Full Name : KEY
Student ID:

Istanbul Şehir University
Math 104, Midterm 3
(20 November 2014, Time: 11:30-12:45)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 5 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	Q5	TOTAL
20 pts	20 pts	20 pts	20 pts	20 pts	100 pts

Q1. (a) Determine the sum of the following series

$$\sum_{n=0}^{\infty} e^{-3n} = \sum_{n=0}^{\infty}\left(\frac{1}{e^3}\right)^n = \frac{1}{1-1/e^3} = \frac{e^3}{e^3-1}$$

, a geometric series $\frac{1}{e^3}<1$

(b) Apply the Integral Test to determine if the following series converges or diverges

$$\sum_{n=2}^{\infty} \frac{\ln n^2}{2n}$$

$$\sum \frac{2\ln n}{2n} = \sum \frac{\ln n}{n}$$

$$\int_2^{\infty} \frac{\overset{u}{\ln x}\,\overbrace{dx}^{du}}{x} = \int u\,du = \frac{1}{2}u^2 = \frac{1}{2}(\ln x)^2\Big|_2^{\infty}$$

$= \infty$ div. by int. test

Full Name :
Student ID:

Q2. (a) Use the Limit Comparison Test to determine if the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{n-1}{n^3+1}$$

compare with $\Sigma \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{\frac{n-1}{n^3+1}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^3-n^2}{n^3+1} = \lim_{n\to\infty} \frac{1-1/n}{1+1/n^3} = 1$$

The series is comparable with $\Sigma \frac{1}{n^2}$, which is a p-series with $p=2$.

Therefore the series converges by the comparison test.

(b) Determine if the following series converges or diverges

$$\sum_{n=1}^{\infty} \left(1-\frac{3}{n}\right)^n$$

$$y = (1-3/x)^x$$

$$\ln y = x \ln(1-3/x)$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(1-3/x)}{1/x} = \lim_{x\to\infty} \frac{\frac{+3/x^2}{1-3/x}}{-1/x^2}$$

$$= -3 \lim_{x\to\infty} \frac{1/x^2}{1-3/x} \cdot \frac{1}{1/x^2} = -3$$

$$\lim_{x\to\infty} \ln y = -3$$

$$\lim_{x\to\infty} y = e^{-3}$$

$$\lim_{x\to\infty} (1-3/x)^x = e^{-3} = 1/e^3$$

The series converges.

Full Name : KEY
Student ID:

Q3. (a) Find the following series' interval of convergence. Do not test the end values in the series.

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{2^n(2n)!}x^n$$

$$\rho = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\dfrac{[(n+1)!]^2}{2^{n+1}[(2n+2)!]}x^{n+1}}{\dfrac{(n!)^2}{2^n(2n)!}x^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(n+1)^{\cancel{2}}\cancel{(n!)^2}\,\cancel{x^n}\cdot x}{2\cdot\cancel{2^n}\cdot 2\cancel{(n+1)}(2n+1)\cancel{[(2n)!]}}\cdot\frac{\cancel{2^n}\cancel{(2n)!}}{\cancel{(n!)^2}\,\cancel{x^n}}\right| = \lim_{n\to\infty}\left|\frac{(n+1)x}{4(2n+1)}\right|$$

$$= \frac{|x|}{4}\lim_{n\to\infty}\frac{n+1}{2n+1} = \frac{|x|}{4}\lim_{n\to\infty}\frac{1}{2} = \frac{|x|}{8}$$

$$\rho = \frac{|x|}{8} < 1 \longrightarrow \left.\begin{array}{l}\frac{x}{8}<1\\ -\frac{x}{8}<1\end{array}\right\} \quad \underline{-8 < x < 8}$$

(b) For what values of x does the following series converge?

$$\sum_{n=1}^{\infty} n!(x-1)^n$$

$$\rho = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!\,(x-1)^{n+1}}{n!\,(x-1)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(n+1)\cancel{n!}\,(x-1)\cancel{(x-1)^n}}{\cancel{n!}\,\cancel{(x-1)^n}}\right|$$

$$= |x-1|\lim_{n\to\infty}(n+1)$$

Only if $x-1=0 \Rightarrow x=1$, then $\rho=0$, the series converges.

Full Name :
Student ID:

Q4. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ if $\quad f(x,y) = x^2 - xy + y^2$

$$\frac{\partial f}{\partial x} = 2x - y$$

$$\frac{\partial f}{\partial y} = -x + 2y$$

Q5. Evaluate $\dfrac{dw}{dt}$ if $w = z - \sin xy$ *and* $\quad x = t,\ y = \ln t,\ z = e^{t-1}$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$= -y\cos xy - x\cos xy \cdot \frac{1}{t} + e^{t-1}$$

$$= -\ln t \cdot \cos(t\ln t) - \frac{\cancel{t}}{\cancel{t}}\cos(t\ln t) + e^{t-1}$$

$$= -\ln t\cos(t\ln t) - \cos(t\ln t) + e^{t-1}$$

$$= e^{t-1} - \cos(t\ln t)(1 + \ln t)$$

$$\frac{\partial w}{\partial x} = -\cos xy \cdot y$$

$$\frac{\partial w}{\partial y} = -\cos xy \cdot x$$

$$\frac{\partial w}{\partial z} = 1$$

$$\frac{dx}{dt} = 1$$

$$\frac{dy}{dt} = 1/t$$

$$\frac{dz}{dt} = e^{t-1}$$