristic polynomial of the matrix, using either a cofactor expansion or the special formula for 3×3 determinants. [Note: Finding the characteristic polynomial of a 3×3 matrix is not easy to do with just row operations, because the variable λ is involved.]

$$\begin{bmatrix} 0 & 3 & 1 \\ 3 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

The characteristic polynomial is $-\lambda^3 + 14\lambda + 12$.
(Type an expression using λ as the variable.)

YOU ANSWERED: $-\lambda^3 + 9\lambda + 12$

7. For the matrix, list the real eigenvalues, repeated according to their multiplicities.

$$\begin{bmatrix} 6 & -4 & 0 & 7 \\ 0 & 6 & 4 & 9 \\ 0 & 0 & 4 & 9 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

The real eigenvalues are 4,6,6,7.
(Use a comma to separate answers as needed.)

Student: Huseyin Kerem Mican	**Instructor:** Taylan Sengul	**Assignment:** Section 5.3 Homework
Date: 7/1/21	**Course:** Linear Algebra	

1. Let $A = PDP^{-1}$ and P and D as shown below. Compute A^4.

$$P = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 536 & -260 \\ 910 & -439 \end{bmatrix}$$

(Simplify your answers.)

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2. Use the factorization $A = PDP^{-1}$ to compute A^k, where k represents an arbitrary integer.

$$\begin{bmatrix} a & 4(b-a) \\ 0 & b \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$$

$$A^k = \begin{bmatrix} a^k & 4\left(b^k - a^k\right) \\ 0 & b^k \end{bmatrix}$$

YOU ANSWERED: 1

0

1

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3. Use the factorization $A = PDP^{-1}$ to compute A^k, where k represents an arbitrary positive integer.

$$\begin{bmatrix} 5 & -6 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$$

$$A^k = \begin{bmatrix} 4 \cdot 2^k - 3 & 6 - 6 \cdot 2^k \\ 2 \cdot 2^k - 2 & 4 - 3 \cdot 2^k \end{bmatrix}$$

YOU ANSWERED: $\begin{bmatrix} 5^k & (-6)^k \\ 2^k & (-2)^k \end{bmatrix}$

4. Matrix A is factored in the form PDP^{-1}. Use the Diagonalization Theorem to find the eigenvalues of A and a basis for each eigenspace.

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & -1 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{4} & -\frac{3}{8} \\ \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

Select the correct choice below and fill in the answer boxes to complete your choice.

(Use a comma to separate vectors as needed.)

A. There is one distinct eigenvalue, $\lambda =$ ____. A basis for the corresponding eigenspace is $\{$ ____ $\}$.

B. In ascending order, the two distinct eigenvalues are $\lambda_1 =$ 1 and $\lambda_2 =$ 5. Bases for the corresponding eigenspaces are $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ and $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$, respectively.

C. In ascending order, the three distinct eigenvalues are $\lambda_1 =$ ____, $\lambda_2 =$ ____, and $\lambda_3 =$ ____. Bases for the corresponding eigenspaces are $\{$ ____ $\}$, $\{$ ____ $\}$, and $\{$ ____ $\}$, respectively.

5. Matrix A is factored in the form PDP^{-1}. Use the Diagonalization Theorem to find the eigenvalues of A and a basis for each eigenspace.

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 2 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 4 \\ -1 & 0 & -2 \end{bmatrix}$$

Select the correct choice below and fill in the answer boxes to complete your choice.

(Use a comma to separate vectors as needed.)

A. There is one distinct eigenvalue, $\lambda =$ ____. A basis for the corresponding eigenspace is $\{$ ____ $\}$.

B. In ascending order, the two distinct eigenvalues are $\lambda_1 =$ 2 and $\lambda_2 =$ 3. Bases for the corresponding eigenspaces are $\left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \right\}$ and $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$, respectively.

C. In ascending order, the three distinct eigenvalues are $\lambda_1 =$ ____, $\lambda_2 =$ ____, and $\lambda_3 =$ ____. Bases for the corresponding eigenspaces are $\{$ ____ $\}$, $\{$ ____ $\}$, and $\{$ ____ $\}$, respectively.

6. Diagonalize the following matrix, if possible.

$$\begin{bmatrix} 8 & 0 \\ 10 & -8 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** For P = $\begin{bmatrix} 8 & 0 \\ 5 & 1 \end{bmatrix}$, D = $\begin{bmatrix} 8 & 0 \\ 0 & -8 \end{bmatrix}$
- **B.** For P = ______, D = $\begin{bmatrix} 8 & 0 \\ 0 & 5 \end{bmatrix}$
- **C.** For P = ______, D = $\begin{bmatrix} 10 & 0 \\ 0 & -10 \end{bmatrix}$
- **D.** The matrix cannot be diagonalized.

7. Diagonalize the following matrix, if possible.

$$\begin{bmatrix} 6 & -2 \\ 2 & 10 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** For P = ______, D = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$
- **B.** For P = ______, D = $\begin{bmatrix} 8 & 0 \\ 0 & -8 \end{bmatrix}$
- **C.** For P = ______, D = $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$
- **D.** The matrix cannot be diagonalized.

8. Diagonalize the following matrix. The real eigenvalues are given to the right of the matrix.

$$\begin{bmatrix} 3 & 3 & -6 \\ -3 & 13 & -18 \\ -1 & 3 & -2 \end{bmatrix}; \lambda = 4,6$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** For P = ______, D = $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$
  (Simplify your answer.)
- **B.** For P = $\begin{bmatrix} 3 & -6 & 1 \\ 1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, D = $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$.
  (Simplify your answer.)
- **C.** The matrix cannot be diagonalized.

9. Diagonalize the following matrix.

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** For P = ______ , $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

  (Type an integer or simplified fraction for each matrix element.)

- **B.** For P = ______ , $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

  (Type an integer or simplified fraction for each matrix element.)

- **C.** The matrix cannot be diagonalized.

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10. Diagonalize the following matrix.

$$\begin{bmatrix} 6 & -4 & 0 & 12 \\ 0 & 3 & 1 & -7 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** For $P = \begin{bmatrix} -1 & 4 & 4 & 1 \\ -1 & 7 & 3 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$

- **B.** The matrix cannot be diagonalized.

11. Show that if A is both diagonalizable and invertible, then so is A^{-1}.

What does it mean if A is diagonalizable?

- **A.** If A is diagonalizable, then $A = PDP^{-1}$ for some invertible P and diagonal D. ✔
- **B.** If A is diagonalizable, then $A^k = PDP^{-1}$ for some invertible P and diagonal D.
- **C.** If A is diagonalizable, then $A = PD$ for some invertible P and diagonal D.
- **D.** If A is diagonalizable, then A must be a triangular matrix.

What does it mean if A is invertible?

- **A.** A has no more than three eigenvalues, so the diagonal entries in D are not zero, so D is invertible.
- **B.** A has no less than three eigenvalues, so the diagonal entries in D are not zero, so D is invertible.
- **C.** Zero is not an eigenvalue of A, so the diagonal entries in D are not zero, so D is invertible. ✔
- **D.** Zero must be an eigenvalue of A, so at least one of the diagonal entries in D is zero, so D is invertible.

What is the inverse of A?

- **A.** $A^{-1} = PD^{-1}P^{-1}$ ✔
- **B.** $A^{-1} = PDP^{-1}$
- **C.** $A^{-1} = P^{-1}D^{-1}P$
- **D.** $A^{-1} = P^{-1}D^{-1}$

Therefore, A^{-1} is also diagonalizable.