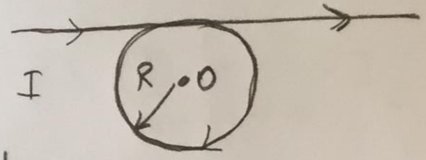
Ch. 30 Problems

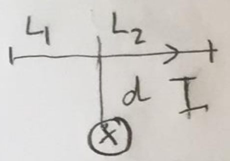
$I = 1\,A \qquad R = 15\text{ cm}$

7. 

What is B at the center

Remember:

We had found out that the B due to a straight wire of length L, carrying a current I, at a distance d from the wire:



$$B = -\frac{\mu_0 I}{4\pi a}\left[\frac{L_1}{(L_1^2+a^2)^{1/2}} + \frac{L_2}{(L_2^2+a^2)^{1/2}}\right]$$

where $L = L_1 + L_2$

for $L_1 = L_2 = L/2$

$$B = -\frac{\mu_0 I}{4\pi a}\left[\frac{2(L/2)}{\left(\frac{L^2}{4}+a^2\right)^{1/2}}\right]$$

$$B = -\frac{\mu_0 I}{4\pi a}\left[\frac{L}{\frac{L}{2}\left(1+\frac{4a^2}{L^2}\right)^{1/2}}\right]$$

for $L \gg a$

$$\frac{a^2}{L^2} \to 0$$

$$B = -\frac{\mu_0 I}{4\pi a}(2) = -\frac{\mu_0 I}{2\pi a}$$ (for an infinitely long wire)

B is into the page!

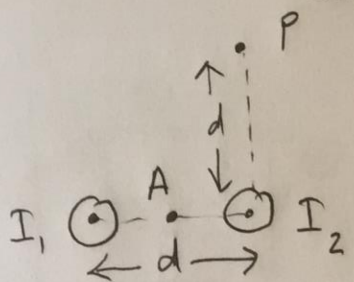
So for the infinitely long wire B at the center of the circle is: $B = \frac{\mu_0 I}{2\pi R}$ (into the page)

For a circle of radius R with current in the clockwise direction:

$$B = \frac{\mu_0 I}{4\pi R^2}\int_0^{2\pi R} ds = \frac{\mu_0 I}{2R}$$ (into the page).

Total B at the center O: $B = \frac{\mu_0 I}{2\pi R} + \frac{\mu_0 I}{2R}$ (in)

P. 20. ②



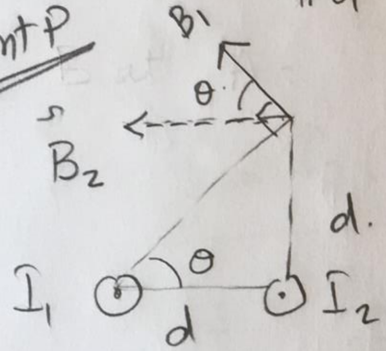
Find total B at point A: wires are infinitely long so, as we have determined in the previous problem

$$B = \frac{\mu_0 I}{2\pi r} \; : \; B_1 = \frac{\mu_0 I_1}{2\pi \frac{d}{2}} \text{ up} \cdot \; B_2 = \frac{\mu_0 I_1}{2\pi \frac{d}{2}} \text{ down}$$

→ Total B at A: $\vec{B} = \vec{B}_1 + \vec{B}_2$, upward direction as y:

$$\vec{B} = \frac{\mu_0}{\pi d}\left[I_1 - I_2\right]\hat{j}$$

Point P



$$\vec{B}_T = \vec{B}_{TX} + \vec{B}_{Ty}$$

$$\vec{B}_{TX} = -B_1 \cos\theta - B_2$$

$$B_{Ty} = B_1 \sin\theta$$

$$B_1 = \frac{\mu_0 I_1}{2\pi d\sqrt{2}} \qquad B_2 = \frac{\mu_0 I_2}{2\pi d}. \qquad \theta = 45^\circ \quad \cos 45^\circ = \frac{\sqrt{2}}{2}$$

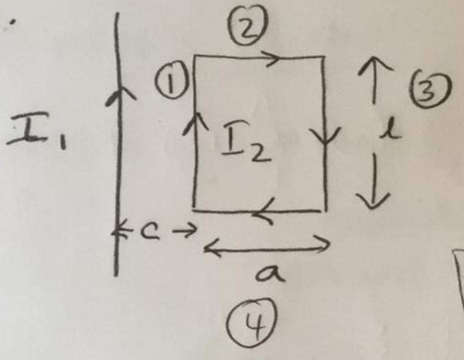
$$B_{TX} = \frac{\mu_0 I_1}{2\pi d\cancel{\sqrt{2}}}\frac{\cancel{\sqrt{2}}}{2} + \frac{\mu_0 I_2}{2\pi d}$$

$$B_{TX} = \frac{\mu_0\left[I_1 + 2I_2\right]}{4\pi d} \qquad \vec{B}_{TX} = -\frac{\mu_0}{4\pi d}\left[I_1 + 2I_2\right]$$

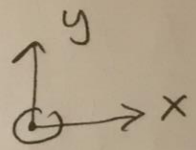
$$B_{Ty} = \frac{\mu_0 I_1}{2\pi d\sqrt{2}}\frac{\sqrt{2}}{2} = \frac{\mu_0 I_1}{4\pi d}.$$

$$\vec{B}_P = -\frac{\mu_0}{4\pi d}\left[(I_1 + 2I_2)\hat{i} - I_1\hat{j}\right]$$

26.



$$B = \frac{\mu_0 I_1}{2\pi x}$$



$$\vec{F}_1 = I_2 \ell \hat{j} \times \left(-\frac{\mu_0 I_1}{2\pi c}\hat{k}\right)$$

$$\boxed{\vec{F}_1 = -I_1 I_2 \ell \frac{\mu_0}{2\pi c}\hat{i}}$$

$$\vec{F}_2 = I_2 \int_{c}^{c+a} (dx\,\hat{i}) \times \left(-\frac{\mu_0 I_1}{2\pi x}\hat{k}\right)$$

$$\vec{F}_2 = \frac{\mu_0 I_1 I_2}{2\pi} \int_{c}^{c+a} \left(\frac{dx}{x}\right)\hat{j} = \boxed{\frac{\mu_0 I_1 I_2}{2\pi}\left(\ln \frac{c+a}{c}\right)\hat{j}}$$

$$F_3 = I_2(-\ell\hat{j}) \times \left(-\frac{\mu_0 I_1}{2\pi(c+a)}\hat{k}\right)$$

$$\boxed{F_3 = \frac{\mu_0}{2\pi}\frac{I_1 I_2 \ell}{(c+a)}\hat{i}}$$

$$F_4 = I_2 \int_{c}^{c+a} (-dx\,\hat{i}) \times \left(-\frac{\mu_0 I_1}{2\pi x}\hat{k}\right)$$

$$F_4 = -\frac{\mu_0 I_1 I_2}{2\pi} \int_{c}^{c+a} \frac{dx}{x}(-\hat{j})$$

$$\boxed{F_4 = -\frac{\mu_0 I_1 I_2}{2\pi} \ln\left(\frac{c+a}{c}\right)(-\hat{j})}$$

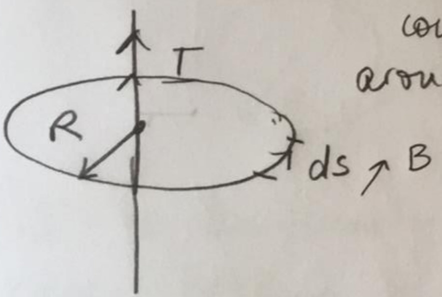
$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 \qquad \vec{F}_2 + \vec{F}_4 = 0$$

$$\boxed{\vec{F}_{net} = \frac{\mu_0 \ell I_1 I_2}{2\pi}\left[\frac{1}{c+a} - \frac{1}{c}\right]\hat{i}}$$

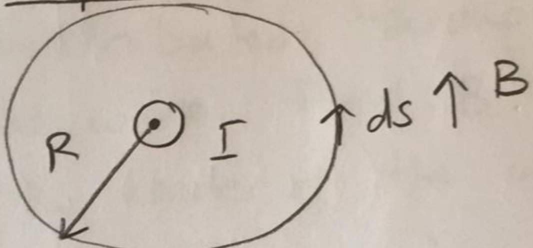
Ampere's Law

B due to an infinitely long wire : $B = \frac{\mu_0 I}{2\pi r}$

consider a loop around the wire.



Top view



The direction of B due to I :
Tangent to the CCW circle. So at any point B is parallel to ds.

$$\vec{B} \cdot \vec{ds} = \frac{\mu_0 I}{2\pi R} \, ds \, (\cos 0)$$

Around the circle :

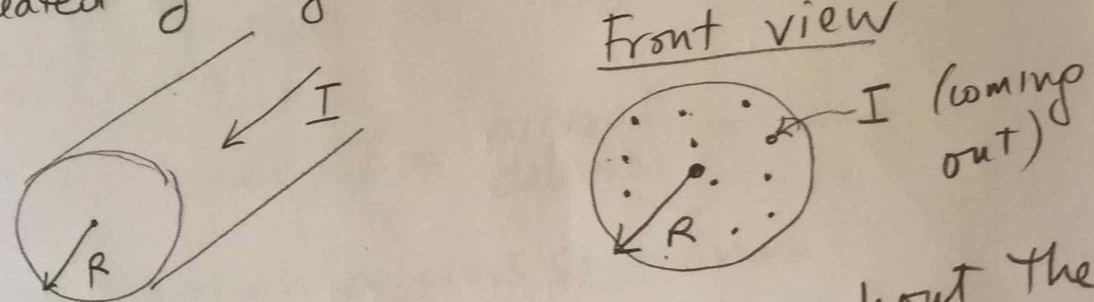
$$\oint \vec{B} \cdot \vec{ds} = \int \frac{\mu_0 I}{2\pi R} \, ds = \frac{\mu_0 I}{2\pi R} (2\pi R)$$

$$\oint ds = 2\pi R$$

$$\boxed{\oint \vec{B} \cdot \vec{ds} = \mu_0 I}$$

Line integral around a closed path of $\vec{B} \cdot \vec{ds}$ equals to μ_0 times the current I passing through any surface inside the closed path.

$\vec{B}$ created by a long wire with current I:

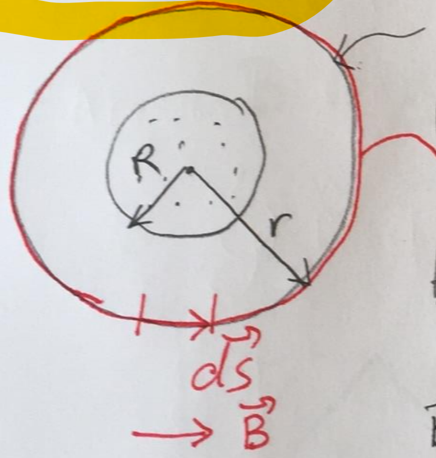


I is uniformly distributed throughout the cross-section of the wire. Find B at a distance r from the center of the wire

for $r < R$ (inside the wire)

$r > R$ (outside " ")

For $r > R$:



Choose an integration loop with $r > R$

$$\boxed{\oint \vec{B} \cdot ds = \mu_0 I}$$

$\vec{B} \parallel \vec{ds}$ $\quad \vec{B} \cdot \vec{ds} = B\,ds$

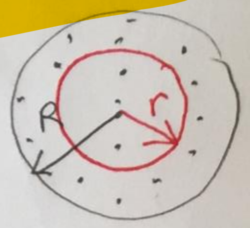
B is const. on the loop:

$$\oint \vec{B} \cdot \vec{ds} = B \oint ds \qquad \oint ds = 2\pi r$$

$$B(2\pi r) = I \mu_0$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$r < R$.



Let us say that the current inside our integration loop is I'.

Let $J = \text{current density} = \frac{I}{A}$

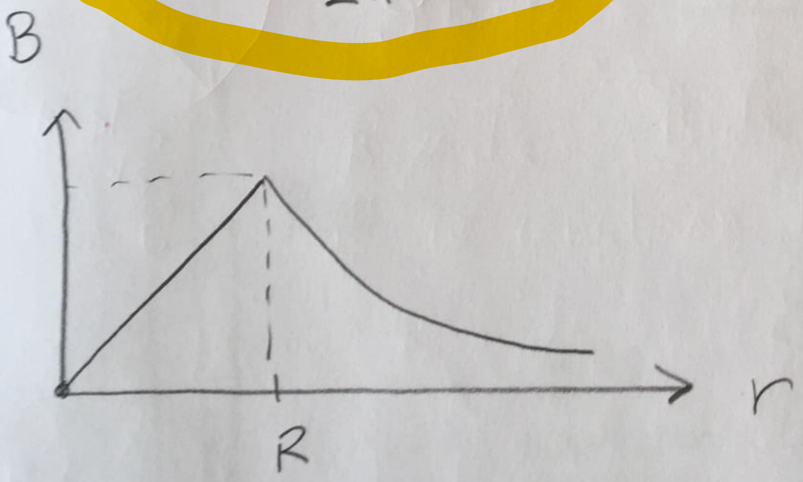
I goes through a cross sectional area $= \pi R^2$

I' goes through cross sectional area $= \pi r^2$

$$J = \frac{I}{\pi R^2} = \frac{I'}{\pi r^2} \qquad I' = I \frac{r^2}{R^2}$$

$$\oint \vec{B} \cdot \vec{ds} = B(2\pi r) = \mu_0 I \frac{r^2}{R^2}$$

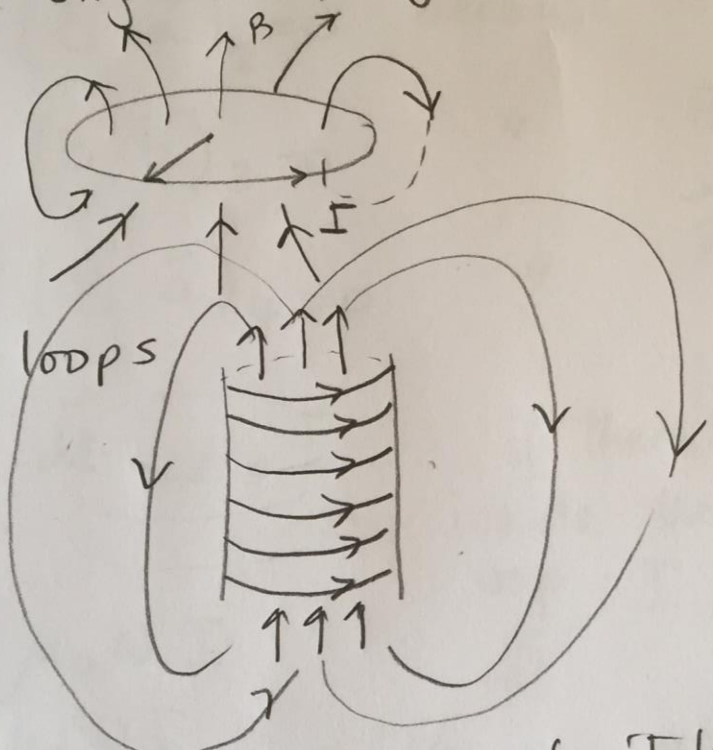
$$B = \frac{\mu_0 I r}{2\pi R^2}$$



B due to a Solenoid

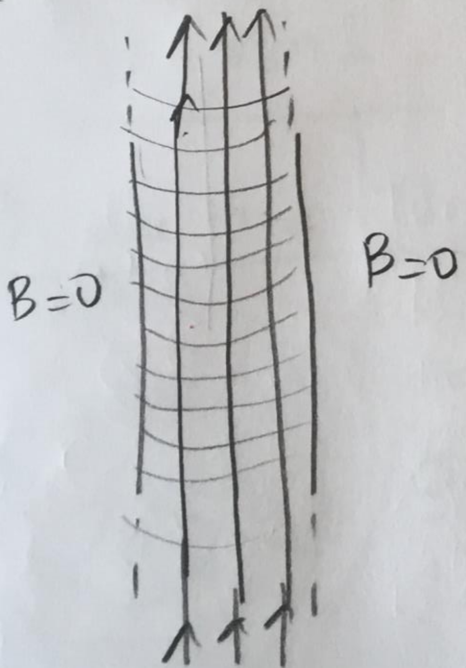
For a single loop of current I

Many loops

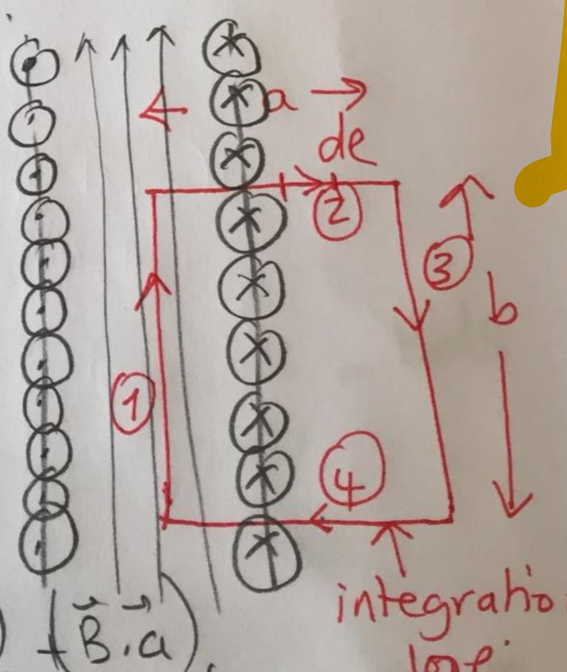


and if we have an infinitely long coil

(a solenoid)



Cross section



B is uniform inside & 0 outside.

$$\oint \vec{B}\cdot d\vec{\ell} = (\vec{B}\cdot\vec{b})_1 + (\vec{B}\cdot\vec{a})_2 + (\vec{B}\cdot\vec{b})_3 + (\vec{B}\cdot\vec{a})_4$$

$$\oint \vec{B}\cdot d\vec{l} = Bb + 0 + 0 + 0 = Bb$$

Note: $(\vec{B}\cdot\vec{a})_2 = 0$ because $\vec{B} \perp \vec{a}$

$(B\cdot b)_3 = 0$ " $B = 0$

$(\vec{B}\cdot\vec{a})_4 = 0$ " $\vec{B} \perp \vec{a}$.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

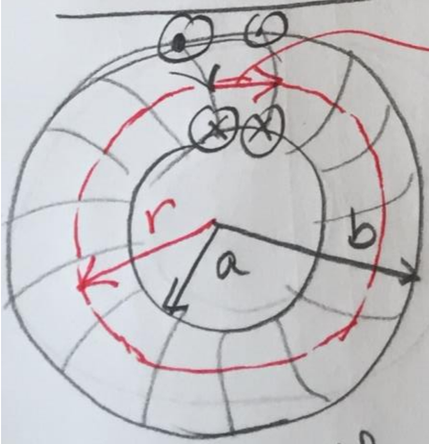
if there are N turns inside the integration loop: $I_{in} = NI$

$$Bb = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{b} \qquad \frac{N}{b} = n = \#\text{ of turns per length.}$$

$$\boxed{B = \mu_0 n I}$$

B due to a Toroid



$d\vec{s}$, $\vec{B}$ (integration loop)

total turns (N)

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 N I \qquad \vec{B}\cdot d\vec{s} = Bds$$

B is // to ds, so

B const around the loop

$$\oint \vec{B}\cdot d\vec{s} = B\oint ds = B\,2\pi r = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$

Direction: tangent to clockwise circle.