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Submitted: 07/02/21 9:55am
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Course: Linear Algebra
Assignment: Final Test

1. Diagonalize the following matrix, if possible.

$$\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. For P = ______, $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

B. For P = ______, $D = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$

C. For $P = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

D. The matrix cannot be diagonalized.

2. Determine if **y** is in the subspace of $\mathbb{R}^4$ spanned by the columns of A.

$$\mathbf{y} = \begin{bmatrix} -3 \\ -9 \\ 5 \\ -5 \end{bmatrix}, A = \begin{bmatrix} 8 & -4 & -7 \\ 36 & 6 & -5 \\ -13 & -8 & 2 \\ 14 & -3 & -9 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. The vector **y** is in the subspace spanned by the columns of A because **y** can be written as a linear combination of these columns as follows.

$$\mathbf{y} = \begin{bmatrix} -3 \\ -9 \\ 5 \\ -5 \end{bmatrix} = \left(-\frac{1}{7} \right) \begin{bmatrix} 8 \\ 36 \\ -13 \\ 14 \end{bmatrix} + \left(-\frac{2}{7} \right) \begin{bmatrix} -4 \\ 6 \\ -8 \\ -3 \end{bmatrix} + \left(\frac{3}{7} \right) \begin{bmatrix} -7 \\ -5 \\ 2 \\ -9 \end{bmatrix}$$

(Type integers or simplified fractions.)

B. The vector **y** is not in the subspace spanned by the columns of A.

3. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$, find $\begin{vmatrix} a & b & c \\ 4d+a & 4e+b & 4f+c \\ g & h & i \end{vmatrix}$.

$$\begin{vmatrix} a & b & c \\ 4d+a & 4e+b & 4f+c \\ g & h & i \end{vmatrix} = 8$$ (Simplify your answer.)

4. Find the characteristic polynomial of the matrix, using either a cofactor expansion or the special formula for 3×3 determinants. [Note: Finding the characteristic polynomial of a 3×3 matrix is not easy to do with just row operations, because the variable λ is involved.]

$$\begin{bmatrix} 0 & 2 & 4 \\ 2 & 0 & 3 \\ 4 & 3 & 0 \end{bmatrix}$$

The characteristic polynomial is $-\lambda^3 + 29\lambda + 48$.
(Type an expression using λ as the variable.)

5. For the subspace below, (a) find a basis, and (b) state the dimension.

$$\left\{ \begin{bmatrix} 3a + 6b - c \\ 12a - 4b - 4c \\ -9a + 5b + 3c \\ -3a + b + c \end{bmatrix} : \text{a, b, c in } \mathbb{R} \right\}$$

a. Find a basis for the subspace.

A basis for the subspace is $\left\{ \begin{bmatrix} 3 \\ 12 \\ -9 \\ -3 \end{bmatrix}, \begin{bmatrix} 6 \\ -4 \\ 5 \\ 1 \end{bmatrix} \right\}$.
(Use a comma to separate vectors as needed.)

b. State the dimension.

The dimension is 2.

6. Find the vector **x** determined by the given coordinate vector $[\mathbf{x}]_B$ and the given basis B.

$$B = \left\{ \begin{bmatrix} -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -5 \\ 3 \end{bmatrix} \right\}, \ [\mathbf{x}]_B = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} -19 \\ 29 \end{bmatrix}$$

(Simplify your answers.)

7. The set $B = \{1 + t^2, -t + t^2, 1 - 2t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = -3 - 7t + 2t^2$ relative to B.

$$[\mathbf{p}]_B = \begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix}$$

(Simplify your answers.)

8. Assume that A is row equivalent to B. Find bases for Nul A and Col A.

$$A = \begin{bmatrix} -2 & 6 & -2 & -6 \\ 2 & -9 & -2 & 4 \\ -3 & 12 & 1 & -7 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 5 & 5 \\ 0 & 3 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A basis for Col A is $\left\{ \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 6 \\ -9 \\ 12 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

A basis for Nul A is $\left\{ \begin{bmatrix} -5 \\ -\frac{4}{3} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -\frac{2}{3} \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)