Math No:

Full Name :

KEY

İSTANBUL ŞEHİR UNIVERSITY

Math 104 4th Midterm Exam
(10 May 2016, 19:00-2:00)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
20 pts	25 pts	20 pts	35 pts	100 pts

Q1. Find the Maclaurin series of the function $f(x) = xsinxcosx$.

(Hint: To solve this question, you may use Taylor or Maclaurin series that you know.)

$$f(x) = x\sin x\cos x = \frac{1}{2}x\sin 2x$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\Rightarrow \sin 2x = \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{(2n+1)!}$$

$$\Rightarrow \boxed{f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n+2}}{(2n+1)!}}$$

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Q2. Find the Taylor series of the function $f(x) = \frac{1}{x^2}$ about the point $a = 1$.

n	$f^{(n)}(x)$
0	x^{-2}
1	$-2x^{-3}$
2	$2 \cdot 3\, x^{-4}$
3	$-2 \cdot 3 \cdot 4\, x^{-5}$
$\vdots$	
n	$(-1)^n (n+1)!\, x^{-(n+2)}$

$$\Rightarrow f^{(n)}(1) = (-1)^n (n+1)!$$

$$f(x) \sim \sum_{n=0}^{\infty} a_n (x-1)^n, \qquad a_n = \frac{f^{(n)}(1)}{n!}$$

$$\Rightarrow f(x) \sim \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)!}{n!} (x-1)^n$$

$$= \boxed{\sum_{n=0}^{\infty} (-1)^n (n+1) (x-1)^n}$$

(You may solve this question by using a change of variable in the geometric series also)

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Q3. Determine whether the series given below converges or diverges:

$$\sum_{n=2}^{\infty}(-1)^n \frac{3}{(lnn)^2}$$

Alternating series ✓

$\lim_{n\to\infty} a_n = \lim \frac{(-1)^n 3}{(\ln n)^2} = 0$ ✓

$|a_n| = \frac{3}{(\ln n)^2}$ is decreasing ✓

$\therefore$ $\boxed{\text{Converges}}$, by the Alternating Series Test.

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Q4. Given the power series

$$\sum_{n=0}^{\infty} \frac{(x-2)^{2n}}{9^n}$$

(a) Find the interval of convergence.
(b) Find the sum of this series.

(a) Generalized Ratio Test:

$$\rho = \lim_{n\to\infty} \left| \frac{\frac{(x-2)^{2(n+1)}}{9^{n+1}}}{\frac{(x-2)^{2n}}{9^n}} \right| = \frac{1}{9}|x-2|^2$$

converges if $\rho < 1$, diverges if $\rho > 1$.

$$\frac{(x-2)^2}{9} < 1 \Rightarrow (x-2)^2 < 9 \Rightarrow -3 < x-2 < 3$$

Converges absolutely for $-1 < x < 5$

$x = 5 \Rightarrow \sum \frac{3^{2n}}{9^n} = \sum 1 = 1+1+1\cdots$ diverges

$x = -1 \Rightarrow \sum \frac{(-3)^{2n}}{9^n} = 1+1+1+\cdots$ diverges

$\therefore$ Interval of convergence is

$$\boxed{-1 < x < 5}$$

(b) This is a geometric series: $\sum_{n=0}^{\infty} \left(\left(\frac{x-2}{3}\right)^2\right)^n$

Since $\sum_{0}^{\infty} r^n = \frac{1}{1-r}$, this series converges to

$$f(x) = \frac{1}{1-\frac{(x-2)^2}{9}} = \frac{9}{9-x^2+4x-4}$$

$$= \boxed{\frac{9}{5-x^2+4x}}$$