Math No:

Full Name :

ISTANBUL ŞEHİR UNIVERSITY

KEY

Math 104, Midterm 4
(16 May 2015, Time: 11:30-12:30)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
20 pts	25 pts	20 pts	35 pts	100 pts

Q1. Find the Maclaurin series of the ***hyperbolic cosine function coshx*** defined as

$$\cosh x = \frac{e^x + e^{-x}}{2}.$$

(Hint: To solve this question, you may use Taylor or Maclaurin series that you know.)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\Rightarrow e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$$

$$\Rightarrow e^x + e^{-x} = 2 + 2\cdot\frac{x^2}{2!} + 2\cdot\frac{x^4}{4!} + \cdots$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}}$$

Since the series for e^x converges for all x, so does the series for $\cosh x$.

Math No:

Full Name :

Q2. Determine whether the series given below converges absolutely, converges conditionally, or diverges:

$$\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,n!}$$

Ratio Test:

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\dfrac{[2(n+1)]!}{(n+1)!\,(n+1)!}}{\dfrac{(2n)!}{n!\,n!}}$$

$$= \lim_{n\to\infty} \frac{(2n+2)!}{(2n)!} \cdot \frac{n!\; n!}{(n+1)!\,(n+1)!}$$

$$= \lim_{n\to\infty} \frac{(2n+2)(2n+1)\cancel{(2n)!}}{\cancel{(2n)!}} \cdot \frac{\cancel{n!}\;\cancel{n!}}{(n+1)\cancel{n!}\,(n+1)\cancel{n!}}$$

$$= \lim_{n\to\infty} \frac{(2n+2)(2n+1)}{(n+1)^2} = \lim_{n\to\infty} \frac{4n^2+4n+2}{n^2+2n+1}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^2}\left(4 + 4/n + 2/n^2\right)}{\cancel{n^2}\left(1 + 2/n + 1/n^2\right)}$$

$= 4 > 1 \Rightarrow$ The series diverges.

Math No:

Full Name :

ISTANBUL ŞEHİR UNIVERSITY

Q3. Determine whether the series given below converges or diverges:

$$\sum_{n=2}^{\infty}(-1)^n\frac{1}{lnn}$$

Alternating Series Test

This is an alternating series.

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(-1)^n}{\ln n} = 0$ because $\ln n \to \infty$

$\{|a_n|\} = \left\{\frac{1}{\ln n}\right\}$ is decreasing because ln is an increasing function

$\therefore$ The series converges.

Math No:

Full Name :

Q4. Given the power series

$$\sum_{n=0}^{\infty} \frac{n(x-2)^n}{5^n}$$

Generalized Ratio Test

(a) Find the radius of convergence.
(b) Find the interval of convergence.

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$\rho = \lim_{n\to\infty} \left| \frac{\frac{(n+1)(x-2)^{n+1}}{5^{n+1}}}{\frac{n(x-2)^n}{5^n}} \right| = \lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{(x-2)^n} \cdot \frac{5^n}{5^{n+1}} \cdot \frac{n+1}{n} \right|$$

$$= \frac{|x-2|}{5} \underbrace{\lim_{n\to\infty} \frac{n+1}{n}}_{1} = \frac{|x-2|}{5}$$

The series converges absolutely if $\rho < 1 \Rightarrow$

$|x-2| < 5 \Rightarrow -5 < x-2 < 5 \Rightarrow -3 < x < 7$

The series diverges if $\rho > 1 \Rightarrow x > 7$ or $x < -3$

$\rho = 1$ no info for $x = -3, 7$.

$$x = 7 \Rightarrow \sum_{n=0}^{\infty} \frac{n \cdot 5^n}{5^n} = \sum_{n=0}^{\infty} n = 1+2+3+\cdots$$

diverges

$$x = -3 \Rightarrow \sum_{n=0}^{\infty} \frac{n(-5)^n}{5^n} = \sum_{n=0}^{\infty} \frac{n(-1)^n 5^n}{5^n} = \sum_{n=0}^{\infty} (-1)^n n$$

diverges because $\lim_{n\to\infty} a_n$ does not exist (nth term test)

$\therefore$ The interval of convergence is $-3 < x < 7$

The radius of convergence is $= 5$