

MATH2059 Numerical Methods
HOMEWORK FOR FINAL EXAM

Spring 2019-2020

Start 10:00 – End 13:00

Name:
Student ID:

Q1	/ 40
Q2	/ 40
Q3	/ 40
Q4	/ 40
Total	/ 160

- Please use A4 size white blank sheets to write your answers with handwriting. You can print this booklet and write your answers on it, if you have a printer.
- Please write your name and student ID at the top of all your answer sheets. Please copy the above grading table to the first page as well.
- Please copy one of the sentences below to the first page of your answer sheet in handwriting and sign under the sentence.

“Sorular tarafımca çözülmüş olup, hiçbir yolla başka bir şahıstan yardım alınmamış ve başka bir şahısla paylaşılmamıştır.”

“The questions have been answered by myself, and I have neither given nor received any unauthorized help in any way.”

- There are 4 questions and 5 pages in this handout.
- You have 3 hours to complete your answers and upload them to UES in a single pdf document. The document should be named as “**name_surname.pdf**”. You should pay attention to the following:
 - Use a scanner or a **scanner application on your mobile phone** (Adobe Scan etc.) to scan your answers into a single pdf file. **Do not take photos** of the pages.
 - Scan each page one by one in upright **portrait orientation** (NOT landscape).
 - Make sure that all pages are clear and **readable**.
 - Do not delay your submission to the last minute as there may be unexpected problems. Try to begin uploading your file **at least 10 minutes** before the deadline (17:00).
- **Chrome browser** is recommended to minimize any problems.
- **SHOW ALL YOUR WORK CLEARLY AND STEP BY STEP TO RECEIVE CREDIT.** Good luck!

QUESTION 1 - Optimization

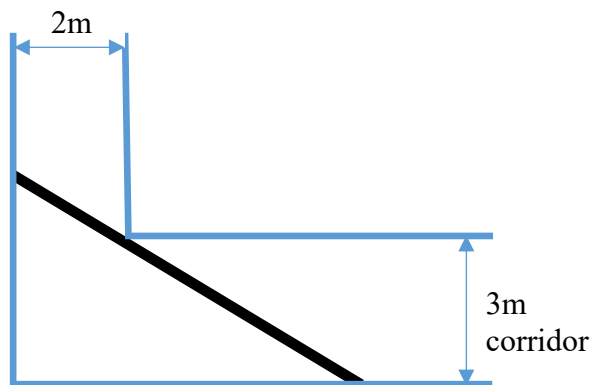
We would like to carry a metal piece of length L (ignore the thickness) in horizontal position around a corner from a corridor of 2 meters wide into a corridor of 3 meters wide as shown in the figure below. In the figure, the metal piece is shown with black color.

What is the maximum length that the metal piece can have?

- Write down the function that needs to be optimized.
- Optimize the function you have written in (a) using one of the **numerical** methods you have learned. Explain your solution in detail including **initialization** and **stopping criterion**. You may use a table to show your parameters at each iteration.

Note 1: If you wish, you may use Matlab/Octave if it is necessary for your method of choice. If you do so, you must also submit your code.

Note 2: If you wish, you can also solve the problem analytically and compare with your numerical solution.



QUESTION 2 – Linear Equations

Solve the following linear system equations using **LU decomposition with partial pivoting**. Clearly indicate your solution and determine the matrices L , U and P , where P describes the pivoting. Show all the steps of the solution.

$$\begin{aligned}10x_1 - 7x_2 &= 7 \\ -3x_1 + 2x_2 + 6x_3 &= 4 \\ 5x_1 - x_2 + 5x_3 &= 6\end{aligned}$$

Solution:

$$x_1 =$$

$$x_2 =$$

$$x_3 =$$

$$L =$$

$$U =$$

$$P =$$

QUESTION 3 – Regression/Interpolation/Extrapolation

Suppose you are given the following data (or measurements), which consists of (x, y) pairs:

x	0	2	4	6	9	11	12	15	17	19
y	5	6	7	6	9	8	7	10	12	12

- (a) You are asked to estimate the value of y at the point $x = 13$. Suggest **two different methods** and apply these methods to perform the estimation.
- (b) You are asked to estimate the value of y at the point $x = 21$. Suggest a method and apply this method to perform the estimation.

QUESTION 4 – Numerical Integration

Estimate $\int_0^{2.5} f(x)dx$ from the data given below, use **two different numerical integration** methods of your choice. Which method is expected to be more accurate and why?

x	0	0.5	1.0	1.5	2.0	2.5
$f(x)$	1.5000	2.0000	2.0000	1.6364	1.2500	0.9565