Math No:

Full Name :

Math 104 1st Midterm Exam
(4 March 2019, 19:00-20:00)

IMPORTANT
1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
20 pts	25 pts	25 pts	30 pts	100 pts

Q1. Evaluate the following integral:

$$\int (1+\sin\theta)^9 \cos\theta \, d\theta$$

$u = 1+\sin\theta \Rightarrow du = \cos\theta \, d\theta$

$$\therefore \int (1+\sin\theta)^9 \cos\theta \, d\theta = \int u^9 \, du$$

$$= \frac{u^{10}}{10} + C$$

$$= \frac{(1+\sin\theta)^{10}}{10} + C$$

Q2. Evaluate the following integral:

$$\int x^2\sqrt{x-1}\,dx$$

Let $u = x-1 \Rightarrow du = dx$ and $x = u+1$

$$\int x^2\sqrt{x-1}\,dx = \int (u+1)^2 u^{1/2}\,du$$

$$= \int (u^2+2u+1)u^{1/2}\,du$$

$$= \int (u^{5/2} + 2u^{3/2} + u^{1/2})\,du$$

$$= \frac{u^{7/2}}{7/2} + 2\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$$

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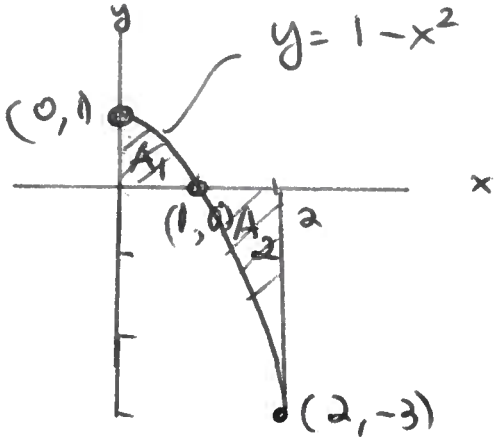
Q3. Find the area of the region between the curve $y = 1 - x^2$ and the x-axis, for $0 \le x \le 2$.



$A = A_1 + A_2$

$$A = \int_0^1 (1-x^2)\,dx + \int_1^2 (x^2-1)\,dx$$

$$= x - \frac{x^3}{3}\Big|_0^1 + \left(\frac{x^3}{3} - x\right)\Big|_1^2$$

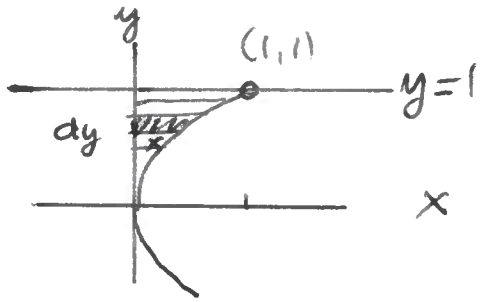
$$= \left(1 - \frac{1}{3}\right) + \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 1\right)$$

$$= 2 \cdot \frac{2}{3} + \frac{8}{3} - 2$$

$$= \frac{12}{3} - 2 = 4 - 2 = \boxed{2}$$

Q4. The region bounded by the line $y = 1$, the curve $x = y^2$, and the y-axis is rotated about the x-axis.

a) Find the volume of the solid of revolution obtained, using the shell method.

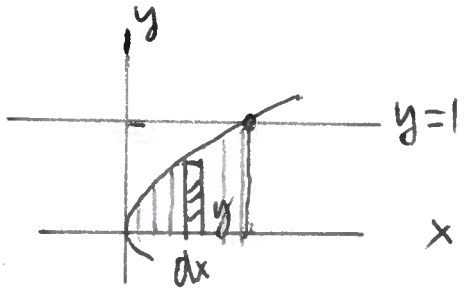


$$V = 2\pi \int_0^1 xy\,dy$$

$$= 2\pi \int_0^1 y^3\,dy$$

$$= \frac{2\pi y^4}{4} = \boxed{\frac{\pi}{2}}$$

b) Find the volume of the same solid of revolution, using the disk (washer) method.



$V = V_{cylinder} - V_{inside}$

$$V = \pi \cdot 1^2 \cdot 1 - \pi \int_0^1 y^2\,dx$$

$$= \pi - \pi \int_0^1 x\,dx$$

$$= \pi - \pi \frac{x^2}{2}\Big|_0^1 = \pi - \frac{\pi}{2} = \boxed{\frac{\pi}{2}}$$