Math No:

Full Name : KEY

İSTANBUL ŞEHİR UNIVERSITY

Math 104 – 3rd Midterm Exam
(17 December 2018, Time: 17:00-18:00)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones, smart watches and electronic devices are to be kept shut and out of sight. All cell phones and smart watches are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
6 pts	6 pts	6 pts	6 pts	24 pts

Q1. The following sums describe geometric series. (a) Write down the geometric series in terms of the sum sign and its nth term(s), that is, $\sum_{n=0}^{\infty}(n^{th}\ term)$. (b) Do the series converge or diverge? Give reasons for your answers.

(i) $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots = \left(\frac{1}{2}\right)^0+\left(\frac{1}{2}\right)^1+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\cdots=\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$

$\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n = \frac{1}{1-1/2} = 2,\quad \frac{1}{2}<1$

conv.

(ii) $\pi - e + \frac{e^2}{\pi} - \frac{e^3}{\pi^2} + \ldots = \pi\left(1-\frac{e}{\pi}+\frac{e^2}{\pi^2}-\frac{e^3}{\pi^3}+\cdots\right)=$

$\pi\sum_{n=0}^{\infty}\left(-e/\pi\right)^n = \frac{\pi}{\pi+e/\pi} = \frac{\pi^2}{\pi+e},\quad |-e/\pi|<1$

conv.

(iii) $1+2^{1/2}+2+2^{3/2}+\ldots = (\sqrt{2})^0+(\sqrt{2})^1+(\sqrt{2})^2+(\sqrt{2})^3+\cdots$

$=\sum_{n=0}^{\infty}(\sqrt{2})^n$; diverges since $\sqrt{2}>1$

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Q2. Apply an appropriate test if the following series converge or diverge? Give reasons for your answers.

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{1+\sqrt{n}}$

By the Comparison Test:

$$\lim_{n\to\infty}\frac{1/\sqrt{n}}{\frac{1}{1+\sqrt{n}}}=\lim_{n\to\infty}\frac{1+\sqrt{n}}{\sqrt{n}}=\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}}+1\right)=1$$

Since $\sum\frac{1}{\sqrt{n}}$ is $p=1/2$ series and diverges,

The series diverges by the Comparison Test!

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{n-1}{n^3-2n+1}$

By the Comparison Test:

$$\lim_{n\to\infty}\frac{1/n^2}{\frac{n-1}{n^3-2n+1}}=\lim_{n\to\infty}\frac{n^3-2n+1}{n^3-n^2}=\lim_{n\to\infty}\left(\frac{1-\frac{2}{n^2}+1/n^3}{1-1/n}\right)$$

$$=1$$

Since $\sum\frac{1}{n^2}$ is a $p=2$ series and converges,

the series converges by the Comparison Test!

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Q3. Apply appropriate tests if the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

By the Ratio Test: $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$, if $\rho < 1$ conv. if $\rho > 1$ div.

$$\lim_{n\to\infty} \frac{3^{n+1}/(n+1)!}{3^n/n!} = \lim_{n\to\infty} \frac{\cancel{3^n}\cdot 3}{(n+1)\cdot \cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{3^n}}$$

$$= 3 \lim_{n\to\infty} \frac{1}{n+1} = 0 < 1$$

It <u>converges</u> by the Ratio Test!

(a) $\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

By the n-Root Test: $\rho = \lim_{n\to\infty} \sqrt[n]{a_n}$, if $\rho < 1$ conv. if $\rho > 1$ div.

$$\lim_{n\to\infty} \sqrt[n]{\frac{4^n}{(3n)^n}} = \lim_{n\to\infty} \frac{4}{3n}$$

$$= \frac{4}{3} \lim_{n\to\infty} \frac{1}{n} = 0 < 1$$

It <u>converges</u> by the n-Root Test!

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Q4. For what values of x does the following series converge? $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4 2^{2n}} x^n$

$$\rho = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$$

$$\lim_{n \to \infty} \left| \frac{x^{n+1}}{(n+1)^4 \cdot 2^{2(n+1)}} \cdot \frac{n^4 \cdot 2^{2n}}{x^n} \right| =$$

$$\lim_{n \to \infty} \left| \frac{\cancel{x^n} \cdot x}{(n+1)^4 \cdot \cancel{2^{2n}} \cdot 2^2} \cdot \frac{n^4 \cdot \cancel{2^{2n}}}{\cancel{x^n}} \right| =$$

$$= \frac{|x|}{4} \underbrace{\lim_{n \to \infty} \left(\frac{n}{n+1} \right)^4}_{1} < 1$$

$$\frac{|x|}{4} < 1$$

$$|x| < 1$$

$$x < 4, \quad x > -4$$

$$-4 < x < 4$$

We need to test the end values:

For $x = -4$: $\sum_{n=1}^{\infty} (-1)^n (-4)^n / n^4 2^{2n} = \sum_{n=1}^{\infty} \frac{1}{n^4}$, $p=4$ series, converges!

For $x = 4$: $\sum_{n=1}^{\infty} (-1)^n (4)^n / n^4 2^{2n} = \sum_{n=1}^{\infty} (-1) \frac{1}{n^4}$, $p=4$ series, converges absolutely!

$$\boxed{-4 \leq x \leq 4}$$