Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 4.3 Homework

1. Determine whether the set $\left\{\begin{bmatrix}7\\7\\0\end{bmatrix},\begin{bmatrix}7\\7\\7\end{bmatrix},\begin{bmatrix}7\\0\\0\end{bmatrix}\right\}$ is a basis for $\mathbb{R}^3$. If the set is not a basis, determine whether the set is linearly independent and whether the set spans $\mathbb{R}^3$.

Which of the following describe the set? Select all that apply.

- [x] **A.** The set spans $\mathbb{R}^3$.
- [x] **B.** The set is linearly independent.
- [x] **C.** The set is a basis for $\mathbb{R}^3$.
- [ ] **D.** None of the above are true.

2. Determine whether the set $\left\{\begin{bmatrix}1\\0\\3\end{bmatrix},\begin{bmatrix}-3\\1\\-11\end{bmatrix},\begin{bmatrix}8\\-2\\28\end{bmatrix}\right\}$ is a basis for $\mathbb{R}^3$. If the set is not a basis, determine whether the set is linearly independent and whether the set spans $\mathbb{R}^3$.

Which of the following describe the set? Select all that apply.

- [ ] **A.** The set is a basis for $\mathbb{R}^3$.
- [ ] **B.** The set is linearly independent.
- [ ] **C.** The set spans $\mathbb{R}^3$.
- [x] **D.** None of the above

3. Determine if the set of vectors shown to the right is a basis for $\mathbb{R}^3$. If the set of vectors is not a basis, determine whether it is linearly independent and whether the set spans $\mathbb{R}^3$.

$$\left\{\begin{bmatrix}1\\-5\\0\end{bmatrix},\begin{bmatrix}-4\\8\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\end{bmatrix},\begin{bmatrix}0\\-8\\6\end{bmatrix}\right\}$$

Which of the following describe the set? Select all that apply.

- [ ] **A.** The set is linearly independent.
- [ ] **B.** The set is a basis for $\mathbb{R}^3$.
- [x] **C.** The set spans $\mathbb{R}^3$.
- [ ] **D.** None of the above are true.

4. Determine if the set of vectors shown to the right is a basis for $\mathbb{R}^3$. If the set of vectors is not a basis, determine whether it is linearly independent and whether the set spans $\mathbb{R}^3$.

$$\left\{\begin{bmatrix} 2 \\ 3 \\ -12 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 8 \end{bmatrix}\right\}$$

Which of the following describe the set? Select all that apply.

- [x] **A.** The set is linearly independent.
- [ ] **B.** The set spans $\mathbb{R}^3$.
- [ ] **C.** The set is a basis for $\mathbb{R}^3$.
- [ ] **D.** None of the above

5. Find a basis for the null space of the matrix $\begin{bmatrix} 1 & 0 & -5 & 7 \\ 0 & 1 & -2 & 4 \\ 5 & -13 & 1 & -17 \end{bmatrix}$.

A basis for the null space is $\left\{\begin{bmatrix} 5 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ -4 \\ 0 \\ 1 \end{bmatrix}\right\}$.

(Use a comma to separate vectors as needed.)

6. Find a basis for the set of vectors in $\mathbb{R}^2$ on the line $y = 7x$.

A basis for the set of vectors in $\mathbb{R}^2$ on the line $y = 7x$ is $\left\{\begin{bmatrix} 1 \\ 7 \end{bmatrix}\right\}$.

(Use a comma to separate vectors as needed.)

7. Assume that A is row equivalent to B. Find bases for Nul A and Col A.

$$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -2 & 1 \\ -3 & 8 & 1 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 5 & 5 \\ 0 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A basis for Col A is $\left\{\begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix}\right\}$.

(Use a comma to separate vectors as needed.)

A basis for Nul A is $\left\{\begin{bmatrix} -5 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -\frac{3}{2} \\ 0 \\ 1 \end{bmatrix}\right\}$.

(Use a comma to separate vectors as needed.)

8. Find a basis for the space spanned by the given vectors.

$$\begin{bmatrix}1\\0\\0\\1\end{bmatrix},\begin{bmatrix}6\\0\\0\\-6\end{bmatrix},\begin{bmatrix}2\\-3\\1\\-1\end{bmatrix},\begin{bmatrix}8\\-9\\3\\-10\end{bmatrix},\begin{bmatrix}13\\-6\\2\\-11\end{bmatrix}$$

A basis for the space spanned by the given vectors is $\left\{\begin{bmatrix}1\\0\\0\\1\end{bmatrix},\begin{bmatrix}6\\0\\0\\-6\end{bmatrix},\begin{bmatrix}2\\-3\\1\\-1\end{bmatrix}\right\}$.

(Use a comma to separate answers as needed.)

9. Let $\mathbf{v}_1=\begin{bmatrix}7\\-9\\4\end{bmatrix}$, $\mathbf{v}_2=\begin{bmatrix}4\\3\\-8\end{bmatrix}$, $\mathbf{v}_3=\begin{bmatrix}23\\-16\\-4\end{bmatrix}$, and $H=\text{Span}\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$. It can be verified that $7\mathbf{v}_1+5\mathbf{v}_2-3\mathbf{v}_3=\mathbf{0}$. Use this information to find a basis for H.

A basis for H is $\left\{\begin{bmatrix}7\\-9\\4\end{bmatrix},\begin{bmatrix}4\\3\\-8\end{bmatrix}\right\}$.

(Type an integer or decimal for each matrix element. Use a comma to separate vectors as needed.)

10. Consider the polynomials $\mathbf{p}_1(t)=4+t^2$ and $\mathbf{p}_2(t)=4-t^2$. Is $\{\mathbf{p}_1,\mathbf{p}_2\}$ a linearly independent set in $\mathbb{P}_3$? Why or why not?

Choose the correct answer below.

- **A.** The set $\{\mathbf{p}_1,\mathbf{p}_2\}$ is a linearly dependent set because both polynomials have degree less than 3.
- **B.** The set $\{\mathbf{p}_1,\mathbf{p}_2\}$ is a linearly independent set because there are only two elements in this set but $\mathbb{P}_3$ has dimension 3.
- **C.** The set $\{\mathbf{p}_1,\mathbf{p}_2\}$ is a linearly independent set because neither polynomial is a multiple of the other polynomial.
- **D.** The set $\{\mathbf{p}_1,\mathbf{p}_2\}$ is a linearly dependent set because $\mathbf{p}_1(t)+\mathbf{p}_2(t)$ does not contain the variable t.