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1. Determine if the vectors are linearly independent.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The vectors are not linearly independent because if $c_1 =$ 2 and $c_2 = 1$, both not zero, then $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 = \mathbf{0}$.

B. The vectors are linearly independent because the vector equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 = \mathbf{0}$ has only the trivial solution.

2. Determine if the columns of the matrix form a linearly independent set.

$$\begin{bmatrix} 1 & 2 & -3 & 3 \\ 2 & 5 & -3 & 0 \\ 2 & 7 & 5 & -16 \end{bmatrix}$$

Select the correct choice below and fill in the answer box to complete your choice.

A. The columns are linearly independent because the reduced row echelon form of $\begin{bmatrix} A & \mathbf{0} \end{bmatrix}$ is ______.

B. The columns are not linearly independent because the reduced row echelon form of $\begin{bmatrix} A & \mathbf{0} \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 0 & -3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \end{bmatrix}$.

3. Use the following vectors to answer parts (a) and (b).

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ 16 \\ -8 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 4 \\ 5 \\ h \end{bmatrix}$$

(a) For what values of h is $\mathbf{v}_3$ in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$?

(b) For what values of h is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly dependent?

(a) For what values of h is $\mathbf{v}_3$ in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** h = ______ (Use a comma to separate answers as needed.)
- **B.** All values of h
- ✔ **C.** No values of h

(b) For what values of h is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly dependent? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** h = ______ (Use a comma to separate answers as needed.)
- ✔ **B.** All values of h
- **C.** No values of h

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4. Find the value(s) of h for which the vectors are linearly dependent.

$$\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -5 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ h \\ -4 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The vectors are linearly dependent if h = ______ because the related matrix will have a free variable. (Type an integer or a simplified fraction.)
- ✔ **B.** The vectors are linearly dependent for all values of h because the related matrix always has a free variable.
- **C.** The vectors are linearly independent for all values of h because the related matrix never has a free variable.

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5. Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 9 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The set is linearly independent because at least one of the vectors is a multiple of another vector.
- **B.** The set is linearly independent because there are four vectors in the set but only two entries in each vector.
- **C.** The set is linearly dependent because at least one of the vectors is a multiple of another vector.
- ✔ **D.** The set is linearly dependent because there are four vectors but only two entries in each vector.

6. Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 3 \\ 4 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The set of vectors is linearly independent because ______ times the first vector is equal to the second vector.
(Type an integer or a simplified fraction.)
- **B.** The set of vectors is linearly dependent because ______ times the first vector is equal to the third vector.
(Type an integer or a simplified fraction.)
- **C.** The set of vectors is linearly dependent because one of the vectors is the zero vector.
- **D.** The set of vectors is linearly independent because none of the vectors are multiples of the other vectors.

7. In parts (a) to (d) below, mark the statement True or False.

a. The columns of a matrix A are linearly independent if the equation A**x** = **0** has the trivial solution. Choose the correct answer below.

- **A.** True. If a matrix equation has the trivial solution then there do not exist nonzero weights for the columns of A such that $c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + \cdots + c_p\mathbf{a}_p = \mathbf{0}$.
- **B.** True. If the columns are linearly independent, then A**x** = **0** has the trivial solution.
- ✔ **C.** False. For every matrix A, A**x** = **0** has the trivial solution. The columns of A are independent only if the equation has no solution other than the trivial solution.
- **D.** False. The columns of a matrix A are linearly independent only if the matrix equation A**x** = **0** has some solution other than the trivial solution.

b. If S is a linearly dependent set, then each vector is a linear combination of the other vectors in S. Choose the correct answer below.

- **A.** True. If an indexed set of vectors, S, is linearly dependent, then at least one of the vectors can be written as a linear combination of other vectors in the set. Using the basic properties of equality, each of the vectors in the linear combination can also be written as a linear combination of those vectors.
- **B.** False. If S is linearly dependent then there is at least one vector that is not a linear combination of the other vectors, but the others may be linear combinations of each other.
- **C.** True. If S is linearly dependent then for each j, $\mathbf{v}_j$, a vector in S, is a linear combination of the preceding vectors in S.
- ✔ **D.** False. If an indexed set of vectors, S, is linearly dependent, then it is only necessary that one of the vectors is a linear combination of the other vectors in the set.

c. The columns of any 4×5 matrix are linearly dependent. Choose the correct answer below.

- **A.** False. If a matrix has more rows than columns then the columns of the matrix are linearly dependent.
- ✔ **B.** True. A 4×5 matrix has more columns than rows, and if a set contains more vectors than there are entries in each vector, then the set is linearly dependent.
- **C.** False. If A is a 4×5 matrix then the matrix equation A**x** = **0** is inconsistent because the reduced echelon augmented matrix has a row with all zeros except in the last column.
- **D.** True. When a 4×5 matrix is written in reduced echelon form, there will be at least one row of zeros, so the columns of the matrix are linearly dependent.

d. If **x** and **y** are linearly independent, and if {**x**, **y**, **z**} is linearly dependent, then **z** is in Span{**x**, **y**}. Choose the correct answer below.

- **A.** False. If **x** and **y** are linearly independent, and {**x**, **y**, **z**} is linearly dependent, then **z** must be the zero vector. So **z** cannot be in Span{**x**, **y**}.
- **B.** True. If {**x**, **y**, **z**} is linearly dependent and **x** and **y** are linearly independent, then **z** must be the zero vector. So **z** is in Span{**x**, **y**}.
- ✔ **C.** True. If {**x**, **y**, **z**} is linearly dependent, then **z** must be a linear combination of **x** and **y** because **x** and **y** are linearly independent. So **z** is in Span{**x**, **y**}.
- **D.** False. Vector **z** cannot be in Span{**x**, **y**} because **x** and **y** are linearly independent.

8. Describe the possible echelon forms of the following matrix.

A is a 3×3 matrix with linearly independent columns.

Select all that apply. (Note that leading entries marked with an X may have any nonzero value and starred entries (*) may have any value including zero.)

- [x] **A.** $\begin{bmatrix} X & * & * \\ 0 & X & * \\ 0 & 0 & X \end{bmatrix}$
- [ ] **B.** $\begin{bmatrix} X & * & * \\ 0 & 0 & X \\ 0 & 0 & 0 \end{bmatrix}$
- [ ] **C.** $\begin{bmatrix} X & * & * \\ * & X & * \\ * & * & X \end{bmatrix}$
- [ ] **D.** $\begin{bmatrix} 0 & X & * \\ 0 & 0 & X \\ 0 & 0 & 0 \end{bmatrix}$

9. Describe the possible echelon forms of the following matrix.

A is a 5×2 matrix, $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 \end{bmatrix}$, and $\mathbf{a}_2$ is not a multiple of $\mathbf{a}_1$.

Select all that apply. (Note that leading entries marked with an X may have any nonzero value and starred entries (*) may have any value including zero.)

- [ ] **A.** $\begin{bmatrix} X & * & 0 & 0 & 0 \\ 0 & X & 0 & 0 & 0 \end{bmatrix}$
- [ ] **B.** $\begin{bmatrix} X & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
- [x] **C.** $\begin{bmatrix} 0 & X \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
- [x] **D.** $\begin{bmatrix} X & * \\ 0 & X \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

10. Suppose A is a 7×5 matrix. How many pivot columns must A have if its columns are linearly independent? Why?

Select the correct answer below.

- ○ **A.** The matrix must have ______ pivot columns. The statements "A has a pivot position in every row" and "the columns of A are linearly independent" are logically equivalent.
- ○ **B.** The matrix must have ______ pivot columns. If A had fewer pivot columns, then the equation $A\mathbf{x} = \mathbf{0}$ would have only the trivial solution.
- ◉ **C.** The matrix must have 5 pivot columns. Otherwise, the equation $A\mathbf{x} = \mathbf{0}$ would have a free variable, in which case the columns of A would be linearly dependent.
- ○ **D.** None of the columns of A are pivot columns. Any column of A that is a pivot column is linearly dependent with the other pivot columns.

11. Suppose A is a 5×7 matrix. How many pivot columns must A have if its columns span $\mathbb{R}^5$? Why?

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The matrix must have ______ pivot columns. If A had fewer pivot columns, then the equation $A\mathbf{x} = \mathbf{0}$ would have only the trivial solution.
- **B.** The matrix must have ______ pivot columns. Otherwise, the equation $A\mathbf{x} = \mathbf{0}$ would have a free variable, in which case the columns of A would not span $\mathbb{R}^5$.
- **C.** The matrix must have 5 pivot columns. The statements "A has a pivot position in every row" and "the columns of A span $\mathbb{R}^5$" are logically equivalent.
- **D.** The columns of a 5×7 matrix cannot span $\mathbb{R}^5$ because having more columns than rows makes the columns of the matrix dependent.

12. Given $A = \begin{bmatrix} 3 & 2 & 5 \\ -5 & 2 & -3 \\ -2 & -2 & -4 \\ 3 & 0 & 3 \end{bmatrix}$, observe that the third column is the sum of the first and second columns. Find a nontrivial solution of $A\mathbf{x} = 0$ without performing row operations. [Hint: Write $A\mathbf{x} = \mathbf{0}$ as a vector equation.]

$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

13. The statement is either true in all cases or false. If false, construct a specific example to show that the statement is not always true.

If $\mathbf{v}_1, \ldots, \mathbf{v}_4$ are in $\mathbb{R}^4$ and $\mathbf{v}_3 = 2\mathbf{v}_1 + \mathbf{v}_2$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** True. If $c_1 = 2$, $c_2 = 1$, $c_3 = 1$, and $c_4 = 0$, then $c_1\mathbf{v}_1 + \cdots + c_4\mathbf{v}_4 = \mathbf{0}$. The set of vectors is linearly dependent.
- **B.** True. The vector $\mathbf{v}_3$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$, so at least one of the vectors in the set is a linear combination of the others and set is linearly dependent.
- **C.** True. Because $\mathbf{v}_3 = 2\mathbf{v}_1 + \mathbf{v}_2$, $\mathbf{v}_4$ must be the zero vector. Thus, the set of vectors is linearly dependent.
- **D.** False. If $\mathbf{v}_1 =$ ______, $\mathbf{v}_2 =$ ______, $\mathbf{v}_3 =$ ______, and $\mathbf{v}_4 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$, then $\mathbf{v}_3 = 2\mathbf{v}_1 + \mathbf{v}_2$ and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent.

14. Determine if **v** is in the set spanned by the columns of B.

$$B = \begin{bmatrix} 2 & -3 & 2 \\ 6 & -6 & 8 \\ 6 & -3 & 12 \\ 6 & 0 & 18 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} -14 \\ -18 \\ 6 \\ 30 \end{bmatrix}$$

Choose the correct answer below and, if necessary, fill in the answer box(es) to complete your choice.

- **A.** Vector **v** is not in the set spanned by the columns of B because the columns of B, $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$ are linearly independent.
- **B.** Vector **v** is not in the set spanned by the columns of B because the reduced echelon form of the matrix formed by writing B with a fourth column equal to **v** is _____.
- **C.** Vector **v** is in the set spanned by the columns of B because 5 $\mathbf{b}_1$ + 8 $\mathbf{b}_2$ + 0 $\mathbf{b}_3 = \mathbf{v}$.
- **D.** Vector **v** is in the set spanned by the columns of B because the columns of B span $\mathbb{R}^4$.

15. Balance the following chemical equation.

$H_3O + CaCO_3 \rightarrow H_2O + Ca + CO_2$

Assume the coefficient of CO_2 is 1. What is the balanced equation?

2 H_3O + 1 $CaCO_3 \rightarrow$ 3 H_2O + 1 $Ca + CO_2$