Math | No:

Full Name :

KEY

Math 104 3rd Midterm Exam
(16 April 2016, 11:30-12:30)

**IMPORTANT**
**1.** Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

| Q1 | Q2 | Q3 | Q4 | TOT |
|---|---|---|---|---|
| | | | | |
| 20 pts | 25 pts | 25 pts | 30 pts | 100 pts |

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**Q1.** Let $\{s_n\}$ be the sequence of partial sums of the series

$\sum_{n=1}^{\infty}(\tan(n) - \tan(n-1))$.

a) Find a formula for $s_n$.
b) Use this formula to determine whether the series converges or diverges. If the series converges, find the sum.

(a) $s_n = (\cancel{\tan 1} - \overset{=0}{\tan 0}) + (\cancel{\tan 2} - \cancel{\tan 1}) + (\cancel{\tan 3} - \cancel{\tan 2}) + \cdots (\tan n - \cancel{\tan(n-1)})$

$\boxed{s_n = \tan n}$

(b) $\lim_{n\to\infty} s_n = \lim_{n\to\infty} \tan n$ does not exist.

$\therefore$ $\boxed{\text{The series diverges.}}$

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**Q2.** Determine whether the given sequence converges or diverges. If it converges, find the limit:

$$\left\{\frac{(\ln n)^3}{\sqrt{n}}\right\}$$

$\infty/\infty$

$$\lim_{x\to\infty} \frac{(\ln x)^3}{x^{1/2}} \underset{\text{L'Hospital}}{=} \lim_{x\to\infty} \frac{3(\ln x)^2/x}{1/2x^{1/2}}$$

$\infty/\infty$

$$= \lim_{x\to\infty} \frac{3(\ln x)^2 \cdot 2x^{1/2}}{x} = \lim_{x\to\infty} \frac{6(\ln x)^2}{x^{1/2}}$$

$$\underset{\text{L'Hospital}}{=} \lim_{x\to\infty} \frac{12\ln x/x}{1/2x^{1/2}} = \lim_{x\to\infty} \frac{12\ln x}{x} \cdot 2x^{1/2}$$

$\infty/\infty$

$$= \lim_{x\to\infty} \frac{24\ln x}{x^{1/2}} \underset{\text{L'Hospital}}{=} \lim_{x\to\infty} \frac{24/x}{1/2x^{1/2}}$$

$$= \lim_{x\to\infty} \frac{48}{x} \cdot x^{1/2} = \lim_{x\to\infty} \frac{48}{x^{1/2}} = \boxed{0}$$

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**Q3.** Evaluate the following integral:

$$\int_0^1 xlnxdx$$ Improper integral

$$\int x\ln x\,dx = \frac{x^2\ln x}{2} - \int \frac{x\,dx}{2} = \frac{x^2\ln x}{2} - \frac{x^2}{4} + c$$

$u = \ln x$ $\quad dv = x\,dx$ $\quad$ Integration by parts

$du = \frac{dx}{x}$ $\quad v = \frac{x^2}{2}$

$$\therefore \int_0^1 x\ln x\,dx = \lim_{t\to 0^+} \left. \frac{x^2\ln x}{2} - \frac{x^2}{4} \right|_t^1$$

$$= \frac{\overset{=0}{\ln 1}}{2} - \frac{1}{4} - \lim_{t\to 0^+}\left(\frac{t^2\ln t}{2} - \frac{t^2}{4}\right)$$

$$= -\frac{1}{4} - \lim_{t\to 0^+} \frac{t^2\ln t}{2} \quad 0\cdot\infty$$

$$= -\frac{1}{4} - \lim_{t\to 0^+} \frac{\ln t}{2t^{-2}} \quad \infty/\infty \quad \overset{=}{\text{L'Hospital}}$$

$$= -\frac{1}{4} - \frac{1}{2}\lim_{t\to 0^+} \frac{1/t}{-2t^{-3}}$$

$$= -\frac{1}{4} + \frac{1}{4}\lim_{t\to 0^+} \frac{t^3}{t} = -\frac{1}{4} + \underbrace{\frac{1}{4}\lim_{t\to 0^+} t^2}_{0}$$

$$= \boxed{-\frac{1}{4}}$$

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**Q4.** Determine whether the following series converge or diverge:

a) $\sum_{n=1}^{\infty} \frac{\sqrt{n}+3}{n^{\pi}+5}$

Limit Comparison Test with

$\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p = \pi - \frac{1}{2} > 1$ (convergent)

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n}+3}{n^{\pi}+5}}{\frac{1}{n^{\pi-1/2}}} = \lim_{n\to\infty} \frac{n^{\pi-1/2}\left(n^{1/2}\right)\left(1+\underbrace{3n^{-1/2}}_{\to 0}\right)}{n^{\pi}\left(1+\underbrace{5n^{-\pi}}_{\to 0}\right)}$$

$$= \lim_{n\to\infty} 1 = 1 \neq 0, \infty$$

$\therefore$ The series $\boxed{\text{converges.}}$

b) $\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$.

Ratio Test

$$\rho = \lim_{n\to\infty} \frac{\frac{2^{n+1}(n+1)!\,(n+1)!}{(2(n+1))!}}{\frac{2^n n! n!}{(2n)!}}$$

$$= \lim_{n\to\infty} \frac{2^n \cdot 2\,(n+1)^2\, n!\, n!}{(2n+2)(2n+1)\cdot(2n)!} \cdot \frac{(2n)!}{2^n\, n!\, n!}$$

$$= \lim_{n\to\infty} \frac{2(n+1)^2}{(2n+2)(2n+1)} = \lim_{n\to\infty} \frac{2(n+1)^2}{2(n+1)(2n+1)}$$

$$= \lim_{n\to\infty} \frac{n+1}{2n+1} = \frac{1}{2} < 1$$

$\therefore$ The series $\boxed{\text{converges.}}$