General Instructions:

- This midterm exam is an online exam.
- Textbooks, slides and notes are open.
- Please write your answers on clean A4 sheets.
- Do not forget to write your name on each sheet.
- After answering the questions, scan your sheets and combine them into one PDF file and name it as “yourname_surname.pdf”.
- You have 120 minutes for the exam. This duration covers both the time for answering the exam questions and the time for scanning sheets, combining them into one PDF file and uploading.
- You have to upload your file until 11:00 via https://ues.marmara.edu.tr.
    - In case of last minute problems (in case of not being able to upload to UES), they should be sent by e-mail (falkaya@marmara.edu.tr) within the time limit.
- No late submissions will be accepted.
- Please write the following Declaration of Honor on your first answer sheet and sign it.
    - “On my honor, I have neither given nor received any unauthorized and/or inappropriate assistance for this exam. The work done on this exam is totally my own. I understand that by the school code, violation of these principles will lead to a zero grade and is subject to harsh discipline issues.”

Show the necessary details of your answer clearly to get proper credit for your work. To prove a statement, provide a general argument showing and justifying all necessary steps. To disprove, you need to give a counter example.

1. (20 Pts) Prove or disprove the following. (DO NOT use Venn Diagrams!)
   a) $A \cap B = A \cap C$ and $A \cup B = A \cup C$ imply $B = C$.
   b) $A \cap B = A \cap C$ implies B=C for all A, B, C.
   c) $f \circ g(x) = g \circ f(x)$ where f and g are functions from $\mathbb{R} \to \mathbb{R}$ and $\circ$ is the functional composition.

2. (20 Pts)
   a) Let $f: \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ and $f(m, n) = 2^m 3^n$. Is f one-to-one? Is f onto? Prove or disprove.
   b) Consider the functions $\boldsymbol{u}$ and $\boldsymbol{s}$ mapping $\boldsymbol{Z}$ into $\boldsymbol{Z}$, where $u(n) = n+1$ for $n \in \boldsymbol{Z}$, and $\boldsymbol{s}$ is the characteristic function $\aleph_E$ where $E = \{n \in \boldsymbol{Z}, n \text{ is odd}\}$. Find $u \circ s$ and $s \circ s$.

3. (10 Pts) Determine the truth values of each of the following.

   a) $\forall x \exists y[x + y = 0]$ $\quad x, y \in \mathbb{R}$
   b) $\exists y \forall x[x + y = 0]$ $\quad x, y \in \mathbb{R}$
   c) $\exists n \exists m[m < n]$ $\quad m, n \in \mathbb{Z}$
   d) $\forall n \forall m[m < n]$ $\quad m, n \in \mathbb{Z}$
   e) $\forall n \exists m[m < n]$ $\quad m, n \in \mathbb{Z}$
   f) $\exists m \forall n[m < n]$ $\quad m, n \in \mathbb{Z}$
   g) $\forall m \exists n[2n = m]$ $\quad m, n \in \mathbb{N}$
   h) $\exists n \forall m[2m = n]$ $\quad m, n \in \mathbb{N}$
   i) $\forall m \exists n[2m = n]$ $\quad m, n \in \mathbb{N}$
   j) $\exists n \forall m[2n = m]$ $\quad m, n \in \mathbb{N}$

4. (10 Pts) Prove the following (Use the rules given in the following page.)
   Conclude $\neg p$ from the hypotheses $p \to (q \wedge r),\ r \to s \text{ and } \neg(q \wedge s)$.

5. (10 Pts) Construct the truth table for $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r)$
   What is the result and what does it mean? (i.e. can you simplify it?)

6. (10 Pts) Solve the linear congruence $7x \equiv 13 \pmod{19}$. (Show your work. That is, firstly find the necessary inverse by finding the Bezout coefficients, and then use it to solve the linear congruence.)

7. (15 Pts) Prove that for any $n \in \mathbb{Z}^+, 5n + 3$ and $7n + 4$ are relatively prime.

8. (15 Pts) Find the solutions to the system $x \equiv 2\ (mod\ 4), x \equiv 1\ (mod\ 5), x \equiv 3\ (mod\ 7)$ and $x \equiv 2\ (mod\ 3)$. (Show your work).

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ID: Duration: 100 minutes

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 7 Logical Equivalences Involving Conditional Statements.

1 $p \to q \equiv \neg p \vee q$

2 $p \to q \equiv \neg q \to \neg p$

3 $p \vee q \equiv \neg p \to q$

4 $p \wedge q \equiv \neg(p \to \neg q)$

5 $\neg(p \to q) \equiv p \wedge \neg q$

6 $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

7 $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

8 $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

9 $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

10 $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

11 $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

12 $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

13 $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \wedge (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution

Different Ways of Expressing $p \to q$

if p**, then** q	p **implies** q
if p**,** q	p **only if** q
q unless $\neg p$	q **when** p
q **if** p	
q **whenever** p	p **is sufficient for** q
q **follows from** p	q **is necessary for** p

a necessary condition for p **is** q

a sufficient condition for q **is** p