MATH 104 TUTORIAL 1

In exercises below, find an antiderivative for each function. Check your answers by differentiation.

1- **a.** $2x^{-3}$ **b.** $\frac{x^{-3}}{2} + x^2$ **c.** $-x^{-3} + x - 1$

2- **a.** $\frac{4}{3}\sqrt[3]{x}$ **b.** $\frac{1}{3\sqrt[3]{x}}$ **c.** $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$

3- **a.** $\pi \cos \pi x$ **b.** $\frac{\pi}{2} \cos \frac{\pi x}{2}$ **c.** $\cos \frac{\pi x}{2} + \pi \cos x$

4- In exercises, find the most general antiderivative or indefinite integral.

a) $\int \left(\frac{1}{5} - \frac{2}{x^3} + 2x\right) dx$ b) $\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right) dx$ c) $\int x^{-3}(x + 1)\, dx$

d) $\int (-5 \sin t)\, dt$ e) $\int 3 \cos 5\theta\, d\theta$ f) $\int \frac{2}{5} \sec \theta \tan \theta\, d\theta$

g) $\int \frac{1}{2}(\csc^2 x - \csc x \cot x)\, dx$ h) $\int (2 + \tan^2 \theta)\, d\theta$ I) $\int \frac{\csc \theta}{\csc \theta - \sin \theta}\, d\theta$

5- Use finite approximations to estimate the area under the graph of the function using

$f(x) = 1/x$ between $x = 1$ and $x = 5$.

a. a lower sum with two rectangles of equal width.
b. a lower sum with four rectangles of equal width.
c. an upper sum with two rectangles of equal width.
d. an upper sum with four rectangles of equal width.

6- Write the sums in exercises without sigma notation. Then evaluate them.

a) $\sum_{k=1}^{3} \frac{k-1}{k}$ b) $\sum_{k=1}^{4} (-1)^k \cos k\pi$

7- Evaluate the sums in Exercises

a. $\sum_{k=1}^{n} \left(\frac{1}{n} + 2n\right)$ **b.** $\sum_{k=1}^{n} \frac{c}{n}$ **c.** $\sum_{k=1}^{n} \frac{k}{n^2}$ **d)** $\sum_{k=1}^{6} (k^2 - 5)$

8- For the functions below , find a formula for the Riemann sum obtained by dividing the interval [a, b] into n equal subintervals and using the right-hand endpoint for each c_k. Then take the limit of these sums as n-> inf. to calculate the area under the curve over [a, b] .

a) $f(x) = 3x^2$ over the interval [0, 1] **b)** $f(x) = 2x^3$ over the interval [0, 1].

9- Evaluate the integrals in exercises

a) $\int_0^{\sqrt{2}} \left(t - \sqrt{2}\right) dt$ b) $\int_1^0 (3x^2 + x - 5)\, dx$