

KEY

Istanbul Şehir University

Math 104

Date: 15 March 2014	Full Name:
Time: 11:00-12:30	
	Student ID:
Fall 2014 First Exam	

IMPORTANT

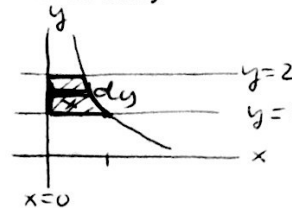
1. Write down your name and surname on top of each page. 2. The exam consists of 4 questions, some of which have multiple parts. 3. Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. You may continue your solutions on the back of the sheets. 4. Show all your work. Correct answers without justification will not get credit. 5. Unless otherwise specified, you may use any method from classwork to solve the problems. 6. Calculators are not allowed. 7. All cellphones and electronic devices are to be kept shut and out of sight.

Q1	Q2	Q3	Q4	TOTAL
20 pts	30 pts	20 pts	30 pts	100 pts

- 1) Find the volume of the solid generated by revolving the region between the curve  $y = 1/x$  and the lines  $y = 1$ ,  $y = 2$  and  $x = 0$  about the y-axis.

Disk method:

$$\begin{aligned}
 V &= \pi \int_1^2 x^2 dy, \quad x = 1/y \\
 &= \pi \int_1^2 y^{-2} dy \\
 &= -\frac{\pi}{y} \Big|_1^2 \\
 &= -\pi \left( \frac{1}{2} - 1 \right) \\
 &= \boxed{\frac{\pi}{2}}
 \end{aligned}$$



NAME:

2) Evaluate each of the following limits, if it exists:

(a)  $\lim_{x \rightarrow \infty} x^{\tan(1/x)} \quad \infty^0$  Let  $y = x^{\tan(1/x)}$

$\ln y = \tan \frac{1}{x} \ln x \quad 0 \cdot \infty$

$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \tan \frac{1}{x} \ln x = \lim_{x \rightarrow \infty} \frac{\ln x}{\cot 1/x} \quad \infty/\infty$

$= \lim_{x \rightarrow \infty} \frac{1/x}{-\csc^2 1/x (-1/x^2)} \quad \text{L'Hospital} = \lim_{x \rightarrow \infty} \frac{\sin^2 1/x}{1/x} \quad \text{Let } t = 1/x$

$= \lim_{t \rightarrow 0^+} \frac{\sin^2 t}{t} = \lim_{t \rightarrow 0^+} \frac{\sin t}{t} \cdot \sin t = 1 \cdot 0 = 0$

$\ln y \rightarrow 0 \Rightarrow \lim y = e^0 = \boxed{1}$

(b)  $\lim_{x \rightarrow 0^+} (\ln \frac{1}{x})^x$  (There are other ways to solve (a))

$\infty^0$  Let  $y = (\ln \frac{1}{x})^x$

$\ln y = x \ln \ln \frac{1}{x} \quad 0 \cdot \infty$

$\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} x \ln \ln \frac{1}{x} = \lim_{x \rightarrow 0^+} \frac{\ln \ln \frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{\ln 1/x} \cdot \frac{1}{1/x} \cdot (-1/x^2)}{-1/x^2} \quad \text{L'Hospital}$

$= \lim_{x \rightarrow 0^+} \frac{x}{\ln 1/x} \quad 0/0 \Rightarrow \ln y \rightarrow 0 \Rightarrow \lim y = e^0 = \boxed{1}$

(There are other ways to solve (b) too.)

NAME:

3) Differentiate each of the following functions:

(a)  $y = \ln \tan^{-1} x$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\tan^{-1} x} \frac{d}{dx} \tan^{-1} x \\ &\text{Chain Rule} \\ &= \boxed{\frac{1}{\tan^{-1} x \cdot (x^2 + 1)}} \end{aligned}$$

(b)  $y = \sin^{-1}(e^{-x})$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sqrt{1 - (e^{-x})^2}} \frac{d}{dx} e^{-x} \\ &\text{Chain Rule} \\ &= \boxed{\frac{-e^{-x}}{\sqrt{1 - e^{-2x}}}} \end{aligned}$$

NAME:

4) Evaluate the following integrals:

$$\begin{aligned} \text{(a)} \int \frac{x dx}{\sqrt{16-x^4}} &= \frac{1}{4} \int \frac{x dx}{\sqrt{1-x^4/16}} = \frac{1}{4} \int \frac{x dx}{\sqrt{1-(x/4)^2}} \\ \left[ \begin{aligned} \text{Let } u &= \frac{x^2}{4} \Rightarrow du = \frac{2x}{4} dx \Rightarrow x dx = 2 du \\ \rightarrow &= \frac{1}{4} \int \frac{2 du}{\sqrt{1-u^2}} = \frac{1}{2} \sin^{-1} u + C \\ &= \boxed{\frac{1}{2} \sin^{-1} \frac{x^2}{4} + C} \end{aligned} \right. \end{aligned}$$

$$\begin{aligned} \text{(b)} \int e^{-x} \tan(e^{-x}) dx \quad \text{Let } u &= e^{-x} \Rightarrow du = -e^{-x} dx \\ \rightarrow &= \int -\tan u du \\ &= \int -\frac{\sin u du}{\cos u} = \ln |\cos u| + C \\ &= \boxed{\ln \cos e^{-x} + C} \end{aligned}$$