Student: Huseyin Kerem Mican **Date:** 5/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 1.9 Homework

1. Assume that T is a linear transformation. Find the standard matrix of T.

   T: $\mathbb{R}^2 \to \mathbb{R}^4$, $T(\mathbf{e}_1) = (8, 1, 8, 1)$, and $T(\mathbf{e}_2) = (-6, 3, 0, 0)$, where $\mathbf{e}_1 = (1,0)$ and $\mathbf{e}_2 = (0,1)$.

   $A = \begin{bmatrix} 8 & -6 \\ 1 & 3 \\ 8 & 0 \\ 1 & 0 \end{bmatrix}$ (Type an integer or decimal for each matrix element.)

2. Assume that T is a linear transformation. Find the standard matrix of T.

   T: $\mathbb{R}^2 \to \mathbb{R}^2$, rotates points (about the origin) through $\frac{7\pi}{4}$ radians.

   $A = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

   (Type an integer or simplified fraction for each matrix element. Type exact answers, using radicals as needed.)

   YOU ANSWERED: $\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$

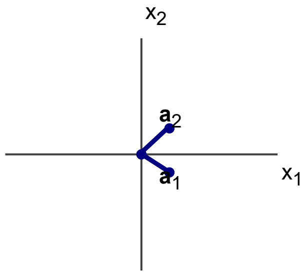
3. Assume that T is a linear transformation. Find the standard matrix of T.

   T: $\mathbb{R}^2 \to \mathbb{R}^2$ is a vertical shear transformation that maps $\mathbf{e}_1$ into $\mathbf{e}_1 - 17\mathbf{e}_2$ but leaves the vector $\mathbf{e}_2$ unchanged.

   $A = \begin{bmatrix} 1 & 0 \\ -17 & 1 \end{bmatrix}$

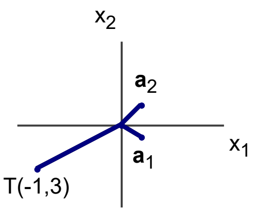
   (Type an integer or simplified fraction for each matrix element.)

4. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with standard matrix $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 \end{bmatrix}$, where $\mathbf{a}_1$ and $\mathbf{a}_2$ are the vectors shown in the figure. Using the figure, draw the image of $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ under the transformation T.

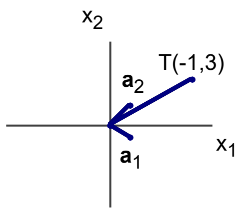


Choose the correct graph below.

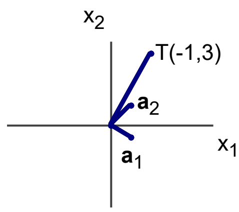
A.



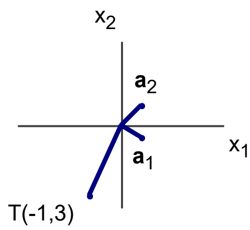
B.



C.



D.



5. Fill in the missing entries of the matrix, assuming that the equation holds for all values of the variables.

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 - 6x_2 \\ 7x_1 - 5x_3 \\ -4x_2 + 7x_3 \end{bmatrix}$$

Fill in the missing entries of the matrix below.

$$\begin{bmatrix} 2 & -6 & 0 \\ 7 & 0 & -5 \\ 0 & -4 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 - 6x_2 \\ 7x_1 - 5x_3 \\ -4x_2 + 7x_3 \end{bmatrix}$$

6. Show that T is a linear transformation by finding a matrix that implements the mapping. Note that $x_1, x_2, \ldots$ are not vectors but are entries in vectors.

$$T(x_1,x_2,x_3,x_4) = (x_1 + 7x_2,\ 0,\ 4x_2 + x_4,\ x_2 - x_4)$$

$A = \begin{bmatrix} 1 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$ (Type an integer or decimal for each matrix element.)

7. Show that T is a linear transformation by finding a matrix that implements the mapping. Note that $x_1, x_2, \ldots$ are not vectors but are entries in a vector.

$T(x_1,x_2,x_3,x_4) = 3x_1 + 2x_2 - 4x_3$ $\qquad$ $(T: \mathbb{R}^4 \to \mathbb{R})$

$A = \begin{bmatrix} 3 & 2 & -4 & 0 \end{bmatrix}$

8. Determine if the specified linear transformation is (a) one-to-one and (b) onto. Justify your answer.

$$T\left(x_1,x_2,x_3,x_4\right) = (x_1+x_2,x_2+x_3,x_2+x_3,0)$$

a. Is the linear transformation one-to-one?

- **A.** T is one-to-one because the column vectors are not scalar multiples of each other.
- **B.** T is one-to-one because T(**x**) = **0** has only the trivial solution.
- **C.** T is not one-to-one because the columns of the standard matrix A are linearly independent.
- **D.** T is not one-to-one because the standard matrix A has a free variable.

b. Is the linear transformation onto?

- **A.** T is not onto because the fourth row of the standard matrix A is all zeros.
- **B.** T is onto because the columns of the standard matrix A span $\mathbb{R}^4$.
- **C.** T is not onto because the columns of the standard matrix A span $\mathbb{R}^4$.
- **D.** T is onto because the standard matrix A does not have a pivot position for every row.

9. Determine if the specified linear transformation is (**a**) one-to-one and (**b**) onto. Justify each answer.

$$T\left(x_1,x_2,x_3\right) = \left(x_1-3x_2+3x_3,\ x_2-9x_3\right)$$

(**a**) Is the linear transformation one-to-one?

- **A.** T is not one-to-one because the columns of the standard matrix A are linearly dependent.
- **B.** T is not one-to-one because the columns of the standard matrix A are linearly independent.
- **C.** T is one-to-one because the column vectors are not scalar multiples of each other.
- **D.** T is one-to-one because T(x) = 0 has only the trivial solution.

(**b**) Is the linear transformation onto?

- **A.** T is not onto because the columns of the standard matrix A span $\mathbb{R}^2$.
- **B.** T is onto because the columns of the standard matrix A span $\mathbb{R}^2$.
- **C.** T is not onto because the standard matrix A does not have a pivot position for every row.
- **D.** T is onto because the standard matrix A does not have a pivot position for every row.

10. Describe the possible echelon forms of the standard matrix for a linear transformation T where T: $\mathbb{R}^3 \to \mathbb{R}^4$ is one-to-one.

Give some examples of the echelon forms. The leading entries, denoted $\blacksquare$, may have any nonzero value. The starred entries, denoted $*$, may have any value (including zero). Select all that apply.

A. $\begin{bmatrix} 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$

C. $\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \\ 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$

E. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \end{bmatrix}$

F. $\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & * \\ 0 & 0 & \blacksquare \end{bmatrix}$

11. Describe the possible echelon forms of the standard matrix for a linear transformation T where T: $\mathbb{R}^4 \to \mathbb{R}^3$ is onto.

Give some examples of the echelon forms. The leading entries, denoted $\blacksquare$, may have any nonzero value; the starred entries, denoted $*$, may have any value (including zero). Select all that apply.

A. $\begin{bmatrix} 0 & \blacksquare & * & * \\ 0 & \blacksquare & * & * \\ 0 & \blacksquare & * & * \end{bmatrix}$

B. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$

D. $\begin{bmatrix} 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$

E. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare \end{bmatrix}$

F. $\begin{bmatrix} 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & \blacksquare \\ 0 & 0 & 0 & 0 \end{bmatrix}$

G. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$

H. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

I. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \end{bmatrix}$

12. Let T be the linear transformation whose standard matrix is given. Decide if T is a one-to-one mapping. Justify your answer.

$$\begin{bmatrix} -6 & 7 & -5 & -25 \\ 8 & 4 & -6 & -6 \\ 3 & 10 & 3 & -14 \\ -3 & 1 & 9 & 4 \end{bmatrix}$$

Choose the correct answer below.

A. The transformation T is not one-to-one because the equation $T(\mathbf{x}) = \mathbf{0}$ has only the trivial solution.

B. The transformation T is one-to-one because the equation $T(\mathbf{x}) = \mathbf{0}$ has a nontrivial solution.

C. The transformation T is not one-to-one because the equation $T(\mathbf{x}) = \mathbf{0}$ has a nontrivial solution.

D. The transformation T is one-to-one because the equation $T(\mathbf{x}) = \mathbf{0}$ has only the trivial