Math No:

Full Name : KEY

İSTANBUL ŞEHİR UNIVERSITY

Math 104 – Final Exam
(9 January 2015, Time: 15:00-16:00)

**IMPORTANT**

**1.** Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which may have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

| Q1 | Q2 | Q3 | Q4 | TOT |
|---|---|---|---|---|
| | | | | |
| 5 pts | 5 pts | 4 pts | 5 pts | 19 pts |

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**Q1.** If $f(u,v,w)$ is differentiable and $u = x - y$, $v = y - z$, and $w = z - x$. Make use of <u>*The Chain Rule*</u> to find the following sum: $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z} = ?$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} + \frac{\partial f}{\partial w}\cdot\frac{\partial w}{\partial x}$$

$$= \frac{\partial f}{\partial u}\cdot 1 + \frac{\partial f}{\partial v}\cdot 0 + \frac{\partial f}{\partial w}\cdot(-1)$$

$$= \frac{\partial f}{\partial u} - \frac{\partial f}{\partial w} \quad //$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial y}$$

$$= \frac{\partial f}{\partial u}(-1) + \frac{\partial f}{\partial v}\cdot 1 + \frac{\partial f}{\partial w}\cdot 0$$

$$= -\frac{\partial f}{\partial u} + \frac{\partial f}{\partial v} \quad //$$

$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial z} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial z} + \frac{\partial f}{\partial w}\cdot\frac{\partial w}{\partial z}$$

$$= \frac{\partial f}{\partial u}\cdot 0 + \frac{\partial f}{\partial v}\cdot(-1) + \frac{\partial f}{\partial w}\cdot 1$$

$$= -\frac{\partial f}{\partial v} + \frac{\partial f}{\partial w} \quad //$$

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = \cancel{\frac{\partial f}{\partial u}} - \cancel{\frac{\partial f}{\partial w}} - \cancel{\frac{\partial f}{\partial u}} + \cancel{\frac{\partial f}{\partial v}} - \cancel{\frac{\partial f}{\partial v}} + \cancel{\frac{\partial f}{\partial w}} = 0 \quad //$$

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**Q2.** Find the Maclaurin series for $\sin 3x$.

[*Hint*: The Taylor Series is given by $\sum_{k=0}^{\infty}\frac{f^{(k)}(a)}{k!}(x-a)^k$, $a$=0 for Maclaurin series. ]

$$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + \cdots$$

$$f(x) = \sin 3x \rightarrow f(0) = 0$$

$$f'(x) = 3\cos 3x \rightarrow f'(0) = 3$$

$$f''(x) = -9\sin 3x \rightarrow f''(0) = 0$$

$$f'''(x) = -27\cos 3x \rightarrow f'''(0) = -27$$

$$f^{(4)}(x) = +27.3\sin 3x \rightarrow f^{(4)}(0) = 0$$

$$f^{(5)}(x) = 27.3.3.\cos 3x \rightarrow f^{(5)}(0) = 27.3.3$$

$$f(x) = \sin 3x = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + f^{(4)}(0)\frac{x^4}{4!} + \cdots$$

$$f(x) = \sin 3x = 3x - \frac{27}{3!}x^3 + \frac{27.3.3}{5!}x^5 - \frac{27.3.3.3.3}{7!}x^7 + \cdots$$

$$= 3x - \frac{3^3}{3!}x^3 + \frac{3^5}{5!}x^5 - \frac{3^7}{7!}x^7 + \frac{3^9}{9!}x^9 - \cdots$$

$$= 3x - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} - \frac{(3x)^7}{7!} + \frac{(3x)^9}{9!} - \cdots$$

$$= \sum_{n=0}^{\infty}(-1)^n\frac{(3x)^{2n+1}}{(2n+1)!}$$

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**Q3.** Evaluate the following integral: $\int e^{-x} \cos x \, dx$

$$u = \cos x \quad \Big| \quad dv = e^{-x} dx$$
$$du = -\sin x \, dx \quad \Big| \quad v = -e^{-x}$$

$$\int e^{-x} \cos x \, dx = uv - \int v \, du$$
$$= -e^{-x} \cos x - \int e^{-x} \sin x \, dx \qquad ; \quad u = \sin x \;\; du = \cos x \, dx \;\Big|\; v = -e^{-x}$$
$$= -e^{-x} \cos x - \left[ uv - \int v \, du \right]$$
$$= -e^{-x} \cos x - \left[ -e^{-x} \sin x + \int e^{-x} \cos x \, dx \right]$$
$$= e^{-x} (\sin x - \cos x) - \int e^{-x} \cos x \, dx$$

$$2 \int e^{-x} \cos x \, dx = e^{-x} (\sin x - \cos x) + c$$

$$\int e^{-x} \cos x \, dx = \frac{e^{-x}}{2} (\sin x - \cos x) + c \,//$$

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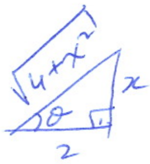
**Q4.** Evaluate the following integral: $\int \frac{dx}{\sqrt{4+x^2}}$

$$x = 2\tan\theta \Rightarrow \tan\theta = \frac{x}{2}$$




$$dx = 2\sec^2\theta \, d\theta$$

$$\int \frac{dx}{\sqrt{4+x^2}} = \int \frac{2\sec^2\theta \, d\theta}{\sqrt{4+4\tan^2\theta}} = \int \frac{\cancel{2}\sec^2\theta \, d\theta}{\cancel{2}\sqrt{1+\tan^2\theta}} = \int \frac{\sec^{\cancel{2}}\theta \, d\theta}{\cancel{\sec\theta}} = \int \sec\theta \, d\theta$$

$$= \int \sec\theta \cdot \left( \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} \right) d\theta = \int \frac{du}{u} = \ln|u| + C$$

$$u = \sec\theta + \tan\theta$$
$$du = \sec^2\theta + \sec\theta\tan\theta$$

$$\int \frac{dx}{\sqrt{4+x^2}} = \ln|\sec\theta + \tan\theta| + C$$
$$= \ln\left| \frac{\sqrt{4+x^2}}{2} + \frac{x}{2} \right| + C \quad //$$