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**Course:** Linear Algebra

**Assignment:** Section 3.1 Homework

1. Compute the determinant using a cofactor expansion across the first row. Also compute the determinant by a cofactor expansion down the second column.

$$\begin{vmatrix} 2 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -2 \end{vmatrix}$$

Compute the determinant using a cofactor expansion across the first row. Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- ☐ A. Using this expansion, the determinant is  $-(2)(-16) + (0)(-4) - (4)(10) =$  \_\_\_\_\_.
- ☐ B. Using this expansion, the determinant is  $(0)(-4) - (3)(-4) + (5)(-4) =$  \_\_\_\_\_.
- ☒ C. Using this expansion, the determinant is  $(2)(-16) - (0)(-4) + (4)(10) =$  8.
- ☐ D. Using this expansion, the determinant is  $-(0)(-4) + (3)(-4) - (5)(-4) =$  \_\_\_\_\_.

Compute the determinant using a cofactor expansion down the second column. Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- ☐ A. Using this expansion, the determinant is  $-(2)(-16) + (0)(-4) - (4)(10) =$  \_\_\_\_\_.
- ☐ B. Using this expansion, the determinant is  $(2)(-16) - (0)(-4) + (4)(10) =$  \_\_\_\_\_.
- ☒ C. Using this expansion, the determinant is  $-(0)(-4) + (3)(-4) - (5)(-4) =$  8.
- ☐ D. Using this expansion, the determinant is  $(0)(-4) - (3)(-4) + (5)(-4) =$  \_\_\_\_\_.

YOU ANSWERED: C.: 0

2. Compute the determinant using a cofactor expansion across the first row. Also compute the determinant by a cofactor expansion down the second column.

$$\begin{vmatrix} 1 & -1 & 6 \\ 6 & 5 & 1 \\ 5 & 6 & -5 \end{vmatrix}$$

Write the expression for the determinant using a cofactor expansion across the first row. Choose the correct answer below.

- ☐ A. Using this expansion, the determinant is  $(1)(-19) + (-1)(41) + (6)(61)$ .
- ☐ B. Using this expansion, the determinant is  $(1)(-31) + (-1)(-35) + (6)(11)$ .
- ☒ C. Using this expansion, the determinant is  $(1)(-31) - (-1)(-35) + (6)(11)$ .
- ☐ D. Using this expansion, the determinant is  $(1)(-19) - (-1)(41) + (6)(61)$ .

Write the expression for the determinant using a cofactor expansion down the second column. Choose the correct answer below.

- ☐ A. Using this expansion, the determinant is  $(-1)(-25) + (5)(25) + (6)(37)$ .
- ☒ B. Using this expansion, the determinant is  $-(-1)(-35) + (5)(-35) - (6)(-35)$ .
- ☐ C. Using this expansion, the determinant is  $-(-1)(-25) + (5)(25) - (6)(37)$ .
- ☐ D. Using this expansion, the determinant is  $(-1)(-35) + (5)(-35) + (6)(-35)$ .

The determinant is 0.

(Simplify your answer.)

3. Compute the determinant using a cofactor expansion down the first column.

$$A = \begin{bmatrix} 6 & -5 & 2 \\ 8 & 1 & 3 \\ 0 & 4 & -2 \end{bmatrix}$$

Determine the value of the first term in the cofactor expansion. Substitute the value for  $a_{11}$  and complete the matrix for  $C_{11}$  below.

$$a_{11}C_{11} = (\underline{6}) \det \begin{bmatrix} \underline{1} & \underline{3} \\ \underline{4} & \underline{-2} \end{bmatrix}$$

Determine the value of the second term in the cofactor expansion. Substitute the value for  $a_{21}$  and complete the matrix for  $C_{21}$  below.

$$a_{21}C_{21} = -(\underline{8}) \det \begin{bmatrix} \underline{-5} & \underline{2} \\ \underline{4} & \underline{-2} \end{bmatrix}$$

Determine the value of the third term in the cofactor expansion. Substitute the value for  $a_{31}$  and complete the matrix for  $C_{31}$  below.

$$a_{31}C_{31} = (\underline{0}) \det \begin{bmatrix} \underline{-5} & \underline{2} \\ \underline{1} & \underline{3} \end{bmatrix}$$

Complete the cofactor expansion to compute the determinant.

$$\det A = \underline{-100}$$

4. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 1 & 0 & 0 & 5 \\ 4 & 7 & 3 & -7 \\ 3 & 0 & 0 & 0 \\ 7 & 3 & 1 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 & 5 \\ 4 & 7 & 3 & -7 \\ 3 & 0 & 0 & 0 \\ 7 & 3 & 1 & 3 \end{vmatrix} = \underline{-30} \text{ (Simplify your answer.)}$$

5. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 4 & 8 & -6 & 3 \\ 0 & -3 & 1 & -1 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 8 & -6 & 3 \\ 0 & -3 & 1 & -1 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 2 \end{vmatrix} = \underline{-24} \text{ (Simplify your answer.)}$$

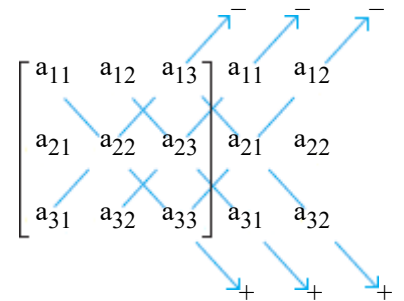
6. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 3 & 0 & -7 & 3 & -5 \\ 0 & 0 & 3 & 0 & 0 \\ 8 & 4 & -6 & 5 & -9 \\ 4 & 0 & 6 & 2 & -4 \\ 0 & 0 & 7 & -2 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 0 & -7 & 3 & -5 \\ 0 & 0 & 3 & 0 & 0 \\ 8 & 4 & -6 & 5 & -9 \\ 4 & 0 & 6 & 2 & -4 \\ 0 & 0 & 7 & -2 & 2 \end{vmatrix} = \underline{-48} \text{ (Simplify your answer.)}$$

7. The expansion of a  $3 \times 3$  determinant can be remembered by this device. Write a second copy of the first two columns to the right of the matrix, and compute the determinant by multiplying entries on six diagonals. Add the downward diagonal products and subtract the upward products. Use this method to compute the following determinant.

$$\begin{vmatrix} 3 & 1 & 0 \\ -2 & 3 & 4 \\ 0 & -2 & -4 \end{vmatrix}$$



$$\begin{vmatrix} 3 & 1 & 0 \\ -2 & 3 & 4 \\ 0 & -2 & -4 \end{vmatrix} = \underline{-20}$$

8. Explore the effect of an elementary row operation on the determinant of a matrix. State the row operation and describe how it affects the determinant.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} a + kc & b + kd \\ c & d \end{bmatrix}$$

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What is the elementary row operation?

- ☐ A. Row 2 is replaced with the sum of itself and k times row 1.
- ☐ B. Row 2 is multiplied by k.
- ☐ C. Rows 1 and 2 are interchanged.
- ☐ D. Row 1 is multiplied by k.
- ☒ E. Row 1 is replaced with the sum of itself and k times row 2.

How does the row operation affect the determinant?

- ☐ A. It changes the sign of the determinant.
- ☐ B. It increases the determinant by k.
- ☐ C. It multiplies the determinant by k.
- ☒ D. It does not change the determinant.

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9. Explore the effects of an elementary row operation on the determinant of a matrix. State the row operation and describe how it affects the determinant.

$$\begin{bmatrix} 5 & 5 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 5 & 5 \\ 3 + 5k & 1 + 5k \end{bmatrix}$$

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What is the elementary row operation?

- ☐ A. Replace row 2 with k times row 1.
- ☐ B. Replace row 2 with row 1 plus k times row 2.
- ☒ C. Replace row 2 with k times row 1 plus row 2.
- ☐ D. Replace row 2 with k times row 2.

How does the row operation affect the determinant?

- ☐ A. The determinant is increased by 50k.
- ☐ B. The determinant is decreased by 25k.
- ☐ C. The determinant is increased by 25k.
- ☒ D. The determinant does not change.
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10. Explore the effect of an elementary row operation on the determinant of a matrix. State the row operation and describe how it affects the determinant.

$$\begin{bmatrix} 3 & 2 & 4 \\ a & b & c \\ 8 & 5 & 1 \end{bmatrix}, \begin{bmatrix} a & b & c \\ 3 & 2 & 4 \\ 8 & 5 & 1 \end{bmatrix}$$

What is the elementary row operation?

- ☐ A. Row 1 is replaced with the sum of rows 1 and 2.
- ☐ B. Row 1 is replaced with the sum of rows 1 and 3.
- ☒ C. Rows 1 and 2 are interchanged.
- ☐ D. Rows 1 and 3 are interchanged.
- ☐ E. Row 2 is replaced with the sum of rows 2 and 3.
- ☐ F. Rows 2 and 3 are interchanged.
- ☐ G. Row 2 is replaced with the sum of rows 1 and 2.

How does the row operation affect the determinant?

- ☒ A. It changes the sign of the determinant.
- ☐ B. It increases the determinant by 1.
- ☐ C. It multiplies the determinant by 2.
- ☐ D. It does not change the determinant.

11. Let  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  and let  $k$  be a scalar. Find a formula that relates  $\det(kA)$  to  $k$  and  $\det(A)$ .

Find  $\det(A)$ .

$\det(A) = \underline{ad - bc}$  (Simplify your answer.)

Find  $\det(kA)$ .

$\det(kA) = \underline{k^2 ad - k^2 bc}$  (Simplify your answer.)

Use the preceding steps to find a formula for  $\det(kA)$ . Select the correct choice below and fill in the answer box(es) to complete your choice.  
(Simplify your answer.)

- ☒ A.  $\det(kA) = \underline{k^2} \cdot \det(A)$
- ☐ B.  $\det(kA) = \underline{\hspace{2cm}} + \det(A)$
- ☐ C.  $\det(kA) = \underline{\hspace{2cm}} - \det(A)$
- ☐ D.  $\det(kA) = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} \cdot \det(A)$