Math No:

Full Name : KEY

ISTANBUL ŞEHİR UNIVERSITY

Math 104 – 4th Midterm Exam
(26 December 2015, Time: 11:30-12:30)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which may have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
5 pts	5 pts	5 pts	4 pts	19 pts

Q1. $f(x,y)=(xy-1)^2$

(a) Find $\dfrac{\partial f}{\partial x}$

$$\frac{\partial f}{\partial x} = 2(xy-1)\underbrace{\frac{\partial}{\partial x}(xy-1)}_{y}$$

$$= 2y(xy-1)$$

(b) Find $\dfrac{\partial f}{\partial y}$

$$\frac{\partial f}{\partial x} = 2(xy-1)\underbrace{\frac{\partial}{\partial y}(xy-1)}_{x}$$

$$= 2x(xy-1)$$

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Q2. $xe^{y} + \sin xy + y - \ln 2 = 0$

Assuming that the above equation defines y as a differentiable function of x. Find dy/dx at point $(0, \ln 2)$. [*Hint*: Use the Theorem $\frac{dy}{dx} = -\frac{\partial F/\partial x}{\partial F/\partial y}$]

$$\frac{\partial F}{\partial x} = e^{y} + \cos xy \cdot \underbrace{\frac{\partial}{\partial x}(xy)}_{y} = e^{y} + y\cos xy$$

$$\frac{\partial F}{\partial y} = xe^{y} + \cos xy \cdot \underbrace{\frac{\partial}{\partial y}(xy)}_{x} + 1 = xe^{y} + x\cos xy + 1$$

$$\frac{dy}{dx} = -\frac{\partial F/\partial x}{\partial F/\partial y} = -\frac{e^{y} + y\cos xy}{xe^{y} + x\cos xy + 1}$$

$$\left.\frac{dy}{dx}\right|_{(0,\ln 2)} = -\frac{e^{\ln 2} + \ln 2 \cdot \cos(0)}{1} \quad ; \quad e^{\ln 2} = 2$$

$$= -(2 + \ln 2) \;//$$

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Q3. For the exercise $z = e^x \ln y$, $x = \ln(u\cos v)$, $y = u\sin v$

(a) Use the *Chain Rule* to express $\dfrac{\partial z}{\partial u}$ as functions of u and v. Then evaluate $\dfrac{\partial z}{\partial u}$ at the given point $(u, v) = (2, \pi/4)$.

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$$= e^x \ln y \cdot \frac{\cancel{\cos v}}{u\cancel{\cos v}} + \frac{e^x}{y} \cdot \frac{\cancel{\sin v}}{u\cancel{\sin v}}$$

$$\frac{\partial z}{\partial u} = \frac{e^x}{u}(\ln y + 1)$$

$$x(2,\pi/4) = \ln\left(2\cdot\frac{\sqrt{2}}{2}\right) = \ln\sqrt{2}$$

$$y(2,\pi/4) = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$$

$$e^{\ln\sqrt{2}} = \sqrt{2}$$

$$\left(\frac{\partial z}{\partial u}\right)_{(2,\pi/4)} = \frac{e^{\ln\sqrt{2}}}{2}(\ln\sqrt{2} + 1)$$

$$= \frac{\sqrt{2}}{2}(\ln\sqrt{2} + 1)$$

(b) Use the *Chain Rule* to express $\dfrac{\partial z}{\partial v}$ as functions of u and v. Then evaluate $\dfrac{\partial z}{\partial v}$ at the given point $(u, v) = (2, \pi/4)$.

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

$$= e^x \ln y \cdot \frac{-\cancel{u}\sin v}{\cancel{u}\cos v} + \frac{e^x}{y} \cdot \frac{\cancel{u}\cos v}{\cancel{u}\sin v}$$

$$= e^x\left(-\ln y \cdot \frac{\sin v}{\cos v} + \frac{\cos v}{\sin v}\right)$$

$$\left.\frac{\partial z}{\partial v}\right|_{(2,\pi/4)} = e^{\ln\sqrt{2}}\left(-\ln\sqrt{2}\cdot\frac{\cancel{\sqrt{2}/2}}{\cancel{\sqrt{2}/2}} + \frac{\cancel{\sqrt{2}/2}}{\cancel{\sqrt{2}/2}}\right)$$

$$= \sqrt{2}(-\ln\sqrt{2} + 1)$$

$$= \sqrt{2}(1 - \ln\sqrt{2}) \quad //$$

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Q4. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$

(a) Find the above power series' interval of convergence.

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x-1)^{n+1}}{n+1} \cdot \frac{n}{(x-1)^n} \right| = \lim_{n\to\infty} \left| \frac{(x-1)\cancel{(x-1)^n}}{n+1} \cdot \frac{n}{\cancel{(x-1)^n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n}{n+1}(x-1) \right| = |x-1| \underbrace{\lim_{n\to\infty} \frac{n}{n+1}}_{=1 \text{ (L'Hop. R)}} = |x-1|$$

$$\rho = |x-1| < 1 \to x > 0, \quad x < 2$$

$$\underline{0 < x < 2}$$

(b) For what values of *x* does the series converge absolutely or conditionally.

For $x = 0$: $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(0-1)^n}{n} = -\sum_{n=1}^{\infty} \frac{1}{n}$, diverges. It's p-series with $p=1$

For $x = 2$: $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2-1)^n}{n} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$, converges conditionally since it is an alternating series.