" On my honor, I have neither given nor received any unauthorized and/or inappropriate assistance for all sessions of this exam. The work done on this exam is totally my own. I understand that by the school code, violation of these principles will lead to a zero grade and is subject to harsh discipline issues. "

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Q1) A security company would like to purchase an Artificial Intelligence-based object detection durations (in seconds) as follows:

6.1, 5.9, 5.9, 5.1, 6.4, 4.9, 4.7, 5.6

To use the software in the real environment, the variance of object detection should be no more than 0.25 seconds. Should the software be returned? Perform the appropriate test of hypothesis using $\alpha = 0.05$.

a) State the null hyphothesis to be tested and the alternative hyphothesis.

H_0:

H_a:

b) Conduct a statistical test of the null hyphothesis and State your conclusion test at $a = 0.05$;

c) Find the Critical Value (s). 1.895

d) Conclusion and interpretation;

e) find the approximate p-value;

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a) Mean: $\frac{6.1 + 5.3 + 5.9 + 5.1 + 6.4 + 4.9 + 4.7 + 5.6}{8} = \underline{5.5}$

$s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} = \frac{37.21 + 28.09 + 34.81 + 26.01 + 40.96 + 24.01 + 22.09 + 31.36}{}$

$\frac{244.54 + \frac{(44)^2}{8}}{7}$ $= 69.50...$ $SE = 0.5635$

$s = 8.34..$

~~$0.6 / -0.2 + 0.4 ~~0.4~~ + 0.9 - 0.6 - 0.8 + 0.1$~~

$t = \frac{5.5 - 0.25}{\frac{0.5635}{8}} = 26.3518$

$p = z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$ $z = \frac{5.5 -}{1.895 / \sqrt{8}}$

a) H_0: 0.5635 < 0.25

effect size is large

H_a:

H_0 is rejected

b) $t = \dfrac{55 - 0.25}{\dfrac{0.5635}{8}} = 26.3578$

c) 1,895

d)

e)

Binomial Probability Distr.

$$P(x=k) = C_k^n \cdot p^k \cdot q^{n-k} = \frac{n!}{k!(n-k)!} p^k \cdot q^{n-k}$$

Mean: $\mu = np$

Variance: $\sigma^2 = npq$

Standard deviation: $\sigma = \sqrt{npq}$

Hypergeometric Probability Distr.

$$P(x=k) = \frac{C_k^M \cdot C_{n-k}^{N-M}}{C_n^N}$$

$M \to$ successes

$N-M \to$ failures

Mean: $\mu = n\left(\frac{M}{N}\right)$

n – size of the random sample space

Variance: $\sigma^2 = n\left(\frac{M}{N}\right)\left(\frac{N-M}{N}\right)\left(\frac{N-n}{N-1}\right)$

Poisson Probability Distr

$$P(x=k) = \frac{\mu^k \cdot e^{-\mu}}{k!}$$

M: $E(x) = \mu$

Variance: $\sigma^2 = \mu$

Standard deviation: $\sigma = \sqrt{\mu}$

Variance of a Sample mean

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$$

Correlation Coefficien

$$r = \frac{s_{xy}}{s_x s_y}$$

Variance of Population

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$$

Standardizing the value of $\bar{x}$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Standardizing the value $\hat{p}$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Statistical Process Control

LCL: $\bar{\bar{x}} - 3\frac{s}{\sqrt{n}}$ UCL: $\bar{\bar{x}} + 3\frac{s}{\sqrt{n}}$

LCL: $\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ UCL: $\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

Confidence interval for a population mean μ

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$$

Confidence interval for a population proportion p

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Point estimator of population mean μ: $\bar{x}$

The 95 % margin of error ($n \geq 30$) $\pm 1.96 \frac{s}{\sqrt{n}}$

Point estimator of population proportion p: $\hat{p} = x/n$

The 95 % margin of error ($n \geq 30$): $\pm 1.96 \sqrt{\frac{\hat{p}\hat{q}}{n}}$

Standardizing the value of a test statistic

$$\left\{ \begin{matrix} z \\ t \end{matrix} \right\} = \frac{(\text{test\_statistic}) - (\text{population parameter})}{SE}$$

Standard Error (SE)

$$SE = \frac{\sigma}{\sqrt{n}}, \text{ or } \quad SE = \sqrt{\frac{\hat{p}\hat{q}}{n}}, \text{ or}$$

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \text{ or } \quad SE = \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \text{ or}$$

$$SE = \sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$ (for two small samples with common variance)

Pooled estimate for the common value of p (large samples)

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Common Variance for two samples (small samples)

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$f = \frac{s_1^2}{s_2^2}$$

Variance of a Sample

Bc sample [illegible]

$$s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n-1} = \frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1}$$

(test statistic

Variance of Population

$$\sigma^2 = \frac{\Sigma (x_i - \mu)^2}{N}$$

Parameter	Test Statistic
μ	$z = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$
p	$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$
$\mu_1 - \mu_2$	$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
$p_1 - p_2$	$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ or $z = \frac{(\hat{p}_1 - \hat{p}_2) - D_0}{\sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}}$

Bound, B

$$z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < B, \text{ or } \quad z_{\alpha}\left(\frac{\sigma}{\sqrt{n}}\right) < B \text{ or}$$

$$z_{\alpha/2}\left(\frac{\sqrt{pq}}{n}\right) < B \text{ or } \quad z_{\alpha}\left(\sqrt{\frac{pq}{n}}\right) < B$$

Confidence Interval for σ^2

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}$$

Confidence Interval for σ_1^2 / σ_2^2

$$\left(\frac{s_1^2}{s_2^2}\right)\frac{1}{F_{df_1, df_2}} < \frac{\sigma_1^2}{\sigma_2^2} < \left(\frac{s_1^2}{s_2^2}\right) F_{df_2, df_1}$$