Math No:

Full Name : KEY

ISTANBUL SEHİR UNIVERSITY

Math 104 1st Midterm Exam
(30 November 2017, 18:00-19:00)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 4 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cell phones and electronic devices are to be kept shut and out of sight. All cell phones are to be left on the instructor's desk prior to the exam.

Q1	Q2	Q3	Q4	TOT
6 pts	6 pts	6 pts	6 pts	24 pts

Q1. Evaluate the following integrals.

a) $\int_0^{\pi/2} \cos x \sqrt[3]{\sin x}\, dx = \int_{u(0)}^{u(\pi/2)} u^{1/3}\, du = \frac{3}{4} u^{4/3} \Big|_{u(0)}^{u(\pi/2)} = \frac{3}{4}[1-0] = 3/4$

$u = \sin x \quad \Big| \quad u(0) = 0$
$du = \cos x\, dx \quad \Big| \quad u(\pi/2) = 1$

b) $\int [(2x-1)^3 + \cos 5x]\, dx = \int (2x+1)^3\, dx + \int \cos 5x\, dx =$

$u = 2x+1 \quad \Big| \quad v = 5x$
$\frac{1}{2} du = dx \quad \Big| \quad \frac{1}{5} dv = dx$

$\frac{1}{2}\int u^3\, du + \frac{1}{5}\int \cos v\, dv = \frac{1}{2}\cdot\frac{1}{4} u^4 + \frac{1}{5}\sin v + C$

$= \frac{1}{8}(2x-1)^4 + \frac{1}{5}\sin 5x + C$

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Q2. Calculate the area between $y = 3x$ and $y = 2 + x^2$. Show your work on finding the integral limits.

$3x = 2 + x^2 \Rightarrow x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0 \Rightarrow x_1 = 1,\ x_2 = 2$

$$\int_1^2 (3x - 2 - x^2)\, dx =$$

$$\frac{3}{2}x^2 - 2x - \frac{x^3}{3}\Big|_1^2 = 1/6$$

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Q3. The region between $y = x$ and $y = x^3$, for $0 \le x \le 1$, is rotated about the x-axis. Find the volume generated.

We could apply the washer technique as

$$\int_0^1 \left(\pi(x)^2 - \pi(x^3)^2\right) dx =$$

$$\pi \frac{x^3}{3} - \pi \frac{x^7}{7} \Big|_0^1 =$$

$$\frac{\pi}{3} - \frac{\pi}{7} =$$

$$\frac{4\pi}{21} \text{ //}$$

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Q4. For the following function $f(x) = 5 - 4x$

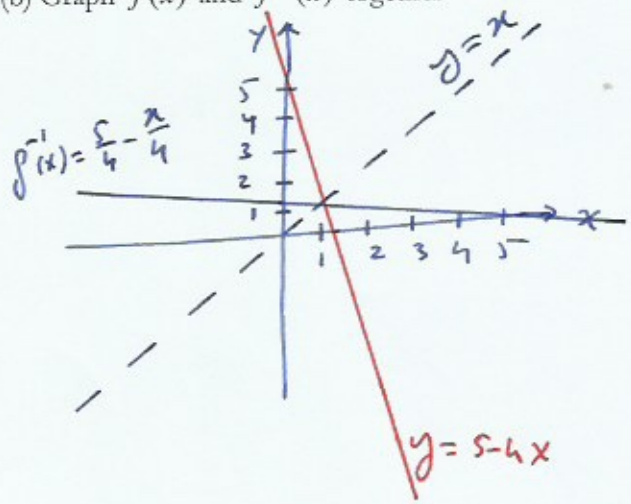
(a) Find $f^{-1}(x)$

$f(x) = y = 5 - 4x$

$x = \frac{5}{4} - \frac{y}{4}$

$f^{-1}(x) = \frac{5}{4} - \frac{x}{4}$

(b) Graph $f(x)$ and $f^{-1}(x)$ together



(c) Evaluate $\frac{df}{dx}$ and $\frac{df^{-1}}{dx}$ at $x=1/2$.

$\left.\frac{df}{dx}\right|_{x=1/2} = -4$, which is constant for any x.

$\left.\frac{df^{-1}}{dx}\right|_{x=1/2} = -\frac{1}{4}$, which is constant for any x.