Math No:

Full Name : KEY

Math 104 3rd Midterm Exam
(29 December 2016, 18:00-19:00)

**IMPORTANT**

**1.** Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

| Q1 | Q2 | Q3 | Q4 | TOT |
|---|---|---|---|---|
| | | | | |
| 6 pts | 6 pts | 6 pts | 6 pts | 24 pts |

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**Q1.** (a) Find a formula for the general term $a_n$ of the sequence

$$\{3/5, -4/25, 5/125, -6/625, 7/3125, ...\}$$

$$a_n = (-1)^{n+1}\frac{n+2}{5^n}$$

or

$$\left\{(-1)^{n+1}\frac{n+2}{5^n}\right\}_{n=1}^{\infty}$$

(b) Is the following geometric series convergent or divergent? $\sum_{n=0}^{\infty} 2^{2n}3^{1-n}$

$$\sum_{n=0}^{\infty} (2^2)^n \cdot 3 \cdot 3^{-n} = 3\sum_{n=0}^{\infty} (4/3)^n, \quad 4/3 > 1$$

the series is divergent!

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**Q2.** Determine whether the series given below converges or diverges:

$$\sum_{n=0}^{\infty}(-1)^{n+1}\frac{n^2 2^n}{n!}$$

$\rho = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$, if $\rho < 1$, the series converges absolutely

$$= \lim_{n\to\infty} \frac{\frac{(n+1)^2 2^{n+1}}{(n+1)!}}{\frac{n^2 2^n}{n!}}$$

$$= \lim_{n\to\infty} \frac{(n+1)^{\cancel{2}}\,2.\cancel{2^n}}{\cancel{(n+1)}\,\cancel{n!}} \cdot \frac{\cancel{n!}}{n^2 \cancel{2^n}}$$

$$= 2\underbrace{\lim_{n\to\infty}\frac{n+1}{n^2}}_{0}$$

$\rho = 0 < 1$

The series converges absolutely.

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**Q3.** Given the power series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$

(a) Find the radius of convergence.

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-3)^{n+1} \cdot x^{n+1}}{\sqrt{n+2}} \cdot \frac{\sqrt{n+1}}{(-3)^n x^n} \right|$$

$$\rho = \lim_{n\to\infty} \left| \frac{(-3)\cdot\cancel{(-3)^n} \cdot x \cdot \cancel{x^n}}{\sqrt{n+2}} \cdot \frac{\sqrt{n+1}}{\cancel{(-3)^n} \cdot \cancel{x^n}} \right|$$

$$= 3|x| \underbrace{\lim_{n\to\infty} \frac{\sqrt{1+1/n}}{\sqrt{1+2/n}}}_{1} = 3|x| < 1, \text{ condition to converge}$$

$$|x| < 1/3 \Rightarrow -\frac{1}{3} < x < \frac{1}{3} \ ; \quad R = 1/3$$

(b) Test the end points and find the interval of convergence.

$$\underline{\underline{x = -\frac{1}{3}}} \Rightarrow \sum_{n=0}^{\infty} \frac{(-3)^n(-1/3)^n}{\sqrt{n+1}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}}$$

By Comparison Test: let us compare it with $\sum \frac{1}{n^{1/2}}$, which is a divergent p-series with $p = 1/2$.

$$\lim_{n\to\infty} \frac{1/n^{1/2}}{1/\sqrt{n+1}} = \lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n}} = \sqrt{1+\cancel{1/\infty}_{0}} = 1, \text{ so they are comparable.}$$

Since $\sum \frac{1}{\sqrt{n}}$ diverges, $\sum \frac{1}{\sqrt{n+1}}$ diverges too.

$$\underline{\underline{x = +\frac{1}{3}}} \Rightarrow \sum_{n=0}^{\infty} \frac{(-3)^n(1/3)^n}{\sqrt{n+1}} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}}$$

The result satisfies the conditions for an alternating series: (i) all $a_n$ are positive, (ii) Non-increasing, (iii) $\lim_{n\to\infty} a_n \to 0$ ; therefore the series converges conditionally

$$-\frac{1}{3} < x \leq 1/3$$

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**Q4.** Find the Maclaurin series of the function $f(x) = x\cos 2x$.

(Hint: To solve this question, you may use Taylor or Maclaurin series that you know.)

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\cos 2x = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n}}{(2n)!}$$

$$f(x) = x\cos 2x = \sum_{n=0}^{\infty} (-1)^n \frac{x(2x)^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n} x^{2n+1}}{(2n)!} \quad //$$

or

$$= \sum_{n=0}^{\infty} (-1)^n \frac{4^n x^{2n+1}}{(2n)!}$$