# Chapter 2

## Limits and Continuity

The average rate change of $y(x) = f(x)$ over interval $[x_1, x_2]$ is

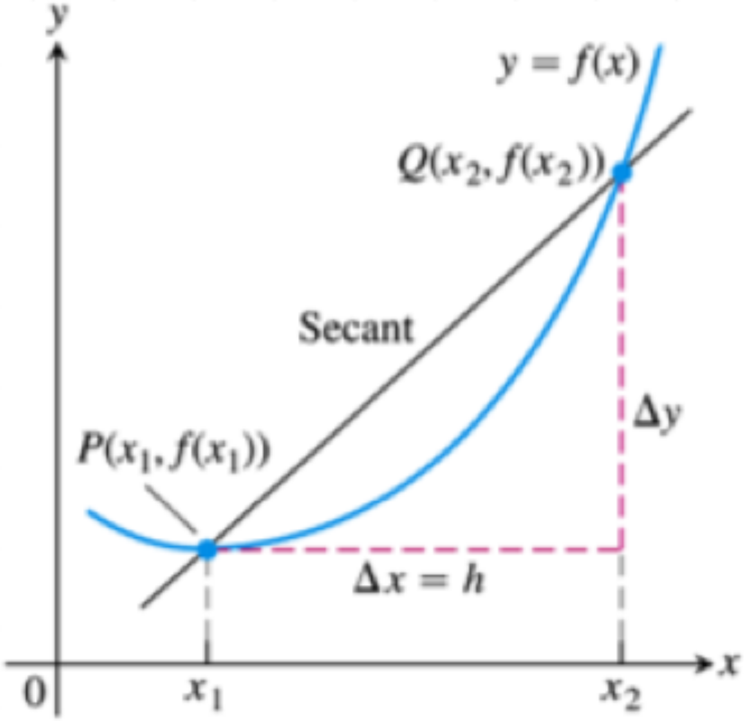
$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \; ; \; h = x_2 - x_1$$

$$= \frac{f(x_2) - f(x_1)}{h} \; ; \; h \neq 0$$

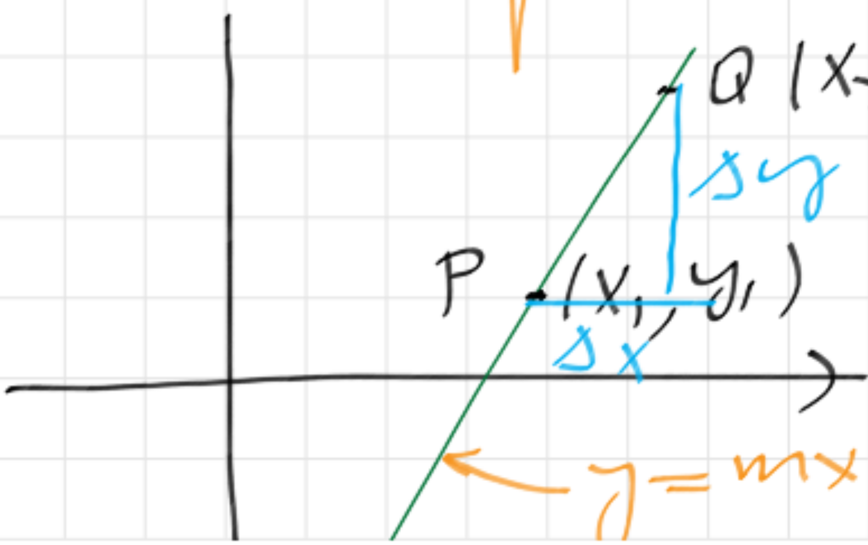
Slope $= m$

$$\frac{\Delta y}{\Delta x}$$

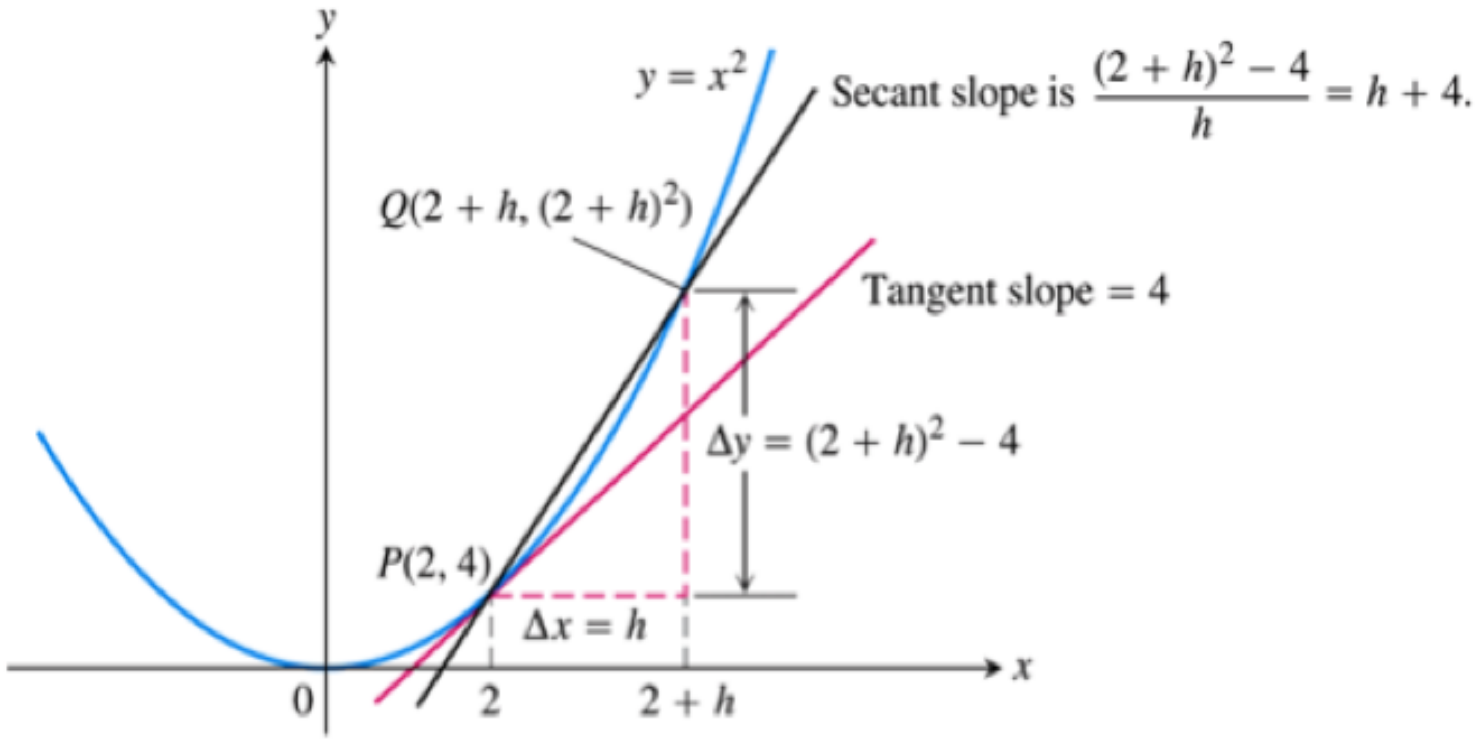
slope of the line (secant) thru P and Q




Slope of a line




$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$




$$y(2+h) = f(2+h) = (2+h)^2$$

Slope of the secant $= \frac{\Delta y}{\Delta x}$

$$\frac{\Delta y}{\Delta x} = \frac{f(2+h) - f(2)}{2+h-2}$$

$$f(2) = 2^2 = 4$$

$$f(2+h) = (2+h)^2 = 4 + 2h + h^2$$

$$\frac{\Delta y}{\Delta x} = \frac{f(2+h)-f(2)}{\cancel{2}+h-\cancel{2}}$$

$$= \frac{\cancel{4}+4h+h^2-\cancel{4}}{h}$$

$$\frac{\Delta y}{\Delta x} = \frac{\cancel{h}(4+h)}{\cancel{h}}$$

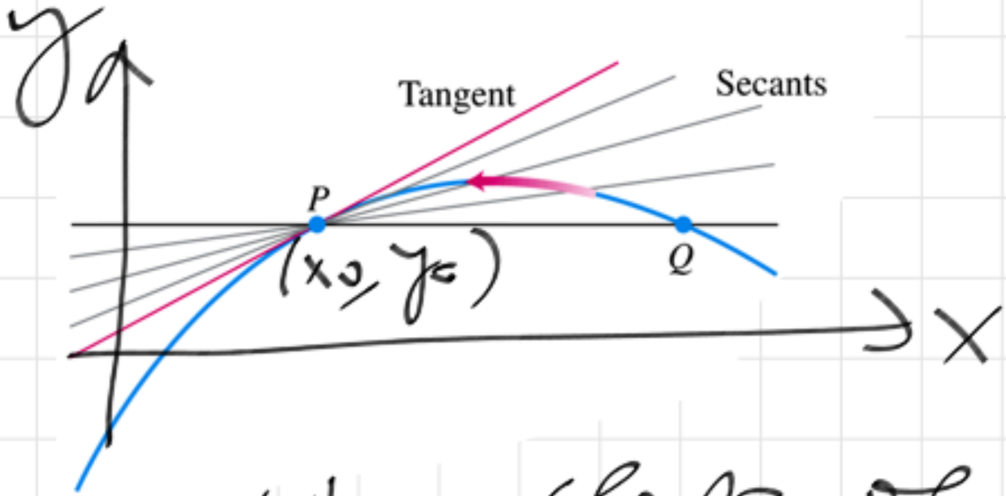
$$\frac{\Delta y}{\Delta x} = 4+h$$

when the secant line touches the curve at a single, it becomes tangent.

The slope of the tangent line at $x=2$, that is $h \to 0$, is

$$\frac{\Delta y}{\Delta x} = 4 + \underset{0}{h} = 4$$

Tangent means the slope of a single pt. on the graph (curve).




The slope of the tangent line to $y(x) = f(x)$ at $P(x_0, y_0)$ is

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x_0) = \left.\frac{dy}{dx}\right|_{x_0}$$

derivative of $f(x)$ at $x_0$.

$f'(x_0)$ means the instantaneous rate change of $f(x)$ at $x_0$.

$$\left.\frac{df}{dx}\right|_{x_0} = f'(x_0)$$

$\frac{\Delta y}{\Delta x}$ is the average rate change of $f(x)$

Ex $y = mx + b \rightarrow y' = m$

$y = c \longrightarrow y' = 0$