KEY

Istanbul Şehir University

Math 104

Date: 12 June 2014	Full Name:
Time: 9:00-11:00	
	Student ID:
Spring 2014 Final Exam	

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 5 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. You may continue your solutions on the back of the sheets. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cellphones and electronic devices are to be kept shut and out of sight.

Q1	Q2	Q3	Q4	Q5	TOTAL
20 pts	20 pts	30 pts	25 pts	25 pts	120 pts

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1) Let $w(x,y) = e^{xy}\ell nx$ be a function of independent variables x and y. Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$.

$$\frac{\partial w}{\partial x} = y e^{xy} \ln x + \frac{e^{xy}}{x} \quad \Big\} \text{ 10 pts}$$

$$\frac{\partial w}{\partial y} = x e^{xy} \ln x \quad \Big\} \text{ 10 pts}$$

(10 pts) 2) Evaluate the following limits, if they exist:

a) $\lim_{x\to 0^+} x^{2x}$ $\qquad 0^0$, indeterminate

$y = x^{2x} \Rightarrow \ln y = 2x \ln x$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} 2x\ln x \;\; (0\cdot -\infty) = \lim_{x\to 0^+} \frac{2\ln x}{1/x} \;\; (-\infty/\infty)$

$\underset{\text{L'Hospital}}{=} \lim_{x\to 0^+} \frac{2/x}{-1/x^2} = \lim_{x\to 0^+} -2x = 0$

$\ln y \to 0 \Rightarrow y \to e^0 = \boxed{1.}$

(10 pts) b) $\lim_{x\to 0^+} (sinx)^{tanx}$. $\qquad 0^0$, indeterminate

$y = (\sin x)^{\tan x} \Rightarrow \ln y = \tan x \ln \sin x$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \tan x \ln \sin x \;\; (0\cdot -\infty)$

$= \lim_{x\to 0^+} \frac{\ln \sin x}{\cot x} \;\; (-\infty/\infty)$

$\underset{\text{L'Hospital}}{=} \lim_{x\to 0^+} \frac{\frac{\cos x}{\sin x}}{-\csc^2 x}$

$= \lim_{x\to 0^+} -\frac{\cos x}{\cancel{\sin x}} \cdot \sin^{\cancel{2}} x = -\lim_{x\to 0^+} \cos x \sin x$

$= 0$

$\ln y \to 0 \Rightarrow y \to e^0 = \boxed{1}$

(15pts) 3) Evaluate the following integrals:

a) $\int \tan^4\theta d\theta$

$$\underbrace{\tan^4\theta}_{\tan^2\theta(\sec^2\theta-1)} = \int \underbrace{\tan^2\theta\sec^2\theta d\theta}_{\substack{u=\tan\theta \\ du=\sec^2\theta d\theta}} - \int \underbrace{\tan^2\theta}_{\sec^2\theta-1} d\theta$$

$$\downarrow$$

$$\int u^2 du = u^3/3$$

$$\Rightarrow \int \tan^4\theta d\theta = \boxed{\frac{\tan^3\theta}{3} - \tan\theta + \theta + C}$$

$$\int u\,dv = uv - \int v\,du$$

(15pts) b) $\int \frac{\ell n x}{x^{11}} dx.$ Integration by parts

$u = \ln x \qquad dv = x^{-11}dx$

$du = dx/x \qquad v = -\frac{x^{-10}}{10}$

$$\Rightarrow \int \frac{\ln x}{x^{11}} dx = -\frac{1}{10}\frac{\ln x}{x^{10}} + \frac{1}{10}\int \frac{1}{x}\cdot x^{-10} dx$$

$$= -\frac{1}{10}x^{-10}\ln x + \frac{1}{10}\int x^{-11}dx$$

$$= -\frac{1}{10}x^{-10}\ln x - \frac{1}{10}\cdot\frac{1}{10}\cdot x^{-10} + C$$

$$= \boxed{-\frac{1}{10}x^{-10}\ln x - \frac{1}{100}x^{-10} + C}$$

(12 pts) 4) Determine whether the following series converge or diverge:

a) $\sum_{n=1}^{\infty} \frac{n^6}{n!}$ Ratio Test:

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^6}{(n+1)!}}{\frac{n^6}{n!}}$$

$$= \lim_{n\to\infty} \frac{\cancel{n!}}{\cancel{(n+1)!}} \left(\frac{n+1}{n}\right)^6$$

$$= \underbrace{\left(\lim_{n\to\infty} \frac{1}{n+1}\right)}_{0} \underbrace{\left(\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^6\right)}_{1} = 0 < 1$$

$\therefore$ $\boxed{\text{Converges}}$

(13 pts) b) $\sum_{n=1}^{\infty} ne^{-2n^2}$

Integral Test

$$\int x e^{-2x^2} dx = -\frac{1}{4}\int e^u du = -\frac{1}{4} e^{-2x^2} + C$$

$u = -2x^2$

$du = -4x\,dx$

$$\therefore \int_1^{\infty} x e^{-2x^2} dx = \lim_{b\to\infty} -\frac{1}{4} e^{-2x^2} \Big|_1^b$$

$$= -\frac{1}{4}\left\{\underbrace{\lim_{b\to\infty} e^{-2b^2}}_{0} - 1\right\} = \frac{1}{4}$$

$\therefore$ $\boxed{\text{Converges}}$

(This can be solved by the Ratio Test too)

5) Given the power series $\sum_{n=0}^{\infty} \frac{n}{2^n} x^n$

(a) Find the radius of convergence.
(b) Find the interval of convergence.

Generalized Ratio Test:

$$\rho = \lim_{n\to\infty} \left| \frac{\frac{(n+1)\,x^{n+1}}{2^{n+1}}}{\frac{n\,x^n}{2^n}} \right| = \lim_{n\to\infty} \frac{\cancel{2^n}^{\,1}}{\cancel{2^{n+1}}_{\,2}} \frac{n+1}{n} |x|$$

15 pts

$$= \frac{|x|}{2} \underbrace{\lim_{n\to\infty} \frac{n+1}{n}}_{1} = \frac{|x|}{2}$$

Converges when $\frac{|x|}{2} < 1 \Rightarrow -2 < x < 2$

Diverges when $\frac{|x|}{2} > 1 \Rightarrow x > 2$ or $x < -2$

Check the endpts:

5 pts

$$x = 2 \Rightarrow \sum_{n=0}^{\infty} \frac{n}{2^n} 2^n = \sum_{n=0}^{\infty} n = 0 + 1 + 2 + \cdots$$

diverges

5 pts

$$x = -2 \Rightarrow \sum_{n=0}^{\infty} \frac{n}{2^n} (-2)^n = \sum_{n=0}^{\infty} \frac{(-1)^n n \cancel{2^n}}{\cancel{2^n}}$$

diverges

Both diverge by nth term test, $\lim a_n \neq 0$.

(a) $\boxed{\text{Radius} = 2}$

(b) Interval $(-2, 2)$, or $\boxed{-2 < x < 2}$