Math

ISTANBUL ŞEHİR UNIVERSITY

No:

Full Name :

KEY

Math 104 Final Exam
(11 June 2018, 13:00-14:30)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 5 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones, smart watches and electronic devices are to be kept shut and out of sight. All cell phones and smart watches are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	Q5	TOT
20 pts	25 pts	20 pts	30 pts	25 pts	120 pts

Q1. Find the MacLaurin series of the function $f(x) = x^2\cos(x^2)$.

(Hint: To solve this question, you may use Taylor or MacLaurin series that you know.)

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$ converges for all x

$$\Rightarrow \cos x^2 = \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$$

$$\Rightarrow x^2\cos x^2 = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n)!} \quad \text{all } x}$$

Math

No:

Full Name :

Q2. Consider the region bounded between the parabola $x = y^2 - 12$ and the line $y = x$.

a) Sketch the region.

b) Set up an integral (or the sum of integrals) for the area of this region.

DO NOT EVALUATE THE INTEGRAL.

(a) Pts of intersection :

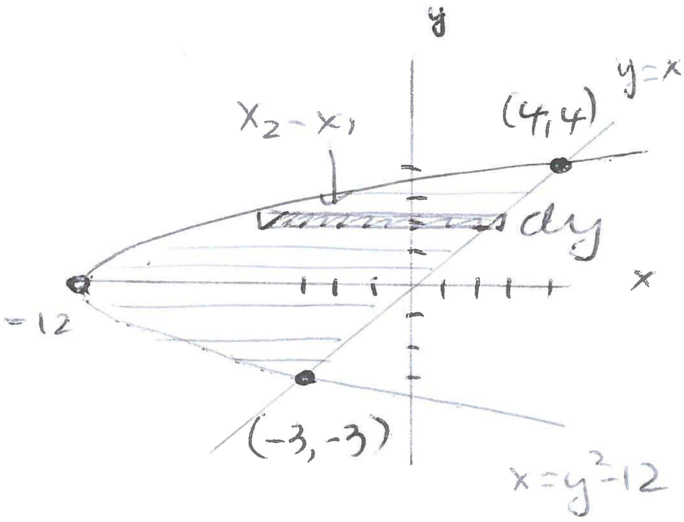
$y^2 - 12 = y$

$y^2 - y - 12 = 0$

$(y-4)(y+3) = 0$

$y = 4, -3$

$(-3,-3)$ & $(4,4)$



(b) $A = \int_{-3}^{4} (x_2 - x_1)\,dy$

$= \int_{-3}^{4} [y - (y^2 - 12)]\,dy$

$= \boxed{\int_{-3}^{4} (y - y^2 + 12)\,dy}$

Math

No:

Full Name :

Q3. Evaluate the following limit, if it exists:

$$\underbrace{\lim_{x\to\infty}\left(1+\sin\frac{3}{x}\right)^{x}}_{y} \qquad 1^{\infty}$$

$$\ln y = \ln\left(1+\sin\frac{3}{x}\right)^{x} = x\ln\left(1+\sin\frac{3}{x}\right)$$

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty} x\ln\left(1+\sin\frac{3}{x}\right) \qquad \infty\cdot 0$$

$$= \lim_{x\to\infty}\frac{\ln\left(1+\sin 3/x\right)}{1/x} \qquad 0/0$$

$$\underset{\text{L'Hospital}}{=} \lim_{x\to\infty}\frac{\frac{1}{1+\sin 3/x}\cdot\cos 3/x\cdot\left(-3/x^2\right)}{-1/x^2}$$

$$= 3\lim_{x\to\infty}\frac{\cos 3/x}{1+\sin 3/x}$$

$$= 3\cdot\frac{1}{1} = 3$$

$$\ln y \to 3 \Rightarrow \lim y = \boxed{e^3}.$$

Math No:

Full Name :

Q4. Evaluate the following integrals:

a) $\int sin^2 2\theta cos^3 2\theta d\theta$ $= \int \sin^2 2\theta \cdot \underbrace{\cos^2 2\theta}_{1-\sin^2 2\theta} \cdot \cos 2\theta d\theta$

$= \frac{1}{2}\int (\sin^2 2\theta - \sin^4 2\theta) 2\cos 2\theta d\theta$

$= \frac{1}{2}\int (u^2 - u^4) du$ $\left\{ \begin{array}{l} u = \sin 2\theta \\ du = 2\cos 2\theta d\theta \end{array} \right.$

$= \frac{1}{2}\left(\frac{u^3}{3} - \frac{u^5}{5} \right) + C = \boxed{\frac{1}{6}\sin^3 2\theta - \frac{1}{10}\sin^5 2\theta + C}$

b) $\int \sqrt{x}\, e^{\sqrt{x}} dx$ $t = \sqrt{x} \Rightarrow t^2 = x \Rightarrow 2t\,dt = dx$

$= \int 2t\, e^t \cdot t\,dt = 2\int t^2 e^t dt$

Integration by parts $u = t^2$ $du = 2t\,dt$ $dv = e^t dt$ $v = e^t$

$= 2\left\{ t^2 e^t - 2\int t e^t dt \right\}$

Integration by parts $u = t$ $du = dt$ $dv = e^t dt$ $v = e^t$

$= 2\left\{ t^2 e^t - 2\left(t e^t - \int e^t dt \right) \right\}$

$= 2t^2 e^t - 4t e^t + 4e^t + C$

$= \boxed{2x e^{\sqrt{x}} - 4\sqrt{x} e^{\sqrt{x}} + 4e^{\sqrt{x}} + C}$

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Q5. Given the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}$

(a) Find the radius of convergence.
(b) Find the interval of convergence.

Generalized Ratio Test

$$\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+2}(x+2)^{n+1}}{(n+1)2^{n+1}}}{\frac{(-1)^{n+1}(x+2)^n}{n2^n}} \right|$$

$$= \frac{|x+2|}{2} \underbrace{\lim_{n\to\infty} \frac{n}{n+1}}_{1} = \frac{|x+2|}{2}$$

converges absolutely when

$$\frac{|x+2|}{2} < 1 \Rightarrow |x+2| < 2$$

$$\Rightarrow -2 < x+2 < 2 \Rightarrow -4 < x < 0$$

Diverges when $x > 0$ or $x < -4$

(a) $\therefore$ $\boxed{\text{Radius of convergence} = 2}$

Check the end pts :

$$x = 0 \Rightarrow \sum \frac{(-1)^{n+1} 2^n}{n 2^n} = \sum \frac{(-1)^{n+1}}{n}$$

alternating harmonic series, converges (done in class)

$$x = -4 \Rightarrow \sum \frac{(-1)^{n+1}(-2)^n}{n 2^n} = \sum -\frac{1}{n}$$

harmonic series, diverges

(b) $\therefore$ $\boxed{\text{Interval of convergence} \quad -4 < x \le 0}$