MATH 259
Numerical Methods
Introduction to MATLAB
Part 2

Original Slides by
Gerald W. Recktenwald

The latest version of this PDF file, along with other supplemental material for the book, can be found at `www.prenhall.com/recktenwald`.

Version 0.97 August 28, 2001

Overview

- Script m-files
  - ▷ Creating
  - ▷ Side effects
- Function m-files
  - ▷ Syntax of I/O parameters
  - ▷ Text output
  - ▷ Primary and secondary functions
- Flow control
  - ▷ Relational operators
  - ▷ Conditional execution of blocks
  - ▷ Loops
- Vectorization
  - ▷ Using vector operations instead of loops
  - ▷ Preallocation of vectors and matrices
  - ▷ Logical and array indexing
- Programming tricks
  - ▷ Variable number of I/O parameters
  - ▷ Indirect function evaluation
  - ▷ Inline function objects
  - ▷ Global variables

Preliminaries

- Programs are contained in m-files
  - ▷ Plain text files – not binary files produced by word processors
  - ▷ File must have ".m" extension
- m-file must be in the path
  - ▷ MATLAB maintains its own internal path
  - ▷ The path is the list of directories that MATLAB will search when looking for an m-file to execute.
  - ▷ A program can exist, and be free of errors, but it will not run if MATLAB cannot find it.
  - ▷ Manually modify the path with the **path**, **addpath**, and **rmpath** built-in functions.

  - ▷ . . . or use interactive Path Browser

Script Files

- Not really programs
  - ▷ No input/output parameters
  - ▷ Script variables are part of workspace
- Useful for tasks that never change
- Useful as a tool for documenting homework:
  - ▷ Write a *function* that solves the problem for *arbitrary* parameters
  - ▷ Use a `script` to run function for specific parameters required by the assignment

Free Advice: Scripts offer no advantage over functions.
Functions have many advantages over scripts.
Always use functions instead of scripts.

Script to Plot $\tan(\theta)$ (1)

Enter statements in file called `tanplot.m`

1. Choose **New** -> **Script** from **File** menu
2. Enter lines listed below

Contents of `tanplot.m`:

```
theta = linspace(1.6,4.6);
tandata = tan(theta);
plot(theta,tandata);
xlabel('\theta    (radians)');
ylabel('tan(\theta)');
grid on;
axis([min(theta) max(theta) -5 5]);
```

3. Choose **Save As** from **File** menu

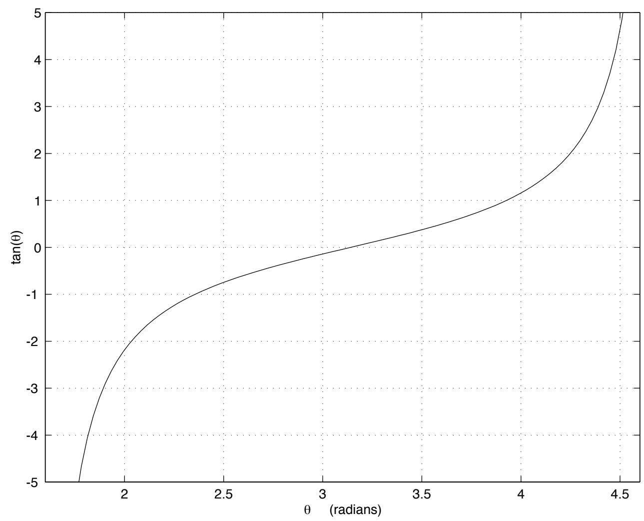
   Save as `tanplot.m`

4. Run it

```
>> tanplot
```

Script to Plot $\tan(\theta)$ (2)

Running `tanplot` produces the following plot:



If the plot needs to be changed, edit the `tanplot` script and rerun it. This saves the effort of typing in the commands. The `tanplot` script also provides written documentation of how to create the plot.

Example: Put a % character at beginning of the line containing the `axis` command, then rerun the script

Script Side-Effects (1)

All variables created in a script file are added to the workplace. This may have undesirable effects because

- Variables already existing in the workspace may be overwritten
- The execution of the script can be affected by the state variables in the workspace.

Example: *The* `easyplot` *script*

```
% easyplot:  Script to plot data in file xy.dat

%  Load the data
D = load('xy.dat');        %  D is a matrix with two columns
x = D(:,1);  y = D(:,2);   %  x in 1st column, y in 2nd column

plot(x,y)                  %  Generate the plot and label it
xlabel('x axis, unknown units')
ylabel('y axis, unknown units')
title('Plot of generic x-y data set')
```

Script Side-Effects (2)

The `easyplot` script affects the workspace by creating three variables:

```
>> clear
>> who
                              (no variables show)
>> easyplot
>> who

Your variables are:

D          x          y
```

The `D`, `x`, and `y` variables are left in the workspace. These generic variable names might be used in another sequence of calculations in the same MATLAB session.

Function m-files (1)

- Functions are subprograms:
  - ▷ Functions use input and output parameters to communicate with other functions and the command window
  - ▷ Functions use *local* variables that exist only while the function is executing. Local variables are distinct from variables of the same name in the workspace or in other functions.
- Input parameters allow the same calculation procedure (same algorithm) to be applied to different data. Thus, function m-files are *reusable*.
- Functions can call other functions.
- Specific tasks can be encapsulated into functions. This modular approach enables development of structured solutions to complex problems.

Function m-files (2)

Syntax:

The first line of a function m-file has the form:

```
function [outArgs] = funName(inArgs)
```

outArgs are enclosed in `[ ]`

- *outArgs* is a comma-separated list of variable names
- `[ ]` is optional if there is only one parameter
- functions with no *outArgs* are legal

inArgs are enclosed in `( )`

- *inArgs* is a comma-separated list of variable names
- functions with no *inArgs* are legal

Function Input and Output (1)

Examples: *Demonstrate use of I/O arguments*

- `twosum.m` — two inputs, no output
- `threesum.m` — three inputs, one output
- `addmult.m` — two inputs, two outputs

Function Input and Output (2)

twosum.m

```
function twosum(x,y)
% twosum  Add two matrices
%         and print the result
x+y
```

threesum.m

```
function s = threesum(x,y,z)
% threesum  Add three variables
%           and return the result
s = x+y+z;
```

addmult.m

```
function [s,p] = addmult(x,y)
% addmult  Compute sum and product
%          of two matrices
s = x+y;
p = x*y;
```

Function Input and Output Examples (3)

Example: *Experiments with* `twosum`*:*

```
>> twosum(2,2)
ans =
      4

>> x = [1 2];  y = [3 4];
>> twosum(x,y)
ans =
     4     6

>> A = [1 2; 3 4];  B = [5 6; 7 8];
>> twosum(A,B);
ans =
     6     8
    10    12

>> twosum('one','two')
ans =
   227   229   212
```

Note: The result of the addition inside `twosum` is exposed because the `x+y` expression does not end in a semicolon.

Function Input and Output Examples (4)

Example: *Experiments with* `twosum`*:*

```
>> clear
>> x = 4; y = -2;
>> twosum(1,2)
ans =
     3

>> x+y
ans =
     2

>> disp([x y])
     4    -2

>> who

Your variables are:

ans       x         y
```

In this example, the `x` and `y` variables defined in the workspace are distinct from the `x` and `y` variables defined in `twosum`. The `x` and `y` in `twosum` are *local* to `twosum`.

Function Input and Output Examples (5)

Example: *Experiments with* **`threesum`***:*

```
>> a = threesum(1,2,3)
a =
     6

>> threesum(4,5,6)
ans =
   15

>> b = threesum(7,8,9);
```

Note: The last statement produces no output because the assignment expression ends with a semicolon. The value of 24 is stored in `b`.

Function Input and Output Examples (6)

Example: *Experiments with* **`addmult`***:*

```
>> [a,b] = addmult(3,2)
a =
     5
b =
     6

>> addmult(3,2)
ans =
     5

>> v = addmult(3,2)
v =
     5
```

Note: `addmult` *requires* two return variables. Calling `addmult` with no return variables or with one return variable causes undesired behavior.

Summary of Input and Output Parameters

- Values are communicated through input arguments and output arguments.
- Variables defined inside a function are *local* to that function. Local variables are invisible to other functions and to the command environment.
- The number of return variables should match the number of output variables provided by the function.

Text Input and Output

It is usually desirable to print results to the screen or to a file. On rare occasions it may be helpful to prompt the user for information not already provided by the input parameters to a function.

Inputs to functions:

- **input** function can be used.
- Input parameters to functions are preferred.

Text output from functions:

- **disp** function for simple output
- **fprintf** function for formatted output.

Prompting for User Input

The **input** function can be used to prompt the user for numeric or string input.

```
>> x = input('Enter a value for x');

>> yourName = input('Enter your name','s');
```

Prompting for input betrays the MATLAB novice. It is a nuisance to competent users, and makes automation of computing tasks impossible.

Free Advice: Avoid using the **input** function. Rarely is it necessary. All inputs to a function should be provided via the input parameter list.

Text Output with disp and fprintf

Output to the command window is achieved with either the **disp** function or the **fprintf** function. Output to a file requires the **fprintf** function.

disp	Simple to use. Provides limited control over appearance of output.
fprintf	Slightly more complicated than `disp`. Provides total control over appearance of output.

The disp function (1)

Syntax:

```
disp(outMatrix)
```

where *outMatrix* is *either* a string matrix or a numeric matrix.

Examples: *Numeric output*

```
>> disp(5)
     5

>> x = 1:3;  disp(x)
     1     2     3

>> y = 3-x;  disp([x; y])
     1     2     3
     2     1     0

>> disp([x y])
     1     2     3     2     1     0

>> disp([x' y])
???  All matrices on a row in the bracketed expression
 must have the same number of rows.
```

Note: The last statement shows that the input to **disp** must be a legal matrix.

The disp function (2)

Examples: *String output*

```
>> disp('Hello, world!)
Hello, world!

>> s = 'MATLAB 6 is built with LAPACK';  disp(s)
MATLAB 6 is built with LAPACK

>> t = 'Earlier versions used LINPACK and EISPACK';
>> disp([s; t])
???  All rows in the bracketed expression
 must have the same number of columns.

>> disp(char(s,t))
MATLAB 6 is built with LAPACK
Earlier versions used LINPACK and EISPACK
```

The `disp[s; t]` expression causes an error because `s` has fewer elements than `t`. The built-in **char** function constructs a string matrix by putting each input on a separate row and padding the rows with blanks as necessary.

```
>> S = char(s,t);
>> length(s),  length(t),  length(S(1,:))
ans =
    29
ans =
    41
ans =
    41
```

The num2str function (1)

The **num2str** function is often used to with the **disp** function to create a labeled output of a numeric value.

Syntax:

```
stringValue = num2str(numericValue)
```

converts *numericValue* to a string representation of that numeric value.

Examples:

```
>> num2str(pi)
ans =
3.1416

>> A = eye(3)
A =
     1     0     0
     0     1     0
     0     0     1

>> S = num2str(A)
S =
1  0  0
0  1  0
0  0  1
```

The num2str function (2)

Although A and S appear to contain the same values, they are not equivalent. A is a numeric matrix, and S is a string matrix.

```
>> clear
>> A = eye(3);  S = num2str(A);  B = str2num(S);
>> A-S
??? Error using ==> -
Matrix dimensions must agree.

>> A-B
ans =
     0     0     0
     0     0     0
     0     0     0

>> whos
  Name      Size         Bytes  Class

  A         3x3             72  double array
  B         3x3             72  double array
  S         3x7             42  char array
  ans       3x3             72  double array

Grand total is 48 elements using 258 bytes
```

Using num2str with disp (1)

Combine **num2str** and **disp** to print a labeled output of a numeric value

```
>> x = sqrt(2);
>> outString = ['x = ',num2str(x)];
>> disp(outString)
x = 1.4142
```

or, build the input to `disp` on the fly

```
>> disp(['x = ',num2str(x)]);
x = 1.4142
```

Using num2str with disp (2)

The

```
disp(['x = ',num2str(x)]);
```

construct works when `x` is a row vector, but not when `x` is a column vector or matrix

```
>> z = y';
>> disp(['z = ',num2str(z)])
???  All matrices on a row in the bracketed expression
 must have the same number of rows.
```

Instead, use two `disp` statements to display column of vectors or matrices

```
>> disp('z = ');  disp(z)
z =
     1
     2
     3
     4
```

Using num2str with disp (3)

The same effect is obtained by simply entering the name of the variable with no semicolon at the end of the line.

```
>> z                    (enter z and press return)
z =
     1
     2
     3
     4
```

The format function

The **format** function controls the precision of **disp** output.

```
>> format short
>> disp(pi)
      3.1416

>> format long
>> disp(pi)
      3.14159265358979
```

Alternatively, a second parameter can be used to control the precision of the output of **num2str**

```
>> disp(['pi = ',num2str(pi,2)])
pi = 3.1

>> disp(['pi = ',num2str(pi,4)])
pi = 3.142

>> disp(['pi = ',num2str(pi,8)])
pi = 3.1415927
```

The fprintf function (1)

Syntax:

```
fprintf(outFormat,outVariables)
fprintf(fileHandle,outFormat,outVariables)
```

uses the *outFormat* string to convert *outVariables* to strings that are printed. In the first form (no *fileHandle*) the output is displayed in the command window. In the second form, the output is written to a file referred to by the *fileHandle* (more on this later).

Notes to C programmers:

1. The MATLAB **fprintf** function uses single quotes to define the format string.
2. The **fprintf** function is vectorized. (See examples below.)

Example:

```
>> x = 3;
>> fprintf('Square root of %g is %8.6f\n',x,sqrt(x));

The square root of 3 is 1.732051
```

The fprintf function (2)

The *outFormat* string specifies how the *outVariables* are converted and displayed. The *outFormat* string can contain any text characters. It also must contain a *conversion code* for each of the *outVariables*. The following table shows the basic conversion codes.

Code	Conversion instruction
`%s`	format as a string
`%d`	format with no fractional part (integer format)
`%f`	format as a floating-point value
`%e`	format as a floating-point value in scientific notation
`%g`	format in the most compact form of either `%f` or `%e`
`\n`	insert newline in output string
`\t`	insert tab in output string

The fprintf function (3)

In addition to specifying the type of conversion (e.g. `%d`, `%f`, `%e`) one can also specify the width and precision of the result of the conversion.

Syntax:

```
%wd
%w.pf
%w.pe
```

where w is the number of characters in the width of the final result, and p is the number of digits to the right of the decimal point to be displayed.

Examples:

Format String	Meaning
`%14.5f`	use floating point format to convert a numerical value to a string 14 characters wide with 5 digits after the decimal point
`%12.3e`	use scientific notation format to convert numerical value to a string 12 characters wide with 3 digits after the decimal point. The 12 characters for the string include the e+00 or e−00 (or e+000 or e−000 on Windows™)

The fprintf function (4)

More examples of conversion codes

Value	%8.4f	%12.3e	%10g	%8d
2	2.0000	2.000e+00	2	2
sqrt(2)	1.4142	1.414e+00	1.41421	1.414214e+00
sqrt(2e-11)	0.0000	4.472e-06	4.47214e-06	4.472136e-06
sqrt(2e11)	447213.5955	4.472e+05	447214	4.472136e+05

The fprintf function (5)

The **fprintf** function is vectorized. This enables printing of vectors and matrices with compact expressions. It can also lead to some undesired results.

Examples:

```
>> x = 1:4;  y = sqrt(x);
>> fprintf('%9.4f\n',y)
   1.0000
   1.4142
   1.7321
   2.0000
```

The `%9.4f` format string is reused for each element of `y`. The recycling of a format string may not always give the intended result.

```
>> x = 1:4;  y = sqrt(x);
>> fprintf('y = %9.4f\n',y)
y =    1.0000
y =    1.4142
y =    1.7321
y =    2.0000
```

The fprintf function (6)

Vectorized **fprintf** cycles through the `outVariables` *by columns*. This can also lead to unintended results

```
>> A = [1 2 3; 4 5 6; 7 8 9]
A =
     1     2     3
     4     5     6
     7     8     9

>> fprintf('%8.2f  %8.2f  %8.2f\n',A)
    1.00      4.00      7.00
    2.00      5.00      8.00
    3.00      6.00      9.00
```

How to print a table with fprintf (1)

Many times a tabular display of results is desired.

The `boxSizeTable` function listed below, shows how the **fprintf** function creates column labels and formats numeric data into a tidy tabular display. The **for** loop construct is discussed later in these slides.

```
function boxSizeTable
% boxSizeTable  Demonstrate tabular output with fprintf

% --- labels and sizes for shiping containers
label = char('small','medium','large','jumbo');
width = [5; 5; 10; 15];
height = [5; 8; 15; 25];
depth = [15; 15; 20; 35];
vol = width.*height.*depth/10000;   %  volume in cubic meters

fprintf('\nSizes of boxes used by ACME  Delivery Service\n\n');
fprintf('size          width     height    depth    volume\n');
fprintf('               (cm)      (cm)      (cm)     (m^3)\n');
for i=1:length(width)
  fprintf('%-8s  %8d  %8d  %8d   %9.5f\n',...
          label(i,:),width(i),height(i),depth(i),vol(i))
end
```

Note: **length** is a built-in function that returns the number of elements in a vector. `width`, `height`, and `depth` are local variables in the `boxSizeTable` function.

How to print a table with fprintf (2)

Example: *Running* `boxSizeTable` *gives*

```
>> boxSizeTable

Sizes of boxes used by ACME Delivery Service

size          width     height     depth     volume
               (cm)       (cm)      (cm)      (m^3)
small            5          5         15     0.03750
medium           5          8         15     0.06000
large           10         15         20     0.30000
jumbo           15         25         35     1.31250
```

The fprintf function (3)

File Output with **fprintf** requires creating a *file handle* with the **fopen** function. All aspects of formatting and vectorization discussed for screen output still apply.

Example: *Writing contents of a vector to a file.*

```
x = ...                             %  content of x
fout = fopen('myfile.dat','wt');    %  open myfile.dat
fprintf(fout,'   k      x(k)\n');
for k=1:length(x)
   fprintf(fout,'%4d     %5.2f\n',k,x(k));
end
fclose(fout)                          % close myfile.dat
```

Flow Control (1)

To enable the implementation of computer algorithms, a computer language needs control structures for

- Repetition: looping or iteration
- Conditional execution: branching
- Comparison

We will consider these in reverse order.

Comparison
Comparison is achieved with *relational operators*. Relational operators are used to test whether two values are equal, or whether one value is greater than or less than another. The result of a comparison may also be modified by *logical operators*.

Relational Operators (1)

Relational operators are used in comparing two values.

Operator	Meaning
<	less than
<=	less than or equal to
>	greater than
>=	greater than or equal to
~=	not equal to

The result of applying a relational operator is a logical value, i.e. the result is either *true* or *false*.

In MATLAB any nonzero value, including a non-empty string, is equivalent to *true*. Only zero is equivalent to *false*.

Note: The <=, >=, and ~= operators have "=" as the *second* character. =<, => and =~ are not valid operators.

Relational Operators (2)

The result of a relational operation is a true or false value.

Examples:

```
>> a = 2;  b = 4;
>> aIsSmaller = a < b
aIsSmaller =
     1

>> bIsSmaller = b < a
bIsSmaller =
     0
```

Relational operations can also be performed on matrices of the same shape, e.g.,

```
>> x = 1:5;   y = 5:-1:1;
>> z = x>y
z =
     0     0     0     1     1
```

Logical Operators

Logical operators are used to combine logical expressions (with "and" or "or"), or to change a logical value with "not"

Operator	Meaning
&	and
|	or
~	not

Examples:

```
>> a = 2;  b = 4;
>> aIsSmaller = a < b;
>> bIsSmaller = b < a;
>> bothTrue = aIsSmaller & bIsSmaller
bothTrue =
     0

>> eitherTrue = aIsSmaller | bIsSmaller
eitherTrue =
     1

>> ~eitherTrue
ans =
     0
```

Logical and Relational Operators

Summary:

- Relational operators involve comparison of two values.
- The result of a relational operation is a logical (True/False) value.
- Logical operators combine (or negate) logical values to produce another logical value.
- There is always more than one way to express the same comparison

Conditional Execution

Conditional Execution or Branching:

As the result of a comparison, or another logical (true/false) test, selected blocks of program code are executed or skipped.

Conditional execution is implemented with `if`, `if...else`, and `if...elseif` constructs, or with a `switch` construct.

There are three types of `if` constructs

1. Plain `if`
2. `if...else`
3. `if...elseif`

if Constructs

Syntax:

```
if expression
    block of statements
end
```

The *block of statements* is executed only if the *expression* is true.

Example:

```
if a < 0
   disp('a is negative');
end
```

One line format uses comma after `if` *expression*

```
if a < 0,  disp('a is negative'); end
```

if. . . else

Multiple choices are allowed with **if. . . else** and **if. . . elseif** constructs

```
if x < 0
   error('x is negative; sqrt(x) is imaginary');
else
   r = sqrt(x);
end
```

if. . . elseif

It's a good idea to include a default **else** to catch cases that don't match preceding **if** and **elseif** blocks

```
if x > 0
   disp('x is positive');
elseif x < 0
   disp('x is negative');
else
   disp('x is exactly zero');
end
```

The switch Construct

A switch construct is useful when a test value can take on discrete values that are either integers or strings.

Syntax:

```
switch  expression
  case value1,
    block of statements
  case value2,
    block of statements
    ⋮
  otherwise,
    block of statements
end
```

Example:

```
color = '...';    %  color is a string
switch color
    case 'red'
        disp('Color is red');
    case 'blue'
        disp('Color is blue');
    case 'green'
        disp('Color is green');
    otherwise
        disp('Color is not red, blue, or green');
end
```

Flow Control (3)

Repetition or Looping

A sequence of calculations is repeated until *either*

1. All elements in a vector or matrix have been processed

or

2. The calculations have produced a result that meets a predetermined termination criterion

Looping is achieved with `for` loops and `while` loops.

for loops

`for` loops are most often used when each element in a vector or matrix is to be processed.

Syntax:

```
for index = expression
    block of statements
end
```

Example: *Sum of elements in a vector*

```
x = 1:5;             %  create a row vector
sumx = 0;            %  initialize the sum
for k = 1:length(x)
   sumx = sumx + x(k);
end
```

for loop variations

Example: *A loop with an index incremented by two*

```
for k = 1:2:n
   ...
end
```

Example: *A loop with an index that counts down*

```
for k = n:-1:1
   ...
end
```

Example: *A loop with non-integer increments*

```
for x = 0:pi/15:pi
   fprintf('%8.2f  %8.5f\n',x,sin(x));
end
```

Note: In the last example, `x` is a *scalar* inside the loop. Each time through the loop, `x` is set equal to one of the *columns* of `0:pi/15:pi`.

while loops (1)

`while` loops are most often used when an iteration is repeated until some termination criterion is met.

Syntax:

```
while expression
   block of statements
end
```

The *`block of statements`* is executed as long as *`expression`* is true.

Example: *Newton's method for evaluating* $\sqrt{x}$

$$r_k = \frac{1}{2}\left(r_{k-1} + \frac{x}{r_{k-1}}\right)$$

```
r = ...                % initialize
rold = ...
while abs(rold-r) > delta
   rold = r;
   r = 0.5*(rold + x/rold);
end
```

while loops (2)

It is (almost) always a good idea to put a limit on the number of iterations to be performed by a `while` loop.

An improvement on the preceding loop,

```
maxit = 25;
it = 0;
while abs(rold-r) > delta & it<maxit
   rold = r;
   r = 0.5*(rold + x/rold);
   it = it + 1;
end
```

while loops (3)

The **break** and **return** statements provide an alternative way to exit from a loop construct. **break** and **return** may be applied to `for` loops or `while` loops.

break is used to escape from an enclosing `while` or `for` loop. Execution continues at the end of the enclosing loop construct.

return is used to force an exit from a **function**. This can have the effect of escaping from a loop. Any statements following the loop that are in the function body are skipped.

The break command

Example: *Escape from a* `while` *loop*

```
function k = breakDemo(n)
% breakDemo  Show how the "break" command causes
%            exit from a while loop.
%            Search a random vector to find index
%            of first element greater than 0.8.
%
% Synopsis:  k = breakDemo(n)
%
% Input:     n = size of random vector to be generated
%
% Output:    k = first (smallest) index in x such that x(k)>0.8
x = rand(1,n);
k = 1;
while k<=n
   if x(k)>0.8
     break
   end
   k = k + 1;
end
fprintf('x(k)=%f   for k = %d   n = %d\n',x(k),k,n);

%  What happens if loop terminates without finding x(k)>0.8 ?
```

The return command

Example: *Return from within the body of a function*

```
function k = returnDemo(n)
% returnDemo  Show how the "return" command
%             causes exit from a function.
%             Search a random vector to find
%             index of first element greater than 0.8.
%
% Synopsis:  k = returnDemo(n)
%
% Input:     n = size of random vector to be generated
%
% Output:    k = first (smallest) index in x
%                such that x(k)>0.8
x = rand(1,n);
k = 1;

while k<=n
   if x(k)>0.8
     return
   end
   k = k + 1;
end

%  What happens if loop terminates without finding x(k)>0.8 ?
```

Comparison of break and return

break is used to escape the current `while` or `for` loop.

return is used to escape the current function.

```
function k = demoBreak(n)

...

while k<=n
   if x(k)>0.8
     break;
   end
   k = k + 1;
end
```

jump to end of enclosing "`while ... end`" block

```
function k = demoReturn(n)

...

while k<=n
   if x(k)>0.8
     return;
   end
   k = k + 1;
end
```

return to calling function

Vectorization

Vectorization is the use of vector operations (MATLAB expressions) to process all elements of a vector or matrix. Properly vectorized expressions are equivalent to looping over the elements of the vectors or matrices being operated upon. A vectorized expression is more compact and results in code that executes faster than a non-vectorized expression.

To write vectorized code:

- Use vector operations instead of loops, where applicable
- Pre-allocate memory for vectors and matrices
- Use vectorized indexing and logical functions

Non-vectorized code is sometimes called "*scalar code*" because the operations are performed on scalar elements of a vector or matrix instead of the vector as a whole.

Free Advice: Code that is slow and correct is always better than code that is fast and incorrect. **Start with scalar code, then vectorize as needed.**

Replace Loops with Vector Operations

Scalar Code

```
for k=1:length(x)
   y(k) = sin(x(k))
end
```

Vectorized equivalent

```
y = sin(x)
```

Preallocate Memory

The following loop increases the size of s on each pass.

```
y = ...      %  some computation to define y
for j=1:length(y)
   if y(j)>0
     s(j) = sqrt(y(j));
   else
     s(j) = 0;
   end
end
```

Preallocate s before assigning values to elements.

```
y = ...      %  some computation to define y
s = zeros(size(y));
for j=1:length(y)
   if y(j)>0
     s(j) = sqrt(y(j));
   end
end
```

Vectorized Indexing and Logical Functions (1)

Thorough vectorization of code requires use of **array indexing** and **logical indexing**.

Array Indexing:

Use a vector or matrix as the "subscript" of another matrix:

```
>> x = sqrt(0:4:20)
x =
       0    2.0000    2.8284    3.4641    4.0000    4.47210

>> i = [1 2 5];
>> y = x(i)
y =
     0     2     4
```

The `x(i)` expression selects the elements of `x` having the indices in `i`. The expression `y = x(i)` is equivalent to

```
k = 0;
for i = [1 2 5]
  k = k + 1;
  y(k) = x(i);
end
```

Vectorized Indexing and Logical Functions (2)

Logical Indexing:

Use a vector or matrix as the mask to select elements from another matrix:

```
>> x = sqrt(0:4:20)
x =
     0    2.0000    2.8284    3.4641    4.0000    4.47210

>> j = find(rem(x,2)==0)
j =
     1     2     5

>> z = x(j)
z =
     0     2     4
```

The `j` vector contains the indices in `x` that correspond to elements in `x` that are integers.

Vectorized Indexing and Logical Functions (3)

Example: *Vectorization of Scalar Code*

We just showed how to pre-allocate memory in the code snippet:

```
y = ...      %  some computation to define y
s = zeros(size(y));
for j=1:length(y)
   if y(j)>0
     s(j) = sqrt(y(j));
   end
end
```

In fact, the loop can be replaced entirely by using logical and array indexing

```
y = ...                 %  some computation to define y
s = zeros(size(y));
i = find(y>0);          %  indices such that y(i)>0
s(y>0) = sqrt(y(y>0))
```

If we don't mind redundant computation, the preceding expressions can be further contracted:

```
y = ...      %  some computation to define y
s = zeros(size(y));
s(y>0) = sqrt(y(y>0))
```

Vectorized Copy Operations (1)

Example: *Copy entire columns (or rows)*

Scalar Code

```
[m,n] = size(A);  %  assume A and B have
                  %  same number of rows
for i=1:m
    B(i,1) = A(i,1);
end
```

Vectorized Code

```
B(:,1) = A(:,1);
```

Vectorized Copy Operations (2)

Example: *Copy and transform submatrices*

Scalar Code

```
for j=2:3
   B(1,j) = A(j,3);
end
```

Vectorized Code

```
B(1,2:3) = A(2:3,3)'
```

Deus ex Machina

MATLAB has features to solve some recurring programming problems:

- Variable number of I/O parameters
- Indirect function evaluation with **feval**
- In-line function objects (MATLAB version 5.x)
- Global Variables

Variable Input and Output Arguments (1)

Each function has internal variables, `nargin` and `nargout`.

Use the value of `nargin` at the beginning of a function to find out how many input arguments were supplied.

Use the value of `nargout` at the end of a function to find out how many input arguments are expected.

Usefulness:

- Allows a single function to perform multiple related tasks.
- Allows functions to assume default values for some inputs, thereby simplifying the use of the function for some tasks.

Variable Input and Output Arguments (2)

Consider the built-in **plot** function

	Inside the `plot` function	
	`nargin`	`nargout`
`plot(x,y)`	2	0
`plot(x,y,'s')`	3	0
`plot(x,y,'s--')`	3	0
`plot(x1,y1,'s',x2,y2,'o')`	6	0
`h = plot(x,y)`	2	1

The values of `nargin` and `nargout` are determined when the `plot` function is invoked.

Indirect Function Evaluation (1)

The **feval** function allows a function to be evaluated indirectly.

Usefulness:

- Allows routines to be written to process an arbitrary $f(x)$.
- Separates the reusable algorithm from the problem-specific code.

Indirect Function Evaluation (2)

```
>> fsum('sin',0,pi,5)
ans =
    2.4142

>> fsum('cos',0,pi,5)
ans =
    0
```

Use of feval

```
function s = fsum(fun,a,b,n)
%  FSUM  Computes the sum of function values, f(x), at n equally
%        distributed points in an interval a <= x <= b
%
%  Synopsis:   s = fsum(fun,a,b,n)
%
%  Input:  fun = (string) name of the function to be evaluated
%          a,b = endpoints of the interval
%          n   = number of points in the interval

x = linspace(a,b,n);     %  create points in the interval
y = feval(fun,x);        %  evaluate function at sample points
s = sum(y);              %  compute the sum
```

```
function y = sincos(x)
%  SINCOS   Evaluates sin(x)*cos(x) for any input x
%
%  Synopsis:  y = sincos(x)
%
%  Input:     x = angle in radians, or vector of angles in radians
%
%  Output:    y = value of product sin(x)*cos(x) for each element in x

y = sin(x).*cos(x);
```

Inline Function Objects

MATLAB version 5.x introduced object-oriented programming extensions. Though OOP is an advanced and somewhat subtle way of programming, in-line function objects are simple to use and offer great program flexibility.

Instead of

```
function y = myFun(x)
y = x.^2 - log(x);
```

Use

```
myFun = inline( 'x.^2 - log(x)' );
```

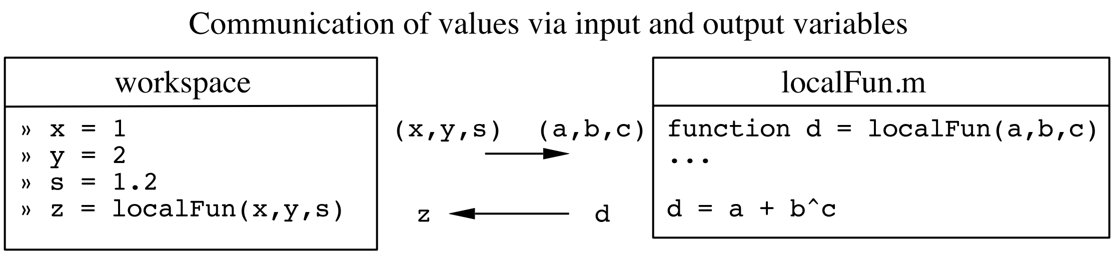
Both definitions of `myFun` allow expressions like

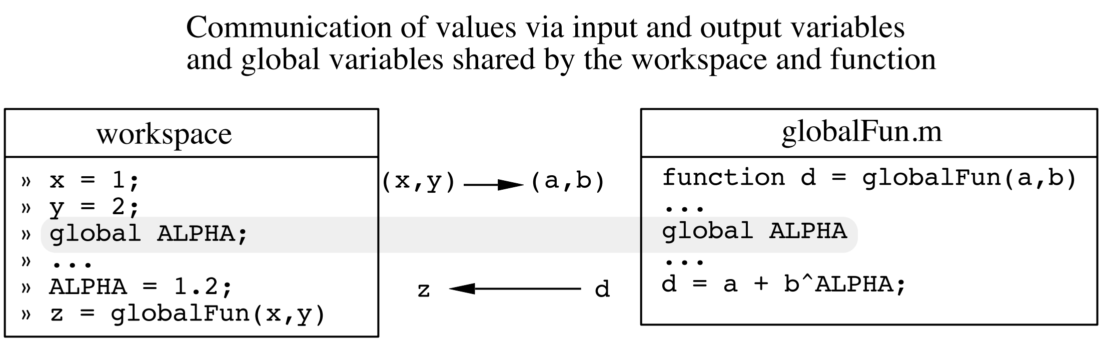
```
z = myFun(3);

s = linspace(1,5);
t = myFun(s);
```

Usefulness:

- Eliminates need to write separate m-files for functions that evaluate a simple formula.
- Useful in all situations where **feval** is used.

Global Variables





Usefulness:

- Allows bypassing of input parameters if no other mechanism (such as pass-through parameters) is available.
- Provides a mechanism for maintaining program state (GUI applications)