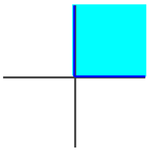
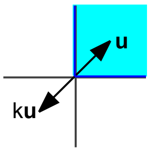
Student: Huseyin Kerem Mican **Date:** 5/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 2.8 Homework

1. A set in $\mathbb{R}^2$ is displayed to the right. Assume the set includes the bounding lines. Give a specific reason why the set H is not a subspace of $\mathbb{R}^2$. (For instance, find two vectors in H whose sum is not in H, or find a vector in H with a scalar multiple that is not in H. Draw a picture.)

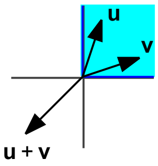


Let **u** and **v** be vectors and let k be a scalar. Select the correct choice below and, if necessary, fill in the answer box within your choice.

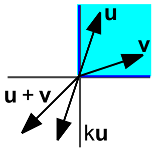
A. The set is not a subspace because it is closed under sums, but not under scalar multiplication. For example, -1 multiplied by (1,1) is not in the set.



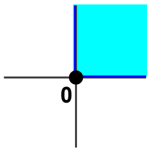
B. The set is not a subspace because it is closed under scalar multiplication, but not under sums. For example, the sum of (3,1) and (1,3) is not in the set.



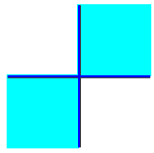
C. The set is not a subspace because it is not closed under either scalar multiplication or sums. For example, ___ multiplied by (1,3) is not in the set, and the sum of (3,1) and (1,3) is not in the set.



D. The set is not a subspace because it does not include the zero vector.

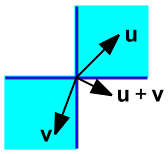


2. A set in $\mathbb{R}^2$ is displayed to the right. Assume the set includes the bounding lines. Give a specific reason why the set H is not a subspace of $\mathbb{R}^2$. (For instance, find two vectors in H whose sum is not in H, or find a vector in H with a scalar multiple that is not in H. Draw a picture.)

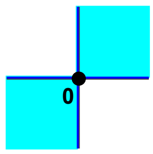


Let **u** and **v** be vectors and let k be a scalar. Select the correct choice below and, if necessary, fill in the answer box within your choice.

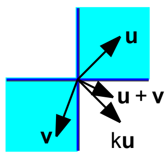
A. The set is not a subspace because it is closed under scalar multiplication, but not under sums. For example, the sum of (2,2) and $(-1,-3)$ is not in the set.



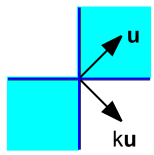
B. The set is not a subspace because it does not include the zero vector.



C. The set is not a subspace because it is not closed under either scalar multiplication or sums. For example, ____ multiplied by (0,1) is not in the set, and the sum of (2,2) and $(-1,-3)$ is not in the set.



D. The set is not a subspace because it is closed under sums, but not under scalar multiplication. For example, ____ multiplied by (0,1) is not in the set.



3. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 5 \\ -6 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ -6 \\ 11 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -3 \\ 1 \\ 16 \end{bmatrix}$. Determine if **w** is in the subspace of $\mathbb{R}^3$ generated by $\mathbf{v}_1$ and $\mathbf{v}_2$.

Is **w** is in the subspace of $\mathbb{R}^3$ generated by $\mathbf{v}_1$ and $\mathbf{v}_2$?

Yes

No

4. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 6 \\ -3 \\ 9 \\ 15 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 5 \\ 13 \\ 3 \\ 6 \end{bmatrix}$, and $\mathbf{u} = \begin{bmatrix} -1 \\ -8 \\ 0 \\ 3 \end{bmatrix}$. Determine if **u** is in the subspace of $\mathbb{R}^4$ generated by $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$.

Is **u** in the subspace of $\mathbb{R}^4$ generated by $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$?

No

Yes

YOU ANSWERED: No

5. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -4 \\ 24 \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} 3 \\ -1 \\ 20 \end{bmatrix}$, and $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}$.

a. How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?
b. How many vectors are in Col A?
c. Is **p** in Col A? Why or why not?

a. How many vectors are in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Select the correct choice below and, if necessary, fill in the answer box within your choice.

- ✔ **A.** 3 (Type a whole number.)
- **B.** There are infinitely many vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

b. How many vectors are in Col A? Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** ____ (Type a whole number.)
- ✔ **B.** There are infinitely many vectors in Col A.

c. Is **p** in Col A? Why or why not?

- **A.** **p** is not in Col A, because A has too few pivot positions.
- ✔ **B.** **p** is in Col A, because the system [A **p**] is consistent.
- **C.** **p** is not in Col A, because the system [A **p**] is not consistent.
- **D.** **p** is in Col A, because A has pivot positions in every row.

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6. Let $\mathbf{v}_1 = \begin{bmatrix} -3 \\ 0 \\ 6 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ -6 \\ 3 \end{bmatrix}$, and $\mathbf{p} = \begin{bmatrix} 4 \\ 8 \\ -2 \end{bmatrix}$. Determine if **p** is in Nul A, where $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}$.

Is **p** in Nul A?

- **A.** Yes, because A**p** is equal to the zero vector.
- **B.** Yes, because the augmented matrix [A **p**] is consistent.
- ✔ **C.** No, because A**p** is not equal to the zero vector.
- **D.** No, because the augmented matrix [A **p**] is not consistent.

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7. Let $\mathbf{v}_1 = \begin{bmatrix} -2 \\ 0 \\ 6 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 3 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ -4 \\ 4 \end{bmatrix}$, and $\mathbf{u} = \begin{bmatrix} -4 \\ 4 \\ 3 \end{bmatrix}$. Determine if **u** is in Nul A, where $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}$.

Is **u** in Nul A?

- No
- ✔ Yes

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8. Give integers p and q such that Nul A is a subspace of $\mathbb{R}^p$ and Col A is a subspace of $\mathbb{R}^q$.

$$A = \begin{bmatrix} 3 & 2 & 1 & -5 \\ -9 & -4 & 1 & 7 \\ 9 & 2 & -5 & 1 \end{bmatrix}$$

Nul A is a subspace of $\mathbb{R}^p$ for p = 4 and Col A is a subspace of $\mathbb{R}^q$ for q = 3 .

9. For the matrix A below, find a nonzero vector in Nul A and a nonzero vector in Col A.

$$A = \begin{bmatrix} 3 & 2 & -5 & 3 \\ -9 & -4 & 19 & -15 \\ 6 & 0 & -18 & 18 \end{bmatrix}$$

Find a nonzero vector in Nul A.

$$\begin{bmatrix} 3 \\ 0 \\ 3 \\ 2 \end{bmatrix}$$

Find a nonzero vector in Col A.

$$\begin{bmatrix} 3 \\ -9 \\ 6 \end{bmatrix}$$

YOU ANSWERED: $\begin{bmatrix} 3 \\ 1 \\ 2 \\ 3 \end{bmatrix}$

10. Determine if the set is a basis for $\mathbb{R}^2$. Justify your answer.

$$\begin{bmatrix} -12 \\ 3 \end{bmatrix}, \begin{bmatrix} -96 \\ 1 \end{bmatrix}$$

Is the given set a basis for $\mathbb{R}^2$?

- **A.** Yes, because these vectors form the columns of an invertible 2×2 matrix. By the invertible matrix theorem, the following statements are equivalent: **A** is an invertible matrix, the columns of **A** form a linearly independent set, and the columns of **A** span $\mathbb{R}^n$.
- **B.** Yes, because these vectors form the columns of a 2×2 matrix that is not invertible. By the invertible matrix theorem, the following statements are equivalent: **A** is a singular matrix, the columns of **A** form a linearly independent set, and the columns of **A** span $\mathbb{R}^n$.
- **C.** No, because these vectors form the columns of an invertible 2×2 matrix. By the invertible matrix theorem, the following statements are equivalent: **A** is a singular matrix, the columns of **A** form a linearly independent set, and the columns of **A** span $\mathbb{R}^n$.
- **D.** No, because these vectors form the columns of a 2×2 matrix that is not invertible. By the invertible matrix theorem, the following statements are equivalent: **A** is an invertible matrix, the columns of **A** form a linearly independent set, and the columns of **A** span $\mathbb{R}^n$.

11. Determine if the set is a basis for $\mathbb{R}^3$. Justify your answer.

$$\begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ 4 \end{bmatrix}$$

Is the given set a basis for $\mathbb{R}^3$?

- **A.** Yes, because these three vectors form the columns of an invertible 3×3 matrix. By the invertible matrix theorem, the following statements are equivalent: **A** is an invertible matrix, the columns of **A** form a linearly independent set, and the columns of **A** span $\mathbb{R}^n$.
- **B.** Yes, because these three vectors form the columns of a 3×3 matrix that is not invertible. By the invertible matrix theorem, the following statements are equivalent: **A** is a singular matrix, the columns of **A** form a linearly independent set, and the columns of **A** span $\mathbb{R}^n$.
- **C.** No, because these three vectors form the columns of a 3×3 matrix that is not invertible. By the invertible matrix theorem, the following statements are equivalent: **A** is an invertible matrix, the columns of **A** form a linearly independent set, and the columns of **A** span $\mathbb{R}^n$.
- **D.** No, because these three vectors form the columns of an invertible 3×3 matrix. By the invertible matrix theorem, the following statements are equivalent: **A** is a singular matrix, the columns of **A** form a linearly independent set, and the columns of **A** span $\mathbb{R}^n$.

YOU ANSWERED: B.

12. Determine if the set is a basis for $\mathbb{R}^3$. Justify your answer.

$$\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix}$$

Is the given set a basis for $\mathbb{R}^3$?

- **A.** Yes, because these vectors form the columns of an invertible 3×3 matrix. By the invertible matrix theorem, the following statements are equivalent: **A** is an invertible matrix, the columns of **A** form a linearly independent set, and the columns of **A** span $\mathbb{R}^n$.
- **B.** Yes, because these two vectors are linearly independent.
- **C.** No, because these vectors form a matrix with only 2 pivot columns. Therefore, these vectors form a basis for a two-dimensional subspace of $\mathbb{R}^3$.
- **D.** No, because these two vectors are linearly dependent.

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13. Determine if the set is a basis for $\mathbb{R}^3$. Justify your answer.

$$\begin{bmatrix} -1 \\ 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 9 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 11 \end{bmatrix}$$

Is the given set a basis for $\mathbb{R}^3$?

- **A.** No, because these vectors form the columns of a 3×3 matrix that is not invertible. By the invertible matrix theorem, the following statements are equivalent: **A** is a singular matrix, the columns of **A** form a linearly independent set, and the columns of **A** span $\mathbb{R}^n$.
- **B.** Yes, because these vectors form the columns of an invertible 3×3 matrix. A set that contains more vectors than there are entries is linearly independent.
- **C.** Yes, because these vectors form the columns of an invertible 3×3 matrix. By the invertible matrix theorem, the following statements are equivalent: **A** is an invertible matrix, the columns of **A** form a linearly independent set, and the columns of **A** span $\mathbb{R}^n$.
- **D.** No, because these vectors do not form the columns of a 3×3 matrix. A set that contains more vectors than there are entries is linearly dependent.

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14. A matrix A and an echelon form of A are shown below. Find a basis for Col A and a basis for Nul A.

$$A = \begin{bmatrix} 4 & 10 & 6 & -10 \\ 7 & 14 & 0 & -7 \\ 2 & 4 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 3 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for Col A.

$$\left\{ \begin{bmatrix} 4 \\ 7 \\ 2 \end{bmatrix}, \begin{bmatrix} 10 \\ 14 \\ 4 \end{bmatrix} \right\}$$

(Simplify your answer. Use a comma to separate answers as needed.)

Find a basis for Nul A.

$$\left\{ \begin{bmatrix} 6 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(Simplify your answer. Use a comma to separate answers as needed.)

15. A matrix A and an echelon form of A are shown below. Find a basis for Col A and a basis for Nul A.

$$A = \begin{bmatrix} 1 & 12 & 13 & -1 & -5 \\ -1 & 6 & 11 & 3 & 11 \\ -2 & -9 & -6 & 7 & 35 \\ 3 & 18 & 15 & -3 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & 9 & 9 & 0 & 2 \\ 0 & 3 & 4 & 0 & -1 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for Col A.

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 12 \\ 6 \\ -9 \\ 18 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 7 \\ -3 \end{bmatrix} \right\}$$

(Use a comma to separate answers as needed. Type an integer or simplified fraction for each matrix element.)

Find a basis for Nul A.

$$\left\{ \begin{bmatrix} 3 \\ -\frac{4}{3} \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ \frac{1}{3} \\ 0 \\ -6 \\ 1 \end{bmatrix} \right\}$$

(Use a comma to separate answers as needed. Type an integer or simplified fraction for each matrix element.)