KEY

Istanbul Şehir University

Math 104

Date: 5 April 2014	Full Name:
Time: 10:00-11:30	
	Student ID:
Spring 2014 Second Exam	

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cellphones and electronic devices are to be kept shut and out of sight.

Q1	Q2	Q3	Q4	TOTAL
20 pts	30 pts	30 pts	20 pts	100 pts

Q1. Evaluate the improper integral

$$\int_1^{\infty} \frac{dx}{\sqrt{x}(x+1)}$$

Let $u = \sqrt{x} \Rightarrow u^2 = x$

$2u\,du = dx$

$x = 1 \Rightarrow u = 1, \quad x = \infty \Rightarrow u = \infty$

$$= \int_1^{\infty} \frac{2u\,du}{u(u^2+1)}$$

$$= 2\,\text{Arctan}\,u \Big|_1^{\infty} = 2 \lim_{b\to\infty} \text{Arctan}\,u \Big|_1^{b}$$

$$= \lim_{b\to\infty} 2\,\text{Arctan}\,b - 2\,\text{Arctan}\,1$$

$$= 2 \cdot \frac{\pi}{2} - 2 \cdot \pi/4$$

$$= 2 \cdot \frac{\pi}{4} = \boxed{\frac{\pi}{2}}$$

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Q2. Evaluate the following integrals:

(a) $\int \frac{dx}{x^2+4x}$

Partial fractions:

$$\frac{1}{x^2+4x} = \frac{1}{x(x+4)} = \frac{A}{x} + \frac{B}{x+4}$$

$$1 = A(x+4) + Bx$$

$x = 0 \quad 1 = 4A \quad A = \frac{1}{4} = -B$

$x = -4 \quad 1 = -4B$

$$\therefore \int \frac{dx}{x^2+4x} = \frac{1}{4}\int \frac{dx}{x} - \frac{1}{4}\int \frac{dx}{x+4}$$

$$= \frac{1}{4}\ln|x| - \frac{1}{4}\ln|x+4| + C$$

$$= \boxed{\frac{1}{4}\ln\left|\frac{x}{x+4}\right| + C}$$

(b) $\int \tan^{-3/4}\theta \sec^4\theta \, d\theta$

$\sec^2\theta \cdot \sec^2\theta \qquad 2 - \frac{3}{4} = \frac{5}{4}$

$(1+\tan^2\theta)$

$$\Rightarrow = \int \tan^{-3/4}\theta \sec^2\theta \, d\theta + \int \tan^{5/4}\theta \sec^2\theta \, d\theta$$

Let $u = \tan\theta \Rightarrow du = \sec^2\theta \, d\theta$

$$= \int u^{-3/4} du + \int u^{5/4} du$$

$$= \frac{u^{1/4}}{1/4} + \frac{u^{9/4}}{9/4} + C$$

$$= \boxed{4\tan^{1/4}\theta + \frac{4}{9}\tan^{9/4}\theta + C}$$

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Q3. Evaluate the following integrals:

(a) $\int x\sin x\,dx$ Integration by parts

$u = x \qquad dv = \sin x\,dx$

$du = dx \qquad v = -\cos x$

$\therefore \int x\sin x\,dx = -x\cos x + \int \cos x\,dx$

$= \boxed{-x\cos x + \sin x + C}$

(b) $\int x^{-2/3}\ln x\,dx$ Integration by parts

$u = \ln x \qquad dv = x^{-2/3}dx$

$du = \frac{dx}{x} \qquad v = \frac{x^{1/3}}{1/3} = 3x^{1/3}$

$\therefore \int x^{-2/3}\ln x\,dx = 3x^{1/3}\cdot\ln x - 3\int \frac{x^{1/3}}{x}dx$

$= 3x^{1/3}\ln x - 3\int x^{-2/3}dx$

$= 3x^{1/3}\ln x - 3\cdot\frac{x^{1/3}}{1/3} + C$

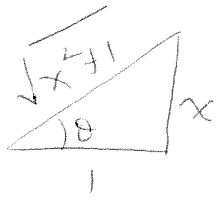
$= \boxed{3x^{1/3}\ln x - 9x^{1/3} + C}$

NAME:

Q4. Evaluate the following integral:

$$\int \frac{dx}{(x^2+1)^{3/2}}$$

Trig Substitution



$\tan\theta = x$

$\sec^2\theta\, d\theta = dx$

$\sec\theta = \sqrt{x^2+1}$

$$= \int \frac{\sec^2\theta\, d\theta}{\sec^3\theta}$$

$$= \int \frac{d\theta}{\sec\theta}$$

$$= \int \cos\theta\, d\theta = \sin\theta + c$$

$$= \boxed{\frac{x}{\sqrt{x^2+1}} + c}$$