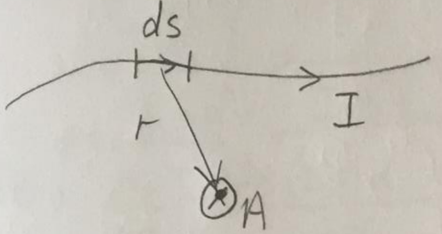
Chapter 30 - Sources of Magnetic Field

Magnetic Field (B) created by a current carrying wire is given by the BIOT-SAVART LAW:



The segment $\vec{ds}$ creates a magnetic field $d\vec{B}$ at point A:

$$\boxed{d\vec{B} = \frac{\mu_0 I \, \vec{ds} \times \hat{r}}{4\pi \, r^2}}$$

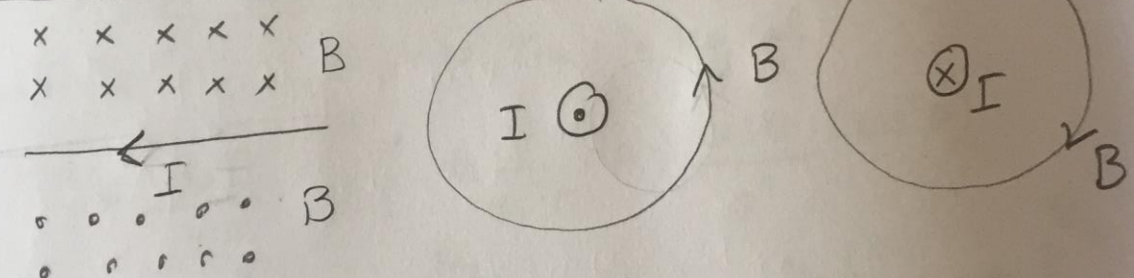
where μ_0 is the permeability of free space $= 4\pi \times 10^{-7}$ T.m/A

Using the right hand rule, $d\vec{B}$ is out $\odot$ at A.

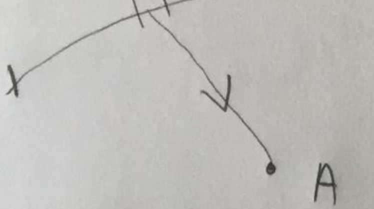
$\underline{\underline{r}}$ is the distance from ds to point A.

Examples for the direction of B (P.2 Ch.30)

a)

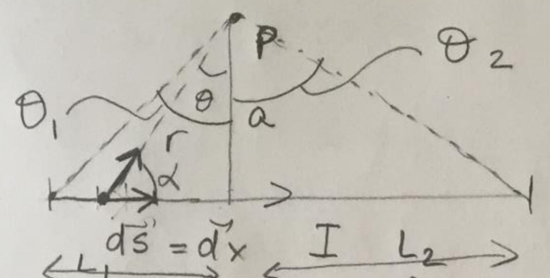


If we want to find $\vec{B}$ at point A created by a current carrying wire (I) of length L



$$B = \frac{\mu_0 I}{4\pi} \int_0^L \frac{\vec{ds} \times \hat{r}}{r^2}$$

If we have a straight wire of Length L current I : Find B at P.



The direction of B at P is out

⊙ $L = L_1 + L_2$

2

$$dB = \frac{\mu_0 I}{4\pi} \frac{dx(\sin\alpha)}{r^2}$$

$\cos\theta = \frac{a}{r}$ $\boxed{r = \frac{a}{\cos\theta}}$

1 $\boxed{\sin\alpha = \cos\theta}$ $\tan\theta = \frac{-x}{a}$

$x = -a\tan\theta$ $dx = -a\frac{d}{d\theta}(\tan\theta)d\theta$

$$\frac{dx}{d\theta} = -a\frac{d}{d\theta}\left(\frac{\sin\theta}{\cos\theta}\right) = -a\left[\frac{\cos^2\theta + (-\sin\theta)\sin\theta}{\cos^2\theta}\right]$$

3 $\boxed{dx = -a\frac{1}{\cos^2\theta}d\theta.}$

Substitute 1, 2, 3 into the integral :

$$B = \frac{\mu_0 I}{4\pi}\int\left(-a\frac{1}{\cos^2\theta}d\theta\right)\frac{\cos\theta\,(\cos^2\theta)}{a^2}$$

$$B = -\frac{\mu_0 I}{4\pi}\frac{1}{a}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = -\frac{\mu_0 I}{4\pi a}\sin\theta\Big|_{\theta_1}^{\theta_2}$$

$$\boxed{B = \frac{\mu_0 I}{4\pi a}\left[\sin\theta_1 - \sin\theta_2\right]}$$

$\sin\theta_2 = \frac{L_2}{(L_2^2+a^2)^{1/2}}$

$\sin\theta_1 = \frac{L_1}{(L_1^2+a^2)^{1/2}}$

$$B = \frac{\mu_0 I}{4\pi a}\left[\frac{L_1}{(L_1^2+a^2)^{1/2}} - \frac{L_2}{(-L_2^2+a^2)^{1/2}}\right]$$

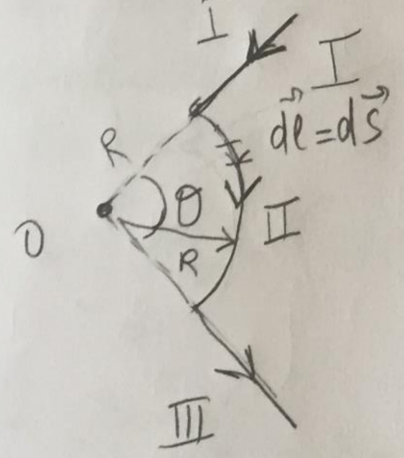
Now assume that the wire is infinitely long. In this case $\theta_1 = \frac{\pi}{2}$ $\theta_2 = -\frac{\pi}{2}$

$\sin\frac{\pi}{2} = 1$ $\sin\left(-\frac{\pi}{2}\right) = -1$

So : $B = \frac{\mu_0 I}{4\pi a}\left[1-(-1)\right] = \frac{\mu_0 I}{4\pi a}(2)$

$$\boxed{B = \frac{\mu_0 I}{2\pi a}}$$ for an infinitely long wire carrying current I.

B due to a curved Wire Segment



Find B at O :

We consider the wire in three segments : I, II, III

$$\vec{B}_0 = \vec{B}_I + \vec{B}_{II} + \vec{B}_{III}$$

$$B_I = \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl}\times\hat{r}}{r^2}$$

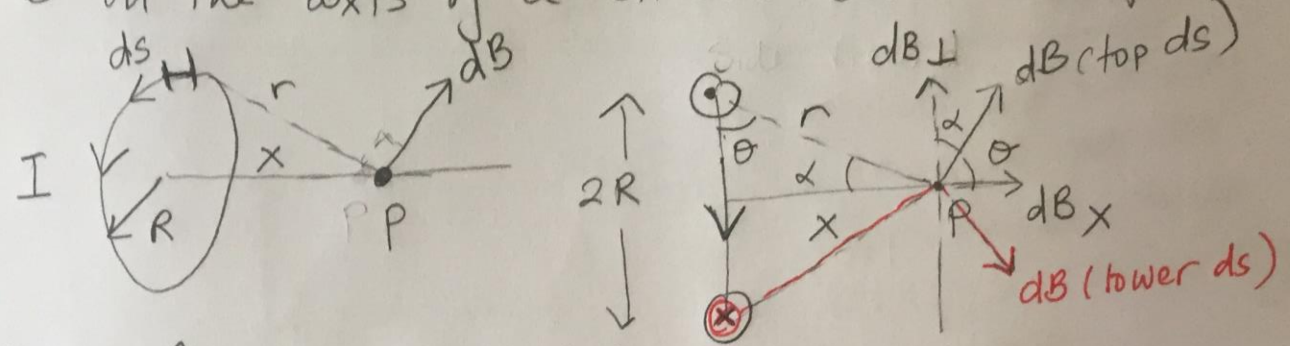
Because dl and r are parallel for segments I + III , $\theta = 0$ $\sin\theta = 0$

$B_I = B_{III} = 0$

$$B_0 = B_{II} = \frac{\mu_0 I}{4\pi}\int\frac{ds}{R^2} = \frac{\mu_0 I}{4\pi R^2}\int_0^\theta R\,d\theta$$

$$\vec{B}_0 = \frac{\mu_0 I}{4\pi R}\theta \otimes \text{ in.}$$

B on the axis of a Circular Current Loop:



On the figures dB created by the top segment ds is shown. (and bottom ds by "red" line)

Side View

$$dB = \frac{\mu_0 I}{4\pi} \int \frac{ds}{r^2} \qquad r = (R^2+x^2)^{1/2} \qquad \cos\theta = \frac{R}{r}$$

If you consider all ds around the circular current loop $\vec{dB}$ lines form a "cone". The ⊥ (perpendicular) components of all dB's cancel out. The parallel or x components of dB add.

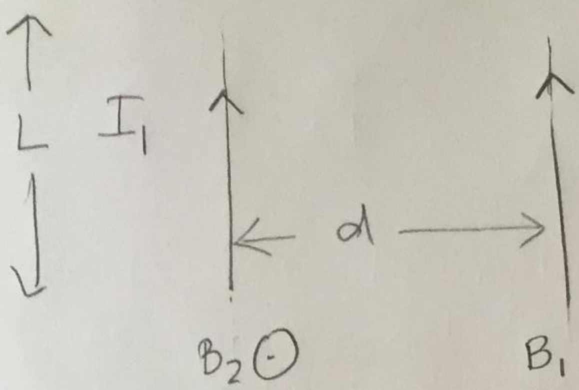
$$B = \int dB_x = \int dB\cos\theta = \frac{\mu_0 I}{4\pi} \int \frac{ds}{(R^2+x^2)^{1/2}} \frac{R}{r}$$

$$B = \frac{\mu_0 I R}{4\pi (R^2+x^2)^{3/2}} \int ds \qquad \int ds = 2\pi R$$

$$\vec{B} = \frac{\mu_0 I R (\cancel{2}\cancel{\pi} R)}{\underset{2}{\cancel{4}}\cancel{\pi} (R^2+x^2)^{3/2}} \uparrow = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} \uparrow$$

(5)

Force between Current Carrying Wires



We assume L is very long, so we can approximate these wires as infinitely long wires.

So we have two wires carryin currents I_1 and I_2. B due to an infinitely long wire at d:

$$B = \frac{\mu_0 I}{2\pi d} \quad (x) \text{ in}$$

B due to I_1 at a distance d (where I_2 is)

$$\boxed{B_1 = \frac{\mu_0 I_1}{2\pi d}}$$

Force on the second wire: $F_2 = I_2 L \times B_1$

since $L \perp B_1$ $\quad F_2 = I_2 L \frac{\mu_0 I_1}{2\pi d}$

direction of F_2 is $-\hat{i}$: $\boxed{\vec{F}_2 = \frac{\mu_0}{2\pi} \frac{L I_1 I_2}{d}(-\hat{i})}$

B due to I_2 at the location of wire 1:

$$B_2 = \frac{\mu_0 I_2}{2\pi d} \odot \text{ out} \quad F_1 = I_1 L \times B_2 \quad L \perp B_2$$

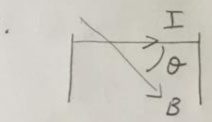
$$F_1 = I_1 L \frac{\mu_0 I_2}{2\pi d}$$

direction of F_1 is $+\hat{i}$

$$\vec{F}_1 = \frac{\mu_0}{2\pi} \frac{L I_1 I_2}{d}(\hat{i})$$

So, the forces are equal in magnitude and opposite in direction (attractive)

41.



$L = 58\ m \quad I = 2.2\ kA$

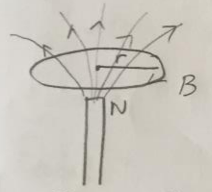
$B = 5\times10^{-5}\ T \quad \theta = 65°$

a) $\vec{F} = I\,\vec{L}\times\vec{B} = (2.2\times10^{3})(58)(5\times10^{-5})\sin 65$

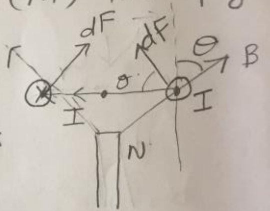
$F = 5.78\ N$

b) The direction of F is (in) to the page.

42.



When looked from sideways



$\vec{dF} = I\,\vec{ds}\times\vec{B} \qquad dF = I\,ds\,B\sin 90 = I\,ds\,B$

To find the total F we have to add all dF's.
The horizontal components of dF's cancel out.
The vertical components add. $dF_{vertical} = dF\sin\theta$

$$F = \int (I\,ds\,B)\sin\theta = IB\sin\theta \int ds$$

$$\int ds = 2\pi r \qquad F = IB(2\pi r)\sin\theta$$

$$\vec{F} = 2\pi r I B \sin\theta\ \hat{j}$$