On my honor, I have neither given nor received any unauthorized assistance on this examination. The work done on this exam is totally my own. I understand that by the school code, violation of these principles will lead to a zero grade and is subject to harsh discipline issues.

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1) a) ~~Say $A=\{1,2\}$, $B=\{2,3\}$, $C=\{1,2,3\}$~~

$A\cap B = A \cap C$ Say $x \in B$ $x \in C$ $x \in A \cup B$ so $x \in A$

$A \cup B = A \cup C$ $x \in A \cap B$ so $\boxed{B \subseteq C \;,\; C \subseteq B \quad B = C}$

b) $A \cap B = A \cap C$

Say $B=\{1,2\}$, $C=\{1,3\}$, $A=\{1\}$ It doesn't make $B=C$

c) $f(g(x)) = g(f(x))$

Say $f(x) = 2x+8$ $f(g(x)) = 6x+8$ $6x+8 \neq 6x+24$

$g(x) = 3x$ $g(f(x)) = 3(2x+8) = 6x+24$ not equal

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2) a) $f(m,n) = 2^m \cdot 3^n$ $2^m \cdot 3^n = y$ say $n=0$ $2^m = y$ $m = \log_2 y$

$f(\log_2 y, 0) = 2^{\log_2 y} \cdot 1 = y$ thus this func is not onto

$2^m \cdot 3^n$ does not give "0" so it's not one-to-one

b)

3) a) True b) false c) True d) False e) True

f) false g) False h) false i) True j) false

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4)

Step	Reason
1. $\neg r \vee s$	premise
2. $\neg q \vee \neg s$	premise
3. $\neg r \vee \neg q$	resolution 1 and 2
4. $\neg(q \wedge r)$	de morgan's law 3
5. $p \to (q \wedge r)$	premise
6. $\neg p$	Modus tollens 4 and 5

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5)

p	q	r	$((\neg p \wedge (\neg q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r)))$
F	F	F	F
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	T
T	T	F	F
T	T	T	T

result is r because values (results) are the same as you can see in truth table

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6) find inverse

$7x \equiv 13 \pmod{19}$ say

$19 = 7.2 + 5$

$7 = 5.1 + 2$

$5 = 2.2 + 1 \rightarrow \gcd(19,7) = 1$

$2 = 1.2 + 0$

$= 4.19 + 7(-4)$

we will multiply by this

$(-4).\ 7x \equiv 13.(-4) \pmod{19}$ therefore

$x \equiv 5 \bmod (19)$

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7) lets say $5n+3 = 7n+4 = x$

$x \equiv 3 \pmod 5$

$x \equiv 4 \pmod 7$

they are relatively prime. we'll use chinese theorem.

$M_1 = M_2 . 1 = 7$ $M_2 = M_1 . 1 = 5$

$7 \equiv 2 \pmod 5 \rightarrow z.2 \equiv 1 \pmod 5$ $z = 3$

$5 \equiv 5 \pmod 7 \rightarrow y.5 \equiv 1 \pmod 7$ $y = 3$

$x = 3.M_1.z + 4.M_2.y = 3.7.3 + 4.3.5 = 123 \pmod{(M_1.M_2)}$ 35

$x = 123 \pmod{35}$ $123 - 35.3 = 18$ $x = 18 \pmod{35}$

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8) $x = 2 \pmod 4$

$x = 1 \pmod 5$ We will use chinese remainder

$x = 3 \pmod 7$ theorem again

$x = 2 \pmod 3$

$M_1 = M_2 \cdot M_3 \cdot M_4 = 5 \cdot 7 \cdot 3 = 105$

$M_2 = M_1 \cdot M_3 \cdot M_4 = 4 \cdot 7 \cdot 3 = 84$

$M_3 = M_1 \cdot M_2 \cdot M_4 = 4 \cdot 5 \cdot 3 = 60$

$M_4 = M_1 \cdot M_2 \cdot M_3 = 4 \cdot 5 \cdot 7 = 140$

$y_1 \rightarrow \quad 105 \equiv 2 \pmod 4 \quad y_1 \cdot 1 \equiv 1 \pmod 4 \quad y_1 = 1$

$y_2 \rightarrow \quad 84 \equiv 1 \pmod 5 \quad y_2 \cdot 4 \equiv 1 \pmod 4 \quad y_2 = 4$

$y_3 \rightarrow \quad 60 \equiv 3 \pmod 7 \quad y_3 \cdot 4 \equiv 1 \pmod 7 \quad y_3 = 2$

$y_4 \rightarrow \quad 140 \equiv 2 \pmod 3 \quad y_4 \cdot 2 \equiv 1 \pmod 3 \quad y_4 = 2$

$x = M_1 \cdot y_1 \cdot a_1 + M_2 \cdot y_2 \cdot a_2 + M_3 \cdot y_3 \cdot a_3 + M_4 \cdot y_3 \cdot a_3$

$= 105 \cdot 1 \cdot 2 + 84 \cdot 4 \cdot 1 + 60 \cdot 2 \cdot 3 + 140 \cdot 2 \cdot 2$

$= 1466 \pmod{420}$

$\boxed{x \equiv 206 \pmod{420}}$