Math No:

Full Name :

Math 104 Final Exam
(13 May 2019, 15:00-16:00)

**IMPORTANT**
**1.** Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

| Q1 | Q2 | Q3 | Q4 | TOT |
|---|---|---|---|---|
| | | | | |
| 20 pts | 30 pts | 25 pts | 35 pts | 110 pts |

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**Q1.** Find the MacLaurin series of the function given below, and determine the interval of convergence:

$$f(x) = \frac{x^2}{1+3x}$$

(Hint: To solve this question, you may use Taylor or MacLaurin series that you know.)

$$f(x) = x^2 \cdot \frac{1}{1-(-3x)} = x^2 \sum_{n=0}^{\infty} (-3x)^n, \text{ for } |-3x|<1,$$

using the geometric series $\sum_{0}^{\infty} x^n = \frac{1}{1-x}$.

$$\therefore f(x) = x^2 \sum_{n=0}^{\infty} (-1)^n 3^n x^n$$

$$= \boxed{\sum_{n=0}^{\infty} (-1)^n 3^n x^{n+2}},$$ with interval of convergence $\boxed{|x| < 1/3}$.

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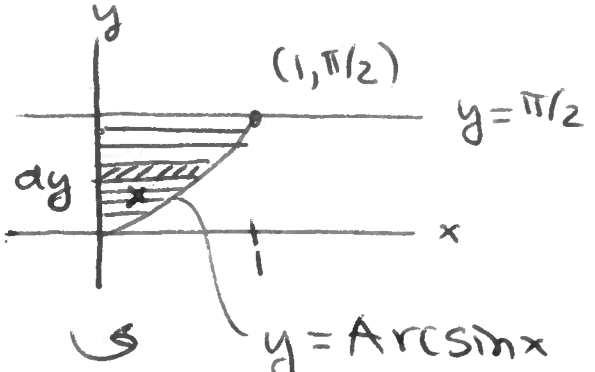
**Q2.** The region in the first quadrant bounded by the curve $y = Arcsinx$ and the lines $y = \pi/2$ and $x = 0$ is revolved about the y-axis. Find the volume of the solid of revolution generated.




Disk method

$$V = \pi \int_0^{\pi/2} x^2 \, dy$$

$$\Downarrow$$

$x = \sin y$

$$\therefore V = \pi \int_0^{\pi/2} \underbrace{\sin^2 y}_{\frac{1-\cos 2y}{2}} \, dy$$

$$= \frac{\pi}{2} \left\{ y - \frac{\sin 2y}{2} \right\}_0^{\pi/2}$$

$$= \frac{\pi}{2} \left\{ \frac{\pi}{2} - \frac{\sin \pi}{2} - 0 \right\}$$

$$= \boxed{\frac{\pi^2}{4}}$$

(This question can be solved by other methods too)

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**Q3.** Evaluate the following integral:

$$\int \sin(\ln x)\, dx$$

Integration by parts

$u = \sin(\ln x)$ $\qquad dv = dx$

$du = \frac{\cos(\ln x)}{x}\,dx$ $\qquad v = x$

$$I = \int \sin(\ln x)\,dx = x\sin(\ln x) - \int \cos(\ln x)\,dx$$

$u = \cos(\ln x)$ $\qquad dv = dx$

$du = -\frac{\sin(\ln x)}{x}\,dx$ $\qquad v = x$

$$I = x\sin(\ln x) - x\cos(\ln x) - \underbrace{\int \sin(\ln x)\,dx}_{I}$$

$$2I = x\sin(\ln x) - x\cos(\ln x)$$

$$\therefore \quad I = \boxed{\frac{x}{2}\left(\sin(\ln x) - \cos(\ln x)\right) + C}$$

**Q4.** Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{x^n}{2^n \ln n}$$

Generalized Ratio Test:

$$\rho = \lim_{n\to\infty} \left| \frac{\frac{x^{n+1}}{2^{n+1}\ln(n+1)}}{\frac{x^n}{2^n \ln n}} \right| = \frac{|x|}{2} \lim_{n\to\infty} \frac{\ln n}{\ln(n+1)} \quad \infty/\infty$$

$$\underset{\text{L'Hospital}}{=} \frac{|x|}{2} \lim_{n\to\infty} \frac{1/n}{1/(n+1)} = \frac{|x|}{2} \lim \underbrace{\frac{n+1}{n}}_{1}$$

$$= \frac{|x|}{2}$$

$\therefore$ Series converges absolutely if

$\frac{|x|}{2} < 1 \Rightarrow |x| < 2$

and diverges if $|x| > 2$.

Answer

$$\boxed{\begin{array}{c}\text{Interval} \\ -2 \le x < 2 \\ \text{radius} \\ = 2\end{array}}$$

$x = 2 \Rightarrow \sum_{n=2}^{\infty} \frac{2^n}{2^n \ln n} = \sum \frac{1}{\ln n}$

$\ln n < n \Rightarrow \frac{1}{\ln n} > \frac{1}{n}$, $\sum \frac{1}{n}$ harmonic, diverges

$\therefore$ series diverges at $x = 2$, by Comparison Test

$x = -2 \Rightarrow \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

Alternating ✓

$\left\{ \left| \frac{(-1)^n}{\ln n} \right| \right\} = \left\{ \frac{1}{\ln n} \right\}$ is decreasing ✓

$\lim \frac{(-1)^n}{n} = 0$ ✓

$\therefore$ series converges at $x = -2$ by Alternating Series Test