Math

ISTANBUL ŞEHİR UNIVERSITY

No:

Full Name :

Math 104 1st Midterm Exam
(5 March 2016, 11:30-12:30)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
20 pts	20 pts	30 pts	30 pts	100 pts

Q1. Let $f(x) = x^4 - 4x - 5$, for $x > 1$.

a) Show that f is invertible.

b) Find $\frac{df^{-1}}{dx}$ at the point $x = 3$. [Hint: $3 = f(2)$.]

(a) (10pts) $f'(x) = 4x^3 - 4 = 4(x^3 - 1) = 4(x-1)(\underbrace{x^2+x+1}_{\text{always} > 0})$

$\therefore f'(x) > 0$ if $x > 1$

f is monotone increasing, $\therefore$ invertible.

(b) (10 pts)
$$\left.\frac{df^{-1}}{dx}\right|_{x=3} = \frac{1}{\left.\frac{df}{dx}\right|_{x=2}} = \left.\frac{1}{4(x^3-1)}\right|_{x=2}$$

$$= \frac{1}{4 \cdot 7} = \boxed{\frac{1}{28}}$$

Q2. The region between the curve $y = \sqrt{\frac{x}{x^2+1}}$ and the x-axis, for $0 \le x \le 1$, is rotated about the x-axis. Find the volume generated.

Disk method:

$$V = \pi \int_0^1 y^2 dx = \pi \int_0^1 \frac{x}{x^2+1} dx$$

$$u = x^2+1 \Rightarrow du = 2x\,dx \Rightarrow \frac{du}{2} = x\,dx$$

$$\therefore \int \frac{x\,dx}{x^2+1} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln(x^2+1) + C$$

$$\therefore V = \frac{\pi}{2}\ln(x^2+1)\Big|_0^1$$

$$= \frac{\pi}{2}(\ln 2 - \ln 1)$$

$$= \boxed{\frac{\pi}{2}\ln 2}$$

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Q3.

a) Differentiate the function $y = x^6 + 6^x$.

(10 pts)

$$y' = \boxed{6x^5 + 6^x \ln 6}$$

b) Find the derivative of the function $y = (cosx)^{lnx}$ at the point $x = 1$.

(20 pts)

$$\ln y = \ln x \ln \cos x$$

$$\frac{1}{y} y' = \frac{1}{x} \ln \cos x + \ln x \cdot \frac{-\sin x}{\cos x}$$

$$y' = y \left(\frac{\ln \cos x}{x} - \ln x \cdot \tan x \right)$$

$$= (\cos x)^{\ln x} \left(\frac{\ln \cos x}{x} - \ln x \tan x \right)$$

$$y'(1) = (\cos 1)^{\ln 1} \left(\frac{\ln \cos 1}{1} - \underbrace{\ln 1}_{0} \tan 1 \right)$$

$$= \boxed{\ln \cos 1}$$

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Q4. Evaluate the following integrals:

a) $\int_1^{e^2} \frac{dx}{x\sqrt{\ln x}}$

(15 pts) $u = \ln x \Rightarrow du = \frac{dx}{x}$

$$\int \frac{dx}{x\sqrt{\ln x}} = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{\ln x} + C$$

$$\therefore \int_1^{e^2} \frac{dx}{x\sqrt{\ln x}} = 2\sqrt{\ln x}\,\Big|_1^{e^2} = 2\left\{\sqrt{\ln e^2} - \sqrt{\ln 1}\right\}$$

$$= 2\sqrt{2\ln e} = \boxed{2\sqrt{2}}$$

b) $\int e^x \cot(e^x)dx$

(15 pts) $u = e^x \Rightarrow du = e^x dx$

$$\int e^x \cot(e^x)\,dx = \int \cot u\,du$$

$$= \ln|\sin u| + C$$

$$= \boxed{\ln|\sin e^x| + C}$$