CSE3038 PS – Chapter 1

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[Adopted from Morgan Kauffman Slides & Book]

Problem Sessions

- Teaching Assistant: Lokman ALTIN
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- Problem Sessions @ M1 Z01 (ZOOM link on Canvas)
  - Friday 10:30-12:30
- Office Hours:
  - You can send an email for an appointment.
- If you have any problem, please contact me!

Performance

- Performance = 1/Execution Time
- "X is n times faster than Y"

$$\text{Performance}_X / \text{Performance}_Y$$
$$= \text{Execution time}_Y / \text{Execution time}_X = n$$

- Elapsed Time
  - Total response time, including all aspects
- CPU Time
  - Time spent processing a given job

CPU Time

- CPU Time

$$\text{CPU Time} = \text{CPU Clock Cycles} \times \text{Clock Cycle Time}$$
$$= \frac{\text{CPU Clock Cycles}}{\text{Clock Rate}}$$

- Clock Cycle Time (period) = seconds per cycle
- Clock Rate (frequency) = cycles per second (1 Hz = 1 cycle/sec)

- Instruction Count & CPI

$$\text{Clock Cycles} = \text{Instruction Count} \times \text{Cycles per Instruction}$$
$$\text{CPU Time} = \text{Instruction Count} \times \text{CPI} \times \text{Clock Cycle Time}$$
$$= \frac{\text{Instruction Count} \times \text{CPI}}{\text{Clock Rate}}$$

$$\text{CPU Time} = \frac{\text{Instructions}}{\text{Program}} \times \frac{\text{Clock cycles}}{\text{Instruction}} \times \frac{\text{Seconds}}{\text{Clock cycle}}$$

Q1.
(Exercise 1.5 from 5th edition of the Book)

1.5 [4] <§1.6> Consider three different processors P1, P2, and P3 executing the same instruction set. P1 has a 3 GHz clock rate and a CPI of 1.5. P2 has a 2.5 GHz clock rate and a CPI of 1.0. P3 has a 4.0 GHz clock rate and has a CPI of 2.2.

a. Which processor has the highest performance expressed in instructions per second?

b. If the processors each execute a program in 10 seconds, find the number of cycles and the number of instructions.

c. We are trying to reduce the execution time by 30% but this leads to an increase of 20% in the CPI. What clock rate should we have to get this time reduction?

Q1.a

- Execution Time = CPU clock cycles / clock rate
  = (IC x CPI) / clock rate

- Performance (intructions/sec.) = IC / E.T.
  = clock rate / CPI

- Performance of P1 = $3 \times 10^9 / 1.5 = 2 \times 10^9$
- Performance of P2 = $2.5 \times 10^9 / 1.0 = 2.5 \times 10^9$
- Performance of P3 = $4 \times 10^9 / 2.2 = 1.8 \times 10^9$

$$P2 > P1 > P3$$

Q1.b

- E. T. = CC / clock rate = 10 sec.
- CC = E.T. x clock rate
  - CC of P1 = $10 \times 3 \times 10^9 = 30 \times 10^9$
  - CC of P2 = $10 \times 2.5 \times 10^9 = 25 \times 10^9$
  - CC of P3 = $10 \times 4 \times 10^9 = 40 \times 10^9$

- CC = IC x CPI
- IC = CC / CPI
  - IC of P1 = $30 \times 10^9 / 1.5 = 20 \times 10^9$
  - IC of P2 = $25 \times 10^9 / 1.0 = 25 \times 10^9$
  - IC of P3 = $40 \times 10^9 / 2.2 = 18.18 \times 10^9$

Q1.c

- ET_new = 0.7 x ET_old = 0.7 x 10 = 7 sec.
- CPI_new = 1.2 x CPI_old
  - CPI of P1 = 1.5 x 1.2 = 1.8
  - CPI of P2 = 1.0 x 1.2 = 1.2
  - CPI of P3 = 2.2 x 1.2 = 2.6
- ET_new = (IC x CPI_new) / clock rate
- Clock rate = (IC x CPI_new) / ET_new
  - Clock rate of P1 = (20 x 10^9 x 1.8) / 7 = 5.14 GHz
  - Clock rate of P2 = (25 x 10^9 x 1.2) / 7 = 4.28 GHz
  - Clock rate of P3 = (18.18 x 10^9 x 2.6) / 7 = 6.75 GHz

CPI in More Detail

- If different instruction classes take different numbers of cycles

$$\text{Clock Cycles} = \sum_{i=1}^{n} (\text{CPI}_i \times \text{Instruction Count}_i)$$

- Weighted average CPI

$$\text{CPI} = \frac{\text{Clock Cycles}}{\text{Instruction Count}} = \sum_{i=1}^{n} \left(\text{CPI}_i \times \underbrace{\frac{\text{Instruction Count}_i}{\text{Instruction Count}}}_{\text{Relative frequency}} \right)$$

Q2.
(Exercise 1.6 from 5th edition of the Book)

1.6 [20] <§1.6> Consider two different implementations of the same instruction set architecture. The instructions can be divided into four classes according to their CPI (class A, B, C, and D). P1 with a clock rate of 2.5 GHz and CPIs of 1, 2, 3, and 3, and P2 with a clock rate of 3 GHz and CPIs of 2, 2, 2, and 2.

Given a program with a dynamic instruction count of 1.0E6 instructions divided into classes as follows: 10% class A, 20% class B, 50% class C, and 20% class D, which implementation is faster?

a. What is the global CPI for each implementation?

b. Find the clock cycles required in both cases.

Q2. a

- Total Instructions = 1.0×10^6
  - 10 % class A
  - 20 % class B
  - 50 % class C
  - 20 % class D

- Weighted average CPI

$$CPI = \frac{\text{Clock Cycles}}{\text{Instruction Count}} = \sum_{i=1}^{n} \left(CPI_i \times \underbrace{\frac{\text{Instruction Count}_i}{\text{Instruction Count}}}_{\text{Relative frequency}} \right)$$

- CPI_global of P1:
  - CPI_global = (0.1 x 1 + 0.2 x 2 + 0.5 x 3 + 0.2 x 3)
  - CPI_global = 0.1 + 0.4 + 1.5 + 0.6
  - CPI_global = 2.6
- CPI_global of P2:
  - CPI_global = (0.1 x 2 + 0.2 x 2 + 0.5 x 2 + 0.2 x 2)
  - CPI_global = 0.2 + 0.4 + 1.0 + 0.4
  - CPI_global = 2.0

Q2. b

- Clock Cycle = Instruction Count x CPI
- CC of P1 = IC x CPI_global
- CC of P1 = $1.0 \times 10^6 \times 2.6$
- CC of P1 = 26×10^5

- CC of P2 = IC x CPI_global
- CC of P2 = $1.0 \times 10^6 \times 2.0$
- CC of P2 = 2×10^6

- E.T. = Clock Cycle / Clock Rate
- E.T. of P1 = $26 \times 10^5 / 2.5 \times 10^9 = 10.4 \times 10^{-4}$ sec.
- E.T. of P2 = $2 \times 10^6 / 3 \times 10^9 = 6.66 \times 10^{-4}$ sec.
- Perf. of P2 / Perf. of P1 = E.T. of P1 / E.T of P2
  $= 10.4 \times 10^{-4} / 6.66 \times 10^{-4}$
  $= 1.56$
- P2 is 1.56 times faster than P1

Q3.
(Exercise 1.7 from 5th edition of the Book)

1.7 [15] <§1.6> Compilers can have a profound impact on the performance of an application. Assume that for a program, compiler A results in a dynamic instruction count of 1.0E9 and has an execution time of 1.1 s, while compiler B results in a dynamic instruction count of 1.2E9 and an execution time of 1.5 s.

a. Find the average CPI for each program given that the processor has a clock cycle time of 1 ns.

b. Assume the compiled programs run on two different processors. If the execution times on the two processors are the same, how much faster is the clock of the processor running compiler A's code versus the clock of the processor running compiler B's code?

c. A new compiler is developed that uses only 6.0E8 instructions and has an average CPI of 1.1. What is the speedup of using this new compiler versus using compiler A or B on the original processor?

Q3.a

- IC of compiler A = 10^9
- E.T. of compiler A = 1.1 sec.
- IC of compiler B = 1.2×10^9
- E.T. of compiler B = 1.5 sec.
- Clock cycle time = 1 ns.
- E.T. = IC x CPI x CCT
- CPI = E.T. / (IC x CCT)
  - CPI of A = $1.1 / (10^9 \times 10^{-9})$ = 1.1
  - CPI of B = $1.5 / (1.2 \times 10^9 \times 10^{-9})$ = 1.25

Q3.b

- E.T. of P1 = E.T of P2
- E.T. = (IC x CPI) / Clock rate
- Clock rate = (IC x CPI) / E.T.
  - Clock rate of P1 = (10^9 x 1.1) / E.T.
  - Clock rate of P2 = (1.2 x 10^9 x 1.25) / E.T.
  - Clock of P2 / Clock of P1 = (1.25 x 1.2) / 1.1
    = 1.36

  - *Clock of P2 is 1.36 times faster than clock of P1*

Q3.c

- E.T. of compiler A = 1.1 sec.
- E.T. of compiler B = 1.5 sec.
- E.T. of *new* compiler = IC x CPI x CCT
  $= 6 \times 10^{8} \times 1.1 \times 10^{-9}$
  $= 0.66$ sec.

- **Speedup** = Perf. of. New compiler / Perf. of. Comp. A
  = E.T. of comp. A / E.T. of *new* comp.
  - Speedup1 = E.T. of comp. A / E.T. of *new* comp. = 1.1 / 0. 66
    = 1.67
  - Speedup2 = E.T. of comp. B / E.T. of *new* comp. = 1.5 / 0. 66
    = 2.27

Q4.

(Exercise 1.14 from 5th edition of the Book)

1.14 Assume a program requires the execution of 50 × 106 FP instructions, 110 × 106 INT instructions, 80 × 106 L/S instructions, and 16 × 106 branch instructions. The CPI for each type of instruction is 1, 1, 4, and 2, respectively. Assume that the processor has a 2 GHz clock rate.

1.14.1 [10] <§1.10> By how much must we improve the CPI of FP instructions if we want the program to run two times faster?

1.14.2 [10] <§1.10> By how much must we improve the CPI of L/S instructions if we want the program to run two times faster?

1.14.3 [5] <§1.10> By how much is the execution time of the program improved if the CPI of INT and FP instructions is reduced by 40% and the CPI of L/S and Branch is reduced by 30%?

Q4.1

- 50×10^6 FP instructions
- 110×10^6 INT instructions
- 80×10^6 L/S instructions
- 16×10^6 Branch instructions

- E.T. = CC / clock rate
- CC = $\sum_{i=1}^{n} (CPI_i \, x \, IC_i)$

$= (1 \times 50 \times 10^6) + (1 \times 110 \times 10^6) + (4 \times 80 \times 10^6) + (2 \times 16 \times 10^6)$

$= 512 \times 10^6$ clock cycles

- ET_old $= 512 \times 10^6 / (2 \times 10^9) = 0.256$ sec.

Q4.1 (cont.)

- ET_new = ET_old / 2 = 0.128 sec.
- ET_new = [(CPI_new x 50 x 10^6) + (1 x 110x 10^6) + (4 x 80 x 10^6) + (2 x 16 x 10^6)] / (2 x 10^9)
- 256 x 10^6 = (CPI_new x 50 x 10^6) + (462 x 10^6)
- CPI_new = [(256 - 462) x 10^6] / (50x 10^6)
- CPI_new < 0
- Not possible !
- *We cannot improve CPI of FP instructions to run the program two times faster !*

Q4.2

- ET_new = ET_old / 2 = 0.128 sec.
- ET_new = $[(1 \times 50 \times 10^6) + (1 \times 110 \times 10^6) + (CPI\_new \times 80 \times 10^6) + (2 \times 16 \times 10^6)] / (2 \times 10^9)$
- $256 \times 10^6 = (CPI\_new \times 80 \times 10^6) + (192 \times 10^6)$
- $CPI\_new = [(256 - 192) \times 10^6] / (80 \times 10^6)$
- CPI_new = 0.8
- CPI_old = 4

Q4.3

- CPI_new = 0.6 x 1 = 0.6 for FP instr.
- CPI_new = 0.6 x 1 = 0.6 for INT instr.
- CPI_new = 0.7 x 4 = 2.8 for L/S intr.
- CPI_new = 0.7 x 2 = 1.4 for Branch instr.
- ET_new = $[(0.6 \times 50 \times 10^6) + (0.6 \times 110 \times 10^6) + (2.8 \times 80 \times 10^6) + (1.4 \times 16 \times 10^6)] / (2 \times 10^9)$
- ET_new = 0.171 sec.
- ET_old = 0.256 sec.

Q5. (Exercise 1.15 from 5th edition of the Book)

1.15 [5] <§1.8> When a program is adapted to run on multiple processors in a multiprocessor system, the execution time on each processor is comprised of computing time and the overhead time required for locked critical sections and/or to send data from one processor to another.

Assume a program requires t = 100 s of execution time on one processor. When run *p* processors, each processor requires t/p s, as well as an additional 4 s of overhead, irrespective of the number of processors. Compute the per-processor execution time for 2, 4, 8, 16, 32, 64, and 128 processors. For each case, list the corresponding speedup relative to a single processor and the ratio between actual speedup versus ideal speedup (speedup if there was no overhead).

Q5.

- E.T. = 100 sec. for one processor
- For p processors;
  - Each requires
    - E.T. / p sec. → for processing
    - 4 sec. → overhead (independent from p)
- For 2 processors;
  - Each requires
    - 100 / 2 = 50 sec. for processing
    - 4 sec. for overhead
    - 54 sec. in total
    - Speedup = 100 / 54 = 1.85
    - Actual Speedup / Ideal Speedup = (100 / 54) / (100/50) = 0.93

Q5. (cont.)

# of processors	E.T. / processor	E.T. w/overhead	Speedup	Actual speedup / Ideal speedup
1	100			
2	50	54	100/54 = 1.85	1.85 / 2 = 0.9
4	25	29	100/29 = 3.44	3.44 / 4 = 0.86
8	12.5	16.5	100/16.5 = 6.06	6.06 / 8 = 0.75
16	6.25	10.25	100/10.25 = 9.76	9.76/16 = 0.61
32	3.125	7.125	100/7.125 =14.03	14.03 /32 = 0.44
64	1.56	5.56	100/5.56=17.99	17.99/64 = 0.28
128	0.78	4.78	100/4.78=20.92	20.92/128 = 0.16

Example Question 1

1. Consider two different implementations, I1 and I2, of the same instruction set. There are three classes of instructions (A, B, and C) in the instruction set. I1 has a clock rate of 6 GHz, and I2 has a clock rate of 3 GHz. The average number of cycles for each instruction class on Il and I2 is given in the following table:

Class	CPI on Ml	CPI on M2	C1 Usage	C2 Usage	C3 Usag
A	2	1	40%	40%	50%
B	3	2	40%	20%	25%
C	5	2	20%	40%	25%

The table also contains a summary of average proportion of instruction classes generated by three different compilers. Cl is a compiler produced by the makers of I1, C2 is produced by the makers of I2, and the other compiler is a third-party product. Assume that each compiler uses the same number of instructions for a given program but that the instruction mix is as described in the table. Using Cl on both I1 and I2, how much faster can the makers of I1 claim I1 is compared to I2? Using C2, how much faster can the makers of I2 claim that I2 is compared to I1? If you purchase I1, which compiler would you use? If you purchased I2, which compiler would you use? Which computer and compiler would you purchase if all other criteria are identical, including cost?

Solution 1

Solution:

Using C1, the average CPI for I1 is (.4 * 2 + .4 * 3 + .2 * 5) = 3, and the average CPI for I2 is (.4 * 1 + .2 * 2 + .4 * 2) = 1.6. Thus, with C1, I1 is ((6 x 10^9 cycles/second)/ 3 cycles/instruction)) / ((3 x 10^9 cycles/second)/ 1.6 cycles/instruction)) = 16/15 times as fast as I2.

Using C2, the average CPI for I1 is (.4 * 2 + .2 * 3 + .4 * 5) = 3.4, and the average CPI for I2 is (.4 * 1 + .4 * 2 + .2 * 2) = 1.6. So with C2, 12 is faster than I1 by factor of ((3 x 10^9 cycles/second) / 1.6 cycles/instruction)) / ((6 x 10^9 cycles/second) / 3.4 cycles/instruction)) = 17/16.

For the rest of the questions, it will be necessary to have the CPIs of I1 and I2 on programs compiled by C3. For I1, C3 produces programs with CPI (.5 * 2 + .25 * 3 + .25 * 5) = 3. I2 has CPI (.5 * 1+ .25 * 2 + .25 * 2) = 1.5.

The best compiler for each machine is the one which produce programs with the lowest average CPI. Thus, if you purchased either I1 or I2, you would use C3.

The performance of I1 in comparison to I2 using their optimal compiler (C3) is ((6x10^9 cycles/second) / 3 cycles/instruction)) / ((3x10^9 cycles/second)/ 1.5 cycles/instruction)) = 1

Since both implementations (I1 and I2) have same performance with Compiler 3, any of them can be purchased.

Example Question 2

Assume that multiply instructions take 12 cycles and account for 15% of the instructions in a typical program, and the other 85% of the instructions require an average of 4 cycles for each instruction. What percentage of time does the CPU spend doing multiplication?

Solution 2

Solution:

The average CPI is .15 * 12 cycles/instruction + .85 * 4 cycles/instruction = 5.2 cycles/instructions, of which .15 * 12 = 1.8 cycles/instruction of that is due to multiplication instructions. This means that multiplications take up 1.8/5.2 = 34.6% of the CPU time.

Example Question 3

A pitfall given in the book is expecting to improve the overall performance of a computer by improving only one aspect of the computer. This might be true, but not always.
Consider a computer running programs with CPU times shown in the following table.

	FP Instr.	INT Instr.	L/S Instr.	Branch Instr.	Total Time
a.	70 s	85 s	55 s	40 s	250 s
b.	40 s	90 s	60 s	20 s	210 s

a) How much is the total time reduced if the time for FP operations is reduced by 20%?

b) How much is the time for INT operations reduced if the total time is reduced by 20%?

c) Can the total time can be reduced by 20% by reducing only the time for branch instructions?

Solution 3

a) How much is the total time reduced if the time for FP operations is reduced by 20%?

a.T(fp)= 70*0.8 =56   T=250-70*0.2=236 reduction=5.6%

b.T(fp)=40*0.8=32   T=210-8=202 reduction=3.8%

b) How much is the time for INT operations reduced if the total time is reduced by 20%?

a. T=250*0.8=200 50/85 = 58.8%

b. T=210*0.8=168 42/90=46.7%

c) Can the total time can be reduced by 20% by reducing only the time for branch instructions?

a.It require a reduction of 50 s, which is not possible (50 > 40)

b. It requires a reduction of 42s, not possible (42>20)

Example Question 3 cont'd

The following table shows the instruction type breakdown per processor of given applications executed in different numbers of processors.

	Processors	FP Instr.	INT Instr.	L/S Instr.	Branch Instr.	CPI (FP)	CPI (INT)	CPI (L/S)	CPI (Branch)
a.	2	280×10^6	1000×16^6	640×10^6	128×10^6	1	1	4	2
b.	16	50×10^6	110×10^6	80×10^6	16×10^6	1	1	4	2

Assume that each processor has a 2 GHz clock rate.

d)How much must we improve the CPI of FP instructions if we want the program to run two times faster?

e) How much must we improve the CPI of L/S instructions if we want the program to run two times faster?

f) How much is the execution time of the program improved if the CPI of INT and FP instructions is reduced by 40% and the CPI of L/S and Branch is reduced by 30%?

Solution 3 cont'd

d)How much must we improve the CPI of FP instructions if we want the program to run two times faster?

i. Clock cyles = $(280* 10^6 * 1) + (1000 * 10^6 * 1)+ (640 * 10^6 * 4)+ (128 *10^6 * 2) = 4096*10^6$ cycles
$Tcpu=4096*10^6 /2 * 10^6 = 2048s$

CPI_FP_improved = (2048-(1000+2560+256))/280<0 Not possible

ii. Clock cyles = $(50* 10^6 * 1) + (110 * 10^6 * 1)+ (80 * 10^6 * 4)+ (16 *10^6 * 2) = 512*10^6$ cycles
$Tcpu=512*10^6 /2 * 10^6 = 256s$
CPI_FP_improved = (256-(110+320+32))/50 <0 not possible

e) How much must we improve the CPI of L/S instructions if we want the program to run two times faster?

i. CPI_L/S_improved = (2048-(280+1000+256))/640=0.8
ii. CPI_L/S_improved = (256-(50+110+32))/80= 0.8

Solution 3 cont'd

f) How much is the execution time of the program improved if the CPI of INT and FP instructions is reduced by 40% and the CPI of L/S and Branch is reduced by 30%?

CPI_int=0.6, CPI_fp=0.6, CPI_L/S=2.8 CPI_branch=1.4

i. Clock cyles = $(280 * 10^6 * 0.6) + (1000 * 10^6 * 0.6) + (640 * 10^6 * 2.8) + (128 * 10^6 * 1.4) = 2739.2 * 10^6$ cycles

$Tcpu = 2739.2 * 10^6 / 2 * 10^6 = 1369.6s$

ii. Clock cyles = $(50 * 10^6 * 0.6) + (110 * 10^6 * 0.6) + (80 * 10^6 * 2.8) + (16 * 10^6 * 1.4) = 342.4 * 10^6$ cycles

$Tcpu = 342.4 * 10^6 / 2 * 10^6 = 171.2s$