KEY

İstanbul Şehir University
Math 104

Date: 28 March 2015	Full Name:
Time: 11:30-12:30	
	Student ID:
Spring 2015 First Exam	Math number:

IMPORTANT

1. Write down your name and surname on top of each sheet. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cellphones and electronic devices are to be kept shut and out of sight.

Q1	Q2	Q3	Q4	TOTAL
10 pts	30 pts	30 pts	30 pts	100 pts

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1) Find the derivative of the function $y = x^{\sqrt{x}}$.

$$\ln y = \sqrt{x}\ln x$$

$$\Rightarrow \frac{1}{y}y' = \frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}} = \frac{1}{\sqrt{x}}\left(1 + \frac{\ln x}{2}\right)$$

$$= \frac{2+\ln x}{2\sqrt{x}}$$

$$\Rightarrow \boxed{y' = \frac{x^{\sqrt{x}}(2+\ln x)}{2\sqrt{x}}}$$

2) Evaluate the following limits, if they exist:

a) $\lim_{x\to 0} \frac{Arctan(x^2)}{x^2}$ $\quad 0/0$

$$= \lim_{x\to 0} \frac{\frac{2x}{1+x^4}}{2x}$$

L'Hospital

$$= \lim_{x\to 0} \frac{1}{1+x^4} = \boxed{1}$$

b) $\lim_{x\to\infty} (\ln x)^{1/x}$ $\quad \infty^0$

$$y = (\ln x)^{1/x} \Rightarrow \ln y = \frac{1}{x} \ln \ln x$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln\ln x}{x} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{\frac{1}{x\ln x}}{1}$$

L'Hospital

$$= \lim_{x\to\infty} \frac{1}{x \ln x} = \frac{1}{\infty} = 0$$

$$\ln y \to 0 \Rightarrow y \to e^0 = \boxed{1}$$

3) Evaluate the integrals given below:

a) $\int xe^{-2x}dx$

Integration by parts $\int u\,dv = uv - \int v\,du$

$u = x \qquad dv = e^{-2x}dx$

$du = dx \qquad v = -\frac{e^{-2x}}{2}$

$$\therefore \int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}dx$$

$$= \boxed{-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C}$$

b) $\int \sin^3\theta \cos^{-3/2}\theta d\theta$

$\sin^2\theta \cdot \sin\theta$

$1-\cos^2\theta$

$$= \int \left(\cos^{-3/2}\theta - \cos^{1/2}\theta\right)\sin\theta\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$= \int \left(-u^{-3/2} + u^{1/2}\right)du$$

$$= -\frac{u^{-1/2}}{-1/2} + \frac{u^{3/2}}{3/2} + C$$

$$= \boxed{2\cos^{-1/2}\theta + \frac{2}{3}\cos^{3/2}\theta + C}$$

4) Evaluate the integrals given below:

$u = 1 + e^x \Rightarrow du = e^x dx$

$\therefore dx = \frac{du}{e^x} = \frac{du}{u-1}$

a) $\int \frac{dx}{1+e^x} = \int \frac{du}{u(u-1)}$

Partial fractions

$\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$

$1 = A(u-1) + Bu$

$u=0 \quad 1 = -A$

$u=1 \quad 1 = B$

$= -\int \frac{du}{u} + \int \frac{du}{u-1}$

$= -\ln|u| + \ln|u-1| + C$

$= \ln\left|\frac{u-1}{u}\right| + C$

$= \boxed{\ln\left|\frac{e^x}{e^x+1}\right| + C}$

b) $\int_0^4 (x^2+16)^{-3/2}dx =$

$\int_0^{\pi/4} (4\sec\theta)^{-3} 4\sec^2\theta \, d\theta$

$= \frac{1}{16}\int_0^{\pi/4} \frac{d\theta}{\sec\theta}$

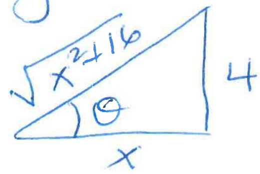
$= \frac{1}{16}\int_0^{\pi/4} \cos\theta \, d\theta$

$= \frac{1}{16} \sin\theta \Big|_0^{\pi/4}$

$= \frac{1}{16}\left(\frac{1}{\sqrt{2}} - 1\right)$

$= \boxed{\frac{1}{16\sqrt{2}}}$

Trig substitution



$\tan\theta = \frac{x}{4}$

$x = 4\tan\theta$

$dx = 4\sec^2\theta \, d\theta$

$\sqrt{x^2+16} = 4\sec\theta$

$x=0 \Rightarrow \theta = 0$

$x=4 \Rightarrow \theta = \pi/4$