Math No:

Full Name : KEY

ISTANBUL ŞEHİR UNIVERSITY

Math 104 2nd Midterm Exam
(14 November 2015, Time: 11:30-12:30)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which may have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
4 pts	5 pts	5 pts	5 pts	19 pts

Q1. Evaluate the following integral using integration by parts: $\int x\sec^2 x\, dx$

$u = x$ | $\sec^2 x\, dx = dv$

$du = dx$ | $\int d(\tan x) = \int dv$

$\tan x = v$

$$\int x \sec^2 x\, dx = uv - \int v\, du$$

$$= x\tan x - \int \tan x\, dx$$

$$= x\tan x - \int \frac{\sin x\, dx}{\cos x}$$

$-du$, u

$$= x\tan x + \int \frac{du}{u}$$

$$= x\tan x + \ln|\cos x| + C$$

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Q2. Evaluate the following integral using integration by parts: $\int x(\ln x)^3 dx$

$$\int x(\ln x)^3 dx = uv - \int v\, du$$

$u = (\ln x)^3$ | $x\,dx = dv$

$du = 3(\ln x)^2 \frac{dx}{x}$ | $\frac{1}{2}x^2 = v$

$$= \frac{1}{2}x^2(\ln x)^3 - \int \frac{1}{2}x^{\cancel{2}}\, 3(\ln x)^2 \frac{dx}{\cancel{x}}$$

$$= \frac{1}{2}x^2(\ln x)^3 - \frac{3}{2}\int x(\ln x)^2 dx$$

$u = (\ln x)^2$ | $x\,dx = dv$

$du = 2(\ln x)\frac{dx}{x}$ | $\frac{1}{2}x^2 = v$

$$= \frac{1}{2}x^2(\ln x)^3 - \frac{3}{2}\left\{\frac{1}{2}x^2(\ln x)^2 - \int \frac{1}{\cancel{2}}x^{\cancel{2}}\, \cancel{2}\ln x \cdot \frac{dx}{\cancel{x}}\right\}$$

$u = \ln x$ | $x\,dx = dv$

$du = dx/x$ | $\frac{1}{2}x^2 = v$

$$= \frac{1}{2}x^2(\ln x)^3 - \frac{3}{2}\left\{\frac{1}{2}x^2(\ln x)^2 - \left[\frac{1}{2}x^2\ln x - \underbrace{\int \frac{1}{2}x^{\cancel{2}} \cdot \frac{dx}{\cancel{x}}}_{\frac{1}{4}x^2}\right]\right\}$$

$$= \frac{1}{2}x^2(\ln x)^3 - \frac{3}{4}x^2(\ln x)^2 - \frac{3}{4}x^2\ln x + \frac{3}{8}x^2 + C$$

$$= \frac{1}{2}x^2\left[(\ln x)^3 - \frac{3}{2}(\ln x)^2 - \frac{3}{2}\ln x + \frac{3}{4}\right] + C$$

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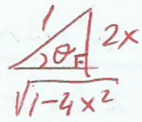
Q3. Evaluate the following integral using a trigonometric substitution: $\int \frac{dx}{x\sqrt{1-4x^2}}$

$$\int \frac{dx}{x\sqrt{1-4x^2}} = \int \frac{dx}{x\sqrt{1-(2x)^2}}$$

$2x = \sin\theta$



$dx = \frac{\cos\theta \, d\theta}{2}$

$$= \int \frac{\cancel{(1/2)}\cos\theta \, d\theta}{\frac{\sin\theta}{\cancel{2}}\sqrt{1-\sin^2\theta}}$$

$$= \int \frac{\cancel{\cos\theta} \, d\theta}{\sin\theta \, \cancel{\cos\theta}}$$

$$= \int \frac{1}{\sin\theta} d\theta$$

$$= \int \csc\theta \, d\theta \cdot \frac{\csc\theta - \cot\theta}{\csc\theta - \cot\theta}$$

$u = \csc\theta - \cot\theta$

$du = (-\csc\theta\cot\theta + \csc^2\theta)\, d\theta$

$$= \int \frac{du}{u}$$

$$= \ln|u| + c$$

$$= \ln|\underbrace{\csc\theta}_{(1/2x)} - \underbrace{\cot\theta}_{\frac{\sqrt{1-4x^2}}{2x}}| + c$$

from the above right triangle

$$= \ln\left|\frac{1-\sqrt{1-4x^2}}{2x}\right| + c$$

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Q4. Evaluate the following integral using partial fractions: $\int \frac{2+3x+x^2}{x(x^2+1)}dx$

$$\int \frac{x^2+3x+2}{x(x^2+1)}dx = \int \frac{A}{x}dx + \int \frac{Bx+C}{x^2+1}dx$$

$$\frac{x^2+3x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$x^2+1 \qquad x$

$$x^2+3x+2 = (A+B)x^2 + Cx + A$$

$A+B=1 \Rightarrow B=-1$

$C=3$

$A=2$

$$\int \frac{x^2+3x+2}{x(x^2+1)}dx = 2\int \frac{dx}{x} + \int \frac{-x+3}{x^2+1}dx$$

$$= 2\ln x - \int \frac{x\,dx}{x^2+1} + 3\int \frac{dx}{x^2+1}$$

$$= 2\ln x - \frac{1}{2}\ln|x^2+1| + 3\,\mathrm{Arctan}\,x + C$$

$$= \ln x^2 - \ln(x^2+1)^{1/2} + 3\,\mathrm{Arctan}\,x + C$$

$$= \ln\left|\frac{x^2}{\sqrt{1+x^2}}\right| + 3\,\mathrm{Arctan}\,x + C$$