FUNCTIONS

A function $f: D \to Y$ is a rule that assigns a unique element $f(x) \in Y$ to each $x \in D$.

$y = f(x)$ ← x is independent variable
↑ is dependent variable

Ex Area of a square of side x is

$$A(x) = x^2$$

inputs = domain of f → [f] → output = range of f

Ex $y = x^2$ domain: all $x \in \mathbb{R}$
range: $y \geq 0$

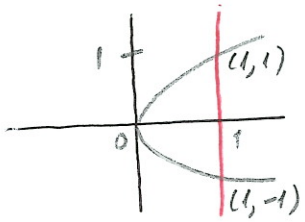
Ex $y = \sqrt{2-x}$ domain: $x \leq 2$
range: $y \geq 0$

Ex $y = x^2$

x	y
0	0
1	1
2	4
-1	1
-2	4

graph of f

Ex $x = y^2$ is NOT a function

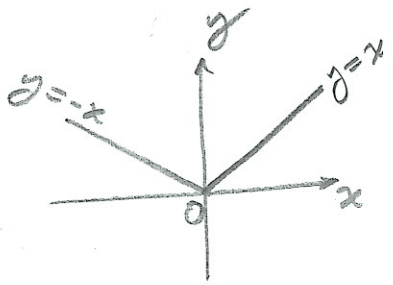


$1 \to \pm 1$

vertical line test

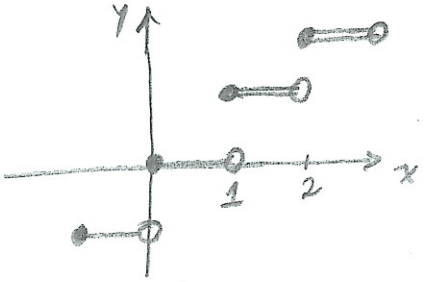
Absolute Value Function

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



The Greatest Integer Function

$\llbracket x \rrbracket = n \quad n \in \mathbb{Z}, \quad n \leq x < n+1$



Linear Functions

$y = mx + b$ (b: y-intercept, m: slope)

Power Functions

$y = x^3, \quad y = x^{1/2}, \quad y = x^{-1/3}$

Polynomials

$P(x) = a_n x^n + \cdots + a_1 x + a_0, \quad a_n \in \mathbb{R}$

Rational Functions

$f(x) = \frac{p(x)}{q(x)}$, p, q polynomial

Trigonometric Functions

$\sin x, \cos x \ldots$

Exponential Functions

$y = 2^x$

Logarithmic Functions

} in Math 104

Def $y = f(x)$ is <u>increasing</u> (<u>decreasing</u>) on interval I if

$$x_1 < x_2 \Rightarrow f(x_1) \underset{>}{<} f(x_2)$$

Symmetry

Def. $y = f(x)$ is an <u>even function</u> if $f(x) = f(-x)$

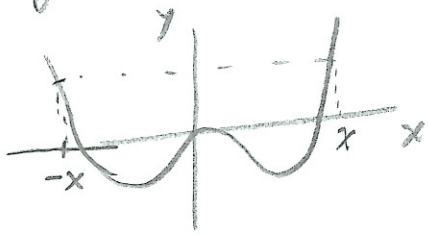
<u>odd function</u> if $f(x) = -f(-x)$

Ex $y = x^4$, even fn.

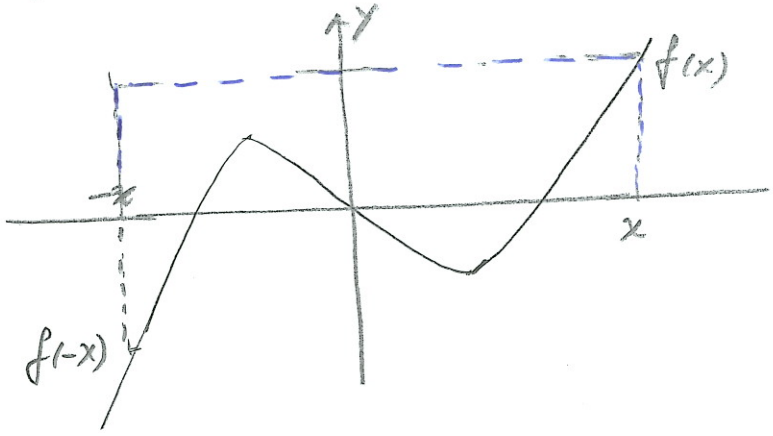
$y = |x|$, even fn.

$y = x^3$, odd fn.

An even fn. is symmetric wrt the y-axis



An odd fn is symmetric wrt to the origin



Combining Functions: Binary Operations on fns.

$$(f \pm g)(x) = f(x) \pm g(x)$$

$$(fg)(x) = f(x)\,g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$$

$$(cf)(x) = c\,f(x)$$

$$(f \circ g)(x) = f(g(x)) \quad \text{composition of } f \text{ by } g.$$

Ex

$$f(x) = x^3$$

$$g(x) = 2x - 1$$

$$(f+g)(x) = x^3 + 2x - 1$$

$$(fg)(x) = x^3(2x-1) = 2x^4 - x^3$$

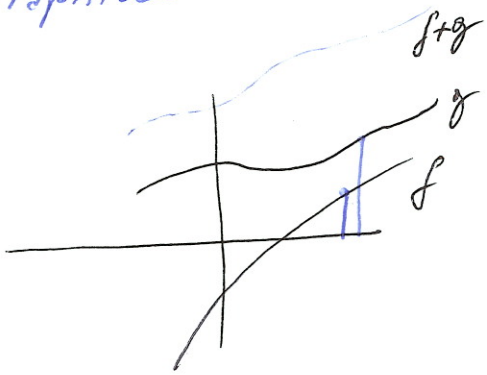
$$\left(\frac{f}{g}\right)(x) = \frac{x^3}{2x-1}$$

$$(f \circ g)(x) = (2x-1)^3$$

$$(g \circ f)(x) = 2(x)^3 - 1 = 2x^3 - 1$$

$$f \circ g \neq g \circ f$$

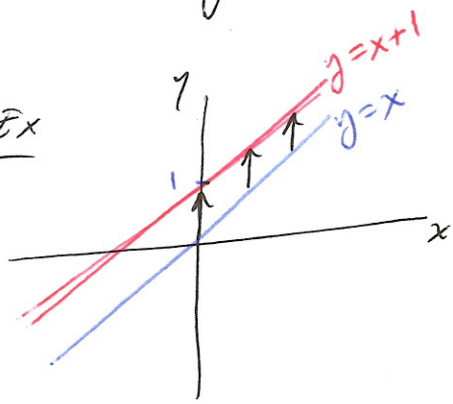
Graphical Addition

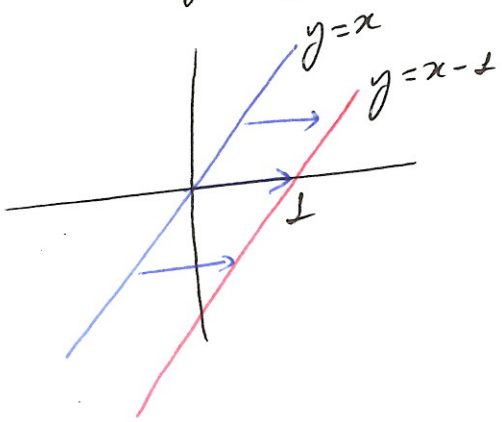


$y = f(x) + k$ shift up/down by k

$y = f(x+h)$ shift left/right by h (left if $h > 0$)

Ex





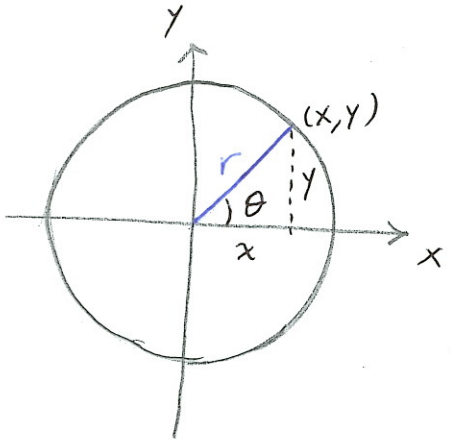
$y = c f(x)$ vertical stretch/compression

$y = f(cx)$ horizontal " "

$y = -f(x)$ reflection across x-axis

$y = f(-x)$ " " y-axis

Trigonometric Functions

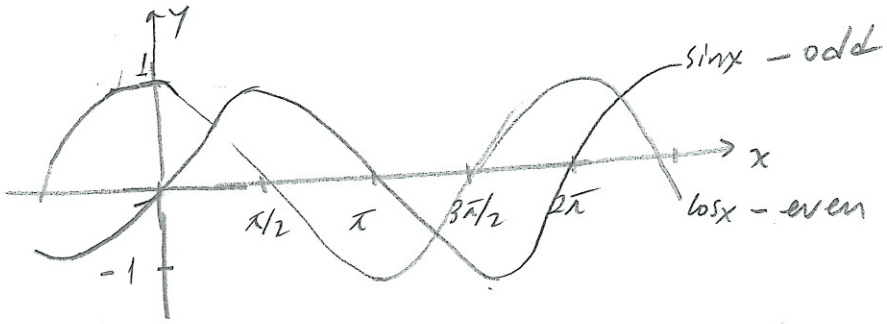


$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y} = \frac{1}{\tan\theta}$$

Def $y = f(x)$ is <u>periodic</u> with <u>period p</u> if $f(x+p) = f(x)$ $\forall x$; p is the smallest such no.

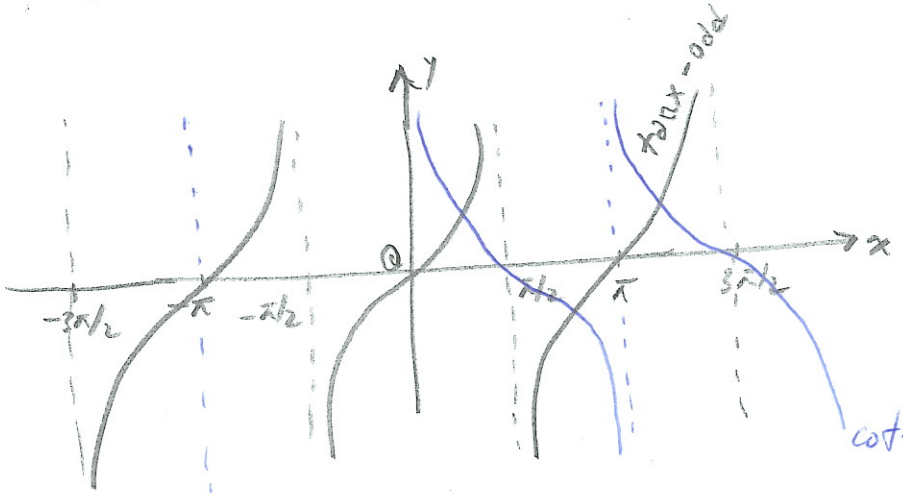


$\cos x = \sin\left(\frac{\pi}{2} - x\right)$

period 2π

Domain all x

Range $-1 \le y \le 1$



Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

Range: All y

cotx – odd

period: π

Trigonometric Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

Addition Formulas

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

Double-Angle Formulas

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

Half-Angle Formulas

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

Law of cosines

$$\vec{a} = \vec{b} + \vec{c}$$

$$\vec{c} = \vec{a} - \vec{b}$$

$$\vec{c} \cdot \vec{c} = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$c^2 = a^2 + b^2 - 2\vec{a} \cdot \vec{b}$$

$$ab\cos\theta$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$