On my honor, I have neither given nor received any unauthorized aid/or inappropriate assistance for all sessions of this exam. The work done on the exam is totally my own. I understand that by the school code, violation of these principles will lead to a zero grade and is subject to forth discipline issues.

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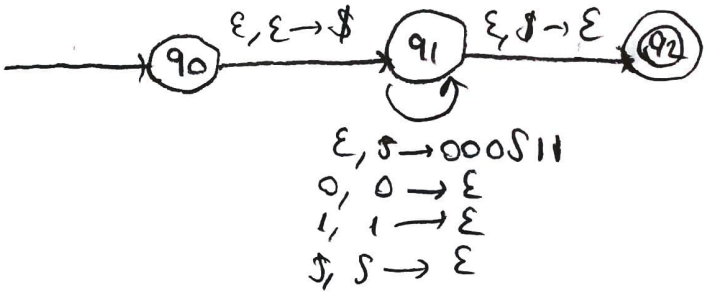
Q1) Design a PDA for the following language (25)

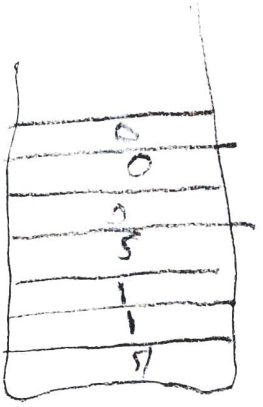
$$L_1 = \{ 0^{3k} 1^{2k} \mid k \geq 0 \}$$

Solution:

I wrote CFG definition and drew it.

G: $S \rightarrow 000S11 \mid \varepsilon$





Q2) $S \rightarrow XY \mid \varepsilon$

$X \rightarrow aY$

$Y \rightarrow Sb$

a) draw the parse tree of aabbbb (10)

b) what is the language generated by this grammar (15)

Solutions are in the next page

Q2) a)

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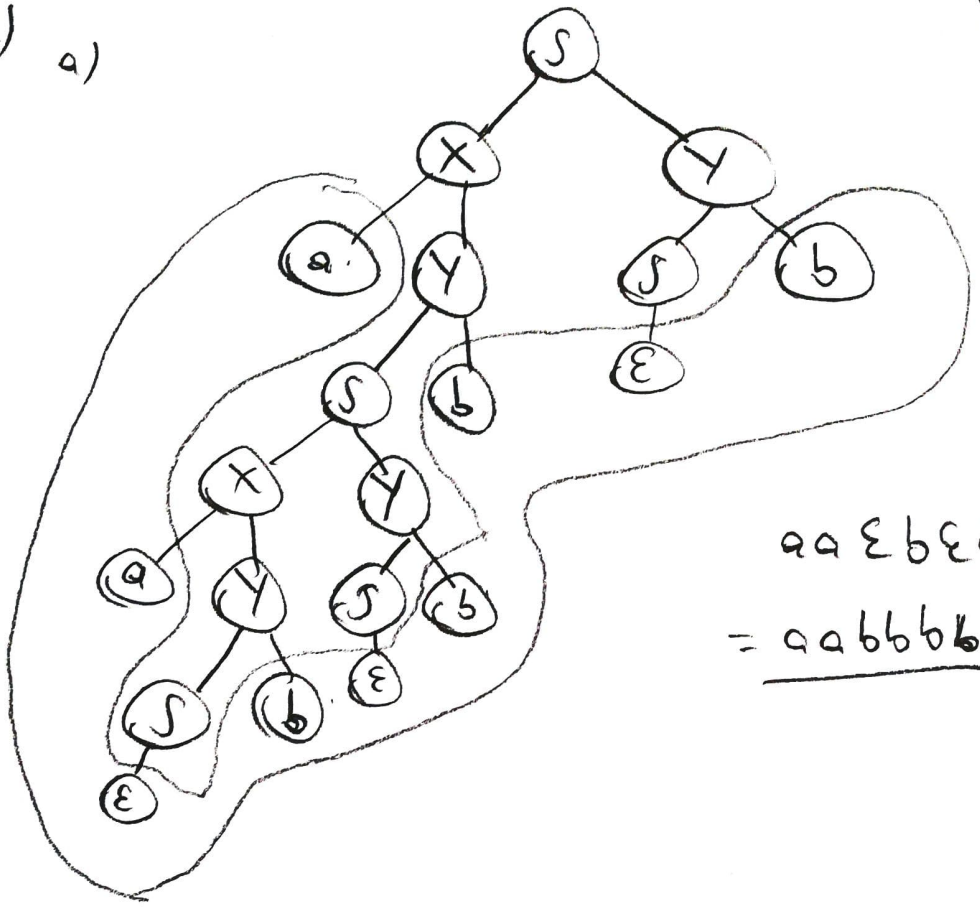

aaεbεbbε

= aabbbb

b)

S → XY → aYY → aSbY → abY → abSb → abb ①

S → XY → XSb → XXYb → XXSbb → XXbb → XaYbb → XaSbbb → XabbbX

→ aXbbb → aSbabbb → ababbb

L = { Language starts with "a" and ends with "b" and number of b's is equal to 2 times number of a's }

$2n_a = n_b$

XXbb → aYXbb → aYaYbb → aSaSbbb → abaXYbbb → abaaYYbbb

abaabbbbb ✓

Q1) (Alternative)

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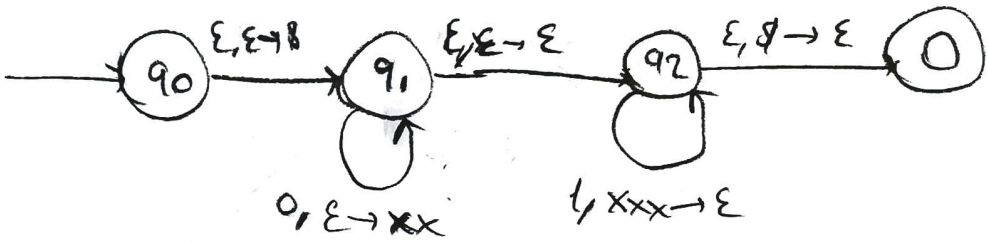
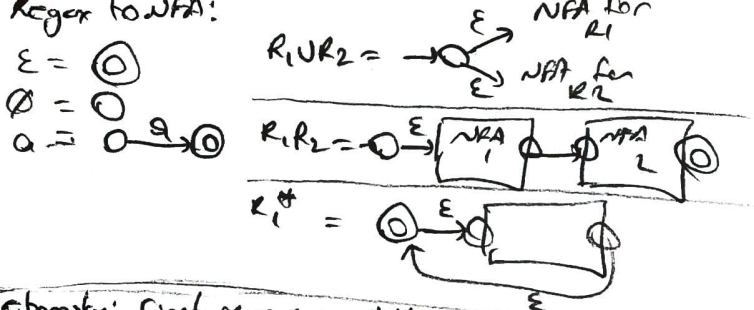


Her bir 0 için 2 tane x push
Her bir 1 için 2 tane x pop

Regex to NFA:



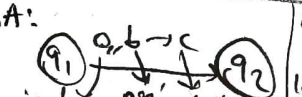
Chomsky: First remove: nullable vars, unit prods, useless prods
then for every symbol a: $T_a \to a$
construct new intermediate variables: $V_1, V_2 \ldots$

$A \to BC$ or $A \to a$

$S \to ABa$, $A \to aab$, $B \to Ac$ ⇒ $S \to AbTa$, $A \to TaTaTb$, $B \to ATc$, $T_a \to a$, $T_b \to b$, $T_c \to c$ ⇒ $S \to AV_1$, $V_1 \to BT_a$ ∴ (generalizing)

$S \to AV_1$, $V_1 \to BT_a$ ⇒ $A \to T_a V_2$, $V_2 \to T_a T_b$

PDA:



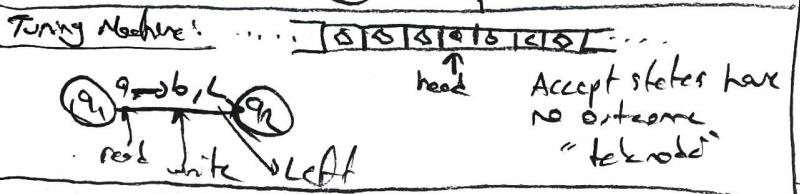
All input consumed AND last state is accepting

PDA Ex: $L(M) = \{vv^R : v \in \{a,b\}^*\}$

1 path enough

(q, u, s) → stack
current (removing) state input

Turing Machine:



Accept states have no outcome "take root"

Variations of Turing:

Stay option, Sem: infinite tape, Multitape, Multidimensional, Nondeterministic

Countable sets:

- The set of strings $\delta = \{a,b,c\}^*$ is countable
  - produce all strings of length 1
  - " " " " " 2
  - ⋮
- The set of all Turing machines are countable.

Uncountable Sets:

If S is an infinite countable set, then 2^S is uncountable

Proof: S is countable $S = \{s_1, s_2, s_3, \ldots\}$

$2^S = \{\emptyset, \{s_1, s_3\}, \ldots\}$

Let's assume (for contradiction) 2^S is countable

$2^S = \{t_1, t_2, t_3, \ldots\}$

Powerset element	Binary
t_1	1 0 0 0 0 ..
t_2	0 1 0 0 0 ..
t_3	1 1 0 1 0 ..
t_4	1 1 0 0 1 ..
⋮	

$t = 0011$

$t \neq t_i$ for every i since they differ in ith bit

$t \in 2^S \Rightarrow t = t_i$ for some i:
Contradiction.

Decidable: A language L is decidable if there is a Turing machine (decider) M which accepts L and halts on every input string.

- Every decidable lang is Turing-Acceptable (not vice versa)
- Decider has 1 accept and 1 reject state.

Ex: is number x prime?

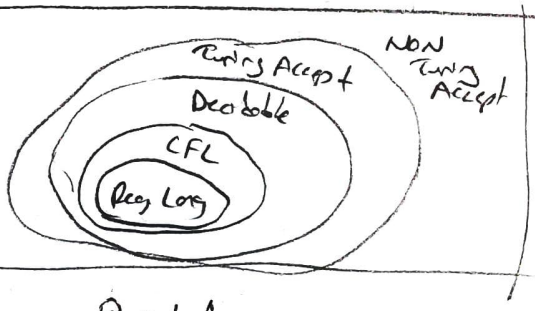
Soln: On input number x =
Divide x with all possible numbers between 1 and $\sqrt{x}$
If any of them divides x
Then reject
Else accept

Ex: Does DFA M accept a finite language?

FINITEDFA = $\{\langle M \rangle : M$ is a DFA that accepts a finite lang$\}$

On input M:
check if there is a walk with a cycle from the initial to an accepting state
if it is
then reject
else accept

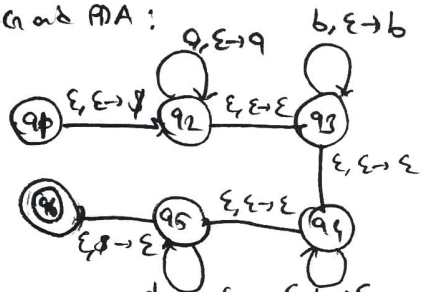
- If a lang is decidable, then its complement is is decidable.



Q and A

Q1) $a^n b^m c^m d^n \mid n,m \geq 0$ CFG and PDA:

$S \to aSb \mid T$
$T \to bTc \mid \varepsilon$
} CFG



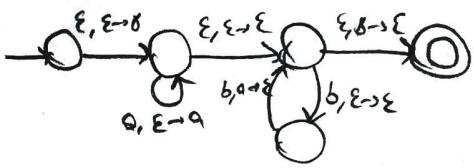
Q2) Define the lang L as: $\langle A, k \rangle$ where A is an NFA, k is a integer and A rejects all strings of length $\leq k$
Show that L is decidable.

1- Simulate A on TM
2- Mark start state
3- for i := 1 to k
  Mark all states that can be reached from marked states in one step
  If a accept state is marked, Return Reject
End for
4- Return Accept.

Q3) Give CFG of $\{w \mid w$ is of even length and starts and ends with the same symbol$\}$

$S \to 0A0 \mid 1A1 \mid \varepsilon$
$A \to 00A \mid 01A \mid 10A \mid 11A \mid \varepsilon$

Q4) PDA $\{a^n b^{2n}\}$

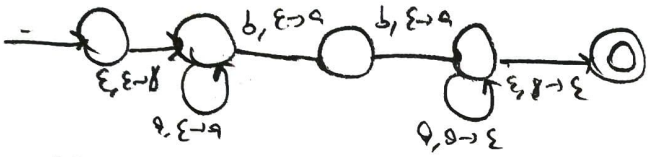


Q5) Let $E_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \emptyset\}$ show that this is decidable.

→ On input A where A is a DFA:

1. Mark the accept state of A. If there is no accept state, ACCEPT
2. Repeat until no new states get marked
3. If a state has an arrow pointing to a marked state, mark it.
4. If start state is marked, REJECT otherwise ACCEPT

Q6) PDA $\{a^n b^{2n} a^{n+2} \mid n \geq 0\}$



CFG:

$S \rightarrow aSa \mid b^2 a^2$

Q7) Describe a Turing machine gives output $x_m, x_{m-1}, \ldots x_2, x_1$ for input $x_1, x_2 \ldots x_{m-1}, x_m$

→ 1. Move to start, find the first element that is not crossed. Call it "a". Cross it. If not found, Go To 4

2. Move right until meeting blank or cross. Move left. Read the element. Call it "b"

3. Swap a and b if they are different. Cross a and b. Go to 1.

4. Sweep from left to right. Restore all crossed elements. Return

Q8) Let B the lang. $\{a,b\}^*$ contains more a's than b

1. Move head to start. Search for a.
2. If there's a a,
   Cross it
   move head to start
   search for b
   If there is a b,
      Cross it
      Go to 1
   Else
      Accept
3. If there is no a,
   Reject.

Stops after finitely many steps. It is a decider

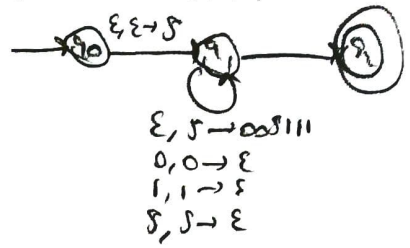
Q9) Let B be the set of all finite strings based on alphabet $\{0,1,\ldots 9\}$ show that B is countable.

0
1
2
⋮
9
00
01
⋮
99
000
001
⋮

Q10) Design a PDA for

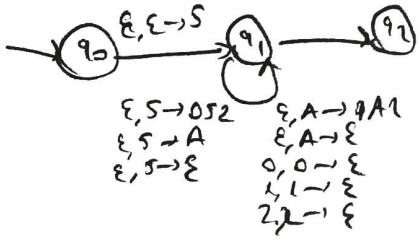
a) $L_1 = \{0^{2k} 1^{3k} \mid k \geq 0\}$

$G_1 = S \rightarrow 00S111 \mid \varepsilon$



b) $L_2 = \{0^a 1^b 2^c \mid a+b=c\}$

$G_2: S \rightarrow 0S2 \mid A1 \mid \varepsilon$
$A \rightarrow 1A2 \mid \varepsilon$



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