Student: Huseyin Kerem Mican **Date:** 7/1/21	**Instructor:** Taylan Sengul **Course:** Linear Algebra	**Assignment:** Section 4.6 Homework

1. Assume that the matrix A is row equivalent to B. Without calculations, list rank A and dim Nul A. Then find bases for Col A, Row A, and Nul A.

$$A = \begin{bmatrix} 1 & -4 & 8 & -3 \\ -1 & 2 & -3 & -1 \\ 6 & -8 & 8 & 14 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 0 & -2 & 5 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rank A = 2

dim Nul A = 2

A basis for Col A is $\left\{ \begin{bmatrix} 1 \\ -1 \\ 6 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -8 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

A basis for Row A is $\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -5 \\ 4 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

A basis for Nul A is $\left\{ \begin{bmatrix} 2 \\ \frac{5}{2} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

2. Assume that the matrix A is row equivalent to B. Without calculations, list rank A and dim Nul A. Then find bases for Col A, Row A, and Nul A.

$$A = \begin{bmatrix} 1 & -2 & -2 & 1 & 2 \\ -2 & 4 & 6 & 0 & -14 \\ -3 & 6 & 2 & -7 & 6 \\ 2 & -4 & -10 & -4 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 & -2 & 1 & 2 \\ 0 & 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rank A = 3

dim Nul A = 2

A basis for Col A is $\left\{ \begin{bmatrix} 1 \\ -2 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \\ 2 \\ -10 \end{bmatrix}, \begin{bmatrix} 2 \\ -14 \\ 6 \\ 0 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

A basis for Row A is $\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

A basis for Nul A is $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

3. If a 7×6 matrix A has rank 2, find dim Nul A, dim Row A, and rank A^T.

dim Nul A = 4

dim Row A = 2

rank A^T = 2

4. Suppose a 7×9 matrix A has six pivot columns. What is dim Nul A? Is Col A = $\mathbb{R}^6$? Why or why not?

dim Nul A = 3 (Simplify your answer.)

Is Col A = $\mathbb{R}^6$? Why or why not?

- **A.** No, because Col A is a subspace of $\mathbb{R}^9$.
- **B.** No, because Col A is a subspace of $\mathbb{R}^7$.
- **C.** Yes, because the number of pivot positions in A is 6.
- **D.** Yes, because dim Col A = rank A = 6.

5. If the null space of a 9×12 matrix A is 6-dimensional, what is the dimension of the column space of A?

dim Col A = 6 (Simplify your answer.)

6. If the null space of a 3×9 matrix A is 7-dimensional, what is the dimension of the row space of A?

dim Row A = 2

7. If A is a 6×5 matrix, what is the largest possible rank of A? If A is a 5×6 matrix, what is the largest possible rank of A? Explain your answers.

Select the correct choice below and fill in the answer box(es) to complete your choice.

- **A.** The rank of A is equal to the number of non-pivot columns in A. Since there are more rows than columns in a 6×5 matrix, the rank of a 6×5 matrix must be equal to ______. Since there are 5 rows in a 5×6 matrix, there are a maximum of 5 pivot positions in A. Thus, there is 1 non-pivot column. Therefore, the largest possible rank of a 5×6 matrix is ______.
- **B.** The rank of A is equal to the number of pivot positions in A. Since there are only 5 columns in a 6×5 matrix, and there are only 5 rows in a 5×6 matrix, there can be at most 5 pivot positions for either matrix. Therefore, the largest possible rank of either matrix is 5.
- **C.** The rank of A is equal to the number of columns of A. Since there are 5 columns in a 6×5 matrix, the largest possible rank of a 6×5 matrix is ______. Since there are 6 columns in a 5×6 matrix, the largest possible rank of a 5×6 matrix is ______.

8. Is it possible that all solutions of a homogeneous system of fourteen linear equations in seventeen variables are multiples of one fixed nonzero solution? Discuss.

Consider the system as $A\mathbf{x} = \mathbf{0}$, where A is a 14×17 matrix. Choose the correct answer below.

- **A.** No. Since A has 14 pivot positions, rank A = 14. By the Rank Theorem, dim Nul A = 14 − rank A = 0. Since Nul A = 0, it is impossible to find a single vector in Nul A that spans Nul A.
- **B.** Yes. Since A has at most 14 pivot positions, rank A ≤ 14. By the Rank Theorem, dim Nul A = 17 − rank A ≥ 3. Since there is at least one free variable in the system, all solutions are multiples of one fixed nonzero solution.
- **C.** Yes. Since A has 14 pivot positions, rank A = 14. By the Rank Theorem, dim Nul A = 14 − rank A = 0. Thus, all solutions are multiples of one fixed nonzero solution.
- **D.** No. Since A has at most 14 pivot positions, rank A ≤ 14. By the Rank Theorem, dim Nul A = 17 − rank A ≥ 3. Thus, it is impossible to find a single vector in Nul A that spans Nul A.