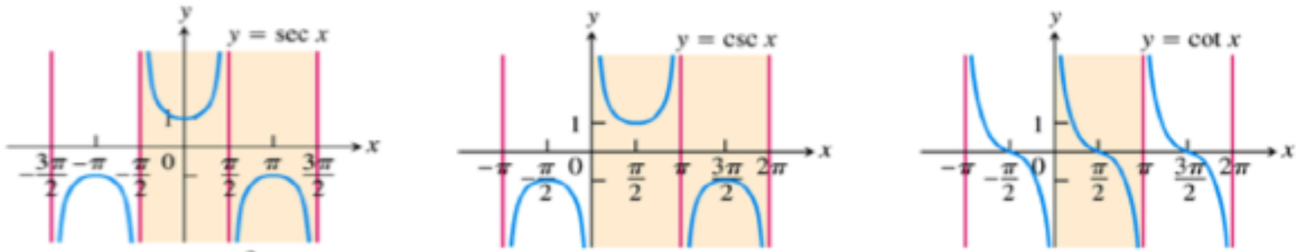
7.6 Inverse Trigonometric Fns.

Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π
(a)

Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Period: 2π
(b)

Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $-\infty < y < \infty$
Period: π
(c)



Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
Range: $y \le -1$ or $y \ge 1$
Period: 2π

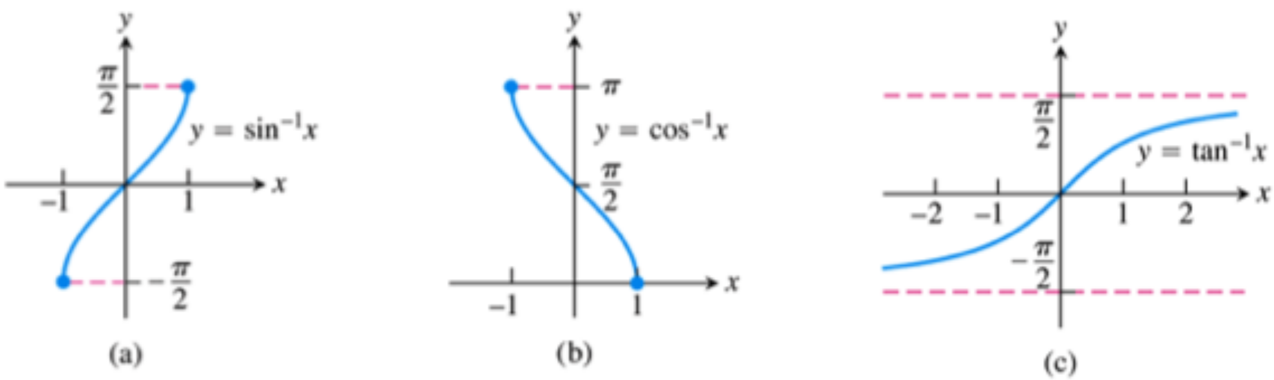
Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $y \le -1$ or $y \ge 1$
Period: 2π

Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$
Range: $-\infty < y < \infty$
Period: π

Domain: $-1 \le x \le 1$
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

Domain: $-1 \le x \le 1$
Range: $0 \le y \le \pi$

Domain: $-\infty < x < \infty$
Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$



Domain: $x \le -1$ or $x \ge 1$
Range: $0 \le y \le \pi, y \neq \frac{\pi}{2}$

Domain: $x \le -1$ or $x \ge 1$
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \neq 0$

Domain: $-\infty < x < \infty$
Range: $0 < y < \pi$

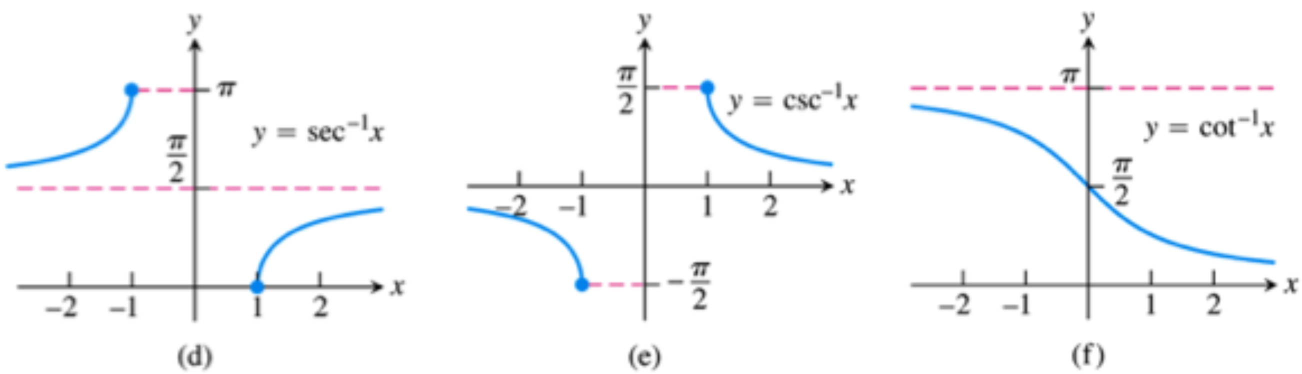


FIGURE 7.15 Graphs of the six basic inverse trigonometric functions.

Ex Simplify $\sin(\underbrace{\tan^{-1}x}_{y})$

$$y = \tan^{-1}x$$

$$x = \tan y$$

$$\sin y = \frac{x}{\sqrt{1+x^2}}$$

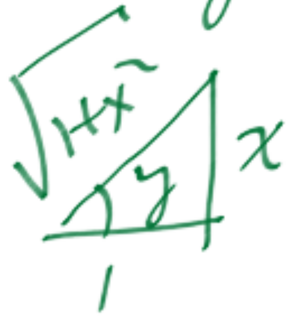
$$\sin(\tan^{-1}x) = \frac{x}{\sqrt{1+x^2}}$$

Ex

Simplify $\cos(2\underbrace{\tan^{-1}x}_{y})$

$y = \tan^{-1}x$

$x = \tan y$



$\cos y = \dfrac{1}{\sqrt{1+x^2}}$

$$\cos(2y) = \cos^2 y - \sin^2 y$$
$$= \cos^2 y - (1 - \cos^2 y)$$
$$= 2\cos^2 y - 1$$

$$\cos(2\tan^{-1}x) = \frac{2}{1+x^2} - 1$$

Ex

$$\cos(\underbrace{\mathrm{Arctan}(-\tfrac{1}{2})}_{y})$$

$$y = \mathrm{Arctan}(-\tfrac{1}{2})$$

$$\tan y = \frac{-1}{2}$$

$$\cos y = +\frac{2}{\sqrt{5}}$$

Ex $\sin(\underbrace{\mathrm{Arctan}(-1/2)}_{y})$

$$\tan y = -\frac{1}{2}$$

$$\sin y = -1/\sqrt{5}$$

$\operatorname{ArcSin} x + \operatorname{ArcCos} x = \pi/2$ for $x > 0$

Derivative of Inverse Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\underbrace{\operatorname{ArcSin} x}_{y} = \,?$$

$y = \operatorname{ArcSin} x$

$x = \sin y$

$\dfrac{dx}{dy} = \cos y$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$$

$\frac{d}{dx} \underbrace{\text{Arctan}\,x}_{y}$

$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

$y = \tan^{-1} x$

$x = \tan y$

$\frac{dx}{dy} = 1 + \underbrace{\tan^2 y}_{x^2}$

$\frac{dx}{dy} = 1 + x^2$

$\frac{d}{dx} \underbrace{\text{ArcSec}\,x}_{y} = \frac{dy}{dx}$;

$y = \text{ArcSec}\,x$

$x = \text{Sec}\,y = (\cos y)^{-1}$

$\frac{dx}{dy} = \frac{\sin y}{\cos^2 y} = \sin y \cdot \sec^2 y$

$= \sqrt{1 - \cos^2 y}\; \sec^2 y$

$\frac{1}{\sec^2 x}$

$$\frac{dx}{dy} = \sqrt{1-\frac{1}{x^2}}\, x^2 = \frac{\sqrt{x^2-1}}{\cancel{x}} \cdot x^{\cancel{2}}$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d\,\mathrm{Arcsec}x}{dx} = \frac{1}{x\sqrt{x^2-1}} \;//$$

$$\Rightarrow \int \frac{1}{x^2+a^2}\,dx = \mathrm{Arctan}\frac{x}{a} + C$$

$$\int \frac{dx}{x\sqrt{x^2-1}} = \mathrm{Arcsec}\,x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \mathrm{ArcSin}\,x + C$$

Ex

$$\frac{d}{dx}\underbrace{\sin^{-1}\left(\cos\frac{1}{x}\right)}_{y} = \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$\cos\frac{1}{x} = \sin y$$

$$\frac{d}{dx}\left(\cos\frac{1}{x}\right) = \frac{d}{dx}\left(\sin y\right)$$

$$-\sin\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right) = \cos y\cdot\frac{dy}{dx}$$

$$\frac{dx}{dy} = \cos y\cdot\frac{x^2}{\sin\frac{1}{x}}$$

$$\frac{d}{dx}\sin^{-1}\left(\cos\frac{1}{x}\right) = \frac{\sin\frac{1}{x}}{x^2\cos y} \qquad \cos y = \sqrt{1-\sin^2 y}$$

$$\cos\frac{1}{x}$$

$$= 1/x^2$$

Find inv. fn. of $f(x)$, if it exists

Ex $f(x) = \ln(x+1) - \ln(x-1)$

Ex $f(x) = \ln(e^{2x} - 1)$

$y = \ln(e^{2x} - 1)$

$e^y = e^{2x} - 1$

$e^y + 1 = e^{2x}$

$\ln(e^y + 1) = 2x \underbrace{\ln e}_{1}$

$\frac{1}{2}\ln(e^y + 1) = x$

$f^{-1}(x) = \frac{1}{2}\ln(e^x + 1)$

Ex $\displaystyle\int \frac{4^x - 3^x}{2^x}\,dx =$

$$\int \left[2^x - \left(\frac{3}{2}\right)^2\right] dx =$$

$$\frac{1}{\ln 2}\,2^x - \frac{1}{\ln\frac{3}{2}}\left(\frac{3}{2}\right)^2 + C_1$$

Ex $\displaystyle\int 3^{2x} \cdot 5^x\,dx =$

$$\int (9 \cdot 5)^x\,dx = \int 45^x\,dx$$

$$= \frac{1}{\ln 45}\,45^x + C$$

Ex $\displaystyle\int \frac{e^{-x}}{1+e^{-2x}}\,dx =$ $u = e^{-x}$

$du = -e^{-x}\,dx$

$$-\int \frac{du}{1+u^2} = -\operatorname{Arctan} e^{-x} + C$$

Ex $\displaystyle\int \frac{1}{x(1+\ln^2 x)}\,dx =$ $u = \ln x$

$du = \frac{dx}{x}$

$$\int \frac{du}{1+u^2} = \operatorname{Arctan} \ln x + C$$

HW Ex $\displaystyle\int_1^2 \frac{dx}{x^2+2} = ?$ $\displaystyle\int \frac{x^2\,dx}{1+x^6} = ?$ $u = x^3$

Differentiate

$$y = \frac{\sqrt{9-x^2}}{x} + \operatorname{ArcSin} \frac{x}{3}$$