

Math No:

Full Name : KEY



Math 104 1st Midterm Exam
(27 October 2016, 18:00-19:00)

IMPORTANT

1. Write down your name and surname on top of each page. 2. The exam consists of 4 questions, some of which have multiple parts. 3. Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. 4. Show all your work. Correct answers without justification will not get credit. 5. Unless otherwise specified, you may use any method from classwork to solve the problems. 6. Calculators are not allowed. 7. All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
6 pts	6 pts	6 pts	6 pts	24 pts

Q1. Evaluate the following integrals.

$$a) \int \left(1 - \sin \frac{x}{2}\right)^{1/3} \cos \frac{x}{2} dx = -2 \int u^{1/3} du = -2 \cdot \frac{3}{4} u^{4/3} + C = -\frac{3}{2} \left(1 - \sin \frac{x}{2}\right)^{4/3} + C$$

$$u = 1 - \sin \frac{x}{2}$$

$$du = -\frac{1}{2} \cos \frac{x}{2} dx$$

$$-2du = \cos \frac{x}{2} dx$$

$$b) \int_{e^2}^e \frac{dx}{x(\ln x)^2} = \int \frac{du}{u^2} = \int u^{-2} du = -\frac{1}{u} \Big|_{u(e^2)}^{u(e)} = -\left[\frac{1}{u(e)} - \frac{1}{u(e^2)}\right] = -\left[1 - \frac{1}{2}\right] = -\frac{1}{2}$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$u(e) = \ln e = 1$$

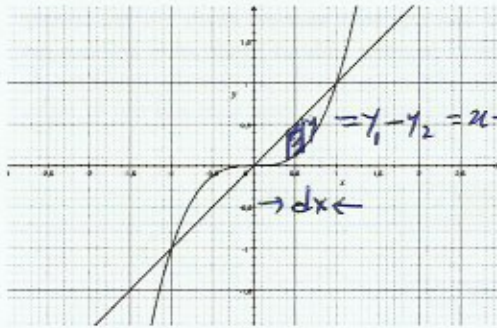
$$u(e^2) = \ln e^2 = 2 \ln e = 2$$

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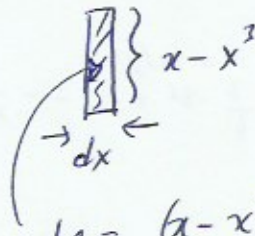
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Q2. Calculate the area between $y_1 = x$ and $y_2 = x^3$. You must show your work on finding the integral limits.



$$\begin{aligned}y_1 &= y_2 \\x &= x^3 \\x^3 - x &= 0 \\x(x^2 - 1) &= 0 \\x &= 0, \quad x = \pm 1\end{aligned}$$

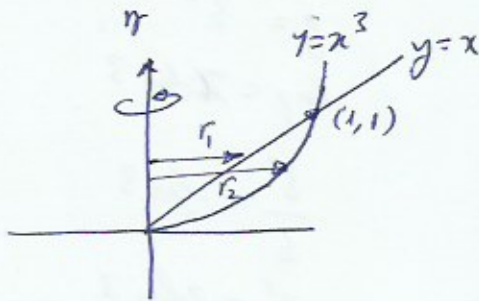

$$\begin{aligned}dA &= (x - x^3) dx \\A &= 2 \int_0^1 (x - x^3) dx \\&= 2 \int_0^1 (x - x^3) dx \\&= 2 \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 \\&= (x^2 - \frac{1}{2}x^4) \Big|_0^1 \\&= 1 - \frac{1}{2} \\&= \frac{1}{2}\end{aligned}$$

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Q3. The region between $y = x$ and $y = x^3$, for $0 \leq x \leq 1$, is rotated about the y -axis. Find the volume generated.



Using the washer method:



$$dV = (\pi r_2^2 - \pi r_1^2) dy$$

$$r_2 = x_2, \quad r_1 = x_1$$

$$r_2 = y^{1/3}, \quad r_1 = y$$

$$V = \int_0^1 (\pi (y^{1/3})^2 - \pi y^2) dy$$

$$= \pi \int_0^1 (y^{2/3} - y^2) dy$$

$$= \pi \left(\frac{3}{5} y^{5/3} - \frac{y^3}{3} \right) \Big|_0^1$$

$$= \pi \left(\frac{3}{5} - \frac{1}{3} \right)$$

$$= \frac{4\pi}{15}$$

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Q4.

a) Differentiate the function $y = 3^x - x^3$.

$$\frac{dy}{dx} = \frac{d}{dx}(3^x) - \frac{d}{dx}(x^3)$$

$$y' = 3^x \ln 3 - 3x^2$$

$$z = 3^x$$
$$\ln z = \ln 3^x$$

$$\frac{z'}{z} = \ln 3$$

$$z' = z \ln 3$$

$$z' = 3^x \ln 3$$

b) Find the derivative of the function $y = (\sin x)^{\ln x}$ at the point $x = 1$. Leave $\sin 1$ as it is.

$$\ln y = \ln x \ln(\sin x)$$

$$\frac{y'}{y} = \frac{1}{x} \ln(\sin x) + \ln x \cdot \frac{\cos x}{\sin x}$$

$$y' = (\sin x)^{\ln x} \left[\frac{1}{x} \ln(\sin x) + \ln x \cdot \frac{\cos x}{\sin x} \right]$$

$$y'(1) = \ln(\sin 1) //$$