Gauss Law in Magnetism

Remember we talked about ϕ (flux) when we studied Electric field: $\phi_E = \vec{E} \cdot \vec{A}$ or $\phi_E = \int \vec{E} \cdot d\vec{A}$

Similarly we can talk about magnetic flux:

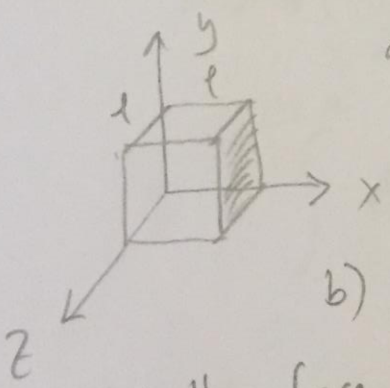
$\phi_B = \vec{B} \cdot \vec{A}$ or $\phi_B = \int \vec{B} \cdot d\vec{A}$

The magnetic field lines begin at the north pole & end at the south pole. Since each "magnet" has a south & north pole, the B lines close onto themselves.

$\therefore$ ϕ through any closed surface is zero.

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Ex: P. 47 : $B = 5\hat{i} + 4\hat{j} + 3\hat{k}$ $\quad \ell = 2.5$ cm



a) ϕ through the shaded face? (right face)

$\vec{A} = \ell^2 \hat{i}$ $\quad \phi = (5\hat{i} + 4\hat{j} + 3\hat{k}) \cdot \ell^2 \hat{i}$

$\phi = 5\ell^2$

b) top face: $\vec{A} = \ell^2 \hat{j}$ $\quad \vec{B} = 4\ell^2$

bottom face: $\vec{A} = -\ell^2 \hat{j}$ $\quad \Phi = -4\ell^2$

left face: $\vec{A} = -\ell^2 \hat{i}$ $\quad \phi = -5\ell^2$

back face: $\vec{A} = -\ell^2 \hat{k}$ $\quad \phi = -3\ell^2$

front face: $A = \ell^2 \hat{k}$ $\quad \phi = 3\ell^2$

Total flux through the cube?

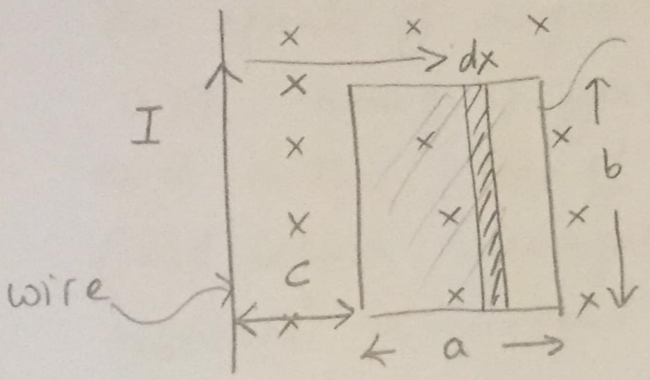
$$\phi_T = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5$$

$$\phi_T = 5\ell^2 + 4\ell^2 - 4\ell^2 - 5\ell^2 - 3\ell^2 + 3\ell^2$$

$$\phi_T = 0$$

Ex. 30.7 ②

Magnetic Flux through a rectangular loop



Find the flux through the loop.

B is created by The current in the wire.

$$B = \frac{\mu_0 I}{2\pi x}$$

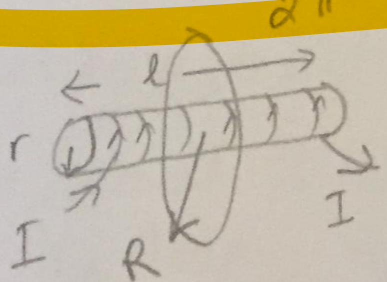
So B is not uniform; it decreases as we go away from the wire. The flux through the shaded area segment $dA = b\,dx$

$$d\phi = \left(\frac{\mu_0 I}{2\pi x}\right)(b\,dx) \quad (\vec{B} \text{ is parallel to } d\vec{A})$$

$$\phi = \int_c^{c+a} \frac{\mu_0 I b}{2\pi x}\,dx = \frac{\mu_0 I b}{2\pi}\int_c^{c+a}\frac{dx}{x}$$

$$\phi = \frac{\mu_0 I b}{2\pi} \ln \frac{c+a}{c}$$

P.48/



a) $B = \mu_0 n I$ $\quad n = \frac{N}{\ell}$ $\quad R = 5\,cm$

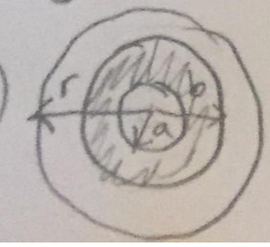
$r = 1.25\,cm$ $\quad \ell = 30\,cm$ $\quad N = 300$

$I = 12\,A$

$n = \frac{N}{\ell} = \frac{300}{0.3} = 1000$ $\quad B = (4\pi\times10^{-7})(10^{3})(12) = 48\pi\times10^{-4}$

$\phi = \vec{B}\cdot\pi r^2 = 48\,\pi^2\times10^{-4}\,(1.25\times10^{-2})^2 = 7.4\times10^{-6}$ Web.

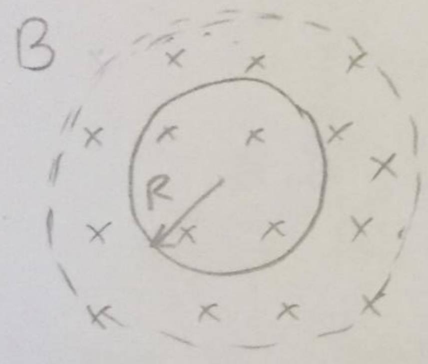
b)



$\phi = B\,(\pi b^2 - \pi a^2)$

Faraday's Law: In this chapter we study the effects of magnetic fields that vary (change) with time: If you have a closed loop of wire and if this loop of wire is in a magnetic field which changes with time, there "appears" a current in the loop, hence an EMF ε (like a battery). This is an induced ε.

Let us see how this happens step by step.



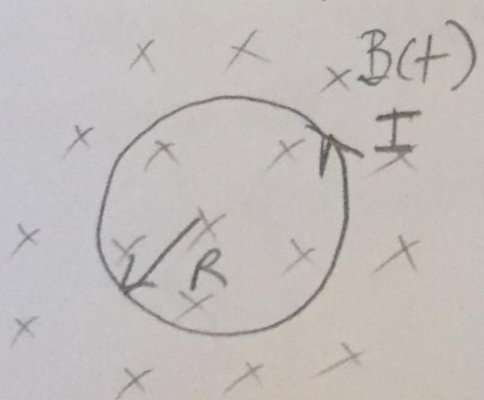
We have a loop of wire, no current in the loop. area of the loop: πR^2

$\phi = B \pi R^2$

assume B is created inside a solenoid, where $B = \mu_0 n I$. if I is time dependent $I = I(t)$

$B(t) = \mu_0 n I(t)$, so, $\phi(t) = B(t) \pi R^2$

as soon as I is $I(t)$, there is a current in the loop:



Since there is a current in the loop, there must be a power supply, an ε in the loop.

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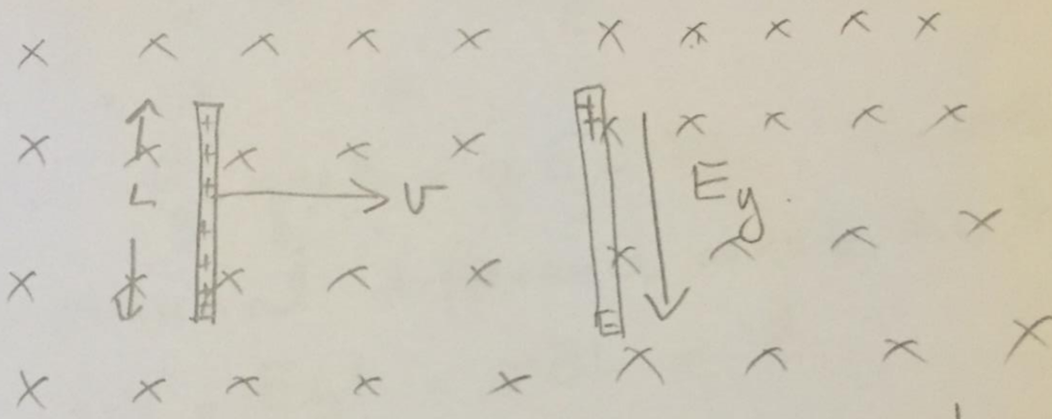
This is the induced EMF: $\mathcal{E}$

$$\mathcal{E} = -\frac{d\phi(t)}{dt} \quad \text{Faraday's Law}$$

$\phi = \vec{B}\cdot\vec{A} = BA\cos\theta$. So here the magnitude of B or A or θ may change with time and all of these cases will create $\mathcal{E}$.

Now consider a conductor bar moving in a B.



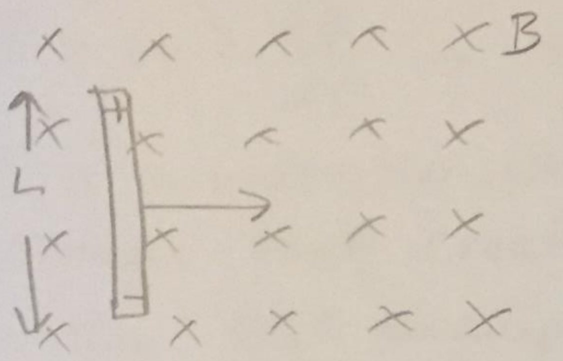
Assume we have + charges in the bar. Since these charges have a velocity in B, there is a force on them: $\vec{F} = q\vec{v}\times\vec{B}$ which is upward. So + charges move up. So, (−) charges stay at the bottom. This creates an electrical field downwards, which in turn creates a new force on + charges downward.

$\uparrow F = qvB$

$+q$

$\downarrow F_y = qE_y$

When these forces are equal the net force on the charges is zero, so charges stop moving. But now we have net positive charge at the top of the bar & net negative charge at the bottom.



So this moving metal bar is like a battery. There is an induced $\mathcal{E}$.

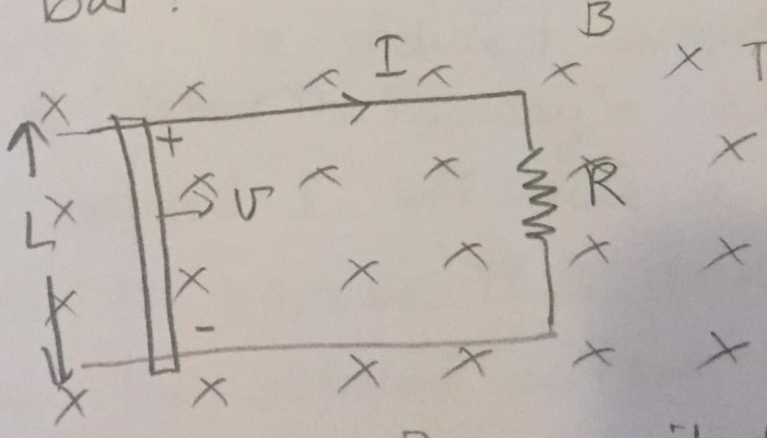
$$F_B = qvB = qE_y = F_c \qquad E_y = vB$$

The potential difference across the bar:

$$\Delta V = E_y L = vBL = \mathcal{E}$$

$$\boxed{\mathcal{E} = vBL}$$

Now if we connect a loop to this moving metal bar:



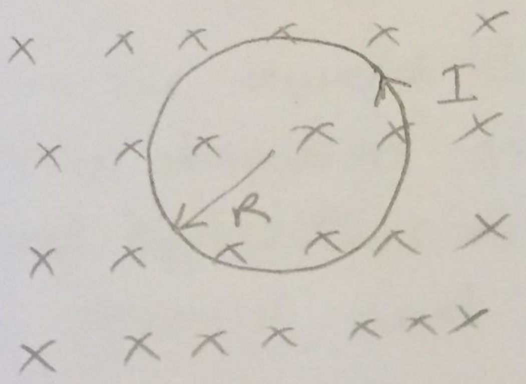
There will be a current in the loop. The direction of the current is clockwise and its magnitude:

$$\mathcal{E} = vBL = IR \qquad I = \frac{vBL}{R}$$

another way of looking at this induction
process is to consider the flux through
the loop: $\phi = BA$. as the bar moves to the
right the ϕ decreases since the area decreases.
Now the (−) sign in the Faraday's Law:

$$\mathcal{E} = (-)\frac{d\phi}{dt} \rightarrow$$ means that the induced

$\mathcal{E}$ is in a direction which is opposite to the
change that created it. For example
in the first example we had, with a loop



in a time dependent
B. We had assumed
that B was increasing
in time: $\frac{dB(t)}{dt} > 0$
Now we saw that this

induced an $\mathcal{E}$ in the loop and hence a
current I. B is increasing, so ϕ in the loop
is increasing. The induced current will create
another B. What the Faraday's Law says is
that the induced I will create a B in a
direction to oppose this increase in ϕ.
Therefore the induced current should create
a B outward. Hence I is counterclockwise

Ex. 31.2 if in the above example, $B = B_{max} e^{-at}$. So B decreases exponentially. What is the induced $\mathcal{E}$ & the direction of the current? B is $\perp$ to A.

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -A\frac{dB}{dt}$$

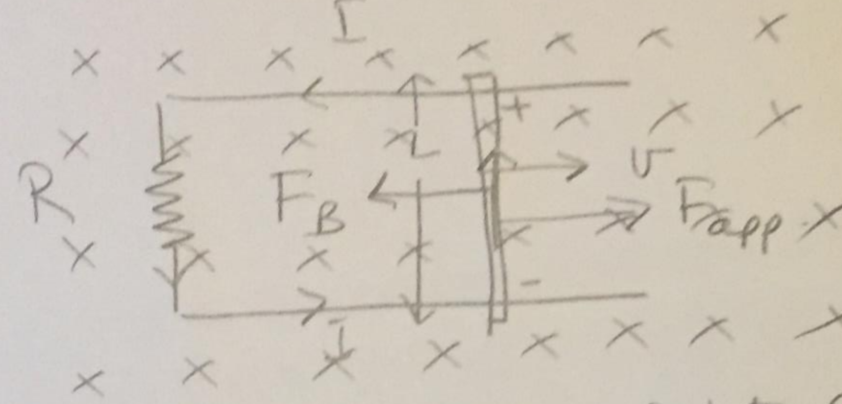
$$\frac{dB}{dt} = \frac{d}{dt}\left(B_{max} e^{-at}\right) = -aB$$

$$\mathcal{E} = -aAB$$

Sin the inward B is <u>decreasing</u>, I should be in a direction as to create a B_{in} which would oppose this decrease. Hence Bin should be <u>inward</u> and I is <u>clockwise</u>.

Note: if there is more than one loop with the same area, $\mathcal{E} = -N\frac{d\phi}{dt}$ where N is the # of loops.

Magnetic Force acting on a Sliding Bar



The sliding bar has length L and mass m. With what force should the bar be pulled to the right so that it moves with constant v to the right?

Since v is to the right, the area and Φ is increasing - B_{in} should be outward

I is counterclockwise.

$$\mathcal{E} = vBL = IR \qquad I = \frac{vBL}{R}$$

this current at the bar is upward.

Now the bar is carrying a current I in B, so there is a magnetic force on it.

$\vec{F}_B = I \vec{L} \times \vec{B}$ → which is towards left. So applied force should be equal to this to the right.

$$\vec{F}_{app} = \frac{vBL}{R} LB \,\hat{i}$$

$$\vec{F}_{app} = \frac{vB^2L^2}{R} \hat{i}$$