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**Course:** Linear Algebra

**Assignment:** Section 4.5 Homework

1. For the subspace below, (a) find a basis for the subspace, and (b) state the dimension.

$$\left\{ \begin{bmatrix} s - 7t \\ s + t \\ 8t \end{bmatrix} : s, t \in \mathbb{R} \right\}$$

(a) Find a basis for the subspace.

A basis for the subspace is  $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 1 \\ 8 \end{bmatrix} \right\}$ .

(Use a comma to separate answers as needed.)

(b) State the dimension.

The dimension is 2.

2. For the subspace below, (a) find a basis, and (b) state the dimension.

$$\left\{ \begin{bmatrix} 6a + 12b - 2c \\ 3a - b - c \\ -12a + 5b + 4c \\ -3a + b + c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$$

a. Find a basis for the subspace.

A basis for the subspace is  $\left\{ \begin{bmatrix} 6 \\ 3 \\ -12 \\ -3 \end{bmatrix}, \begin{bmatrix} 12 \\ -1 \\ 5 \\ 1 \end{bmatrix} \right\}$ .

(Use a comma to separate vectors as needed.)

b. State the dimension.

The dimension is 2.

3. For the subspace below, (a) find a basis for the subspace, and (b) state the dimension.

$$\left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : a - 5b + 6c = 0 \right\}$$

(a) Find a basis for the subspace.

A basis for the subspace is  $\left\{ \begin{bmatrix} 5 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ .

(Use a comma to separate matrices as needed.)

(b) State the dimension.

The dimension is 3.

4. Find the dimension of the subspace of all vectors in  $\mathbb{R}^6$  whose first and fifth entries are equal.

The dimension is 5.

(Type a whole number.)

5. Find the dimension of the subspace spanned by the given vectors.

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -15 \\ 4 \\ -26 \end{bmatrix}, \begin{bmatrix} 11 \\ -3 \\ 19 \end{bmatrix}$$

The dimension of the subspace spanned by the given vectors is 2.

6. Determine the dimensions of Nul A and Col A for the matrix shown below.

$$A = \begin{bmatrix} 1 & 9 & 9 & 1 & 4 \\ 0 & 1 & -1 & -1 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The dimension of Nul A is 3, and the dimension of Col A is 2.

7. Determine the dimensions of Nul A and Col A for the matrix shown below.

$$A = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The dimension of Nul A is 0, and the dimension of Col A is 4.

8. Let  $B$  be the basis of  $\mathbb{P}_3$  consisting of the Hermite polynomials  $1$ ,  $2t$ ,  $-2 + 4t^2$ , and  $-12t + 8t^3$ ; and let  $\mathbf{p}(t) = -2 + 4t^2 - 8t^3$ . Find the coordinate vector of  $\mathbf{p}$  relative to  $B$ .
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$$[\mathbf{p}]_B = \begin{bmatrix} 0 \\ -6 \\ 1 \\ -1 \end{bmatrix}$$