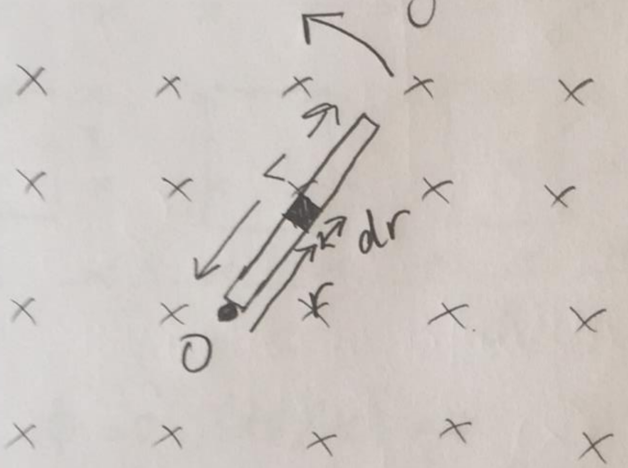
FARADAY'S LAW – II

Motional $\mathcal{E}$ (EMF) induced in a rotating bar:

Conside a conducting bar of length L in a B. The bar is rotating around a fixed axis O:

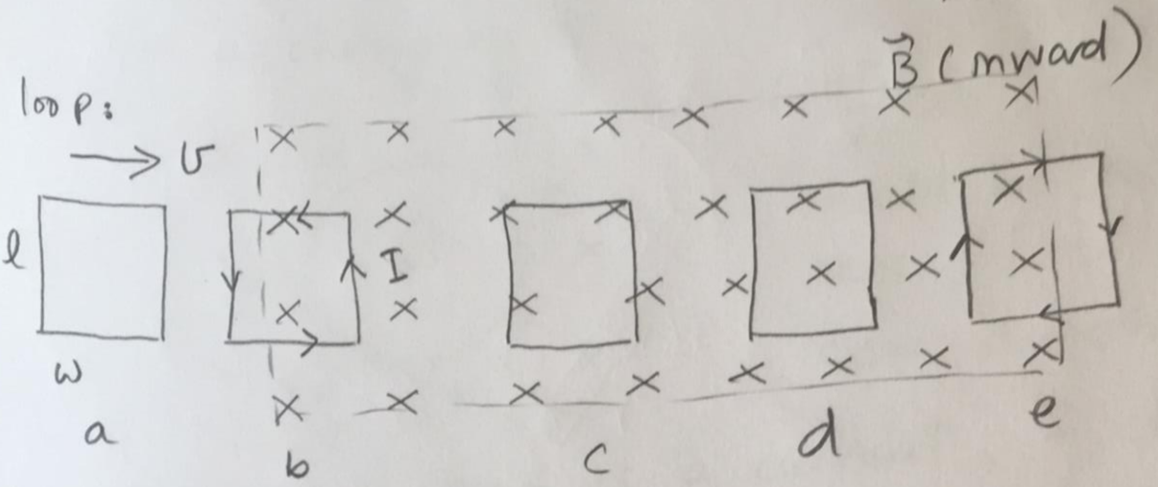


assume the rotation is with constant angular speed ω.

We have seen that when a conducting bar moves in a B, there appears a potential difference ΔV between the two ends of it. This is the induced emf ($\mathcal{E}$): $\mathcal{E} = \Delta V$ and this $\mathcal{E} = vBL$. Now in a rotating bar not all parts of the bar have the same speed: $V = \omega r$, where r is the distance from O to the point with v. So we have to consider the bar in segments of length dr with speed $V = \omega r$. Therefore $d\mathcal{E}$ induced in the segment dr is: $d\mathcal{E} = vB\,dr = \omega r B\,dr$. Total $\mathcal{E}$ is the sum of all $d\mathcal{E}$'s:

$$\mathcal{E} = \int_0^{\mathcal{E}} d\mathcal{E} = \int_0^L B\omega r\,dr \qquad \mathcal{E} = B\omega L^2/2 \text{ Volts}$$

Let us look once again in the direction of the $\mathcal{E}$ or I induced : This is the Lenz's Law.



a. $I = 0 \quad \phi = 0 \quad (d\phi/dt) = 0$

b. ϕ increasing. B induced should be opposite to this, so it should be outwards I is ccw.

c. ϕ is constant, $\phi = B(lw) \quad d\phi/dt = 0 \quad \mathcal{E} = 0$
$I = 0$

d. " " " " "

e. ϕ is decreasing. B should be inwards. I is cw.

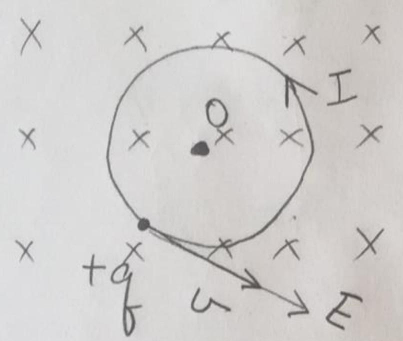
f: $\phi = 0 \quad I = 0$

If you want to pull this loop to the right with constant speed v, you should apply a force to the right, $F_{app} = F_L = IlB$. If R is the resistence of the loop, $\mathcal{E} = IR \quad \mathcal{E} = lBv$.

$$F_{app} = \frac{Blv}{R}(lB) \qquad \vec{F}_{app} = \frac{B^2 l^2 v}{R} \uparrow$$

Induced electric field

We have studied the induced current in a loop in a changing B:



if $\frac{dB}{dt} > 0$ the induced B in should be outward, hence I is in the ccw direction.

Now since there is a current, + charges must also be moving in the ccw direction.

Since a charge moves due to an electrical force, $\vec{F} = q\vec{E}$, there must be an E present.

This is also an induced E.

Consider the line integral around the loop:

$\oint \vec{E} \cdot \vec{ds}$, the work done by this field, in taking a positive charge around a complete loop:

$dW = F \cdot ds = q\vec{E} \cdot ds$

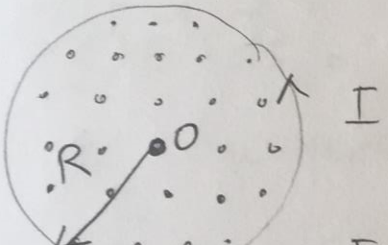
$W = q \oint \vec{E} \cdot \vec{ds}$ $\quad W = q\Delta V = q\mathcal{E} = q\left(-\frac{d\phi}{dt}\right)$

$\cancel{q}\left(-\frac{d\phi}{dt}\right) = \cancel{q} \oint \vec{E} \cdot \vec{ds}$:

$$\boxed{\oint \vec{E} \cdot \vec{ds} = -\frac{d\phi}{dt}}$$

Notice that this induced E is not a conservative field, since the work done by this field over a closed path does not equal to zero. (Rember the definition of a conservative force).

Ex: Assume B created by a solenoid.



$B = \mu_0 n I \qquad n = \frac{N}{L}$

if $I = I(t)$ with $\frac{dI}{dt} > 0$

Find the induced $\vec{E}$ inside and outside of the solenoid of radius R.

a) E at r $(r < R) = ?$ inside

b) E " " $(r > R) = ?$ outside.

Let us assume $\frac{dI}{dt} = k$ where k is a positive constant

$$\frac{d\phi}{dt} = \frac{d}{dt}(AB) = A\frac{dB}{dt} = A\frac{d}{dt}(\mu_0 n I)$$

$$\frac{d\phi}{dt} = A\mu_0 n k$$

a) Now consider a circle with radius $r < R$.

take the line integral around the loop of radius r:

$$\oint \vec{E} \cdot \vec{ds} = -\frac{d\phi}{dt}$$

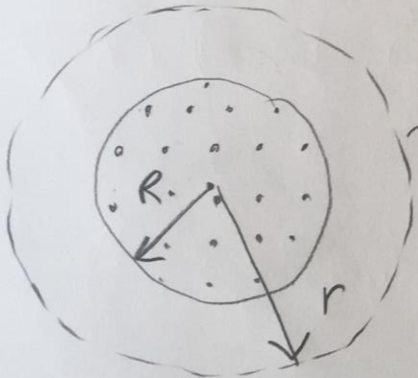
We expect $\vec{E}$ to be parallel to $\vec{ds}$, so $\vec{E} \cdot \vec{ds} = E\,ds$ and also E to have a constant magnitude around the loop

$$\oint \vec{E} \cdot \vec{ds} = E \oint ds = E(2\pi r) = -\pi r^2 \mu_0 n k$$

where $A = \pi r^2$

$$E = -\frac{r}{2} \mu_0 n k$$

b) E outside the solenoid $(r > R)$



$$\oint \vec{E} \cdot \vec{ds} = E(2\pi r) = -\frac{d\phi}{dt}$$

$$\frac{d\phi}{dt} = \frac{d}{dt}(AB) \qquad A = \pi R^2$$

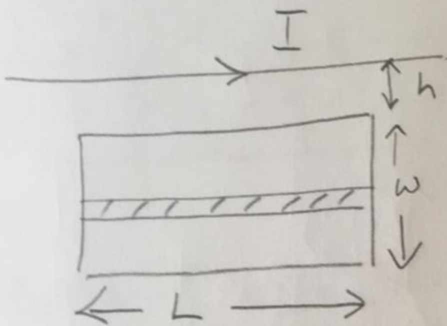
$$\frac{dB}{dt} = \mu_0 n k$$

$$E(2\pi r) = -\pi R^2 \mu_0 n k$$

$$E = -\frac{R^2}{2r} \mu_0 n k.$$

PROBLEMS FROM CH. 31

7.



a) Flux through the loop:

$$B = \frac{\mu_0 I}{2\pi y}$$

$$\phi = \int B \cdot dA = \int_h^{h+w} \frac{\mu_0 I}{2\pi y} L\,dy$$

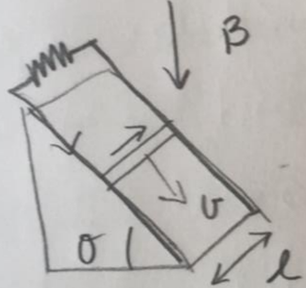
$$\phi = \frac{\mu_0 I L}{2\pi} \ln \frac{h+w}{h}$$

b) $I = a + bt$, a & b are constants.
ε induced in the loop?

$$\varepsilon = -\frac{d\phi}{dt} \qquad \frac{d\phi}{dt} = \frac{\mu_0 L}{2\pi} \ln \frac{h+w}{h} \frac{dI}{dt}$$

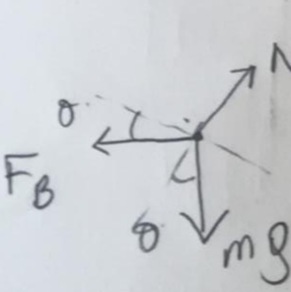
$$\varepsilon = -\frac{\mu_0 L}{2\pi} \ln \frac{h+w}{h} (b)$$

31.

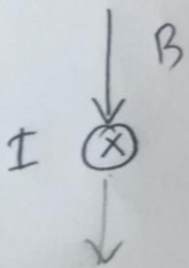


$m = 0.2$ kg $\quad \theta = 25°$ $\quad R = 1\ \Omega$
$B = 0.5$ Tesla $\quad l = 1.2$ m $\quad v = ?$

$$\varepsilon = vBl. \qquad I = \frac{\varepsilon}{R} = \frac{vBl}{R}$$



ϕ is increasing so
induced I is in the CCW direction.
F_B is to the left

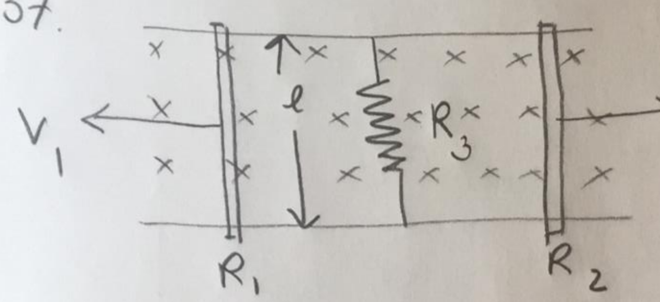


$$mg\sin\theta = F_B\cos\theta \qquad F_B = IlB$$

$$mg\tan\theta = IlB = vB^2l^2/R.$$

$$\boxed{v = Rmg\tan\theta / B^2 l^2}$$

37.

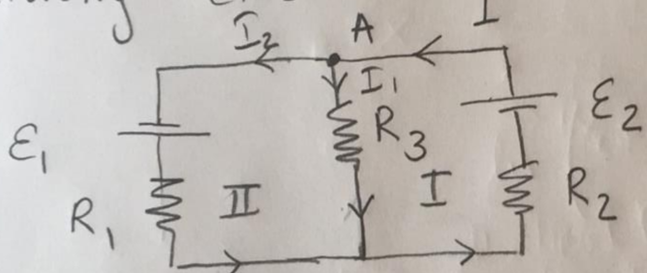


Determine The current in R_3.

Motions of both rods increase the ϕ. So both loops have current in the CCW direction.

$\mathcal{E}_1 = V_1 B \ell \qquad \mathcal{E}_2 = V_2 B \ell$

The above figure is equivelant to the following circuit:



Circuit Eqns:

Loop I: starting at A & going CCW:

① $-I_1 R_3 - I R_2 + \mathcal{E}_2 = 0$

Loop II: starting at A + going CCW.

② $\mathcal{E}_1 - I_2 R_1 + I_1 R_3 = 0 \qquad I_1 + I_2 = I$

$I_2 = I - I_1 \rightarrow$ Substitute in ②

$\mathcal{E}_1 - I R_1 + I_1 R_1 + I_1 R_3 = 0$ ②

$\mathcal{E}_1 - I R_1 + I_1 (R_1 + R_3) = 0$ ② Multiply by R_2

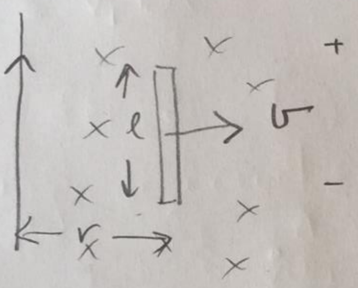
$\mathcal{E}_2 - I R_2 - I_1 R_3 = 0$ ① " " R_1 and subtract

$\mathcal{E}_1 R_2 - \mathcal{E}_2 R_1 + I_1 R_2 (R_1 + R_3) + I_1 R_1 R_3 = 0$

$$I_1 = \frac{\mathcal{E}_1 R_2 - \mathcal{E}_2 R_1}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$I_1 = \frac{B\ell (v_1 R_2 - v_2 R_1)}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

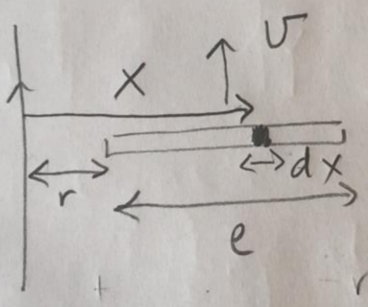
68.



$$B = \frac{\mu_0 I}{2\pi r}$$

$$\mathcal{E} = vB\ell = \frac{\mu_0 I v \ell}{2\pi r}$$

74.



$$d\mathcal{E} = vB\,dx$$

$$d\mathcal{E} = \frac{\mu_0 I v\,dx}{2\pi x}$$

$$\mathcal{E} = \frac{\mu_0 I v}{2\pi} \int_{r}^{r+\ell} \frac{dx}{x} = \frac{\mu_0 I v}{2\pi} \ln\left(\frac{r+\ell}{r}\right.$$