KEY

İstanbul Şehir University

Math 104

Date: 7 March 2015	Full Name:
Time: 11:30-12:30	
	Student ID:
Spring 2015 First Exam	Math number:

IMPORTANT

1. Write down your name and surname on top of each sheet. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cellphones and electronic devices are to be kept shut and out of sight.

Q1	Q2	Q3	Q4	TOTAL
15 pts	30 pts	25 pts	30 pts	100 pts

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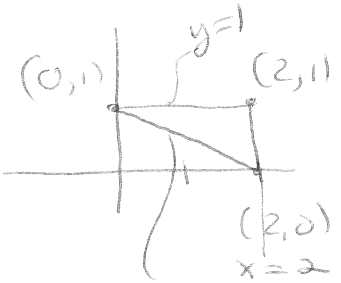
1) Let $f(x) = x^5 + 5x^3 + 2$. Find the derivative of the inverse function f^{-1} at the point -4.

[*Hint*: $f(-1) = -4$]

$$\left.\frac{dx}{dy}\right|_{-4} = \frac{1}{\left.\frac{dy}{dx}\right|_{-1}} = \frac{1}{\left.5x^4+15x^2\right|_{-1}} = \frac{1}{5+15} = \boxed{\frac{1}{20}}$$

2) Consider the region enclosed by the triangle with vertices $(0,1)$, $(2,0)$ and $(2,1)$. Set up (DO NOT EVALUATE!) an integral for the volume of the solid generated by revolving this region

(a) about the y-axis.



Shell method

$$V = 2\pi \int_0^2 xy\,dx = \boxed{2\pi \int_0^2 \frac{1}{2}x^2\,dx}$$

$1-\left(-\frac{1}{2}x+1\right) = \frac{1}{2}x$

$y = -\frac{1}{2}x+1$

$\downarrow$

$2y = -x+2$

$x = 2-2y$

or

Disk method

$$V = \pi \cdot 2^2 \cdot 1 - \pi \int_0^1 x^2\,dy = \boxed{4\pi - \pi \int_0^1 (2-2y)^2\,dy}$$

(b) about the x-axis.

Shell method

$$V = 2\pi \int_0^1 xy\,dy = \boxed{2\pi \int_0^1 2y^2\,dy}$$

$2-(2-2y) = 2y$

or

Disk method

$$V = \pi \cdot 1^2 \cdot 2 - \int_0^2 \pi y^2\,dx = \boxed{2\pi - \pi \int_0^2 \left(-\frac{1}{2}x+1\right)^2 dx}$$

(c) about the line $x = -1$.

Shell method

$$V = 2\pi \int_0^2 (x+1)\,y\,dx = \boxed{2\pi \int_0^2 \frac{1}{2}x(x+1)\,dx}$$

or

Disk method

$$V = \pi \cdot 3^2 \cdot 1 - \pi \int_0^1 x^2\,dy = \boxed{9\pi - \pi \int_0^1 (3-2y)^2\,dy}$$

$\downarrow$

$x = 1+2-2y$

NAME:

3) Let $f(x) = \frac{x+4}{x-1}$.

(a) Show that the function f is one-to-one.
(b) Find the formula for the inverse function f^{-1}. Based on your answer, what can you say about the symmetry properties of the graph of f?

(a) $f'(x) = \frac{x-1-(x+4)}{(x-1)^2} = -\frac{5}{(x-1)^2} < 0$ always

$\therefore f$ is monotone decreasing $\Rightarrow f$ is 1-1.

(b) $y = \frac{x+4}{x-1}$

$xy - y = x + 4$

$xy - x = y + 4$

$x(y-1) = y+4$

$x = \frac{y+4}{y-1}$

$\therefore$ $f^{-1}(x) = \frac{x+4}{x-1}$

Since $f = f^{-1}$, the graph of f is symmetric with respect to the line $y = x$.

4) (a) Find the derivative of the function $y = \ln \frac{1}{x^3\sqrt{x+1}}$. Simplify your answer.

$$y = -\ln\left(x^3 \cdot (x+1)^{1/2}\right) = -\ln x^3 - \ln(x+1)^{1/2}$$

$$= -3\ln x - \frac{1}{2}\ln(x+1)$$

$$\Rightarrow \boxed{\frac{dy}{dx} = -\frac{3}{x} - \frac{1}{2(x+1)}}$$

(b) Evaluate the following integral: $\int_0^{\pi/4} \frac{\sec^2\theta d\theta}{\tan\theta+2}$

$$u = \tan\theta + 2 \Rightarrow du = \sec^2\theta d\theta$$

$$\int \frac{\sec^2\theta d\theta}{\tan\theta+2} = \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|\tan\theta+2| + C$$

$$\therefore \int_0^{\pi/4} \frac{\sec^2\theta d\theta}{\tan\theta+2} = \ln|\tan\theta+2|\Big|_0^{\pi/4}$$

$$= \ln\left(\tan\frac{\pi}{4}+2\right) - \ln(\tan 0+2)$$

$$= \ln 3 - \ln 2 = \boxed{\ln\frac{3}{2}}$$