

Reading

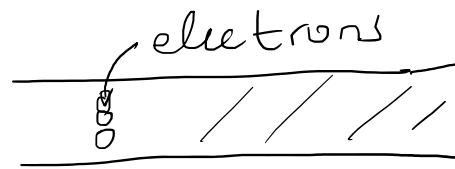
Fundamentals of Electric Circuits 5th Edition, McGrawHill, 2015
By Charles K. Alexander and Matthew N. O. Sadiku

- Chapter 1
- Chapter 2
- Chapter 6

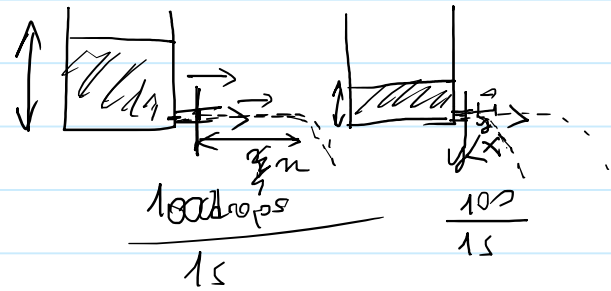
Applications



Charge and Current



$$i \triangleq \frac{dq}{dt} \longleftrightarrow Q \triangleq \int_{t_0}^t i dt$$



1 Coulomb of electrons $\approx 6,241,509,074,460,762,607$ electrons

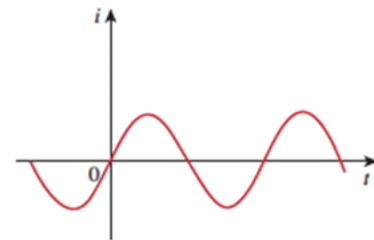
$$i = \frac{\Delta Q}{\Delta t} \quad 6 \times 10^{18}$$

$$\frac{1 \text{ Coulomb}}{1s} = 1A$$

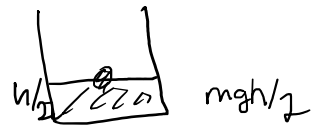
DC



AC



Voltage



$$v_{ab} \triangleq \frac{dw}{dq}$$

$$1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ newton-meter/coulomb}$$

Power

$$p \triangleq \frac{dw}{dt}$$

$$\int dw = \int p \cdot dt$$
$$w = \int p dt$$

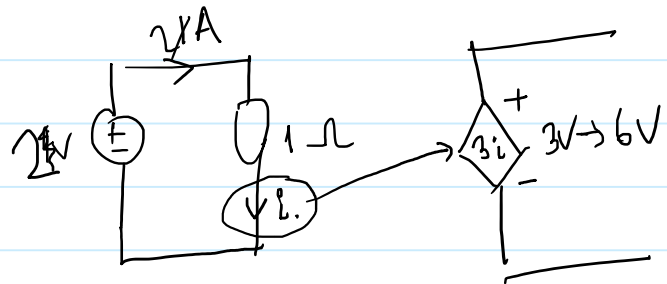
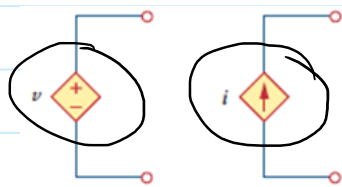
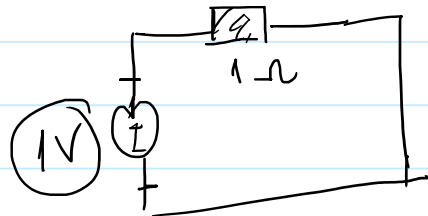
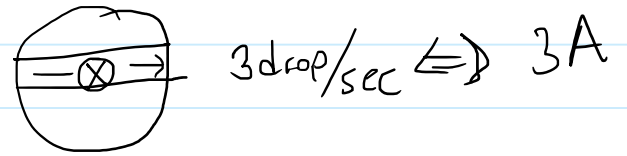
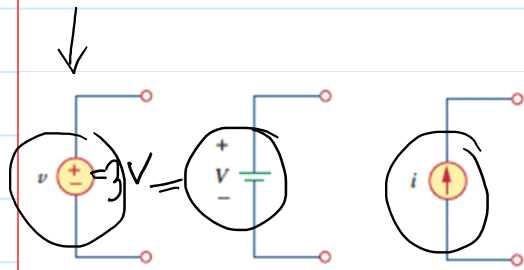
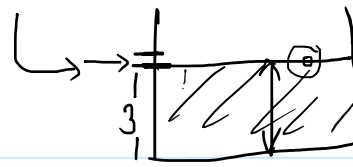
$$w = \int_{t_0}^t p dt = \int_{t_0}^t v i dt$$

$$= \frac{\frac{dw}{dq}}{\frac{dq}{dt}} = \frac{dw}{dt}$$

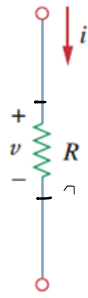
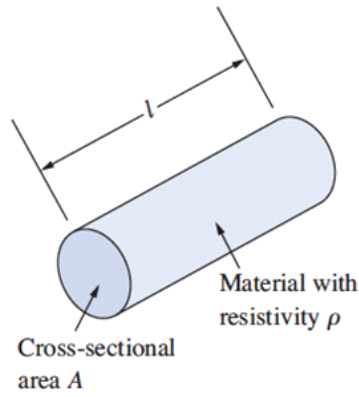
$V \quad I$

$$\sum p = 0$$

Active Circuit Elements



Ohm's Law

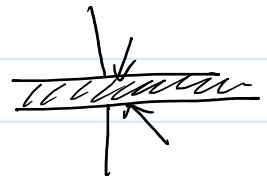


$$v = iR$$

$$R = \frac{l \cdot \rho}{A}$$

Resistivities of common materials.

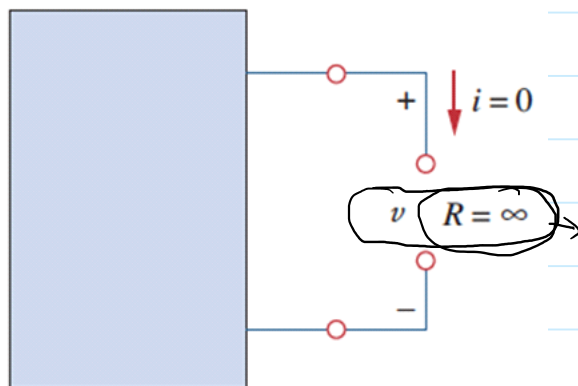
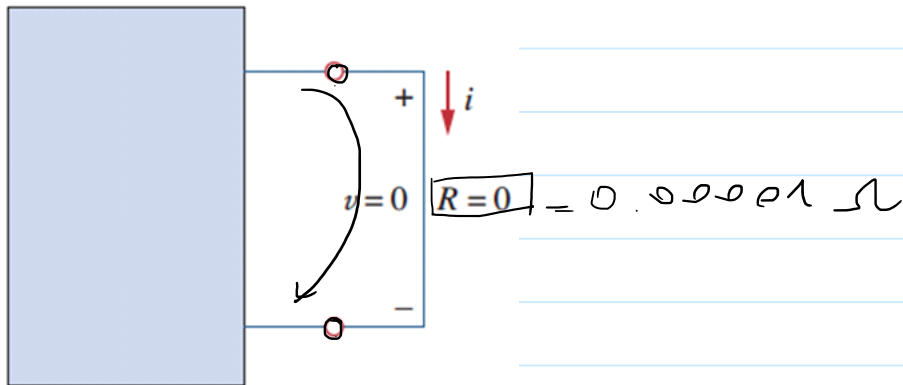
Material	Resistivity ($\Omega \cdot m$)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^2	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator



Short and Open Circuit

$$v = iR \leftarrow 0$$

\uparrow \uparrow
 0 0

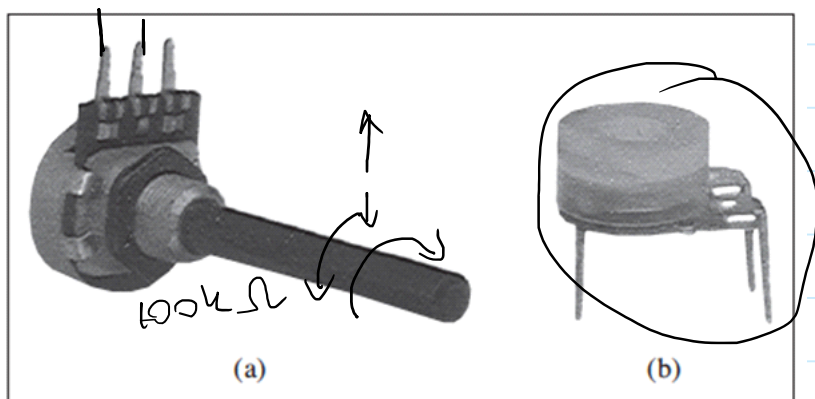
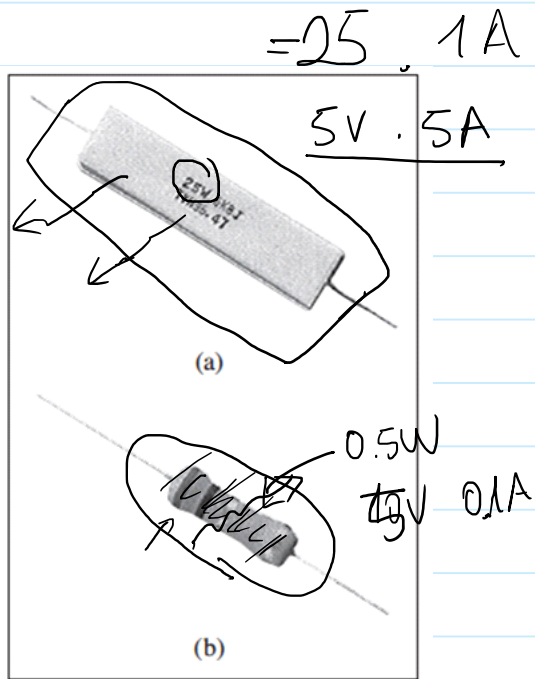


$$v = iR$$

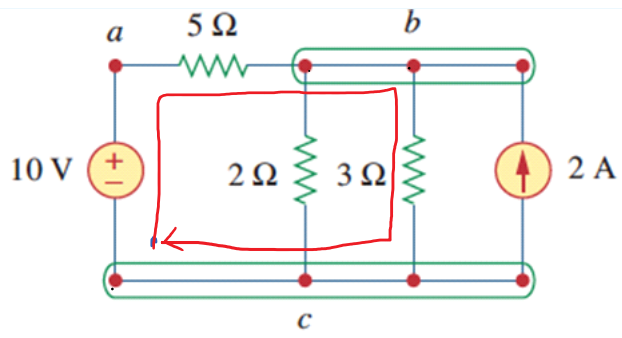
\uparrow \uparrow
 0 $0 \rightarrow \infty$

$v \ll 1000$

Types of Resistors



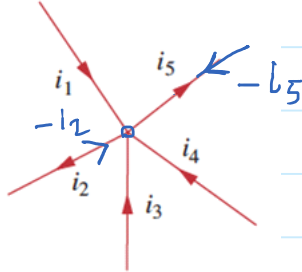
Branch, Node and Loop



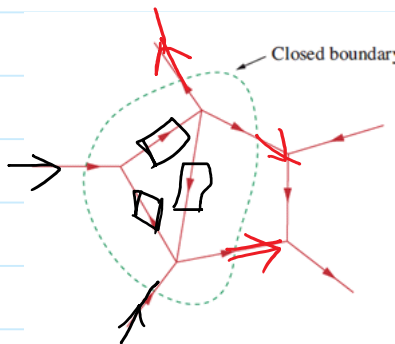
Kirchhoff's Laws - KCL

$$Q = \int i \cdot dt$$

$$\int i_1 + i_4 + i_3 dt = \int i_2 + i_5 dt$$

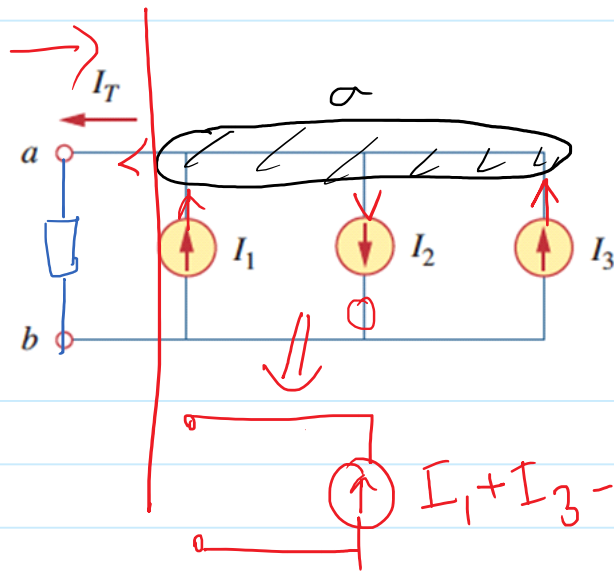


$$\sum_{n=1}^N i_n = 0$$



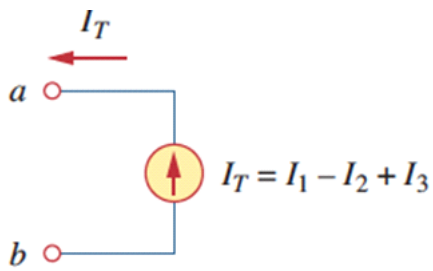
KCL

Kirchhoff's Laws - KCL



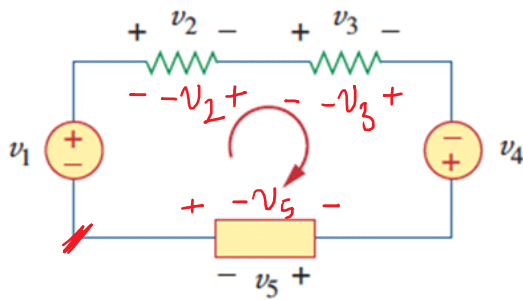
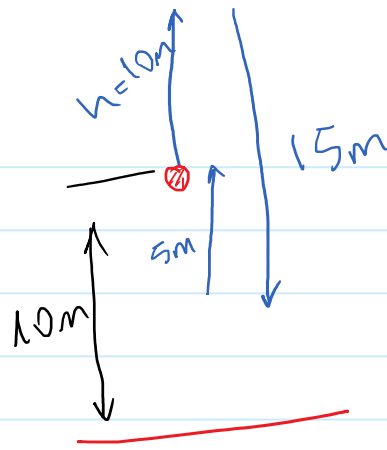
$$I_1 + I_3 = I_T + I_2$$

$$I_T = I_1 + I_3 - I_2$$



Kirchhoff's Laws - KVL

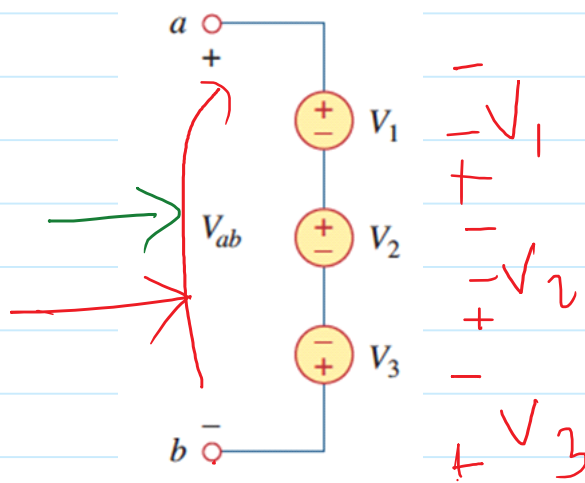
$$\sum_{m=1}^M v_m = 0$$



$$v_4 + v_1 = v_2 + v_3 + v_5$$

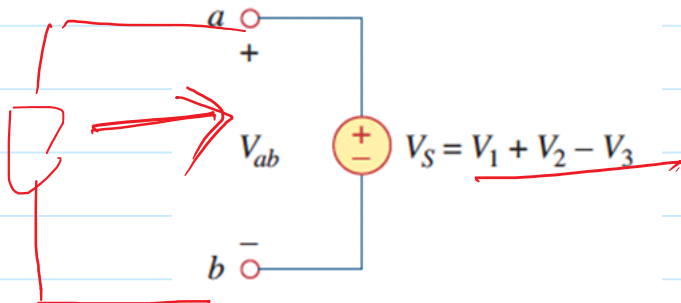
$$0 = v_1 + (-v_2) + (-v_3) + v_4 + (-v_5)$$

Kirchhoff's Laws - KVL



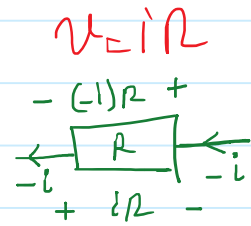
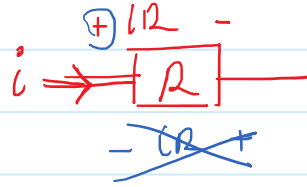
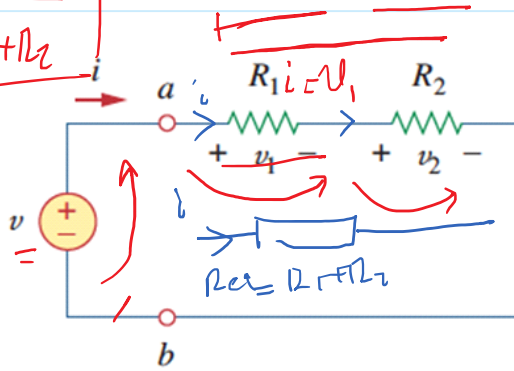
$$V_{ab} + (-V_1) + (-V_2) + V_3 = 0$$

$$V_{ab} = V_1 + V_2 - V_3$$

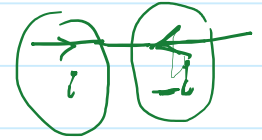


Series Resistors and Voltage Division

$$V = v \frac{R_1}{R_1 + R_2}$$



$$R_{eq} = R_1 + R_2$$



$$v + (-v_1) + (-v_2) = 0$$

$$v = v_1 + v_2$$

$$v = iR_1 + iR_2 = i(R_1 + R_2)$$

$$i = \frac{v}{R_1 + R_2}$$

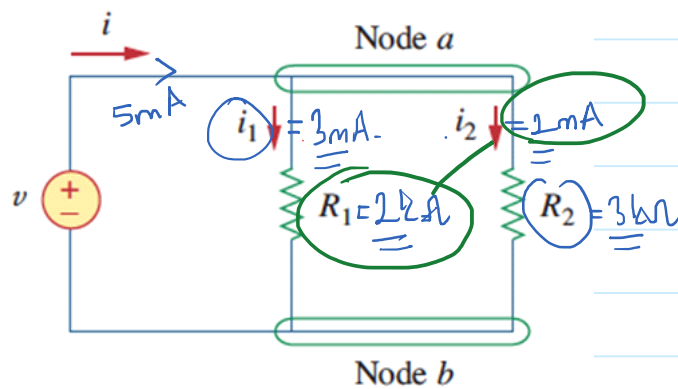
$$v_1 = \frac{R_1}{R_1 + R_2} v, \quad v_2 = \frac{R_2}{R_1 + R_2} v$$

$$v_1 = R_1 i = R_1 \frac{v}{R_1 + R_2}$$

$$v_1 = v \cdot \frac{R_1}{R_1 + R_2}$$

$$R_{eq} = R_1 + R_2 + \dots + R_N = \sum_{n=1}^N R_n$$

Parallel Resistors and Current Division

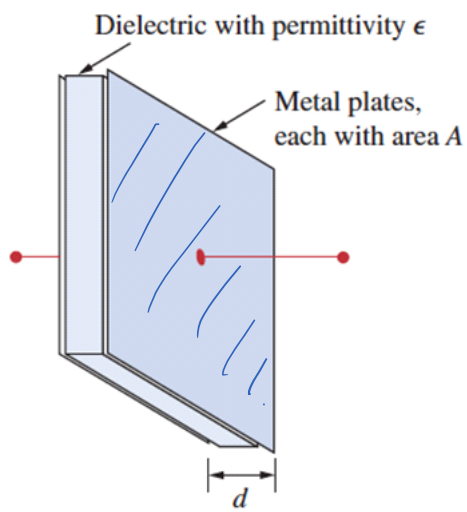


$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}$$

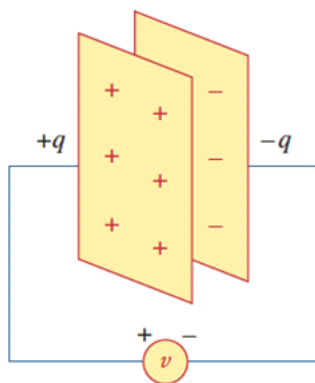
$$i_1 = \frac{R_2 i}{R_1 + R_2}, \quad i_2 = \frac{R_1 i}{R_1 + R_2}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

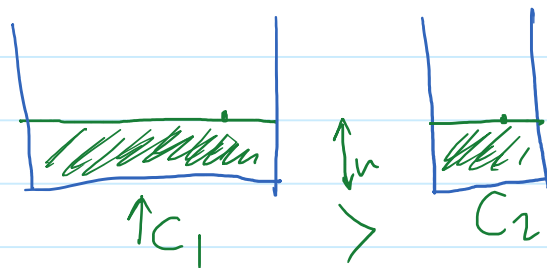
Capacitance



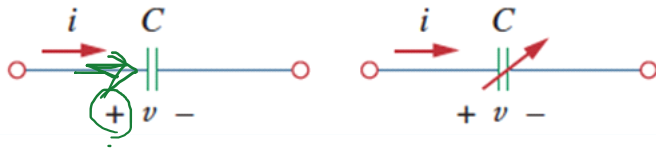
$$C = \frac{\epsilon A}{d}$$



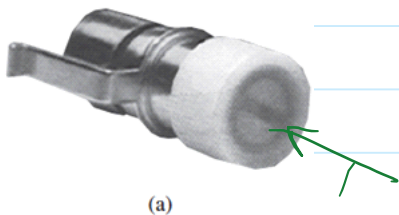
$$q = C_v$$



Types of Capacitors



$$i = C \frac{dv}{dt}$$



(a)



(b)

Capacitors

$$v = iR$$

$$i = \frac{1}{R} \frac{dv}{dt}$$

$$q = Cv$$

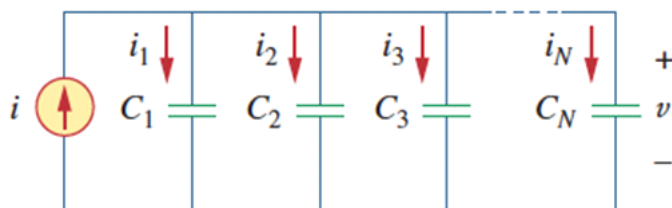
$$\frac{dq}{dt} = C \frac{dv}{dt}$$

$$i = C \frac{dv}{dt}$$

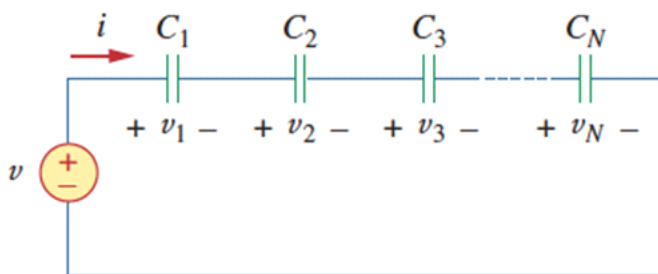
$$w = \int p dt = v \cdot C \cdot dv$$

$$w = \frac{1}{2} C v^2$$

$$w = \int_{-\infty}^t p(\tau) d\tau = C \int_{-\infty}^t v \frac{dv}{d\tau} d\tau = C \int_{v(-\infty)}^{v(t)} v dv = \frac{1}{2} C v^2 \Big|_{v(-\infty)}^{v(t)}$$

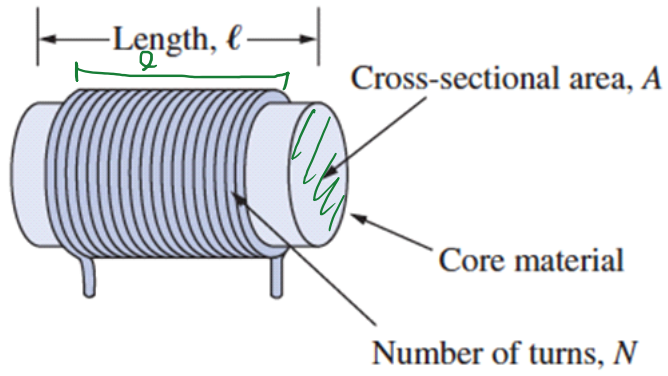


$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_N$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}$$

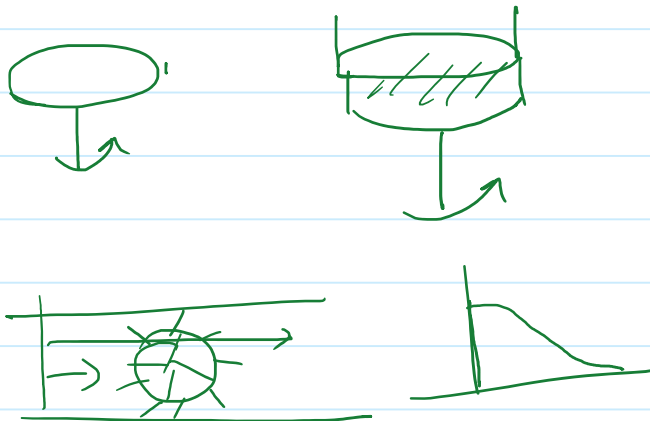
Inductance



$$L = \frac{N^2 \mu A}{\ell}$$

$$v = L \frac{di}{dt}$$

$$i = C \frac{dv}{dt}$$



Types of Inductors



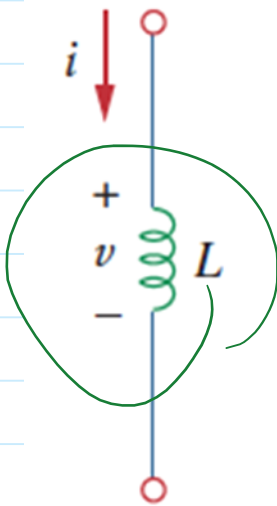
(a)



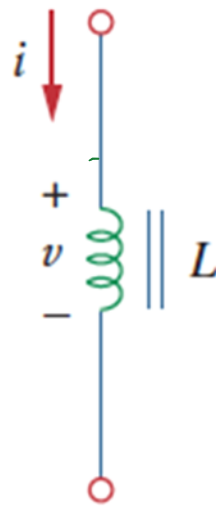
(b)



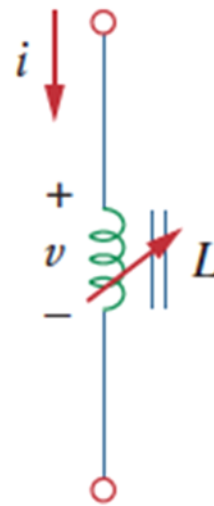
(c)



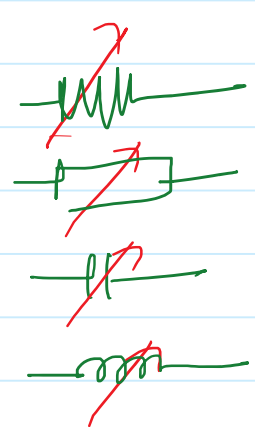
air-core



iron-core



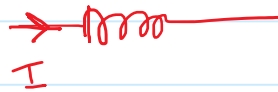
variable iron-core



Inductor

$$V = Ri$$

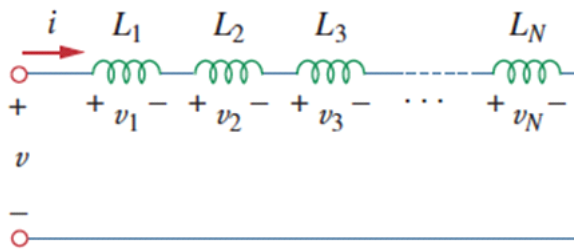
$$v = L \frac{di}{dt}$$



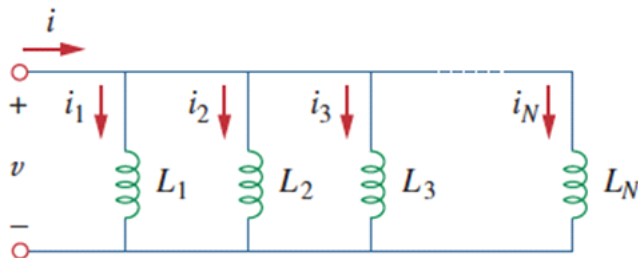
$$\frac{1}{2} I^2 L$$

$$w = \int_{-\infty}^t p(\tau) d\tau = L \int_{-\infty}^t \frac{di}{d\tau} i d\tau = L \int_{-\infty}^t i di = \frac{1}{2} Li^2(t) - \frac{1}{2} Li^2(-\infty)$$

$$w = \frac{1}{2} Li^2$$



$$L_{eq} = L_1 + L_2 + L_3 + \dots + L_N$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_N}$$

Summary

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
$v-i$:	$v = iR$	$v = \frac{1}{C} \int_{t_0}^t i(\tau) d\tau + v(t_0)$	$v = L \frac{di}{dt}$
$i-v$:	$i = v/R$	$i = C \frac{dv}{dt}$	$i = \frac{1}{L} \int_{t_0}^t v(\tau) d\tau + i(t_0)$
p or w :	$p = i^2 R = \frac{v^2}{R}$	$w = \frac{1}{2} C v^2$	$w = \frac{1}{2} L i^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i