Logarithmic Differentiation

Ex Diff. $y = \dfrac{(3x^2-1)^{1/2}(x^3+1)^{2/5}}{(x+2)^3}$

$$\ln y = \frac{1}{2}\ln(3x^2-1) + \frac{2}{5}\ln(x^3+1) - 3\ln(x+2)$$

$$\frac{y'}{y} = \frac{1}{2}\cdot\frac{6x}{3x^2-1} + \frac{2}{5}\cdot\frac{3x^2}{x^3+1} - \frac{3}{x+2}$$

$$y' = y\Big(\qquad\qquad \Big)$$

Ex Diff. $y = \sqrt[4]{\dfrac{x^2+1}{x^2-1}} = \left(\dfrac{x^2+1}{x^2-1}\right)^{1/4}$

$$\ln y = \frac{1}{4}\left[\ln(x^2+1) - \ln(x^2-1)\right]$$

$$\frac{y'}{y} = \frac{1}{4}\left[\frac{2x}{x^2+1} - \frac{2x}{x^2-1}\right]$$

$$y' = \frac{2xy}{4}\left[\frac{\cancel{x^2}-1-\cancel{x^2}-1}{x^4-1}\right]$$

$$= \frac{xy}{2} \cdot \frac{-2}{x^4-1}$$

Exponential Fns.

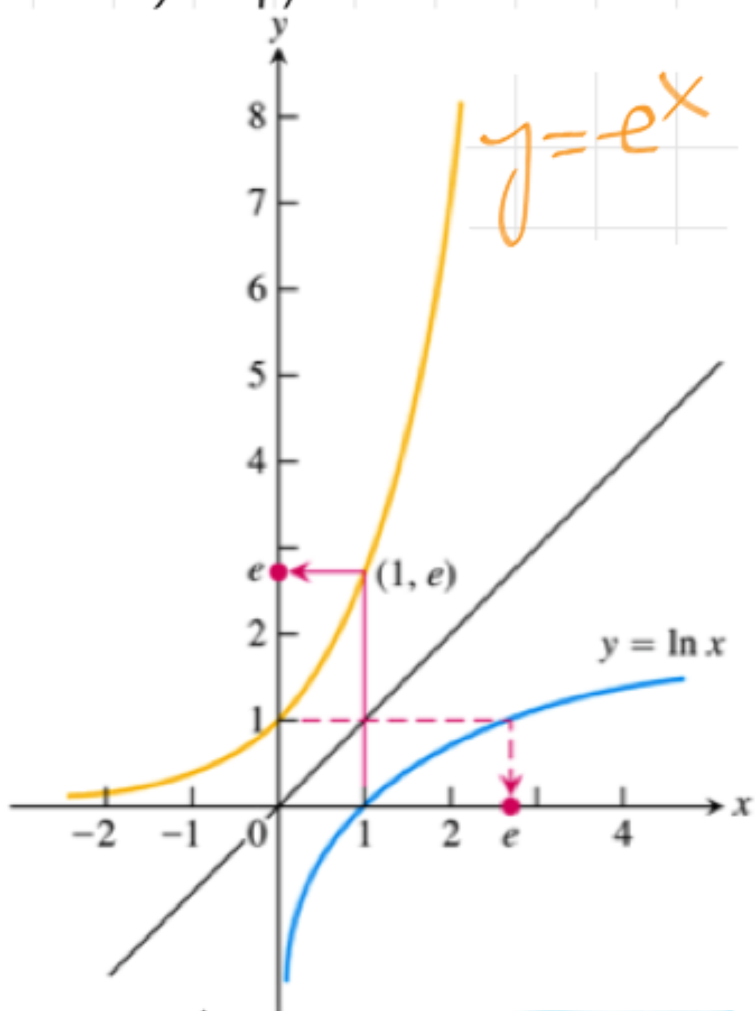
$y=\ln x$ is monotone increasing
$\therefore$ it is invertable.

The inverse fn. of $y=\ln x$
is $y=e^x$

$$\lim_{x\to\infty} e^x=\infty$$

$$\lim_{x\to-\infty} = 0$$

$$\ln e = 1$$



$$\ln e^r = r\ln e = r$$

$$e^r = \exp r = \ln^{-1} r$$

$$\boxed{e^{\ln x}=x,\ \ln e^x = x}$$

ln has meaning for all $x>0$,
exp. does too.

The Derivative & Integral of e^x

$$\ln e^x = x$$

$$y = \ln e^x = x \underbrace{\ln e}_{1} = x$$

$$y = \ln e^x$$

$$x = \ln e^x$$

$$\frac{d}{dx}(\ln e^x) = \frac{d}{dx}(x) = 1$$

$$\frac{1}{e^x} \frac{d}{dx}(e^x) = 1 \quad e^x$$

$$\boxed{\frac{d}{dx} e^x = e^x}$$

$$\int \cancel{d} (e^x) = \int e^x dx$$

$$c + e^x = \int e^x dx$$

$$\boxed{\int e^x dx = e^x + c}$$

Ex

$$\int e^{x^2} x \, dx =$$

$$u = x^2$$

$$du = 2x \, dx$$

$$\frac{1}{2} du = x \, dx$$

$$\frac{1}{2} \int e^u du = \frac{1}{2} e^u + c$$

$$= \frac{1}{2} e^{x^2} + c$$

Law of Exponents

$$e^{a+b} = e^a \cdot e^b$$

$$\ln e^{a+b} = a\ln e + b\ln e = a+b$$

$$e^{ar} = (e^a)^r$$

$$\ln e^{ar} = ar\ln e = ar$$

$$\frac{e^a}{e^b} = e^{a-b}$$

$$e^{-a} = \frac{1}{e^a}$$

The general power rule for diff.

$$\frac{d}{dx} x^n = n x^{n-1} \; ; n \in \mathbb{R}$$

when $x \leq 0$, the above formula also holds, whenever it has meaning.

Ex Diff $y = x^7$

$$\frac{d}{dx} x^7 = 7x^6$$

what about $\frac{d}{dx}(7^x) = ?$

$y = 7^x$

$\ln y = x \ln 7$

$y = e^{x \ln 7}$

$\uparrow$

$7^x = e^{x \ln 7}$

generally,

$$\boxed{a^x = e^{x \ln a}}$$

let us now diff a^x:

$$\frac{d}{dx} a^x = \frac{d}{dx} \left(e^{x \ln a} \right)$$

$$\frac{d}{dx}a^x = \frac{d}{dx}e^{x\ln a}$$

$$= e^{x\ln a}\frac{d}{d}(x\ln a)$$

$$= \ln a \underbrace{e^{x\ln a}}_{a^x}$$

$$\boxed{\frac{d}{dx}a^x = a^x \ln a}$$

Ex $\frac{d}{dx}7^x = 7^x \ln 7$

Ex $y = x^{\sin x}$

$\ln y = \sin x \ln x$

$y'/y = \cos x \ln x + \sin x \cdot \frac{1}{x}$

Ex

Diff.

- $y^{2/3} = \dfrac{(x^2+1)(3x+4)^{1/2}}{\sqrt[5]{(2x-3)(x^2-4)}}$

- $y = (2x)^{\sqrt{2}}$

- $y = \dfrac{\sin e^x}{e^{\sin x}}$

- $y = \sin e^x + e^{\sin x}$

- $y = (\ln x)^{\ln x}$

logarithms with base a

$a^x = e^{x \ln a}$ is a monotone

so has inverse

the logarithm

with base a,

written as

$\log_a x$

$y = a^x$

$\ln y = x \ln a \Rightarrow x = \dfrac{\ln y}{\ln a}$

$x = \log_a y$

$\log_{10}$ is called the common logarithm.

$$\boxed{\log_a x = \frac{\ln x}{\ln a}}$$ change of base

$$\log x = \frac{\ln x}{\ln 10}$$

$$\log_2 x = \frac{\ln x}{\ln 2}$$

since $\log_a x$ is a multiple $\frac{1}{\ln a}$ of $\ln x$, the rules for $\ln$ also hold for $\log_a$

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a \frac{1}{y} = -\log_a y$$

$$\log_a x^y = y \log_a x$$

$$\frac{d}{dx}\ln x\Big|_{x=1} = \frac{1}{x}\Big|_{x=1} = 1$$

$$f(x) = \ln x, \quad f'(1) = 1$$

$$1 = f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$1 = \lim_{x\to 0} \frac{\ln(1+x) - \cancel{\ln 1}^{\,0}}{x}$$

$$1 = \lim_{x\to 0} \frac{1}{x}\ln(1+x) = \lim_{x\to 0} \ln(1+x)^{1/x}$$

$$1 = \ln\Big[\underbrace{\lim_{x\to 0}(1+x)^{1/x}}_{e}\Big] = \ln e$$

$$\boxed{\lim_{x\to 0}(1+x)^{1/x} = e}, \quad e = 2.7182\ldots$$