Math

İSTANBUL ŞEHİR UNIVERSITY

No:

KEY

Full Name :

Math 104 1st Midterm Exam
(27 February 2017, 17:00-18:00)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
10 pts	40 pts	20 pts	30 pts	100 pts

Q1. Find the derivative given below:

$$\frac{d}{dx}\int_2^x \frac{sin\theta d\theta}{\theta} = \boxed{\frac{\sin x}{x}}$$, by the Fundamental Thm of Calculus

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Q2. Evaluate the following integrals:

a) $\int x(x-1)^{19}\,dx$

Let $u = x-1 \Rightarrow du = dx$

$\Downarrow$

$x = u+1$

$$= \int (u+1)u^{19}\,du$$

$$= \int (u^{20} + u^{19})\,du = \frac{u^{21}}{21} + \frac{u^{20}}{20} + C$$

$$= \boxed{\frac{(x-1)^{21}}{21} + \frac{(x-1)^{20}}{20} + C}$$

b) $\int \frac{\sin^2\theta}{\cos^4\theta}\,d\theta$

$$= \int \tan^2\theta \sec^2\theta\, d\theta$$

$u = \tan\theta \Rightarrow du = \sec^2\theta\, d\theta$

$$\Rightarrow \int u^2\,du = \frac{u^3}{3} + C$$

$$= \boxed{\frac{\tan^3\theta}{3} + C}$$

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Q2. Evaluate the following integral: 3

$$\int \sqrt{\frac{x^3-3}{x^{11}}}\,dx = \int \sqrt{\frac{x^3(1-3/x^3)}{x^{11}}}\,dx = \int \sqrt{\frac{1-3/x^3}{x^8}}\,dx$$

$$= \int \frac{\sqrt{1-3x^{-3}}}{x^4}\,dx$$

Let $u = 1-3x^{-3} \Rightarrow du = 9x^{-4}dx$

$$\Rightarrow \frac{1}{9}\int \frac{9\sqrt{1-3x^{-3}}}{x^4}\,dx = \frac{1}{9}\int u^{1/2}\,du$$

$$= \frac{1}{9}\cdot\frac{u^{3/2}}{3/2} + C$$

$$= \boxed{\frac{2}{29}\left(1-3/x^3\right)^{3/2} + C}$$

Q4. Consider the region in the first quadrant bounded by the curve $y = x^2$, the line $x + y = 2$, and the x-axis.

a) Sketch the region.
b) Write an integral (or a sum of integrals) for the area of this region.
c) Evaluate this integral.

Points of intersection:

$x^2 = 2 - x$

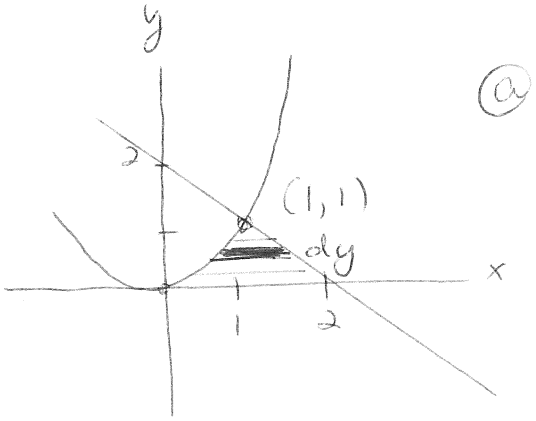
$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = 1 \Rightarrow y = 1$

$x = -2 \Rightarrow y = 4$

(a)



(b) $A = \int_0^1 (x_2 - x_1)\,dy$

$= \int_0^1 (2 - y - \sqrt{y})\,dy$

(c) $A = 2y - \frac{y^2}{2} - \frac{y^{3/2}}{3/2}\Big|_0^1$

$= 2 - \frac{1}{2} - \frac{2}{3} - 0$

$= \frac{12 - 3 - 4}{6}$

$= \boxed{\frac{5}{6}}$