Q:

1. (Q-M) Give the state diagram of a DFA recognizing the following language (Σ={a,b}):

   L = {*w* | *w* contains at least three **b**s and at most one **a**}

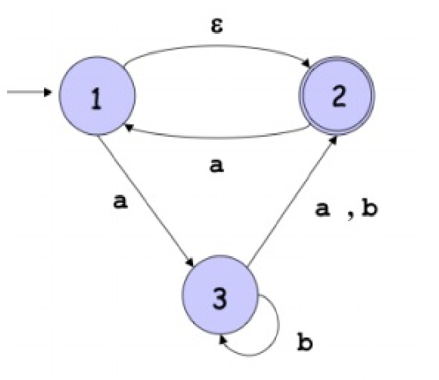
2. (Q-M) Construct an NFA recognizing the language of the following regular expression:

   $$(01)*(1* \cup 0000)*(0101 \cup \varepsilon)$$

3. (Q-M) Design an NFA for the following language over an alphabet Σ = {0,1,2}:
   L = { y2z | y, z $\in$ {0, 1}*, the last symbols of both y and z are 1, and both y and z contain 010 as substring}

4. (Q-M) Given two regular languages L_1 and L_2 over an alphabet Σ = {0,1,2}, prove or disprove that the following languages are regular:
   a. $L_3 = \{ w \in \Sigma^* \mid w \in L_1 \text{ but } w \notin L_2 \}$
   b. $L_4 = \{ w \in \Sigma^* \mid w \text{ is in exactly one of } L_1 \text{ and } L_2 \}$

5. (Q-M) Convert the following NFA to an equivalent DFA following the steps described in class (see Theorem 1.39 in Sipser).



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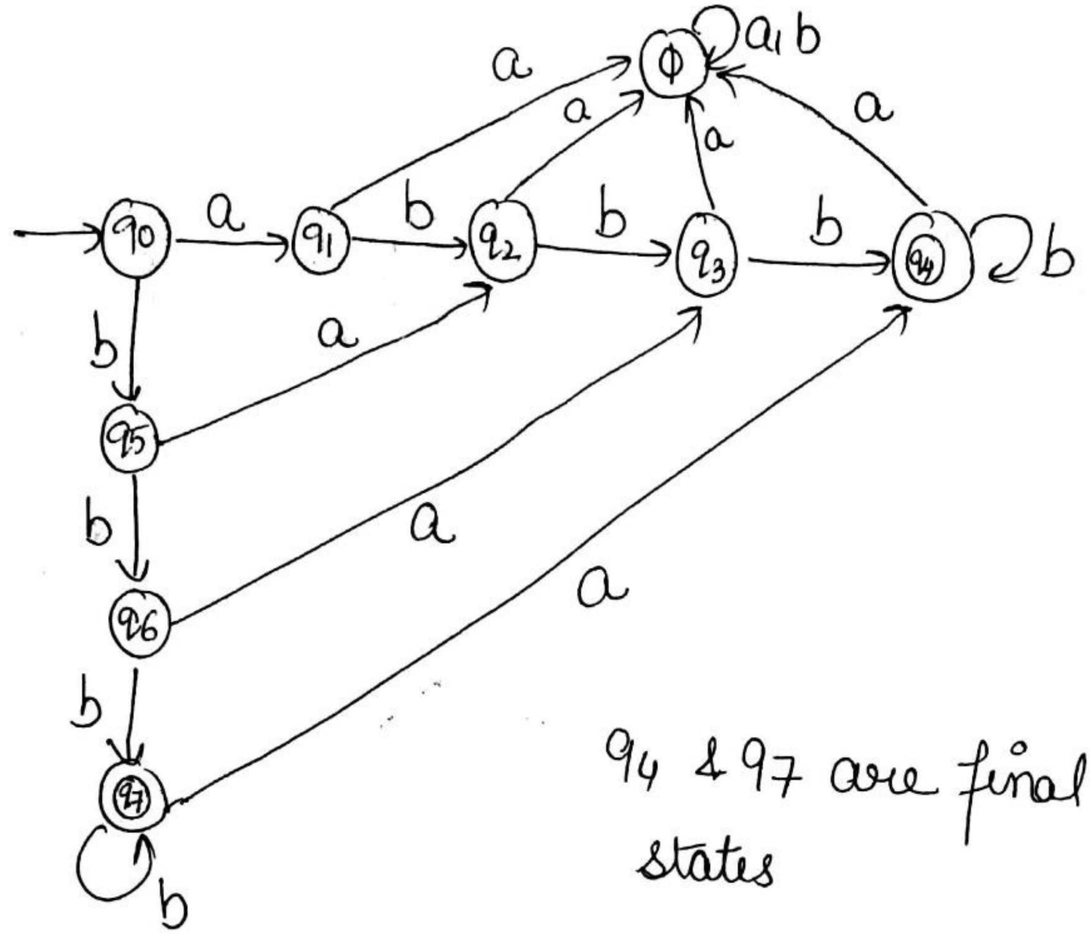
A:

Ans 1:

Ques 1 DFA for

L = {w | w contains at least three b's and at most one a}.

Sol = { bbb, abbb, babb, bbab, bbba, abbbb, bbabbb - - - }



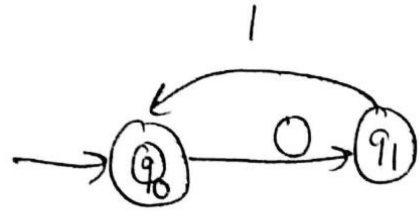
q4 & q7 are final states

Here minimum strings are bbb, abbb, babb, bbab, bbba first accept these strings.
then mark all pending a's to hang state & b's as loop because there can be any no of b's

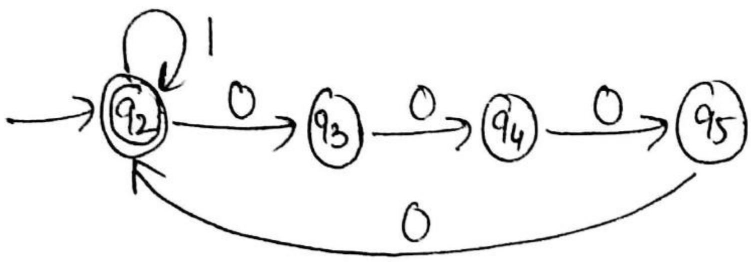
Ans 2:

Ques 2 $(01)^* (1^* \cup 0000)^* (0101 \cup \epsilon)$

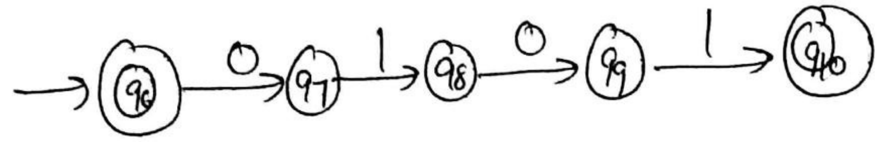
NFA for $(01)^*$



NFA for $(1^* \cup 0000)^*$

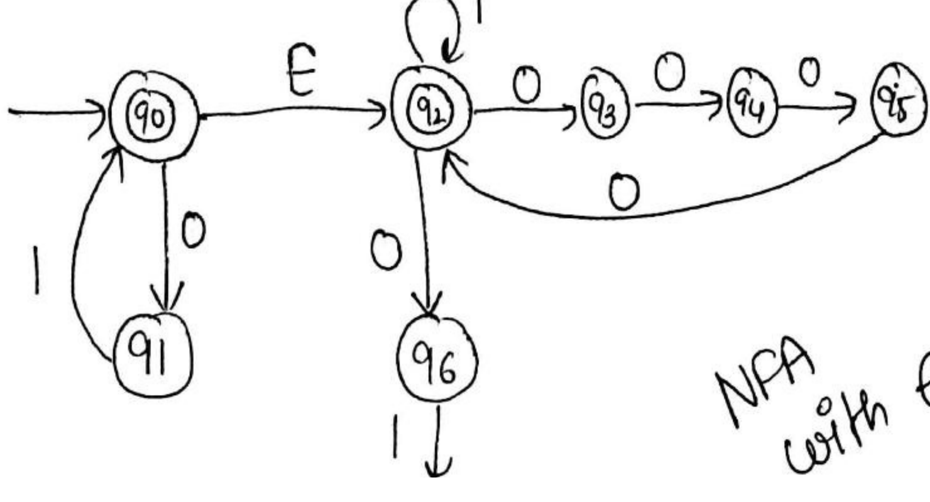


NFA for $(0101 \cup \epsilon)$

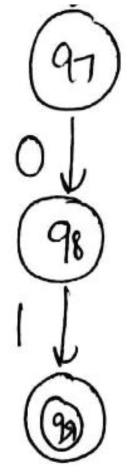


$L = \{\epsilon, 0101, 0101, 1111, 0000, 1000011000, 01000010000\,0101 - - - \}$.

Now combine all of these.

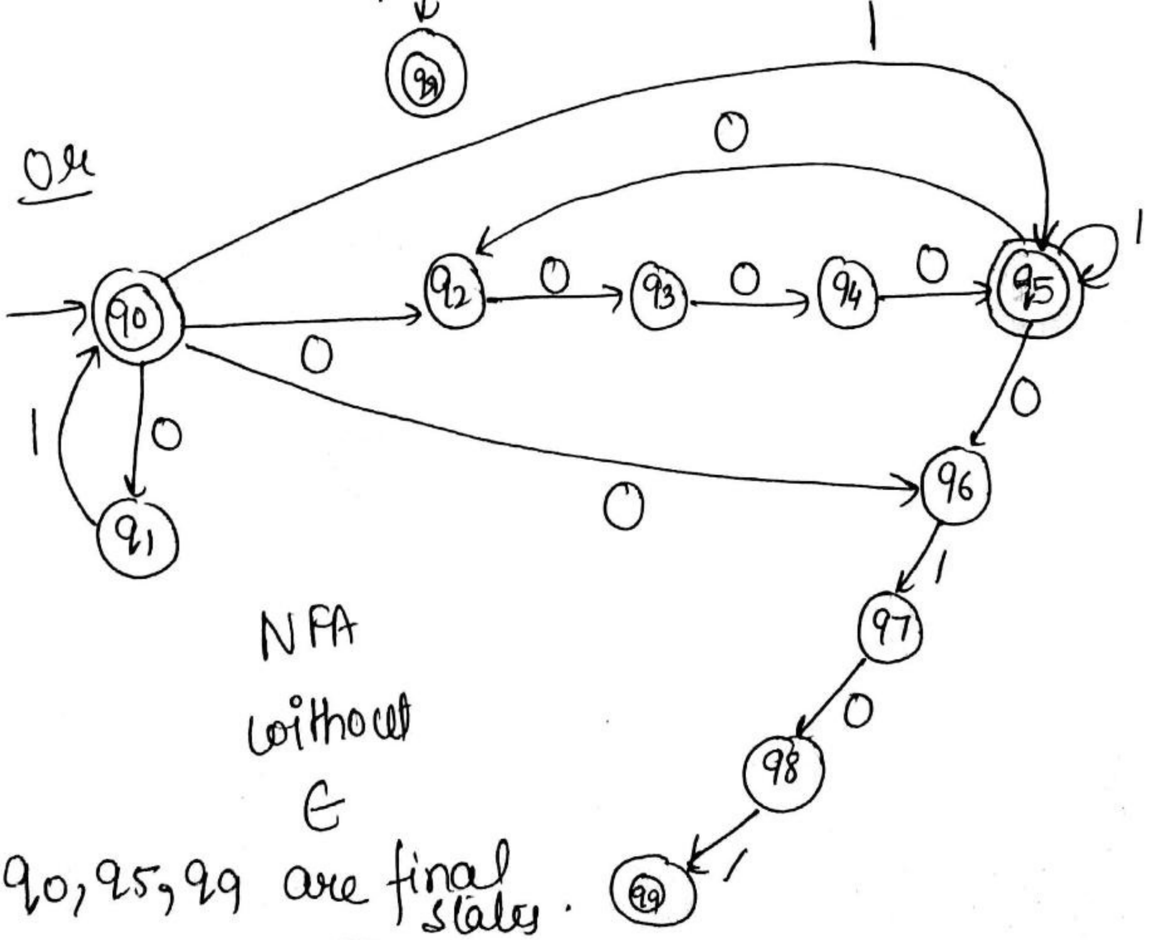


NFA with ε



q0, q2, q9 are final states

Or



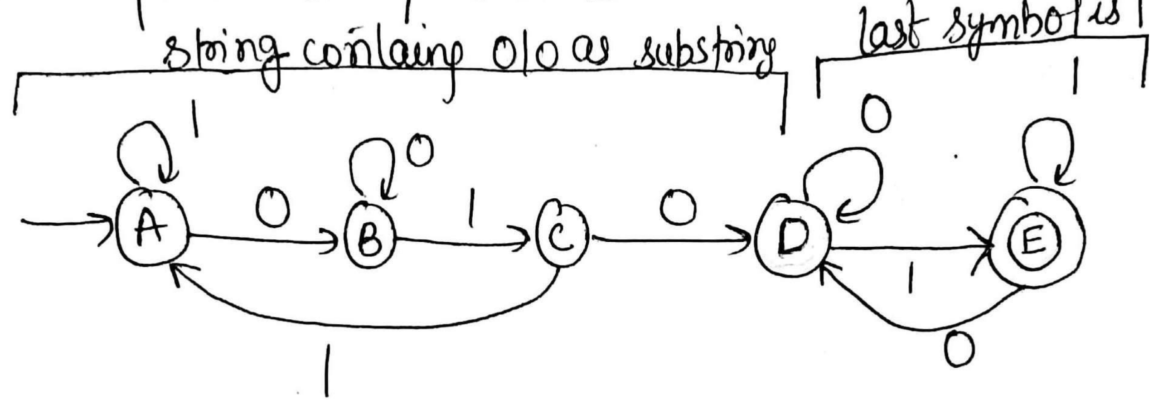
Ans 3:

Que 3 $L = \{ y2z \mid y, z \in \{0,1\}^* \}$

the last symbols of both y & z are 1, and both y & z contain 010 as substring.

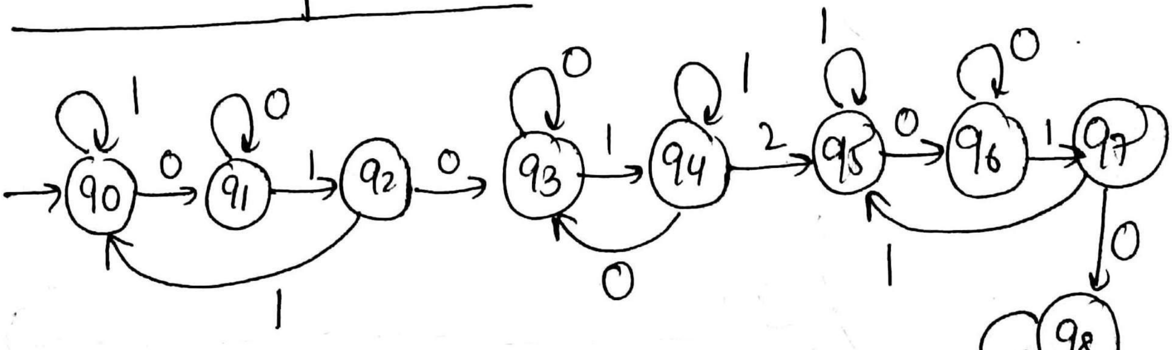
Sol

NFA for the y and z are -

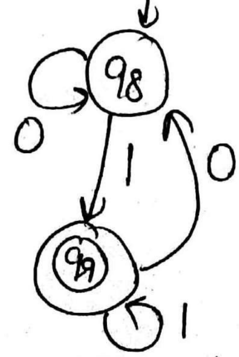


accepts all the strings contains 010 as substring & last symbol is 1

Now NFA for L is



The input 2 of all the states is in sink state.
q9 is final state.



And 4:

Ques 4

(a) $L_3 = \{ w \in \Sigma^* \mid w \in L_1 \text{ but } w \notin L_2 \}$

$w \in L_1$ hence L_3 is the subset of L_1.

(It can be proper subset or improper subset).

The language L_3 can be non regular or can be regular. we can not say subset of a regular language is always regular.

eg $a^* b^*$ is a regular Language.

$a^n b^n$ is the subset of $a^* b^*$.

but $a^n b^n$ is not a regular language.

Hence L3 can be regular or non regular.

Ques 4

(b) $L_4 = \{ w \in \Sigma^* \mid w \text{ is in exactly one of } L_1 \text{ and } L_2$.

$w \in$ to both L_1 and L_2

Hence L4 is the intersection of L_1 & L_2.

$$\boxed{L_4 = L_1 \cap L_2.}$$

The intersection of 2 regular languages are regular. **Hence L4 is regular language.**

Proof $L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$

L_1 is regular, complement of $\overline{L_1}$ is also regular.
L_2 is regular, complement of $\overline{L_2}$ is also regular.

The $\cup$ of 2 regular languages is regular
$\therefore \overline{L_1} \cup \overline{L_2}$

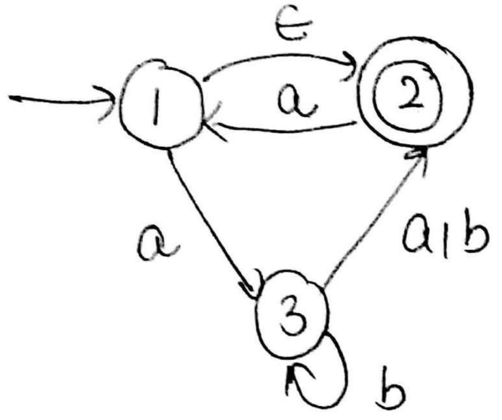
Now the complement of a regular language is regular $\overline{\overline{L_1} \cup \overline{L_2}}$

Now $\overline{\overline{L_1} \cup \overline{L_2}}$ is regular hence $L_1 \cap L_2$ is also regular.

Ans 5:

Ques 5

Convert NFA with ϵ to DFA.



Sol

Step 1 The initial state of DFA is = ϵclosure(initial state of NFA)

$= \epsilon$ closure (1)

$= \{1, 2\}$ say A

ϵ closure (q) = is set of all the states which are reachable from q with ϵ inputs, including the state q. So ϵclosure(1) = {1,2}

Step2 Find transitions from A with input a & b.

$\delta(A,a) = \epsilon closure(\delta(A,a))$

$= \epsilon closure(\delta(1,2,a))$

$= \epsilon closure(3,1)$

$= \{1,3\}$ say B

$\delta(A,b) = \epsilon closure(\delta(A,b))$

$= \epsilon closure(\delta(1,2,b))$

$= \epsilon closure(\phi)$

$= \phi$

Step3 Find transition from B with input a and b.

$\delta(B, a) = \epsilon closure(\delta(B, a))$
$= \epsilon closure(\delta(1,3, a))$
$= \epsilon closure(3, 2)$
$= \{2, 3\}$ say C

$\delta(B, b) = \epsilon closure(\delta(B, b))$
$= \epsilon closure(\delta(1, 3, b))$
$= \epsilon closure(2, 3)$
$= \{2, 3\}$ say C

Step4 Find transition from C with input a & b.

$\delta(C, a) = \epsilon closure(\delta(C, a))$
$= \epsilon closure(\delta(2, 3, a))$
$= \epsilon closure(2, 1)$
$= \{1, 2\}$ say A

Step5

$\delta(C, b) = \epsilon closure(\delta(C, b))$
$= \epsilon closure(\delta(2, 3, b))$
$= \epsilon closure(2, 3)$
$= \{2, 3\}$ say C

Now from the above transitions prepare transition table:-

Here

A : {1, 2}

	a	b
→(A)	B	ϕ
B	C	C
(C)	A	C

A : {1,2}

B : {1,3}

C : {2,3}

Here A & C are final states because set A & C contains State 2 (Final state of NFA).

DFA Transition Diagram