Q3) $L = \{ a^x b^y c^z \mid x,y,z \geq 0$ and $x + 2y = z \}$

Answer: $G = (V, \Sigma, R, S)$ with set of variables $V = \{S, X\}$, where S is the start variable; and rule:

$$S \rightarrow aSc \mid X$$

$$X \rightarrow bXcc \mid \varepsilon$$

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Q4) $L = \{ a^x b^y c^z \mid x,y,z \geq 0$ and $x + 2y = z \}$

→ Suppose that L is regular language. Let P be the pumping length.
Consider the string $w = a^P b^P c^{3P}$. Note that $w \in L$.
Since $|w| \geq P$, the pumping lemma will hold. Thus we can split string into 3 parts $w = xyz$, satisfying these conditions:

(i) $xy^iz \in L$ for each $i \geq 0$

(ii) $|y| \geq 1$

(iii) $|xy| \leq P$

Since the first P symbols of w are all a's, the third condition implies that x and y consist only a's. So z will be the rest of the first set of a's, followed by $b^P c^{3P}$. The second condition states that $|y| \geq 1$, so y has at least one a. We can say that:

$x = a^j$ for some $j \geq 0$

$y = a^k$ for some $k \geq 1$

$z = a^m b^P c^{3P}$ for some $m \geq 0$

→ Since $a^P b^P c^{3P} = w = xyz = a^j a^k a^m b^P c^{3P} = a^{j+k+m} b^P c^{3P}$, we must have that $j + k + m = P$. The first condition implies that $xy^2z \in L$ but

$$xy^2z = a^j a^k a^k a^m b^P c^{3P} = a^{P+k} b^P c^{3P}$$

Since $j+k+m = P$. Hence, $xy^2z \notin L$ because $k \geq 1$ and integer so $P + k + 2P = 3P$ cannot be correct.

And we got a contradiction.

Therefore L is not regular language.

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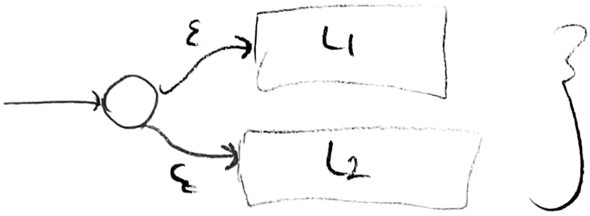
Q5)

Regular languages are closed under union, concatenation, star operations.

For example:



L1 and L2 is regular.
Then this language is also regular.

Since we can not show non-regular languages as NFA or DFA, we cannot build new regular language by using non-regular language.