Math

No:

Full Name :

Math 104 2nd Midterm Exam
(25 April 2018, 18:00-19:00)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones, smart watches and electronic devices are to be kept shut and out of sight. All cell phones and smart watches are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
25 pts	25 pts	25 pts	25 pts	100 pts

Q1. Evaluate the following limit, if it exists:

$$\lim_{x\to 0^+}\frac{\ln(e^x-1)}{\ln x} \quad \infty/\infty$$

$$\underset{\text{L'Hospital}}{=} \lim_{x\to 0^+}\frac{\frac{e^x}{e^x-1}}{\frac{1}{x}}$$

$$= \lim_{x\to 0^+}\frac{xe^x}{e^x-1}$$

$$= \lim_{x\to 0^+}\frac{x}{1-e^{-x}} \quad 0/0$$

$$\underset{\text{L'Hospital}}{=} \lim_{x\to 0^+}\frac{1}{e^{-x}}$$

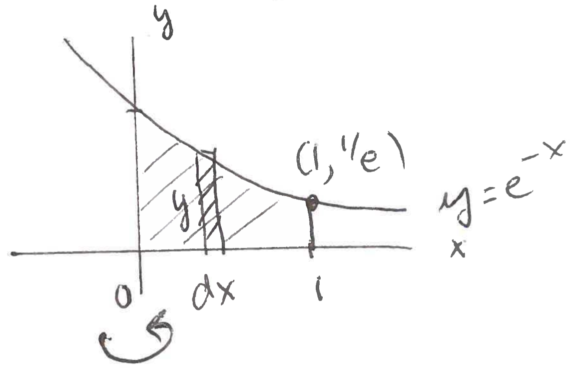
$$= \boxed{1}$$

Math

No:

Full Name :

Q2. Find the volume of the solid of revolution generated by rotating the region between the curve $y = e^{-x}$ and the x-axis, for $0 \le x \le 1$, about the y-axis.



Shell Method:

$$V = 2\pi \int_0^1 x\, y\, dx$$

$$= 2\pi \int_0^1 x e^{-x}\, dx$$

Integration by parts

$u = x$ $\qquad dv = e^{-x}dx$

$du = dx$ $\qquad v = -e^{-x}$

$$V = 2\pi \left\{ -xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}dx \right\}$$

$$= 2\pi \left\{ -e^{-1} - e^{-x}\Big|_0^1 \right\}$$

$$= 2\pi \left(-\frac{1}{e} - \frac{1}{e} + 1 \right)$$

$$= \boxed{\frac{2\pi}{e}(e-2).}$$

(You can solve this question by using the disk method also, but it takes more effort!)

Math

No:

Full Name :

Q3. Evaluate the following integral:

$$\int Arcsinx dx$$

Integration by parts:

$u = \text{Arcsin}x \qquad dv = dx$

$du = \dfrac{dx}{\sqrt{1-x^2}} \qquad v = x$

$\int u dv = uv - \int v du$

$$= x\text{Arcsin}x - \int \frac{x dx}{\sqrt{1-x^2}}$$

Substitution

$t = 1 - x^2 \Rightarrow dt = -2x dx$

$$= x\text{Arcsin}x + \frac{1}{2}\int t^{-1/2} dt$$

$$= x\text{Arcsin}x + \frac{1}{2} \cdot \frac{t^{1/2}}{1/2} + C$$

$$= \boxed{x\text{Arcsin}x + \sqrt{1-x^2} + C}$$

(You can also solve this question by the change of variable $\text{Arcsin}x = \theta \Rightarrow \sin\theta = x$)

Math

İSTANBUL ŞEHİR UNIVERSITY

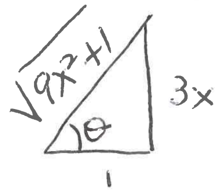
No:

Full Name :

Q4. Evaluate the following integral:

$$\int \frac{6dx}{(9x^2+1)^2}$$

Trigonometric substitution



$$\tan\theta = 3x$$

$$\sec^2\theta\, d\theta = 3dx$$

$$\sec\theta = \sqrt{9x^2+1}$$

$$= \frac{6}{3}\int \frac{\sec^2\theta\, d\theta}{\sec^4\theta}$$

$$= 2\int \cos^2\theta\, d\theta = 2\int \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= \theta + \frac{\sin 2\theta}{2} + C$$

$$= \theta + \frac{2\sin\theta\cos\theta}{2} + C$$

$$= \operatorname{Arctan} 3x + \frac{3x}{\sqrt{9x^2+1}} \cdot \frac{1}{\sqrt{9x^2+1}} + C$$

$$= \boxed{\operatorname{Arctan} 3x + \frac{3x}{9x^2+1} + C}$$