Math No:

Full Name : KEY

Math 104 2nd Midterm Exam
(2 December 2016, 16:30-17:30)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
6 pts	6 pts	6 pts	6 pts	24 pts

Q1. Evaluate the following integrals.

a) $\int \sqrt{\sin x}\cos^5 x dx = \int \sqrt{\sin x}\,(\cos^2 x)^2 \cos x\, dx = \int \sqrt{\sin x}\,(1-\sin^2 x)^2 \cos x\, dx =$

$u = \sin x, \quad du = \cos x\, dx$

$\int u^{1/2}(1-u^2)^2 du = \int u^{1/2}(1-2u^2+u^4)\, du = \int (u^{1/2} - 2u^{5/2} + u^{9/2})\, du =$

$\frac{2}{3}u^{3/2} - 2\cdot\frac{2}{7}u^{7/2} + \frac{2}{11}u^{11/2} + C =$

$\frac{2}{3}\sin^{3/2}x - \frac{4}{7}\sin^{7/2}x + \frac{2}{11}\sin^{11/2}x + C$

b) $\int \cos^4 x\, dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\, dx =$

$\frac{1}{4}\left[x + \sin 2x + \int \left(\frac{1+\cos 4x}{2}\right) dx\right] = \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}\int (1+\cos 4x)\, dx =$

$\frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}\left[x + \frac{1}{4}\sin 4x\right] + C =$

$\frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

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Q2. Evaluate the following integrals.

(a) $\int x\ln(4+x^2)dx = \frac{1}{2}\int \ln t \, dt$

$t = 4+x^2, \quad \frac{1}{2}dt = x\,dx$

$\frac{1}{2}\int \ln t \, dt = \frac{1}{2}[uv - \int v\,du] = \frac{1}{2}[t\ln t - \int \cancel{t}\frac{dt}{\cancel{t}}]$ (t)

$= \frac{1}{2}[(4+x^2)\ln(4+x^2) - (4+x^2)] + C$

$u = \ln t, \quad dv = dt$

$du = \frac{dt}{t}, \quad v = t$

$= \frac{4+x^2}{2}[\ln(4+x^2) - \underbrace{1}_{\ln e}] + C$

$= \left(\frac{4+x^2}{2}\right)\ln\left(\frac{4+x^2}{e}\right) + C$

$= \ln\left(\frac{4+x^2}{e}\right)^{\frac{4+x^2}{2}} + C$

(b) $\int \cos x \ln(\sin x)dx = uv - \int v\,du =$

$u = \ln(\sin x)$ | $\int \cos x\,dx = \int dv$

$du = \frac{\cos x}{\sin x}dx$ | $\sin x = v$

$\sin x \ln(\sin x) - \int \cancel{\sin x}\frac{\cos x}{\cancel{\sin x}}dx =$

$\sin x \ln(\sin x) - \int \cos x\,dx =$

$\sin x \ln(\sin x) - \sin x + C =$

$\sin x\,[\ln(\sin x) - \ln e] + C =$

$\sin x \cdot \ln\left(\frac{\sin x}{e}\right) + C = \ln\left(\frac{\sin x}{e}\right)^{\sin x} + C$

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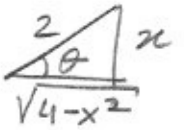
Q3. Evaluate the following integrals

(a) $\int \frac{dx}{x^2\sqrt{4-x^2}} =$

$\sin\theta = \frac{x}{2}$



$dx = 2\cos\theta\, d\theta$

$$\int \frac{2\cos\theta\, d\theta}{4\sin^2\theta\sqrt{2^2(1-\sin^2\theta)}} =$$

$$\int \frac{\cancel{2}\cancel{\cos\theta}\, d\theta}{4\sin^2\theta \cdot \cancel{2}\cancel{\cos\theta}} = \frac{1}{4}\int \csc^2\theta\, d\theta = \frac{1}{4}\int d(-\cot\theta) =$$

$$-\frac{1}{4}\cot\theta + c = -\frac{\sqrt{4-x^2}}{4x} + C$$

(b) $\int \frac{x-19}{x^2-3x-10}dx$

$$\frac{x-19}{x^2-3x-10} = \frac{A}{(x+2)} + \frac{B}{x-5}$$

$2 \quad -5$

$$x-19 = A(x-5) + B(x+2)$$

$x=5: \ -14 = 7B \Rightarrow \boxed{B=-2}$

$x=-2: \ -21 = -7A \Rightarrow \boxed{A=3}$

$$\int \frac{x-19}{x^2-3x-10}dx = 3\int \frac{dx}{x+2} - 2\int \frac{dx}{x-5}$$

$$= 3\ln(x+2) - 2\ln(x-5) + C$$

$$= \ln\frac{(x+2)^3}{(x-5)^2} + C$$

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Q4. Determine the following limits

a) $\lim_{x\to 0^+}\left(1+\frac{1}{x}\right)^x$.

$y=(1+1/x)^x$

$\ln y = x\ln(1+1/x)$

$\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\frac{\ln(1+1/x)}{1/x}=\lim_{x\to 0^+}\frac{\frac{\cancel{-1/x^2}}{1+1/x}}{\cancel{-1/x^2}}$ L'Hospt. Rule

$\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\frac{1}{1+1/x}=0$

$\lim_{x\to 0^+}y=e^0=1$

$\therefore \lim_{x\to 0^+}(1+1/x)^x=1$

b) $\lim_{x\to\infty}(1+2x)^{\frac{1}{2\ln x}}$

$y=(1+2x)^{\frac{1}{2\ln x}}$

$\ln y=\frac{1}{2\ln x}\ln(1+2x)$

$\lim_{x\to\infty}\ln y=\lim_{x\to\infty}\frac{\ln(1+2x)}{2\ln x}=\frac{1}{2}\lim_{x\to\infty}\frac{\frac{2}{1+2x}}{\frac{1}{x}}$ (L'Hosp. R!)

$\lim_{x\to\infty}\ln y=\frac{\cancel{2}}{\cancel{2}}\lim_{x\to\infty}\frac{x}{1+2x}=\lim_{x\to\infty}\frac{1}{2}=\frac{1}{2}$ (L'Hosp.R) again

$\lim_{x\to\infty}y=e^{1/2}$

$\lim_{x\to\infty}(1+2x)^{\frac{1}{2\ln x}}=e^{1/2}$