Ch 15 – Constrained Optimization

<u>Case 1</u>: objective function and constraints are linear.
(Linear Programming)

<u>Case 2</u>: Nonlinear constrained optimization.

Linear Programming

<u>objective function</u>:

$$\text{maximize} \quad z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

(profit)

c_j: payoff of each unit of j'the activity.

x_j: magnitude of j^{th} activity.

z: total payoff due to n activities.

<u>constraints</u>: $a_{i1} x_1 + a_{i2} x_2 + \dots + a_{in} x_n \leq b_i$

b_i: amount of the i^{th} resource that is available (Resources are limited)

a_{ij}: amount of the i^{th} resource that is consumed for each unit of j^{th} activity.

<u>constraint</u>: $x_i \geq 0$

<u>EX</u> <u>Gas</u>-processing plant: <u>regular quality</u> (grade) or <u>premium quality</u>

Resource	Product: Regular	Product: Premium	Resource Availability
Raw Gas	$7\ m^3$/tonne	$11\ m^3$/tonne	$77\ m^3$/week
Production Time	10 hr/tonne	8 hr/ton	80 hr/week
Storage	9 ton	6 ton	
Profit	150/ton	175/ton	

- Only one of the grades can be produced at a time
- Both grades are guaranteed to sell.

Develop a LP formulation to maximize the profit for this operation.

<u>Sol^n</u> How much of each gas should be produced to maximize the profit?

Total profit $= 150x_1 + 175x_2 = z$

$\boxed{\text{maximize } z = 150x_1 + 175x_2}$ objective function

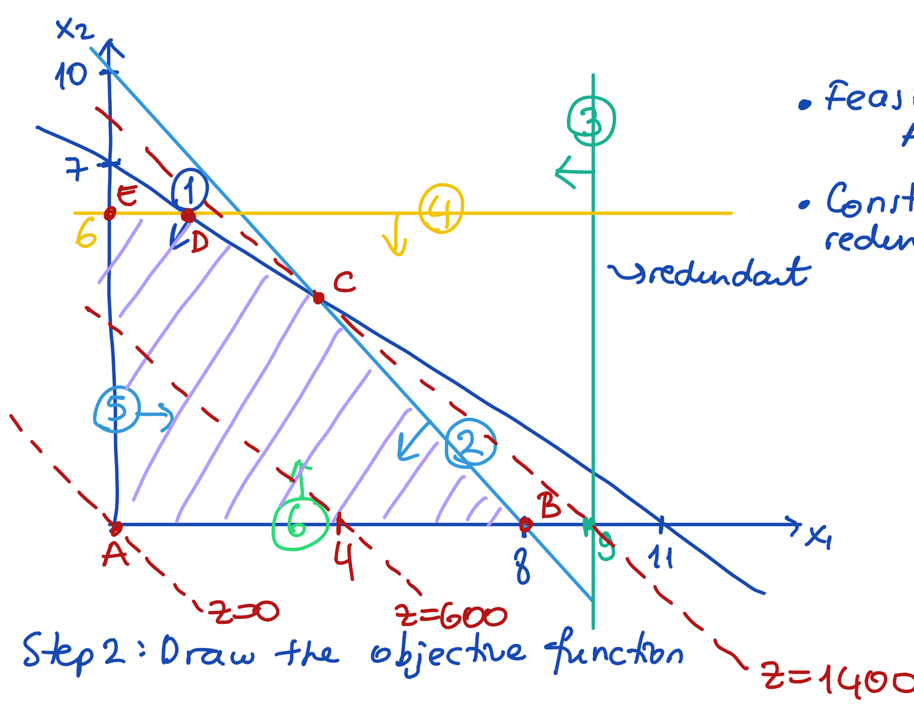
Total gas used $= \begin{cases} 7x_1 + 11x_2 \leq 77 & (1) \text{ constraint (material)} \\ 10x_1 + 8x_2 \leq 80 & (2) \text{ time constraint} \\ x_1 \leq 9 & (3) \text{ regular storage constraint} \\ x_2 \leq 6 & (4) \text{ premium " "} \\ x_1 \geq 0 & (5) \\ x_2 \geq 0 & (6) \end{cases}$

constraints

(5), (6): positivity constraints

Graphical Solution

Step 1: Draw the constraints



- Feasible solution space: ABCDE
- Constraint (3) is redundant

Step 2: Draw the objective function

At $z=0 = 150x_1 + 175x_2 \Rightarrow x_2 = \frac{-150}{175}x_1$

at $z=600 = 150x_1 + 175x_2$

$$x_2 = \frac{600}{175} - \frac{150}{175}x_1$$

at $z=1400 = 150x_1 + 175x_2 \Rightarrow \left.\begin{array}{l} x_1 = 4.9 \\ x_2 = 3.9 \end{array}\right\}$ max. profit is 1400

Substitute the answer into constraints:

$\left.\begin{array}{r} 7(4.9) + 11(3.9) \cong 77 \\ 10(4.9) + 8(3.9) \cong 80 \end{array}\right\}$ just meet the resource and time constraints. (binding constraints)

$\left.\begin{array}{l} 4.9 \leqslant 9 \\ 3.9 \leqslant 6 \end{array}\right\}$ non-binding

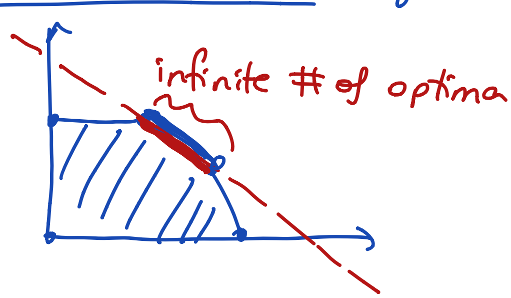
<u>Conclusions</u>

- can increase profit by by increasing rawgas and production time resources.
- increasing storage will have no impact on profit.

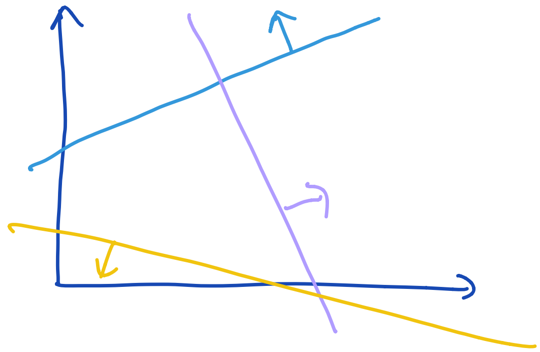
* Four possible outcomes in a linear programming problem

1) <u>unique solution</u>: a single pt (prev. example)

2) <u>alternate solutions</u>: objective function is parallel to one of the constraints.



3) No feasible solution : problem is over constrained



4) Unbounded Problems : problem is underconstrained