Full Name : KEY
Student ID:

Istanbul Şehir University
Math 104, Midterm I
(18 October 2014, Time: 11:00-12:30)

IMPORTANT

1. Write down your name and surname on top of each page. **2.** The exam consists of 5 questions, some of which have multiple parts. **3.** Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. You may continue your solutions on the back of the sheets. **4.** Show all your work. Correct answers without justification will not get credit. **5.** Unless otherwise specified, you may use any method from classwork to solve the problems. **6.** Calculators are not allowed. **7.** All cellphones and electronic devices are to be kept shut and out of sight.

Q1	Q2	Q3	Q4	Q5	TOTAL
20 pts	20 pts	20	20	20 pts	100 pts

Q1. Find the volume of the solid generated when the region between the graphs of the equations $f(x)=\frac{1}{2}+x^2$ and $g(x)=x$ over the interval [0, 2] is revolved about the x -axis. [*Hint: You may use the washer method*]

$$V=\int_a^b \pi\left([f(x)]^2-[g(x)]^2\right)=\int_0^2 \pi\left(\left[\tfrac{1}{2}+x^2\right]^2-x^2\right)dx$$

$$=\int_0^2 \pi\left(\tfrac{1}{4}+x^4\right)dx=\pi\left[\frac{x}{4}+\frac{x^5}{5}\right]_0^2=\frac{69\pi}{10}$$

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Q2. (a) Find a formula for $f^{-1}(x)$ if $f(x)=\sqrt[5]{4x+2}$.

$y^5 = 4x+2$

$y^5 - 2 = 4x$

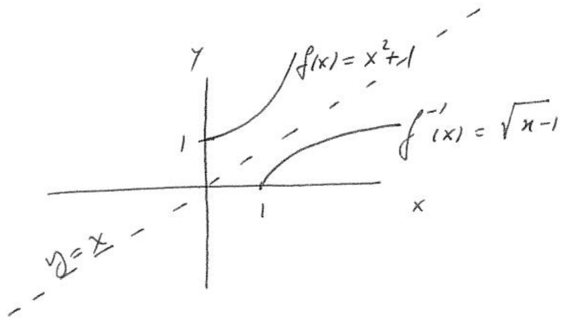
$\frac{1}{4}(y^5 - 2) = x$

$\hookrightarrow f^{-1}(x) = \frac{1}{4}(x^5 - 2)$

(b) Find a formula for $f^{-1}(x)$ if $f(x)=x^2+1$, $x \geq 0$. Plot both $f(x)$ and $f^{-1}(x)$ on the same graph.

$y = x^2 + 1$

$\sqrt{y-1} = x \Rightarrow f^{-1}(x) = \sqrt{x-1}$



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Q3. (a) Differentiate $f(x)=\dfrac{(x^2-1)(x^2-2)(x^2-3)}{(x^2+1)(x^2+2)(x^2+3)}$ and find $f'(1)$.

$$\ln y = \ln(x^2-1)+\ln(x^2-2)+\ln(x^2-3)-\ln(x^2+1)-\ln(x^2+2)-\ln(x^2+3)$$

$$\frac{y'}{y} = \frac{2x}{x^2-1}+\frac{2x}{x^2-2}+\frac{2x}{x^2-3}-\frac{2x}{x^2+1}-\frac{2x}{x^2+2}-\frac{2x}{x^2+3}$$

$$y' = 2xy\left(\frac{1}{x^2-1}+\frac{1}{x^2-2}+\frac{1}{x^2-3}-\frac{1}{x^2+1}-\frac{1}{x^2+2}-\frac{1}{x^2+3}\right)$$

$$y' = 4xy\left(\frac{1}{x^4-1}+\frac{2}{x^4-4}+\frac{3}{x^4-9}\right)$$

(b) At what points does the graph $y(x)=x^2e^{-x^2}$ have a horizontal tangent line (that is, $dy/dx=0$)

$$\ln y = 2\ln x + (-x^2)\underbrace{\ln e}_{1}$$

$$\frac{y'}{y} = 2\cdot\frac{1}{x} - 2x$$

$$y' = y\left(\frac{2}{x}-2x\right)$$

$$y' = x^2 . e^{-x^2}\left(\frac{2}{x}-2x\right)$$

$$y' = e^{-x^2}(2x-2x^3)$$

$$y' = 0$$

$$2x-2x^3=0$$

$$2x(1-x^2)=0 \qquad \boxed{x_1=0 \quad x_2=1 \quad x_3=-1}$$

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Q4. Evaluate the following limits

(a) $\lim_{x\to\infty} \dfrac{x^2}{e^x}$ $= \lim_{x\to\infty} \dfrac{2x}{e^x} = \lim_{x\to\infty} \dfrac{2}{e^x} = \dfrac{2}{\infty} = 0$

(b) $\lim_{x\to 0^+} x^a \ln x$, where $a > 0$.

$$\lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^a}} = \lim_{x\to 0^+} \frac{1/x}{-a \cdot x^{-a-1}} = \cancel{\lim_{x\to 0^+} \frac{1/x}{-a \cdot \frac{1}{x} \cdot \frac{1}{x^a}}} = \frac{\lim_{x\to 0} x^a}{-a} = \frac{0}{-a} = 0$$

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Q5. Find the limits for the following problems:

(a) $\lim_{x\to 1^+} x^{\frac{1}{(1-x)}}$

$$y = x^{\frac{1}{1-x}}$$

$$\ln y = \frac{1}{1-x} \ln x$$

$$\lim_{x\to 1^+} \ln y = \lim_{x\to 1^+} \frac{\ln x}{1-x} = \lim_{x\to 1^+} \frac{1/x}{-1} = \frac{1}{-1} = -1$$

$$\lim_{x\to 1^+} \ln y = -1$$

$$\lim_{x\to 1^+} y = e^{-1}$$

$$\lim_{x\to 1^+} x^{\frac{1}{1-x}} = 1/e$$

(b) $\lim_{x\to e^+} (\ln x)^{\frac{1}{(x-e)}}$

$$y = (\ln x)^{\frac{1}{x-e}}$$

$$\ln y = \frac{1}{x-e} \ln(\ln x)$$

$$\lim_{x\to e^+} \ln y = \lim_{x\to e^+} \frac{\ln(\ln x)}{x-e} = \lim_{x\to e^+} \frac{\frac{1/x}{\ln x}}{1} = 1/e$$

$$\lim_{x\to e^+} y = e^{1/e}$$

$$\lim_{x\to e^+} (\ln x)^{\frac{1}{x-e}} = e^{1/e}$$