Sec 2.5 Continuity

Let x_0 be an interior pt of D_f. We say f is continuous at x_0 if

$$\lim_{x \to x_0} f(x) = f(x_0)$$

Let x_0 be the left/right end pts. of D_f; then, if

$$\lim_{\substack{x \to x_0^- \\ x \to x_0^+}} f(x) = f(x_0)$$

$f(x)$ is cont.

Otherwise we say $f(x)$ is discontinuous at x_0.

We say f is cont. if it is cont. at all pts. on its domain.

A cont. function can be graphed without lifting the pen!

<u>Jump Discontinuity</u>

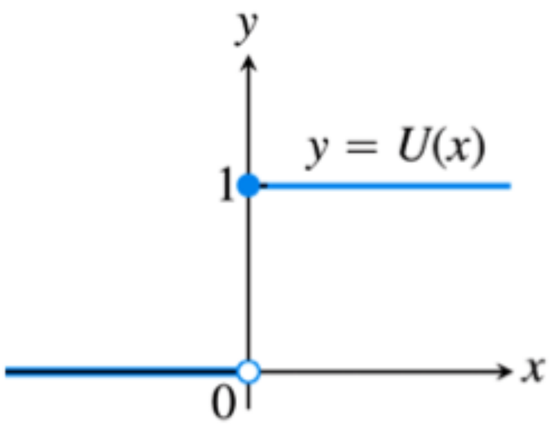


FIGURE 2.38 A function that has a jump discontinuity at the origin

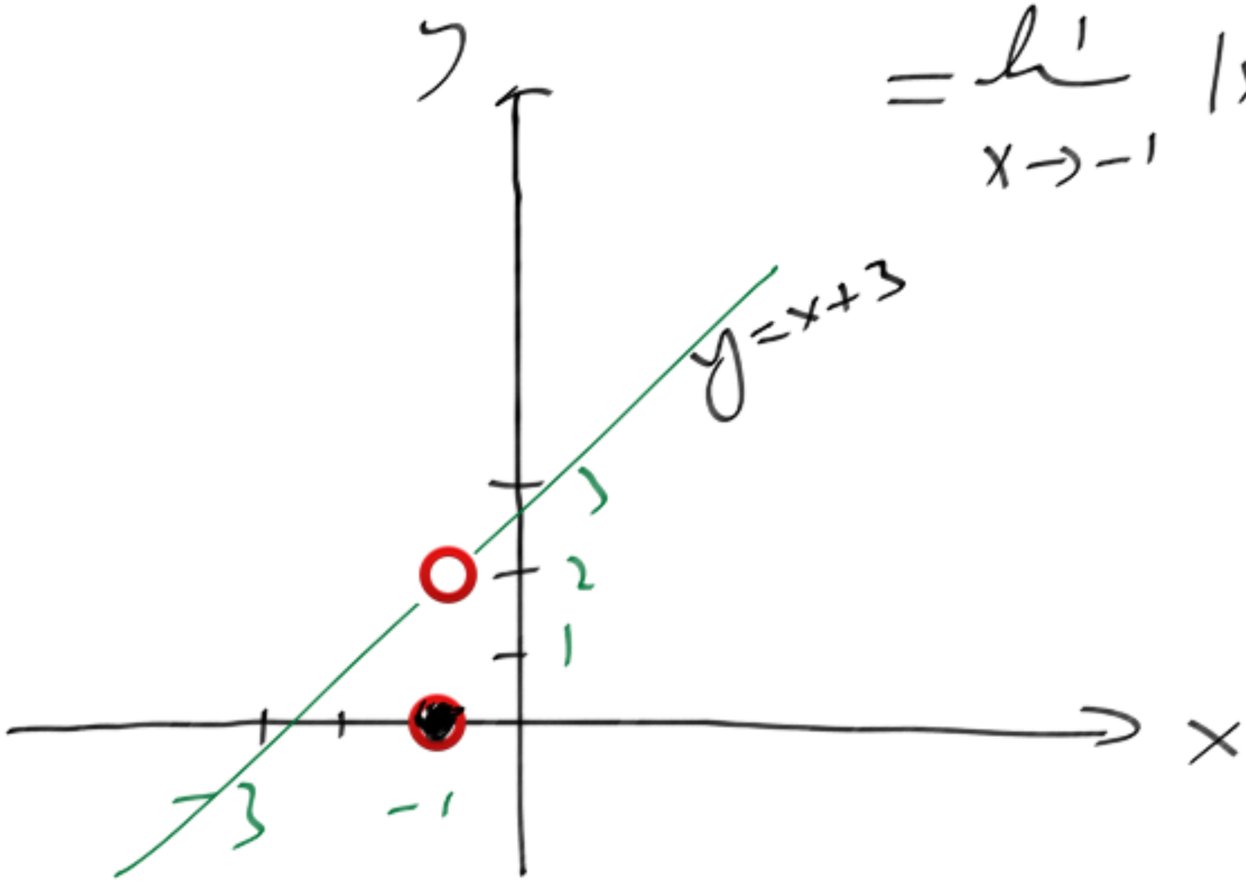
At $x=0$, left & right limits exist, but are not equal!

Removable Discontinuity

$$f(y) = \begin{cases} \dfrac{x^2+4x+3}{x+1} & x \neq -1 \\ 0 & x = -1 \end{cases}$$

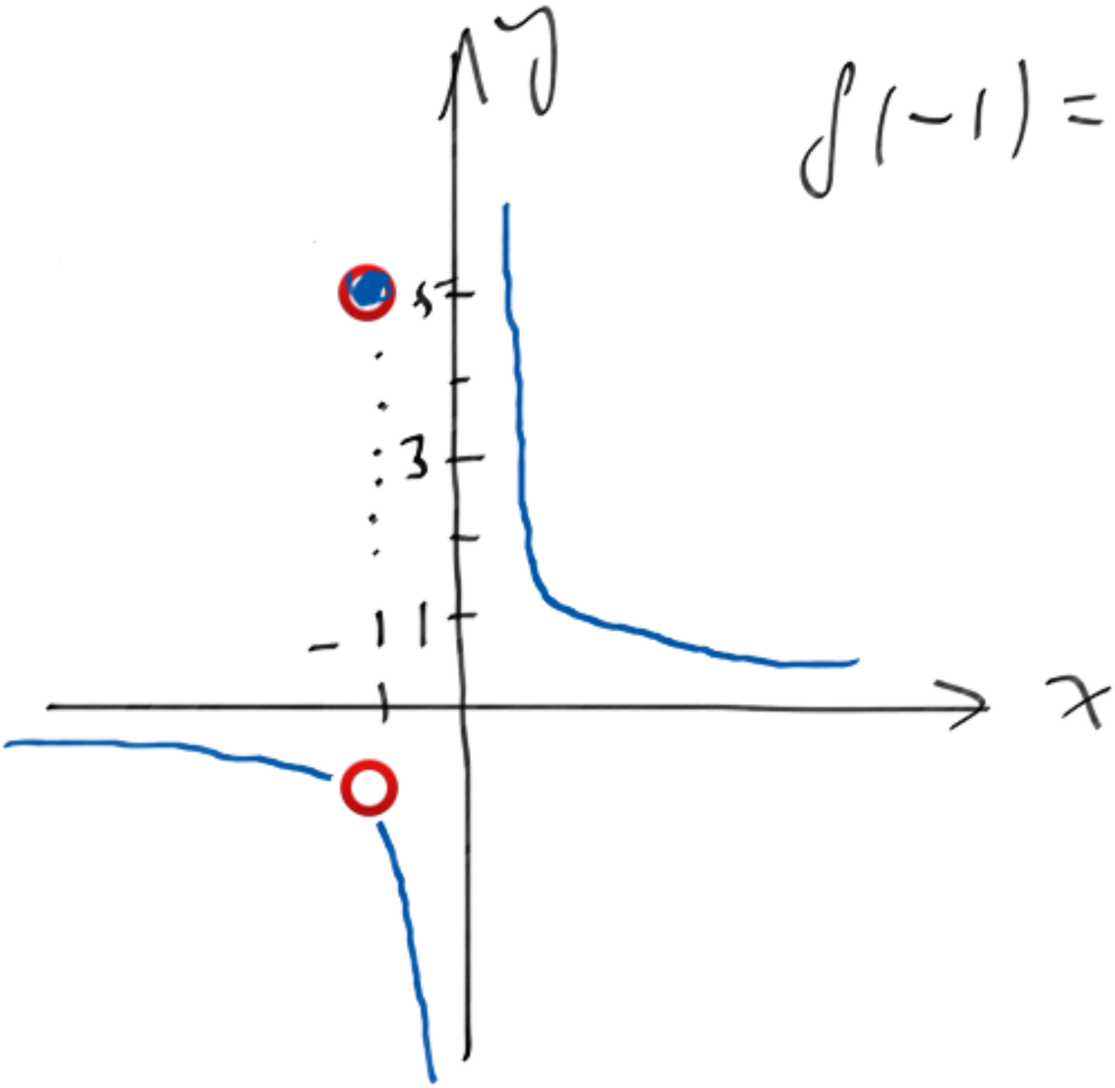
$$\lim_{x\to -1} \frac{x^2+4x+3}{x+1} = \lim_{x\to -1} \frac{\cancel{(x+1)}(x+3)}{\cancel{x+1}}$$

$$= \lim_{x\to -1} (x+3) = 2$$



Essential Discontinuity

$$f(x) = \begin{cases} \frac{1}{x} & x \neq -1 \\ 5 & x = -1 \end{cases}$$



$$f(-1) = \frac{1}{-1} = -1$$

THEOREM —Properties of Continuous Functions If the functions f and g are continuous at $x = c$, then the following combinations are continuous at $x = c$.

1. *Sums:* $f + g$
2. *Differences:* $f - g$
3. *Constant multiples:* $k \cdot f$, for any number k
4. *Products:* $f \cdot g$
5. *Quotients:* f/g, provided $g(c) \neq 0$
6. *Powers:* f^n, n a positive integer
7. *Roots:* $\sqrt[n]{f}$, provided it is defined on an open interval containing c, where n is a positive integer

Polinomials are cont. everywhere
Rational fns. are cont. everywhere,
except when the denominator
is zero.

THEOREM —Composite of Continuous Functions If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c.

Ex

Find and classify all pts of discontinuity of the fn.

$$f(x)=\begin{cases} x & 0\leq x\leq 1 \\ -x^2+7x & 1\leq x<2 \\ x-3 & 2\leq x<4 \\ \dfrac{1}{x-5} & x\geq 4 \end{cases}$$

<u>Jump</u> disc. at $x=2$ & $x=4$

<u>Essential</u> disc. at $x=5$.

For what values of a and b is

$$f(x) = \begin{cases} -2, & x \leq -1 \\ ax - b, & -1 < x < 1 \\ 3, & x \geq 1 \end{cases}$$

continuous at every x?

$$\lim_{x \to -1^+} ax - b = \lim_{x \to -1^-} -2$$

$$-a - b = -2$$

$$\boxed{a + b = 2}$$

$$\lim_{x \to 1^-} ax - b = \lim_{x \to 1^+} 3$$

$$\boxed{a - b = 3}$$

$$\left.\begin{aligned} a + \cancel{b} &= 2 \\ a - \cancel{b} &= 3 \end{aligned}\right\} 2a = 5 \Rightarrow a = 5/2$$

$$b = -\frac{1}{2}$$

. For what values of a and b is

$$g(x) = \begin{cases} ax + 2b, & x \le 0 \\ x^2 + 3a - b, & 0 < x \le 2 \\ 3x - 5, & x > 2 \end{cases}$$

continuous at every x?

$$\lim_{x\to 0^-} ax+2b = \lim_{x\to 0^+} x^2+3a-b$$

$$2b = 3a-b$$

$$\boxed{a=b}$$

$$\lim_{x\to 2^-} x^2+3a-b = \lim_{x\to 2^+} 3x-5$$

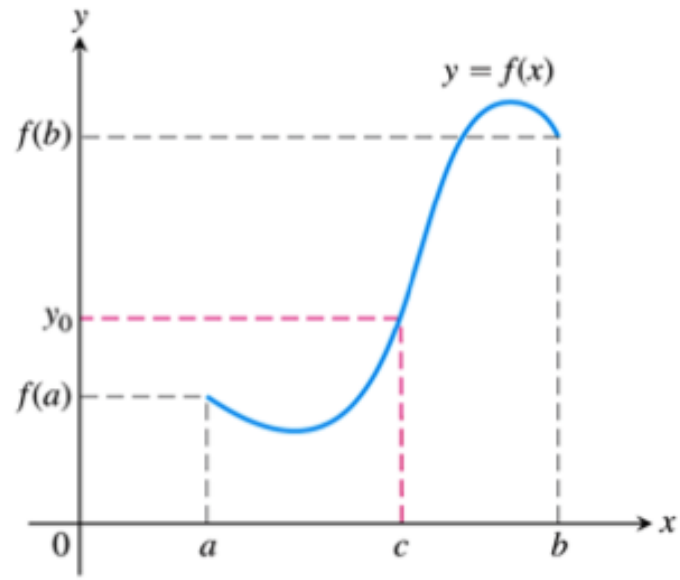
$$4+3a-\cancel{b} = 6-5$$

$$a$$

$$2a = -3$$

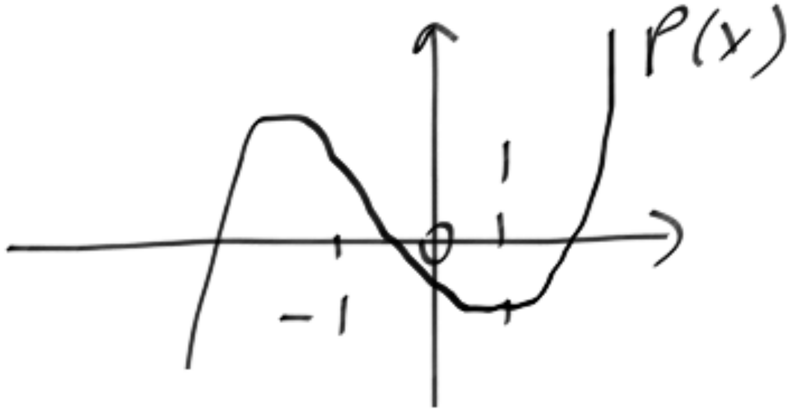
$$a=b=-3/2 \quad //$$

THEOREM ·**The Intermediate Value Theorem for Continuous Functions** If f is a continuous function on a closed interval $[a, b]$, and if y_0 is any value between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.



Ex Show that

$P(x) = x^3 - 3x - 1$ has a root.



$P(0) = -1$

$P(-1) = 1$

a root between $x=0$ & $x=-1$.