Math

No:

Full Name :

Math 100 3rd Midterm Exam
(25 April 2017, 19:00-20:00)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
20 pts	20 pts	20 pts	40 pts	100 pts

Q1. Determine whether the given sequence converges or diverges. If it converges, find the limit:

$$\left\{\frac{n+(-1)^n}{n}\right\}$$

$$\frac{n-1}{n} \leq \frac{n+(-1)^n}{n} \leq \frac{n+1}{n}$$

$$\frac{n-1}{n} \to 1, \qquad \frac{n+1}{n} \xrightarrow{\lim_{n\to\infty}} 1$$

$\therefore$ By the Sandwich thm, the given sequence $\boxed{\text{converges to } 1.}$

OR

$$\frac{n+(-1)^n}{n} = 1 + \frac{(-1)^n}{n}$$

$\lim_{n\to\infty} \frac{(-1)^n}{n} = 0$, so the given sequence converges to 1.

Math No:

Full Name :

Q2. Evaluate the following integral:

$$\int \sqrt{tan\theta}\, sec^4\theta d\theta$$

$$= \int \tan^{1/2}\theta \cdot \underbrace{\sec^2\theta}_{1+\tan^2\theta} \cdot \sec^2\theta\, d\theta$$

$$= \int \left(\tan^{1/2}\theta + \tan^{5/2}\theta\right)\sec^2\theta\, d\theta$$

$$u = \tan\theta \Rightarrow du = \sec^2\theta\, d\theta$$

$$= \int \left(u^{1/2} + u^{5/2}\right) du$$

$$= \frac{2}{3}u^{3/2} + \frac{2}{7}u^{7/2} + C$$

$$= \boxed{\frac{2}{3}\tan^{3/2}\theta + \frac{2}{7}\tan^{7/2}\theta + C}$$

Math

No:

Full Name :

Q3. Evaluate the following integral:

$$\int \frac{3x^2 + x + 8}{x^3 + 4x} dx \quad = \int \frac{3x^2+x+8}{x(x^2+4)}$$

Partial fractions:

$$\frac{3x^2+x+8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$3x^2+x+8 = A(x^2+4) + Bx^2 + Cx$$

$x=0$ $\quad 8 = 4A \Rightarrow A = 2$

$x=1$ $\quad 12 = 5A + B + C \Rightarrow B + C = 2$

$\quad 12 = 10 + B + C$

$x=-1$ $\quad 10 = 5A + B - C \Rightarrow B - C = 0$

$\Downarrow$

$B = C = 1$

$$\therefore I = 2\int \frac{dx}{x} + \frac{1}{2}\int \frac{2x}{x^2+4}dx + \int \frac{dx}{x^2+4}$$

$$= 2\ln|x| + \frac{1}{2}\ln(x^2+4) + \frac{1}{4}\int \frac{dx}{(x/2)^2+1}$$

$u = x/2 \Rightarrow du = dx/2$

$$= 2\ln|x| + \frac{1}{2}\ln(x^2+4) + \frac{1}{2}\int \frac{du}{1+u^2}$$

$$= \boxed{2\ln|x| + \frac{1}{2}\ln(x^2+4) + \frac{1}{2}\operatorname{Arctan}\frac{x}{2} + C}$$

Math

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Q4. Determine whether the following series converge or diverge:

a) $\sum_{n=1}^{\infty} n\tan\frac{1}{n}$

$\infty \cdot 0$ $\quad$ $0/0$

$$\lim_{x\to\infty} x\tan\frac{1}{x} = \lim_{x\to\infty} \frac{\tan\frac{1}{x}}{\frac{1}{x}}$$

$$\underset{\text{L'Hospital}}{=} \lim_{x\to\infty} \frac{\sec^2\frac{1}{x}\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}} = \lim \sec^2\frac{1}{x} = 1$$

$\lim = 1 \neq 0$

$\therefore$ The series diverges by the nth term test.

b) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$

Limit Comparison Test with $\sum \frac{1}{\sqrt{n}}$, p-series, $p=\frac{1}{2}<1$, divergent.

$$\lim_{n\to\infty} \frac{\sqrt{\frac{n+1}{n^2+2}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \sqrt{\frac{n(n+1)}{n^2+2}}$$

$$= \sqrt{\lim_{n\to\infty} \frac{n(n+1)}{n^2+2}} = \sqrt{1} = 1 \quad \neq 0, \infty$$

$\therefore$ The given series diverges.