Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 5.3 Homework

1. Let $A = PDP^{-1}$ and P and D as shown below. Compute A^4.

$$P = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 536 & -260 \\ 910 & -439 \end{bmatrix}$$

(Simplify your answers.)

2. Use the factorization $A = PDP^{-1}$ to compute A^k, where k represents an arbitrary integer.

$$\begin{bmatrix} a & 4(b-a) \\ 0 & b \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$$

$$A^k = \begin{bmatrix} a^k & 4\left(b^k - a^k\right) \\ 0 & b^k \end{bmatrix}$$

YOU ANSWERED: 1

0

1

3. Use the factorization $A = PDP^{-1}$ to compute A^k, where k represents an arbitrary positive integer.

$$\begin{bmatrix} 5 & -6 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$$

$$A^k = \begin{bmatrix} 4 \cdot 2^k - 3 & 6 - 6 \cdot 2^k \\ 2 \cdot 2^k - 2 & 4 - 3 \cdot 2^k \end{bmatrix}$$

YOU ANSWERED: $\begin{bmatrix} 5^k & (-6)^k \\ 2^k & (-2)^k \end{bmatrix}$

4. Matrix A is factored in the form PDP^{-1}. Use the Diagonalization Theorem to find the eigenvalues of A and a basis for each eigenspace.

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & -1 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{4} & -\frac{3}{8} \\ \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

Select the correct choice below and fill in the answer boxes to complete your choice.

(Use a comma to separate vectors as needed.)

- **A.** There is one distinct eigenvalue, $\lambda =$ ______ . A basis for the corresponding eigenspace is { ______ }.
- **B.** In ascending order, the two distinct eigenvalues are $\lambda_1 =$ 1 and $\lambda_2 =$ 5 . Bases for the corresponding eigenspaces are $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ and $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$, respectively.
- **C.** In ascending order, the three distinct eigenvalues are $\lambda_1 =$ ______ , $\lambda_2 =$ ______ , and $\lambda_3 =$ ______ . Bases for the corresponding eigenspaces are { ______ }, { ______ }, and { ______ }, respectively.

5. Matrix A is factored in the form PDP^{-1}. Use the Diagonalization Theorem to find the eigenvalues of A and a basis for each eigenspace.

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 2 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 4 \\ -1 & 0 & -2 \end{bmatrix}$$

Select the correct choice below and fill in the answer boxes to complete your choice.

(Use a comma to separate vectors as needed.)

- **A.** There is one distinct eigenvalue, $\lambda =$ ______ . A basis for the corresponding eigenspace is { ______ }.
- **B.** In ascending order, the two distinct eigenvalues are $\lambda_1 =$ 2 and $\lambda_2 =$ 3 . Bases for the corresponding eigenspaces are $\left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \right\}$ and $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$, respectively.
- **C.** In ascending order, the three distinct eigenvalues are $\lambda_1 =$ ______ , $\lambda_2 =$ ______ , and $\lambda_3 =$ ______ . Bases for the corresponding eigenspaces are { ______ }, { ______ }, and { ______ }, respectively.

6. Diagonalize the following matrix, if possible.

$$\begin{bmatrix} 8 & 0 \\ 10 & -8 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** For P = $\begin{bmatrix} 8 & 0 \\ 5 & 1 \end{bmatrix}$, D = $\begin{bmatrix} 8 & 0 \\ 0 & -8 \end{bmatrix}$
- **B.** For P = ____, D = $\begin{bmatrix} 8 & 0 \\ 0 & 5 \end{bmatrix}$
- **C.** For P = ____, D = $\begin{bmatrix} 10 & 0 \\ 0 & -10 \end{bmatrix}$
- **D.** The matrix cannot be diagonalized.

7. Diagonalize the following matrix, if possible.

$$\begin{bmatrix} 6 & -2 \\ 2 & 10 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** For P = ____, D = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$
- **B.** For P = ____, D = $\begin{bmatrix} 8 & 0 \\ 0 & -8 \end{bmatrix}$
- **C.** For P = ____, D = $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$
- **D.** The matrix cannot be diagonalized.

8. Diagonalize the following matrix. The real eigenvalues are given to the right of the matrix.

$$\begin{bmatrix} 3 & 3 & -6 \\ -3 & 13 & -18 \\ -1 & 3 & -2 \end{bmatrix}; \lambda = 4,6$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** For P = ____, D = $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

  (Simplify your answer.)
- **B.** For P = $\begin{bmatrix} 3 & -6 & 1 \\ 1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, D = $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$.

  (Simplify your answer.)
- **C.** The matrix cannot be diagonalized.

9. Diagonalize the following matrix.

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** For P = ______, D = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.
  (Type an integer or simplified fraction for each matrix element.)
- **B.** For P = ______, D = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.
  (Type an integer or simplified fraction for each matrix element.)
- **C.** The matrix cannot be diagonalized.

10. Diagonalize the following matrix.

$$\begin{bmatrix} 6 & -4 & 0 & 12 \\ 0 & 3 & 1 & -7 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** For P = $\begin{bmatrix} -1 & 4 & 4 & 1 \\ -1 & 7 & 3 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, D = $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$
- **B.** The matrix cannot be diagonalized.

11. Show that if A is both diagonalizable and invertible, then so is A^{-1}.

What does it mean if A is diagonalizable?

- **A.** If A is diagonalizable, then $A = PDP^{-1}$ for some invertible P and diagonal D. ✓
- **B.** If A is diagonalizable, then $A^k = PDP^{-1}$ for some invertible P and diagonal D.
- **C.** If A is diagonalizable, then $A = PD$ for some invertible P and diagonal D.
- **D.** If A is diagonalizable, then A must be a triangular matrix.

What does it mean if A is invertible?

- **A.** A has no more than three eigenvalues, so the diagonal entries in D are not zero, so D is invertible.
- **B.** A has no less than three eigenvalues, so the diagonal entries in D are not zero, so D is invertible.
- **C.** Zero is not an eigenvalue of A, so the diagonal entries in D are not zero, so D is invertible. ✓
- **D.** Zero must be an eigenvalue of A, so at least one of the diagonal entries in D is zero, so D is invertible.

What is the inverse of A?

- **A.** $A^{-1} = PD^{-1}P^{-1}$ ✓
- **B.** $A^{-1} = PDP^{-1}$
- **C.** $A^{-1} = P^{-1}D^{-1}P$
- **D.** $A^{-1} = P^{-1}D^{-1}$

Therefore, A^{-1} is also diagonalizable.