Full Name : KEY
Student ID:

Istanbul Şehir University
Math 104, Midterm II
(15 November 2014, Time: 11:30-13:00)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 5 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	Q5	TOTAL
20 pts	20 pts	20 pts	20 pts	20 pts	100 pts

Q1. Evaluate the following integral

$$\int z^2(\ln z)^2 dz = uv - \int v\,du = \frac{z^3}{3}(\ln z)^2 - \frac{2}{3}\int \cancel{z^3}^{\,2} \ln z \frac{dz}{\cancel{z}}$$

$$\boxed{\begin{array}{l|l} u = (\ln z)^2 & dv = z^2 dz \\ du = 2\ln z \dfrac{dz}{z} & v = \dfrac{1}{3} z^3 \end{array}}$$

$$\int z^2(\ln z)^2 dz = \frac{z^3}{3}(\ln z)^2 - \frac{2}{3}\int z^2 \ln z\, dz \Rightarrow \boxed{\begin{array}{l} u = \ln z \Rightarrow du = \dfrac{dz}{z} \\ dv = z^2 dz \Rightarrow v = z^3/3 \end{array}}$$

$$= \frac{z^3}{3}(\ln z)^2 - \frac{2}{3}\left\{ \frac{z^3}{3}\ln z - \frac{1}{3}\int \cancel{z^3}^{\,2} \frac{dz}{\cancel{z}} \right\}$$

$$= \frac{z^3}{3}(\ln z)^2 - \frac{2}{3}\left\{ \frac{z^3}{3}\ln z - \frac{1}{3}\cdot\frac{1}{3}z^3 \right\} + C$$

$$= \frac{z^3}{3}\left[(\ln z)^2 - \frac{2}{3}\ln z + \frac{2}{9} \right] + C$$

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Q2. Evaluate the following trigonometric integral

$$\int \cos^5 x \, dx = \int (\cos^2 x)^2 \cos x \, dx$$

$$= \int (1 - \sin^2 x)^2 \cos x \, dx$$

$u = \sin x$
$du = \cos x \, dx$

$$= \int (1 - u^2)^2 \, du$$

$$= \int (1 - 2u^2 + u^4) \, du$$

$$= u - \frac{2}{3} u^3 + \frac{1}{5} u^5 + C$$

$$= \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C$$

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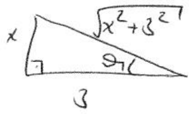
Q3. Use an appropriate substitution and then a trigonometric substitution to evaluate the following integral

$$\int_0^{\ln 4} \frac{e^t dt}{\sqrt{e^{2t}+9}} \qquad x = e^t \Rightarrow dx = e^t dt$$

$$\int_9^{\ln 4} \frac{dx}{\sqrt{x^2+3^2}} \qquad \begin{array}{l} x = 3\tan\theta \\ dx = 3\sec^2\theta\, d\theta \end{array}$$



$$\int \frac{3\sec^2\theta\, d\theta}{\sqrt{3^2\tan^2\theta + 3^2}} = \int \frac{\cancel{3}\sec^{\cancel{2}}\theta\, d\theta}{\cancel{3}\underbrace{\sqrt{1+\tan^2\theta}}_{\cancel{\sec\theta}}} = \int \sec\theta\, d\theta$$

$$= \int \sec\theta \left(\frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} \right) d\theta = \int \frac{\overbrace{(\sec^2\theta + \sec\theta\tan\theta)}^{du}\, d\theta}{\underbrace{(\sec\theta + \tan\theta)}_{u}}$$

$$= \int \frac{du}{u} = \ln u = \ln(\sec\theta + \tan\theta)$$

$$= \ln\left(\frac{\sqrt{x^2+9}}{3} + \frac{x}{3} \right) = \ln\left(\frac{\sqrt{e^{2t}+9}}{3} + \frac{e^t}{3} \right) \Bigg|_0^{\ln 4}$$

$$= \ln\left(\frac{\sqrt{e^{2\ln 4}+9}}{3} + \frac{e^{\ln 4}}{3} \right) - \ln\left(\frac{\sqrt{e^0+9}}{3} + \frac{e^0}{3} \right)$$

$$= \ln\left(\frac{5}{3} + \frac{4}{3} \right) - \ln\left(\frac{\sqrt{10}}{3} + \frac{1}{3} \right)$$

$$= \ln\frac{9}{3} - \ln\left(\frac{\sqrt{10}+1}{3} \right) = \ln\frac{9}{\sqrt{10}+1}$$

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Q4. Evaluate the following integral

$$\int \frac{2x+1}{x^2-7x+12}dx = \int \frac{2x+1}{(x-3)(x-4)}dx$$

$-3 \quad -4$

$$\frac{2x+1}{(x-3)(x-4)} = \frac{A}{x-3} + \frac{B}{x-4}$$

$$2x+1 = (x-4)A + (x-3)B$$

$$= \int \frac{2x+1}{(x-3)(x-4)}dx = \int \frac{A\,dx}{x-3} + \int \frac{B\,dx}{x-4}$$

$x=3 \Rightarrow 7 = -A \Rightarrow A = -7$

$x=4 \Rightarrow 9 = B$

$$= -7\int \frac{dx}{x-3} + 9\int \frac{dx}{x-4}$$

$$= -7\ln(x-3) + 9\ln(x-4) + C$$

$$= \ln\left[\frac{(x-4)^9}{(x-3)^7}\right] + C$$

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Q5. Evaluate the following *improper* integral

$$\int_1^{\infty} \frac{\arctan x}{1+x^2}\,dx =$$

$$y = \arctan x \Rightarrow x = \tan y$$

$$\frac{dx}{dy} = (1+\tan^2 y)$$

$$\frac{dx}{dy} = 1 + x^2$$

$$\frac{dx}{1+x^2} = dy$$

$$\frac{dx}{1+x^2} = d(\arctan x)$$

$$\int_1^{\infty} \frac{\overset{y}{\arctan x}}{1+x^2}\,\overset{dy}{dx} =$$

$$\int y\,dy = \frac{1}{2}y^2 = \frac{1}{2}(\arctan x)^2\Big|_1^{\infty}$$

$$= \frac{1}{2}\lim_{x\to b} \arctan^2 x\Big|_1^{b}$$

$$= \frac{1}{2}\left[\arctan^2(\infty) - \arctan^2(1)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2}\right)^2 - \left(\frac{\pi}{4}\right)^2\right]$$

$$= \frac{1}{2}\left[\frac{\pi^2}{4} - \frac{\pi^2}{16}\right]$$

$$= \frac{3\pi^2}{32}$$