Math No:

Full Name : KEY

Math 104 Final Exam
(10 January 2017, 15:00-16:00)

**IMPORTANT**

**1.** Write down your name and surname on top of each page. **2.** The exam consists of 5 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

| Q1 | Q2 | Q3 | Q4 | Q5 | TOT |
|---|---|---|---|---|---|
| | | | | | |
| 5 pts | 5 pts | 5 pts | 5 pts | 5 pts | 25 pts |

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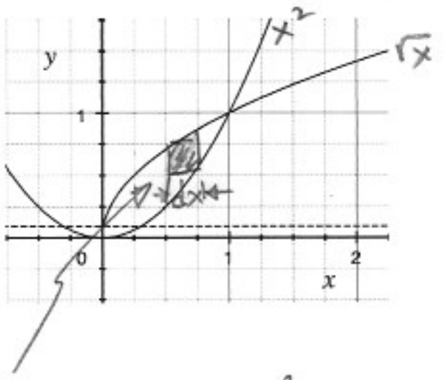
**Q1.** Calculate the area between $y=\sqrt{x}$ and $y=x^2$.




$\sqrt{x} = x^2$

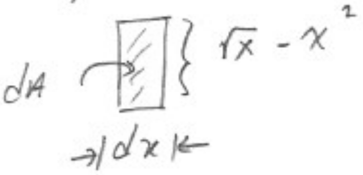
$(\sqrt{x})^2 = (x^2)^2$

$x^4 - x = 0$

$x(x^3-1) = 0$

$\underline{x=0}, \quad x^3 = 1 \Rightarrow \underline{x=1}$

$y = 1$




$dA = (\sqrt{x} - x^2)\, dx$

$A = \int_0^1 (x^{1/2} - x^2)\, dx$

$= \left. \frac{2}{3} x^{3/2} - \frac{1}{3} x^3 \right|_0^1$

$= \frac{2}{3} - \frac{1}{3}$

$A = 1/3$

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**Q2.** Evaluate the following integral $\int \sin x \ln(\cos x)\,dx$

$$u = \ln(\cos x) \qquad dv = \sin x\,dx$$

$$v = -\cos x$$

$$du = -\frac{\sin x}{\cos x}\,dx$$

$$\int \sin x \ln(\cos x)\,dx = uv - \int v\,du$$

$$= -\cos x \ln(\cos x) - \int \cancel{\cos x}\,\frac{\sin x}{\cancel{\cos x}}\,dx$$

$$= -\cos x \ln(\cos x) + \cos x + C$$

$$= \cos x\,(1 - \ln(\cos x)) + C$$

**Q3.** Determine the following limit $\lim_{x\to 1^+}\left(1+\frac{1}{x-1}\right)^{x-1} = \infty^0$

$$y = \left(1+\frac{1}{x-1}\right)^{x-1}$$

$$\ln y = (x-1)\ln\left(1+\frac{1}{x-1}\right)$$

$$\lim_{x\to 1^+}\ln y = \lim_{x\to 1^+}\frac{\ln\left(1+\frac{1}{x-1}\right)}{\frac{1}{x-1}} = \lim_{x\to 1^+}\frac{\frac{\cancel{-(x-1)^{-2}}}{1+\frac{1}{x-1}}}{\cancel{-(x-1)^{-2}}} = 0$$

(L' Hospt. Rule)

$$\lim_{x\to 1^+}\ln y = 0$$

$$\lim_{x\to 1^+} y = e^0 = 1$$

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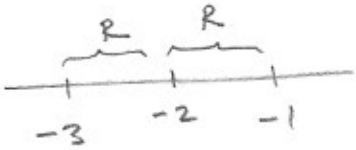
**Q4.** Find the following series' radius and interval of convergence. Test the end values to decide whether the series converges absolutely or conditionally?

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n}$$

$$\rho = \lim_{n\to\infty} \left| \frac{\frac{(x+2)^{n+1}}{n+1}}{\frac{(x+2)^n}{n}} \right| = \lim_{n\to\infty} \left| \frac{(x+2)\cancel{(x+2)^n}}{n+1} \cdot \frac{n}{\cancel{(x+2)^n}} \right|$$

$$\rho = |x+2| \underbrace{\lim_{n\to\infty} \frac{n}{n+1}}_{1} = |x+2| < 1 \text{, to converge.}$$

$$\left.\begin{array}{l} x+2 < 1 \Rightarrow x < -1 \\ -(x+2) < 1 \Rightarrow x+2 > -1 \Rightarrow x > -3 \end{array}\right\} -3 < x < -1$$




For $\underline{x=-3}$ : $\sum_{n=1}^{\infty} \frac{1}{n}$, diverges ($p=1$, series)

For $\underline{x=-1}$ : $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$, converges conditionally since it is an alternating series.

Interval of convergence is

$$-3 < x \leq -1$$

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**Q5.** Find the Maclaurin series for the following functions $f(x) = e^{-x/3}$.

[*Hint: The Taylor Series is given by* $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$]

Let us consider $f(x) = e^x$ for Maclaurin series:

$$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + f^{(4)}(0)\frac{x^4}{4!} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

Now, we substitute $x$ with $-x/3$ in the above series:

$$e^{-x/3} = 1 + (-x/3) + \frac{(-x/3)^2}{2!} + \frac{(-x/3)^3}{3!} + \frac{(-x/3)^4}{4!} + \frac{(-x/3)^5}{5!} + \cdots$$

$$e^{-x/3} = 1 - \frac{x}{3} + \frac{1}{2!}\frac{x^2}{3^2} - \frac{1}{3!}\frac{x^3}{3^3} + \frac{1}{4!}\frac{x^4}{3^4} - \frac{1}{5!}\frac{x^5}{3^5} + \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!\,3^n}$$