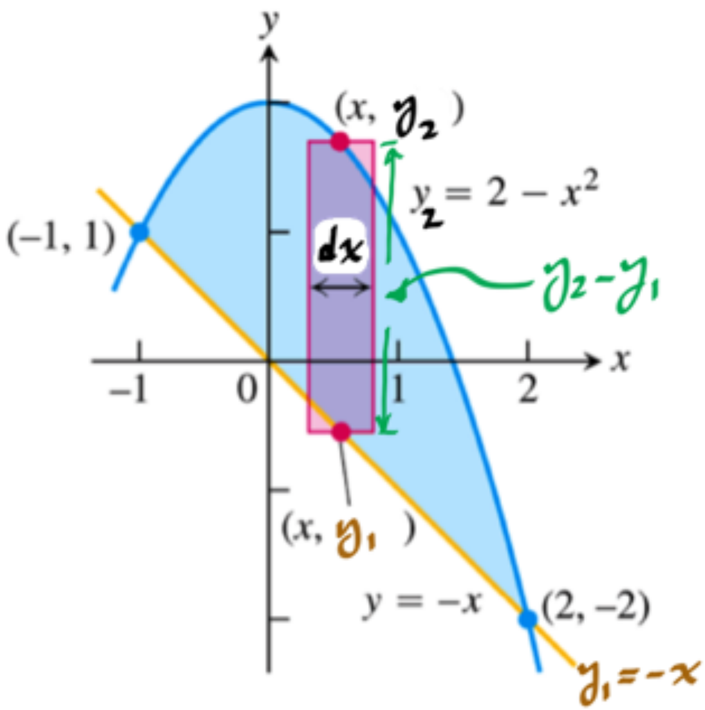
Area between curves

Ex

Find the area of the region between the parabola $y = 2 - x^2$ and the line $y = -x$



$$A = \int_a^b f(x)\,dx$$

$$A = \int_{-1}^{2} (y_2 - y_1)\,dx = \int_{-1}^{2} \left[(2 - x^2) - (-x)\right] dx$$

$$= \int_{-1}^{2} (2 - x^2 + x)\,dx$$

$$= \left| 2x - \frac{x^3}{3} + \frac{x^2}{2} \right|_{-1}^{2}$$

$$= 2.2 - \frac{2^3}{3} + \frac{2^2}{2} - \left(2.(-1) - \frac{(-1)^3}{3} + \frac{(-1)^2}{2}\right)$$

$$= 9/2$$

$y_1 = -x$, $y_2 = 2 - x^2$

$y_1 = y_2$

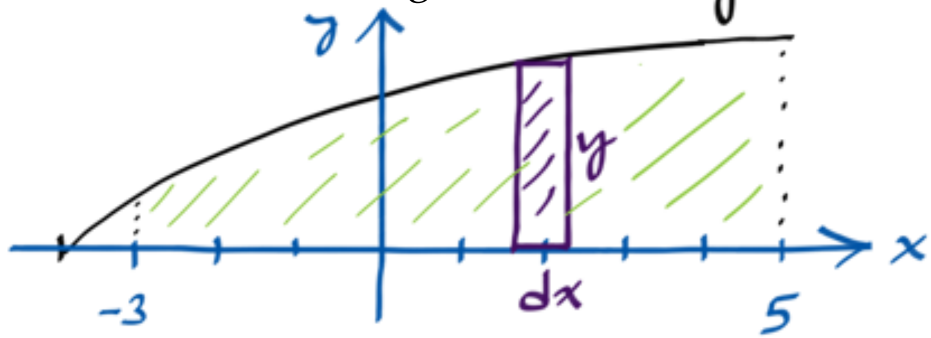
$-x = 2 - x^2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0 \Rightarrow x = -1, 2$

$(-1, 1)$, $(2, 2)$

Find the are of the region between $y=\sqrt{x+4}$ and the x-axis, $-3 \leq x \leq 5$



$$A = \int_{-3}^{5} y\,dx = \int_{-3}^{5} \sqrt{x+4}\,dx$$

$u(x) = x+4$

$du = dx$

$u(-3) = -3+4 = 1$

$u(5) = 5+4 = 9$

$$= \int_{u(-3)}^{u(5)} u^{1/2}\,du$$

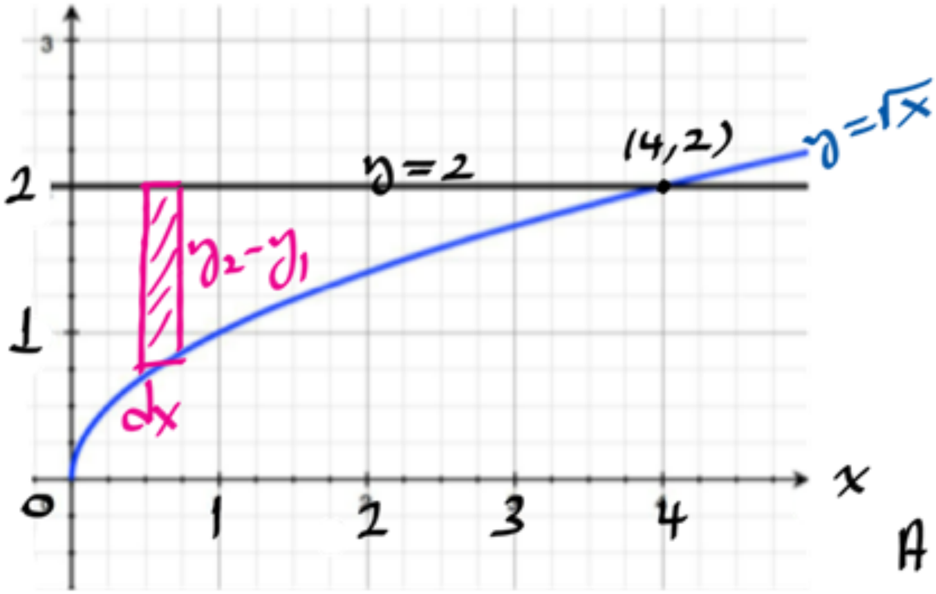
$$= \frac{2}{3} u^{3/2} \Big|_{u(-3)}^{u(5)}$$

$$= \frac{2}{3}\left[u(5)^{3/2} - u(-3)^{3/2} \right]$$

$$= \frac{2}{3}\left[(9)^{3/2} - (1)^{3/2} \right]$$

$$= 52/3 \quad \text{unit}^2$$

Find the are of the region between $y=\sqrt{x}$ and the y-axis, $0 \le y \le 2$



$y_1=\sqrt{x}\,,\quad y_2=2$

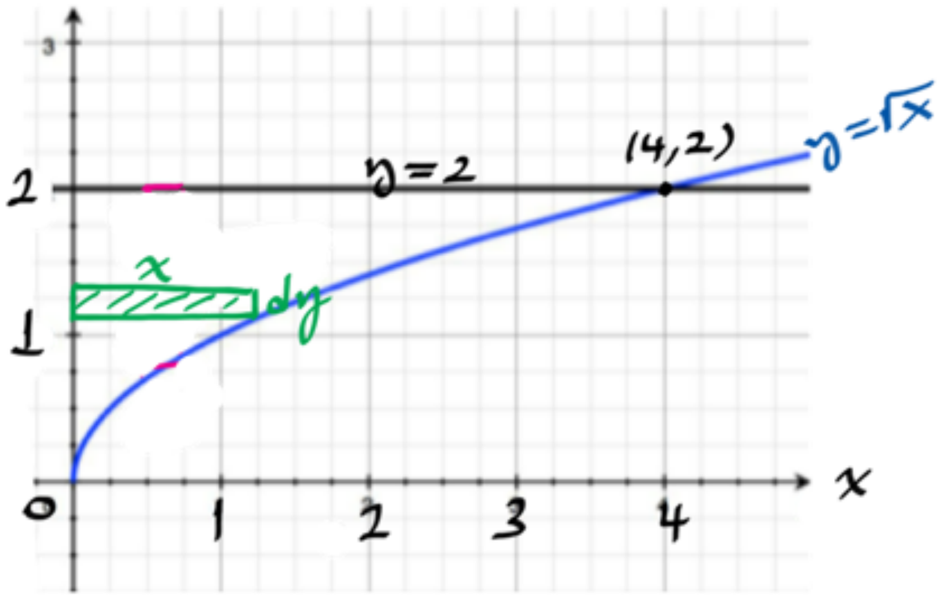
$y_1=y_2$

$(\sqrt{x})^2=(2)^2 \Rightarrow x=4 \quad \Rightarrow (2,4)$

$$A=\int_0^4 (y_2-y_1)\,dx=\int_0^4 (2-\sqrt{x})\,dx$$

$$=2x-\frac{2}{3}x^{3/2}\Big|_0^4=\frac{8}{3}$$

methot-II :

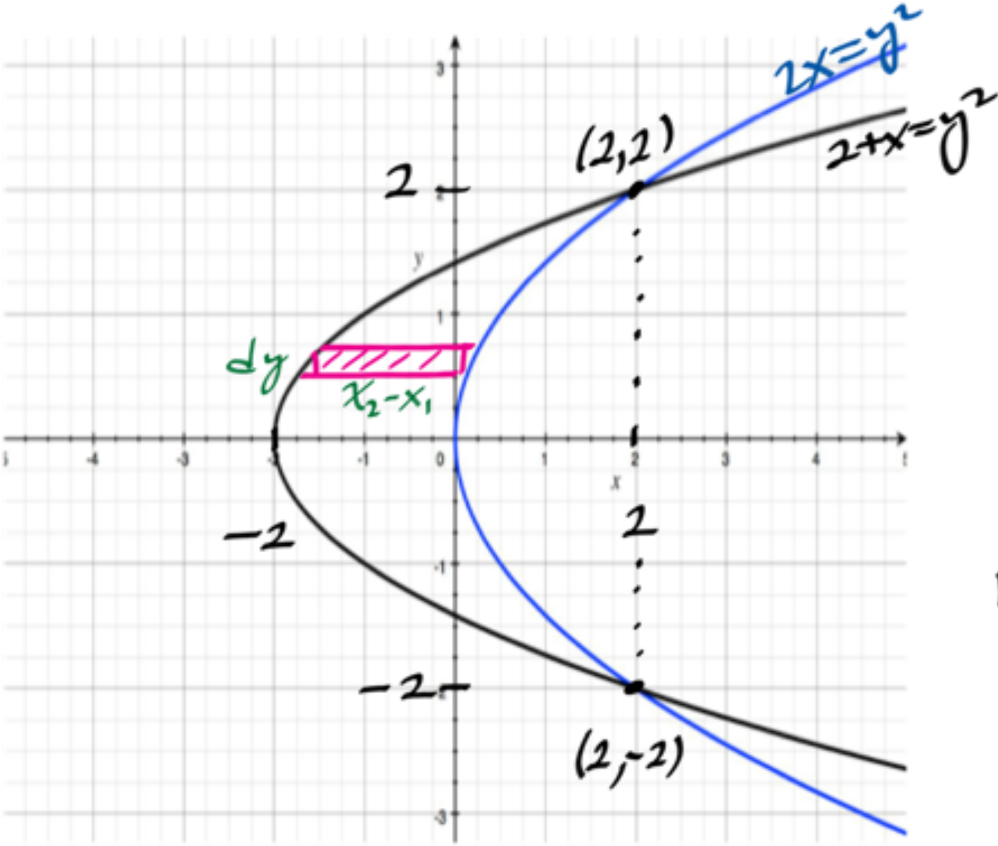


$$A=\int_0^2 x\,dy=\int_0^2 y^2\,dy \quad ; \; y=\sqrt{x},\; y^2=x$$

$$=\frac{1}{3}y^3\Big|_0^2$$

$$=\frac{1}{3}2^3-0=\frac{8}{3}$$

Find the are of the region between the parabolas $2x = y^2$ and $x + 2 = y^2$



$$2x = x + 2$$

$$x = 2 \Rightarrow (2,2)$$

$$(2,-2)$$

$$y^2 = 4$$

$$y = \pm 2$$

$$A = 2\int_0^2 (x_2 - x_1)\,dy = 2\int_0^2 \left(\tfrac{1}{2}y^2 - (y^2 - 2)\right)dy$$

$$= 2\int_0^2 \left(-\tfrac{1}{2}y^2 + 2\right)dy$$

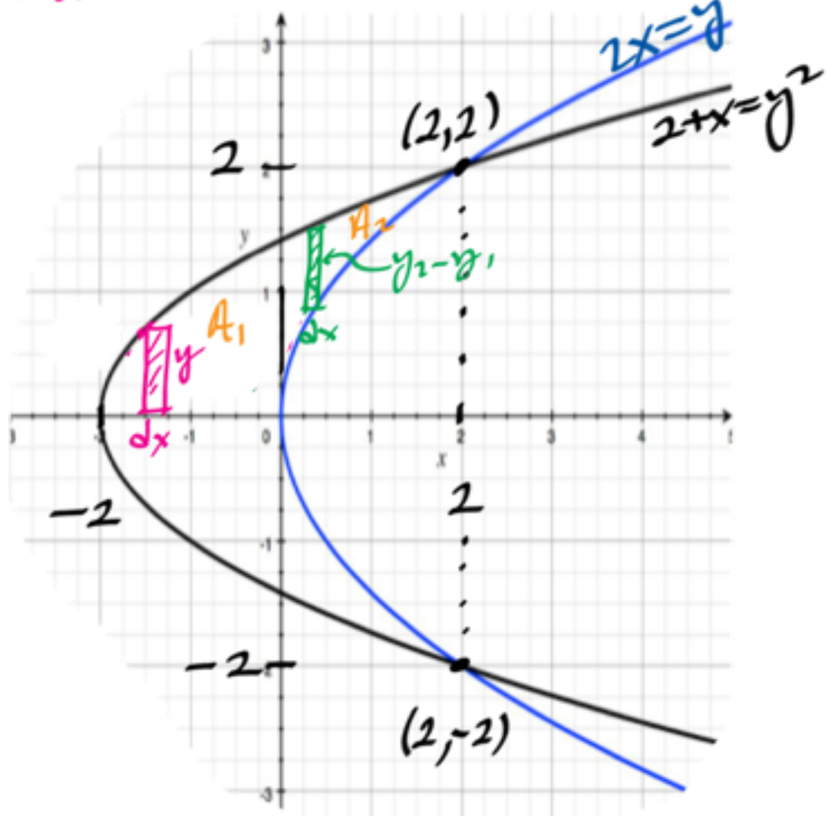
$$= 2\left(-\frac{1}{2}\frac{y^3}{3} + 2y\right)\Big|_0^2$$

$$= 2\left(2y - y^3/6\right)\Big|_0^2$$

$$= 2\left(2.2 - 2^3/6\right) - 0$$

$$= 16/3$$

Method-2:



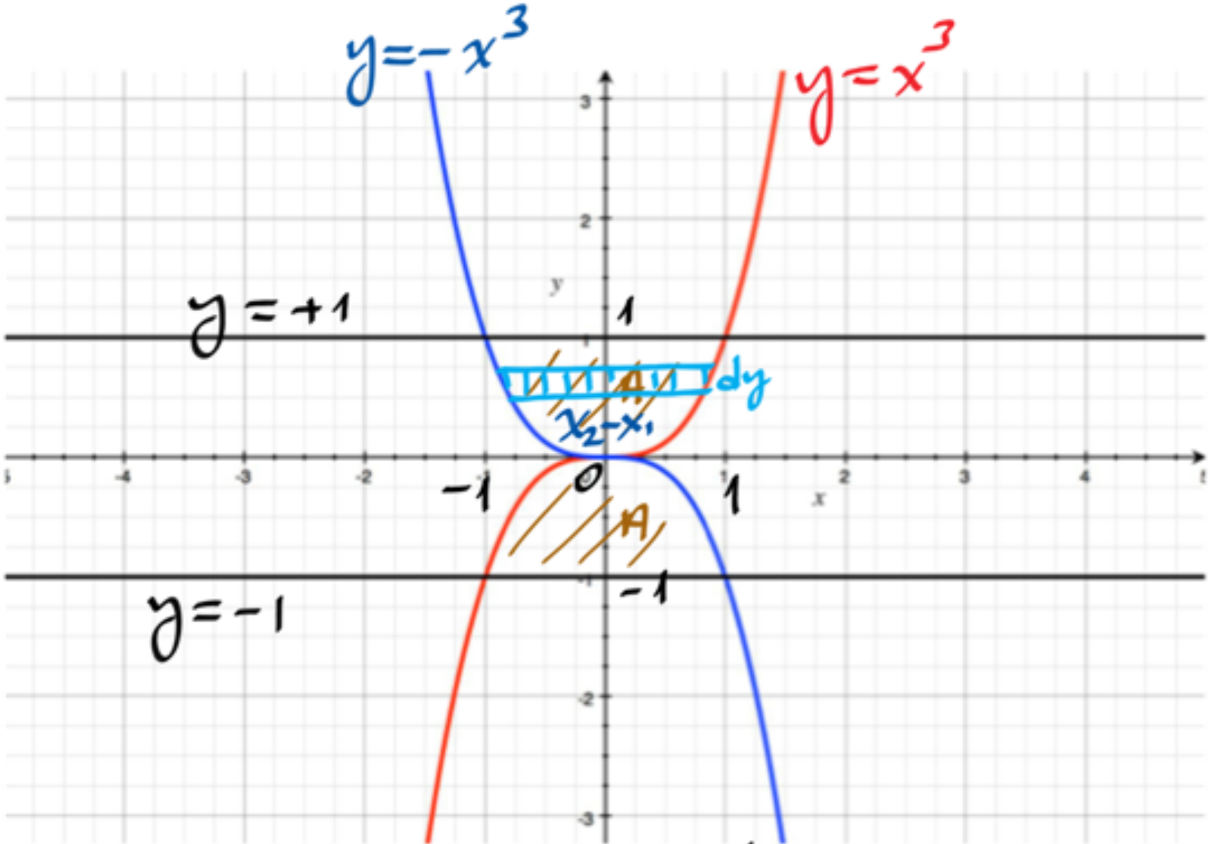
$$y_2^2 = 2+x \Rightarrow y_1 = \sqrt{2+x}$$

$$y_1^2 = 2x \Rightarrow y_2 = \sqrt{2x}$$

$$A = 2(A_1 + A_2), \quad A_1 = \int_{-2}^{0} y_2\,dx \;; \quad A_2 = \int_0^2 (y_2 - y_1)\,dx$$

$$A_1 = \int_{-2}^{0} \sqrt{x+2}\,dx \;; \quad A_2 = \int_0^2 (\sqrt{x+2} - \sqrt{2x}\,)\,dx$$

$$A = 2A_1 + 2A_2 = 2\int_{-2}^{0} \sqrt{x+2}\,dx + 2\int_0^2 (\sqrt{x+2} - \sqrt{2x})\,dx = 16/3$$



Find the area of the dashed region.

$$A_T = 2A = \int_0^1 (x_2 - x_1)\,dy = \int_0^1 \left(\sqrt[3]{y} - (-\sqrt[3]{y})\right)dy = 4\int_0^1 \sqrt[3]{y}\,dy$$

$$= \cancel{4}\cdot\frac{3y^{4/3}}{\cancel{4}}\Big|_0^1$$

$$= 3$$